

**Non-cooperative Games Among Groups**

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# Non-cooperative Games Among Groups

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### **Abstract**

A model of group interaction that combines the theory of Nash equilibrium (across groups) and the theory of social choice (within groups) is investigated. If group preferences are acyclic, and if group aggregation rules are continuous, in an appropriate sense, then "group Nash" equilibria exist. If the sets of alternatives facing the groups are one-dimensional and individual induced preferences are single-peaked, then pure strategy group Nash equilibria exist. Connections to models of legislatures and to social choice theory are explored.

# 1 Introduction

Consider competition among a finite number of groups, each of which must make some collective choice from a set of group alternatives. An individual member of a group has preferences over vectors of group decisions, allowing for externalities across groups. Within a group, members may have different preferences over vectors of decisions and, therefore, different induced preferences over the alternatives facing the group. Suppose that, due to low communication costs and the availability of commitment devices, coalitions may form freely within groups but that groups cannot easily form coalitions among themselves. Examples are competition among firms, among governments, among political parties, among legislative committees, or among academic departments.

I propose a game-theoretic analysis that is non-cooperative across groups but cooperative within groups. Specifically, each group is endowed with an aggregation rule that maps the heterogeneous induced preferences of group members into a “social preference” relation on the set of group alternatives. This is used to define a group choice set, namely, the maximal elements of the group preference relation, a set referred to as the “core” of the group. Because individuals have preferences over *vectors* of group decisions, members’ preferences over group alternatives, and therefore the group core, will depend on the choices of other groups, generating strategic interdependence among the groups. A “group Nash” equilibrium is a vector of group decisions such that each group, given the decisions of other groups, is choosing from its core.

This paper contains two results. The first is a general existence result for group Nash equilibria in “mixed strategies,” assuming acyclic group preferences and a condition on aggregation rules combining continuity and monotonicity. The latter is satisfied, for example, whenever a group’s aggregation rule is “simple,” i.e., whenever group preferences are determined by the winning coalitions within the group. The second is a pure strategy equi-

librium existence result, which assumes that each group's action space is one-dimensional, that induced preferences of individuals are single-peaked, and that aggregation rules are simple.

Section 2 sets forth the model, and Section 3 contains the existence results. The paper closes with a discussion of some connections to Arrowian social choice, to structure-induced equilibria in models of legislatures, and to models of bargaining and elections.

## 2 The Model

Consider a collection  $G$  of  $m$  groups, indexed  $j = 1, \dots, m$ . Each group  $j$  is described by four elements.

(i) A set  $N_j$  of  $n_j$  individual members, indexed  $i = 1, \dots, n_j$ .

(ii) A compact metric space  $X_j$  of alternatives, denoted  $x_j$ . Let  $X = \prod_{j \in G} X_j$  denote the space of vectors of group decisions, denoted  $x = (x_1, \dots, x_m)$ . Let  $X_{-j} = \prod_{k \neq j} X_k$  denote the space of vectors of decisions of groups other than  $j$ . Let  $\mathcal{P}(X_j)$  denote the space of Borel probability measures on  $X_j$ , with generic element  $\mu_j$ , which is to be interpreted as a "mixed" decision by the group. This space is endowed with the topology of weak convergence. Given  $\mu_k$  for all  $k \neq j$ ,  $\mu_{-j}$  denotes the product measure

$$\mu_1 \otimes \cdots \otimes \mu_{j-1} \otimes \mu_{j+1} \otimes \cdots \otimes \mu_m$$

induced by those mixtures. Let  $\mathcal{P}(X)$  denote the space of Borel probability measures on  $X$ , with generic element  $\mu$ , also endowed with the weak topology.

(iii) A complete, transitive relation on  $\mathcal{P}(X)$  for each group member  $i$ , denoted  $\succeq_i$ , which is interpreted as  $i$ 's weak preference relation over lotteries on  $X$ . Assume  $\succeq_i$  is *continuous*, i.e., the upper and lower contour sets,

$$\{\mu' \in \mathcal{P}(X) \mid \mu' \succeq_i \mu\} \quad \text{and} \quad \{\mu' \in \mathcal{P}(X) \mid \mu \succeq_i \mu'\},$$

are closed in the weak topology for all  $\mu$ . Given probability measures  $\mu_k$  on  $X_k$  for all other groups, the ordering  $\succeq_i$  induces a preference relation on  $X_j$ , denoted  $R_i(\mu_{-j})$ , defined as follows:

$$x_j R_i(\mu_{-j}) x'_j \quad \text{if and only if} \quad (\delta_{x_j} \otimes \mu_{-j}) R_i(\mu_{-j}) (\delta_{x'_j} \otimes \mu_{-j}),$$

where  $\delta_{x_j}$  is the point mass on  $x_j$ . Note that  $R_i(\mu_{-j})$  is complete, transitive, and continuous. Let

$$\mathcal{D}_j = \{(R_1(\mu_{-j}), \dots, R_{n_j}(\mu_{-j})) \mid \mu_k \in \mathcal{P}(X_k), k \neq j\}$$

denote the space of all profiles of member preferences induced by other groups' mixtures.

(iv) A preference aggregation rule. Letting  $\mathcal{R}(X_j)$  denote the space of all complete and continuous relations on  $X_j$ , this is denoted by

$$F_j: \mathcal{D}_j \rightarrow \mathcal{R}(X_j),$$

where  $F_j(R_1, \dots, R_{n_j})$  is interpreted as the weak preference relation of group  $j$ . For example, an aggregation rule may be defined by a collection  $\mathcal{W}_j$  of subsets of  $N_j$  that is proper ( $C \in \mathcal{W}_j$  implies  $N \setminus C \notin \mathcal{W}_j$ ) and monotonic ( $C \in \mathcal{W}_j$  and  $C \subseteq C'$  implies  $C' \in \mathcal{W}_j$ ) such that

$$F_j(R_1, \dots, R_{n_j}) = \bigcap_{C \in \mathcal{W}_j} \bigcup_{i \in C} R_i.$$

Call such an aggregation rule *simple*. In general, say  $C \subseteq N_j$  is *winning* for  $j$  if, for all  $(R_1, \dots, R_n) \in \mathcal{D}_j$  and all  $x_j, x'_j \in X_j$ ,

$$[x_j F_j(R_1, \dots, R_n) x'_j] \quad \text{implies} \quad [x_j R_i x'_j \text{ for at least one } i \in C].$$

If  $F_j$  is simple, then  $\mathcal{W}_j$  consists of precisely the winning coalitions.

In analyzing interaction among groups, the perspective of Nash equilibrium is adopted: groups choose independently, taking the choices of other groups as given. In analyzing interaction within groups, the perspective of

cooperative game theory is adopted, specifically using the concept of the core. Given a weak preference relation  $R$  for a group  $j$ , define the *core* for the group, denoted  $K_j$ , as

$$K_j(R) = \{x_j \in X_j \mid x_j R x'_j \text{ for all } x'_j \in X_j\}.$$

This captures the idea that communication costs within groups are small, facilitating the formation of coalitions to overturn any alternative not in  $K$ . Given a product measure  $\mu_{-j}$ , define the *strategic core* for the group as

$$K_j(\mu_{-j}) = K(F_j(R_1(\mu_{-j}), \dots, R_{n_j}(\mu_{-j}))),$$

the viable group decisions when members' preferences over  $X_j$  are induced by the mixtures used by other groups.

The profile  $(\mu_1, \dots, \mu_m)$  is a *group Nash equilibrium* if, for every group  $j$ ,

$$\mu_j(K_j(\mu_{-j})) = 1,$$

i.e., if every group is mixing over alternatives in its strategic core, given the mixed strategies of other groups. The profile is a *pure strategy* group Nash equilibrium if each  $\mu_j$  is degenerate on some element of the group's strategic core.

### 3 Results

At every group Nash equilibrium, it is clear that each group's strategic core must be non-empty. Non-emptiness will not hold for all aggregation rules and all profiles of member preferences, as the well-known Condorcet paradox demonstrates. When  $X_j$  is finite, however, a sufficient condition for non-emptiness of the strategic core is that  $F_j$  is *negatively acyclic*, i.e., for every  $(R_1, \dots, R_{n_j}) \in \mathcal{D}_j$  and every finite collection  $x_j^1, \dots, x_j^k \in X_j$ ,

$$[\text{not } x_j^k F_j(R_1, \dots, R_{n_j}) x_j^1] \text{ implies } \left[ \begin{array}{l} x_j^h F_j(R_1, \dots, R_{n_j}) x_j^{h+1} \\ \text{for some } h \leq k-1 \end{array} \right].$$

This is equivalent to the restriction that the asymmetric part of  $F_j(R_1, \dots, R_{n_j})$  is acyclic in the usual sense.

There are two ways in which negative acyclicity may be ensured. First, the preference aggregation rule may be restricted. If  $F_j$  is simple, for example, then a sufficient condition for negative acyclicity is that

$$\bigcap \mathcal{W}_j \neq \emptyset,$$

i.e., there is some group member who belongs to all winning coalitions. Second, restrictions may be imposed on the dimensionality of  $X_j$  and the domain  $\mathcal{D}_j$  of member preferences. Suppose  $X_j \subseteq \mathbb{R}$  and each  $(R_1, \dots, R_n) \in \mathcal{D}_j$  is *convex*, i.e., for all  $i \in N_j$  and all  $x_j \in X_j$ , the weak upper contour set  $\{x'_j \in X_j \mid x'_j R_i x_j\}$  is convex. A sufficient condition for negative acyclicity is then that  $F_j$  is simple, with no further restrictions. In fact, the strategic core then equal to the set of “generalized medians.”

When  $X_j$  is infinite, continuity properties must be imposed on group preferences to ensure non-emptiness of the strategic core. To define the following continuity condition, let  $\text{ls}Y_k$  denote the topological limit superior of a sequence of sets  $\{Y_k\}$ .<sup>1</sup> Say  $F_j$  is *monotonically continuous* if, for every sequence  $\{(R_1^k, \dots, R_{n_j}^k)\}$  in  $\mathcal{D}_j$  and for every profile  $(R_1, \dots, R_{n_j}) \in \mathcal{D}_j$  with  $\text{ls}R_i^k \subseteq R_i$  for all  $i \in N_j$ , we have

$$\text{ls}F_j(R_1^k, \dots, R_{n_j}^k) \subseteq F_j(R_1, \dots, R_{n_j}).$$

Roughly, this condition says that, for every sequence of profiles of member preferences, any limit of group preferences along the sequence must hold for the group at limiting member preferences. Here, a “limiting preference” refers to any relation for a member that contains all limits of preferences along the sequence. The condition is somewhat strong, implying that  $F_j(R_1, \dots, R_{n_j})$  is continuous, in the sense that weak upper and lower contour sets are closed. It also implies, as the name suggests, that  $F_j$  is monotonic, in the following

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<sup>1</sup>Thus,  $\text{ls}Y_k$  consists of the points that can be approximated by elements from any subsequence of  $\{Y_k\}$ .



weak sense: if  $x_j F_j(R_1, \dots, R_{n_j}) y_j$ , and if  $(R'_1, \dots, R'_{n_j})$  satisfies  $R_i \subseteq R'_i$  for all  $i \in N_j$ , then  $x_j F_j(R'_1, \dots, R'_{n_j}) y_j$

Monotonic continuity holds, for example, if  $F_j$  is simple. To see this, take any sequence  $\{(R_1^k, \dots, R_{n_j}^k)\}$ , take any  $(R_1, \dots, R_{n_j})$  such that  $\text{ls}R_i^k \subseteq R_i$  for all  $i \in N_j$ , and take any  $(x_j, y_j) \in \text{ls}F_j(R_1^k, \dots, R_{n_j}^k)$ . Thus, there is a subsequence  $\{(R_1^{k_l}, \dots, R_{n_j}^{k_l})\}$  of preference profiles and a sequence of pairs  $\{(x_j^l, y_j^l)\}$  such that

- (i)  $(x_j^l, y_j^l) \in F_j(R_1^{k_l}, \dots, R_{n_j}^{k_l})$  for all  $l$ ,
- (ii)  $(x_j^l, y_j^l) \rightarrow (x_j, y_j)$ .

By (i), for each  $l$  and each  $C \in \mathcal{W}$ , there exists  $i \in C$  with  $x_j^l R_i^{k_l} y_j^l$ . Since  $N_j$  is finite, this implies that, for each  $C$ , there is some  $i_C \in C$  for whom this weak preference holds for infinitely many  $l$ . By (ii), this implies  $(x_j, y_j) \in \text{ls}R_{i_C}$ . Since  $\text{ls}R_i^k \subseteq R_i$ , we have  $(x_j, y_j) \in R_{i_C}$ . Finally, because  $F_j$  is simple, this implies that  $x_j F_j(R_1, \dots, R_{n_j}) y_j$ , as required.

The following result establishes existence of group Nash equilibria under general conditions.

**Theorem 1** *Assume, for all  $j \in G$ , that  $F_j$  is negatively acyclic and monotonically continuous. There exists a group Nash equilibrium.*

*Proof:* Consider any group  $j$ , and define the correspondence

$$\psi_j: \prod_{k \in G} \mathcal{P}(X_k) \rightarrow X_j$$

by

$$\psi_j(\mu_1, \dots, \mu_m) = K(F_j(R_1(\mu_{-j}), \dots, R_{n_j}(\mu_{-j}))),$$

Because  $F_j$  is negatively acyclic and  $F_j(R_1, \dots, R_{n_j})$  is continuous for all  $(R_1, \dots, R_{n_j}) \in \mathcal{D}_j$  (in particular, it has closed upper contour sets), the

strategic core for group  $j$  is non-empty, i.e.,  $\psi_j$  has non-empty values. To see that it has closed graph, take any sequence  $\{(\mu_1^k, \dots, \mu_m^k)\}$  converging to some  $(\mu_1, \dots, \mu_m)$ , and take any sequence  $\{x_j^k\}$  with  $x_j^k \in \psi_j(\mu_1^k, \dots, \mu_m^k)$  for all  $k$  and with limit  $x_j$ . It must be shown that  $x_j \in \psi_j(\mu_1, \dots, \mu_m)$ . Take any  $y_j \in X_j$ . For each  $k$ , we have  $x_j^k F_j(R_1(\mu_{-j}^k), \dots, R_{n_j}(\mu_{-j}^k))y_j$ . Note that connectedness of  $\mathcal{P}(X)$  and continuity and transitivity of  $\succeq_i$  implies closed graph of  $\succeq_i$ . Thus, for all  $i \in N_j$ , we have

$$\text{ls}R_i(\mu_{-j}^k) \subseteq R_i(\mu_{-j}).$$

By monotonic continuity of  $F_j$ , we then have

$$\text{ls}F_j(R_1(\mu_{-j}^k), \dots, R_{n_j}(\mu_{-j}^k)) \subseteq F_j(R_1(\mu_{-j}), \dots, R_{n_j}(\mu_{-j})).$$

And since  $(x_j^k, y_j) \in F_j(R_1(\mu_{-j}^k), \dots, R_{n_j}(\mu_{-j}^k))$  for all  $k$  and  $(x_j^k, y_j) \rightarrow (x_j, y_j)$ , this implies  $x_j F_j(R_1(\mu_{-j}), \dots, R_{n_j}(\mu_{-j}))y_j$ . Thus,  $\psi_j$  has closed graph. Now define the correspondence

$$\Psi_j: \prod_{k \in G} \mathcal{P}(X_k) \rightarrow \mathcal{P}(X_j)$$

by

$$\Psi_j(\mu_1, \dots, \mu_m) = \mathcal{P}(\psi_j(\mu_1, \dots, \mu_m)),$$

where  $\mathcal{P}(\cdot)$  denotes the Borel probability measures on a set. This correspondence clearly has non-empty, convex values. And since  $\psi_j$  has closed graph, so does  $\Psi_j$ . Finally, define

$$\Psi: \prod_{j \in G} \mathcal{P}(X_j) \rightarrow \prod_{j \in G} \mathcal{P}(X_j)$$

by

$$\Psi(\mu_1, \dots, \mu_m) = \prod_{j \in G} \Psi_j(\mu_1, \dots, \mu_m).$$

As the product of correspondence with non-empty, convex values and closed graph,  $\Psi$  inherits these properties. Since the domain of the correspondence is convex and compact in the weak topology, Glicksberg's theorem yields a fixed point, which is a group Nash equilibrium.  $\blacksquare$

Because the above existence result uses mixed group strategies, there is some question about how those mixtures come about. In games among individual players, mixing takes place over strategies toward which a player is indifferent, implying a willingness to randomize. In the case of group competition, mixing takes place over alternatives in the group core, and, though no two elements of the core dominate each other, it is probably unjustified to claim that the group is indifferent in the same sense: there need not, for example, be a decisive group all of whose members are indifferent between the alternatives. Moreover, outcomes in the core may be generated as equilibria of an underlying non-cooperative game among the group members (see Section 4), and, in that case, one should not imagine a group randomization over equilibria.

An alternative interpretation of mixed strategies, one that carries over to games among groups, is that they represent the beliefs of others about how a player will play the game. In the present context, if a group core has multiple elements, then a single alternative must be selected from the core by some process, and a mixture  $\mu_j$  may simply represent beliefs about how group  $j$  will make that selection. Then requiring that  $\mu_j$  put probability one on the group core in equilibrium is to demand that beliefs be consistent with common knowledge of individual preferences, group aggregation procedures, and the beliefs of others.

Restricting the sets of group alternatives, individual preferences, and aggregation rules, the next result establishes the existence of pure strategy group Nash equilibria.

**Theorem 2** *Assume, for all  $j \in G$ , that  $X_j \subseteq \mathbb{R}$ , that each  $(R_1, \dots, R_{n_j}) \in \mathcal{D}_j$  is convex, and that  $F_j$  is simple. There exists a pure strategy group Nash equilibrium.*

The proof follows the proof of Theorem 1, with the following difference.

Because the strategic core of a group is now the generalized medians, define

$$\psi_j: \prod_{k \in G} X_k \rightarrow X_j$$

by

$$\psi_j(x_1, \dots, x_m) = K(F_j(R_1(x_{-j}), \dots, R_{n_j}(x_{-j}))),$$

where  $R_i(x_{-j})$  is member  $i$ 's induced preference relation over  $X_j$  when other groups choose alternatives  $x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_m$ . In this case,  $\psi_j$  has non-empty, convex values and closed graph. Defining

$$\psi: \prod_{j \in G} X_j \rightarrow \prod_{j \in G} X_j$$

by  $\psi(x) = \psi_1(x) \times \dots \times \psi_m(x)$ , the correspondence  $\psi$  has a fixed point, which is a pure strategy group Nash equilibrium.

## 4 Connections

### Arrovian aggregation

In his address at the 2000 Meetings of the Public Choice Society, Kenneth Arrow recalled studying the problem of strategic interaction between nations prior to conducting his seminal work in social choice theory (Arrow (1963)). To define a game among national actors, he naturally endowed each with a payoff function, implicitly assuming complete and transitive national preferences, and only later did he come to question the logical foundations of those assumptions. The answer is the well-known impossibility theorem: assuming at least three alternatives, there is no way to define complete and transitive social preferences while respecting Pareto optimality and independence of irrelevant alternatives over the universal domain of individual preferences. Though his conditions are interesting on purely normative grounds, Arrow's recollection points to an interesting *positive* motivation for them.

While transitivity of social preferences is a necessary condition for the existence of a social payoff function, it is not necessary for non-emptiness of group choice sets nor, therefore, for the existence of group “best responses” to other groups. In fact, the perspective offered by Arrow provides even more compelling motivation for the weaker rationality condition of acyclicity, which, combined with continuity, yields non-empty group cores. This condition has, of course, been the subject of an extensive literature in social choice theory, but only in the context of the decision of a single group.

This perspective also provides a rationale for aggregation over a *domain* of group member preferences, a consideration that is absent in non-cooperative games of complete information: there, individual preferences are specified as primitives, and equilibrium outcomes are determined without reference to “hypothetical” preferences. If we are considering aggregation within a single group, then we could similarly argue that group choices will be determined by actual preferences of individuals, making references to hypothetical preferences within some domain, as in the impossibility theorem, moot on positive grounds. In this paper, individual preferences  $\succeq_i$  over lotteries on vectors  $(x_1, \dots, x_m)$  were indeed fixed, but the issue of aggregation over domains arose naturally from the analysis, because individual *induced* preferences over *group* alternatives varied with the strategies of other groups.

### Legislative models

The model of group competition defined in this paper is particularly well-suited for modeling legislatures, and some of these considerations have arisen before. Kramer (1972), in analyzing a model of sophisticated voting, proves existence of an “issue-by-issue median,” a vector  $(x_1, \dots, x_m)$  such that each  $x_j$  is in the majority core along the  $j$ th dimension. Though Kramer was considering only one group, a legislature, this is technically an implication of Theorem 2, where we set  $N_1 = \dots = N_m$  equal to the legislature and  $X_j \subseteq \mathbb{R}$  for all  $j$ .<sup>2</sup> Because the same group is making decisions on all dimensions

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<sup>2</sup>In this formulation, the set of legislative outcomes is  $X = X_1 \times \dots \times X_m$ . Kramer actually allows for any strictly convex set  $X \subseteq \mathbb{R}^m$ , but Theorems 1 and 2 can be extended

in this model, it does not capture competition by separate committees that can unilaterally decide policy in their assigned dimensions, as under a closed rule, a model proposed by Austen-Smith and Banks (1998, pp.279-280).

Shepsle (1979) explicitly models competition among committees, and he proves existence of a “structure-induced equilibrium,” where each of a finite number of committees is assigned a one-dimensional “jurisdiction.” Because his definition specifies that a committee proposal obtain approval by a majority of the legislature, however, every issue-by-issue median is a structure-induced equilibrium. This is reflected in the proof of his existence result (Theorem 4.1) and in later discussion (Remark 3, p.54). Thus, Shepsle’s model also fails to capture competition among separate committees under closed rule. In contrast, the model of Sections 2 and 3 does capture this as a special case.

### Non-cooperative foundations

The model of group interaction defined above takes group preference aggregation rules as primitives and does not provide a non-cooperative model for how core alternatives are reached. Two possibilities are immediate. First, it may be that the choice within a group is the policy implemented by the winner of an election. Suppose two candidates,  $A$  and  $B$ , are purely office-motivated and commit to platforms,  $x_A$  and  $x_B$ , prior to the election. Now interpret  $F_j$  as a voting rule:  $A$  wins if not  $x_B F_j(R_1, \dots, R_{n_j}) x_A$ ,  $B$  wins if not  $x_A F_j(R_1, \dots, R_{n_j}) x_B$ , and the candidates tie otherwise. It is well-known that a platform pair  $(x_A, x_B)$  is a pure strategy Nash equilibrium of the candidate game if and only if  $x_A$  and  $x_B$  are in the core of the group.

Second, as in Banks and Duggan (2000), members of a group may play a complete information, infinite-horizon bargaining game, defined as follows: in any period  $t$ , an individual is randomly drawn to make a proposal; all individuals then vote on the proposal, which passes if some winning coalition votes for it; in that case, the game ends with the alternatives agreed upon; to this setting using arguments in Banks and Duggan (2001).

otherwise, the game moves to period  $t + 1$ , and the process repeats. Banks and Duggan (2000) prove that, if individuals are perfectly patient, if the group's aggregation rule is simple, and if  $\bigcap \mathcal{W}_j \neq \emptyset$ , then the stationary equilibrium outcomes of bargaining coincide with the group's core. If the condition on winning coalitions is dropped, then the same result holds if the group action space is one-dimensional and members' induced preferences are single-peaked. Thus, under standard conditions for a non-empty core, group Nash equilibria can be viewed as the stationary equilibrium outcomes of simultaneous bargaining within groups, in the presence of externalities across groups.

## References

- [1] K. Arrow (1963) *Social Choice and Individual Values*, 2nd ed., New Haven: Yale University Press.
- [2] D. Austen-Smith and J. Banks (1998) "Social Choice Theory, Game Theory, and Positive Political Theory," *Annual Review of Political Science*, 1: 259-287.
- [3] J. Banks and J. Duggan (2000) "A Bargaining Model of Collective Choice," *American Political Science Review*, 94: 73-88.
- [4] J. Banks and J. Duggan (2001) "Existence of Equilibria on Convex Sets," mimeo.
- [5] G. Kramer (1972) "Sophisticated Voting over Multidimensional Choice Spaces," *Journal of Mathematical Sociology*, 2: 165-180.
- [6] K. Shepsle (1979) "Institutional Arrangements and Equilibrium in Multidimensional Voting Models," *American Journal of Political Science*, 23: 27-59.

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