

**What Constitutions Promote Capital Accumulation?
A Political-Economy Approach**

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Abstract

With the standard neoclassical growth model and an assumption of sequential voting on tax rates, we derive predictions for actual tax outcomes as a function of, on the one hand, the distribution of wealth and, on the other, specific elements of the fiscal and political constitutions in the economy. More precisely, we study how the frequency of elections and the lag between policy decision and policy implementation influence equilibrium tax rates, economic growth, and welfare. We also let the degree of progressivity in the tax code be a parameter of the constitution, and we study how it influences outcomes. We find that constitutional change may lead to large, long-run effects on economic performance. In particular, we find that the more frequently taxes are voted on, and the shorter the policy implementation lag, the higher are taxes in equilibrium, and the lower is growth and welfare. We also find that the more progressive is the tax code, the weaker are the distortions implied by the political transfer system. However, the quantitative effects from changing the progressivity of the tax code are much smaller than those resulting from changing the timing of elections.

1 Introduction

Countries differ widely in their public policy choices. For example, among the OECD countries, 1983 marginal tax rates on capital income varied between -90% and 49.5% ; the average labor income tax varied between 24% and 63% ; and the net total tax burden as measured by public expenditures varied between 26% and 54% of GNP.¹ Even larger differences would be revealed if one included countries on a lower level of development. Similarly, large differences in policies can also be recorded within given countries over time. To the extent one thinks actual policy outcomes affect economic performance, it seems an important task for economists to explore the origins of these wide disparities.

A typical way of building a positive theory of policy assumes that policies are chosen optimally. If they are indeed, then the large policy differences should be derivable from differences in economic primitives such as preferences and technology. In contrast, the approach we employ in this paper is to model different policy-selection procedures/collective choice mechanisms, and to examine their implications for policy outcomes. With this alternative approach, differences in policy outcomes depend not only on mentioned primitives and on differences in population characteristics, but they also depend on the details of the procedure by which policies are selected. We study some aspects of these policy-selection procedures—which we have chosen to label political and fiscal constitutions—and show that they can be crucial in the determination of an economy’s policy and capital accumulation paths. Note, hence, that our work follows the political-economy approach set out in Meltzer & Richard (1981), and that it extends that approach toward a normative analysis of policy more broadly defined.

Indeed, there are substantial differences in both political and fiscal aspects of real-world constitutions. One example of this is the difference across countries in voting/election frequency: among the OECD countries, there is substantial variation in regular governmental/presidential election frequencies.² As for fiscal aspects, there are substantial differences in the progressivity of the tax codes; income taxes on labor were strongly progressive in some of the countries and close to proportional in others. The purpose of our present paper is to develop a dynamic model of policy in order to analyze these potentially important differences in constitutions. We choose a calibrated, multiagent version of the neoclassical growth model

¹See McKee, Visser, & Saunders (1986) for data sources.

²Direct comparisons are of course hard due to other differences in the constitutions: French presidents have 7-year and their governments 4-year terms, whereas Swedish governments are elected every 3 years (and the king sits for life).

as our framework of analysis, and we extend it to include policy selection in a sequential political equilibrium. We focus on taxes on income, and we let the details of when policies are chosen, and how long they are in effect, be a parameter of the political constitution; as for the fiscal constitution, we let agents vote on the level of taxes, but we make the degree of progressivity a constitutional parameter. This framework delivers quantitative answers to our questions about the effects of constitutional change on both short-run and long-run outcomes for policy variables, capital accumulation, and economic welfare.

The politico-economic model has the property that the long-run growth performance—in this case the steady-state level of capital and output—is a function of the initial distribution of wealth. In particular, the lower the wealth of the politically pivotal agent relative to average wealth, the higher the equilibrium taxes and transfers, and hence the slower the accumulation of capital. In particular, it turns out in our model that small initial wealth disparities can cause substantial effects on capital accumulation. Our constitutional experiments, however, go one step further than this: they analyze how given initial wealth disparities lead to different subsequent capital accumulation paths depending on the constitution adopted.

The outcome of these constitutional experiments can be summarized briefly as follows. One of our main findings is that the quantitative effects of differences in constitutions on policy variables and on growth are typically quite large. More specifically, we establish that an increase in the time period between votes/elections will lead to a large positive effect on capital accumulation both in the short and in the long run. Similarly, an increase in the implementation lag of policy decisions also has a positive impact on capital accumulation. It also turns out, somewhat surprisingly, that constitutional change in most of our experiments changes equilibrium welfare in the same direction for all agents.

Why does the frequency of votes have implications for what taxes will be chosen? The reason is that frequent reassessments of the taxes imply that there is little commitment in the political system: when voters think about increasing the tax for a short period, the distortions are much less severe than when the tax increase is in effect for a long period, since agent's savings are much less elastic in the short run. This argument is related to the literature on time-consistency of optimal plans; in fact, in the median-voter case, our politico-economic equilibrium can be viewed as a time-consistent equilibrium where the government is represented by the median voter. In concrete terms, our model provides one interpretation of why the Swedish income tax rates have been so high, despite the relatively low dispersion/skewness in the income distribution in Sweden: governments are reassessed

by voters every three years.³

We also show that progressive tax codes necessarily lead to long-run equalization of income, whereas proportional tax codes never equalize income. In addition, we find that the more progressive is the tax code, the weaker are the distortions implied by the political transfer system. The reason for this is that the higher the progressivity, the higher is the cost of transferring wealth from the point of view of the politically pivotal agent, and hence the lower are the levels of the tax rates chosen in equilibrium. However, the short-run differences between economies with different degrees of progressivity turn out to be quite small, both in terms of absolute differences in tax rates and in terms of welfare.

Our work is also closely related to some recent studies of the role of political economy for economic growth. In particular, we were influenced by Alesina & Rodrik (1994) and Persson & Tabellini (1994), and there are also important connections to the work in Bertola (1993), Boldrin (1993), Glomm & Ravikumar (1992), Krusell & Ríos-Rull (1992), Perotti (1992), Perotti (1993), and Saint-Paul & Verdier (1991). Unlike these papers, our main focus here is on constitutional comparisons, and we use the standard Solowian growth model as a modeling framework. There are also methodological differences between the present work and many of these papers, and we will comment more on these below.⁴

Studies of the role of the frequency of elections include Rogoff (1990), who studies how often elections should be held. Rogoff's analysis focuses on another aspect of election frequency: he studies the tradeoff between the monitoring of leaders and transaction costs of holding elections. Further, Alesina & Roubini (1990) looks at endogenous elections, and presents some empirical evidence.

Our analysis of progressive taxation is related to a series of analyses for static economies in Berliant & Gouveia (1994), Cukierman & Meltzer (1991), and Snyder & Kramer (1988), all of whom regard some aspect of progressivity as something which is voted upon. These papers, in turn, can be viewed as extensions to an earlier literature of voting over linear tax schedules (Roberts (1975), Romer (1975), and Romer (1977)). We fix the degree of progressivity by postulating a specific functional form for total taxes paid and let agents vote on the total tax burden. One interpretation of this analysis is that we explore the effects on capital accumulation and welfare of adopting different degrees of "preferences for

³Note that the argument goes through despite the fact that the Swedish (Social Democrat) government did not change for a long time: what is important is that the incumbent government could not commit to future tax rates.

⁴A more detailed comparison can be found in Krusell, Quadrini, & Ríos-Rull (1994).

equity” in the society. Our assumption is also convenient since it allows us to keep the vote one-dimensional; this implies, in our context, that we avoid Condorcet cycles.

We begin in Section 2 with a discussion of the standard growth model with infinitely-lived agents, exogenous policy and a nondegenerate wealth distribution. We assume that there are a finite number of types of agents who are identical in all respects except in their holdings of capital. For this model, we show conditions on preferences under which the relative wealth distribution does not matter for capital accumulation. This is useful, since with preferences in this family we can isolate the effects of the political choice process: any dependence of the capital accumulation path on the relative wealth distribution has to originate in the political process and the endogeneity of taxes.

In Section 3 we discuss our politico-economic equilibrium concept, and in Section 4 we make a preliminary characterization of steady-state equilibria. The first of these sections introduces recursive equilibrium language, and the second begins the discussion of the consequences of our equilibrium concept. This discussion is conceptually important, since it emphasizes what is different in this paper from both the literature on exogenous policy and most of the existing political-equilibrium literature. What is new here is that voters, who we posit to be fully rational, need to think through the effects on their utility of a whole range of dynamic adjustment paths when considering different alternative current policy choices. In political science theory, there are similar analyses where voters make correct forecasts and are fully rational (see Danzau & Mackay (1981) for a two-period model, and Epple & Kadane (1990) for a multiperiod setup with uncertainty). It has been stressed in these contexts that Condorcet cycles may occur due to a violation of single-peakedness of preferences. Our framework has the added features that the preferences over the policy are induced via the economic equilibrium and not considered primitives, and that the time horizon is infinite. The requirement that voters make rational forecasts and that policy preferences be derived explicitly in an infinite-horizon equilibrium model clearly complicates the analysis, but we view these features as unavoidable if one wishes to address our types of questions in quantitatively reasonable macroeconomic settings. Our approach, therefore, is to proceed with numerical analysis. A natural part of the paper is therefore our reporting throughout of how the choice of model parameters affects the results.

In Section 5 we describe the basic politico-economic mechanics: we show how differences in initial wealth distributions induce differences in tax rates and capital accumulation paths. Aside from its role as a building block in the comparison of constitutions later in the paper, this section restates some of the main results from the political-equilibrium literature in the context of a neoclassical growth model, and in quantitative terms. Earlier contributions to the literature have been restricted to a narrow set of economic environments which in

particular does not include the standard growth model.

The study of constitutions is the focus of Section 6. It consists of two parts, one varying the political and one the fiscal constitution. For each part, we proceed by first showing how the constitution affects the set of steady states. This turns out to be quite informative, and constitutes our version of a “comparative-statics” analysis. Since our analysis is numerical for the most part, the comparative statics serve both as a check on the importance of specific parameter values and as a way of understanding some nontrivial aspects of how the model works. We then conduct normative analysis of constitutions: we answer questions about what would happen to growth and welfare if one undertook a constitutional change and, hence, which constitutions within the class considered can be recommended.

2 The Standard Growth Model with Exogenous Policy

In this section we describe a typical capital accumulation model which allows agent heterogeneity. The policy parameters here are modeled as given exogenously. We first lay out the framework and then review some results on the interactions between the distribution of wealth or capital in the population and aggregate behavior in our framework. These results are important as a background for understanding the model with endogenous policy.

2.1 A Benchmark Framework

We describe the simplest prototype growth model: a neoclassical, complete-markets growth model with a nontrivial distribution of infinitely-lived agents across asset holdings. The neoclassical production technology is such that growth cannot be sustained in the absence of exogenous productivity change.

There is an infinite, discrete number of time periods, and the economy consists of agents indexed by type. An agent’s type i refers to his initial holding of capital. Apart from different holdings of capital, all other characteristics—preferences, time endowments etc.—are identical across all agents. We assume that there is a large number of agents of each type and that the economy is stationary: there is a finite set \mathcal{I} of I types, i.e. $i \in \mathcal{I}$. There is a measure μ_i of each type and the total is normalized to $\sum_i \mu_i = 1$. It is useful to note that our economy will have the property that the decision rules determining next period’s capital holding will be identical across agents, and that this decision rule will be monotonic in the current capital holding of the agent. This fact implies that agents of the same type will make the same consumption and capital accumulation decisions, and it implies that if agent i is richer than agent i' at any date, he will be richer at all other dates as well.

We let all agents have additively separable utility functions of consumption at different

dates, with $u(c)$ being the current utility of consumption and β being the discount factor.

We denote by a the beginning-of-period asset holdings of a given agent. Since the agent needs to keep track of the distribution of asset holdings for the purpose of predicting prices, we also need notation for the current asset holdings of a generic type i agent. For this we use A_i . We also denote by $A \in \mathbf{R}^I$ the vector of capital holdings of the population types. The time endowment of each agent is equal to 1.

Production takes place by combining capital and efficiency units of labor. Let $Y = F(K, N)$ be a homogeneous of degree zero production function of total capital, K , and total labor, N , with standard neoclassical properties, including $\lim_{K \rightarrow \infty} F(K, \cdot) = 0$. Note that we have $K = \bar{A}$, where $\bar{A} \equiv \sum_i \mu_i A_i$. For notational convenience, let $f(A) \equiv F(\bar{A}, 1)$, and let the competitive rental and wage rates be given by the functions $r(A) \equiv F_1(\bar{A}, 1) - \delta$, and $w(A) \equiv F_2(\bar{A}, 1)$. Finally, we use the standard assumption that one unit of output can be transformed into one unit of capital next period, and that capital depreciates geometrically at some rate $\delta \in [0, 1]$.

An individual agent with capital a collects capital income $a r(A)$ and wage income $w(A)$, and, of course, keeps initial capital a . We consider a type-independent tax and transfer scheme with a proportional tax τ on income, and lump-sum transfers tr of the tax receipts back to the agents. The tax rate, and, therefore, the level of the lump sum transfer, is determined one period in advance. We do this to avoid ex post-taxation of existing capital while keeping a negative distorting effect on capital accumulation. The asset accumulation of a given agent thus reads

$$a' = a + [a r(A) + w(A)](1 - \tau) + tr - c.$$

Given our assumptions, the lump-sum transfer to each agent will be

$$tr = \tau Y - \delta \bar{A}$$

We assume in this section that next period's tax rate τ' is an exogenously given function Ψ of the distribution of wealth and of the current tax system τ i.e. $\tau' = \Psi(A, \tau)$. We will focus on equilibria in which the distribution of capital is first-order Markov, i.e. in which it

evolves according to a function $A' = H(A, \tau)$.⁵ We then write the problem of the agent as:

$$\begin{aligned}
v(A, \tau, a; \Psi) &= \max_{c, a'} u(c) + \beta v(A', \tau', a'; \Psi) \quad \text{s.t.} & (1) \\
a' &= a + [ar(A) + w(A)](1 - \tau) + tr - c \\
tr &= \tau[f(A) - \delta \sum_i \mu_i A_i] \\
\tau' &= \Psi(A, \tau) \\
A' &= H(A, \tau; \Psi)
\end{aligned}$$

A solution to this problem gives next period's asset holdings as a function $a' = h(A, \tau, a; \Psi)$.

We define recursive competitive equilibrium in the standard way, given the function Ψ , by a set of functions $\{H, h\}$ such that

$$H(A, \tau; \Psi) = \{h(A, \tau, A_1; \Psi), \dots, h(A, \tau, A_I; \Psi)\}.$$

This condition represents the fixed point of the recursive equilibrium formulation, i.e. it requires that the optimal laws of motion of the individual agents reproduce the aggregate law of motion they perceive when solving their decision problems.⁶

2.2 Does the Distribution of Capital Matter for Capital Accumulation?

Here we will review the properties of the equilibria defined in the previous section. We will show that under a set of (quite standard) assumptions on preferences to which we will later restrict our analysis, the distribution of capital will not matter for capital accumulation, as long as the tax policies themselves do not depend on the distribution of capital. This result enables us to get a direct connection between politics and capital accumulation, since the political determination of policies precisely makes the policies depend on the distribution of capital. This means that to the extent the distribution of capital matters for capital accumulation in the analysis where taxes are endogenous, it is solely due to the political aspects of the economy. Our discussion in this section draws on Chatterjee (1994), who studies a setup similar to the present one.

⁵For some economies, other types of equilibria may exist.

⁶See, for example, Cooley (1995), chapters 1-4, for an exposition and details on this concept as well as on its computation and properties.

Consider the case in which all agents have the same preferences, and these belong to the generalized Bergson class, (see Pollack (1971)) i.e. they are such that discount factors are the same and u can be written as a function v of an affine function of consumption:

$$u(c) = v(\chi_0 + \chi_1 c),$$

with v belonging to the quadratic, exponential, or constant relative risk aversion families and with restrictions on χ_0 and χ_1 such that the first derivative of u is always positive and the second always negative. It is easy to show in each of these cases that the Engel curves are linear, i.e. that the optimal consumption behavior can be described as follows:

$$c_{it} = d(\{p_s\}_{s=t}^{\infty}) + e(\{p_s\}_{s=t}^{\infty})\omega_{it}, \quad (2)$$

where ω_{it} is defined as the life-time wealth of agent i , i.e. the present value of his current and future labor income (human wealth) together with the current holding of capital (non-human wealth), and where p_t is the price of the date t good (in our formulation, $p_t/p_{t+1} = [1 + r(A_{t+1})][1 - \tau_{t+1}]$). This is true since these utility functions give rise to first-order (Euler) conditions which reduce to linear equations in c_{it} and $c_{i,t+1}$.

Assume now that the function Ψ can be written as a function $\bar{\Psi}$ of total capital, i.e. of $K = \sum_i \mu_i A_i$. Under this assumption, we will show that the given consumption behavior allows aggregation. First, we know that the wage and rental rates are also functions of total capital, and therefore total wealth only depends on (present and future) totals of the capital stock. Second, the first-order conditions, as described by a population-weighted summation of the consumption equations (2) across agents, then imply that total consumption in the economy, and hence the total investment level and tomorrow's total capital stock will only depend on a sequence of (present and future) total stocks of capital:

$$F(K_t, 1) - K_{t+1} + (1 - \delta)K_t = d(\{p_s\}_{s=t}^{\infty}) + e(\{p_s\}_{s=t}^{\infty}) \sum_i \mu_i \omega_{it}, \quad \text{with}$$

$$p_s = \prod_{n=0}^{s-1} \frac{1}{1 + (F_1(K_{n+1}, 1) - \delta)(1 - \bar{\Psi}(K_n))}, \quad p_0 = 1, \quad \text{and}$$

$$\sum_i \mu_i \omega_{it} = \sum_i \mu_i \left(A_{it} + \sum_{n=t}^{\infty} (p_n/p_t) w(A_n) \right) = K_t + \sum_i \sum_{n=t}^{\infty} (p_n/p_t) F_2(K_n, 1).$$

A sequence of total capital stocks solving this relation at all points in time will, provided that additional transversality conditions are met, constitute an equilibrium. Now simply observe that the equilibrium path of the total stock of capital has to be independent of the

distribution of wealth/capital, since the derived equilibrium relations only involve totals! In other words, for the given assumption on preferences, and with a tax policy Ψ which does not depend on the distribution of capital, this economy allows aggregation.

Although this class of economies displays no dependence of capital accumulation and growth on the distribution of wealth, there may be a reverse relation: the accumulation of capital may have nontrivial implications for the evolution of wealth. It is possible to show that unless the function v is linear in consumption, i.e. unless the mentioned affine function of consumption has a zero intercept ($\chi_0 = 0$), the evolution of wealth depends on whether capital is below or above steady state. For example, in the case without taxes and with identical labor productivities across agents, it can be shown that if there is a “subsistence” level of consumption, and the economy is accumulating capital toward a steady state, there will be a gradual (but limited) increase in inequality toward this steady state as measured by the distribution of wealth at $t + 1$ being Lorenz dominated by the distribution at t . As poor agents consume more (save less) in relative terms than rich agents, due to the particularly high marginal utilities of consumption close to the subsistence level, richer agents gain a larger and larger share of the total wealth as time goes by.⁷

Finally, consider instead a tax policy function Ψ which does depend on more than the first moment of the distribution of capital. Then it is clear that the path for total capital can no longer be independent of the distribution of capital. In this paper our focus is precisely on how the Ψ function is determined. In the next section we describe a notion of politico-economic equilibrium, and this notion typically leads to a tax that depends on more than the first moment of the distribution of capital. The preference assumptions described above therefore allows us to isolate the effects of politics on growth: any effect of the distribution on the capital accumulation path has to derive solely from the political considerations.

Note that it is also possible to specialize the experiment so that there will be no distributional consequences of the taxes: if we choose preferences such that χ_0 is zero, then in a relative sense there will be no movement in the distribution of wealth over time. This is easy to see: for these preferences, and in a complete markets context, all agents’ marginal rates of substitution between consumption today and tomorrow have to be equal date by date, which for the given preferences implies that the growth rate of consumption (or, in the case of exponential utility, the increase in consumption) is equal across all agents. Observe, however, that the wealth distribution itself is endogenously determined since it depends on

⁷For a detailed analysis of these issues, see Chatterjee (1994).

the relative prices of current (human and nonhuman) wealth and future (human) wealth. In our computational experiments below we will use preferences with $\chi_0 = 0$.

3 Political Equilibrium

In section 2.2 it was argued that the effects of the initial distribution of wealth on capital accumulation depend above all on the form of the tax policy function Ψ . We now describe a positive theory of capital taxation: Ψ is chosen as an outcome of period-by-period voting over the following period's tax rate. We keep the notation of the last section; Ψ describes the political outcome as a function of the current distribution of capital, and the current tax system, and H is the law of motion for this distribution of assets. Of course, the H function will have to satisfy precisely the conditions listed in the previous section where Ψ was treated as exogenous.

An essential input into the political equilibrium is the tax policy preferences for each type of agent. Our point of view is that these preferences are formed by the agent considering the competitive equilibrium consequences of each possible tax rate for prices and, hence, for his utility. The fact that the current tax rate has consequences for all future periods—it determines the amount of total investment, and hence next period's total capital stock—means that it is altogether nontrivial to think this problem through. Add to this the issue of predicting future tax rates, which will typically depend on the future capital stock, and therefore also on the current tax rate, and the consequences of different current tax rates become yet more complex. Our recursive equilibrium definition follows that set out in Krusell & Ríos-Rull (1992), and it provides a way of consistently dealing with these issues.

An essential element into our politico-economic equilibrium is the law of motion for the distribution of assets (A) specified for all possible values of the current tax rate and *next period's* tax rate. This function, given by $A' = \tilde{H}(A, \tau, \tau'; \Psi)$, is key in the political considerations: it summarizes all the equilibrium consequences of changes in next period's tax rate for the evolution of the aggregate state. For example, it tells us that for a next period tax rate of 0.05, whether it is voted in or not, the next period wage would be $w(\tilde{H}(A, \tau, 0.05; \Psi))$, the rental rate would be $r(\tilde{H}(A, \tau, 0.05; \Psi))$, the next period's vote would be $\Psi(\tilde{H}(A, \tau, 0.05; \Psi), 0.05)$, the distribution of capital two periods hence would be $A'' = H(\tilde{H}(A, \tau, 0.05; \Psi), 0.05; \Psi)$, and so on. In particular, \tilde{H} will satisfy $\tilde{H}(A, \tau, \Psi(A, \tau); \Psi) = H(A, \tau; \Psi)$ for all A , and all τ i.e. for the tax rate chosen by the majority the original law of motion H is reproduced.

We showed in the last section how H was determined given Ψ . We now turn first to how \tilde{H} is determined given Ψ , and lastly we show the fixed-point condition for Ψ . Since \tilde{H} is

generated by individual maximization, we need to consider the following problem for a given agent of type i who has wealth a :

$$\begin{aligned}
\tilde{v}(A, \tau, \tau', a; \Psi) &= \max_{c, a'} u(c) + \beta v(A', \tau', a'; \Psi) \quad \text{s.t.:} & (3) \\
a' &= a + [a r(A) + w(A)](1 - \tau) + tr - c \\
tr &= \tau[f(A) - \delta \sum_i \mu_i A_i] \\
A' &= \tilde{H}(A, \tau, \tau'; \Psi)
\end{aligned}$$

In this problem, where next period's tax rate is given as opposed to determined by Ψ , there are two key things to note. The first is that next period's evaluation of capital is given by the value function obtained in the recursive competitive equilibrium associated to problem (1). Second, the law of motion of the distribution of wealth is given by the function \tilde{H} . Its solution delivers decision rules $a' = \tilde{h}(A, \tau, \tau', a; \Psi)$.

Like before, we can define equilibria with the aid of problem (3) in addition to problem (1). Such an object is a standard competitive equilibrium for the sequence of tax rates given by τ in the current period, and those taxes generated by the function Ψ in all consecutive periods. Formally, this is a set of functions $\{H, h, \tilde{H}, \tilde{h}\}$ such that $\tilde{H}(A, \tau, \tau'; \Psi) = (\tilde{h}(A, \tau, \tau', A_1; \Psi), \dots, \tilde{h}(A, \tau, \tau', A_I; \Psi))'$, i.e. with the usual property that individual behavior reproduces the perceived aggregate behavior.⁸ We use \tilde{H} and \tilde{h} to denote such competitive equilibria, accompanied by value functions \tilde{v} .

Now turning to the determination of Ψ , suppose that an agent contemplates the effect of different current tax rates, given this family of \tilde{H} equilibria, on his realized utility. The current tax rate τ will not only affect the current transfer; it will also affect tomorrow's distribution of income—through the function \tilde{H} —and hence later distributions—through the function H —which determine all future policies. It also implicitly takes into account the effect of today's tax rate on future tax rates; unless it happens that $\Psi(H(A, \tau; \Psi)) = \Psi(\tilde{H}(A, \tau, \tau'; \Psi))$, there will be a nontrivial such effect.

The highest utility achievable for the agent then occurs for the tax rate that solves:

$$\max_{\tau'} \tilde{v}(A, \tau, \tau', a; \Psi). \quad (4)$$

We denote the solution to this problem $\psi(A, \tau, a; \Psi)$. This function returns the most preferred

⁸Note that it follows that \tilde{H} and \tilde{h} assume the values of H and h when evaluated at $\tau' = \Psi(A, \tau)$.

current tax rate for the level of wealth of each type of agent i , given that from tomorrow on the tax policy reverts to that given by the function Ψ .⁹

In Krusell & Ríos-Rull (1992) a majority rule was used as a way of aggregating preferences. In that context, there were only two possible policies, and in such cases the majority rule is well-defined and the obvious (democratic) aggregator to use. Here, we have a continuum of possible tax rates and the majority rule is typically not a useful aggregator. In what follows, we aggregate preferences over policies by using the preference of the agent with median wealth. We do not justify this aggregator theoretically in this paper. However, in our numerical work we do make sure for each parameter configuration that (i) the policy preferences ψ are single-peaked for all wealth levels; and (ii) that the preferred policy is monotone in the agent's level of wealth, so that the median voter coincides with the agent with median wealth.

We refer to median agent types as m agents. The median agent in this economy may or may not change over time and m may change as a function of A , but we leave out the notation that would make this updating explicit. The specific economies we study below have the property that the relative wealth positions do not change (this is achieved using $\chi_0 = 0$ in the Bergson class discussed in that section). In that case, the median voter remains the same throughout time. The condition determining Ψ is thus:

$$\Psi(A, \tau) = \psi(A, \tau, A_m; \Psi) \quad \text{for all } A, \tau. \quad (5)$$

This is our final fixed point equation in the function Ψ : a politico-economic equilibrium has to have the property that the chosen tax policy function Ψ is generated by the preferences of the median agent when this agent takes Ψ as given (along with H and \tilde{H} which also depend on Ψ) in his economic as well as political decisions.

4 Steady States

We now make some preliminary characterization of politico-economic equilibria. We will mainly point to some properties of steady states. A steady state of our economy is defined as a wealth distribution $A^{\text{ss}} \in \mathbf{R}^I$ and a tax rate τ^{ss} such that $A^{\text{ss}} = H(A^{\text{ss}}, \tau^{\text{ss}})$, and $\tau^{\text{ss}} = \Psi(A^{\text{ss}}, \tau^{\text{ss}})$. In the case of economies where tax rates are exogenous, a time-independent tax

⁹Of course, as illustrated in Krusell & Ríos-Rull (1992), this need not be a function, but since none of the discussion below depends on (4) having a unique solution we use the simpler notation.

rate τ would generate a set of steady states as the solution to

$$\frac{1}{\beta} = 1 + (1 - \tau)(F_1(\sum_i \mu_i A_i^{\text{ss}}, 1) - \delta). \quad (6)$$

Any such solution is a steady state, since it implies that each agent faces a total return to savings equal to the marginal rate substitution between goods today and goods tomorrow, date by date. As we noted before, this relation implies that any relative distribution of wealth constitutes a steady state—the steady state condition applies only to the total amount of capital. To find the steady state capital level, we simply note that the mentioned condition has a unique solution under the assumptions set out, and that it can be solved for analytically.

In Figure 1 we depict the set of steady states, a line with a slope of $-\mu_1/\mu_2$, and the transitional dynamics for an economy populated with two types of agents with equal measure.

When taxes are determined through the function Ψ , the set of steady states is the set of wealth distributions and tax rates that solve:

$$\begin{aligned} \frac{1}{\beta} &= 1 + (1 - \Psi(A^{\text{ss}}, \tau)) \left(F_1(\sum_i \mu_i A_i^{\text{ss}}, 1) - \delta \right) \\ \tau^{\text{ss}} &= \Psi(A^{\text{ss}}, \tau^{\text{ss}}) \end{aligned} \quad (7)$$

Our first task here is to characterize the set of solutions $(A^{\text{ss}}, \tau^{\text{ss}})$ to (7). This is a considerably more difficult task than in the case of constant and exogenous taxes, since Ψ is endogenous. In turn, Ψ cannot be determined without simultaneously determining the function \tilde{H} (and its special case H). This just means, of course, that it is impossible to claim that a certain pair of assets distribution and tax rate $(A^{\text{ss}}, \tau^{\text{ss}})$ is a steady state without also having solved for the dynamic equilibrium capital paths generated for the alternative current tax rates $\tau \neq \Psi(A^{\text{ss}}, \tau^{\text{ss}})$.

When a steady state exists and Ψ is well-behaved, we have the required elements for discussing some of the properties of the set of steady states. First, as long as Ψ is not a constant, the set of steady states is different than in the case of exogenous taxes. Of course, Ψ will typically not be constant: as the distribution of capital is changed, the median agent will find that a given change in the tax rate generates a different set of current and future transfers. This intuition would suggest, for example, that in an economy in which the richest type is not the median voter, tax rates are increasing in the capital holdings of the richest type. Second, note that as in the case of constant taxes, the set of steady states seems to allow $I - 1$ degrees of freedom: the steady state condition is one equation, and there are I capital stocks to be determined. Granted certain regularity conditions, we could characterize

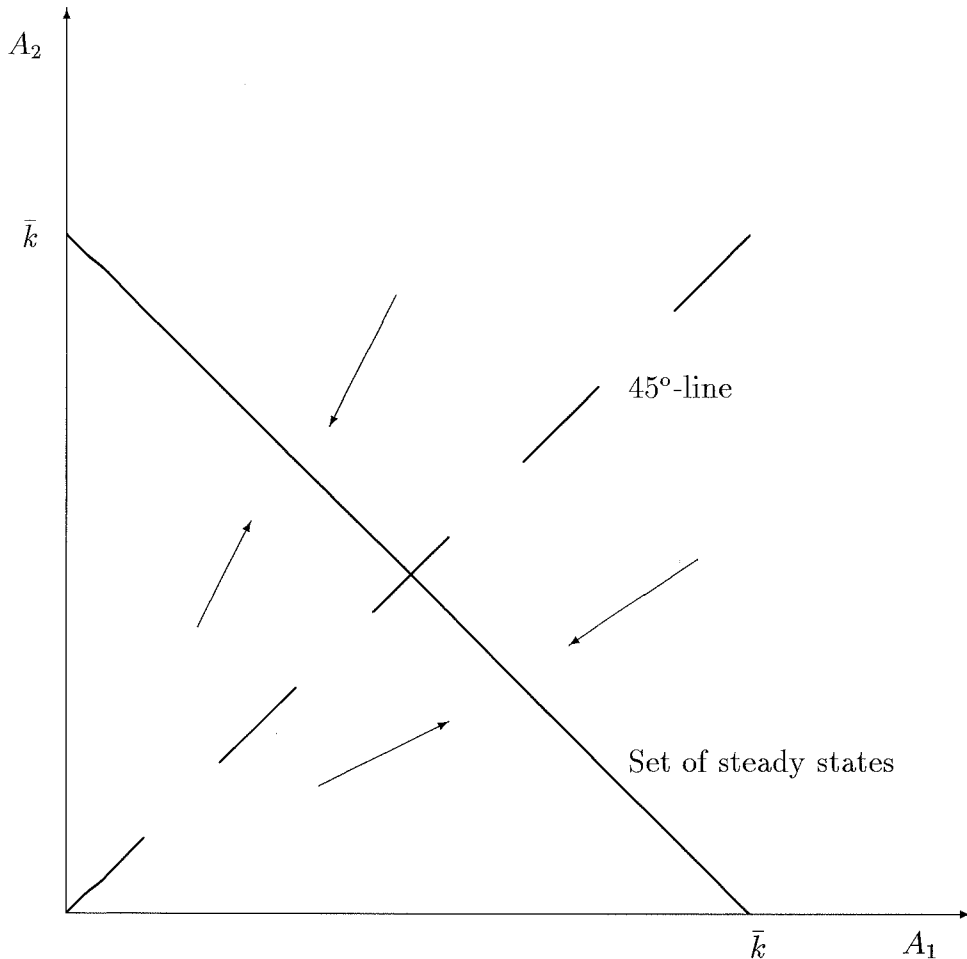


Figure 1: Steady states and dynamics with constant, exogenous taxes.

the set of steady states as an $I - 1$ -dimensional manifold.

In the following we will focus our discussion on a particular, very simple set of steady states, where the mean wealth equals the median wealth. In general, the existence of steady states is difficult to establish analytically; the difficulty derives from the political-equilibrium layer which is superimposed on the standard model.¹⁰ What we can do, however, is to guess the value of Ψ at a specific subspace, and prove that such a subspace is “part” of a politico-

¹⁰To show that equilibria exist in discretized economies (in this case, if there is a finite number of feasible levels of asset holdings and capital), it is possible to apply Fudenberg & Tirole (1991)’s results on the existence of Markov equilibria for dynamic games.

economic equilibrium. The partial nature of the result is the following: we use a recursive definition of equilibrium, as above, with the additional restriction that all equilibrium functions are defined only on a subspace of the domain R^I . In general any such definition is vacuous, since politico-economic considerations may require knowledge about the behavior of the economy outside this subspace: for any given value of the state (the vector of asset holdings) in the subspace, it has to be true that no current tax rate would imply an equilibrium law of motion of the state which takes the economy outside the given subspace. The trick here is that we know that there are subspaces where the rationale for taxation and, therefore, for distortion is not strong enough. We start characterizing the subspace where all agents have identical wealth (this subspace has one dimension). All proofs are contained in Appendix 1.

Theorem 1. *There exists a politico-economic equilibrium restricted to the domain of wealth levels given by $A_i = A$ for all i . This equilibrium has $\Psi = 0$ on its entire domain.*

With stronger assumptions on preferences, we can consider a larger domain.

Theorem 2. *If preferences belong to the Bergson class, there exists a politico-economic equilibrium restricted to the domain of wealth levels given by $A_m = \bar{A} = \sum_i \mu_i A_i$ (median wealth equals mean wealth). This equilibrium has $\Psi = 0$ on its domain.*

In particular, the above theorems imply that zero taxes and equal distribution with A_i solving (6) is a steady state. This steady state can be found due to its very specific characteristics: since redistribution is impossible, preferences of agents over tax rates capture only the pure distortionary nature of the tax system, simplifying the voting choice for agents. In the next section we exploit the knowledge of the existence of the zero-tax steady state by using numerical techniques to extend the domain of the equilibria locally to a set of full measure, and where taxes are not necessarily zero. This allows us to understand the mechanics of politico-economic equilibria and, in particular, to explore the effects on the accumulation of capital of small changes in the initial distribution of capital over the population.

5 Politico-Economic Mechanics

In this section we discuss the mechanics of politico-economic equilibria. In particular, we show how the distribution of asset holdings, which in the absence of endogenous policy determination plays no independent role for capital accumulation, fundamentally changes the course of prices and aggregate capital accumulation via its impact on the level of taxes.

These mechanics are important to understand both qualitatively and quantitatively, since they constitute a key building block for the the constitutional experiments in the following section.

5.1 An economy with two types

Consider an economy populated with two types, with type 2 being the median type. For simplicity let us also assume that the type 2 group is only arbitrarily larger than the type 1 group, i.e. $\mu_1 = \mu_2 = 0.5$. The preference parameters we use are the following: the discount rate, β , is set at 0.96^4 , reflecting the treatment of a period as covering 4 years, and the coefficient of risk aversion, σ is set to 2. The technology paramaters are a labor share α of 0.64 and a depreciation rate satisfying $1 - \delta = (1 - 0.08)^4$, to match observed factor shares and consumption-to-investment ratios in the postwar U.S. data. Further, assume that the holdings of capital of each type are the same, and that the total capital stock equals that of the steady state with an exogenous tax rate of zero, which is approximately equal to 0.70 for this economy.

In this section we restrict the discussion to a linearization around the steady state with zero taxes. The linearization provides a convenient vehicle for the discussion of short- and long-run effects of changes in the initial distribution of assets; we will provide global results in our constitutional experiments below.¹¹ For the present specification, we indeed found that a zero tax is consistent with a steady state of the politico-economic equilibrium. The linearized tax function we found was

$$\Psi(A_1, A_2, \tau) = 3.953A_1 - 3.953A_2,$$

which equals 0 as long as the two types have the same capital stock, in accordance with the above theorems. Furthermore, note that if $A_2 > (<)A_1$, there will be a negative (positive) current tax: if the median is richer (poorer), savings will be subsidized (taxed).¹²

The law of motion of the economy can be described locally by the matrix

$$H = \begin{pmatrix} 0.50 & 0.27 \\ -0.50 & 1.27 \end{pmatrix}.$$

¹¹For a description of the numerical procedure, see Appendix 3.

¹²All the environments described will have a monotonicity property: agents who are richer save more and remain richer.

This matrix applied to any initial deviation $(\Delta A_1, \Delta A_2)'$ from the steady state describes the deviation implied in the next period. It is straightforward to check that this matrix has one eigenvalue equal to one and one positive and less than one. The eigenvalue equal to one indicates the steady state indeterminacy we expected, and the fact that the remaining eigenvalue is less than one says that the system is non-explosive.

It is possible to use the H matrix to find the set of steady states in the neighborhood of the zero tax steady state. It is given by the null space of H , or, equivalently, it can be calculated by using the obtained function Ψ and applying the formula (7). We find, somewhat surprisingly, that these steady states are given by a line

$$\Delta A_2 = 1.85\Delta A_1,$$

This is to be compared to the case where taxes are exogenously set at zero, where we know that the set of steady states is given by a line of slope $-\mu_1/\mu_2 = -1$. A way to see this is to observe that equation (7) has the property that the function Ψ is quantitatively important as changes are made in the two capital stocks: the derivative of the production function and the effect on the interest rate are small in relative terms. Since if F_1 is constant in equation (7), this equation implies precisely a slope of $+1$. Now, we consequently get a number not too far from $+1$. Figure (2) shows the slopes of the steady states for both the economy with zero taxes and the economy with endogenously determined taxes.

It is also useful to compare the dynamics of our politico-economic equilibrium to that of the case with exogenous zero taxes. The law of motion in that case can be described by a matrix \bar{H} , which satisfies

$$\bar{H} = \begin{pmatrix} 0.89 & -0.11 \\ -0.11 & 0.89 \end{pmatrix}.$$

Clearly, the local dynamics are also very different from when taxes are exogenous. In particular, an initial increase in, say, A_1 by one unit will trigger tax increases when type 2 is the median voter, and the economy will move away from the initial steady state. In the case taxes are exogenous, the dynamics following this initial change in type 1's capital are simple: the economy decumulates capital to get back to a new point on the line with slope -1 : one implying that type 1 is permanently richer than type 2 by one unit. What happens in the endogenous-tax economy "in the long run"? To find out, we calculated H_∞ , which when postmultiplied by any vector $(\Delta A_{10}, \Delta A_{20})'$ describes the long-run change, $(\Delta A_{1\infty}, \Delta A_{2\infty})'$, in the two capital stocks, i.e. $H_\infty = \lim_{t \rightarrow \infty} H^t$. In our example, we obtained

$$H_\infty = \begin{pmatrix} -1.2 & 1.2 \\ -2.2 & 2.2 \end{pmatrix}.$$

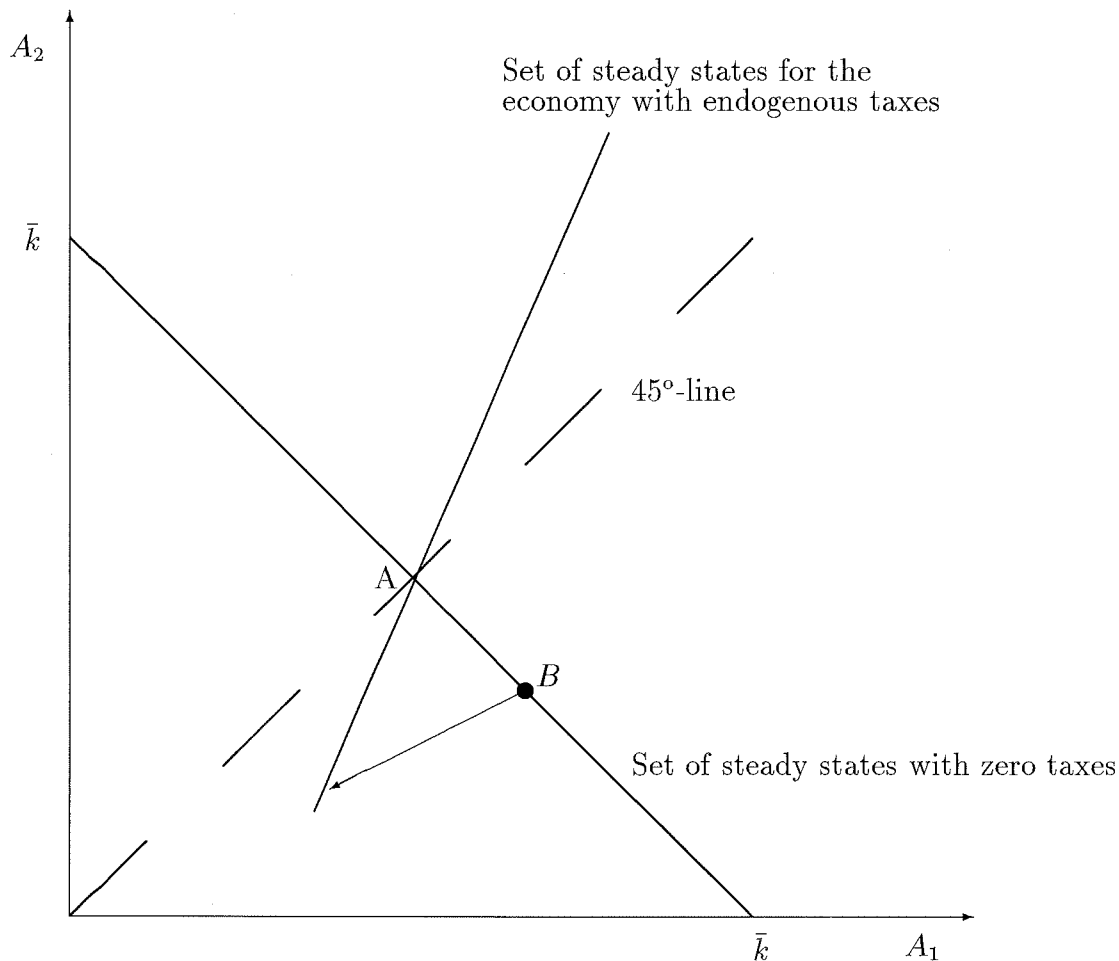


Figure 2: Steady states and dynamics with endogenous taxes.

The changes following initial changes in the asset distribution are dramatic. If type 1 (2) is given 1 additional unit at time zero, the long-run response is for both types to decrease (increase) their capital stock by 2.2 units. The dynamic response takes us back to a new point on the positively sloped steady state line, but we are now far away from the initial equal-capital point. The relative distribution remains intact throughout time: the type given an additional unit will remain richer indefinitely. We see that at least in this example, the capital accumulation path fundamentally changed with a small change in the initial distribution of capital. When we analyze the importance of the redistributive mechanism for the long-run performance of the economy, we will consider the following redistribution exercise: from a position of equal wealth among all types, take away 0.5% of the wealth from the median type and give it to the richest group, creating a 1% initial wealth gap. In Figure

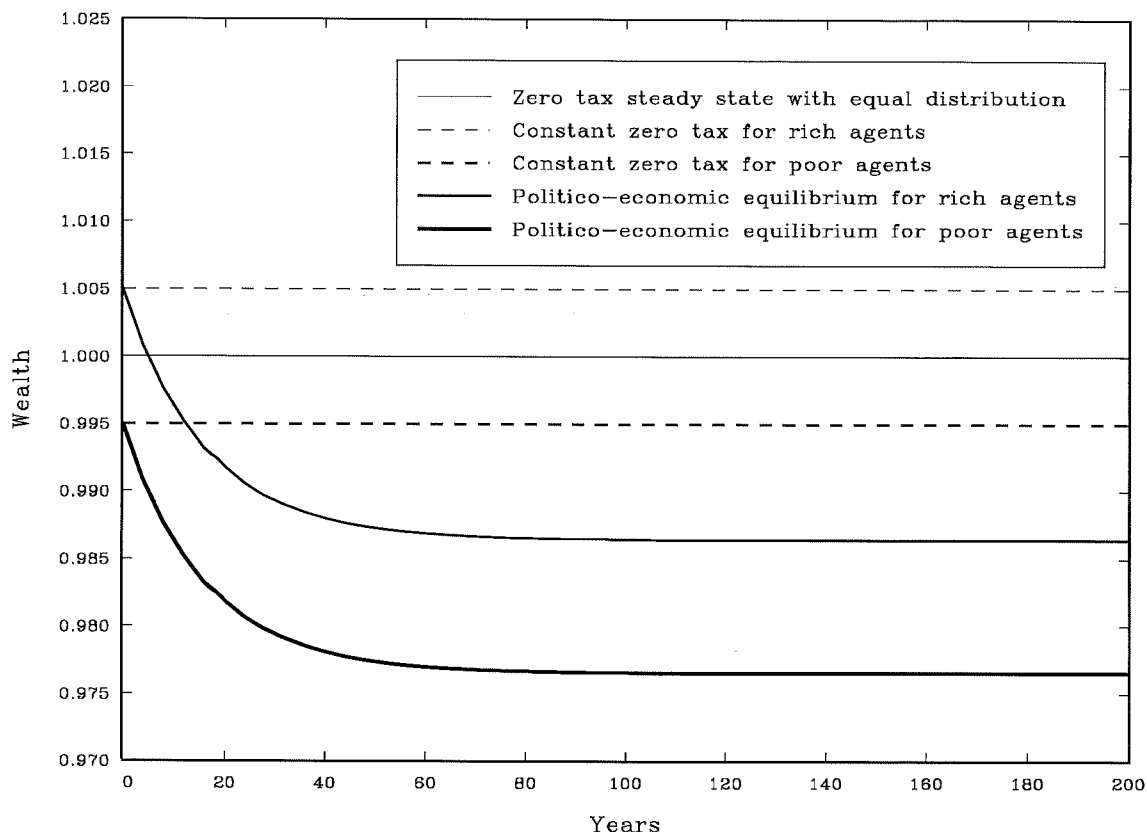


Figure 3: Wealth dynamics with a 1% initial wealth gap.

2 this is represented by moving from point *A* to point *B*. The arrow in the figure denotes the ensuing dynamics, which is to be compared with the dynamics in Figure 1.

Figure 3 shows the path for the distribution of wealth over time, and Figures 4 and 5 show the consumption and tax dynamics, respectively. Given the small initial wealth gap, the quantitative effects of adding the political equilibrium to the model are very large. One summary way to describe our results is to calculate the percentage change in long-run output (i.e. output in the new steady state) which follows the redistribution of capital described—we call this the long-run output to initial capital elasticity. In this case, this elasticity is about 1.22 (the curvature of the production function makes the decrease in output a lot smaller than the reduction in aggregate capital). The corresponding number for the exogenous tax experiment is identically 0: the capital accumulation path is not affected when taxes are set at zero exogenously.

5.2 Changes in parameter values

Our calibration above used specific values for parameters. We will now comment briefly on the sensitivity of the results to changes in these some of the parameter values.

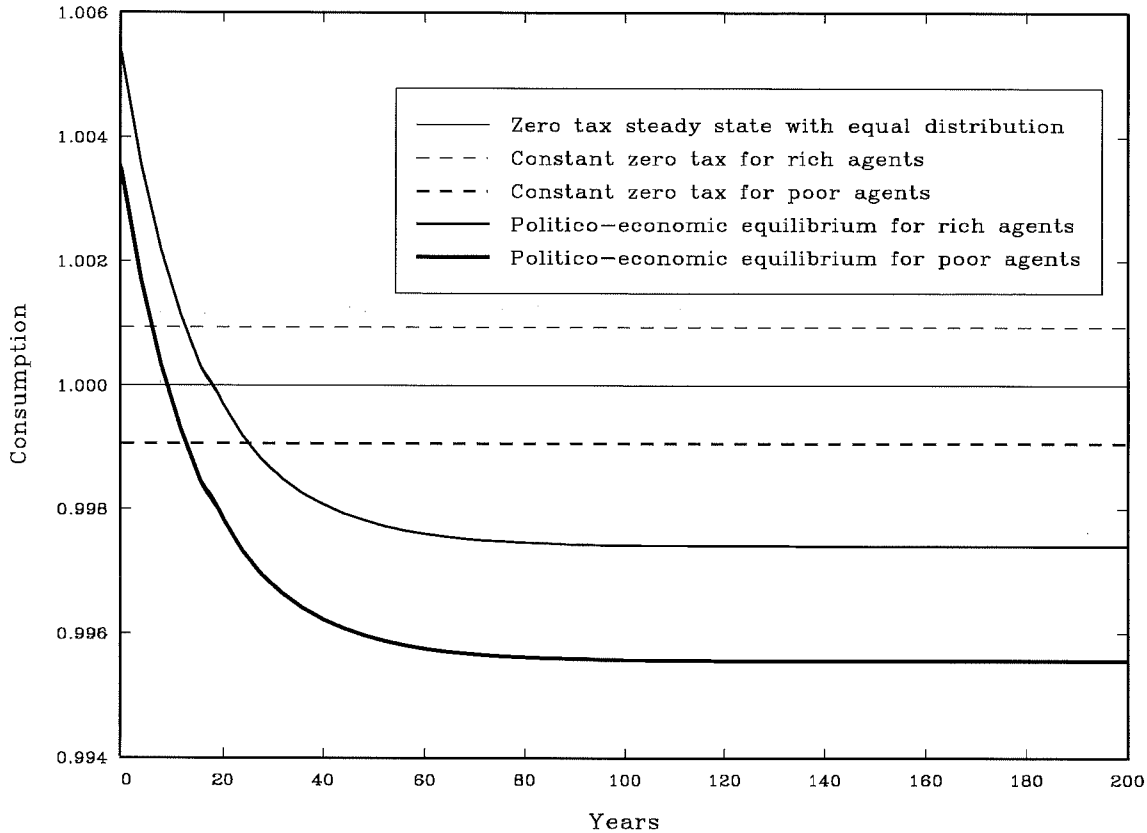


Figure 4: Consumption dynamics with a 1% initial wealth gap.

First, as will become clear in the sections below, the timing is a crucial parameter in the endogenous tax economy, and the key timing parameter in the baseline model is β . If one were to interpret the period as something different than 4 years, β would have to be recalibrated to maintain a reasonable long-run interest rate. Indeed, it will turn out that β has stark quantitative implications in the baseline case, where the period length both represents the length of a base period and the frequency of elections. As we will show below, however, it is the latter which really matters, and for this reason we postpone the analysis of the effects of changes in β until the sections where we conduct constitutional experiments.

We also varied σ , the parameter which governs the intertemporal elasticity of substitution. Figure 6 shows the results: with less willingness to substitute consumption over time (with a higher σ), it is easier to raise tax revenue, since agents' savings are less elastic with respect to the tax rate.¹³ Consequently, the benefits from taxing a little more are perceived as higher by the median voter, and the effects of the political transfer system will be more dramatic.

¹³The set of steady states is not linear in Figure 6, reflecting the fact that the steady states were computed separately, and not as a simple linearization around the equal-wealth point. See Appendix 3 for details.

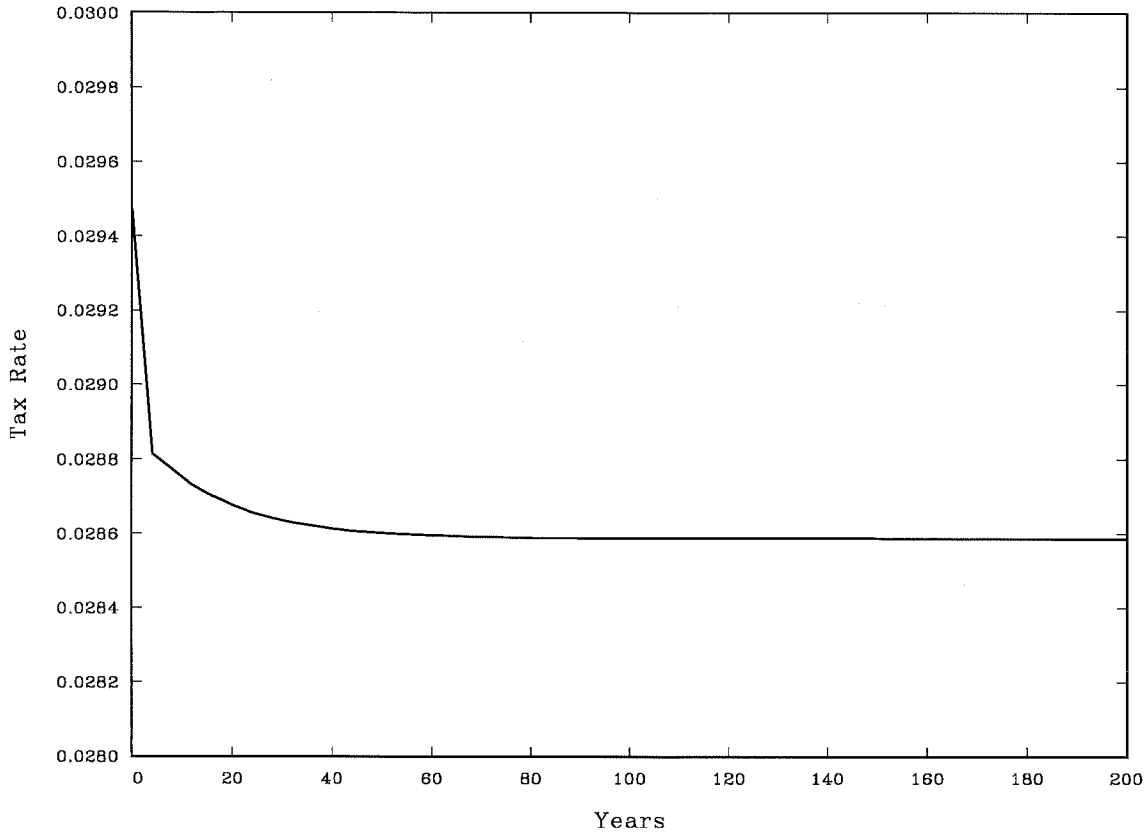


Figure 5: Tax dynamics with a 1% initial wealth gap.

In the figure, this is represented by a narrower set of steady-state wealth distributions: for any given relative wealth level, the tax rate is higher, and the total capital stock is lower; with a higher value of σ .

Varying the remaining parameters α and δ adds little to the above. The share parameter α does change the quantitative results somewhat, but changes no qualitative features; the depreciation parameter δ has effects similar in nature to those of β , since it influences the aspects of timing. For brevity, we omit tables showing these properties explicitly.

5.3 More than two types

It is easy to extend this analysis to economies with more than two types. For the purpose of illustration, consider the following two cases with three agents: one with $\mu_1 = \mu_3 = 0.4$ and $\mu_2 = 0.2$, and another with $\mu_1 = \mu_3 = 0.49$ and $\mu_2 = 0.02$. In both experiments the parameters are as in the baseline case, the initial distribution is equal, and the median agent is type 2.

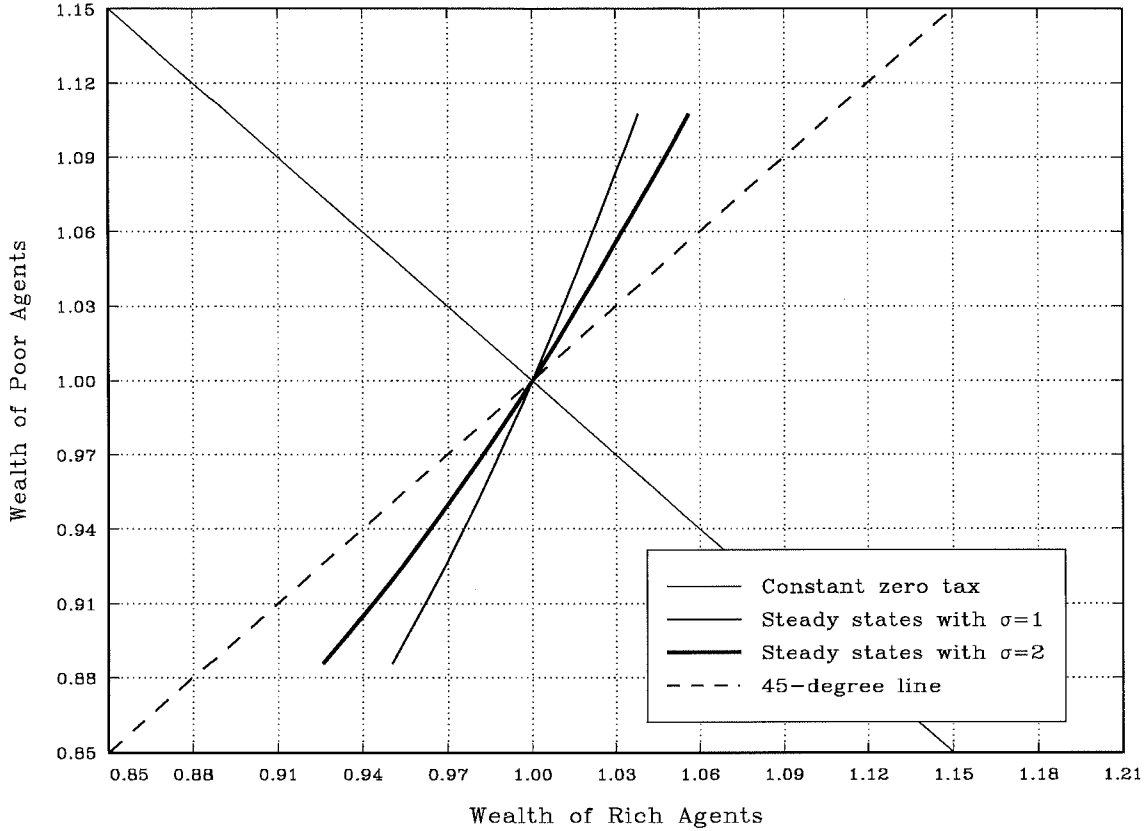


Figure 6: Steady states for different values of σ .

For the $\mu = (0.4, 0.2, 0.4)$ the law of motion becomes

$$H = \begin{pmatrix} 0.60 & 0.57 & -0.40 \\ -0.40 & 1.57 & -0.40 \\ -0.40 & 0.57 & 0.60 \end{pmatrix},$$

and

$$H_\infty = \begin{pmatrix} -0.75 & 2.49 & -1.75 \\ -1.75 & 3.49 & -1.75 \\ -1.75 & 2.49 & -0.75 \end{pmatrix}.$$

The latter matrix is symmetric in the sense that an income shock to type 1 has the exact same effect as an equivalent shock to type 3. The long-run output to initial capital elasticity for this case is 1.22 as above. We see again that there are very large effects on capital accumulation of initial redistribution of capital. Of course, many different redistributions are possible now that there are three agents, and some redistributions have larger effects than others. Redistributions across types 1 and 3 using, say, $(1, 0, -1)'$, do not have any effect other than maintaining the initial implied inequality; the aggregate capital accumulation path is not affected. Thus, one important point to make here is that reshufflings of wealth

such that the relation (in this case difference) between mean wealth and median wealth is preserved have no effect on capital accumulation (recall, however, that this result depends on the preferences having the assumed form).

The case with population weights (0.49,0.02,0.49) gives large effects as well:

$$H = \begin{pmatrix} 0.51 & 0.75 & -0.49 \\ -0.49 & 1.75 & -0.49 \\ -0.49 & 0.75 & 0.51 \end{pmatrix},$$

and

$$H_{\infty} = \begin{pmatrix} -1.14 & 3.28 & -2.14 \\ -2.14 & 4.28 & -2.14 \\ -2.14 & 3.28 & -1.14 \end{pmatrix}.$$

The elasticity long-run output to initial capital is the same as before: the median-to-mean wealth ratio is the same in all our three experiments.

6 The Role of the Constitution

We now explore how the rules of policy selection in a society affect final policy outcomes, and thereby how they affect economic activity. For lack of a better term, we have chosen to label these rules constitutions, and we will study a few political as well as fiscal aspects of these constitutions. The analysis is normative in spirit: whereas traditionally (viz. the optimal taxation literature) the economist analyzing the role of policy chooses across values of the policy itself, we incorporate what we believe are important “political restrictions” by formally studying political equilibria, and then instead choose across constitutions.¹⁴ Our work thus answers the question: “What constitutions lead to good economic outcomes?”, as opposed to “What policies lead to good economic outcomes?”. It seems important to know the answers to both of these questions.

It should also be said that, in reality, rules as well are subject to endogenous choice, which then in turn holds for rules for choosing rules, *ad infinitum*. Unless one adopts a pure mechanism design approach, an exogeneity assumption has to be made on some level. The real issue is practical: what could this kind of policy analysis be used for?¹⁵ In this perspective, it seems clear that exogeneity assumptions on different levels are complementary—it

¹⁴Our work is therefore close in spirit to the rules-vs.-discretion debate; for an overview, see Persson & Tabellini (1990).

¹⁵For mechanism design approaches, see Lagunoff (1992), or Holmstrom & Myerson (1983).

is important to know how agents behave with respect to different kinds of given taxes, but recommendations in the form of policies may well be less effective than recommendations which take into account some elements of the political reality and which are formulated instead in terms of institutional rules.

Because we restrict our analysis to making the rules exogenous, we need to specify a set of allowable rules, just like in traditional policy analysis the set of allowable policies is postulated at the outset of the analysis. In our context, there are of course many more rules to consider than those we look at here. Our analysis emphasizes the intertemporal nature of the economy—we are interested in capital accumulation—so we choose to vary aspects of the rules with interesting and nontrivial dynamic implications.

6.1 The Political Constitution

Even in an environment as simple as the one that we have here—the members of society only have, at each point in time, one policy to apply and disagree over—there is a variety of specific rules that could determine the policy selection.

First, there is a choice of what we have called a political aggregator, which determines the relative political weights of the different agents of the economy. In this paper, the aggregator used is one person one vote (or any mechanism weighting all agents equally), which for our specification makes the agent with median wealth the pivotal agent. Other choices include weighting agents by wealth levels, or reducing the constituency to include only those agents with a minimum level of wealth (which was popular until not too long ago). Changing the aggregator in this way mainly has the effect of displacing the identity of the median voter in a mechanical way. We have therefore chosen not to pursue this type of experiment in detail. Further, more elaborate descriptions of political games can also be made, such as in Besley & Coate (1994), leading to nontrivially different outcomes. We postpone such extensions to future research.

Second, what is important and nontrivial in our model is the issue of timing. There are two aspects of timing. One aspect is how often the decisions over policies are revised; we call this the **frequency of elections**. The other aspect is the length of time between when a policy is decided over and when it is put in place; we call this the **implementation lag**. In other words, we let agents vote on the policy variable every J periods, and we let the policy applied in period t be voted on in period $t - L$, where J is the frequency of elections and L is the implementation lag. For example, if $J = 2$ and $L = 1$ and there is a vote at time t , this vote determines the policies at times $t + 1$ and $t + 2$, and these policies are restricted to be the same; the next vote takes place at $t + 2$, and so on.

It is straightforward to generalize the setup in the earlier section to general values for J and L (there, $J = L = 1$). We include this formalization in Appendix 2. In Appendix 3 we also discuss the numerical methods used in this section, where we no longer restrict ourselves to local analysis.

To analyze the differences between constitutional settings, we start by comparing their steady-state properties: how does the set of steady-state wealth distributions depend on J and L ? In other words, we do not in that context consider adjustment paths which may be necessary in response to changes in the constitutions. This is our version of traditional comparative statics, but it is clear that comparative statics here really involves understanding dynamics as well. Next, we do perform justified constitutional experiments: we ask what different constitutions would imply in terms of policy and capital accumulation paths, and in terms of welfare, for a given initial wealth distribution.

6.1.1 How does the political constitution affect the set of steady states?

We present the results in two parts; we first study the effect of changing the frequency of elections by varying the parameter J , and then analyze implementation lags by varying L .

6.1.1.1 The frequency of elections. Figure 7 shows the steady-state distributions of wealth for economies where the economic decisions are made every quarter but elections are held every two, four and eight years (in these cases $J = 8, 16, 32$ while $L = 1$).¹⁶ The figure shows how the set of steady states changes with J : the longer the time period between elections, the more the steady-state line tilts toward the line associated with exogenous taxes, or, the narrower the wealth distribution has to be to support any given total level of capital.

Why does the election frequency influence the outcomes so much? In our economy, the identity of the median agent does not change over time, which might suggest that at least steady states should not depend on the frequency by which this agent remakes, or reconfirms, the policy choice. But this argument is faulty: the median voter cannot commit to whole sequences of taxes, but takes future taxes as given by the way future median voter, i.e. his future self, behaves as a function of the wealth distribution at that point. In fact, our equilibrium concept can be described as a time-consistent (Markov) equilibrium where all voters take the future “reaction function”—the function Ψ —as given when they figure out the effects of current policy changes. Now, the longer the time over which a

¹⁶The economy with $J = 16$ is the economy of the previous section recalibrated to quarters.

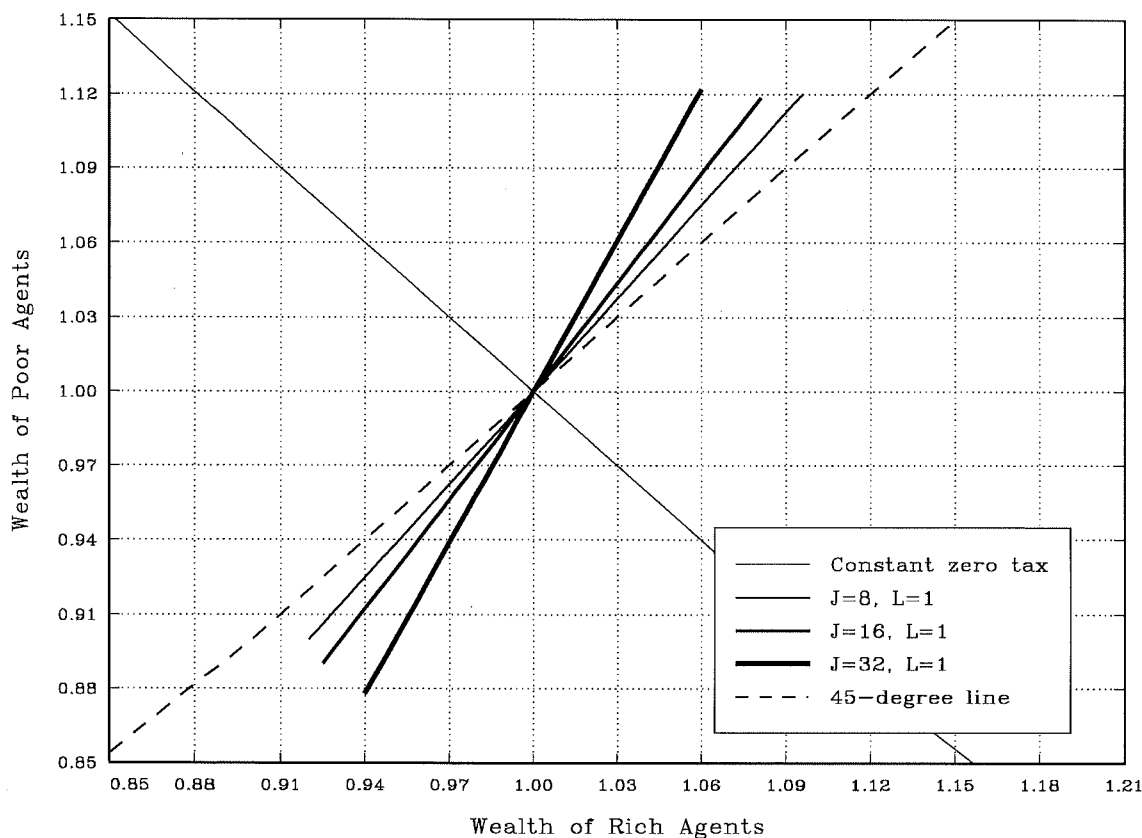


Figure 7: Steady states with different election frequencies.

policy remains intact, or, in our case, the longer the election frequency, the less severe is the commitment problem, since it implies that the tax can be chosen over a longer horizon. To fully understand why the commitment problem is more severe with more frequent policy reconsiderations, it is useful to elaborate further on our constitution. As we shall see, it is not only the number of periods for which you can commit in a given vote that matters to the political outcome but, perhaps more importantly, the time span between the policy decision and the implementation of the policy.

6.1.1.2 Implementation lags. Figure 8 includes the steady states associated to economies with a variety of implementation lags. These are all economies calibrated so that a period corresponds to a quarter and such that elections are held every four years ($J = 16$). The implementation lag ranges from $L = 2$ to $L = 8$. Again, the graph shows that differences are non-trivial. The longer the implementation lag, the larger is the difference in the wealth between rich and poor that can be sustained as a steady state for a given amount of aggregate wealth. Quantitatively, doubling the length of the implementation lag makes the mentioned difference in wealth increase by about 40%.

Why, then, does the implementation lag have such significance? The argument is as

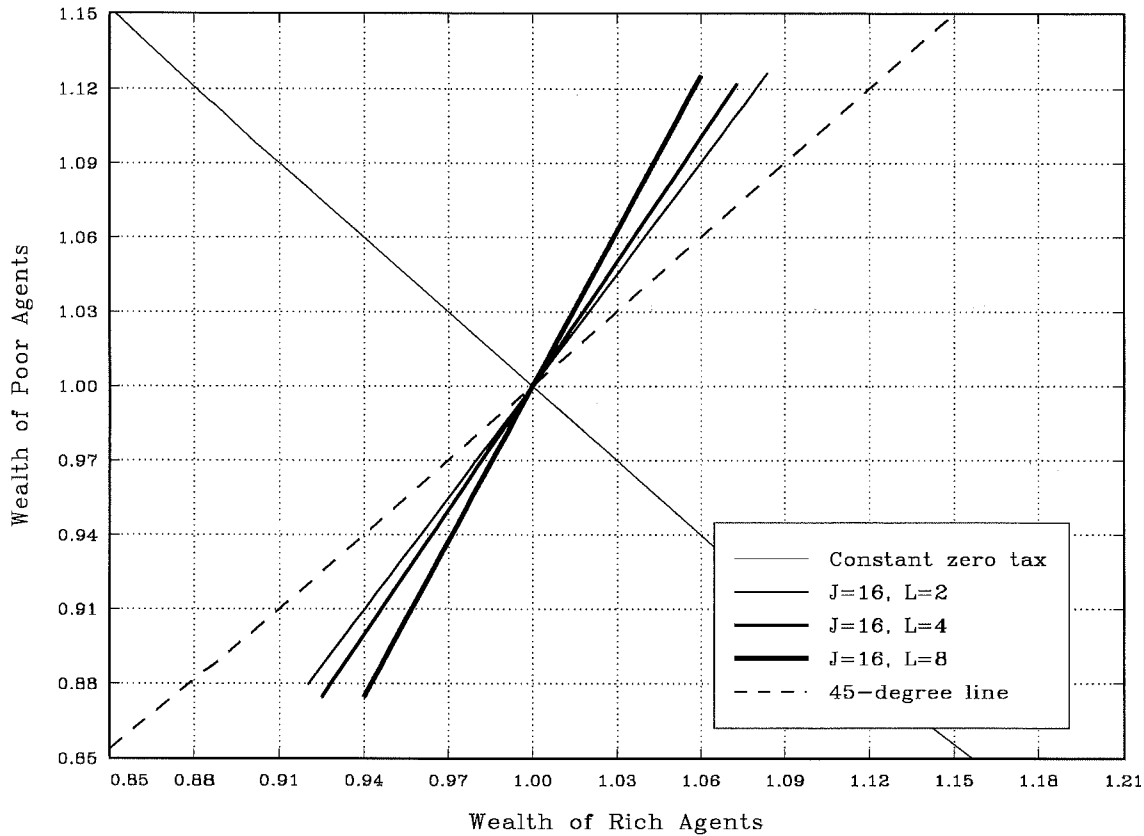


Figure 8: Steady states with different implementation lags.

follows. Raising the tax rate (from an already positive level) increases the distortion, which is perceived as a cost by all agents, and it also generates benefits (losses) in terms of transfers for the poorer (richer) agent. When the tax change occurs in the future, both costs and benefits are discounted. However, there is also a specific sense in which the benefits from a tax increase are lower with a longer implementation lag: the stock of capital far in the future is supplied more elastically, since the agent can draw down on capital holdings in a smooth way in response to the tax increase, so less net transfers are generated for each given tax rate.¹⁷ Therefore, the distortion implied by a tax increase hurts more than if the tax change had occurred sooner. *Ceteris paribus*, this makes all voters vote for lower taxes, and equilibrium taxes are forced downward, or, alternatively, a given steady state tax rate needs to be supported by a larger wealth dispersion.

It is important to note that this explanation is also a key factor behind the effects of changes in the election frequency: a decrease in the election frequency has the property that

¹⁷This type argument can also be applied in the context of optimal taxation to argue in favor of consumption taxes over capital income taxes.

a given election affects tax rates further into the future. An additional aspect of varying the election frequency J , of course, is that the higher J is, the longer the given tax stays in effect, and the larger the wedge between marginal rates of substitution and marginal rates of transformation of consumption at different dates, since taxes on savings accumulate.

6.1.2 What is the effect of changing the political constitution?

Is it a good idea to change the current constitution? This question can only be answered for a given initial condition on the wealth distribution, and by tracing out the transitional dynamics implied by different constitutions. In our experiments, we use an initial wealth distribution where half the population has 1% less wealth than the other half, and where the poorer half contains the median voter. This can be viewed as an initial condition resulting from a one-time, mechanical redistribution of 0.5% of total wealth away from perfect equality. As before, we report separately the results of a change in the election frequency and a change in the implementation lag.

6.1.2.1 A change in the frequency of elections. The differences in the properties of the length of the period in between elections can be seen in Figure 9. The top graph in this figure includes the dynamic path of the assets held by the rich agents, while the bottom graph shows the dynamic path for the poor agents. The first thing to note is that in all cases the asset holdings converge. This, together with our local analysis around steady states, seems to indicate that the economies are stable.

We see again that the dynamic effects are quite dramatic in all cases. For the baseline economy with four years between elections and a one-period implementation lag, $(J, L) = (16, 1)$, the long term drop in wealth for both types is about four times the size of the difference between individual and per-capita initial wealth. For the economy which holds elections every two years, this number increases to eight, and when elections take place only every eight years, the drop in wealth is about twice the initial wealth difference across the groups. The relative size of the drop is hence roughly inversely proportional to the length of time in between elections. Of course, if taxes rates had been exogenous, no drop in wealth would have been recorded for either type.

Regarding the length of the adjustment period, we can see that it is relatively fast: since the tax rate adjusts almost instantaneously, as in Figure 5, in around 6 years half of the adjustment of wealth towards the new steady state is completed, while within 10 years 75% of the adjustments are made. These features hold for all the constitutions we looked at, and we conclude that the speed of adjustment of this economy is very similar to that of the standard growth model.

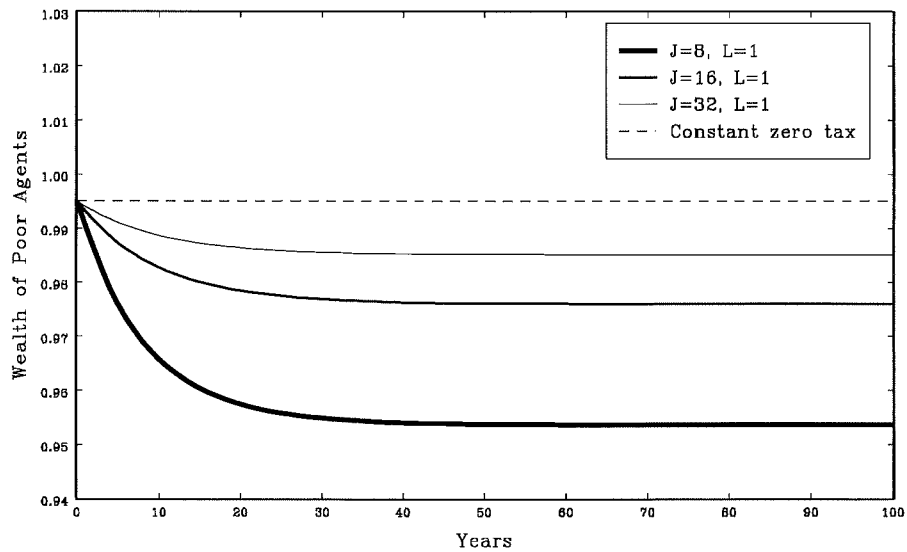
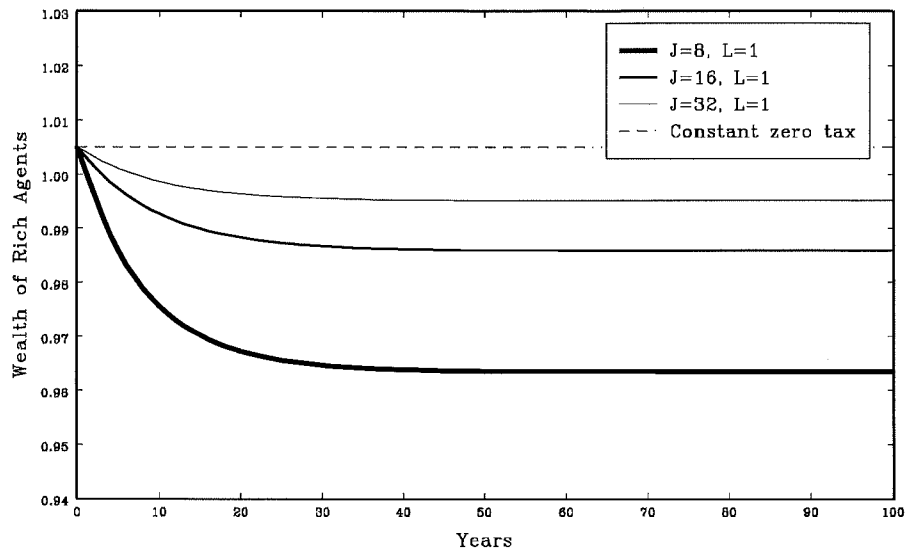


Figure 9: Wealth dynamics with different election frequencies and a 1% initial wealth gap.

Who gains and who loses when we change constitutions, and how much? Table 1 shows the utility changes in the above experiments for the two types. It describes the utility losses (which we always measure as percentage compensating (constant) consumption flows) that occur in the politico-economic equilibrium relative to the situation where the initial distribution would have been maintained.¹⁸ For comparison, the table also includes the value of the asymptotic tax rates for each of the economies, as well as a utility comparison between the two stationary situations without taxes, one with equality and one with a 1% wealth difference. Figure 10 thus shows the points that are being compared. The first rows of the table compare points *B* and *C*, while the last row compares point *B* and *A*.

The first thing to notice in the welfare table is that it is consistent with our previous findings regarding the importance of the length of the period in between elections: the more frequent are the elections, the higher are the distortions in the economy, and the higher are the utility losses. Note also that the asymptotic tax rates are not excessively high, at least not by historical standards.

Relative to the size of the initial wealth dispersion, the differences in utility across constitutions are quite high. As an illustration, imagine that the 1% wealth gap between rich and poor were created with a mechanical time-zero transfer of wealth from the median to the non-median type, so that we can talk about a “dead-weight loss” from this transfer of wealth. In the case of frequent elections, i.e. $J = 8$, the utility loss translates into a fall in wealth of approximately the magnitude of 3.38% of first-period consumption—the present value of 1 unit of consumption forever is around 100, since the per-period interest rate is about 1%. Since the stock of wealth at time zero is 14.7 times per-period consumption, this amount roughly equals 0.23% of initial wealth. If we add that the 1% difference in wealth was achieved by redistributing 0.5% of the wealth, we obtain that from any unit of wealth given to the rich agents, as much as 46% will dissolve through the politico-economic equilibrium distortions.¹⁹

A surprising feature in Table 1 is that the poor agents are losers in the politico-economic equilibrium relative to the environment with zero taxes. The median voter, who is poor, chooses subsequent taxes resulting in positive net transfers, but are still worse off than if he/she had not had the power to tax. The reason for this, of course, is the time-consistency problem: the median voter only has control over the taxes for J periods, but not after that.

¹⁸In other words, this compares the endogenous-policy world with a (zero-tax) exogenous-policy economy for a given initial condition.

¹⁹This number can also be obtained by directly comparing points *A* and *C* using the table.

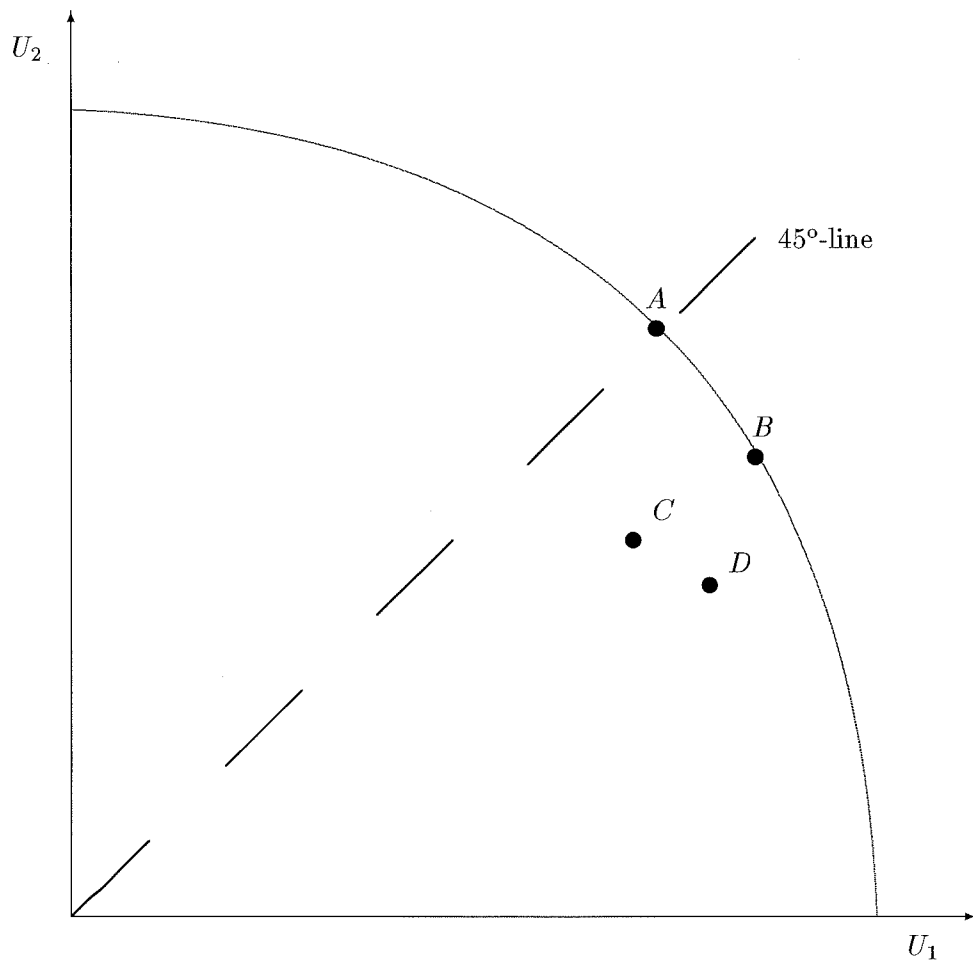


Figure 10: The utility frontier. Point A: utilities of equal-wealth steady state; point B: utilities with maintained 1% wealth gap and zero taxes; point C: utilities of the politico-economic equilibrium; point D: utilities when the taxes are zero for one period, and then given by politico-economic equilibrium. Point D is feasible for the median voter at time zero but not chosen.

Point *D* in Figure 10 illustrates this: this point shows one feasible alternative for the median voter, namely the choice of a zero tax rate for the next period, with taxes after that point determined by future median voters in political equilibrium. As it should, point *D* gives lower welfare for the median voter; in particular, it does not lead to zero taxes other than for the first period, so it is strictly within the Pareto frontier.

6.1.2.2 A change in the implementation lag. Figure 11 is the counterpart of Figure 9 for the implementation lag economies. Again, the top graph in the figure describes the dynamic path of the assets of the agents who become rich after a perturbation away from the equal-wealth steady state, while the bottom describes the path of the agents that become poor.

Again, we see that in all cases the model economies converge to a new steady state. We also see that the differences in outcomes across constitutions are large. Changes in implementation lags of from 2 to 4 or from 4 to 8 quarters induce long-term reductions of wealth of two thirds of a percent. Put differently, the drop in aggregate wealth can be reduced by half by increasing the implementation lag from two to eight quarters. Regarding the speed of convergence, we can see that about half of the wealth reductions have occurred within eight years.

Table 2 reports utility comparisons for a variety of implementation lags in a way parallel to the comparisons in Table 1. We see the same patterns as in the previous experiments: the welfare effects of changing constitutions are sizeable. We again also see the curse of the time consistency problem inherent in the dynamic politico-economic equilibrium: the poor agents, those who vote for and receive the transfers through the tax system, end up worse off in this world than in one without the possibility of taxing.

6.2 The Fiscal Constitution

So far we have looked at a particular family of fiscal policies: there is a proportional income tax on total income net of depreciation and equal per-capita lump-sum rebates, and all that needs to be chosen by the electorate is the level of taxes. In this section we discuss some alternatives.

The focus of our analysis is on capital accumulation and on the evolution of the income/wealth distribution, so the tax schemes we consider far from exploit all possibilities; they mainly include those which we find informative for our focus of analysis. In particular, this means that we rule out pure expropriation schemes, i.e. schemes with lump-sum stealing across arbitrary groups, and more generally we do not look at taxes which depend on individual names. We also limit the analysis to taxes which are one-dimensional, and the

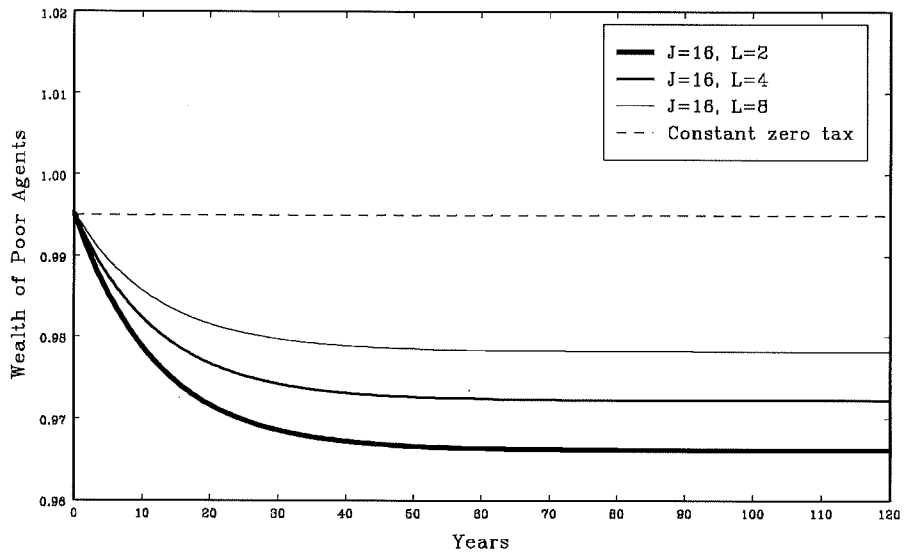
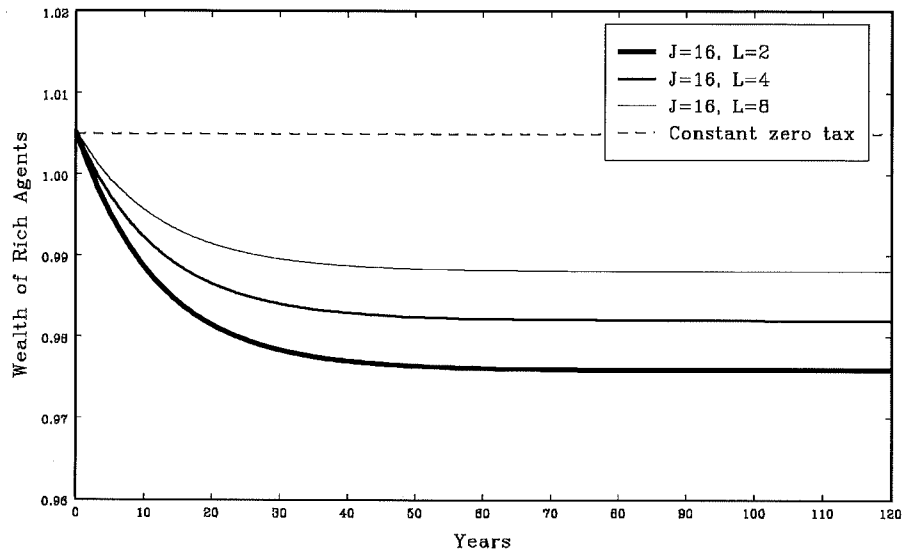


Figure 11: Wealth dynamics with different implementation lags and a 1% initial wealth gap.

reason for this is the familiar problem that such voting environments easily lead to Condorcet cycles.

First, does it matter whether labor is taxed or not, and whether the intertemporal tax falls on capital income or on savings, and whether or not there is a capital consumption allowance? The answer is that, as long as leisure is not valued, these tax structures are all equivalent: for any given tax structure, and for any tax rate and associated competitive equilibrium, there exists for each alternative tax structure a specific tax rate (which in general is different than the first one) and an associated competitive equilibrium with an identical allocation of resources. We therefore do not consider these alternative tax structures in this context.

Second, note that in our case the assumption about proportional taxes leads to extremely uninteresting wealth dynamics, since preferences with constant intertemporal elasticity of substitution imply that relative wealth levels—measured in present-value terms—do not change over time. In particular, it is a property of proportional income tax schemes that asymptotic equalization of income does not occur. It is easy to see this using our analysis in Section 2.2: even though poor agents in majority can use the proportional tax to redistribute income, there can only be a one-time equalization of wealth, and this equalization will not be complete. In order to look at taxation schemes where the relative distribution of wealth does move in a nontrivial way over time, we analyze the case of progressive taxation.

Different societies seem to have made different choices in terms of the progressivity of income taxes. In part, these choices may reflect differences in “social preference for equity”, but our point of view is that they also have nontrivial implications for capital accumulation, especially when the endogeneity of the total level of taxes is taken into account. In this paper, we will specify an average tax rate schedule which allows for different degrees of progressivity, and we will regard this degree of progressivity as a constitutional parameter:

$$\tau(\tau_0, a, A; \alpha) = \tau_0 f^\alpha, \tag{8}$$

where f is short-hand for individual relative to per-capita before-tax total income, and where $\alpha (> -1)$ is the progressivity parameter and τ_0 represents the level of taxes. In other words, we will not let agents vote on the degree of progressivity itself, but only on the total tax level τ_0 . Thus, one way of interpreting this experiment is that we want to report the economic consequences of different “social preferences for equity”.

Could we have chosen to endogenize the whole tax schedule, or more aspects of the tax schedule? Since this amounts to multidimensional voting, such an analysis is difficult even in static models (for examples, see Snyder & Kramer (1988), Cukierman & Meltzer (1991), and

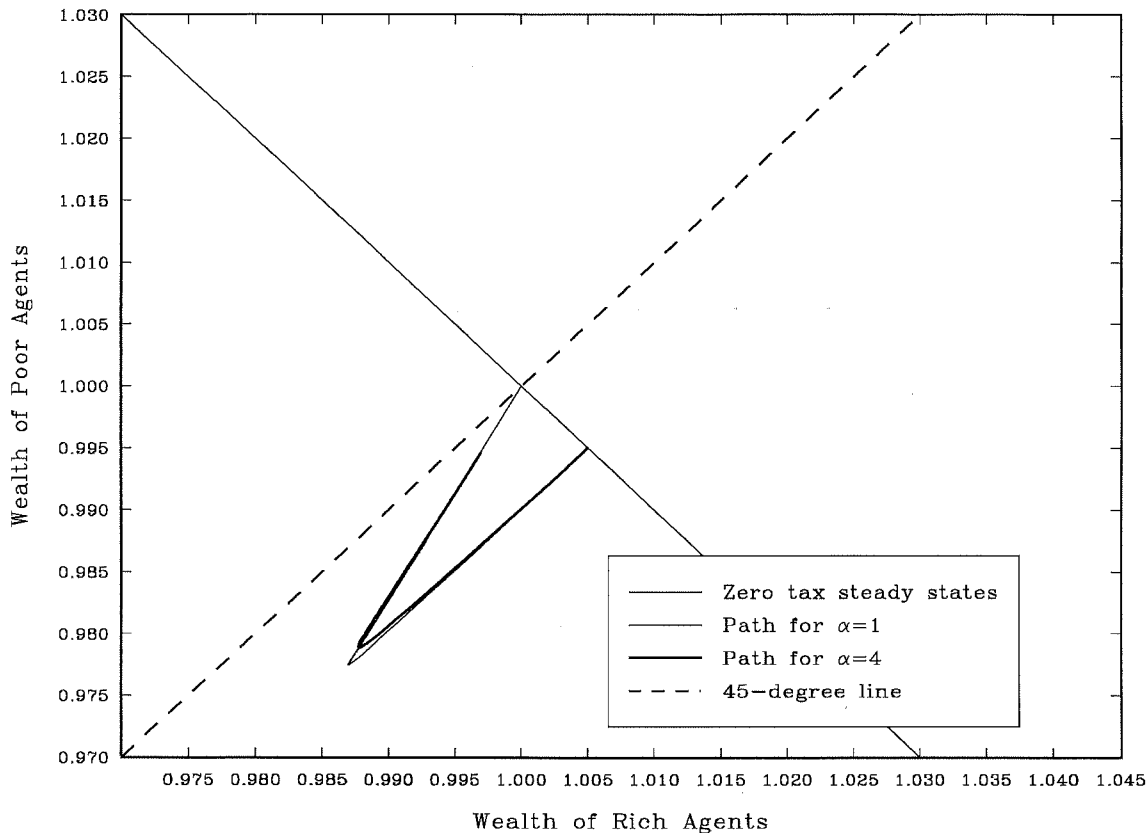


Figure 12: Wealth distribution dynamics with progressive taxes and a 1% initial wealth gap.

Berliant & Gouveia (1994) for some early progress on this issue). One convenient property of the family of progressive tax schedules which we look at here is that the induced preferences over tax levels are (1) single-peaked, and (2) such that the preferred tax rate is monotone in asset holdings—this justifies the median-voter construct.²⁰

We first note that for our family of tax schedules, the following must be true:

Theorem 3. *If the average tax rate schedule is given by (8) and $\alpha \neq 0$, then any steady state for our economy has equal wealth among all agents and zero taxes.*

Thus, at least if our economy is stable, there will be asymptotic equalization of income whenever $\alpha \neq 0$.

We look at the baseline 4-year model appended with values for the degree of progressivity ranging from $\alpha = 0$ to $\alpha = 2$. This is a very large range compared to estimates of the degree of progressivity in the U.S.²¹ For these parameter values, we always found globally stable

²⁰We verify these properties numerically.

²¹See Gouveia & Strauss (1994).

equilibrium paths, and Figure 12 displays the dynamic path of an economy with initial conditions where half of the population has one percent more asset wealth than the other half, but where the total wealth is that of the zero-tax steady state (this is the initial condition we also used in Section 6.1.2). It is clear from the figure that the properties of the wealth path do not vary much at all with α over our range: both economies suffer an initial drop in wealth very much like that observed for the economy with proportional taxation, and the maximum drop in wealth is about 1.5%. Thereafter, total wealth slowly picks up and converges to its initial value.

In Figures 13 and 14 we also provide a more detailed account of how each agent's wealth as well as marginal tax rate develop over time. Figure 13 shows that it takes very long to recover to the new steady state for both types: when $\alpha = 1$ (4), it takes 64 (84) years for the rich agent to reach the minimum wealth level, and it takes 56 (76) years for the poor agent. Note that the differences between the rich and the poor agents are not only wealth-related: the agents face different marginal taxes rates at each point in time. It is also possible to see from the figure that the minimum level of wealth is higher the more progressive is the tax schedule, and that the upward turn toward equalization after the maximum wealth drop has occurred is steeper with a higher degree of progressivity.

Figure 14 shows the dynamic path of the effective marginal tax rates for both types of agents. As we can see, the marginal tax rate is higher for the rich agents. Comparing across degrees of progressivity, we also see that the higher the progressivity level, the lower the marginal tax rates.

To draw on our results for normative purposes still need to compute the welfare associated with the different tax schedules. We see in Table 3 that the welfare is almost identical across all the different fiscal constitutions, with a slight advantage to the more progressive schedules. It seems that the more progressive is the tax code, the more the taxes hurt, and the lower are the effective taxes and the implied distortions in our political equilibrium. To conclude this discussion, our main finding seems to be that the "social preference for equity" has surprisingly small effects on equilibrium tax levels and distortions.

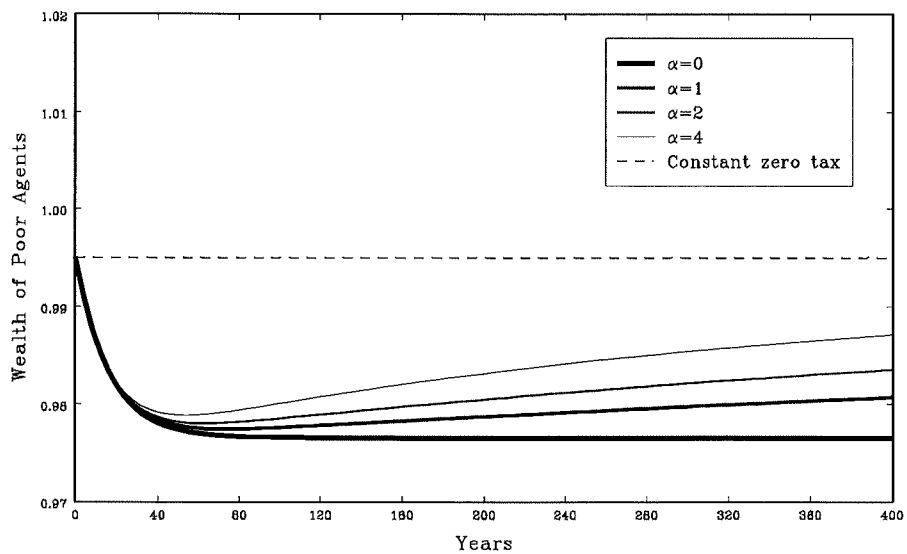
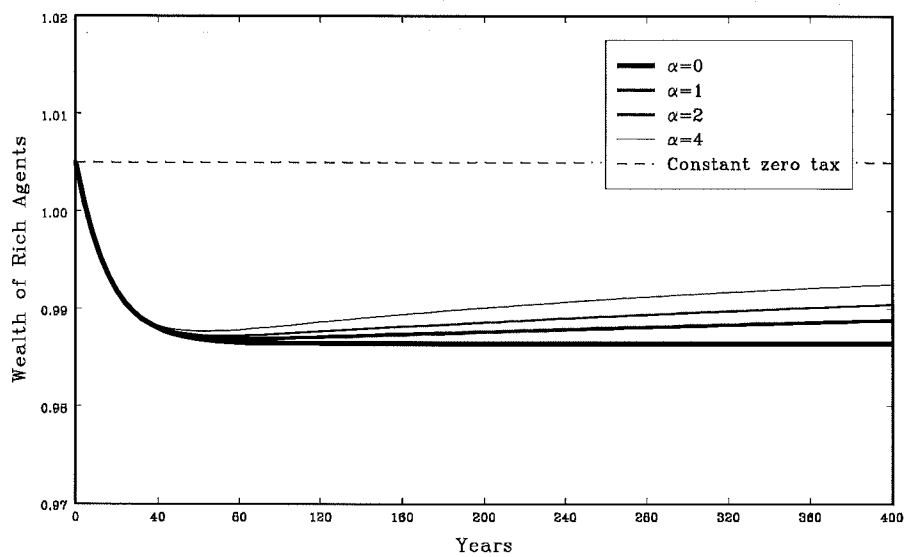


Figure 13: Wealth dynamics with different degrees of progressivity and a 1% initial wealth gap.

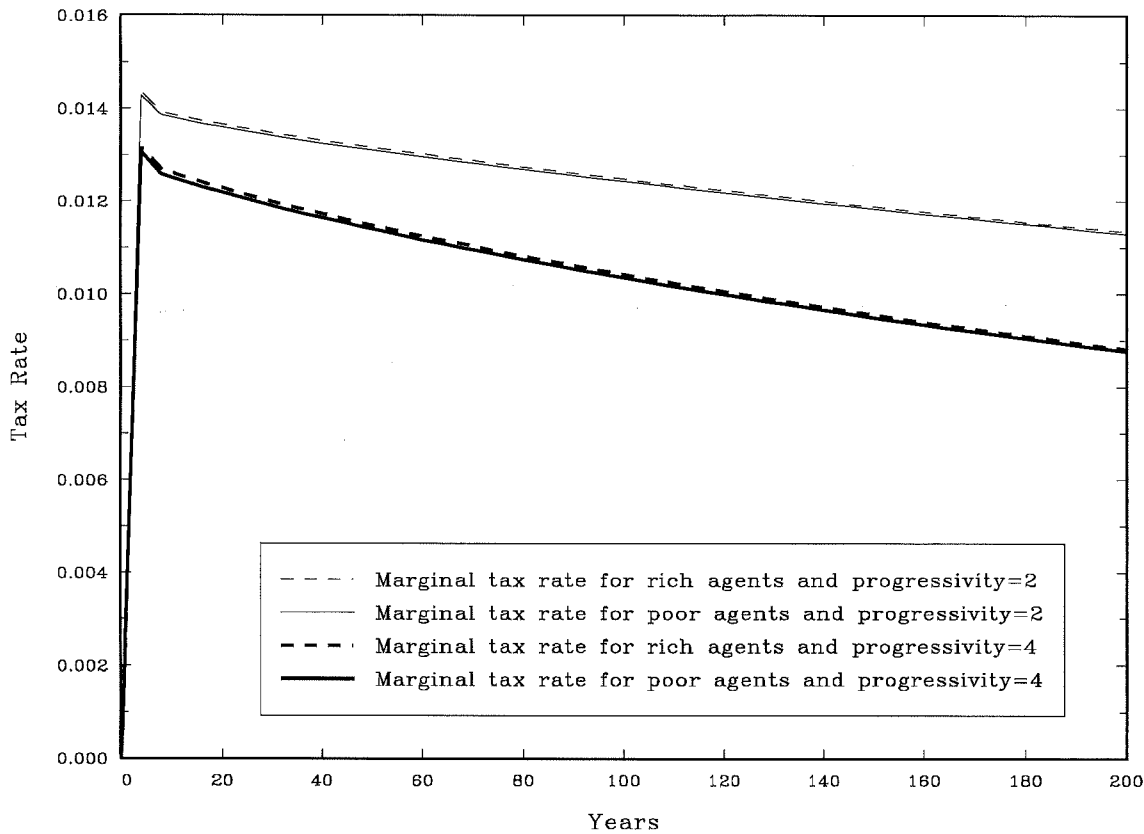


Figure 14: Tax dynamics with different degrees of progressivity and a 1% initial wealth gap.

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Appendix 1 Proofs

Proof of Theorem 1:

When everybody has the same wealth, taxes cannot redistribute wealth as the amount of the tax equals the amount transferred by every person. Since all agents have the same preferences, endowments and wealth, they will act the same, and their per capita utility will be the same. Feasibility requires this utility to be no higher than the solution to the social planner's problem of a representative agent version of this economy. The second welfare theorem guarantees that this utility, v^* , can be achieved through a competitive equilibrium with zero taxes forever. If agents assume that from tomorrow on taxes are set at zero, they can achieve v^* by also setting the taxes to zero for next period, i.e. $\tilde{v}(k, \dots, k, 0, r', k) \leq v^*$ with equality at $r' = 0$. Therefore this will be the tax rate chosen. \square

Proof of Theorem 2:

The proof proceeds by guessing that an equilibrium with this property exists and verifying that when this is the case, the taxes chosen have in fact the property described. There are three steps in the proof. The first step states that if $A_m = \bar{A} = \sum_i \mu_i A_i$, then $A'_m = \sum_i \mu_i A'_i$, i.e. for every tax rate chosen, it will also be true next period that the median wealth equals the mean wealth. The second step argues that from the point of view of a median voter, this economy is identical to one where no other agent types are present. The third shows that in such an economy, a situation of zero taxes is preferred.

- **Step 1.** $A_m = \sum_i \mu_i A_i \Rightarrow A'_m = \sum_i \mu_i A'_i$. Recall that with Bergson's preferences, consumption decisions take the form $c_i = d + e\omega_i$, where d , and e are constants that depend on the relative prices but not on the individuals, and ω_i is the sum of total human and nonhuman wealth. Note that human wealth, η , is the same for all agents, i.e. $\omega_i = a_i + \eta$. This implies that next period's wealth of a typical agent is given by $a'_i = a_i + ra_i + w - (d + e\eta + ea_i)$, where we have used the fact that taxes, and therefore transfers, are zero. This is, of course, also true for the median agents: $A'_m = (1 + r)A_m + w - (d + e\eta + eA_m)$. Aggregating we get $\bar{A}' = \sum_i \mu_i A'_i = w - (d + e\eta) + (1 + r - e)\sum_i \mu_i A_i = w - (d + e\eta) + (1 + r - e)\bar{A}$. These last two equations together complete the argument of this step.
- **Step 2.** Since the agent with median wealth will remain in this position the next period, it is clear that no net transfers will ever be made to or from this agent. Moreover, since preferences are of the Bergson class we know that only the mean level of capital will matter to the allocation. The median voter therefore views this economy as identical to one where there are no other agent types than his own type.
- **Step 3.** The most preferred tax of the median voter is zero. This follows from the same reasoning as in the proof of the previous theorem.

This completes the proof. \square

Proof of Theorem 3:

We have to prove two things: first, that any steady state requires all agents to have the same wealth; and second, that the wealth held by the agents is precisely that of the steady state of the economy without taxes.

- A necessary condition for a steady state is that for all types of agents i (types differ in their wealth levels A_i)

$$[1 + r^{\text{ss}}(1 - \tau'_i)] = 1/\beta$$

where r^{ss} is the steady-state rate of return and τ'_i is the marginal tax rate of agents of type i . If $\alpha \neq 0$, and given the monotonicity of the tax function, τ'_i will be different across any two agents with different asset levels, and thus 6.2 cannot hold for both agents.

- To show that the wealth of the agents has to be equal to that of the steady state of the economy with zero taxes, simply apply Theorem 1.

This completes the proof. \square

Appendix 2

The general model

We now describe the recursive structure of the general model with J periods between elections and L periods before of implementation lags. We consider only the cases $J > L \geq 1$, which are the ones we study. A description of the more general case increases the burden of notation without adding substance. Note that the case $L = 1$ is the baseline case for when we explore only economies with different election frequencies, while the case $J = L = 1$ is the simple case described in Section 4.

Define v_j , for $j = J, J - 1, \dots, 1$, to be the agent's value function when j periods are left before a new election takes place. Therefore v_J is the value function in the election period while v_1 is the value function in the period preceding a new election.

The presence of implementation lags complicates the recursive structure because the state of the economy changes depending on whether we are before or after the change of policy. Before the change of policy, the aggregate state of the economy consists of the vector (A, τ, τ_N) , while after the policy change the state vector is (A, τ) , where A is the vector of asset holdings, τ the current tax rate, and τ_N the new voted tax rate. The periods that include the variable τ_N as a state variable are $j = J - 1, \dots, J - L + 1$, while those that do not include τ_N are $j = J - L, \dots, 1$, and J .

We proceed by specifying the problem of the agent for all different periods. In the election period the programming problem faced by the agent is the following:

$$v_J(A, \tau, a; \Psi) = \max_{c, a'} u(c) + \beta \begin{cases} v_{J-1}(A', \tau, \tau_N, a'; \Psi) & \text{if } J > 1, L > 1 \\ v_{J-1}(A', \tau_N, a'; \Psi) & \text{if } J > 1, L = 1 \end{cases} \quad (9)$$

s.t.

$$\begin{aligned} a' &= a + [a r(A) + w(A)](1 - \tau) + tr - c \\ tr &= \tau[f(A) - \delta \sum_i \mu_i A_i] \\ A' &= H_J(A, \tau; \Psi) \\ \tau_N &= \Psi(A, \tau) \end{aligned}$$

If $L > 1$, there are periods between elections and policy changes, and therefore for $j = J - 1, \dots, J -$

($L - 1$) the agent's problem is:

$$\begin{aligned}
v_j(A, \tau, \tau_N, a; \Psi) &= \max_{c, a'} u(c) + \beta \begin{cases} v_{j-1}(A', \tau, \tau_N, a'; \Psi) & \text{if } j > J - (L - 1) \\ v_{j-1}(A', \tau_N, a'; \Psi) & \text{if } j = J - (L - 1), \end{cases} \quad (10) \\
\text{s.t.} & \\
a' &= a + [a r(A) + w(A)](1 - \tau) + tr - c \\
tr &= \tau[f(A) - \delta \sum_i \mu_i A_i] \\
A' &= H_j(A, \tau, \tau_N; \Psi)
\end{aligned}$$

Finally for $j = J - L, \dots, 1$, i.e. from the change of tax on, the agent's problem reads:

$$\begin{aligned}
v_j(A, \tau, a; \Psi) &= \max_{c, a'} u(c) + \beta \begin{cases} v_{j-1}(A', \tau, a'; \Psi) & \text{if } j > 1 \\ v_J(A', \tau, a'; \Psi) & \text{if } j = 1 \end{cases} \quad (11) \\
\text{s.t.} & \\
a' &= a + [a r(A) + w(A)](1 - \tau) + tr - c \\
tr &= \tau[f(A) - \delta \sum_i \mu_i A_i] \\
A' &= H_j(A, \tau; \Psi)
\end{aligned}$$

To find out the effect of a policy which does not agree with the political outcome, we need to consider one additional maximization problem:

$$\begin{aligned}
\tilde{v}_J(A, \tau, \tau_N, a; \Psi) &= \max_{c, a'} u(c) + \beta \begin{cases} v_{J-1}(A', \tau, \tau_N, a'; \Psi) & \text{if } L > 1, \\ v_{J-1}(A', \tau_N, a'; \Psi) & \text{if } L = 1, \end{cases} \quad (12) \\
\text{s.t.} & \\
a' &= a + [a r(A) + w(A)](1 - \tau) + tr - c \\
tr &= \tau[f(A) - \delta \sum_i \mu_i A_i] \\
A' &= \tilde{H}_J(A, \tau, \tau_N; \Psi)
\end{aligned}$$

The function \tilde{v}_J denotes the utility associated to a change in policy to τ_N L periods later when the current state in an election period is (A, τ) . Note that the law of motion of the economy under this circumstances is not H_J , but \tilde{H}_J . This function is determined by similar equilibrium conditions to those pinning down H_j given the policy function Ψ .

Thus, the problem of the voter with J periods between elections and L periods of implementation lag is:

$$\max_{\tau_N} \tilde{v}_J(A, \tau, \tau_N, a; \Psi) \quad (13)$$

Denoting the solution to this problem $\psi(A, \tau, a; \Psi)$ and assuming that the aggregator function is given by the preferences of the median voter, the function Ψ is given by the fixed point of the following functional equation:

$$\Psi(A, \tau) = \psi(A, \tau, A_m; \Psi) \quad (14)$$

where m denotes the index for the median agent.

Appendix 3

Computational procedure

We first describe the procedures involved in the computation of dynamic political equilibria for the general model described in the previous appendix. We then comment on two aspects of the accuracy of computation, one concerning the accuracy of the quadratic approximations, and another concerning errors introduced by differences in time aggregation.

Our procedure involves linear-quadratic approximations to solve for recursive equilibria for given policies (Ψ functions), and the median voter's problem is then solved given these equilibrium functions, again using linear-quadratic approximations. If the choice of the median voter coincides with the original Ψ function, an equilibrium is found; if not, we update and continue until convergence.

1. Algorithms for finding steady states

We are searching for a $I - 1$ -dimensional subspace of steady states. We first choose a grid on the ratio of asset holdings between the different types of agents around the point of perfect equality. For each point on this grid, the search for a steady state involves a search for a tax rate. The procedure for computing such a tax rate can be described as follows.

- (i) Let $R^0(A, \tau, \tau', a, a')$ be a quadratic function that approximates the utility function in a neighborhood of the steady state (note that the budget constraint has been used here to substitute out consumption). Guess on τ_0 as a value for the tax rate and compute the implied steady state values of the other variables. This involves computing a value for aggregate capital with the property that the after-tax rate of return is the inverse of the discount rate.
- (ii) Fix an initial affine tax policy Ψ^0 .
- (iii) Given Ψ_0 , use standard methods to solve for the equilibrium elements associated with (9), (10), and (11), obtaining linear functions h^0 and H^0 and a quadratic function v^0 .
- (iv) Solve for the functions described in (12). Note that this is a simple problem since we already have obtained functions v_{j-1}^0 ; the key difference is that in this case we do not use Ψ^0 as an update for next period's tax rate, instead leaving the dependence on τ_N explicit. A solution to the problem of the individual delivers decision rules as functions of individual wealth a , the distribution of wealth in this period, A , in the next period, A' , the current tax rate τ , and the new voted tax rate τ_N . The use of the representative-type assumption and the inversion of a matrix delivers the linear law of motion of the economy \tilde{H}_J^0 .
- (v) Substitute the decision rules and the law of motion \tilde{H}_J into (12) to obtain the function \tilde{v}_J^0 .
- (vi) Maximize \tilde{v}_J^0 with respect to τ_N to obtain a function ψ^0 of the distribution of wealth and the own wealth of the agent. Check for the concavity of the function \tilde{v}_J^0 with respect to τ_N , ensuring that the first-order conditions deliver a maximum.
- (vii) Use the representative-type condition on the median agent to obtain the function Ψ^1 by letting $\Psi^1(1, A, \tau) \equiv \psi(1, A, \tau, A_m; \Psi)$.
- (viii) Compare Ψ^1 to Ψ^0 . If these functions are close enough, continue to (ix). If not, redefine Ψ^0 to be a linear combination of its old value and Ψ^1 and go back to step (iii). This updating procedure has been used before and it is necessary in our case for avoiding "overshooting" problems. We found that a very small step (less than .01 in the direction of Ψ^1) works best.

(ix) Verify that the policy function Ψ reproduces the conjectured tax rate. In other words, the following condition has to be verified:

$$\tau_0 = \tau_1 \equiv \Psi(A, \tau_0). \quad (15)$$

If it is not, go back to step (i) and update the guess for τ . We update using $\tau_0 = (\tau_0 + \tau_1)/2$.

One last equilibrium property remains to be verified: whether the majority rule implies that the median voter is pivotal in this economy. This requires (1) that the preferred tax is increasing in the agent's holdings of capital; and (2) that there is single peakedness with respect to τ' of $\tilde{v}(A, \tau, \tau', a)$. The quadratic nature of the value functions used and the strict concavity checked in step (iii) ensure that these conditions are met locally.

In our experiments we use two procedures to characterize the set of steady states. The first consists of performing steps (i) through (ix) described above. The second procedure, which is much simpler and less time-consuming, was already described in the description of the mechanics of the model. It is based on the knowledge that a zero tax and equal distribution constitute a steady state: by using the law of motion for the economy approximated around this point, one can compute the set of steady states by simply finding the set of values for A_1 and A_2 that are reproduced by this law of motion. Clearly this procedure is only strictly valid locally, and it is likely to give lower accuracy further away from the point of perfect equality. Finally, note that this procedure can also be applied to extend locally the set of steady states around any steady-state point found with the first procedure.

2. Algorithms for computing transitional dynamics

As for the case of computing steady states, we employ two alternative procedures to compute transitional dynamics. The first procedure uses the linear law of motion for the zero-tax steady state in order to generate a path for wealth and taxes after a small initial perturbation in asset holdings. The second procedure essentially follows steps (i)-(vii) above, i.e. it also involves a separate linearization around each new point the economy passes through. The slight complication needed is an additional round of iterations “within” step (i): it is necessary to make sure that, at each point on the dynamic path, the point (A, A') around which the linearization is made coincides with the equilibrium outcome. Note that the equilibrium elements take on different forms for each point on the path, and the accuracy of this procedure is hence better the more slowly, or the more linearly, the exact equilibrium evolves.

Numerical accuracy

A comparison of the approximations

As mentioned above, in order to gauge the sensitivity of the results to the numerical procedure, and as a check on the calculations, we used two alternative procedures for computation: one which relied on linearization around the equal-wealth point (“equal-wealth linearization”), and one which performed iterative linearizations away from the equal-wealth point (“iterative linearization”). As an illustration of the differences we obtained between these procedures, we report parallel results in Table 4 for the welfare comparisons of the transitional dynamics for the economies with varying implementation lags. It is clear from the table that the two approximations do not give identical solutions. However, the magnitude of the errors is small, and the errors decrease as the implementation lag gets longer.

Time Aggregation

It is important to make sure that the procedure for time aggregation does not itself bias our comparisons across economies. The models in the constitutional sections have been calibrated so that a period corresponds to a quarter: the real rate of return per period is 1% and the capital-output ratio is about 10. These are standard numbers for the public finance and real-business-cycle literatures. In contrast, the models in the preceding and following sections are calibrated with 4-year periods; there, the real rate of return is 17% and the capital-output ratio about 0.63. This makes the two types of calibrations comparable, except for the problem of time aggregation.²²

The natural counterpart to a model calibrated to 4-year periods with $(J, L) = (1, 1)$ in terms of a model calibrated to quarters is that with $(J, L) = (16, 16)$. We can compare these two models by looking at Tables 2 and 3. The numbers reported in the last row of Table 2 should coincide with the numbers in the first row of the second half of Table 3, as they are associated to economies that have 4 years between elections, a 4-year implementation lag, and the income taxes are proportional. These numbers are not equal: in the model with 4-year periods, the welfare cost is twice as large for both types of agents. The difference in the long-run behavior of tax rates is not, however, substantial. The reason for the discrepancies derive from the calibration: whereas β , δ , and θ are straightforward to pin down, σ , is not. The value of σ , the intertemporal elasticity of substitution for consumption, indeed influences short-run behavior, and therefore affects welfare evaluations of transition paths. We chose to let σ be the same in the two models. The alternative would have been to let σ vary between models, and choose them so that some measure of the short-run behavior of the model (such as the speed of convergence) is constant across economies with different time aggregation.

²²More precisely, if β_0 , δ_0 , σ_0 , and θ_0 are the parameters for the quarterly model and the same letters without subscripts refer to the 4-year model, we have that: $\beta = \beta_0^{16}$, $\delta = 1 - (1 - \delta_0)^{16}$, $\sigma_0 = \sigma$, and $\theta_0 = \theta$.

Table 1: Welfare losses associated with different election frequencies for economies with a 1% initial wealth gap (quarterly model, $\sigma = 2$).

Election frequency	Loss for rich agents:	Loss for poor agents:	Asymptotic tax rate
Points B→C			
$J = 8, L = 1$	0.0338%	0.0278%	10.85%
$J = 16, L = 1$	0.0098%	0.0068%	5.68%
$J = 24, L = 1$	0.0047%	0.0027%	3.84%
$J = 32, L = 1$	0.0028%	0.0014%	2.89%
Comparison with exogenous, zero tax: B→A	-0.0793%	0.0793%	

Table 2: Welfare losses for both types of agents associated with different implementation lags for economies with a 1% initial wealth gap (quarterly model, $\sigma = 2$).

Implementation lag	Loss for rich agents	Loss for poor agents	Asymptotic tax rate
$J = 16, L = 1$	0.0098%	0.0068%	5.68%
$J = 16, L = 2$	0.0079%	0.0053%	5.08%
$J = 16, L = 4$	0.0057%	0.0036%	4.26%
$J = 16, L = 8$	0.0035%	0.0020%	3.29%
$J = 16, L = 16$	0.0016%	0.0008%	2.21%

Table 3: Welfare losses associated with different degrees of progressivity for economies with a 1% initial wealth gap (four-year model, $\sigma = 2$).

Degree of progressivity	Loss for rich agents	Loss for poor agents	Asymptotic tax rate
$\alpha = 0$	0.0031%	0.0016%	2.88%
$\alpha = 1$	0.0031%	0.0016%	0.00%
$\alpha = 2$	0.0031%	0.0016%	0.00%
$\alpha = 4$	0.0031%	0.0016%	0.00%
Zero tax	-0.0894%	0.0894%	

Table 4: Comparisons of numerical procedures, implementation lag economies.

Implementation lag	Loss for rich agents	Loss for poor agents	Asymptotic tax rate
Iterative linearization			
$J = 16, L = 1$	0.0147%	0.0113%	6.42%
$J = 16, L = 2$	0.0102%	0.0075%	5.39%
$J = 16, L = 4$	0.0065%	0.0044%	4.28%
$J = 16, L = 8$	0.0036%	0.0022%	3.19%
Equal-wealth linearization			
$J = 16, L = 1$	0.0098%	0.0068%	5.68%
$J = 16, L = 2$	0.0079%	0.0053%	5.08%
$J = 16, L = 4$	0.0057%	0.0036%	4.26%
$J = 16, L = 8$	0.0035%	0.0020%	3.29%

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