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"The political economy of income taxation"

by

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Abstract. Parties compete on a large policy space of continuous functions. Only a fraction of each voter type will vote for each party, perhaps because of party reputations, or issues not modeled here. Each party's policy makers comprise two factions, one concerned with maximizing the welfare of its constituency, or its core, the other with winning over swing voters. An application to competition over redistribution produces equilibria in which each party proposes a piece-wise linear tax schedule, and these schedules coincide for a possibly large interval of middle-income voters. This appears to conform with recent tax history in the US.

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> The spirit of a people, its cultural level, its social structure, the deeds its policy may prepare—all this and more is written in its fiscal history, stripped of all phrases. He who knows how to listen to its message here discerns the thunder of world history more clearly than anywhere else.[1]

1. Introduction

Formal political-economic analysis of taxation has been in the main of a schematic nature: that is, existing models of income taxation usually assume that taxation is an affine function of income[2]. In reality, income-tax policy is extremely complex, reflecting the fact that many competing interests must be satisfied, or attended to. (For a useful 'short history' of the income tax in the United States, see Brownlee (2004), which contains a full guide to the literature.) In this paper, I attempt to capture this complexity by modeling political competition over the income tax as taking place on an infinite dimensional space of functions. Each political party will propose a function which will define the post-tax-and-transfer income, for every possible realization of pre-tax income, and these functions will be chosen from a large space, constrained only by upper and lower bounds on what the marginal tax rates can be[3].

We will suppose that two parties are competing in a general election, and that the platform of each party consists in a proposal of such a 'post-fisc' function. We attempt to capture what appear to be an important aspect of party competition: that parties are concerned with the base (or core) voters, and with winning the undecided (or swing) voters[4]. A simple way of formalizing these aims of a party is to assume that there are

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[1] Schumpeter (1954 [1918]), as quoted in Brownlee (2004).

[2] A number of papers, for example Dixit and Londregan (1998), study 'pork barrel politics,' in which parties propose payments to each of a finite number of voter types. Here, the policy space is finite dimensional, but could be of high dimension.

[3] I first studied taxation on this policy space in Roemer (2006). The present paper is a substantial variation on some themes developed there.

[4] See Cox (2006) for a recent review of the formal literature which attempts to model parties' concerns with swing and core voters.

intra-party factions concerned, respectively, with these two problems – of satisfying the core constituency, and of appealing to the swing voters. We show that this solves the problem of the existence of a (Nash-type) equilibrium in pure strategies in the game of party competition, even when the parties are choosing strategies from an infinite dimensional space.

Besides modeling parties as complex organizations (in the sense that policy is set by intra-party bargaining), we depart from traditional formal approaches in the study of political competition in another way. The polity consists of a continuum of voter types, where 'type' is defined by the pre-tax income of the agent or household. In many – perhaps most -- formal papers about political competition, parties represent *no* constituencies. This is the case with the Downs model, where each party is only the vehicle of a candidate who seeks election. It is as well essentially the case with the citizen-candidate models of Osborne and Slivinski (1996) and Besley and Coate (1997), where candidates run on their own ideal policies (each 'party' represents a constituency of one type). In models where parties represent non-trivial constituencies – see for instance Dixit and Londregan (1998), Austen-Smith(2000), Levy (2004), and Roemer (1999, 2001,2006) -- it is supposed that each party represents an element of some partition of the polity. Here, we depart from these practices, by recognizing that, in reality, it is never the case that the sets of voters who support the various parties form an easily defined partition of the space of voter types. For instance, in American elections, a substantial fraction of voters *at every income level* supports each of the two parties: see figure 8 below for the details, and McCarty, Poole and Rosenthal (2006, chapter 3). Of course, one can say that, if the *space of types* were modeled as having sufficiently large dimensionality, there would always be characteristics of voters that would enable us to define the set voting for a particular party as an element of a partition of that space. We prefer, however, to take a more statistical approach – to keep the space of types simple, and to say that, from the viewpoint of parties, there is a random element in voting, and therefore, if a party is concerned to represent its constituents, it has to attempt to represent every household type, at least to some extent – because *some* fraction of every type will vote for each party.

We will define an equilibrium concept reflecting these concerns, and then characterize an important class of equilibria in the income-tax competition game. The main characteristics of the equilibria are these:

1. In every equilibrium, there is a 'Left' and a 'Right' party. The Left party puts more weight on the interests of voters the poorer they are, and the Right puts more weight on the interests of voters, the richer they are;
2. In an important sub-class of these equilibria, each party proposes a piece-wise linear post-fisc policy;
3. In every such equilibrium, the policy proposed by Left entails an increasing average rate of taxation on the whole domain of incomes; the policy proposed by the Right entails an average rate of taxation that increases up to a point, and then decreases;
4. There is a two dimensional manifold of these equilibria, where a particular equilibrium can be viewed as being characterized by the relative strength of the 'swing' versus 'core' factions within the parties;
5. In every equilibrium, the two parties propose exactly the same tax treatment for what may be a substantial interval of middle-income voters. The greater the focus the parties place upon swing voters, the larger will be the size of this interval;
6. All equilibrium policies are progressive in the sense that the redistribute income from a class of rich voters to the class of non-rich voters.

Recent empirical work on the history of income taxation in the United States enables us to check some of these predictions. In particular, there appears to be quite striking confirmation of the fifth prediction. One might think of the fifth property as a generalization of the median voter theorem (where, in the Downs model, both candidates propose the same policy for the whole electorate).

In section 2, we propose several concepts of political equilibrium. In section 3, we characterize so-called 'left-right' equilibria in the income-tax setting. In section 4, we examine US income-tax data to see how well reality conforms to the model's predictions. Section 5 concludes. Lengthy proofs are gathered in the appendix.

2. A concept of political equilibrium in two-party politics

Let the policy space be a convex set $\Im$; the generic policy is denoted *X.* The space of voter types is $H \subset \mathbb{R}$; voters are distributed according to a cdf *F* on *H.* There is a continuum of voters of each type. Voters are endowed with preferences represented by a non-negative utility function $v : \Im \times H \to \mathbb{R}_+$, where *v* is concave on $\Im$. Voters behave stochastically. When facing two policies X^a and X^b from parties *a* and *b,* a voter of type *h* votes for policy X^a when

$$S\left(\frac{v(X^a;h)}{v(X^b;h)}\right) > \varepsilon, \qquad (2.1)$$

where ε is a normal variate, with mean zero, whose cdf is *N,* and *S* is a function such that:

(i) $S : \mathbb{R}_+ \to \mathbb{R}$

(ii) *S* is strictly increasing

(iii) $S(1) = 0$, and

(iv) $S(z) = -S(\frac{1}{z})$.

Thus, if $v(X^a;h) = v(X^b;h)$ then one-half the voters of type *h* will vote for each policy, by (iii) and the continuum assumption[5]. Condition (iv) guarantees that the probability that a voter votes for party *a* is one minus the probability that she votes for party *b.*

We propose a concept of political equilibrium in which parties are endogenous, and each party contains political entrepreneurs who adopt different strategies. One strategy is to attempt to represent the *constituency* of the party; the other strategy is to target *swing voters.* The constituency of the party and the swing voters are endogenous.

Note, from (2.1), that given two policies X^a, X^b, the fraction of voters of type *h* who vote for policy X^a is:

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[5] For example, $S(z) = \log(z)$. We can specify a function *S* arbitrarily on the interval [0,1], subject to conditions (ii) and (iii), and then define *S* on $(1,\infty)$ by condition (iv). Nothing hinges on *N*'s being a normal distribution: any symmetric distribution on the real line will do.

$$\theta(h;X^a,X^b)=N\left(S\left(\frac{v(X^a;h)}{v(X^b;h)}\right)\right). \qquad (2.2)$$

We will usually write this simply as $\theta(h)$ when the policies are clear. Given a pair of policies (X^a,X^b) the fraction of the vote share going to party *a* is:

$$\sigma^a(X^a,X^b)=\int\theta(h;X^a,X^b)dF(h). \qquad (2.3)$$

We propose to define the core (constituency) of a party as an *historical* and *statistical* concept. Suppose in the last election, at date $t-1$, the set of voters who voted for the party is characterized by the function θ_{t-1} defined in (2.2). The party identifies its *core* as the voters so described: that is as a fraction $\theta_{t-1}(h)$ of voters of type *h,* for every *h.*

We say the *swing voters* comprise the set of types $\{h \mid \theta(h)=\frac{1}{2}\}$.[6]

We now discuss the behavior of political entrepreneurs, who set policy for the parties. We assume there are two parties. Parties exist for a long time; they build a reputation by representing certain constituencies. With stochastic voting (i.e., a simple space of types), the constituency of a party is hard to define, because one can never be sure exactly who will vote for the party. Nevertheless, from a statistical viewpoint, the constituency of a party may be quite clear, as I have indicated.

We suppose at the present election (date *t*) those politicians who attempt to represent the party's constituency want to choose the policy *X* to maximize the function:

$$\int\theta_{t-1}(h)v(X;h)dF(h)$$

where $\theta_{t-1}(\cdot)$ is the function describing the fraction of votes the party received for each type in the election at the last date ($t-1$). That is, they will attempt to maximize the average welfare of their statistical constituency, by weighting the welfare of every type

[6] Another modeling possibility is to define the core of the party as the set of types $\{h \mid \theta(h) > \frac{1}{2}\}$. This would lead to a somewhat different theory than the present one, but similar in structure.

by the fraction of that type that comprise the constituency of the party in that type[7]. Formally, they desire to represent every income type, but with varying weights. We depart from the more familiar formulation that each party represents a distinct set of voter types.

We model the second faction of politicians, the 'swing faction,' as insisting that the party promise at least as much to the swing voters as the other party is proposing to them. They are battling for the loyalty of the swing voters.

We now propose a concept of a sequence of political equilibria over time. Note, from what I said above, that the party's constituency is defined by the last election. We suppose that the distribution of types, *F*, is unchanging over time (i.e., the distribution of types changes slowly compared to the period of the election cycle).

Definition 1 A *history of political equilibria given a function* $\theta_0 : H \to [0,1]$ is a sequence of policies $\{(X_t^L, X_t^R) \in \Im \times \Im \mid t = 1,2,...\}$, and a sequence of functions $\{\theta_t : H \to [0,1] \mid t = 1,2,...\}$ such that:

(1a) for every *t*=1,2,… policy X_t^L solves the following program:

$$\max_{X \in \Im} \int \theta_{t-1}(h) v(X;h) dF(h)$$

$$\text{subj. to } (\forall h)(\theta_{t-1}(h) = \frac{1}{2} \Rightarrow v(X;h) \ge v(X_t^R;h)) \quad (L1)$$

(1b) for every *t* =1,2,…, policy X_t^R solves the program:

$$\max_{X \in \Im} \int (1 - \theta_{t-1}(h)) v(X;h) dF(h)$$

$$\text{subj. to } (\forall h)(\theta_{t-1}(h) = \frac{1}{2} \Rightarrow v(X;h) \ge v(X_t^L;h)) \; (R2)$$

(2) for every *t*=1,2,…, and for all $h \in H$:

[7] Some aggregation principle (i.e., social welfare function) other than summing up could be used. The key point is that types are weighted by their historical loyalty to the party.

$$\theta_t(h) = N\left(S\left(\frac{v(X_t^L;h)}{v(X_t^R;h)} \right)\right).$$

From (3), the function θ_t gives the fraction of each type that votes for party L in the election at date *t.* The constraints (L1) and (R1) in the two programs are imposed by the factions concerned with swing voters: for instance, (L1) says that party L can propose no policy that provides lower utility to the swing voters than the R party proposes to provide them. In words, a history of political equilibria comprises a sequence of pairs of policies such that, given the party's conception of its statistical constituency from the election held at date $t-1$, the policy of the party at date *t* cannot be dominated by any other policy with respect to weighted average welfare of the party's historical constituency, subject to providing at least as well as the opposition proposes to provide for swing voters.

The datum of the equilibrium concept is the pair of functions (F,θ_0). We may view θ_0 as the initial conjecture of the two parties concerning their statistical constituencies.

One can ask: Why not model the 'swing voter faction' as maximizing vote share, by placing the following constraint in the program of (for example) party *L*:

$$\sigma^L(X, X_t^R) \geq \sigma_*^L \qquad (2.4)$$

for some constant σ_*^L (perhaps equal to one-half)? Doing so would model a bargaining problem in the party between those who attempt to maximize the welfare of the constituency and those who attempt to maximize the party's vote share. Unfortunately, constraint (2.4) would make the optimization problem of the party non-concave, and there is little hope of solving it in our application, in which the policy space is infinite-dimensional. This – indeed – may be why actual politicians appear to focus on swing voters -- a manageable task-- while seeking to maximize vote share in a more precise way is not.

We can expect that there will be many possible histories of political equilibria. If one is interested in modeling general elections to understand the *underlying long-range* political conflicts in a society, then one should be interested in stationary points of these histories. An interest in stationary points must, of course, be justified by a view that the underlying distribution of preferences, represented by F, is changing slowly relative to the frequency of political events, that is, of elections – as we have assumed.

I propose a concept of stationarity which entails that the sequence of functions $\{\theta_t\}$ in a history of political equilibria converges to a function θ_*: thus, the constituency of each party becomes stable. We define a *stationary equilibrium* as a function θ_* and a pair of policies (X_*^L, X_*^R) such that:

(α1a) policy X_*^L solves the program:

$$\max_{X\in\Im} \int \theta_*(h)v(X;h)dF(h)$$
$$\text{subj. to } (\forall h)(\theta_*(h)=\frac{1}{2} \Rightarrow v(X;h)\geq v(X_*^R;h)) \quad (L2)$$

(α1b) policy X_*^R solves the program:

$$\max_{X\in\Im} \int (1-\theta_*(h))v(X;h)dF(h)$$
$$\text{subj. to } (\forall h)(\theta_*(h)=\frac{1}{2} \Rightarrow v(X;h)\geq v(X_*^L;h)) \, (R2)$$

(α2) For all $h \in H$, $\quad \theta_*(h) = N\left(S\left(\frac{v(X_*^L;h)}{v(X_*^R;h)} \right)\right)$

There is a kind of equilibrium which is a refinement of stationary equilibrium and plays an important role in the analysis:

<u>Definition 2</u> A 1-*stationary equilibrium* is a function θ_*, a pair of policies (X_*^L, X_*^R), and an ordered pair $(h_*, y) \in \mathbb{R}_+^2$ such that:

(β1a) X_*^L solves the program:

$$\max_{X\in\Im} \int \theta_*(h)v(X;h)dF(h)$$
$$\text{subj. to } v(X;h^*)\geq y \quad (L3)$$

(β2a) X_*^R solves the program

$$\max_{X\in\Im} \int (1-\theta_*(h))v(X;h)dF(h)$$
$$\text{subj. to } v(X;h^*)\geq y \quad (R3)$$

(β3) For all $h \in H$, $\theta_*(h) = N\left(S\left(\dfrac{v(X_*^L;h)}{v(X_*^R;h)}\right)\right)$

(β4) $v(X_*^L;h_*) = y = v(X_*^R;h_*)$.

In this concept, it is as if the vote-share-seeking faction is concentrating on not losing the loyalty of one swing voter type, namely *h**. What is important is the relationship of 1-stationary equilibrium to stationary equilibrium.

Proposition 1 *Every* 1-*stationary equilibrium is a stationary equilibrium*.

Proof:

Let $(\theta_*, X_*^L, X_*^R, h_*, y)$ be a 1-stationary equilibrium. By (β4), we can write the constraint (L3) as $v(X;h_*) \geq v(X_*^R;h_*)$. But by (β4), it also follows that $\theta_*(h_*) = \frac{1}{2}$. Therefore, constraint (L3) is weaker than constraint (L2). Hence the program in (β1a) has the same objective function but a larger opportunity set than the program in (α1a). However, X_*^L is a member of the opportunity set defined by (L2). It follows that X_*^L solves (α1a). In like manner, X_*^R solves (α1b), proving the claim. ■

Now examine the program in condition (β1a). It is a concave programming problem. There is no interaction between the *L* and *R* policies – so in principle it can be solved. The same goes for the program in (β1b). In other words, the refined equilibrium concept of 1-stationary equilibrium is quite tractable.

I will proceed by applying these equilibrium concepts to the study of income taxation, and will then argue, by looking at the history of income tax reform, that the equilibrium concept appears to explain quite well some of its main features.

3. Income taxation

We assume that the (pre-tax) income distribution is given by a cdf F on $H = \mathbb{R}_+$; the generic pre-tax income is denoted *h,* and the mean pre-tax income is denoted μ. The policy space $\Im$ consists of functions $X : \mathbb{R}_+ \to \mathbb{R}_+$ such that:

$(P0) X$ is continuous,
$(P1) \alpha \le X'(h) \le 1,$ where X is differentiable,
$(P2) \int X(h) dF(h) = \mu$

where α is a number, $0 \le \alpha < 1$[8]. $X(h)$ is interpreted as the post-fisc income of type h; the two conditions (P1) and (P2) state that the derivative of X, where it exists, lies between α and 1, and that X redistributes pre-tax income fully.

The preferences of voters are given by:

$$v(X;h) = X(h);$$

that is, each voter wishes to maximize his/her post-fisc income. If the policy is X, then the net taxes paid by an individual h are $t(h;X) = h - X(h)$. Hence the marginal tax rate for h at policy X is $1 - X'(h)$, which is bounded below and above by zero and $1-\alpha$, respectively.

Leisure is not an argument of the utility function for reasons of tractability: the equilibrium analysis would otherwise become unmanageable[9]. I attempt to recognize the elasticity of labor supply by requiring that the marginal tax rate be at most $1-\alpha$: political parties agree not to consider policies that have very high marginal tax rates, because of the deleterious labor-supply effects (α is a parameter of the model). Alternatively put, we are assuming that when marginal tax rates lie in the interval $[0,1-\alpha]$, labor-supply elasticity is very small and can be ignored.

Notice that the space $\Im$ is infinite dimensional: we allow parties to choose any continuous function as the post-fisc income distribution, subject to the proviso on maximum and minimal marginal rates. The choice to work on this large space is dictated by reality, where income-tax policies are generally very complex.

The first theorem will characterize a two dimensional family of 1-stationary equilibria. To do so, we define two families of piece-wise linear functions. Fix a number $h_* > 0$. The first family is

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[8] More precisely, we can eliminate the requirement that X be continuous and replace the condition (P1) with a Lipschitz condition on X.
[9] If leisure were in the utility function, then each party would be solving a Mirrlees-type optimization problem.

$$M_a(h_*) = \left\{ \begin{array}{l} X \in \Im \mid \exists (x_a, h_1) \in \mathbb{R}^2_+ \text{ such that } h_1 \le h_* \text{ and} \\ X(h) = \left\{ \begin{array}{c} x_a + \alpha h, \text{ if } h \le h_1 \\ x_a + \alpha h_1 + (h - h_1), \text{ if } h_1 < h \le h^* \\ x_a + \alpha h_1 + (h^* - h_1) + \alpha(h - h^*), \text{ if } h > h^* \end{array} \right. \end{array} \right\}.$$

A typical function in the family is graphed in figure 1. $M_a(h_*)$ is a unidimensional family of functions which we may view as being parameterized by the value $y \equiv X(h_*)$; that is, fixing the ordered pair (h_*, y) determines at most one policy in the family $M_a(h_*)$. By construction, the policies $X \in M_a(h_*)$ satisfy (P0) and (P1). The budget- balance condition (P2) gives one equation in the two unknowns (x_a, h_1): hence, the unidimensionality of this family.

The second family is:

$$M_b(h_*) = \left\{ \begin{array}{l} X \in \Im \mid \exists (x_b, h_2) \in \mathbb{R}^2_+ \text{ such that } h_2 \ge h_* \text{ and} \\ X(h) = \left\{ \begin{array}{c} x_b + h, \text{ if } h \le h_* \\ x_b + h_* + \alpha(h - h_*), \text{ if } h_* < h \le h_2 \\ x_b + h_* + \alpha(h_2 - h_*) + (h - h_2), \text{ if } h > h_2 \end{array} \right. \end{array} \right\}.$$

Likewise, $M_b(h_*)$ is a unidimensional family of piece-wise linear policies, which is parameterized by $y \equiv X(h_*)$; a typical policy is also graphed in figure 1.

We will be interested in policy pairs $(X^a, X^b) \in M_a(h_*) \times M_b(h_*)$ which share a common value of $y = X^a(h_*) = X^b(h_*)$. The next proposition tells us exactly what the admissible range is for *y*.

<u>Proposition</u> <u>2</u> *Let* $h_* > 0$, *and let y lie in the interval*

$$\max[(1-\alpha)\mu + \alpha h_*, h_*] \le y \le h_* + (1-\alpha)\int_{h^*}^{\infty} (h - h^*) dF(h). \qquad (3.1)$$

Then:

A. *There exist unique policies* $X_a \in M_a(h_*)$, $X_b \in M_b(h_*)$ *such that*

$$X_a(h_*) = y = X_b(h_*). \qquad (3.2)$$

B. *Conversely, if y does not lie in the interval defined by* (3.1), *then there is no pair of policies in the two families for which* (3.2) *holds.*

C. *The number* x_a *is positive, and the number* x_b *is non-negative, and positive except in a singular case*.

All longer proofs, beginning with this one, appear in the Appendix.

Define:

$$\Gamma = \{(h_*, y) \in \mathbb{R}_+^2 \mid \max[(1-\alpha)\mu + \alpha h_*, h_*] \le y \le h_* + (1-\alpha)\int_{h^*}^{\infty}(h - h^*)dF(h)\}$$

and

$$y_{\min}(h_*) = \max[(1-\alpha)\mu + \alpha h_*, h_*], \quad y_{\max}(h_*) = h_* + (1-\alpha)\int_{h^*}^{\infty}(h - h^*)dF(h).$$

Proposition 2 tells us that for any $(h_*, y) \in \Gamma$, there exists a unique pair of policies $X^a \in M_a(h_*)$ and $X^b \in M_b(h_*)$ such that

$$X^a(h_*) = y = X^b(h_*).$$

To avoid notational complexity, let us fix $(h_*, y) \in \Gamma$ and denote these two functions simply by X^a and X^b. Figure 1 displays the graphs of a typical pair of such functions. Note, in particular, that these two policies coincide on the interval $[h_1, h_2]$. Suppose, now, that these two policies are being proposed by the parties, *a* and *b,* and define the function $\theta(\cdot; X^a, X^b)$ by equation (2.2). We have:

<u>Proposition</u> <u>3</u>. *The function* $\theta(\cdot; X^a, X^b)$ *is decreasing on the interval* $[0, h_1]$*, constant and equal to one-half on the interval* $[h_1, h_2]$*, and decreasing on the interval* (h_2, ∞).

<u>Proof:</u>

Easily verified from the definition of the functions X^a, X^b, and *S.* ■

We may now state our first main result:

<u>Theorem</u> <u>1</u> *Let* $(h_*, y) \in \Gamma$*, and let* $X^a \in M_a(h_*)$, $X^b \in M_b(h_*)$ *such that* $X^a(h_*) = y = X^b(h_*)$. *Let* $\theta(\cdot; X^a, X^b)$ *be defined as above. Then* (θ, X^a, X^b) *is a* 1-*stationary equilibrium.*

Theorem 1 gives us a stationary equilibrium for each $(h_*, y) \in \Gamma$: thus, a two-parameter family of equilibria. We propose an interpretation of the political nature of these various equilibria, which follows from the next result. In Figure 2, we graph the manifold Γ for the case where *F* is the lognormal distribution of income with mean 50 and median 40, an approximation of the US household income distribution in 2000, in units of $1000, and $S(z) = \log(z)$. Note that for $h_* \geq \mu$, the lower envelope of Γ coincides with the 45^0 ray.

Theorem 2 A. *Consider a point $(h_*, y_{\max}(h_*))$ on the upper envelope of the manifold Γ. Let the two policies of the* 1*-stationary equilibrium at this point be denoted X^L and X^R. Then $h_1 = 0, \quad h_2 = \infty$ and $X^L = X^R = X^*$, where X^* is the ideal policy in $\Im$ of voter h_*.*

B. *Consider a point $(h_*, y_{\min}(h_*))$ on the lower envelope of Γ, with its associated* 1-*stationary equilibrium (X^L, X^R). If $h_* \leq \mu$ then X^L is the ideal policy in $\Im$ of Left's constituency*[10]*, and if $h_* \geq \mu$ then X^R is the ideal policy in $\Im$ of Right's constituency.*

Fix the 'pivot type' h_* and begin at the equilibrium on the upper envelope of Γ at h_*. In the stationary equilibrium at this point, both parties propose the ideal policy of the pivot type, h_*. Each party receives half the vote. Here we have politics where the concern for swing voters is very strong in both parties: the factions representing constituent interests have no pull. As we start to move down the manifold Γ, decreasing y and holding h_* fixed, the two policies diverge. The factions concerned with core voters become more powerful in intra-party bargaining. When we reach the equilibrium on the lower envelope of Γ, if $h_* < \mu$, then this faction is entirely dominant in the Left party in the sense that the L party is playing as if it is only concerned with constituent interests; if $h_* > \mu$, then constituent interests are dictating policy in the Right party. In the singular case that $h_* = \mu$, both parties are maximizing over $\Im$ the average utility of their statistical constituencies.

For a policy *X,* define the average tax rate at *h* as:

$$a(h;X) = \frac{h - X(h)}{h}.$$

Define a policy as progressive if it unambiguously redistributes from the rich to the poor, in the following sense:

Definition 3. A policy *X* is *progressive* if there exists $\hat{h}$ such that:

[10] That is, X^L maximizes $\int \theta(h)X(h)dF(h)$, for $X \in \Im$, where $\theta(h) \equiv \theta(h; X^L, X^R)$.

$$h \le \hat{h} \Rightarrow X(h) \ge h$$
$$h > \hat{h} \Rightarrow X(h) \le h$$

and at least one (some) of these inequalities hold(s) strictly for some *h*.

We have:

Proposition 4 *Consider any* 1-*stationary equilibrium* (X^L, X^R) *of theorem* 1. *Then:*

A. $a(\cdot; X^L)$ *is increasing on* $\mathbb{R}_+$*, and* $a(\cdot; X^R)$ *is increasing on* $[0, h_2)$ *and decreasing on* (h_2, ∞)*, except in the singular case that* X^R *is the laissez-faire policy.*

B. *Both policies are progressive, except in the singular case that* X^R *is the laissez-faire policy.*

Proof:

A. The condition $\frac{d}{dh} a(h; X) > 0$ is equivalent to $hX'(h) < X(h)$. It is easy to check (for instance, examine Figure 1) that this condition is true for X^L: formally, this follows from the fact that $x_a > 0$ and $x_b \ge 0$ (see Prop. 2(c)). For X^R, it is also easy to check that the condition holds if and only if $h < h_2$. Now note that if the segment of the graph of X^R on the domain $[h_2, \infty)$ is extended into a line, it passes below the origin. (For if it passed through or above the origin , the policy X^R would dominate the policy $X(h) \equiv h$, and would not be feasible, as it would integrate to more than μ.) This means that $hX' > X$ on $[h_2, \infty)$.

B. This follows immediately from the fact that $x_a > 0$ and $x_b > 0$ (except in the laissez-faire policy). ■

We provide some simulations showing these stationary equilibria. We choose *F* to be the lognormal distribution with mean 50 and median 40. We choose *N* to be the normal variate with mean zero and standard deviation 0.720 and $S(z) = \log z$. Since $N(\log(1.2)) = 0.6$, a voter of any type has a 60% probability of voting for a party that promises her 20% higher post-fisc income than the other party (see (2.1)). In figure 3, we graph the 1-stationary equilibrium of theorem 1 for four values of *y,* holding h_* at the mode of the income distribution. When $y = y_{\max}(h_*)$ (figure 3a), the two policies coincide at the ideal policy of h_*; when $y = y_{\min}(h_*)$ (figure 3c), Left is playing the ideal policy of its average constituency, since $h_* < \mu$. Even on the lower envelope of the manifold, the policies agree for 16% of the polity.

In Figure 4, we plot the average tax rate functions for the Left and Right policies, at a point in Γ. The Right imposes a higher average tax rate up to h_1; of course average tax rates of the two policies coincide on the interval $[h_1, h_2]$; the Left policy imposes a higher average tax rate on (h_2, ∞). Moreover, the Left average tax rate is monotone increasing on the whole domain, while the Right average tax rate is increasing until h_2, and then monotone decreasing asymptotically to zero thereafter.

We next ask what is the effect of a change in h_* on equilibrium policies. Rather than prove a theorem, we graph some examples to show the contrast. In figure 5, we present the average tax rate functions associated with Left and Right policies for two values, $h_* \in \{20, 80\}$. In each case we plot policies for $(h_*, \frac{y_{\min}(h_*) + y_{\max}(h_*)}{2}) \in \Gamma$. We see that the effect of increasing the 'pivot' h_* is to flatten out the average tax rate functions. For large values of the pivot, both parties propose net tax rates of close to zero for middle-income voters.

In American elections, the vote share of each party has been, in recent decades, quite close to one-half. This suggests that we find the policies in Γ where the vote shares are close to 0.5. Figure 6a presents the points in Γ where the Left policy defeats the Right policy by a margin of 0.025 or less. (We randomly generated 600 points in Γ, choosing $h_* \in (20, 80)$ and $y \in (y_{\min}(h_*), y_{\max}(h_*))$, and recorded the ones with this property.) We see that the equilibria are clustered quite close to the upper envelope of Γ[11]. This suggests that, if the model is an accurate description of American politics, we should see the two parties offering tax policies which are quite similar, in the precise sense that they will coincide for a large fraction of the polity – corresponding in the model to the income interval $[h_1, h_2]$.

We next note the central role of 1-stationary equilibria in the theory.

[11] Clearly, vote shares depend intimately on the function *S* and *F*. For *S*=log, in the present example, it appears that the *L* party never has a vote share less than 0.5. Of course, this has to do with the behavior of the ratios $\frac{X^L(h)}{X^R(h)}$.

<u>Definition</u> <u>4</u> A *left-right stationary equilibrium* is a stationary equilibrium where the function θ_* is weakly monotone decreasing, $\theta_*(0) > \frac{1}{2}$, and for sufficiently large *h,* $\theta_*(h) < \frac{1}{2}$.

In other words, such an equilibrium is one where one party (Left) gives more weight to voters the poorer they are, the other (Right) gives more weight to voters, the richer they are. The next theorem tells us how 1-stationary equilibria come about:

<u>Theorem</u> <u>3</u>. *Suppose that* $\theta_0(\cdot)$ *is a weakly monotone decreasing function, and there is a unique income* h_* *such that* $\theta_0(h_*) = \frac{1}{2}$. *Let* $y \in [y_{\min}(h_*), y_{\max}(h_*)]$ *and let* $(\hat{X}^L, \hat{X}^R) \in M_a(h_*) \times M_b(h_*)$ *be the unique policies associated with the ordered pair* $(h_*, y) \in \Gamma$. *Then* $(\hat{X}^L, \hat{X}^R)$ *is a stationary equilibrium reached at date* 1 *beginning from* θ_0. *Conversely, let* (X^L, X^R) *be any equilibrium reached at date* 1 *beginning from* θ_0*, in a sequence of historical equilibria. Then* (X^L, X^R) *are precisely the policies in* $M_a(h_*) \times M_b(h_*)$ *associated with the ordered pair* $(h_*, X^L(h_*)) \in \Gamma$.

If we assume, in a history of political equilibria, that once a stationary equilibrium is reached, it continues to be played at all future dates, then theorem 3 says that all sequences of historical equilibria which begin with a vote share function θ_0 as specified in the premise, end in one period, with a left-right 1-stationary equilibrium, as depicted in figure 1.

We next will describe the stationary equilibria reached by sequences of historical equilibria associated with historical share functions θ_0 which are weakly monotone decreasing and for which there is a non-degenerate interval $[h_*, h_{**}]$ upon which $\theta_0 = \frac{1}{2}$. Let $Z : [h_*, h_{**}] \to \mathbb{R}_+$ be an arbitrary, continuous function such that $\alpha \leq Z'(h) \leq 1$. If the integral (d*F*) of *Z* on $[h_*, h_{**}]$ is neither too small nor too large, then there exists a left-right stationary equilibrium, in which both L and R policies coincide with *Z* on the interval $[h_*, h_{**}]$, and are as depicted in Figure 6b. On the intervals $[0, h_*]$ and $[h_{**}, \infty)$ the two policies behave just as the policies in a 1-equilibrium (see figure 1). Note that if $h_{**} = h_*$, then figure 6b becomes exactly figure 1.

To be precise:

Theorem 4. *Let θ_0 be weakly monotone decreasing such that $\theta_0(h) = \frac{1}{2}$* on $[h_*, h_{**}]$. *Let Z be a continuous function defined on $[h_*, h_{**}]$ such that*

$$(\forall h \in [h_*, h_{**}])(\alpha \le Z'(h) \le 1).$$

Suppose that there exist numbers $h_1 \in [0, h_*]$ and $h_2 \in [h_{**}, \infty)$ *and $x_a \ge 0$, $x_b \ge 0$ such that the functions $\hat{X}^L$* and $\hat{X}^R$*, defined below, are continuous and integrate (*d*F) to* μ:

$$\hat{X}^L(h) = \begin{Bmatrix} x_a + \alpha h,\ 0 \le h \le h_1 \\ x_a + \alpha h_1 + (h - h_1),\ h_1 < h \le h_* \\ Z(h),\ h_* < h \le h_{**} \\ Z(h_{**}) + \alpha(h - h_{**}),\ h > h_{**} \end{Bmatrix}$$

$$\hat{X}^R(h) = \begin{cases} x_b + h,\ 0 \le h \le h_* \\ Z(h),\ h_* \le h \le h_{**} \\ Z(h_{**}) + \alpha(h - h_{**}),\ h_{**} \le h \le h_2 \\ Z(h_{**}) + \alpha(h_2 - h_{**}) + h - h_2,\ h > h_2 \end{cases}$$

Then $(\hat{X}^L, \hat{X}^R)$ is a left-right stationary equilibrium, reached in one date from the historical vote share function θ_0. Conversely, let (X^L, X^R) be any equilibrium beginning at θ_0 which is reached at the first date, and let $Z(h) \equiv X^L(h)$ on $[h_*, h_{**}]$. *Then the functions $(\hat{X}^L, \hat{X}^R)$ can be defined as in the statement, they integrate (*d*F) to μ, and* $(X^L, X^R) = (\hat{X}^L, \hat{X}^R)$.

Theorems 1 and 4 characterize left-right stationary equilibria. Obviously, the 1-stationary equilibria are the simplest; there is a 2-manifold of them. The manifold of equilibria of the form described in theorem 4 is infinite dimensional, for the function *Z* can be specified in an essentially arbitrary way.

We can locate sets of equilibria which lie 'between' the 1-equilibria and the equilibria of theorem 4. Let us define the concept of a 2-stationary equilibrium:

<u>Definition</u> <u>5</u> A 2-*stationary equilibrium* is a tuple $(h_*, y_*, h_{**}, y_{**})$ and a triple of functions (θ_*, X^L, X^R) such that:

(γ1a) X^L solves

$$\max_{X \in \Im} \int \theta_*(h) X(h) dF(h)$$
s.t.
$$X(h_*) \geq y_*$$
$$X(h_{**}) \geq y_{**}$$

(γ2a) X^R solves

$$\max_{X \in \Im} \int (1 - \theta_*(h)) X(h) dF(h)$$
s.t.
$$X(h_*) \geq y_*$$
$$X(h_{**}) \geq y_{**}$$

(γ3) $\theta_*(h) = N\left(S\left(\frac{X^L(h)}{X^R(h)}\right)\right)$

(γ4) $X^L(h_*) = y_* = X^R(h_*)$ and $X^L(h_{**}) = y_{**} = X^R(h_{**})$.

It will not surprise the reader that there is a 4-manifold of 2-stationary equilibria, parameterized by the choice of the vector $(h_*, y_*, h_{**}, y_{**})$; they comprise piece-wise linear policies, where each policy has five pieces. A typical one is depicted in Figure 6c. The slopes of the line segments of each policy alternate between α and 1, beginning with slope α for the L policy and slope 1 for the R policy. Each of these is, of course, an equilibrium of the type described in theorem 4, where the function *Z* is a piece-wise linear function with two pieces. The theorem characterizing 2-stationary equilibria is again proved by the same method as theorem 1.

It is hard to imagine how the general left-right stationary equilibria of figure 4 might come about, if politicians are boundedly rational. In contrast, 1-stationary equilibria are easy to imagine, where the factions concerned with swing voters in each party concentrate on one voter type h_* – perhaps the mode of the income distribution (that's where the votes are, as Willie Sutton might have said[12]). Even 2-stationary equilibria are imaginable, where the swing-voter factions are concentrating not on all the swing voters, but on two prominent income types. We can easily generalize this concept to *n*-stationary equilibrium, where the swing factions concentrate on not losing the loyalty of *n* voter types: this generates stationary equilibria where the policies are each piece-wise linear with $2n+1$ pieces. Thus, in the Eisenhower administration, when the piece-wise linear income-tax schedule in the United States had 17 pieces, we can imagine that eight income types had sufficient clout to convince both parties that their votes were up for grabs.

4. Income tax rates in the United States

In this section, we ask how well the model performs in light of the recent historical record in the United States. We use the data on income taxation assembled by Piketty and Saez (2006). In their research, Piketty and Saez have used the public micro-file tax data of the IRS, and have computed the sum of four federal taxes for US taxpayers: the income tax, the social security and medicare payroll taxes, the estate tax, and the corporate tax. The corporate tax is allocated to households in proportion to their holdings of corporate equity. The authors then compile the distribution of taxes paid, annually for the years 1960 to 2004, and consequently the distribution of post-tax income.

Post-tax income, so computed, does not correspond exactly to the theoretical concept that we used in sections 2 and 3 of *post-fisc* income. Adding in transfer payments and benefits to the Piketty-Saez data, however, would affect the results only for the bottom two quintiles of the income distribution. (It is said that net taxes paid by US households below median income are essentially zero. Although higher quintiles do receive some transfer payments, they constitute a negligible fraction of their net income.) Allocating the value of public goods to citizens is obviously trickier. With these caveats, we proceed to use the Piketty-Saez data to evaluate our theory.

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[12] For non-American readers, the famous bank robber of the 1930s replied to the query why he robbed banks, "Because that's where the money is."

In recent US fiscal history, the main tax reforms were the following[13]:

- In 1981, the Economic Reform Tax Act was passed under R. Reagan, which reduced the top marginal income tax rate from 70 to 50%, and continued to cut rates over three years;
- In 1986, the Tax Reform Act was signed by Reagan;
- In 1993, the top personal income tax rate was raised under B. Clinton to 39.6%;
- In 1997, the Tax Payer Relief Act cut the top rate on capital gains from 28 to 20%;
- In 2001, under George W. Bush, the Economic Growth and Tax Relief Reconciliation Act reduced the tax rate in the lowest bracket to 10%, reduced the highest marginal rate to 35%, and reduced the marriage penalty. In addition, the estate tax was to be reduced over a ten year period to the vanishing point.

Our model supposes that each of the two political parties proposes a tax policy as part of its electoral strategy. This is, of course, a stylization of reality. In order to confront the data, we will assume that when a major tax reform occurs, the policy that is enacted is the equilibrium policy of the president's party. That policy continues to hold until the next major tax reform. Thus, for example, we will assume that the policy in force prior to 1981 was the Democratic Party's equilibrium policy; that the policy after 1986 was the Republican Party's equilibrium policy; that the policy after 1993 was the Democratic Party's equilibrium policy; and that the policy after 2001 was the Republican Party's equilibrium policy. Our method will be to examine the *de facto* changes in the distribution of post-tax income, as given by the Piketty-Saez data, before and after these major tax reforms, identifying the results that we observe with equilibrium party policies as described.

Figure 7 (various panels) presents the average federal tax rates, from Piketty and Saez (2006), but normalized to control for the variation in total taxes collected as a fraction of national income, a number which changes slowly over time. (All tax reforms promise to be revenue-neutral; if they were, taxes as a fraction of total income would be unchanged.) Thus, a height of '1' of a bar in figure 7 represents a tax rate which is equal to the average fraction of national income collected in federal taxes. Groups whose

[13] See Brownlee (2004).

average tax rate is less (greater) than one pay a smaller (larger) fraction of their income in taxes than the average.

In figure 7a, we present the average tax rates for the various quantile groups reported by Piketty and Saez, before (1981) and after (1988) the Reagan tax reforms. We maintain the conventional US color coding [in the electronic version of this paper]: Democratic policies are blue, Republican policies are red. Piketty and Saez are particularly interested in the tax treatment of the very rich; we see that they disaggregate the top decile of the income distribution into six quantile groups, where the top group refers to the top 0.01% of the income distribution.

The main observations from figure 7a are that the Reagan tax reforms substantially reduced the tax rates on the top five quantile groups, from the 95th centile up, that the tax rate on the bottom quintile was substantially increased, and that tax rates on the groups from the 20th centile to the 95th centile stayed about the same. If we interpret the pre- and post- Reagan reform tax rates as associated with Democratic and Republican equilibrium policies, respectively, these characteristics conform to the model's predictions: in particular, there is a large group of middle income voters who receive the same tax treatment by both parties.

While *de jure* income-tax schedules are indeed piece-wise linear, we cannot assert that the tax data from the Piketty-Saez aggregation of several taxes conform to a piece-wise linear rule.

One characteristic of our equilibrium policies that does not conform to the data is the predicted decrease in the average tax rate proposed by the Right party for the upper end of the distribution (incomes greater than h_2). In addition, the equilibrium policies in our model either tax at the minimal marginal tax rate (zero) or the maximal marginal tax rate ($1-\alpha$); this is not a feature of observed tax rates. We do not observe negative net tax rates at the bottom part of the distribution (as the model predicts): of course, were we to add in transfer payments, as we should to conform to the model, we would observe negative average tax rates for at least the bottom quintile. See the *Note* at the end of this section for more on the inclusion of transfer payments.

Figure 7b presents the normalized tax rates by quantile groups in 1988 and 1996, to attempt to capture the effect of the Clinton tax reform of 1993. The most obvious features of this figure are that the Democrats raised the tax rates on the top groups, lowered them on the bottom groups, while the treatment of the middle groupings, from the 60th to the 99th centiles, did not change much. This conforms to our predictions.

Figure 7c presents the tax rates before and after the Bush tax bill of 2001. The Bush tax bill decreased the rates on the rich, as expected, but it also decreased the rates on the bottom two quintiles. This was probably a consequence of the reduction of the bottom marginal rate to 10%. Casual empiricism would suggest that if we included transfer payments, Bush's cuts might well have the total effect of increasing the average taxes on the lowest quintiles, as the theory predicts. Tax rates from the 60th to the 99.5th centile stayed about the same.

In Figure 7d, we present the tax rates before and after a minor tax bill enacted in 1969 under the Nixon administration. This is the one example that does not fit the theory at all. We interpret the 1968 taxes as that of the Kennedy – Johnson administrations. Nixon evidently raised tax rates on the very rich, but taxes on the rest of the distribution remained about the same[14].

As a final contrast, we present in figure 7e tax rates in 1960 and 2004. These years are too far apart for us to interpret the tax treatments as a pair of equilibrium policies. The figure shows that there has been a massive decrease in the taxation of the top centile of the income distribution, and also a significant decrease in the taxation of the

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[14] We quote *Time Magazine* wrote about this bill, to indicate that it was not a very partisan piece of legislation: "The tax bill worked out by a Senate-House conference in a series of exhausting 16-hour sessions last week provides plenty of tax relief but relatively little in the way of long-term reforms. What started out as an effort to close tax loopholes turned into a tax-cutting binge designed to win friends for Congress in an election year. In the short run, the bill will increase federal revenues. Eventually, however, as tax reductions take effect, federal intake will decline sharply, creating what one Treasury man calls "the revenue crunch of the '70s." The bill represents a Democratic attempt to win the affections of Nixon's middle-class constituency by offering ample benefits to middle-income taxpayers. (*Time,* December 26, 1969)

bottom quintile, much of which is probably due to the earned income tax credit, enacted initially in 1986.

In figure 8, we present data on the percentage of voters who voted for the Democratic presidential candidate, for various election years, by income quantile[15]. In other words, the graphs in figure 8 give us a discrete approximation of the function $\theta(\cdot; X^L, X^R)$ for various years. In 2000, 1996, and 1992, we can say that the swing voter types occupied a region between the 34th and 95th centiles of the income distribution; in 1980, swing voter types were between the 17th and 33rd centiles of the income distribution; and in 1968, it appears that swing voter types lay near the very bottom of the income distribution. It certainly appears from these data that the US is characterized by what we have called a left-right equilibrium: the function θ is monotone decreasing. The general observation seems to be that over this period, the swing voter types have become more numerous, and have moved to up in the income distribution. At least for the recent elections (since 1992), the regions of the income distribution where the two parties coincide in their tax treatment (from Figure 7) seem to correspond roughly to where the swing voter types lie.

Note. As mentioned above, the proper test of the model would use data on *post-fisc* (i.e., post tax and transfer) income, not post-tax income. Because the Piketty-Saez definition of income and taxes does not conform to U.S. Government statistics, as found in the available sources, it would require substantial work to amend their data to include transfer payments. Here, I perform a back-of-the-envelope calculation of average tax rates, by quintile, and as defined in the model of this paper, using US Government data for the year 2004.

For this note, let Y_q be average pre-fisc income of quintile *q*, τ_q be average taxes paid, and T_q be average tranfers received by quintile *q*. Then post-fisc income is $X_q = Y_q + T_q - \tau_q$. The Income Survey Branch of the U.S. Census Bureau (2006) reports the fraction of total 'disposable income' owned by the five quintiles, where disposable income is a concept similar to X_q. Denote these fractions, which sum to one, by

[15] I thank Joseph Bafumi for providing me with these data, which he compiled from the American National Election Studies (ANES).

$\{r(q) \mid q = 1,...,5\}$. The same publication also reports median (but not mean) disposable income.

Let mean disposable income be denoted by $\bar{X}$. The average tax rate of quintile q, in the sense of the present paper, is given by

$$t_q = \frac{\tau_q - T_q}{X_q}.$$

Now we have by definition:

$$\frac{Y_q + T_q - \tau_q}{\bar{Y} + \bar{T} - \bar{\tau}} = 5r_q,$$

from which it follows that:

$$t_q = 1 - r_q \frac{(\bar{Y} + \bar{T} - \bar{\tau})}{Y_q}. \qquad (*)$$

The Current Population Survey (2006) gives both mean and median disposable household income; their ratio is 1.30. (I use their 'definition 7' of income.) It also gives the values of $\{Y_q\}$. I apply this ratio to estimate mean disposable income in 2004 from the Census Bureau table referred to above. Hence the right-hand side of (*) can be computed. I compute:

q	t_q
1	-3.657
2	-0.251
3	0.012
4	0.126
5	0.288

Note that these values look much more like the graphs of the average tax rate in figure 5. The lowest two quintiles are net gainers, the third quintile is a small net contributor, and net taxes are paid mainly by the top two quintiles.

5. <u>Conclusion</u>

I have attempted to model political competition in a general election between two parties, incorporating two features of what appears to be American political reality: that parties compete on a very large policy space, and that their leaders appear to conflict internally over whether to represent their core voters, or to appeal to swing voters.

In the application of the model to competition over tax policy, we chose the policy space to consist of all continuous functions restricted only by a budget constraint, and by a requirement that marginal tax rates lie everywhere in an interval $[0,1-\alpha]$. Choosing $\alpha <1$ was our simple strategy for capturing concerns with labor supply elasticity. In the simplest stationary equilibria of the model, the parties propose piece-wise linear post-fisc distributions of income, with the same treatment for what may be a quite large interval of middle-income voters. The more 'swing-voter' concerns dominate in the parties, the larger will this interval be. But even on the lower envelope of the equilibrium manifold, the policies will coincide for a non-negligible fraction of the income distribution.

We characterized *left-right* stationary equilibria. We remark that the characterization was fairly simple, mathematically speaking, because the proofs exploited heavily the monotonicity of the vote share function θ. There may well be non-left-right stationary equilibria, in which the share function is not monotonic: for instance, one party might win more than half the votes of the very poor and the very rich, while the other wins more than half of the middle income voters. These equilibria are much more difficult to study, and I did not attempt to do so here.

Finally, we tested the model's predictions by examining the income-tax data assembled by Piketty and Saez (2006). Some of the features of our equilibria appear to hold, and some do not. Certainly legislated tax policies in the US are piece-wise linear; however, our model produces equilibrium policies with only two marginal tax rates, zero and $1-\alpha$, and three pieces for each party (although different sets of pieces for the two parties). We described how we can generate equilibria with $2n+1$ linear pieces for any positive interger *n*. We do find that Left policies tax the poor more lightly, and the rich more heavily, than Right policies: in this, reality and the model agree. We also appear to find in the data that there is a significant interval of middle-income voters who receive the same tax treatment from both parties. We take this to be the strongest evidence for the model. Finally, both Democratic and Republican tax policies appear to increase the average tax rates on the whole domain: this does not conform with the model, which predicts that the average tax rate should decrease for the Right policy, for very rich voters.

It is perhaps appealing to view certain aspects of tax policy as being due to *simplicity* or *inertia*: thus, one might conjecture that piece-wise linear policies with a small number of pieces are adopted for reasons of simplicity, and that policies do not change much between Left and Right administrations for a large group of middle-income voters for reasons of both simplicity (costly to change the entire tax code) and inertia. We have shown, however, that these characteristics of policies derive from political competition—they are equilibrium characteristics. We need not appeal to simplicity and inertia.

One could point to many ways in which the model simplifies real politics. One of the most important is that actual tax policy is not proposed by parties in general elections: it is the consequence of legislation, and in particular, of legislative bargaining between the parties, and between the Congress and the executive branch. Modeling the problem of tax policy as a legislative bargaining problem could improve the fit of reality to theory. Nevertheless, if we take Schumpeter's dictum seriously, as stated in the paper's epigram, and also Riker's (1982) dictum, that the most important moment of democracy is the general election, then the investigation reported upon here is a recommended undertaking.

Appendix: Proofs

Proof of Proposition 2:

1. Note that if $X_a \in M_a(h_*)$ and $X_b \in M_b(h_*)$ then

$$y = x_a + \alpha h_1 + h_* - h_1 \text{ and } y = x_b + h_* .$$

2. Write the budget constraint for a policy $X \in M_a(h_*)$:

$$x_a + \alpha\int_0^{h_1} h dF(h) + \alpha h_1(1-F(h_1)) + \int_{h_1}^{h_*}(h-h_1)dF(h) +$$
$$(h_* - h_1)(1-F(h_*)) + \alpha\int_{h_*}^{\infty}(h-h_*)dF(h) = \mu \, .$$

We can rewrite this equation as:

$$x_a = (1-\alpha)\left(\int_0^{h_1} h dF(h) + \int_{h_*}^{\infty}(h-h_*)dF(h) + h_1(1-F(h_1))\right).$$

3. Viewing $M_a(h_*)$ as parameterized by h_1, and differentiating the first expression for y in step 1 w.r.t. h_1, we have:

$$\frac{dy}{dh_1} = \frac{dx_a}{dh_1} - (1-\alpha).$$

Now differentiating the expression derived in step 2 for x_a w.r.t. h_1 gives:

$$\frac{dx_a}{dh_1} = (1-\alpha)(1-F(h_1)).$$

These two equations together tell us that:

$$\frac{dy}{dh_1} = (\alpha-1)F(h_1) < 0.$$

Therefore the smallest (largest) value of y compatible with a policy's being in $M_a(h_*)$ is associated with $h_1 = h_*$ (respectively, $h_1 = 0$). Using the equation for x_a in step 2, we have:

$$x_a(h_*) = (1-\alpha)\mu, \quad x_a(0) = (1-\alpha)\int_{h_*}^{\infty}(h-h_*)dF(h),$$

and so these two values of y are given by:

$$y_a(h_*) = (1-\alpha)\mu + \alpha h_*, \quad y_a(0) = h_* + (1-\alpha)\int_{h_*}^{\infty}(h-h_*)dF(h).$$

4. We perform a similar analysis of policies in $M_b(h_*)$. For any such policy, we may rewrite the budget constraint as:

$$x_b = (1-\alpha)\int_{h_*}^{h_2} h\, dF(h) + (1-\alpha)h_2(1-F(h_2)) - (1-\alpha)h_*(1-F(h_*)).$$

Differentiating this equation w.r.t. the parameter h_2 gives:

$$\frac{dx_b}{dh_2} = (1-\alpha)(1-F(h_2)) > 0\,;$$

now using the expression for y in step 1, we have:

$$\frac{dy}{dh_2} = \frac{dx_b}{dh_2} > 0\,.$$

Therefore, the smallest (largest) value of y compatible with a policy's being in $M_b(h_*)$ is associated with $h_2 = h_*$ (respectively, $h_2 = \infty$). These two values of y are:

$$y_b(h_*) = h_*, \quad y_b(\infty) = (1-\alpha)\int_{h_*}^{\infty}(h-h_*)dF(h) + h_*\,.$$

5. To summarize, the number y is associated with a policy in $M_a(h_*)$ if and only if

$$y_a(h_*) \le y \le y_a(0)\,,$$

and y is associated with a policy in $M_b(h_*)$ if and only if

$$y_b(h_*) \le y \le y_b(\infty)\,.$$

Notice that $y_a(0) = y_b(\infty)$; parts A and B of the proposition follow immediately.

6. We prove part C. We have shown that the smallest value of x_a is $(1-\alpha)\int_{h_*}^{\infty}(h-h_*)dF(h)$ which is positive, as long as F has some support on (h_*, ∞). The argument in step 4 above shows that $x_b > 0$ except in the singular case that $h_* = h_2$. In that case, the policy X_b is the laissez-faire policy $X_b(h) = h$. ■

Proof of Theorem 1:

1. The theorem will be proved if we can show that X^a and X^b solve the programs in conditions $(\beta 2)$ and $(\beta 3)$ of definition 2, respectively. We first address $(\beta 2)$. Let the density function of F be denoted f. The numbers h_1 and h_2 come with the functions X^a and X^b.

2. Define the number ρ, the functions $r(h)$ on $[0, h_1]$, $s(h)$ on $[h_1, h_*]$ and $t(h)$ on $[h_*, \infty)$, and the number λ as follows.

(i) $\rho = \dfrac{\int_0^{h_1} \theta(h)dF(h)}{F(h_1)}$,

(ii) $r(0) = 0$ and $r'(h) = (\theta(h) - \rho) f(h)$ on $[0, h_1]$,

(iii) $s(h_1) = 0$ and $s'(h) = (\rho - \theta(h)) f(h)$ on $[h_1, h_*]$,

(iv) $t(h_*) = \rho(1 - F(h_*)) - \int_{h_*}^{\infty} \theta(h)dF(h)$ and
$t'(h) = (\theta(h) - \rho) f(h)$ on $[h_*, \infty)$

(v) $\lambda = s(h_*) + t(h_*)$.

Note that $\rho > 0$. Note, from Proposition 3, that the function r is first increasing and then decreasing. Compute that $r(h_1) = r(0) + \int_0^{h_1} r'(h)dh = 0$. Therefore *r* is a non-negative function on its domain. Note from Proposition 3 that *s* is an increasing function on its domain: since θ is constant on $[h_1, h_*]$, by Proposition 3, we know that $\rho > \theta(h)$ on this interval. Therefore *s* is a non-negative function on its domain, and $s(h_*) = \rho(F(h_*) - F(h_1)) - \int_{h_1}^{h_*} \theta(h)dF(h) > 0$. Note that *t* is decreasing on its domain, and $t(\infty) = t(h_*) + \int_{h_*}^{\infty} t'(h)dh = 0$. Therefore *t* is non-negative on its domain. Finally, note that $\lambda > 0$.

4. Suppose that X^a were not the solution to the program (β2) of definition 2, and that the true solution is some other policy X. Define the function *g* by the equation $X(h) = X^a(h) + g(h)$. Now define the function $\Delta : \mathbb{R} \to \mathbb{R}$ as follows.

$$\begin{aligned}
\Delta(\varepsilon) = &\int_0^{\infty} (X^a(h) + \varepsilon g(h))\theta(h)dF(h) + \int_0^{h_1} \left(X^{a\prime}(h) + \varepsilon g'(h) - \alpha\right) r(h)dh + \\
&\int_{h_1}^{h_*} \left(1 - (X^{a\prime}(h) + \varepsilon g'(h))\right) s(h)dh + \int_{h_*}^{\infty} \left(X^{a\prime}(h) + \varepsilon g'(h) - \alpha\right) t(h)dh + \\
&\lambda\left(X^a(h_*) + \varepsilon g(h_*) - y\right) + \rho\left(\mu - \int_0^{\infty} (X^a(h) + \varepsilon g(h))dF(h)\right)
\end{aligned}.$$

Note that Δ is a linear function, and that $\Delta(0)=\int_0^\infty X^a(h)\theta(h)dF(h)$: this is the objective of program (β2) evaluated at the policy X^a. Note as well that when $\varepsilon=1$, all the terms in the expression defining Δ are non-negative: this follows from the fact that *r,s,t,* λ and ρ are all non-negative functions or numbers, and that $X\in\Im$. Suppose we can show that $\Delta'(0)=0$: then Δ will be equal to a constant, and consequently $\Delta(0)=\Delta(1)$. But this implies that the value of the objective function of (β2) at X^a is at least as large as its value at *X*: a contradiction. Thus we will have proved that X^a solves program (β2) if we can show that $\Delta'(0)=0$.

5. Compute that

$$\begin{aligned}\Delta'(0)&=\int_0^\infty \theta(h)g(h)dF(h)+\int_0^{h_1} g'(h)r(h)dh-\int_{h_1}^{h_*} g'(h)s(h)dh\\&+\int_{h_*}^\infty g'(h)t(h)dh+\lambda g(h_*)-\rho\int_0^\infty g(h)dF(h)\end{aligned}.$$

Hence, integrating three times by parts, we have:

$$\Delta'(0)=\int_0^\infty \theta(h)g(h)dF(h)+g(h)r(h)\Big|_0^{h_1}-\int_0^{h_1} r'(h)g(h)dh-g(h)s(h)\Big|_{h_1}^{h_*}$$
$$+\int_{h_1}^{h_*} s'(h)g(h)dh+g(h)t(h)\Big|_{h_*}^\infty-\int_{h_*}^\infty t'(h)g(h)dh+\lambda g(h_*)-\rho\int_0^\infty g(h)dF(h).$$

We next re-group terms and write:

$$\Delta'(0)=\int_0^{h_1}\big((\theta(h)-\rho)f(h)-r'(h)\big)g(h)dh+\int_{h_1}^{h_*}\big(s'(h)-(\rho-\theta(h))f(h)\big)g(h)dh+$$
$$\int_{h_*}^\infty\big((\theta(h)-\rho)f(h)-t'(h)\big)g(h)dh+g(h_*)(\lambda-s(h_*)-t(h_*))-g(0)r(0)+g(h_1)(r(h_1)+s(h_1))$$
$$+t(\infty)g(\infty).$$

Now check, by the definitions of *r,s,t* and λ that every term on the r.h.s. of this equation vanishes, which proves that $\Delta'(0)=0$.

6. We proceed to prove that X^b is the solution to program $(\beta 3)$ of definition 2. Suppose that the true solution is *X* and now define the function *g* by $X = X^b + g$. We now define functions *R,S,* and *T,* and numbers γ and δ as follows:

(i) $\delta = \int\limits_{h_2}^{\infty}(1-\theta(h))dF(h)/(1-F(h_2)),$

(ii) $R(0) = 0$ and $R'(h) = (\delta - (1-\theta(h))f(h)$ on $[0, h_*]$,

(iii) $S(h_*) = \int\limits_{h_2}^{h_*}(\delta - (1-\theta(h))dF(h)$ and $S'(h) = (1-\theta(h) - \delta)f(h)$ on (h_*, h_2),

(iv) $T(h_2) = 0$ and $T'(h) = (\delta - (1-\theta(h))f(h)$ on (h_2, ∞),

(v) $\gamma = R(h_*) + S(h_*)$.

Since the function $1-\theta(h)$ is (weakly) increasing (see Proposition 3), it follows from the definition of δ that $R' \geq 0, S' \leq 0$, and that $S'(h_2) = 0$. The functions *R,S,* and *T* are non-negative on their domains. As well, $R(h_*), S(h_*)$ and γ are positive.

7. We now define the function Φ by:

$$\Phi(\varepsilon) = \int\limits_0^{\infty}(1-\theta(h))(X^b(h)+\varepsilon g(h))dF(h) + \int\limits_0^{h_*}\left(1-(X^{b\prime}(h)+\varepsilon g'(h)\right)R(h)dh +$$
$$\int\limits_{h_*}^{h_2}\left(X^{b\prime}(h)+\varepsilon g'(h)-\alpha\right)S(h)dh + \int\limits_{h_2}^{\infty}\left(1-(X^{b\prime}(h)+\varepsilon g(h))\right)T(h)dh + \gamma\left(X^b(h_*)+\varepsilon g(h_*)\right)$$
$$+\delta\left(\mu - \int\limits_0^{\infty}(X^b(h)+\varepsilon g(h))dF(h)\right).$$

All the terms on the r.h.s. of this equation are non-negative, and so, as we argued above, if we can demonstrate that $\Phi'(0) = 0$, then we will have proved that X^b solves the program in condition $(\beta 3)$ of definition 2.

8. Compute that

$$\Phi'(0) = \int\limits_0^{\infty}(1-\theta(h))g(h)dF(h) - g(h)R(h)\Big|_0^{h_*} + \int\limits_0^{h_*}R'(h)g(h)dh + g(h)S(h)\Big|_{h_*}^{h_2}$$
$$-\int\limits_{h_*}^{h_2}S'(h)g(h) - g(h)T(h)\Big|_{h_2}^{\infty} + \int\limits_{h_2}^{\infty}T'(h)g(h)dh + \gamma g(h_*) - \delta\int\limits_0^{\infty}g(h)dF(h).$$

Re-grouping terms, we have:

$$\Phi'(0) = \int_0^{h_*} ((1-\theta(h)-\delta)f(h) + R'(h))g(h)dh + \int_{h_*}^{h_2} ((1-\theta(h)-\delta)f(h) - S'(h))g(h)dh +$$

$$\int_{h_2}^{\infty} ((1-\theta(h)-\delta)f(h) + T'(h))g(h)dh + (\gamma - R(h_*) - S(h_*)) - g(0)R(0) + g(h_2)(S(h_2)+T(h_2))$$

$$-g(\infty)T(\infty).$$

From the definitions of the functions R,S,T and the numbers δ and γ, we observe that all terms on the r.h.s. of this equation vanish, which proves the theorem[16]. ■

Proof of Theorem 2:

1. It is clear that the ideal policy for a type h_* -- the policy in $\Im$ that maximizes its (post-fisc) income -- has some value y at h_*, increases as slowly as possible for $h > h_*$, and decreases from the (h_*, y) as rapidly as possible for $h < h_*$. This is the way to spend as few resources as possible on everyone other than h_*. Thus the ideal policy for h_* is defined by:

$$X^*(h) = \begin{cases} x_0 + h, & h \le h_* \\ x_0 + h_* + \alpha(h - h_*), & h > h_* \end{cases}$$

where x_0 is such that this policy integrates to μ. But this is precisely the policy in $M_a(h_*) \cap M_b(h_*)$ when $y = y_{\max}(h_*)$.

2. If $h_* \le \mu$ and $y = (1-\alpha)\mu + \alpha h_*$ then the policy $X^a \in M_a(h_*)$ is a line of slope α such that $x_a = (1-\alpha)\mu$. We can prove, using the variational technique of the proof of theorem 1, that this is the policy that maximizes $\int \theta(h)X(h)dF(h)$ on $\Im$. Moreover, the fact is intuitively clear. Because θ is a decreasing function and the objective functional is linear in *X*, the objective wishes to push resources as much as possible to the poorest. The solution is to maximize what is given to *h*=0, which means to increase as slowly as

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[16] The convention that "$t(\infty) = 0 = T(\infty)$" is short-hand for a transversality condition. The proof can be rigorously completed by checking that $\lim_{h\to\infty} t(h)g(h) = 0$ and $\lim_{h\to\infty} T(h)g(h) = 0$: these claims are true.

possible (that is, at rate α) on the whole positive line, subject to having given just enough to h=0 so that the policy integrates to μ.

3. If $h_* \geq \mu$ and $y = y_{\min}(h_*) = h_*$ then the policy $X^b \in M_b(h_*)$ is the laissez-faire policy $X(h) = h$. It is also intuitively clear that this is the policy that maximizes $\int (1-\theta(h))X(h)dF(h)$: for now, the objective wishes to push resources to the very rich. Once it is decided how much the very rich get, the strategy must be to decrease as fast as possible (i.e., at rate one) for h smaller. This yields in the limit the laissez-faire policy. Of course, this can also be proved using the variational technique of theorem 1. ■

<u>Proof of Theorem 3:</u>

1. In the political contest at date 1, the L party (the one whose constituency is defined by the function θ_0) solves this program:

$$\max_{X \in \Im} \int \theta_0(h)X(h)dF(h)$$
$$\text{s.t.}$$
$$X(h_*) \geq X^R(h_*)$$

since the unique swing type at date 0 is h_*. There is a similar program for the R party. Thus the political equilibrium at date one is exactly a triple (X^L, X^R, y) such that:

(1) X^L maximizes $\int \theta_0(h)X(h)dF(h)$ subject to $X(h_*) \geq y$;

(2) X^R maximizes $\int (1-\theta_0(h))X(h)dF(h)$ subject to $X(h_*) \geq y$;

(3) $X^L(h_*) = y = X^R(h_*)$.

This looks almost like a 1-stationary equilibrium, except that the function θ_0 is different from the function $\theta(h) \equiv N(S\left(\dfrac{X^L(h)}{X^R(h)}\right))$. Now suppose we choose $y \in [y_{\min}(h_*), y_{\max}(h_*)]$ so that $(h_*, y) \in \Gamma$. Note, by examining the proof of theorem 1, that the only fact invoked about the function θ_* was that it was weakly monotone decreasing. So the argument of theorem will apply just as well if we substitute the decreasing function θ_0 for θ_*. Hence, by the same argument as in theorem 1, the functions X^L and X^R which satisfy conditions (1)-(3) above are an ordered pair in the set $M_a(h_*) \times M_b(h_*)$. This proves the first statement in the theorem.

2. Conversely, let (X^L, X^R) be any equilibrium (stationary or not) at date 1 emanating from θ_0. The policies have a common value $y = X^L(h_*) = X^R(h_*)$ since h_* was the unique swing voter. If $(h_*, y) \in \Gamma$, then again the optimization of each party yields inexorably to the ordered pair of policies $M_a(h_*) \times M_b(h_*)$ associated with the income y at the pivot h_*.

Thus, what must be shown is that $y = X^L(h_*) \in [y_{\min}(h_*), y_{\max}(h_*)]$. Define the following two functions:

$$X^{\max}(h) = \begin{cases} (y - \alpha h_*) + \alpha h, & 0 \le h \le h_* \\ y - h_* + h, & h > h_* \end{cases}$$

$$X^{\min}(h) = \begin{cases} y - h_* + h, & 0 \le h \le h_* \\ y + \alpha(h - h_*), & h > h_* \end{cases}$$

Subject to passing through the point (h_*, y) and satisfying conditions (P0) and (P1) of the definition of policies in $\Im$, $X^{\max}$ is the way of allocating income which consumes the maximal amount of resource and $X^{\min}$ is the way of allocating income which consumes the minimal amount of resource. It therefore follows that

$$\int X^{\max}(h)dF(h) \ge \mu \text{ and } \int X^{\min}(h)dF(h) \le \mu .$$

But this is precisely the condition for (h_*, y) to be in Γ. ■

<u>Proof of Theorem 4:</u>

1. We prove the converse part first. Let (X^L, X^R) be the date-1 equilibrium in a sequence of historical equilibrium beginning with the datum θ_0. Define for $h \in [h_*, h_{**}]$, $Z(h) \equiv X^L(h)$. From the definition of equilibrium, it follows that for $h \in [h_*, h_{**}]$, $X^R(h) = Z(h)$ as well. Suppose that the functions $(\hat{X}^L, \hat{X}^R)$ defined in the theorem's statement can be defined and integrate to μ. We show that X^L and X^R are

precisely the functions $\hat{X}^L$ and $\hat{X}^R$. Suppose, to the contrary, that $X^L \neq \hat{X}^L$. Then define the (non-zero) function g by $X^L(h) = \hat{X}^L(h) + g(h)$.

Let the functions $r(\cdot), s(\cdot), t(\cdot),$ and $u(\cdot)$ be some non-negative functions, defined on the intervals given by the limits of the integrals in which they appear below, and let λ_1, λ_2 and ρ be arbitrary non-negative numbers. Now define the function:

$$\Delta(\varepsilon) = \int_0^\infty \theta_0(h)(\hat{X}^L(h) + \varepsilon g(h))dF(h) + \int_0^{h_1} \varepsilon g'(h)r(h)dh - \int_{h_1}^{h_*} \varepsilon g'(h)s(h)dh$$

$$+\lambda_1 \varepsilon g(h_*) + \int_{h_*}^{h_{**}} \varepsilon g(h)t(h)dh + \lambda_2 \varepsilon g(h_{**}) + \int_{h_{**}}^\infty \varepsilon g'(h)u(h)dh - \rho \int_0^\infty \varepsilon g(h)dF(h).$$

Note that $\Delta(0)$ is the value of the L party's objective at date 1 evaluated at $\hat{X}^L$ and $\Delta(1)$ is the value of the L party's objective at X^L plus a series of terms all of which must be non-negative. (E.g., in the interval $[0, h_1]$, $g' \geq 0$ because $(\hat{X}^L)'(h) \equiv \alpha$ on this interval, and so on for all the other terms.)

Suppose we can choose $r, s, t, u, \lambda_1, \lambda_2,$ and ρ so that $\Delta'(0) = 0$. Since Δ is a linear function, it will follow that it is maximized at $\varepsilon = 0$; in particular, $\Delta(0) \geq \Delta(1)$. This implies, *a fortiori,* that the value of the L party's objective is at least as great at $\hat{X}^L$ as at X^L, which will be the desired contradiction.

2. Calculate, using integration by parts, that:

$$\Delta'(0) = \int_0^\infty \theta_0(h)g(h)dF(h) + g(h)r(h)\Big|_0^{h_1} - \int_0^{h_1} g(h)r'(h)dh - g(h)s(h)\Big|_{h_1}^{h_*} +$$

$$\int_{h_1}^{h_*} g(h)s'(h)dh + \lambda_1 g(h_*) + \int_{h_*}^{h_{**}} g(h)t(h)dh + \lambda_2 g(h_{**}) + g(h)u(h)\Big|_{h_{**}}^\infty - \int_{h_{**}}^\infty g(h)u'(h)dh$$

$$-\rho \int_0^\infty g(h)dF(h);$$

now organize the terms above to express:

$$\Delta'(0)=\int_0^{h_1}[\theta_0(h)f(h)-\rho f(h)-r'(h)]g(h)dh+\int_{h_1}^{h_*}[\theta_0(h)f(h)-\rho f(h)+s'(h)]g(h)dh+$$

$$\int_{h_*}^{h_{**}}[\theta_0(h)f(h)-\rho f(h)+t(h)]g(h)dh+\int_{h_{**}}^{\infty}[\theta_0(h)f(h)-\rho f(h)-u'(h)]g(h)dh+r(0)g(0)+$$

$$g(h_1)(r(h_1)+s(h_1))+g(h_*)(\lambda_1-s(h_*))+g(h_{**})(\lambda_2-u(h_{**}))+g(\infty)u(\infty),$$

where $f(h)\equiv dF(h)$.

Therefore, we can annihilate all these terms if the 'Lagrangian functions and multipliers' are chosen to fulfill the following equations:

(a) $r'(h)=(\theta_0(h)-\rho)f(h)$ on $[0,h_1]$

(b) $s'(h)=(\rho-\theta_0(h))f(h)$ on $[h_1,h_*]$

(c) $t(h)=(\rho-\theta_0(h))f(h)$ on $[h_*,h_{**}]$

(d) $u'(h)=(\theta_0(h)-\rho)f(h)$ on $[h_*,h_{**}]$

(e) $r(0)=0=r(h_1)=s(h_1)$

(f) $\lambda_1=s(h_*)$

(g) $\lambda_2=u(h_{**})$

(h) $u(\infty)=0$.

3. Since *r* must be zero at its endpoints (statement (e)), using (a), we define:

$$\rho=\theta^{av}[0,h_1]\equiv\frac{\int_0^{h_1}\theta_0(h)dF(h)}{F(h_1)}.$$

Since θ_0 is a weakly decreasing function, it follows that *r* is non-negative on $[0,h_1]$. Obviously $\rho>0$. It now follows from statement (b) that $s'\geq 0$ on $[h_1,h_*]$, again invoking the fact that θ_0 is decreasing. Hence, $s(h_*)=\int_0^{h_1}s'(h)dh\geq 0$ (here we use the fact that we choose $s(h_1)=0$). Hence from (f), $\lambda_1\geq 0$ and *s* is a non-negative function on its domain. From (c), $t(h)$ is a non-negative function, again invoking the fact that θ_0

is decreasing. Now from (d), u must be a decreasing function on $[h_{**},\infty)$ and must converge to zero at infinity, so we define:

$$u(h_{**}) = \int_{h_{**}}^{\infty} (\rho - \theta_0(h))dh > 0 .$$

Hence from (g), $\lambda_2 > 0$.

Hence the Lagrangian functions and multipliers have been defined, to be non-negative, and to fulfill the conditions (a) – (h), proving the claim[17].

A similar argument shows that $X^R = \hat{X}^R$.

4. Finally, we remark that indeed the functions $(\hat{X}^L, \hat{X}^R)$ *can* be defined and integrate to μ: this argument is just like the one presented in the proof of theorem 3.

5. We have shown that any date-1 equilibrium with the historical vote-share function θ_0 is of the form $(\hat{X}^L, \hat{X}^R)$. The first statement in the theorem is clearly proved by the same technique. That is, if the functions $(\hat{X}^L, \hat{X}^R)$ can be defined (which is true if the integral of Z on its domain is not to small or too large) then they comprise a stationary equilibrium reached at date 1. To show stationarity, we need only observe that the vote share function $\hat{\theta}(\cdot)$ defined by $(\hat{X}^L, \hat{X}^R)$ is itself monotone decreasing, and the same optimization proof works. ■

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[17] As we remarked in the proof of theorem 1, the statement "$u(\infty) = 0$" is short-hand for the statement $\lim_{h\to\infty} g(h)u(h) = 0$, which is true.

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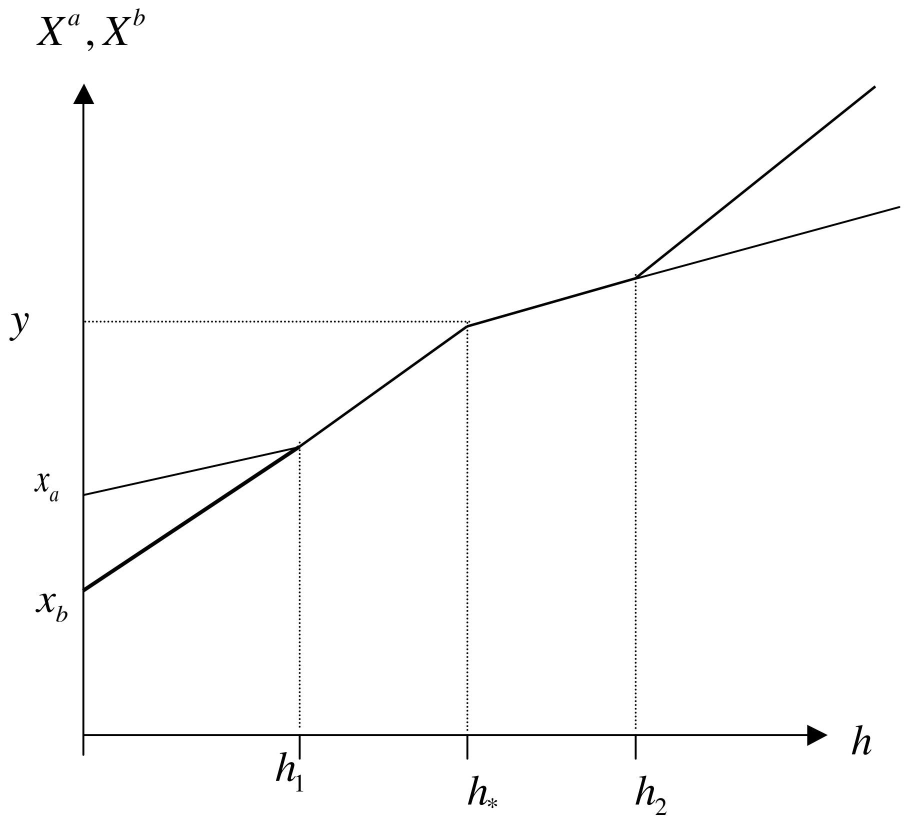
Version 6

Figure 1 Policies $X^a \in M_a(h_*)$ [thin line] and $X^b \in M^b(h_*)$ [bold line] which share a common value y at h_*



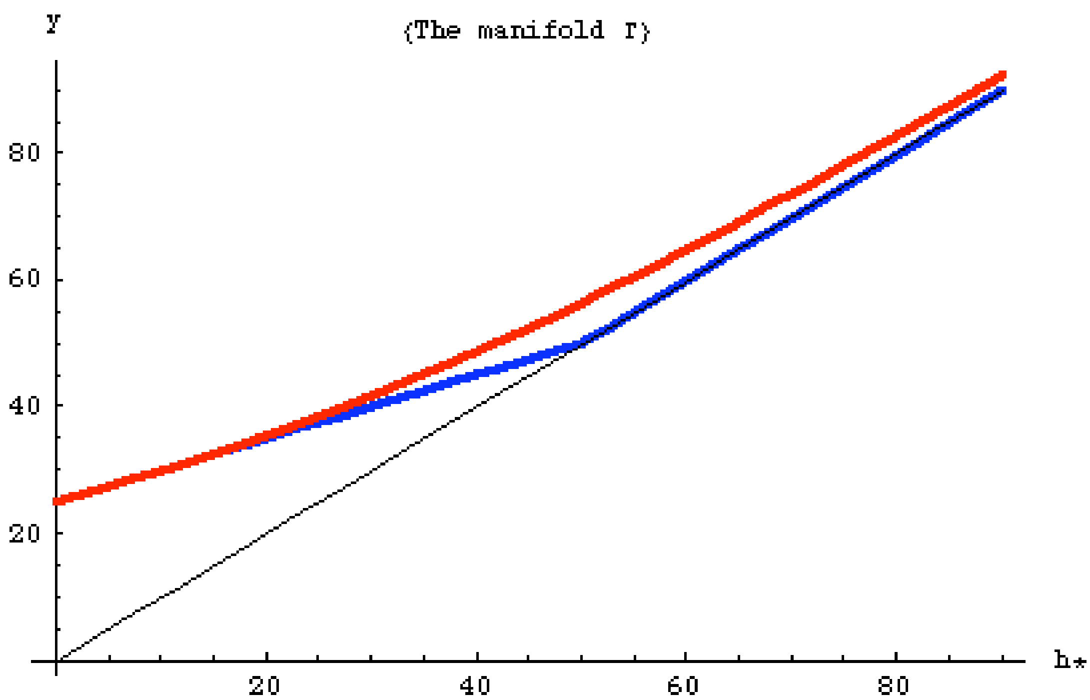


Figure 2 Graphs of the functions $y^{\max}(\cdot), y^{\min}(\cdot)$ and the ray $y = h_*$. The manifold Γ is the set bounded by the two bold curves.

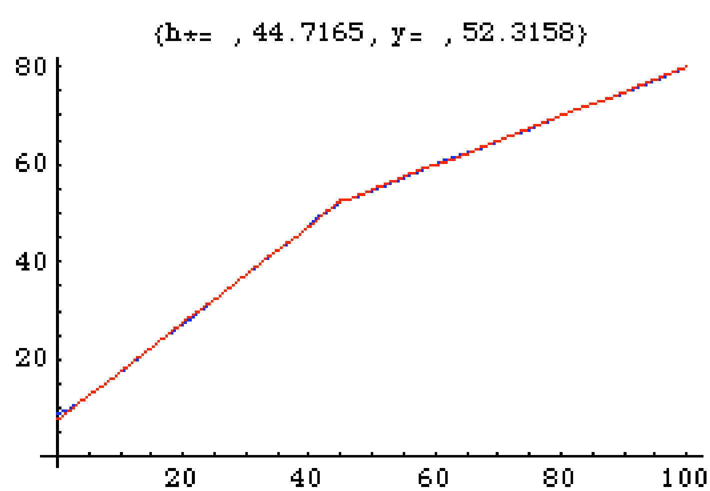


Figure 3a L and R policies on the upper envelope of the manifold Γ

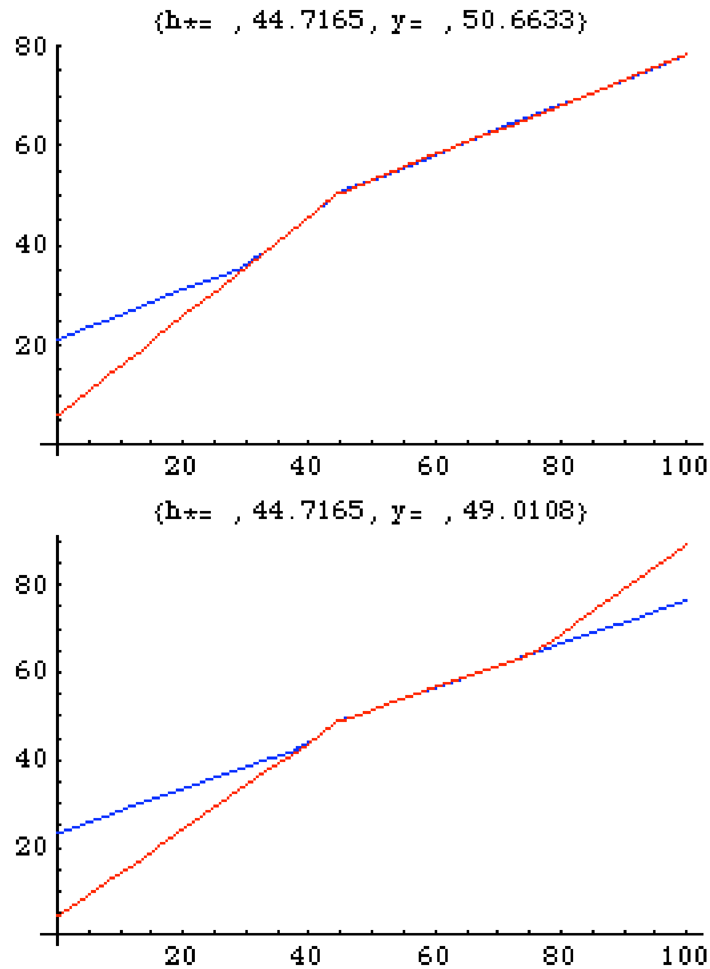


Figure 3b. Graphs of stationary equilibria as y decreases in Γ at constant h_*

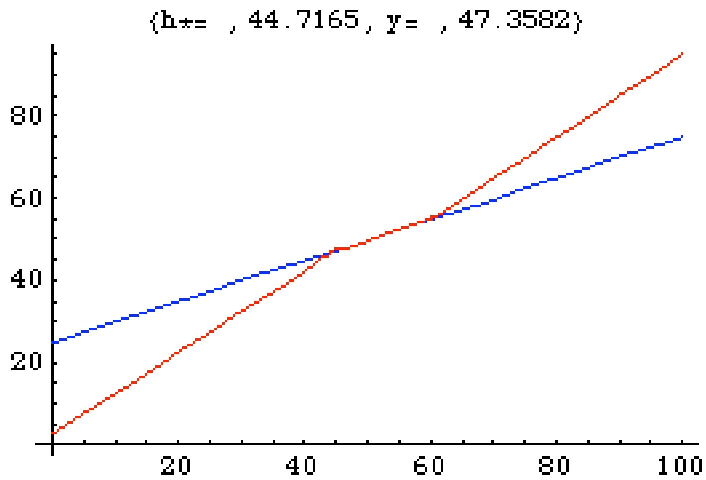


Figure 3c. Stationary equilibrium on the lower envelope of the Γ, where Left plays the ideal policy of its average constituency

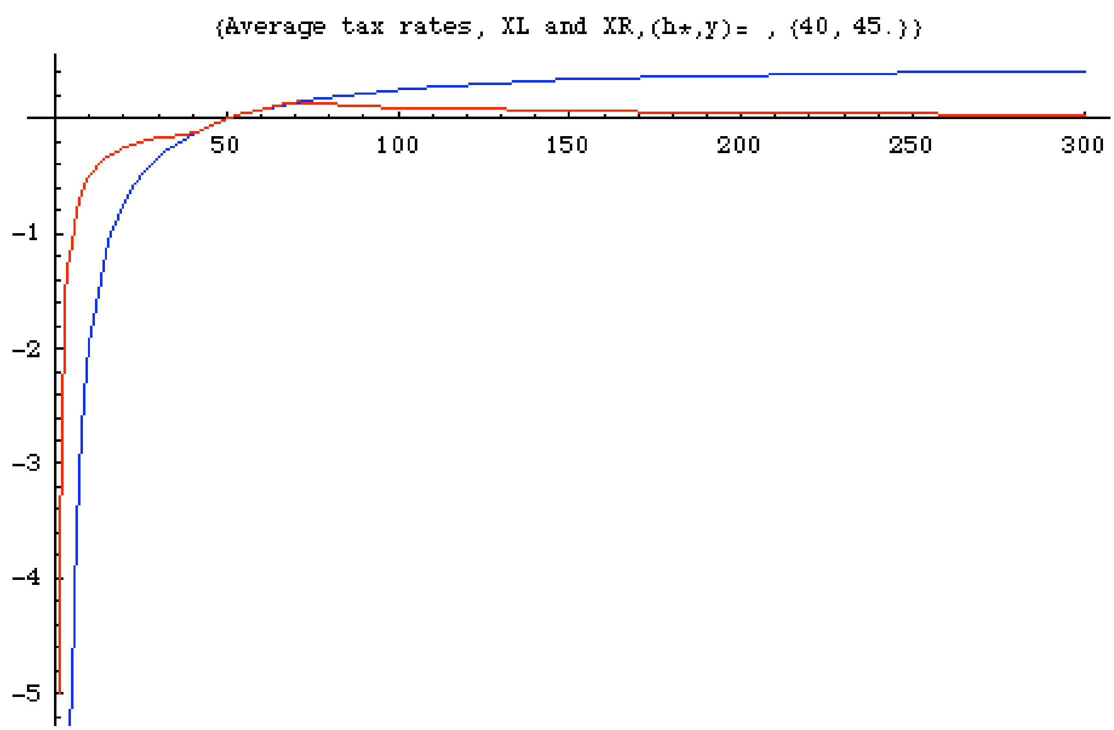


Figure 4 The average tax rate functions for a pair of equilibrium policies

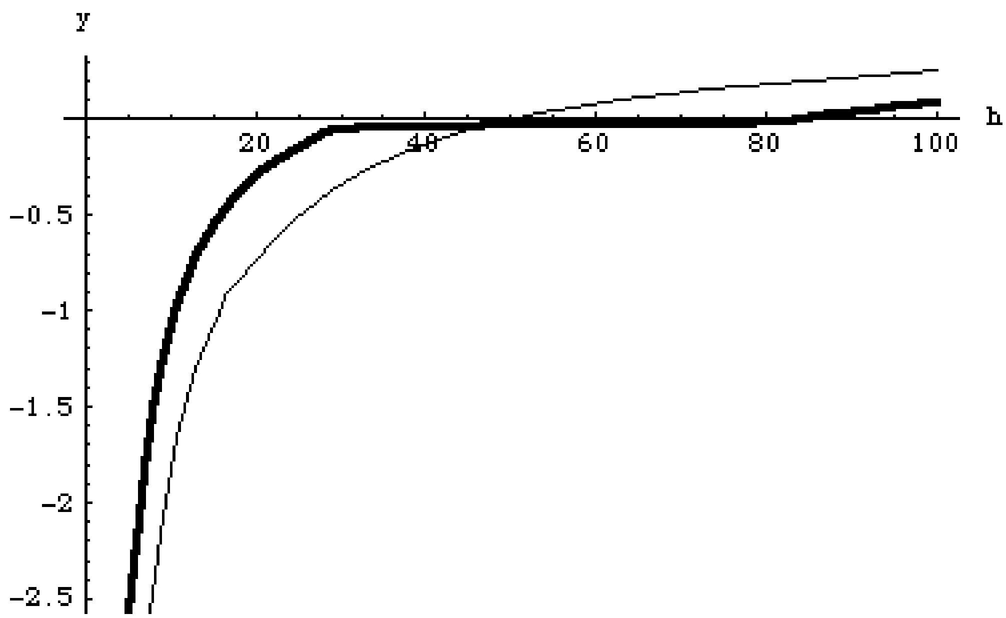


Figure 5a. Average tax rates, Left policy, at $h_* = 20$ (plain) and $h_* = 80$ (bold)

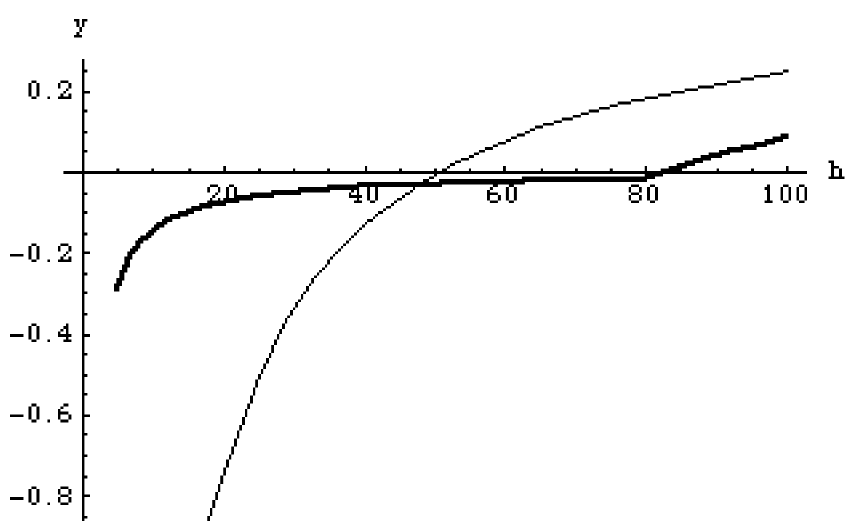


Figure 5b. Average tax rates, Right policy, at $h_* = 20$ (plain) and $h_* = 80$ (bold)

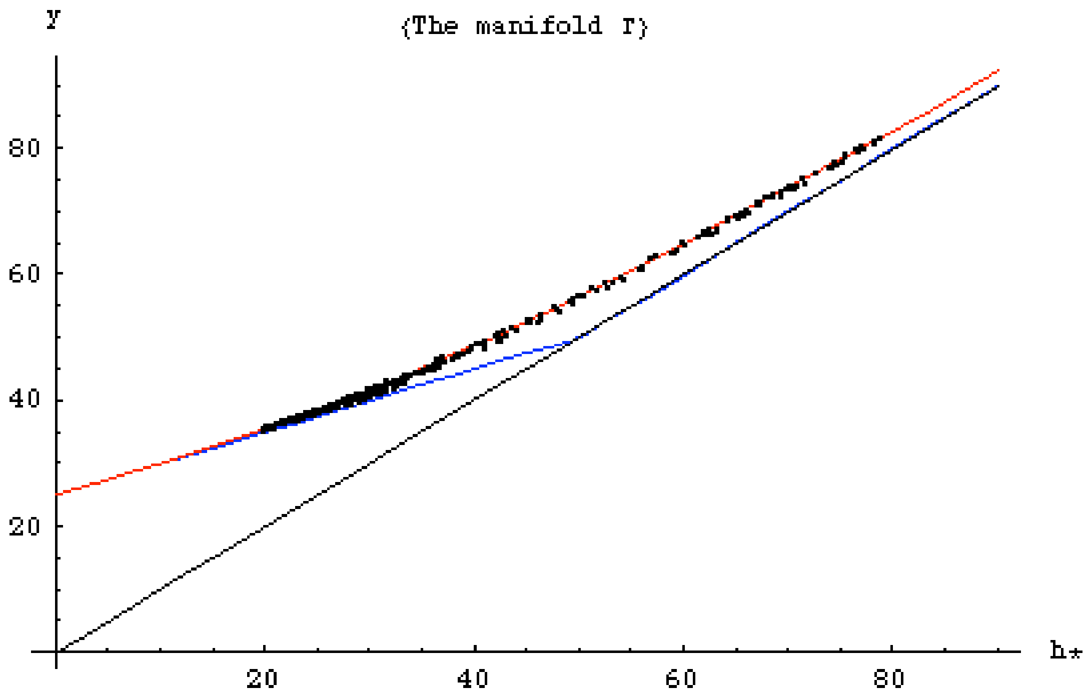


Figure 6a Equilibria where the Left vote share exceeds the Right vote share by less than 0.025, for lognormal distribution with mean 50 and median 40

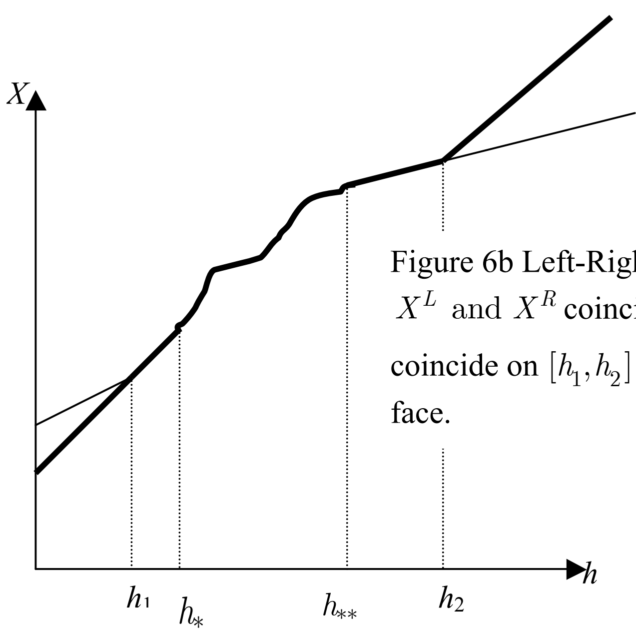


Figure 6b Left-Right stationary equilibrium where X^L and X^R coincide with arbitrary *Z* on $[h_*, h_{**}]$ and coincide on $[h_1, h_2]$. *R* policy in bold, *L* policy in plain face.

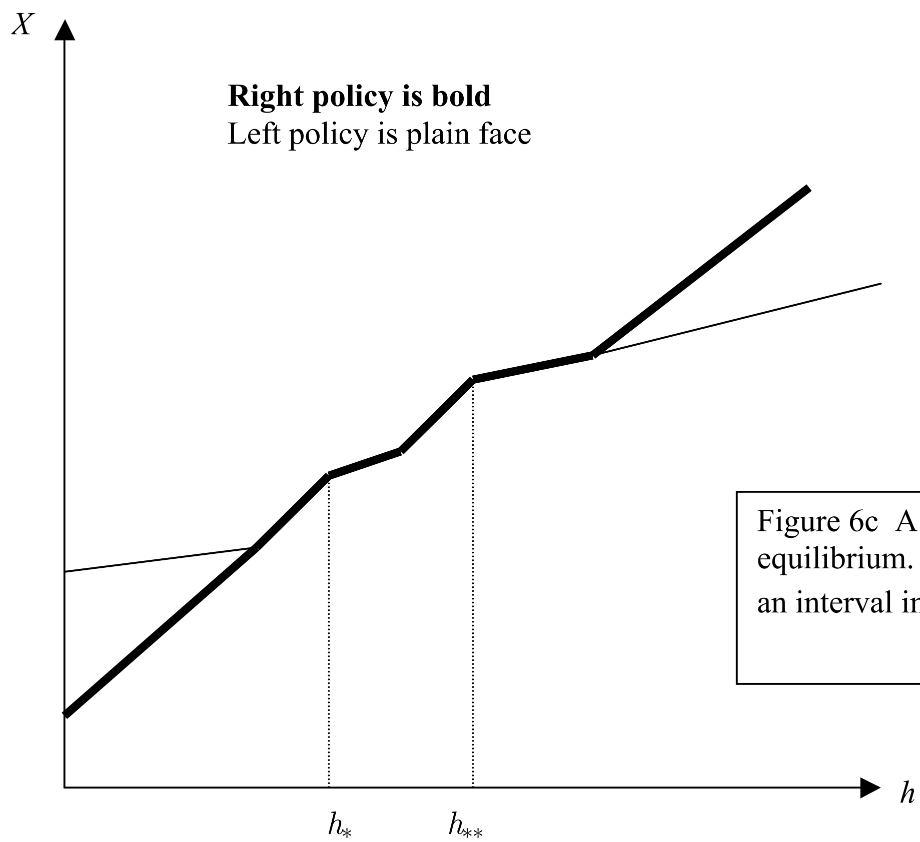


Figure 6c A generic 2-stationary equilibrium. Policies coincide on an interval including $[h_*, h_{**}]$

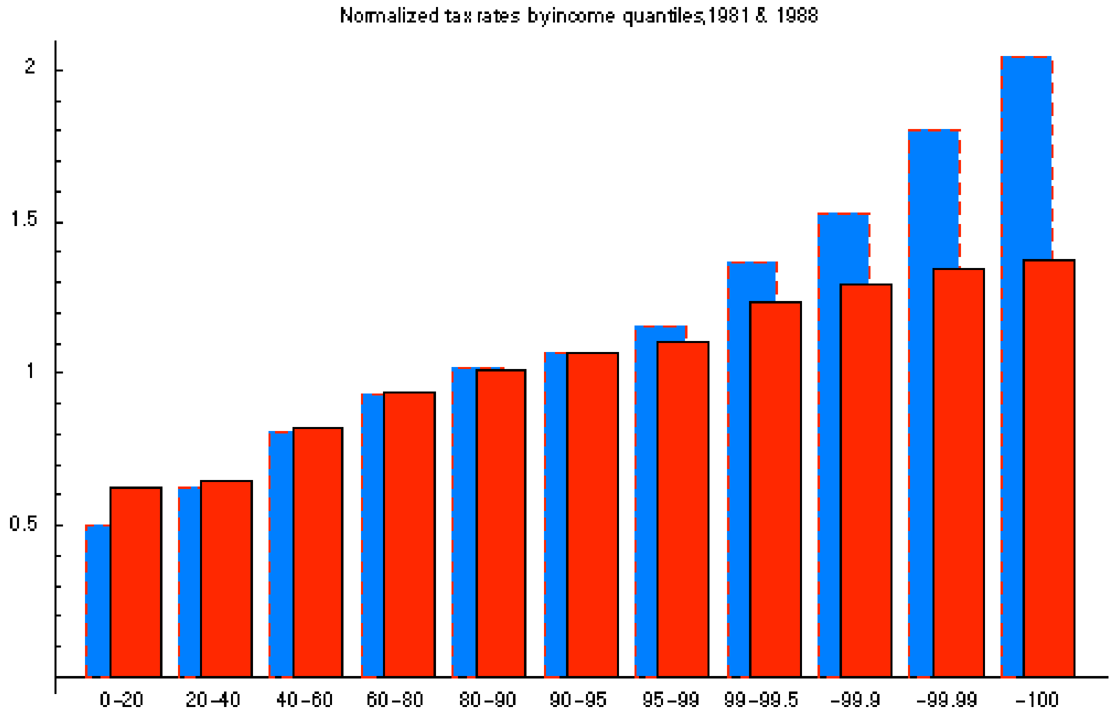


Figure 7a Tax rates, various quantile groups, before and after the Reagan tax reforms: Democrats in blue, Republicans in red

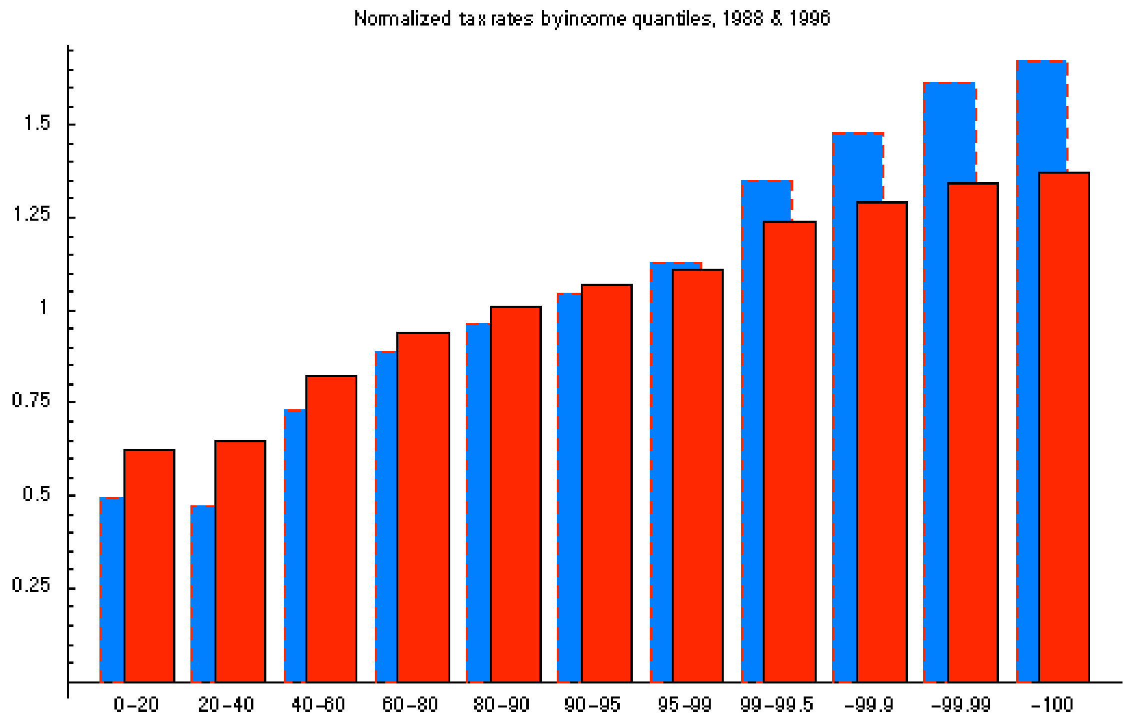


Figure 7b. Tax rates before and after the Clinton tax reform of 1993 (Democrats in blue)

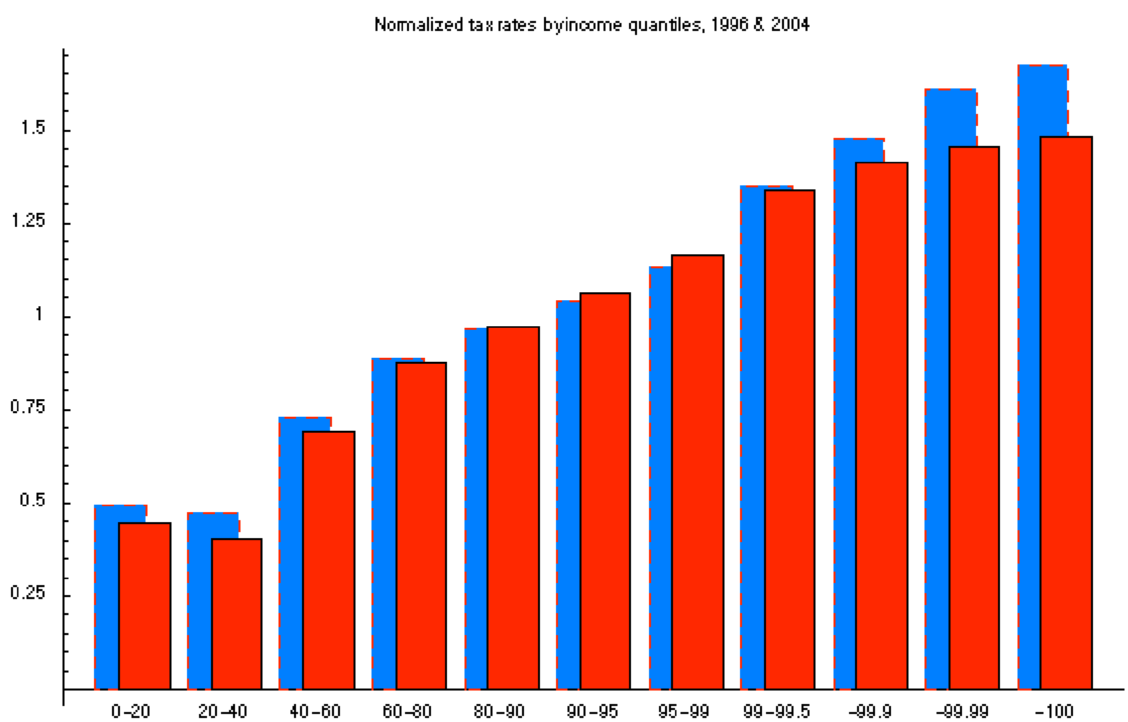


Figure 7c. Tax rates before and after the Bush tax reform of 2001

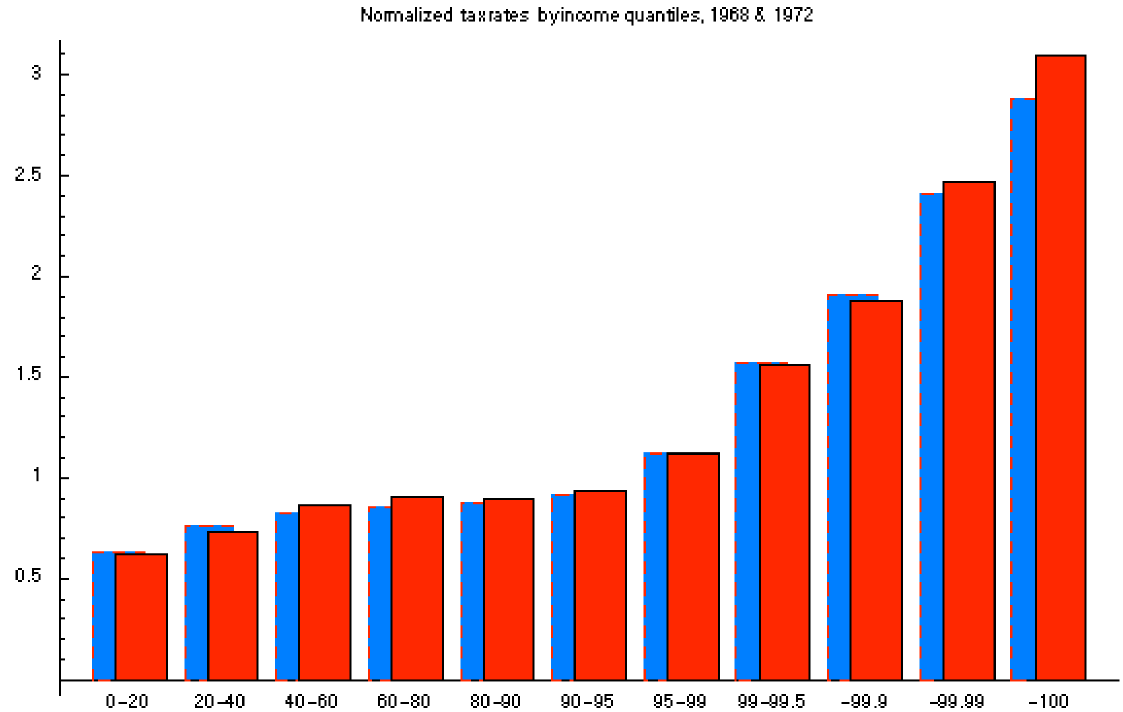


Figure 7d. Tax rates before and after Nixon's 1969 tax bill

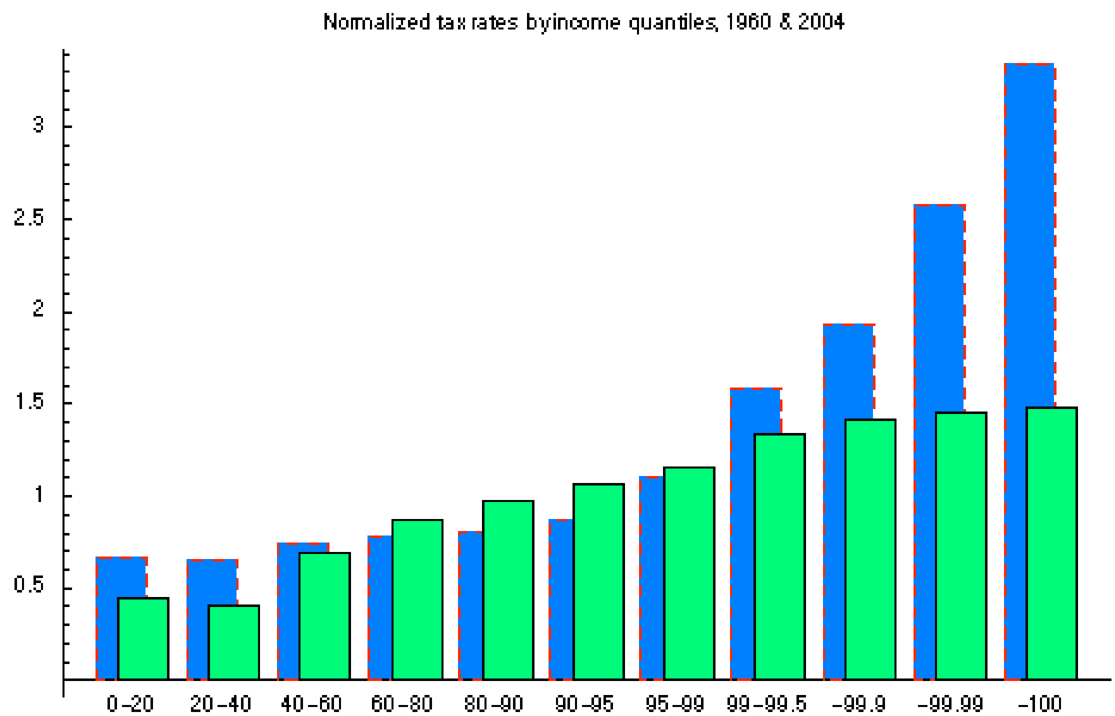
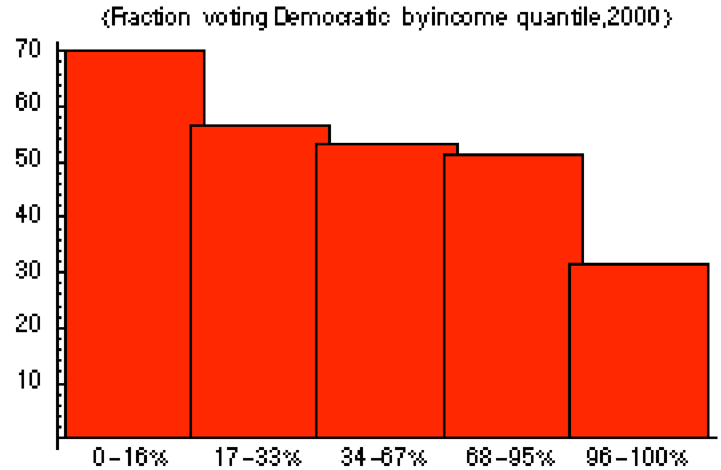
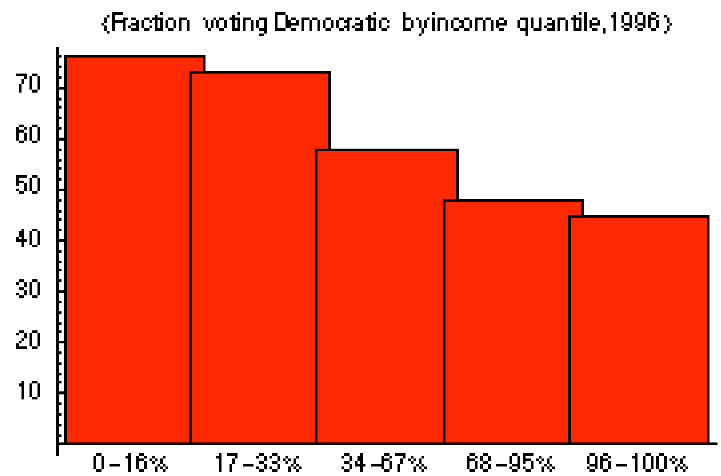
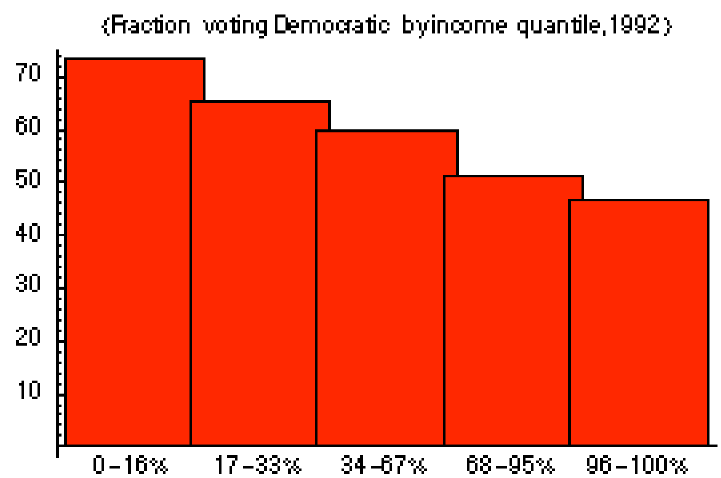


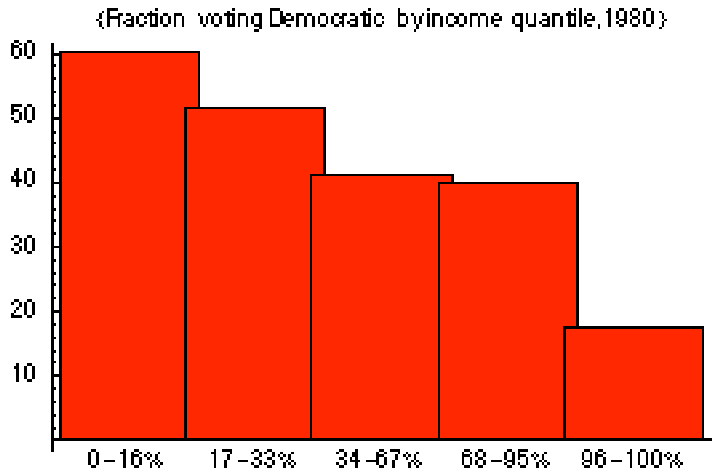
Figure 7e. Tax rates in 1960 and 2004

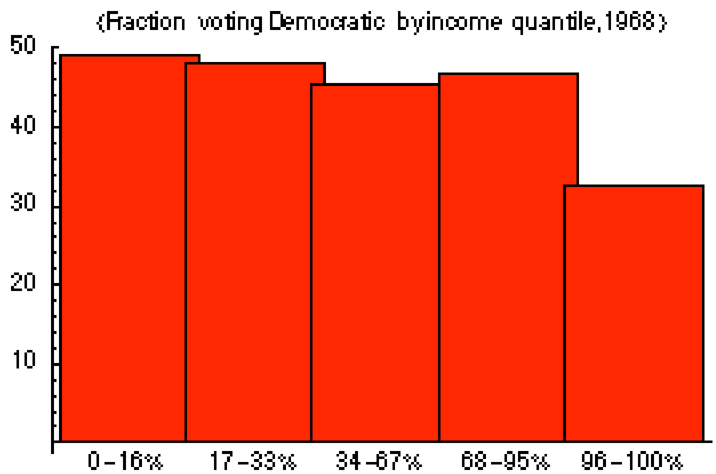
Figure 8 The empirical function θ: Democratic presidential candidate vote share, by voter income quantile, various years








{Fraction voting Democratic by income quantile, 1980}
60
50
40
30
20
10
0-16%
17-33%
34-67%
68-95%
96-100%


{Fraction voting Democratic by income quantile, 1968}
50
40
30
20
10
0-16%
17-33%
34-67%
68-95%
96-100%