Electoral Campaigns as Dynamic Contests*

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Abstract

We develop a model of electoral campaigns as dynamic contests in which two office-motivated candidates allocate their budgets over time to affect their odds of winning. We measure the candidates' evolving odds of winning using a state variable that tends to decay over time, and we refer to it as the candidates' "relative popularity." In our baseline model, the equilibrium ratio of spending by each candidate equals the ratio of their initial budgets; spending is independent of past realizations of relative popularity; and there is a positive relationship between the strength of decay in the popularity process and the rate at which candidates increase their spending over time as election day approaches. We use this relationship to recover estimates of the perceived decay rate in popularity leads in actual U.S. subnational elections.

Key words: campaigns, dynamic allocation problems, contests

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1 Introduction

When looking at patterns of spending on political advertising, a key feature that stands out across electoral campaigns is that candidates tend to increase their spending over time in the run up to election day. This feature is reflected in the upper panel of Figure 1, which shows an increasing pattern in average spending on TV ads by Democrat and Republican candidates in subnational (House, Senate and gubernatorial) American elections in the years 2000-2014. At the same time, the lower panel of the figure shows considerable heterogeneity in the spending paths of individual candidates.

What motivates these spending decisions and the overall patterns that we see in the data? Factors such as unequal budgets, random shocks to inflows of campaign resources, and the candidates' own valuation of winning the campaign against their valuation of money are important drivers of spending decisions. Another factor that plays a critical role in determining spending decisions is the *decay rate* in the effects of campaign spending on the candidates' popularity. Prior work in political advertising has highlighted the positive, but fleeting effects of campaign spending. For example, the results in Gerber et al. (2011) and Hill et al. (2013) suggest that the effects of spending can dissipate in a matter of days, and no more than a couple of weeks.

If the effects of political advertising decay as rapidly as the literature suggests, then a candidate may want to spend more resources closer to the election day than early on. But campaigns are a game between strategic candidates, and it is not obvious how a strategic candidate would optimize her spending plan knowing that her opponent is also spending strategically. For example, if the effects of campaign spending dissipate quickly, and two equally resourced and equally effective candidates save most of their budgets for the final week of the campaign (fully offsetting the effects of each other's spending), then would one of them be better off preempting the other by shifting some resources a little earlier in time, when her opponent is spending less? Presumably, this depends on the impact of campaign spending on the electoral outcome, on the spending decisions by the other candidate, and on how quickly the effect of spending decays. More generally, what is the optimal spending path for each candidate, and how does it compare to actual spending choices?

In this paper, we develop a tractable model to analyze how two strategic candidates allocate campaign resources over time when the effects of campaign spending decay. This model provides a benchmark that can be developed to incorporate other factors that shape spending decisions. The analysis enables us to estimate the decay rate in the effects of campaigning perceived by the candidates, and to assess the extent to which perceived decay rates explain the patterns of spending in actual elections.

Our model captures electoral campaigns as dynamic contests in which two candidates allocate their campaign budgets across time ahead of an election that is held at a fixed future date.[1] In the model, the candidates (called 1 and 2) spend their budgets to influence the evolution of a random variable that we call *relative popularity*. Time runs discretely and in each period each candidate decides how much of her budget to spend to try to raise her relative popularity. The realization of relative popularity in each period measures candidate 1's lead over candidate 2, and thus her odds of eventually winning the election.

Candidates start with one being possibly more popular than the other. In each period, candidate 1's relative popularity may increase or decrease, evolving over time according to an AR(1) process that allows for decay in popularity leads. The candidates' spending decisions in any period affect the drift of this process between the current period and the next. The drift is strictly increasing and strictly concave in candidate 1's spending and strictly decreasing and strictly convex in candidate 2's spending. The candidate that is more popular at time T wins the election.

Our baseline model is a zero-sum game in which the candidates are purely office-motivated and have a fixed budget. This game has a unique equilibrium and the equilibrium path of spending is independent of the realizations of the stochastic process governing the evolution of relative popularity.

If the function that maps the candidates' spending levels to the drift of the popularity process is homogeneous of nonnegative degree, then the equilibrium path features two key properties. The first is an "equal spending ratio" result: the two candidates spend an equal share of their remaining budgets in every period. The second is a "constant spending growth" result: the rate of spending growth is (the same) constant over time.

The homogeneity assumption has an intuitive interpretation. The drift function of the popularity process is like a production function: it takes the candidates' spending levels as inputs and maps them into the next period's (expected) relative popularity output. The homogeneity of the production function implies that when both candidates increase/decrease their inputs by the same proportion, the relative popularity output increases/decreases by

[1] A key premise here is that money spent on advertising influences elections. For recent evidence on this, see Spenkuch and Toniatti (2018) and Martin (2014). On the effect of political advertising and political persuasion more generally, see DellaVigna and Gentzkow (2010), Kalla and Broockman (2018), Jacobson (2015), the references therein, as well as the related literature section below.

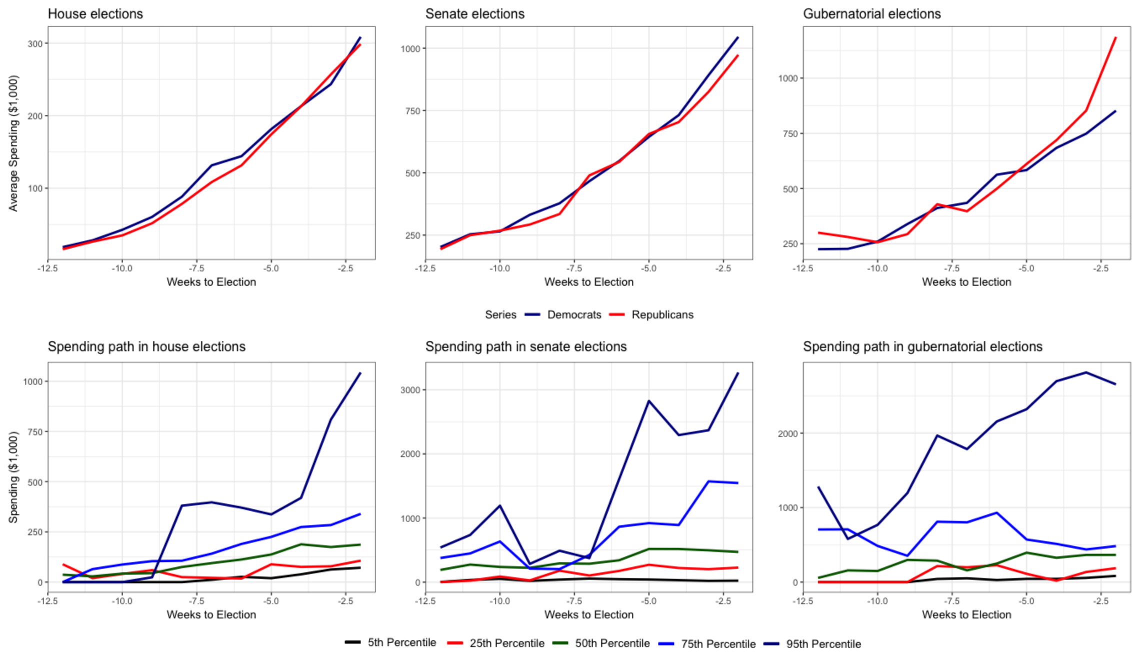

Figure 1: Upper figures are average spending paths by Democrats and Republicans on TV ads in "competitive" House, Senate and gubernatorial races in the period 2000-2014. These are elections in which both candidates spent a positive amount; see Section 4.1 for the source of these data, and more details. Bottom figures are spending paths for 5th, 25th, 50th, 75th, and 95th percentile candidates in terms of total money spent in the corresponding elections of the upper panel.

a fixed proportion.[2] This property then guarantees that the drift of the popularity process depends on the candidates' spending levels only through their ratio.

Under the homogeneity assumption, we fully characterize the equilibrium rate of growth in spending over time. On the equilibrium path, the two candidates increase their spending levels over time when popularity leads tend to decay; the rate of spending growth is increasing in the decay rate; and when there is no decay, they spread their budgets evenly across periods.

The logic behind these results rests on two competing motivations: (i) the incentive to smooth spending over time because the drift of the popularity process is concave and spending only affects this drift, and (ii) the incentive to spend more in later periods because popularity leads decay. When popularity leads do not decay, the solution for each candidate is full smoothing (i.e., even spending across periods) because each period receives equal weight in the candidates' objective functions. When popularity leads decay, later periods

[2]Homogeneity of degree β in these inputs implies that when the candidates increase/decrease their spending by the same factor $\alpha > 0$, the drift of the popularity process is scaled by a factor α^{β}.

receive more weight than earlier ones; thus, the candidates spend more in later periods. Exactly how much more depends both on the decay rate and on the degree of homogeneity of the drift function, which indirectly captures the concavity of the function.

The tractability of our framework enables us to study several variants of the baseline model. First, we allow for the possibility that some voters turn out early, prior to the election date. Early voting has been an increasingly important phenomenon in American elections over the past decade. We characterize the candidates' spending paths under the assumption that voting commences prior to the election date. Early voting gives candidates an incentive to spend more resources in earlier stages. Once early voting starts, the growth rate of spending is no longer constant over time, and spending grows at a rate that is decreasing in the extent of early voting.

Second, we relax the model's zero-sum assumption by having the candidates value money left over at the end of the race. Although election law restricts candidates from personally consuming campaign funds, they may still value money left over. For example, they may want to save money to spend on future elections. To characterize the equilibrium of this variant, we assume that the marginal value of money left over is constant and that the drift of the popularity processes is homogeneous of degree zero in the candidates' spending levels. Homogeneity of degree zero implies that if the amounts spent by the two candidates are scaled by the same factor, the drift of the popularity process does not change. We show that in every period, the ratio of the candidates' spending levels is constant and equal to the inverse ratio of their marginal values for money. However, in this variant, spending levels do vary with relative popularity: if the election is lopsided (in that one candidate develops a large popularity lead over the other), then both candidates spend less.

Third, we look at a variant of our model in which competition is over multiple targetable subpopulations—for example, different media markets within a single district. We assume again that the drift is homogeneous of degree zero in the candidates' spending levels. We show that the equal spending ratio result holds within each subpopulation, and we characterize how resources are allocated not just over time but also across subgroups.

We end the paper by examining patterns of TV ad spending in the actual elections that are aggregated in Figure 1. We first examine the extent to which the predictions of our model are violated in the data. We then fit the model to the data to obtain estimates of the candidates' perceived decay rates. Perceived decay rates are an important quantity of interest in practice because they tell us how candidates view a key factor that drives their spending decisions. They may also be useful as a benchmark for future candidates seeking to optimize their spending. We uncover substantial variation in perceived decay rates across races that is not explained by race characteristics such as incumbency vs. open seat, state-wide races vs. congressional races, and the availability of early voting.

Related Literature— Our paper relates to the prior literature on campaigning. Kawai and Sunada (2015), for example, build on the work of Erikson and Palfrey (1993, 2000) to estimate a model of fund-raising and campaigning. While they assume that candidates allocate resources across different elections, we study the allocation problem across periods in the run-up to a particular election. In de Roos and Sarafidis (2018) candidates that won past races enjoy momentum, which results from a complementarity between prior successes and the current returns to spending.[3] In our setting, on the other hand, prior spending affects the popularity process but popularity leads decay over time.

Meirowitz (2008) studies a static model to show how asymmetries in the cost of effort can explain the incumbency advantage. Polborn and David (2004) and Skaperdas and Grofman (1995) also examine static campaigning models in which candidates choose between positive or negative advertising.[4] In contrast, we study a dynamic allocation model.

In other related work, Iaryczower et al. (2017) estimate a model in which campaign spending weakens electoral accountability, assuming that the cost of spending is exogenous rather than subject to an inter-temporal budget constraint as in our model. Garcia-Jimeno and Yildirim (2017) estimate a dynamic model of campaigning in which candidates decide how to target voters in the presence of strategic media. Gul and Pesendorfer (2012) study a model of campaigning in which candidates provide information to voters over time, and face the strategic timing decision of when to stop. In our setting, by contrast, the date of the election is fixed, and spending affects the drift of the popularity process.

Our work is also related to the literature in marketing and operations research that models advertising as a stochastic control problem.[5] In the seminal work of Nerlove and

[3] Other models of electoral campaigns in which candidates enjoy momentum—such as Callander (2007), Knight and Schiff (2010), Ali and Kartik (2012)—entail sequential voting.

[4] Other static models of campaigning include Prat (2002) and Coate (2004), that investigate how one-shot campaign advertising financed by interest groups affects elections and voter welfare, and Krasa and Polborn (2010), that study a model in which candidates compete on the level of effort that they exert in different policy areas. Prato and Wolton (2018) study the effects of reputation and partisan imbalances on the electoral outcome.

[5] Feichtinger et al. (1994) provide a survey of the literature on stochastic control models in advertising. Several papers in this literature look at advertising for regular consumer goods (in the absence of a product launch), where advertisers use a "pulsing" strategy: short, high-intensity periods of ad spending followed by no spending at all. This pattern of spending is justified through a threshold-based (Dubé et al., 2005) or an S-shaped sales response curve to advertising (Feinberg, 2001, Aravindakshan and Naik, 2015). Using a model in which a stock of goodwill depreciates over time, Bronnenberg et al. (2012) study the long-term effects of marketing and brand images.

Arrow (1962), an agent controls the "stock of goodwill" over time by continuously deciding how much to spend on advertising while goodwill depreciates. More recently, Marinelli (2007) studies a problem similar to ours with a single advertiser facing an exogenous launch date for a product. The stock of goodwill evolves as a Brownian motion that the advertiser controls through spending. In the optimal control strategy the advertiser spends nothing until an intermediate time, and then she spends the maximum amount possible until the launch date. We differ from this literature in that we focus on a strategic setting that involves two players competing to influence the same stochastic process.

The effect of advertising in elections is also studied in the marketing literature (see Gordon et al., 2012, for an early contribution). For example, Gordon and Hartmann (2013) estimate that political advertising impacts the outcome of U.S. presidential elections, but the elasticities of advertising are smaller than for other branded goods.[6] Lovett and Peress (2015) estimate a model of targeted political advertising and find that TV ads target mostly swing voters. Chung and Zhang (2015) estimate the effectiveness of different campaign activities for the two major parties in U.S. presidential elections. Our model contributes to this literature by providing a tractable theoretical framework to study the allocation of advertising resources over time in a two-candidate generic electoral-competition setting.

Kwon and Zhang (2015) study a two-player model of stochastic control and strategic exit motivated by a duopolistic market where market shares are modeled as a diffusion process and the firms can exit at any time. Our approach in which two players simultaneously take actions in pre-determined periods is more tractable and allows us to fully characterize the unique equilibrium spending paths.

Our paper also relates to Kamada and Kandori (2020) who study electoral campaigns through revision games, and to Kamada and Sugaya (2020) who study electoral campaigns as finite-horizon dynamic games in which candidates periodically adjust/clarify their platforms ahead of the election. We differ from this work in that we analyze the dynamic allocation of financial resources ahead of the election.

The focus on the dynamic strategic allocation problem relates our paper to the vast literature on dynamic contests (see Konrad, 2009, and Vojnović, 2016 for reviews). Within this literature, Glazer and Hassin (2000) and Hinnosaar (2018) study contests in which multiple players move sequentially and only once, while we consider a setting in which the same two candidates move repeatedly over multiple periods.

[6] Gordon and Hartmann (2016) also find that electoral colleges skew the allocations of advertising resources toward battleground states and increase overall spending when the election is not tight.

Finally, our model relates to models of strategic races (see Harris and Vickers, 1985, 1987 for seminal contributions).[7] The papers that are most closely related to ours in this literature include Klumpp and Polborn (2006), Konrad and Kovenock (2009) and Klumpp et al. (2019). In particular, Klumpp et al. (2019) study a dynamic contest that is strategically similar to the special case of our baseline model in which there is no decay. They find that in equilibrium resource allocation is constant over time. We show that this finding generalizes in the form of the equal spending ratio result to a variety of settings that fit our application to electoral campaigning.[8]

2 Baseline Model

2.1 Setup

Consider the following complete information *dynamic campaigning game* between two candidates, $i = 1, 2$, ahead of an election. Time is discrete with a finite horizon and indexed by $t = 0, ..., T$. At the start of the game, each candidate is endowed with a budget: $X_0 > 0$ for candidate 1 and $Y_0 > 0$ for candidate 2.[9]

Candidates allocate their budgets across time to influence a state variable that we call *relative popularity*. We identify a *period* with the time $t = 0, 1, ..., T-1$ that candidates make spending decisions, and we use *time* to refer to the dates $t = 0, 1, ..., T$ at which relative popularity is measured. This includes the final date T at which the election takes place. Let x_t be the amount of her remaining budget that candidate 1 spends in period t and y_t be the amount that candidate 2 spends. Candidate 1's remaining budget in period t is $X_t = X_0 - \sum_{t'<t} x_{t'}$ while candidate 2's is $Y_t = Y_0 - \sum_{t'<t} y_{t'}$. In every period t, budget constraints must hold: $x_t \le X_t$ and $y_t \le Y_t$.

Relative popularity at any time t is a random variable $Z_t \in \mathbb{R}$, whose realization we denote z_t. We interpret this random variable as a measure of candidate 1's lead in the polls. If $z_t > 0$, then candidate 1 is ahead of candidate 2; if $z_t < 0$, then candidate 2 is ahead;

[7] Lee and Wilde (1980) and Reinganum (1981, 1982) study races in the presence of uncertainty, but do not cover situations in which one competitor leads or trails against the others.

[8] Our use of the first order approach to characterize the equilibrium behavior also connects our paper to Cornes and Hartley (2005), Kolmar and Rommeswinkel (2013), Choi et al. (2016), Konishi et al. (2019) and Crutzen et al. (2020), who use CES functions in static contests to aggregate individual efforts.

[9] This fixed budget assumption is tantamount to assuming that the candidates can forecast how much money will be available to them, they have access to credit, and they cannot end the race in debt. In actual elections, some large donors make pledges early on and disburse funds over time. In section OA3 of the Online Appendix, we allow candidates' budget to change stochastically over time in response to changes in the popularity process.

and if $z_t = 0$, it is a dead-heat. The campaign starts with relative popularity set to some arbitrary level $z_0 \in \mathbb{R}$.

The winner of the election at time T is the candidate that is more popular. So, if $z_T > 0$, then candidate 1 wins the election; if $z_T < 0$, then candidate 2 wins the election; and if $z_T = 0$, then the election is a tie and we assume that each candidate wins the election with probability 1/2. The winner accrues a payoff of 1 while the loser gets a payoff of 0.

Relative popularity evolves according to the following AR(1) process:

$$Z_{t+1} = p(x_t, y_t) + \delta Z_t + \varepsilon_t \tag{1}$$

Spending levels x_t and y_t thus affect the evolution of popularity through the function $p : \mathbb{R}^2_+ \to \mathbb{R}$. $\delta \in (0, 1]$ is an inverse measure of the decay rate of the popularity process, and $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ is a normally distributed mean 0 random shock with variance σ^2.

We assume throughout that the shocks $\{\varepsilon_t\}$ are iid and that each shock ε_t is realized after the candidates make their period t spending choices. We note that by allowing for $\delta = 1$, we cover the case in which popularity leads do not decay.

Our solution concept is pure strategy subgame perfect equilibrium (SPE), which we refer to succinctly as "equilibrium." In the following section, we introduce an assumption on the popularity process—specifically, on the function p—to establish equilibrium existence and uniqueness, and we show that on-path equilibrium spending levels are independent of the past realizations of relative popularity. In the sections that follow, we strengthen the assumptions on the function p to derive additional properties of the equilibrium path.

2.2 Equilibrium Analysis

Recursive substitution of equation (1) yields the following expression for relative popularity at the time of the election:

$$Z_T = \sum_{t=0}^{T-1} \delta^{T-1-t} p(x_t, y_t) + \delta^T z_0 + \sum_{t=0}^{T-1} \delta^{T-1-t} \varepsilon_t \tag{2}$$

Note that Z_T is the sum of three additively separable terms: the (weighted sum of the) impact of candidates' spending levels, the (discounted) level of the initial popularity, and the (weighted sum of the) normal mean-zero popularity shocks.

In any period t, candidate 1 maximizes $\Pr[Z_T > 0 \mid (z_{t'}, X_{t'}, Y_{t'})_{t' \leq t}]$, while candidate 2 minimizes this. The coefficients of the normal shocks in (2) do not depend on the candidates' choices. The variance of Z_T, and in fact the whole shape of its distribution, are thus independent of the candidates' strategies. Therefore, we can assume that candidate 1 maximizes the expected value of Z_T, while candidate 2 minimizes it. Because the game is zero-sum, given candidate 2's on-path spending levels $(y_0, ..., y_{T-1})$, the on-path equilibrium spending levels $(x_0, ..., x_{T-1})$ of candidate 1 solve the following maximization problem:

$$\max_{x_0,...,x_{T-1}} \sum_{t=0}^{T-1} \delta^{T-1-t} p(x_t, y_t)$$
$$\text{s.t. } x_t \geq 0, \ \forall t = 0, ..., T-1, \text{ and } \sum_{t=0}^{T-1} x_t = X_0 \tag{3}$$

Given candidate 1's on-path spending levels $(x_0, ..., x_{T-1})$, candidate 2's on-path equilibrium spending levels minimize $\sum_{t=0}^{T-1} \delta^{T-1-t} p(x_t, y_t)$ subject to the corresponding constraints. The following assumption ensures equilibrium existence and uniqueness.

Assumption 1. *The function p is twice continously differentiable, and*

(a) $p(\cdot, y)$ is strictly increasing for all y, and $p(x, \cdot)$ is strictly decreasing for all x;

(b) $p(\cdot, y)$ is strictly concave for all y, and $p(x, \cdot)$ is strictly convex for all x;

(c) p satisfies the Inada-0 conditions:

$$\lim_{x \to 0} \frac{\partial p(x, y)}{\partial x} = \infty \text{ for all } y \quad \text{and} \quad \lim_{y \to 0} \frac{\partial p(x, y)}{\partial y} = -\infty \text{ for all } x.$$

Assumption 1(a) states that each candidate's spending has a positive effect on her popularity. Assumption 1(b) implies that each candidate has a unique spending level that maximizes her relative popularity given the spending level of the other candidate. Finally, Assumption 1(c) says that the marginal benefit of spending is very large when a candidate is spending close to zero.

Assumption 1 guarantees that problem (3) for candidate 1 and the corresponding problem for candidate 2 are both concave. The candidates' equilibirum on-path spending levels can thus be found by solving the system of first order conditions to these problems. Our first proposition records this observation and the fact that the equilibrium spending path is independent of past realizations of relative popularity.

Proposition 1. *Suppose Assumption 1 holds. Then,*

(i) the dynamic campaigning game has a unique equilibrium, and the on-path spending levels satisfy the first order conditions of the optimization problem (3)*, and*

(ii) for all periods t, the equilibrium on-path spending levels (x_t, y_t) are independent of the past history of relative popularity $(z_{t'})_{t' \leq t}$.

The intuition behind part (ii) is as follows. In equilibrium, the candidates allocate their budgets based on the marginal rate of substitution between spending in different periods. When a popularity shock occurs at time t, the probability a candidate wins changes, but the marginal benefit of spending in all periods after time t also changes by the same amount. The marginal rate of substitution between spending in different periods is then independent of the shock. This holds because the popularity process in (1) is additively separable.

We note that the additive separability between the candidates' spending decisions and the random shocks in (1) does not rule out strategic effects: the candidates still compete against each other to affect the drift of the popularity process and their best responses are not necessarily constant in the opponent's choices; see the end of Section 2.3 for a more thorough discussion of this point. If the process is not additively separable, a candidate's optimal spending decision at time t could in principle depend not only on the opponent's behavior, but also on the history of popularity shocks up to time t, rendering the model less tractable. However, under some additional assumptions, we can show that the ratio of the candidates' equilibrium spending levels will still be independent of the popularity shocks, and this ratio is enough to study the equilibrium evolution of the electoral competition. This happens when there is a strictly concave, strictly increasing function q such that $p(x, y) \equiv q(x/y)$. When this is the case, even though popularity shocks modify the marginal benefit of spending, these effects cancel out and we can still characterize the equilibrium evolution of the popularity process (see Section OA1 in the Online Appendix for details).

The results of Proposition 1—particularly, the history-independence property reported in part (ii)—have further notable consequences for the robustness of the equilibrium path to changes in the structure of the game. Although the dynamic campaigning game has complete information, the equilibrium path of the game is robust to candidates having incomplete information about the popularity process or to the candidates moving sequentially in each period, as the following remark clarifies.

Remark 1. *The equilibrium of the game has the same path of play as*

(i) any equilibrium of the alternative version of the game where candidates imperfectly (and possibly asymmetrically) observe the realization of the path of relative popularity, and

(ii) every Nash equilibrium of a game where candidates move sequentially within a period with arbitrary (and possibly stochastic) order of moves.

These observations follow from equation (2) and known results in the literature on zero-sum games. In particular, because on-path spending levels do not depend on past realizations of popularity, the candidates' equilibrium spending paths would be the same even if popularity was not fully observable. Furthermore, because the game is zero-sum, the equilibrium path of play is unique and robust to allowing candidates to move sequentially within a period, with arbitrary order of moves (see, for example, Mertens et al., 2015).

2.3 Equilibrium Spending Ratios

To say more about the equilibrium spending paths and the candidates' equilibrium probabilities of winning, we need to impose additional assumptions on how spending levels affect the popularity process. Under the following assumption, we can fully specify the equilibrium evolution of the popularity process.

Assumption 2. *The function p is homogeneous of degree $\beta \geq 0$.*

The function $p(x, y) = \alpha_1 x^\beta - \alpha_2 y^\beta$ satisfies this assumption and further satisfies Assumption 1 when $\beta \in (0, 1)$ and $\alpha_1, \alpha_2 > 0$. Another example that satisfies the assumption is the function $p(x, y) = \alpha (\log x - \log y)$, which is homogeneous of degree 0 and satisfies Assumption 1 when $\alpha > 0$.[10]

We define the *spending ratio* of a candidate in period t to be the ratio between her spending level in period t and the remaining budget available to her that period: in period t, candidate 1's spending ratio is x_t/X_t and candidate 2's is y_t/Y_t. We refer to the ratio of spending in period $t+1$ to spending in period t for a candidate as the *consecutive period spending ratio*, and we use $r_{1,t} := x_{t+1}/x_t$ and $r_{2,t} := y_{t+1}/y_t$ from here on to denote them.

Assumption 2 implies two key results that inform our analysis of spending patterns in actual elections. The first is an equal spending ratio result. It says that the candidates' spending ratios equal each other on the path of play. The second is a constant spending growth result. It says that the candidates' consecutive period spending ratios equal the same constant in all periods.

Proposition 2. *Suppose Assumptions 1 and 2 hold. Then, in the unique equilibrium path of the dynamic campaigning game,*

[10] For this example, the model is not closed since p is not defined when either $x = 0$ or $y = 0$. However, we can close the model by assuming that: (i) if any candidate i spends 0 at any time t, then the game ends immediately with candidate $j \neq i$ winning so long as j spends a positive amount, and (ii) if both candidates simultaneously spend 0, then the game ends with each candidate winning with probability 1/2. The results of Proposition 1 and 2 continue to hold under this amendment.

(i) the candidates' spending ratios equal each other every period: $x_t/X_t = y_t/Y_t$ for all periods t.

(ii) the candidates' consecutive period spending ratios equal each other and are constant through time; in particular, $r_{1,t} = r_{2,t} = \delta^{1/(\beta-1)}$ for all periods $t < T-1$.

The results in Proposition 2 are based on the following reasoning. To maximize the probability of winning the election, both candidates equalize the (decay-weighted) marginal benefit of spending at any period $t < T-1$ with the (decay-weighted) marginal benefit of spending in period $T-1$, just ahead of the election. The homogeneity of function p (Assumption 2) implies that the ratio of candidates' first order conditions depends only on the ratio of their spending levels at time t (x_t/y_t) and at time $T-1$ (x_{T-1}/y_{T-1}). The equal spending ratio result then follows iteratively from the strict concavity of p and from the budget balance condition, which implies

$$\frac{x_{t-1}}{y_{t-1}} = \frac{X_{t-1}}{Y_{t-1}} = \frac{X_t - \sum_{t \le t' < T-1} x_{t'}}{Y_t - \sum_{t \le t' < T-1} y_{t'}}.$$

The equal spending ratio result implies some additional equilibrium properties. For example, it implies that the candidates' on-path consecutive period spending ratios are equal to each other, i.e. $r_{1,t} = r_{2,t}$ for all periods t. In addition, because budgets are fixed, the equal spending ratio result implies that the ratio x_t/y_t of the candidates' spending levels in any period t (which we refer to as the *cross-candidate spending ratio*) is a constant that is equal to the ratio of the starting budgets; that is, $x_t/y_t = X_0/Y_0$ for all periods t. The two properties described above yield part (ii) of Proposition 2.

Given that δ does not exceed 1, Proposition 2 implies that candidates' spending grows over time if both $\beta, \delta < 1$.[11] The expression in the proposition verifies that if $\delta = 1$ (i.e., if popularity leads do not decay), then the candidates spread their budgets evenly across periods. Since p is concave, the candidates want to smooth their spending over time. The lack of decay further implies that this smoothing is full: candidates allocate the same share of their initial budgets to each period. On the other hand, if $\delta < 1$, then spending increases over time, and the fraction of the initial budget each candidate spends at time t is

$$\gamma_t = \frac{x_t}{X_0} = \frac{y_t}{Y_0} = \frac{r-1}{r^T - 1} r^t, \tag{4}$$

[11] Although we assume $\delta \le 1$, none of the above results relies on this assumption. If $\delta > 1$, popularity leads tend to amplify over time; and, on the equilibrium path, the candidates would decrease their spending over time if $\beta < 1$.

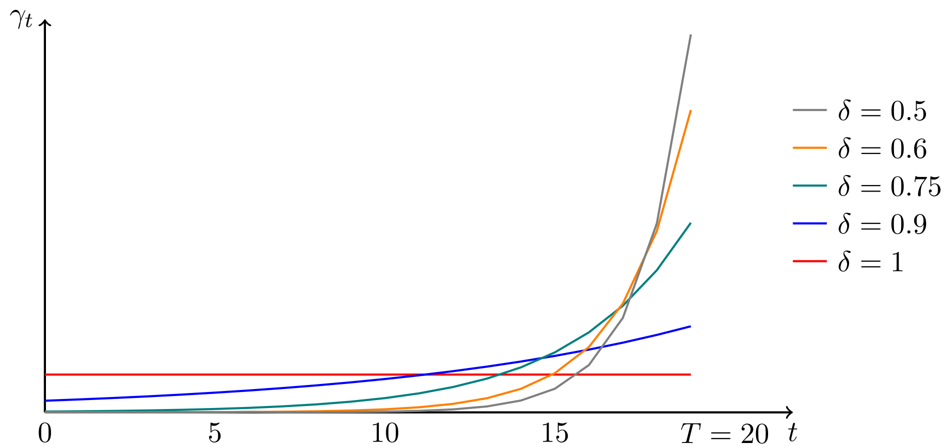

Figure 2: Equilibrium share of the initial budget $\gamma_t = x_t/X_0$ that the candidates spend over time when $T = 20$, $\beta = 0$ and δ takes different values.

where $r = \delta^{1/(\beta-1)}$ is the common consecutive period spending ratio. In this case, the decay of popularity leads generates a force that pushes for greater spending in later periods.

The comparative statics of γ_t reflect these countervailing forces. If β increases, the marginal return to spending decreases at a slower rate within each period. Candidates thus spend even more towards the end of the campaign and less in the early stages. As $\beta \to 1^-$ candidates spend all of their resources in the final period. As δ decreases, popularity leads decay more and candidates have an incentive to invest less in the early stages of the race and more in the later stages. Figure 2 depicts this last feature, plotting γ_t for $\beta = 0$ and different values of δ.

Strategic Spending Considerations. A candidate's optimal spending behavior varies with the spending behavior of the other candidate only if the effects of the spending levels on the drift of the popularity process (i.e., function p) are not additively separable. Suppose $p(x, y) = (x - y)/2(x + y)$, which is not additively separable.[12] Given any behavior by candidate 2, the first order condition for candidate 1 implies that the marginal benefit to spending in period $t < T - 1$ equals the marginal benefit to spending in the final period $T - 1$:

$$\delta^{T-1-t} \frac{y_t}{(x_t + y_t)^2} = \frac{y_{T-1}}{(x_{T-1} + y_{T-1})^2}.$$

[12]To close the model when both candidates spend 0, see footnote 10. In addition, although this function does not satisfy Assumption 1(c), the results of Propositions 1 and 2 hold with $\beta = 0$; in particular, the first order conditions are satisfied at an interior equilibrium, and since the function is homogeneous of degree 0 the common consecutive period spending ratio is $r = 1/\delta$.

Both sides of this equation feature expressions of the form $y/(x+y)^2$, whose partial derivative in y is $(x-y)/(x+y)^3$ and in x is $-2y/(x+y)^3$. With this in mind, suppose that candidate 2 marginally lowers his spending in period t and, to keep his budget balanced, increases his spending in a later period, say $T-1$. The previous observation implies that candidate 1's best response at time t would be to either increase her spending (this happens if $x_t \leq y_t$), or to lower it as well but by a factor smaller than candidate 2's (this happens if $x_t > y_t$).[13] (From this we see that a candidate's optimal response to a change in the opponent's spending depends not just on the direction of the change, but also on its magnitude.) In both cases, the cross-candidate spending ratio x_t/y_t increases. Analogously, if candidate 2 raises her spending in any period t relative to the equilibrium level, and candidate 1 best responds, then the cross-candidate spending ratio x_t/y_t decreases.

Suppose that candidate 2 naively spends almost all of her budget in the final period. The observations above imply that a strategic candidate 1 would best respond by spending a positive amount in all periods and increasing her spending over time at a rate that is faster than the equilibrium rate, i.e. the rate stated in Proposition 2 for $\beta = 0$. On the other hand, if candidate 2 naively allocates his budget evenly across all periods, a strategic candidate 1 would best respond by increasing her spending over time at a slower rate than the equilibrium rate.

2.4 Discussion

Our baseline model provides a useful benchmark to understand how strategic candidates compete against each other in a dynamic setting. To highlight the key forces behind this dynamic contest, it abstracts away from several factors that shape spending in actual elections. Yet, our theoretical framework is flexible enough to accommodate several of these factors. For instance, advantages (or disadvantages) due to incumbency, to prior legislative records, or to a candidate's name recognition can affect the initial lead in relative popularity, z_0, or starting budgets, X_0 and Y_0.

Candidates can also differ in the effectiveness of their campaign spending. These differences may depend on differences in how their campaigns are organized, or on the fact that one candidate is simply better than the other at campaigning. A candidate's policy

[13]To see why, note that if candidate 2 lowers his spending in period t from y_t to αy_t with $\alpha < 1$ and candidate 1 also lowers her spending from x_t to αx_t (or to an even lower amount) then $y_t/(x_t+y_t)^2$ drops to $y_t/[\alpha(x_t+y_t)^2]$ (or even lower) and the FOC is violated. For the FOC to hold, candidate 1's spending level at t must be larger than αx_t.

platform may also be more popular than the one of the other candidate. We can capture these features through asymmetries in the partial first derivatives of p.

Although the payoffs we have assumed imply a winner-take-all electoral rule, our equilibrium analysis immediately extends to the case where the candidates' payoffs are linear (or piecewise linear) in relative popularity on election day, z_T. Therefore, it covers the case in which the margin of victory also matters to the candidates.

Equation (1) assumes that relative popularity evolves according to an AR(1) process. In the Online Appendix, we examine non-separable popularity processes, imposing additional assumptions to guarantee that the first-order approach is still sufficient to characterize the equilibrium evolution of relative popularity.

Our baseline model assumes that candidates have fixed budgets, or equivalently that they can forecast exactly how much money they will have by the end of the campaign, and they are not allowed to finish in debt. In the Online Appendix, we consider a variant of the model in which budgets are uncertain and evolve over time in response to fluctuations in relative popularity. We show for a specification of that model that the equal spending ratio result continues to hold but the constant spending growth result does not. Because spending decisions depend on the candidates' expectations of how their budgets evolve and because these expectations vary with fluctuations in relative popularity, equilibrium spending also evolves stochastically.

Finally, in the Online Appendix we present a model in which we allow voters to react to campaign spending differently, following the approach of the marketing literature. Our model of the electorate gives rise to a popularity process for the candidates that is equivalent to equation (1). We then demonstrate how this approach can be used to derive policy implications; specifically, we study the welfare effects of campaign silence laws and of spending caps.

In the next section, we look at three additional variants of our model.

3 Variants

3.1 Early Voting

In the baseline model, the candidates' payoffs depend only on their relative popularity on election day, i.e., at time T. But in many elections voters can and do cast their votes prior to election day, which suggests that the candidates' payoffs should depend on realizations of relative popularity even prior to time T. We now analyze how early voting affects the candidates' spending decisions.

Consider the baseline model, but now suppose that voters can vote from period $\hat{T} < T$ onwards. Suppose that the difference in votes cast for the two candidates in each period $t \geq \hat{T}$ is proportional to their relative popularity in that period, Z_t, and let the number of votes cast in period $t \geq \hat{T}$ be a proportion $\xi \in (0,1)$ of the number of votes cast in period $t+1$. Turnout thus increases over time at a constant rate. This assumption simplifies the notation, but our analysis extends to other assumptions concerning the evolution of turnout so long as candidates can perfectly forecast turnout rates and cannot manipulate them. As ξ converges to zero, almost all votes are cast at time T and we converge to the baseline model. Finally, we assume that despite early voting, either candidate is still able to eventually win the election if she is sufficiently more popular than her opponent at date T, no matter how low her popularity was in previous periods.[14]

In each period t, candidate 1 thus maximizes $\Pr[\sum_{t=\hat{T}}^{T} \xi^{T-t} Z_t \geq 0 \mid (z_{t'}, X_{t'}, Y_{t'})_{t' \leq t}]$, while candidate 2 minimizes this expression. An analogue to problem (3) in the baseline model holds in this variant as well. In particular, given candidate 2's on path spending levels $\{y_0, ..., y_{T-1}\}$, candidate 1's equilibrium spending path $\{x_0, ..., x_{T-1}\}$ now maximizes

$$\sum_{t=0}^{\hat{T}-1} \sum_{t'=0}^{T-\hat{T}} \xi^{t'} \delta^{T-1-t-t'} p(x_t, y_t) + \sum_{t=\hat{T}}^{T-1} \sum_{t'=0}^{T-1-t} \xi^{t'} \delta^{T-1-t-t'} p(x_t, y_t), \tag{5}$$

subject to the same nonnegativity and budget constraints as in problem (3). Candidate 2's spending path correspondingly minimizes this expression subject to her own nonnegativity and budget constraints.

Proposition 3. *Suppose Assumptions 1 and 2 hold. Then in the unique equilibrium path of the game with early voting,*

(i) $x_t/X_t = y_t/Y_t$ *for all periods* t.

(ii) the consecutive period spending ratios equal each other: $r_{1,t} = r_{2,t} = r_t$ *for all periods* t*, and in particular,* $r_t = \delta^{1/(\beta-1)}$ *if* $t < \hat{T} - 1$*, and*

$$r_t = \left[\delta\left(1 + \frac{1}{\sum_{t'=0}^{T-2-t} \xi^{-(T-1-t-t')} \delta^{T-2-t-t'}}\right)\right]^{1/(\beta-1)} \qquad \text{if } t \geq \hat{T} - 1$$

[14] This condition holds if $\xi(2 - \xi^{T-\hat{T}}) < 1$, which is implied by $\xi < 1/2$. Alternatively, we could also assume that candidate 1 maximizes (and candidate 2 minimizes) the difference in candidate 1 and 2's vote shares, $\sum_{t=\hat{T}}^{T} \xi^{T-t} Z_t$. The results of Proposition 3 extend to this case.

Proposition 3 asserts that under early voting candidates continue to allocate the same share of their budgets on the path of play. But early voting modifies the spending path. Because the term inside the large round bracket in Proposition 3 is larger than 1 (and because $\beta < 1$), the spending path is flatter than in the baseline model ($r_t < r$ when $t \geq \hat{T} - 1$). As some voters vote early, candidates now have a new incentive to allocate a larger share of their budget to earlier periods, relative to the baseline model. Moreover, once early voting begins, the consecutive period spending ratio is decreasing in ξ. As the share of voters who vote in early periods increases (higher ξ), the candidates' spending levels will be more evenly distributed (lower r_t for $t > \hat{T} - 1$).

3.2 Valuing Money Left Over

In the variants studied so far, the two candidates are purely office-motivated and fully deplete their budgets by the end of the race because they do not value money left over. In reality, money left over may be valuable: candidates may want to save for future campaigns, or for investments outside politics—to the extent that this is legally allowed.

To capture this, let X_T and Y_T be money left over at the end of the campaign for candidates 1 and 2 respectively. Assume that in each period t candidate 1 maximizes $\Pr[Z_T \geq 0 \mid (z_{t'}, X_{t'}, Y_{t'})_{t' \leq t}] + \kappa_1 X_T$, while candidate 2 maximizes $(1 - \Pr[Z_T \geq 0 \mid (z_{t'}, X_{t'}, Y_{t'})_{t' \leq t}]) + \kappa_2 Y_T$. The parameter $\kappa_i > 0$ reflects candidate i's marginal value for money. On top of saving money and benefiting from this at rate κ_i, we also assume that each candidate i can overspend his budget by borrowing money at a cost equal to κ_i.[15] Thus, X_T and Y_T can be negative. In addition, for tractability we assume that Assumption 2 holds with $\beta = 0$ and we define the function q so that $p(x, y) = p(x/y, 1) =: q(x/y)$ for $y > 0$. (To close the model in the case of $y = 0$, see footnote 10.)

In this variant of the model, candidates trade off spending on the campaign against not spending on it. The marginal benefit of spending depends on the probability of winning, which is history-dependent as it varies with the popularity shocks. The marginal value of not spending on the race is, on the other hand, history-independent. The marginal rate of substitution between spending in a given period and not spending on the campaign is thus history-dependent. As a result, the candidates' equilibrium paths of spending depend on the realization of the popularity shocks. In the baseline model, by contrast, candidates have no incentive to not spend their money on the race. Popularity shocks thus affect the

[15] To simplify the analysis, we abstract from the time dimension when we model borrowing: a unit of money borrowed at any point during the race has the same cost κ_i.

marginal benefit of spending money in all periods by the same amount and the marginal rate of substitution across periods is independent of these shocks.

When money left over is valuable, spending by both candidates decreases as the race becomes more lopsided. To state this popularity dependence formally, define the following quantity for every time t:

$$\zeta((\varepsilon_{t'})_{t'=0}^{t-1}) = \frac{\sum_{t'=0}^{T-1} \delta^{T-1-t'} q\left(\kappa_1/\kappa_2\right) + \delta^T z_0 + \sum_{t'=0}^{t-1} \delta^{T-1-t'} \varepsilon_{t'}}{\sigma \sqrt{\sum_{t'=t}^{T-1} \delta^{T-1-t'}}} \tag{6}$$

This quantity measures the expected electoral advantage that one candidate has over the other at time t: when one candidate has a large popularity advantage over the other, $|\zeta((\varepsilon_{t'})_{t'=0}^{t-1})|$ is large.

Proposition 4. *Suppose Assumptions 1 and 2 hold with $\beta = 0$. Then in the unique equilibrium path of the game in which candidates' marginal valuations for money left over are $\kappa_1, \kappa_2 > 0$,*

(i) $x_t/y_t = \kappa_2/\kappa_1$ for all periods t,

(ii) x_t and y_t are both decreasing in $|\zeta((\varepsilon_{t'})_{t'=0}^{t-1})|$, and

(iii) if κ_1 and κ_2 decrease by the same factor for both candidates, then x_t and y_t increase for both candidate in all periods t and for all realizations z_t.

Part (i) of Proposition 4 says that all the equilibrium cross-candidate spending ratios x_t/y_t equal the ratio of marginal valuations of money left over, κ_2/κ_1. (Recall that in the baseline model, all the cross-candidate spending ratios equal the ratio of starting budgets X_0/Y_0.) In equilibrium, both candidates equalize the marginal benefit of spending with its opportunity cost, which is now equal to the candidates' marginal value of money left over. When function p is homogeneous of degree 0, the ratio of the candidates' marginal benefits to spending in any given period depends only on the cross-candidate spending ratio. In equilibrium, the cross-candidate spending ratio, x_t/y_t, must then equal the ratio of marginal values of money left over.

Part (ii) of the proposition says that spending by both candidates decreases as the election becomes more lopsided, implying that the candidates' spending levels are no longer independent of relative popularity. In particular, if candidate 1 becomes more popular relative to candidate 2, then candidate 2 prefers to save more of her budget because her probability of winning is now smaller. In equilibrium, this pushes candidate 1 to lower her spending as well. This finding is in line with the "discouragement effect" studied in the dynamic contest literature (see Konrad, 2009 and Fu and Wu, 2019 for reviews). In our setting, the result arises from the existence of an outside option (i.e., saving money for after the campaign) that becomes more appealing for a candidate as her odds of winning worsen.

Finally, part (iii) of the proposition asserts that when the marginal values of money κ_1 and κ_2 decrease proportionally for the two candidates, the spending levels of the two candidates go up uniformly in each period. This implies that high stakes elections (those with lower κ_1 and κ_2) should see on average higher spending.

3.3 Targetable Subpopulations

In any campaign, candidates choose not just when to spend their resources, but also how to target these resources across voters—for example by targeting specific geographic areas or media markets. Suppose that the two candidates compete over a set of targetable subpopulations. The set of subpopulations is $\{1, 2, ..., S\}$ and the payoffs of the candidates depend on how these different subpopulations aggregate.

Popularity in each subpopulation s is represented by the random variable Z_t^s with realizations z_t^s. We assume that $(Z_t^s)_s$ are distributed according to a multivariate normal distribution with arbitrary variance-covariance matrix. For each subpopulation s, the popularity process is

$$Z_{t+1}^s = p(x_t^s, y_t^s) + \delta^s Z_t^s + \varepsilon_t^s, \tag{7}$$

where $\varepsilon_t^s \sim \mathcal{N}(0, (\sigma^s)^2)$ and these shocks are iid over time. Each subpopulation s thus has its own decay parameter δ^s, and its own variance $(\sigma^s)^2$. In addition, as in the previous section, we assume that the function p satisfies Assumptions 1 and 2 with $\beta = 0$, so that $p(x, y) = p(x/y, 1) = q(x/y)$ for some function q.[16]

The aggregation rule for the outcomes in the various subpopulations is arbitrary, but we impose the following assumptions: the candidates' payoffs depend only on the vector $(Z_T^s)_{s=1}^S$, the game is still zero sum, and candidate 1's payoff is strictly increasing in each Z_T^s, while candidate 2's is strictly decreasing in each Z_T^s. More formally, denote candidate 1's payoff $u((Z_T^s)_{s=1}^S)$ so that candidate 2's payoff is $-u((Z_T^s)_{s=1}^S)$, and assume that

$$\frac{\partial u((Z_T^s)_{s=1}^S)}{\partial Z_T^s} > 0, \qquad \text{for every } s. \tag{8}$$

[16] We extend the assumption in footnote 10 as follows: if a candidate spends an amount equal to 0 in some subpopulation, then the game ends and the candidate wins with probability 1/2 if the other candidate is also spending an amount equal to 0 in some subpopulation, and loses with probability 1 otherwise.

For this model, we can show that the equal spending ratio result holds subpopulation by subpopulation, which is stated in part (i) of Proposition 5 below. However, unlike in the baseline model, spending decisions may depend on the history of the popularity processes. If the competition in some subpopulations becomes lopsided (in terms of the candidates' relative popularity), the marginal benefit of spending money in those subpopulations decreases for both candidates. Candidates will then react by concentrating their spending in other, more competitive subpopulations. Relative popularity within different populations thus plays a role in spending decisions.

This popularity-dependence does not arise in the special case in which payoffs are a weighted sum of relative popularity in each district at time T. In this case, candidate 1's marginal benefit of increasing her popularity in a specific subpopulation is constant and it is equal to the marginal benefit of candidate 2. Moreover, under this assumption, we can characterize the consecutive period spending ratios for this model as well as the optimal allocation of resources across districts in each period—results that are stated in parts (ii) and (iii) of Proposition 5 respectively. The following assumption, which strengthens the monotonicity assumption in equation (8), states the condition formally.

Assumption 3. *For weights $\{w^s\}_{s=1}^S$ such that $w^s > 0$ and $\sum_{s=1}^S w^s = 1$,*

$$u\left((Z_T^s)_{s=1}^S\right) = \sum_{s=1}^S w^s Z_T^s.$$

Assumption 3 fits either a setting where candidates allocate resources across multiple media markets, or one in which the candidates are two parties that compete to maximize the number of seats in a legislature, seats are allocated proportionally in each district, and the number of seats assigned to each district depends on the district population reflected in w^s.

To state Proposition 5, let h_t denote histories prior to the candidates choosing their period-t spending levels. Let the consecutive period spending ratios for the two candidates in any district s be $r_{1,t}^s = x_{t+1}^s / x_t^s$ and $r_{2,t}^s = y_{t+1}^s / y_t^s$.

Proposition 5. *Suppose Assumptions 1 and 2 hold with $\beta = 0$. In any equilibrium of this targetable subpopulations extension,*

(i) in each subpopulation s, $x_t^s / X_t = y_t^s / Y_t$ for every t.

(ii) if Assumption 3 holds, then, in each subpopulation, the candidates' consecutive period spending ratios conditional on any on-path history equal each other: $r^s_{1,t} = r^s_{2,t} = 1/\delta^s$ *for all* s *and all histories* h_t*.*

(iii) if Assumption 3 holds, then for all periods t *and any pair of subpopulations* s, s'*,*

$$\frac{x^s_t}{x^{s'}_t} = \frac{y^s_t}{y^{s'}_t} = \frac{w^s}{w^{s'}} \left(\frac{\delta^s}{\delta^{s'}}\right)^{T-t-1}$$

By Proposition 5(iii), the allocation of resources across subpopulations given total spending in a period is independent of the popularity process. Moreover, candidates spend more on subpopulations that have greater electoral weight and for whom popularity leads decay at a slower rate. Finally, the differences in spending due to different decay rates are maximal at the beginning of the campaign and decrease as election day approaches. These results hold even if the candidates' investments in any one subpopulation also affect their relative popularity in other subpopulations.

4 TV Ad Spending in Actual Elections

We now look at actual campaign spending data through the lens of our baseline model. Under Assumptions 1 and 2, the predicted pattern of spending is given by $r_{1,t} = r_{2,t} = r = \delta^{1/(\beta-1)}$ (see Proposition 2 and Figure 2). Our main goal is to use this relationship to recover election-specific estimates of δ from patterns of spending. If candidates compete according to our baseline model, this gives us estimates of how they perceive the decay rate $1-\delta$ when making their spending decisions. Actual spending paths obviously depend on some factors that our baseline model does not account for. Our estimation exercise, nevertheless, still informs us on how much we can explain with a simple competitive environment. As such, it can guide our understanding of what are the factors that are likely missing to get a better fit with the data.

Before proceeding, we introduce the data we use and we investigate the extent to which two important implications of our baseline model are violated in the data: the equal spending ratio result ($x_t/X_t = y_t/Y_t$ for all t) and the constant spending growth result ($r_{1,t} = r_{2,t} = r$ for all t).

4.1 Data

We focus on subnational American elections, namely U.S. House, Senate, and gubernatorial elections in the period 2000 to 2014.

Spending in our model refers to all spending—TV ads, calls, mailers, door-to-door canvasing visits—that directly affects the candidates' relative popularity. But for some of these categories, it is not straightforward to separate out the part of spending that has a direct impact on relative popularity from the part that does not (e.g. fixed administrative costs). For television ads, it is straightforward to do this, so we focus exclusively on TV ad spending. Television advertising constitutes around 35% of the total expenditures by congressional candidates, and around 90% of all advertisement expenditures during the period we study (see, e.g., Albert, 2017). Furthermore, for TV ads, we have access to the exact timing of the candidates' expenditures. We proceed under the assumption that any spending on other types of campaign activities that directly affect relative popularity is proportional to spending on TV ads.

Our TV ad spending data are from the Wesleyan Media Project and the Wisconsin Advertising Database, which draws information directly from TV channels. For each election in which TV ads were bought, the database contains information about the candidate that each ad supports, the date it was aired, and the estimated cost. The dataset does not include information on the source of spending (whether PACs or the candidates themselves), but the vast majority of expenditure on TV advertising is likely to happen through PACs (Martin and Peskowitz, 2018).

For the year 2000, the dataset covers only the 75 largest Designated Market Areas (DMAs), and for years 2002-2004 it covers the 100 largest DMAs. The data from 2006 onward covers all 210 DMAs. We obtain the amount spent on ads from total ads bought and price per ad. Ad price data are missing for 2006, so for that year we estimate prices using ad prices in 2008.[17]

We focus on races where the leading two candidates in terms of vote share are from the Democratic and the Republican party. We label the Democratic candidate as candidate 1 and the Republican candidate as candidate 2, so that x_t, X_0, etc. refer to the Democrat's spending, budget, etc. and y_t, Y_0, etc. refer to the Republican's.

[17] In principle, as election day approaches, TV ad prices can increase. Increases in total spending over time could confound price increases with increased advertising. Federal regulations, however, limit the ability of TV stations to increase ad prices close to elections. TV stations must charge political candidates "the lowest unit charge of the station for the same class and amount of time for the same period" (Chapter 5 of Title 47 of the United States Code 315, Subchapter III, Part 1, Section 315, 1934). These regulations allay some of these concerns.

In our model, spending decisions are made at discrete moments in time defined so that the inter-period decay rate $1 - \delta$ is constant. This raises the question of how to define a period of spending in the data, given that spending data are reported irregularly. To address this issue, in the Online Appendix we examine a continuous time formulation of our model in which candidates make spending decisions at fixed intervals of time and the decay rate is constant. There, we prove an identification result that implies that the level of aggregation of spending is irrelevant: e.g., if candidates make their spending decisions daily but the data are aggregated weekly, then the sum of what they spend over seven days is the same as in a setting in which they make spending decisions weekly. Given this result, we proceed by aggregating spending data at the weekly level.

To restrict attention to general elections, we focus on the 12 weeks leading to election day, though we drop the final week which is typically incomplete since elections are held on Tuesdays.[18] We exclude elections that are clearly not genuine contests to which our model does not apply. These are elections in which one of the candidates did not spend anything for more than half of the period studied. This leaves us with 346 House, 122 Senate, and 133 gubernatorial elections, tabulated in the Appendix. We define the total budgets of the candidates to be the total amount that they spend over these 12 weeks. In the Online Appendix, we replicate our analyses by excluding fewer elections (leaving us with 1163 elections over 14 years), or by allowing for a longer time window for each election (20 weeks instead of 12).

Table 1 reports summary statistics for the elections we consider. There is considerable difference in the amount spent between state-wide and House elections, with another key difference being the time at which candidates start spending positive amounts. For statewide races, candidates spend on average about \$6 million on TV ads, with most candidates already spending positive amounts 12 weeks prior to the election. For House races, they spend \$1.5 million on average and the majority of candidates start spending 9 weeks out.

In addition, there is variation in the amount spent by candidates competing in the same race. The average difference in the amount spent by the candidates competing in the same congressional election is one third of the average total spending for those races, while for gubernatorial elections the same difference exceeds half. Finally, candidates tend to spend more in more competitive elections: the overall amount spent is higher in elections where

[18]In some cases, primaries are held less than 12 weeks before the general election, but ad spending for the general election before the primaries is typically zero. In the rare cases where ad spending for primary elections happens, we exclude it from our analysis.

Table 1: Descriptive Statistics

	N	Open Seat Election	Incumbent Competing	No Excuse Early Voting	Average total spending	Average spending difference
Senate	122	68	54	82	6019 (5627)	1962 (2921)
Governor	133	59	74	92	5980 (9254)	3173 (6,337)
House	346	97	249	223	1533 (1304)	521 (615)
Overall	601	224	377	397	3428 (5581)	1401 (3,461)

Average Spending and Standard Deviations in Parentheses by Week and Election Type

Week	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
Senate	196	250	266	314	357	477	545	652	716	860	1,002
	(291)	(328)	(403)	(487)	(401)	(505)	(577)	(724)	(803)	(947)	(1,047)
Share spending 0	0.270	0.180	0.123	0.082	0.008	0	0	0	0	0	0
Governor	262	253	258	316	420	416	530	597	701	800	1,019
	(632)	(468)	(424)	(581)	(865)	(579)	(1,249)	(1,015)	(1,305)	(1,523)	(1,956)
Share spending 0	0.297	0.207	0.139	0.068	0.030	0	0	0	0	0	0
House	17	27	38	56	83	120	137	177	212	250	303
	(41)	(55)	(57)	(85)	(93)	(134)	(134)	(182)	(219)	(270)	(340)
Share spending 0	0.653	0.545	0.386	0.246	0.095	0	0	0	0	0	0
Early Voting	113	123	128	168	223	262	320	390	449	526	624
	(324)	(256)	(256)	(348)	(488)	(404)	(694)	(663)	(775)	(895)	(1033)
No Early Voting	99	122	144	162	194	250	283	321	373	436	569
	(213)	(246)	(286)	(314)	(259)	(317)	(348)	(394)	(473)	(524)	(866)
Open Seat	164	183	191	217	279	324	362	445	521	602	729
	(404)	(311)	(325)	(352)	(476)	(445)	(485)	(635)	(800)	(892)	(1046)
Incumbent	75	87	99	135	174	219	275	320	366	432	532
	(189)	(202)	(218)	(324)	(386)	(324)	(657)	(550)	(606)	(716)	(931)
Close Election	122	131	154	200	250	292	383	479	544	661	858
	(318)	(236)	(320)	(421)	(539)	(407)	(915)	(791)	(746)	(884)	(1266)
Not Close Election	103	120	125	152	199	245	278	322	376	430	506
	(280)	(259)	(242)	(297)	(369)	(364)	(412)	(476)	(659)	(741)	(820)
Close Budgets	97	118	129	150	196	264	301	362	411	477	587
	(190)	(209)	(255)	(225)	(246)	(339)	(385)	(442)	(488)	(550)	(710)
Not Close Budgets	117	127	137	178	227	254	313	370	434	510	620
	(351)	(282)	(275)	(404)	(525)	(405)	(727)	(680)	(813)	(938)	(1149)

Note: Spending on television advertising for the twelve weeks prior to election dates, excluding the final (partial) week, as elections are held on Tuesdays. The upper panel reports the breakdown of elections that are open seat versus those that have an incumbent running, the number of elections in which voters can vote early without an excuse to do so, average spending levels by the candidates, and the average difference in spending between the two candidates, all by election type. The lower panel presents average spending for each week in our dataset, by election type. Standard deviations are in parentheses. All monetary amounts are in units of $1,000. Close elections are races where the final difference in vote shares between two candidates is less than 5 percentage points. Close budget races are those in which the ratio of budgets of the two candidates lies in the interval $(0.75, 1.25)$.

there is no incumbent, and in elections where the final margin of victory is thin. We will consider these differences in our estimation.

4.2 Diagnostics

How well do the predictions of the baseline model under Assumptions 1 and 2 agree with actual spending patterns in the data?

The prediction in Proposition 1(ii)—that spending is independent of popularity—cannot be tested because publicly available polling data are too sparse.[19] So we proceed to investigate the predictions of Proposition 2. These predictions are the equal spending ratio result and the constant spending growth result.

Equal Spending Ratios. In Table 2 we look at the extent to which the equal spending ratio result is violated in our data. Since spending ratios are defined as the shares of leftover (rather than total) budgets spent, these ratios can take any value between 0 and 1 every week prior to the final week, where, by construction, they equal 100%. So to not bias the results in the direction of fewer and smaller violations of the equal spending ratio result, we exclude this final week from our analysis.

Table 2 reports that the candidates' weekly spending ratios are within 10 percentage points (pp) of each others' in 80% of election-weeks, and within 5 pp of each others' in 56% (see Table 1 in the Online Appendix for disaggregations of the 5pp analysis). Even in the final six weeks of the campaign when candidates spend larger amounts, they are within 10 pp of each others' in 75% of election-weeks, and within 5 pp of each others' in about half.

Violations of the equal spending ratio result do not seem to be more pronounced in open-seat elections, nor in those where voters are able to cast their ballots early without an excuse. This last finding is consistent with our early voting extension in which the equal spending ratio result continues to hold analytically. On the other hand, we do see more pronounced violations in elections that are lopsided in terms of money spent and final vote shares. If these elections are those in which one candidate (e.g. the better-resourced one) has large leads against the other, then these more pronounced violations could be explained by the variant of our model in which candidates value money left over.

[19] To the best of our knowledge, FiveThirtyEight and Pollster provide the largest publicly available database on polls. We collected data from these sources and identified only 24 elections (all state-wide races) with more than 3 weeks of polling data, which constitutes a sample that is too sparse and potentially not representative of the full set of races in our dataset to conduct a systematic analysis of how spending decisions are affected by changes in relative popularity.

Table 2: $x_t/X_t - y_t/Y_t$

Week	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
% $\in$ (-0.1,0.1)	0.963	0.953	0.938	0.902	0.879	0.847	0.829	0.754	0.676	0.622	0.797
Senate	0.943	0.934	0.975	0.926	0.934	0.885	0.844	0.787	0.746	0.648	0.803
Governor	0.932	0.910	0.887	0.820	0.812	0.812	0.767	0.774	0.639	0.624	0.782
House	0.983	0.977	0.945	0.925	0.884	0.847	0.847	0.734	0.665	0.613	0.801
Early Voting	0.970	0.955	0.942	0.912	0.884	0.844	0.816	0.753	0.673	0.612	0.798
No Early Voting	0.951	0.951	0.931	0.882	0.868	0.853	0.853	0.755	0.681	0.642	0.794
Open Seat	0.942	0.933	0.920	0.897	0.857	0.862	0.866	0.795	0.705	0.656	0.804
Incumbent Competing	0.976	0.966	0.950	0.905	0.891	0.838	0.806	0.729	0.658	0.602	0.793
Close Election	0.976	0.965	0.935	0.941	0.947	0.924	0.906	0.882	0.776	0.706	0.788
Not Close Election	0.958	0.949	0.940	0.886	0.852	0.817	0.798	0.703	0.636	0.589	0.800
Close Budgets	0.974	0.974	0.959	0.925	0.914	0.895	0.883	0.812	0.763	0.695	0.838
Not Close Budgets	0.955	0.937	0.922	0.884	0.851	0.809	0.785	0.707	0.606	0.564	0.764
% $\in$ (-0.05,0.05)	0.865	0.815	0.757	0.727	0.661	0.599	0.554	0.468	0.418	0.369	0.562
Average x_t/X_t	0.021	0.028	0.039	0.054	0.075	0.109	0.134	0.184	0.251	0.377	0.728
	(0.032)	(0.036)	(0.044)	(0.051)	(0.054)	(0.067)	(0.073)	(0.085)	(0.095)	(0.108)	(0.076)
Average y_t/Y_t	0.021	0.029	0.038	0.049	0.074	0.105	0.133	0.184	0.249	0.380	0.733
	(0.035)	(0.041)	(0.046)	(0.053)	(0.063)	(0.073)	(0.080)	(0.094)	(0.097)	(0.111)	(0.073)

Note: The table reports the share of elections in which the candidates' spending ratios are within 10 percentage points (or 5 percentage points) of each other for every week, across election types. See the note below Table 1 for definitions of close elections and close budget elections.

Finally, the extent to which our equal spending ratio result appears violated in the data is increasing as the election approaches. One reason for this could be that as election day approaches, spending decisions are more affected by disturbances resulting from factors outside our baseline model.[20] Another possibility is that spending ratios are more likely to be close in percentage points in the early weeks when both candidates spend lower shares of their available budget. To address these possibilities, in the Online Appendix, we also examine the percentage (as opposed to percentage points) differences between the spending ratios of the candidates across weeks. We find that percentage differences tend to decrease (i.e., the equal spending ratio result tends to hold more often) as election day approaches and candidates spend larger amounts.

[20]One such factor is an "October surprise"—the surfacing of new information, like a scandal that creates a wedge between a candidate's forecasted budget (on which some past spending was based) and the budget that actually becomes available. Another factor outside our model is the idea that close to election day, trailing candidates may simply give up because of threshold effects.

Constant Spending Growth. The consecutive period spending ratio (CPSR) is x_{t+1}/x_t for the Democrat and y_{t+1}/y_t for the Republican candidate. In our eleven-week dataset, these variables are defined for ten consecutive week pairs. If the constant spending growth prediction holds, these two ratios should be relatively stable over time. However, since there are candidates who spend zero in some of the earlier weeks, the CPSR cannot be calculated for certain periods. In what follows, we thus calculate CPSRs using two approaches: (i) dropping all elections with zero spending in any week, and (ii) dropping all pairs of consecutive weeks that would include a week with zero spending. Approach (i) leaves us with only 221 (out of the total 601) elections where no zero spending occurs. In approach (ii), instead, we drop 1,692 consecutive week pairs out of a total of 13,222, which is only 12.8%. We also note that in our data there is no instance of zero spending following positive spending: once a candidate starts spending, she continues to do so until the election.

Figure 3 reports the distribution of average CPSRs for every candidate, along with the intervals centered at these averages and width equal to ± 1.96 times the estimated standard deviation. (In the Online Appendix we also report similar plots with the interval defined by the second lowest and the second highest observation for each election.) The distributions obtained from approaches (i) and (ii) are very similar. The reported CPSRs for approach (ii) can be interpreted as growth rates conditional on having started positive spending during an electoral campaign. Approach (ii) discards less data and so we proceed with analyzing the growth rates obtained using such an approach. Hereafter, when we say "growth rates," we refer to growth rates conditional on having started spending positive amounts.

Our baseline model predicts a positive and constant spending growth rate. Looking at Figure 3, the middle 90% of the distribution of CPSR values (5th to 95th percentile) spans $[0.98, 1.9]$. For the candidate with the median value, the average CPSR is 1.16, meaning that her spending increases by 16% on average every week after she starts spending positive amounts. We also find that spending increases from one week to the next for 85% of candidate-weeks. The median standard deviation in candidate CPSRs within an election is 0.814 and more than 75% of candidates have a standard deviation below 2. Variation in CPSR values within an election is typically driven by only a few weeks of volatile growth, rather than by extreme volatility in the entire spending path. Table OA3 in the Online Appendix provides a measure of how the central tendency of candidates' CPSRs within elections varies week by week.

Overall, CPSRs vary within elections, contrary to what our baseline model predicts. One possible explanation for this variation is given by our early voting model in which

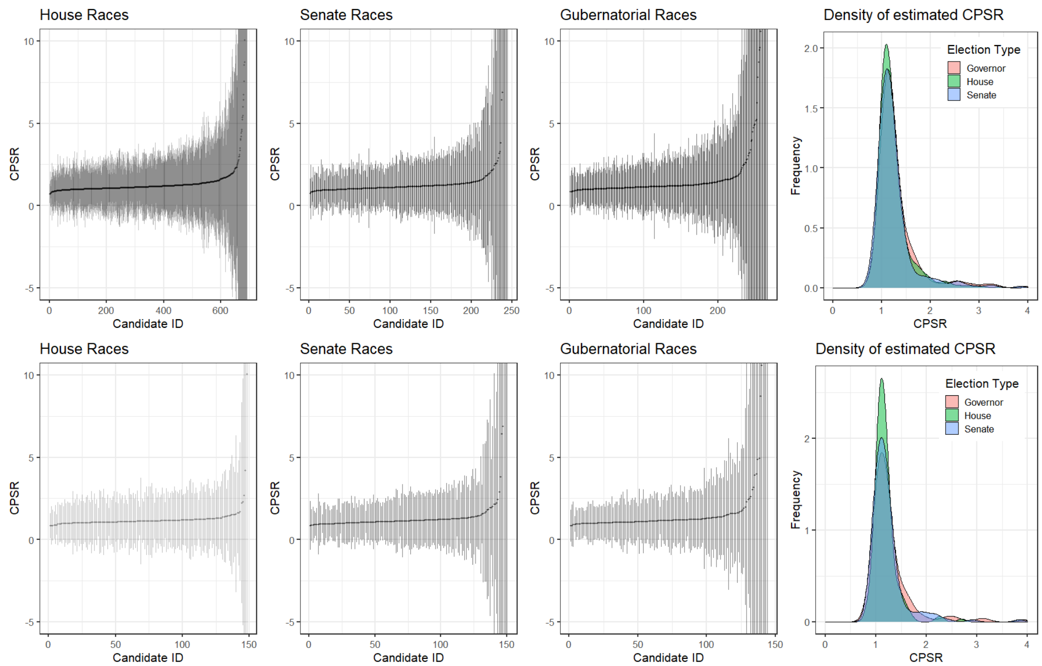

Figure 3: Average CPSR values for candidates in our dataset, along with the interval $[\mu_r - 1.96\sigma_r, \mu_r + 1.96\sigma_r]$, where μ_r and σ_r are the sample average and sample standard deviation of CPSRs. The upper display row depicts the averages that we get by dropping all elections with zero spending. The bottom depicts the averages that we get by dropping all pairs of consecutive weeks that include zero spending. In the first three charts of each row, candidates are sorted based on their average CPSR from lowest to highest. The last chart of each row depicts the densities of average CPSRs across election types from each approach.

spending growth is constant until the time early voting starts, which is typically anywhere from a few days prior to the election to up to eight weeks from election day. Early voting, however, does not appear to be a major driver of violations to the constant spending growth prediction (see Table OA3 in the Online Appendix). Another possible explanation for the deviations from constant spending growth is that candidates value money left over, as in our extension. Though we cannot directly test this, we can reason that if House candidates are more likely to value money left over than Senate or gubernatorial candidates (because the value of office is lower, or their future political ambitions—perhaps to become Senators or governors—are greater, or because they compete more frequently in future elections), this does not appear to be reflected in the disaggregation by election type (again see Table OA3). A third possibility is that the candidates have uncertain budgets that react to their polling performance, as in the evolving budgets model that we present in the Online Appendix. We cannot investigate whether this model can account for the violations from the constant

CPSRs prediction because data on when candidates receive money or pledges from donors are not available.

4.3 Perceived Decay Rates

In our model, the decay rate in popularity leads is $1-\delta$. The perceived decay rate is the value of $1-\delta$ that is "most consistent" with the candidates' spending behavior in an election given that the CPSR in the baseline model is $r=\delta^{1/(\beta-1)}$. Since the perceived decay rate cannot be separately identified from the parameter β using spending data alone, we fix a grid of values of β ranging from 0 to 1 and we report how the distribution of estimated perceived decay rates varies with β.

A straightforward way to estimate the perceived decay rate $1-\delta_j$ in election j, is to let r_j be the mean of the candidates' CPSRs estimated from their actual spending levels in election j (these are given in Figure 3) and then use the relationship $1-\delta_j=1-(r_j)^{\beta-1}$. We perform this estimation using approach (ii) above, namely dropping all candidate-weeks with zero spending. More specifically, δ_j can be estimated directly from the first moment of the distribution of observed CPSRs. Denote

$$r_{j,i,t}=\frac{i\text{'s spending in week } t+1}{i\text{'s spending in week } t},\quad \text{in election } j$$

which is observed for $t=0,1,...,T-2$, for both candidates $i=1,2$ running in election j and can be calculated so long as the candidate spends a positive amount in week t. We compute the first moment of these CPSRs for election j as

$$\hat{r}_j=\frac{1}{|\mathcal{T}|}\sum_{i=1,2}\sum_{t\in\mathcal{T}}r_{j,i,t}$$

where $\mathcal{T}$ is the set of candidate-weeks in election j for which $r_{j,i,t}$ can be computed.[21] Then, as our model predicts $r_{j,i,t}=r_j=(\delta_j)^{1/(\beta-1)}$ for both i and all t, we fix β to some value and estimate the perceived decay rate $1-\delta_j$ from $\hat{r}_j$ as $1-(\hat{r}_j)^{\beta-1}$.

The reason we pool the two candidates' CPSRs to estimate a common perceived decay rate is that this approach increases the precision of our estimates, as it gives us potentially up to 20 total CPSR values (which occurs when there are no weeks with zero spending). In the Online Appendix, we also report candidate-specific decay rates obtained without pooling together the CPSRs of the two candidates. The densities of the estimates we

[21] For example, if both candidates spend positive amounts in all eleven weeks prior to election day, then we have $|\mathcal{T}|=20$.

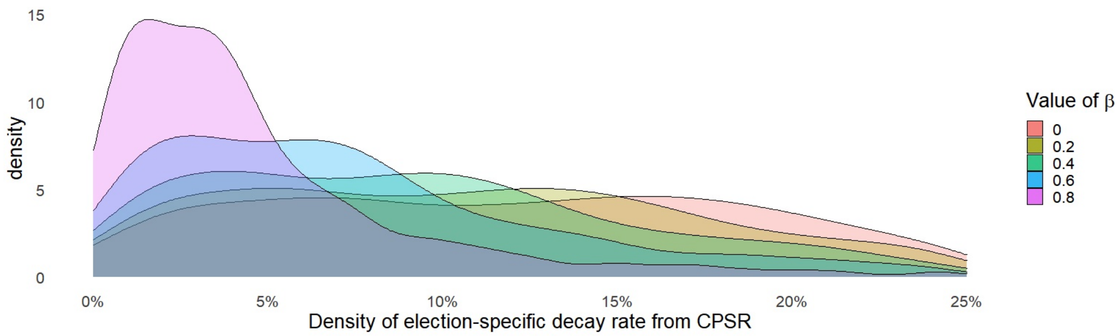

Figure 4: The distributions of our estimates of the candidates' perceived decay rates from their CPSRs. We estimate different distributions for values of β ranging from 0 to 1. The figure depicts only positive values of the decay rates.

obtain for Democrat and Republican candidates do not exhibit any major differences. In addition, although our model assumes that the decay rate is constant over time, in the Online Appendix we also discuss how to generalize our analysis to a setting where decay rates are time-varying. In particular, we produce estimates of these time-varying decay rates using the approach described here.

Figure 4 shows the distributions of the point estimates of the common perceived decay rates for five different values of β, indicating that most of the mass in decay rates is below 25% no matter what value of β we fix. The decay rate estimates along with the ± 1.96 standard deviation intervals are plotted in the Online Appendix.

Recall that a candidate's equilibrium spending path is determined by two parameters: the candidate's starting budget, which determines the level of the spending curve, and the common equilibrium consecutive period spending ratio r, which is constant in time and determines the shape of the spending curve. Since in our data a candidate's budget is fixed as the total spent by the candidate, the candidates' mean observed CPSR alone determines how well our model fits the data. One measure of this fit is to simply take the standard deviation of mean CPSRs; the distribution of these standard deviations is plotted in the Online Appendix, and it shows considerable variation in fit, confirming what we see in Figure 3. As an example, we plot in Figure 5 our predicted spending path to actual spending path in the election with the smallest standard deviation, that with the 10th percentile smallest and that with the 90th percentile. Since the candidate budgets simply scale the level of the

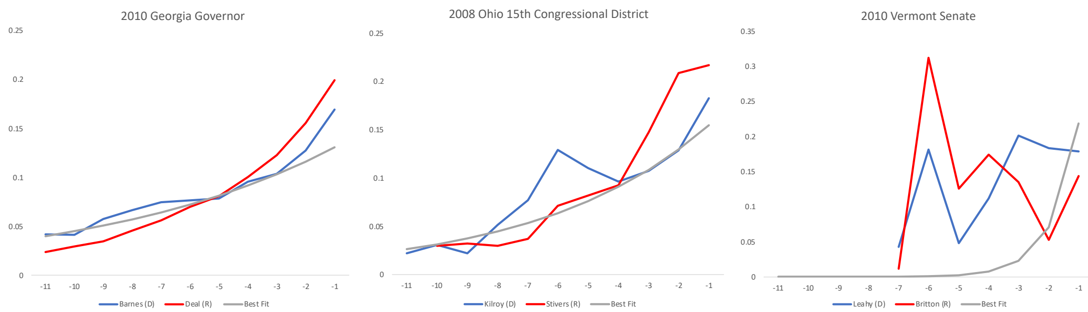

Figure 5: Weekly spending paths for Democrat (blue) and Republican (red) candidates in the election with the lowest standard deviation in candidate CPSRs (2010 Georgia gubernatorial), 90th percentile lowest (2008 Ohio's 15th Congressional district), and 10th percentile lowest (2010 Vermont Senate). Also depicted in gray is the equilibrium spending path corresponding to the fitted decay rate from estimates in Section 4.3. Candidate budgets are normalized to 1 for all series.

graph and do not affect the overall fit (given our approach of fixing the total budget to be total amount spent), we normalize both candidates' budgets to 1 in these figures.

The differences in fit shown in Figure 5 may be due to several factors that our baseline model does not account for. For example, a candidate's spending may change in response to the surfacing of political scandals, random shocks to the available budgets, changes in voter attention, and the candidates themselves experimenting to learn about which campaigning strategies are effective and which are not. Incorporating these factors into our framework and estimating their impacts on campaigning are natural directions for future research.

Our estimates of election-specific decay rates using the approach outlined above are obtained after discarding weeks with zero spending, and for elections with sparse positive spending data the estimates can be quite noisy. In the Online Appendix we thus estimate decay rates using a hierarchial Bayes model that enables us to estimate election-specific decay rates while specifying certain parameters of the model to be common across elections and modeling the odds of observing zero spending in any given week. The estimates obtained through this alternative approach are less noisy and more concentrated on smaller values than the ones discussed here.

Comparison with the Experimental Literature. Previous literature estimates actual (as opposed to perceived) decay rates using survey and experimental data. For example, using survey data and an exponential decay model similar to ours, Hill et al. (2013) recover an average daily decay rate in the persuasive effects of political advertising of 52.4% in

2006 U.S. elections. This corresponds to a 99% weekly decay, though their 95% confidence interval for this estimate covers the $[0, 100\%]$ interval. Similarly, using a field-experimental approach, Gerber et al. (2011) recover a weekly decay rate of 88%, though their estimates vary substantially according to the specification of their model.[22]

Nevertheless, if we take the point estimates from these prior studies at face value, Figure 4 shows that the perceived weekly decay rates—which are typically below 25%—are considerably lower than previous estimates of actual decay rates. Our parameterized baseline model, therefore, suggests that candidates spend more in earlier weeks compared to what the decay rates estimated from the past literature would imply. On the other hand, since our estimates of the perceived decay rates are within the large margins of error of prior estimates of actual decay rates, we can make no conclusive inferences on this.

There are several possible reasons why our estimates are lower than the point estimates found in the experimental literature. One is that candidates are irrationally spending too much money in the early stages of the campaign. Another is that candidates are spending rationally but prior point estimates are off because they measure decay rates only for marginal spending, which could differ substantially from the global average.[23] This is certainly a possibility as prior work in the (non-political) marketing literature finds decay rates that are more in line with our estimates; e.g., Dubé et al. (2005) estimate the weekly decay of goodwill from ads in the frozen food industry to be only around 12%. Yet, another possibility is that candidates are spending rationally, actual decay rates are quite high, but our baseline model is failing to capture the full benefits of early spending. One of these benefits is experimentation—campaigns spend early to try to learn what kind of ad targeting works best given their characteristics and political platforms. Another is the increase in support from donors due to improvements in early polling leads. In the Online Appendix, we estimate perceived decay rates using the model with evolving budgets introduced in Section OA3 of that appendix. We find that estimates of the candidates' perceived decay rate increase with the degree of positive feedback between early polling leads and donor support,

[22] For example, their 3rd order polynomial distributed lag model estimates show that the standing of the advertising candidate increases by 4.07 percentage points in the week that the ad is aired, and the effect goes down to 3.05 percentage points the following week (25% decay). In another specification, the first week effect is 6.48%, and goes down to 0.44% in the second week (94% decay). The volatility of these estimates may be due to data limitations, as well as sensitivity to the parametric specifications; see, e.g., Lewis and Rao (2015).

[23] The political consultant David Shor told one of the present authors that he advises campaigns to perceive a weekly decay rate in ad spending in the ballpark of 15%. Moreover, the timing of the field experiments conducted by the experimental literature varies considerably and does not always coincide with the twelve-week period that we focus on. Decay rates may be different for ad spending that happens even before the general election period starts, as voters pay less sustained attention to political ads.

but more work needs to be done on the fundraising side to determine whether budget concerns provide a quantitatively plausible explanation for the extent of early spending that we observe in the data.

5 Conclusion

We have developed a model of electoral campaigns as dynamic contests and used it to study the optimal allocation of campaign resources over time when popularity leads tend to decay. The model provides a tractable benchmark to analyze the dynamics of campaign spending. In this benchmark, we identify conditions under which spending decisions are independent of popularity and satisfy an equal spending ratio condition.

Our framework is flexible enough to allow for arbitrary initial advantages, early voting, candidates valuing money left over at the end of the campaign, and campaign spending targeting various subpopulations. We have analyzed the main predictions of our baseline model by looking at spending data from U.S. elections, and we recovered estimates of the candidates' perceived rates of decay of popularity leads.

To focus on the strategic aspects of the dynamic budget allocation problem, we have abstracted away from some important considerations in campaigning like the incentives of donors, and the candidates' trade-off between campaigning and fundraising. These considerations are natural complements to our analysis.[24] Embedding the strategic behavior of donors in a model of dynamic campaign spending is a particularly interesting avenue for future research.

In actual elections, some candidates are more popular or simply better known than others. As a result, their campaigns may reach a broader audience and thus be more effective. In these situations, the lesser known candidate may need to build up momentum to gain visibility and improve her returns from later advertising. Studying the dynamic relationship between popularity and the returns to advertising represents another promising direction for future work.

We have also abstracted from the fact that candidates may not know the return to spending or the decay rate of popularity leads at various stages of the campaign. These quantities may be specific to the characteristics of the candidates or to the political envi-

[24]Mattozzi and Michelucci (2017) analyze a two-period dynamic model in which donors decide how much to contribute to each of two possible candidates without knowing ex-ante who is the more likely winner. Bouton et al. (2022b) study the strategic choice of donors who try to affect the electoral outcome and highlight that donor behavior depends on the competitiveness of the election. Bouton et al. (2022a) provide an empirical analysis of small donors' contribution decisions.

ronment, including the "mood" of voters. Candidates thus face an optimal experimentation problem whereby they try to learn about the campaign environment through early spending. There is no doubt that well-run campaigns spend resources to acquire valuable information about how voters are engaging with and responding to their messages over time. These are interesting and important questions that ought to be addressed in subsequent work.

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Other data were obtained from the Wesleyan Media Project, a collaboration between Wesleyan University, Bowdoin College, and Washington State University, and includes media tracking data from Kantar/Campaign Media Analysis Group in Washington, D.C. The Wesleyan Media Project was sponsored by grants from the Sunlight Foundation and The John S. and James L. Knight Foundation. The opinions expressed in this article are those of the author(s) and do not necessarily reflect the views of the Wesleyan Media Project, the Sunlight Foundation, Knight Foundation, or any of its affiliates.

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Appendix

A Proofs

A.1 Proof of Proposition 1

Equilibrium existence follows from Debreu-Fan-Glicksberg Theorem, given the compactness and convexity of the set of candidates' strategies and the continuity and concavity (convexity) of p with respect to x_t (y_t). Uniqueness follows from Assumption 1(b) and the minmax theorem (see Theorem 10 in Rockafellar, 1971).

In equilibrium, spending profiles must be interior: candidates must spend a positive amount at every history. Suppose to the contrary that there exists an equilibrium spending profile in which one of the candidates spends 0 at some history h_t. Assumption 1(a) implies that this candidate spends a positive amount at some history $h_{t'}$ that includes history h_t. By Assumption 1(b)-(c), this candidate will then be better off moving some spending from history $h_{t'}$ to history h_t.

Thus, the equilibrium spending profile from time t onwards must satisfy the set of first-order conditions with respect to x_t and y_t obtained from problem (3). These first order conditions are:

$$\delta^{T-1-t} p_x (x_t, y_t) = p_x (x_{T-1}, y_{T-1})$$
$$\delta^{T-1-t} p_y (x_t, y_t) = p_y (x_{T-1}, y_{T-1})$$

where p_x denotes the partial derivative with respect to the first component and p_y with respect to the second. These conditions do not depend on the past realizations of relative popularity $(z_{t'})_{t'<t}$. These observations establish the claims made in parts (i) and (ii) of the proposition. □

A.2 Proof of Proposition 2

To show part (i), let $h_t = ((x_{t'}, y_{t'}, z_{t'})_{t'<t}, z_t)$ denote the history of candidates' spending decisions up to period $t-1$ and of the relative popularity process up to time t. The budgets available to candidates at history h_t are $X[h_t] = X_0 - \sum_{t'=0}^{t-1} x_{t'}$ and $Y[h_t] = Y_0 - \sum_{t'=0}^{t-1} y_{t'}$. Optimality implies that for any period t and any h_t, candidate 1 maximizes $\Pr[Z_T \geq 0 \mid h_t]$ under the constraint $\sum_{t'=t}^{T} x_{t'} \leq X_t[h_t]$, while candidate 2 minimizes this probability under the constraint $\sum_{t'=t}^{T-1} y_{t'} \leq Y_t[h_t]$.

Using equation (1), we can recast the objective of maximizing $\Pr\left[Z_T \geq 0 \mid h_t\right]$ as problem (3). Under Assumption 2, for every $t < T-1$ and every h_t, the candidates' first order conditions with respect to x_t and y_t are thus respectively:

$$\delta^{T-1-t} p_x(x_t, y_t) = p_x(x_{T-1}, y_{T-1})$$
$$\delta^{T-1-t} p_y(x_t, y_t) = p_y(x_{T-1}, y_{T-1})$$

Taking the ratio of these two first order conditions and noting that the partial derivatives of p are homogeneous of degree $\beta - 1$, we get:

$$\frac{p_x\left(\frac{x_t}{y_t}, 1\right)}{p_y\left(\frac{x_t}{y_t}, 1\right)} = \frac{p_x\left(\frac{x_{T-1}}{y_{T-1}}, 1\right)}{p_y\left(\frac{x_{T-1}}{y_{T-1}}, 1\right)}$$

Assumption 1 implies that equilibrium spending levels are interior and unique. Thus, we must have $x_t/y_t = x_{T-1}/y_{T-1}$ for every period t. Using the candidates' budget constraints, we get that for all periods t, $x_t/X_t = y_t/Y_t$. This immediately implies $x_0/y_0 = X_0/Y_0$. Suppose for the sake of the induction argument that $x_{t'}/y_{t'} = X_0/Y_0$ for every period $t' \leq t$; then

$$\frac{x_{t+1}}{y_{t+1}} = \frac{X_{t+1}}{Y_{t+1}} = \frac{X_t - x_t}{Y_t - y_t} = \frac{X_t - x_t}{\frac{X_t}{x_t} y_t - y_t} = \frac{X_t - x_t}{\frac{X_t - x_t}{x_t} y_t} = \frac{x_t}{y_t} = \frac{X_0}{Y_0}$$

where the first and third equalities hold because $x_t/X_t = y_t/Y_t$ for every t and the last equality holds by the inductive hypothesis. Hence, by induction $x_t/y_t = X_0/Y_0$ for every t. (This also implies that $r_{1,t} = r_{2,t} = r_t$ for every $t < T-1$.)

Now consider part (ii). For any two consecutive periods t and $t+1$ take candidate 1's first order condition among the following pair

$$\delta p_x\left(x_t, y_t\right) = p_x\left(x_{t+1}, y_{t+1}\right)$$
$$\delta p_y\left(x_t, y_t\right) = p_y\left(x_{t+1}, y_{t+1}\right)$$

and note that because the partial derivatives of p are homogeneous of degree $\beta - 1$ we have

$$\delta (x_t)^{\beta-1} p_x(1, y_t/x_t) = (x_{t+1})^{\beta-1} p_x(1, y_{t+1}/x_{t+1})$$

The equal spending ratio result proven above says that that $y_t/x_t = y_{t+1}/x_{t+1} = X_0/Y_0$. Substituting this into the centered equation above and simplifying we get

$$r_{1,t} = x_{t+1}/x_t = \delta^{1/(\beta-1)}.$$

The result for candidate 2 follows from the fact that $r_{1,t} = r_{2,t}$. $\square$

A.3 Proof of Proposition 3

Consider the periods in which voters cast their votes: $\hat{T}$,..., T. We can write the popularity processes at the beginning of these periods as:

$$\begin{aligned}
Z_T &= \sum_{t=0}^{T-1} \delta^{T-1-t} p(x_t, y_t) + \delta^T z_0 + \sum_{t=0}^{T-1} \delta^{T-1-t} \varepsilon_t, \\
Z_{T-1} &= \sum_{t=0}^{T-2} \delta^{T-2-t} p(x_t, y_t) + \delta^{T-1} z_0 + \sum_{t=0}^{T-2} \delta^{T-2-t} \varepsilon_t, \\
&\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \\
Z_{\hat{T}} &= \sum_{t=0}^{\hat{T}-1} \delta^{\hat{T}-1-t} p(x_t, y_t) + \delta^{\hat{T}} z_0 + \sum_{t=0}^{\hat{T}-1} \delta^{\hat{T}-1-t} \varepsilon_t.
\end{aligned}$$

Substituting these expressions into the candidates' objective function, we get:

$$\Pr\left[\sum_{t=\hat{T}}^{T} \xi^{T-t} Z_t \geq 0\right] = \Pr\left[\sum_{t=\hat{T}}^{T} \xi^{T-t} E_t \geq -\sum_{t=\hat{T}}^{T} \xi^{T-t} B_t\right]$$

where $E_t := \sum_{t'=0}^{t-1} \delta^{T-1-t'} \varepsilon_{t'}$ and $B_t := \sum_{t'=0}^{t-1} \delta^{t-1-t'} p(x_{t'}, y_{t'}) + \delta^t z_0$.

Each E_t is the sum of normally distributed shocks with zero mean and with a variance that does not depend on candidates' spending. We can thus assume that candidate 1 maximizes (and candidate 2 minimizes) $\sum_{t=\hat{T}}^{T} \xi^{T-t} B_t$, or equivalently

$$\sum_{t=0}^{\hat{T}-1} \sum_{t'=0}^{T-\hat{T}} \xi^{t'} \delta^{T-1-t-t'} p(x_t, y_t) + \sum_{t=\hat{T}}^{T-1} \sum_{t'=0}^{T-1-t} \xi^{t'} \delta^{T-1-t-t'} p(x_t, y_t)$$

The same steps of the proof of Proposition 2 allow us to show that $x_t/X_t = y_t/Y_t$ for every t, and that the cross-candidate spending ratio, x_t/y_t, is constant over time. In particular, $x_t/y_t = X_0/Y_0$. This establishes part (i).

For part (ii) consider a period $t < \hat{T} - 1$. The same steps used in the proof of Proposition 2 yield that the consecutive period spending ratio for every t is constant across players and it is equal to the one derived for the baseline model; that is, $r_t = \delta^{1/(\beta-1)}$ for all $t < \hat{T} - 1$. Next, consider a period $t \geq \hat{T} - 1$. The result of part (i) implies that even in this case $r_{1,t} = r_{2,t}$, so we can focus on candidate 1's first order conditions. If we equate her first order conditions for two consecutive periods and use the homogeneity of function p we get

$$\left(\frac{x_{t+1}}{x_t}\right)^{\beta-1} = \frac{\sum_{t'=0}^{T-1-t} \xi^{t'} \delta^{T-1-t-t'}}{\sum_{t'=0}^{T-2-t} \xi^{t'} \delta^{T-2-t-t'}}.$$

From this we obtain:

$$r_t = \left[\delta \left(1 + \frac{1}{\sum_{t'=0}^{T-2-t} \xi^{-(T-1-t-t')} \delta^{T-2-t-t'}}\right)\right]^{1/(\beta-1)}.$$

A.4 Proof of Proposition 4

Under Assumptions 1 and 2 the function $q(x/y)$ defined as $p(x/y, 1)$ is strictly increasing and strictly quasiconcave. Pick an arbitrary history h_{T-1} up to period $T-1$ and let $(\hat{x}_t)_{t=0}^{T-2}$ and $(\hat{y}_t)_{t=0}^{T-2}$ be the amounts spent by the candidates along this history. Denote the choice variable for candidate 1's spending at history h_{T-1} by x_{T-1} and for candidate 2 by y_{T-1}. Candidate 1 maximizes $\mathbb{E}[\mathbf{1}_{\{Z_T \geq 0\}} + \kappa_1 X_T \mid h_{T-1}]$ and candidate 2 maximizes $\mathbb{E}[(1 - \mathbf{1}_{\{Z_T \geq 0\}}) + \kappa_2 Y_T \mid h_{T-1}]$. Let

$$L[h_{T-1}] = \sum_{t=0}^{T-2} \delta^{T-1-t} q\left(\frac{\hat{x}_t}{\hat{y}_t}\right) + q\left(\frac{x_{T-1}}{y_{T-1}}\right) + \delta^T z_0 + \sum_{t=0}^{T-2} \delta^{T-1-t} \varepsilon_t.$$

Given that $\varepsilon_{T-1} \sim \mathcal{N}(0, \sigma^2)$, we have that $Z_T \mid h_{T-1} \sim \mathcal{N}(L[h_{T-1}], \sigma^2)$. Hence, the first order conditions of the two candidates candidates are respectively:

$$\phi_{(0,1)}\left(-\frac{\hat{L}[h_{T-1}]}{\sigma}\right)\frac{1}{\sigma} q'\left(\frac{\hat{x}_{T-1}}{\hat{y}_{T-1}}\right)\frac{1}{\hat{y}_{T-1}} = \kappa_1$$

$$\phi_{(0,1)}\left(-\frac{\hat{L}[h_{T-1}]}{\sigma}\right)\frac{1}{\sigma} q'\left(\frac{\hat{x}_{T-1}}{\hat{y}_{T-1}}\right)\frac{\hat{x}_{T-1}}{(\hat{y}_{T-1})^2} = \kappa_2$$

where $\phi_{(0,1)}$ is the pdf of the standard normal, $\hat{x}_{T-1}$ and $\hat{y}_{T-1}$ are equilibrium values of x_{T-1} and y_{T-1} following history h_{T-1}, and $\hat{L}[h_{T-1}]$ is the value that $L[h_{T-1}]$ takes when

$x_{T-1}/y_{T-1} = \hat{x}_{T-1}/\hat{y}_{T-1}$. Taking the ratio of these first order conditions gives $\hat{x}_{T-1}/\hat{y}_{T-1} = \kappa_2/\kappa_1$, which is independent of the history h_{T-1}. Thus

$$\hat{x}_{T-1} = \frac{\kappa_2}{(\kappa_1)^2}\phi_{(0,1)}\left(-\frac{\hat{L}[h_{T-1}]}{\sigma}\right)\left(\frac{1}{\sigma}q'\left(\frac{\kappa_2}{\kappa_1}\right)\right)$$

$$\hat{y}_{T-1} = \frac{1}{\kappa_1}\phi_{(0,1)}\left(-\frac{\hat{L}[h_{T-1}]}{\sigma}\right)\left(\frac{1}{\sigma}q'\left(\frac{\kappa_2}{\kappa_1}\right)\right)$$

Both spending decisions are decreasing in $|\hat{L}[h_{T-1}]|$, which depends on history.

Now assume for the sake of an inductive argument that for all histories h_t with $t \in \{\tilde{t}+1, \tilde{t}+2, ..., T-1\}$, we have that in an interior equilibrium, (i) $\hat{x}_t/\hat{y}_t = \kappa_2/\kappa_1$ where $\hat{x}_t$ and $\hat{y}_t$ are the equilibrium amounts spent following history h_t, and (ii) spending decisions are given by:

$$\hat{x}_t = \frac{\kappa_2}{(\kappa_1)^2}\phi_{(0,1)}\left(-\frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{T-1-t'}}}\right)\left(\frac{\delta^{T-1-t}}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{T-1-t'}}}q'\left(\frac{\kappa_2}{\kappa_1}\right)\right)$$

$$\hat{y}_t = \frac{1}{\kappa_1}\phi_{(0,1)}\left(-\frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{T-1-t'}}}\right)\left(\frac{\delta^{T-1-t}}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{T-1-t'}}}q'\left(\frac{\kappa_2}{\kappa_1}\right)\right),$$

where

$$\hat{L}[h_t] = \sum_{t'=0}^{t-1}\delta^{T-1-t'}\left[q\left(\frac{\hat{x}_{t'}}{\hat{y}_{t'}}\right)+\varepsilon_{t'}\right]+\delta^T z_0 + \sum_{t'=t}^{T-1}\delta^{T-1-t'}q\left(\frac{\kappa_2}{\kappa_1}\right),$$

and $(\hat{x}_{t'})_{t'=0}^{t-1}$ and $(\hat{y}_{t'})_{t'=0}^{t-1}$ are the spending choices of candidates along history h_t. Obviously, spending decisions $\hat{x}_t$ and $\hat{y}_t$ are decreasing in $|\hat{L}[h_t]|$.

Consider period t and pick an arbitrary history h_t. Since $(\varepsilon_{t'})_{t'=0}^{T-1}$ are iid shocks distributed according to $\mathcal{N}(0,\sigma^2)$ and (by the inductive hypothesis) the ratios of spending decision in subsequent periods are history independent and equal to κ_2/κ_1, we have that $Z_T \mid h_t \sim \mathcal{N}(L[h_t], \sigma^2\sum_{t'=t}^{T-1}\delta^{2(T-1-t')})$, where

$$L[h_t] = \sum_{t'=0}^{t-1}\delta^{T-1-t'}\left[q\left(\frac{\hat{x}_{t'}}{\hat{y}_{t'}}\right)+\varepsilon_t\right]+\delta^T z_0 + \delta^{T-1-t}q\left(\frac{x_t}{y_t}\right)+\sum_{t'=t+1}^{T-1}\delta^{T-1-t'}q\left(\frac{\kappa_2}{\kappa_1}\right),$$

and $(\hat{x}_{t'})_{t'=0}^{t-1}$ and $(\hat{y}_{t'})_{t'=0}^{t-1}$ are the amounts spent by candidates along history h_t, and x_t and y_t are the choice variables for the candidates' spending levels at history h_t.

The first order conditions for an interior optimum are

$$\phi_{(0,1)}\quad - \frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{2(T-1-t')}}}\Bigg)\Bigg(\frac{\delta^{T-1-t}}{\alpha\sqrt{\sum_{j=t}^{T-1}\delta^{2(T-1-j)}}}q'\left(\frac{\hat{x}_t}{\hat{y}_t}\right)\frac{1}{\hat{y}_t} = \kappa_1$$

$$\phi_{(0,1)}\quad - \frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{2(T-1-t')}}}\Bigg)\Bigg(\frac{\delta^{T-1-t}}{\alpha\sqrt{\sum_{j=t}^{T-1}\delta^{2(T-1-j)}}}q'\left(\frac{\hat{x}_t}{\hat{y}_t}\right)\frac{\hat{x}_t}{(\hat{y}_t)^2} = \kappa_2$$

where $\hat{L}[h_t]$ is equal to $L[h_t]$ after replacing the ratio x_t/y_t with $\hat{x}_t/\hat{y}_t$. Taking the ratio of these expressions, we get $\hat{x}_t/\hat{y}_t = \kappa_2/\kappa_1$, which is independent of the past, and the candidates' equilibrium spending decisions are

$$\hat{x}_t = \frac{\kappa_2}{(\kappa_1)^2}\phi_{(0,1)}\quad - \frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{2(T-1-t')}}}\Bigg)\Bigg(\frac{\delta^{T-1-t}}{\alpha\sqrt{\sum_{j=t}^{T-1}\delta^{2(T-1-j)}}}q'\left(\frac{\hat{x}_t}{\hat{y}_t}\right)\Bigg)$$

$$\hat{y}_t = \frac{1}{\kappa_1}\phi_{(0,1)}\quad - \frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{2(T-1-t')}}}\Bigg)\Bigg(\frac{\delta^{T-1-t}}{\alpha\sqrt{\sum_{j=t}^{T-1}\delta^{2(T-1-j)}}}q'\left(\frac{\hat{x}_t}{\hat{y}_t}\right)\Bigg)$$

Given the condition $\hat{x}_t/\hat{y}_t = \kappa_2/\kappa_1$, we have $\hat{L}[h_t] = \zeta((\varepsilon_{t'})_{t'=0}^{t-1})$, where $(\varepsilon_{t'})_{t'=0}^{t-1}$ are the shocks along history h_t. Thus, the candidates' equilibrium spending levels are decreasing $|\zeta((\varepsilon_{t'})_{t'=0}^{t-1})|$. The argument that we have given establishes both parts (i) and (ii) of the proposition by induction. Part (iii) follows from the expressions for $\hat{x}_t$, $\hat{y}_t$, and $L[h_t]$ above.

□

A.5 Proof of Proposition 5

We start observing that there cannot be an equilibrium in which both candidates spend an amount equal to 0 on some subgroup in the same period. In this case, footnote 16 implies that either candidate would have an incentive to deviate and spend a positive amount, securing victory with probability 1.

Furthermore, there cannot be an equilibrium in which one of the two candidates spend an amount equal to 0 on a subpopulation, say subgroup s, in a given period. In this case, the candidate would lose with certainty and she would be better off saving a small amount from each of the other subgroups and investing the saved amount in subgroup s. In equilibrium spending must then be interior, satisfying the first order conditions for any subgroup and in any period.

We now prove part (i) of the proposition by induction. Consider the final period. Fix $(z^s_{T-1})^S_{s=1}$ arbitrarily. Suppose candidates 1 and 2 have resource stocks equal to X_{T-1} and Y_{T-1} at the beginning of the last period. Fix an equilibrium strategy profile $(\hat{x}^s_{T-1}, \hat{y}^s_{T-1})^S_{s=1}$. We will show that if the candidates have budgets ϑX_{T-1} and ϑY_{T-1}, then $(\vartheta\hat{x}^s_{T-1}, \vartheta\hat{y}^s_{T-1})^S_{s=1}$ is an equilibrium, which in turn implies that the equilibrium payoff in the last period is determined by $(z^s_{T-1})^S_{s=1}$ and X_{T-1}/Y_{T-1} only. Suppose otherwise. Without loss of generality, assume that there exists $(\tilde{x}^s_{T-1})^S_{s=1}$ satisfying $\sum_{s=1}^S \tilde{x}^s_{T-1} \leq \vartheta X_{T-1}$ that gives a higher probability of winning to candidate 1 given $(z^s_{T-1})^S_{s=1}$ and $(\vartheta\hat{y}^s_{T-1})^S_{s=1}$. The distribution of $(Z^s_T)^S_{s=1}$ is determined by $(z^s_{T-1})^S_{s=1}$ and $(x^s_{T-1}/y^s_{T-1})^S_{s=1}$ only. This means that the distribution of $(Z^s_T)^S_{s=1}$ given $(z^s_{T-1})^S_{s=1}$ and $(\tilde{x}^s_{T-1}/\vartheta\hat{y}^s_{T-1})^S_{s=1}$ is more favorable to candidate 1 than that given $(z^s_{T-1})^S_{s=1}$ and $(\hat{x}^s_{T-1}/\vartheta\hat{y}^s_{T-1})^S_{s=1}$. Obviously, $(\vartheta\hat{x}^s_{T-1}/\vartheta\hat{y}^s_{T-1})^S_{s=1} = (\hat{x}^s_{T-1}/\hat{y}^s_{T-1})^S_{s=1}$ and candidate 1 could spend $(\frac{1}{\vartheta}\tilde{x}^s_{T-1})^S_{s=1}$ when the budgets are (X_{T-1}, Y_{T-1}). Because $(\hat{x}^s_{T-1}, \hat{y}^s_{T-1})^S_{s=1}$ is an equilibrium, the distribution of $(Z^s_T)^S_{s=1}$ given $(z^s_{T-1})^S_{s=1}$ is more favorable to candidate 1 under $(\hat{x}^s_{T-1}/\hat{y}^s_{T-1})^S_{s=1}$ than under $(\frac{1}{\vartheta}\tilde{x}^s_{T-1}/\hat{y}^s_{T-1})^S_{s=1} = (\tilde{x}^s_{T-1}/\vartheta\hat{y}^s_{T-1})^S_{s=1}$. This establishes a contradiction.

Now, we prove the inductive step. The inductive hypotheses are (i) that the continuation payoff for either candidate in period $t' \geq t+1$ can be written as a function of only the budget ratio $X_{t'}/Y_{t'}$ and the vector $(z^s_{t'})^S_{s=1}$, and (ii) second that $x^s_{t'}/X_{t'} = y^s_{t'}/Y_{t'}$ for every subgroup s and every period $t' \geq t+1$. We want to show that $x^s_t/X_t = y^s_t/Y_t$ in each subgroup s and that the continuation value at time t can be written as a function of only the budget ratio X_t/Y_t and the vector $(z^s_t)^S_{s=1}$. For each period t, let $x_t = \sum_s x^s_t$, $y_t = \sum_s y^s_t$ and let $z_t = (z^s_t)^S_{s=1}$. Let $V_{t+1}\left(\frac{X_{t+1}}{Y_{t+1}}, z_{t+1}\right)$ denote the continuation payoff of candidate 1 starting in period $t+1$. Candidate 1's objective is

$$\max_{(x^s_t)^S_{s=1}} \int V_{t+1}\left(\frac{X_t - x_t}{Y_t - y_t}, z_{t+1}\right) \phi_t \left(z_{t+1} \mid \left(\frac{x^s_t}{y^s_t}\right)^S_{s=1}, z_t\right) dz_{t+1}$$

where $\phi_t(\cdot|\cdot)$ is the conditional distribution of the vector z_{t+1}. For each subgroup s, the first order conditions for an interior optimum for candidate 1 is then

$$\begin{aligned}\frac{1}{Y_t - y_t} &\int \frac{\partial V_{t+1}\left((X_t - x_t)/(Y_t - y_t), z_{t+1}\right)}{\partial (x^s_t/y^s_t)} \phi_t \left(z_{t+1} \mid (x^s_t/y^s_t)^S_{s=1}, z_t\right) dz_{t+1} \\ &= \frac{1}{y^s_t} \int V_{t+1}\left(\frac{X_t - x_t}{Y_t - y_t}, z_{t+1}\right) \frac{\partial \phi_t \left(z_{t+1} \mid (x^s_t/y^s_t)^S_{s=1}, z_t\right)}{\partial (x^s_t/y^s_t)} dz_{t+1}.\end{aligned}$$

Similarly, the objective function for candidate 2 is

$$\min_{(y_t^s)_{s=1}^S} \int V_{t+1}\left(\frac{X_t - x_t}{Y_t - y_t}, z_{t+1}\right) \phi_t \left(z_{t+1} \mid \left(\frac{x_t^s}{y_t^s}\right)_{s=1}^S, z_t\right) dz_{t+1}$$

and the corresponding first order condition for each s is

$$\frac{X_t - x_t}{(Y_t - y_t)^2} \int \frac{\partial V_{t+1}\left((X_t - x_t)/(Y_t - y_t), z_{t+1}\right)}{\partial (x_t^s / y_t^s)} \phi_t \left(z_{t+1} \mid (x_t^s/y_t^s)_{s=1}^S, z_t \right) dz_{t+1}$$
$$= \frac{x_t^s}{(y_t^s)^2} \int V_{t+1}\left(\frac{X_t - x_t}{Y_t - y_t}, z_{t+1}\right) \frac{\partial \phi_t \left(z_{t+1} \mid (x_t^s/y_t^s)_{s=1}^S, z_t \right)}{\partial (x_t^s / y_t^s)} dz_{t+1}.$$

Dividing the candidate 1's first order condition by candidate 2's, we have

$$\frac{X_t - x_t}{Y_t - y_t} = \frac{x_t^s}{y_t^s},$$

which implies $x_t^s / y_t^s = X_t / Y_t$ for all s. As a result, the continuation value of candidates in period t is a function of only the budget ratio X_t / Y_t and the vector $(z_t^s)_{s=1}^S$. Part (i) of the proposition follows by induction.

Now for part (ii), note that through iterative substitution we can write:

$$\sum_{s=1}^S w^s Z_T^s = \sum_{s=1}^S w^s \left[\sum_{t=0}^{T-1} (\delta^s)^{T-1-t} p\left(x_t^s, y_t^s\right) + (\delta^s)^T z_0^s \right] + \sum_{s=0}^S w^s \sum_{t=0}^{T-1} (\delta^s)^{T-1-t} \varepsilon_t^s.$$

Since

$$\mathbb{E}\left[\sum_s w^s Z_T^s \right] = \sum_{s=1}^S w^s \sum_{t=0}^{T-1} (\delta^s)^{T-1-t} p\left(x_t^s, y_t^s\right) + (\delta^s)^T z_0^s \text{ for all } \left(x_t^s, y_t^s\right)_{t,s},$$

candidate 1 maximizes

$$\sum_{s=1}^S w^s \sum_{t=0}^{T-1} (\delta^s)^{T-1-t} p\left(x_t^s, y_t^s\right)$$

subject to $\sum_{s=1}^S \sum_{t=0}^{T-1} x_t^s = X_0$, and candidate 2 minimizes the same expression subject to $\sum_{s=0}^S \sum_{t=0}^{T-1} y_t^s = Y_0$. Now, fix a subpopulation s and define $X_{-s} = \sum_{s' \neq s} \sum_{t=0}^{T-1} x_t^{s'}$ and $Y_{-s} = \sum_{s' \neq s} \sum_{t=0}^{T-1} y_t^{s'}$. If we focus on this subpopulation, candidate 1 chooses $(x_0^1, ... x_{T-1}^s)$ to maximize $\sum_{t=0}^{T-1} w^s (\delta^s)^{T-1-t} p\left(x_t^s, y_t^s\right)$ subject to $\sum_t x_t^s = X_0 - X_{-s}$, and player 2 chooses $(y_0^s, ..., y_{T-1}^s)$ to minimize the same expression subject to $\sum_t y_t^s = Y_0 - Y_{-s}$. Given that

$p\left(x_t^s, y_t^s\right) = q\left(x_t^s / y_t^s\right)$, we obtain the following set of first order conditions:

$$\begin{aligned} w^s \left(\delta^s\right)^{T-1-t} q'\left(x_t^s / y_t^s\right) / y_t^s &= w^s \left(\delta^s\right)^{T-1-t'} q'\left(x_{t'}^s / y_{t'}^s\right) / y_{t'}^s \text{ for all } t, t', \\ w^s \left(\delta^s\right)^{T-1-t} q'\left(x_t^s / y_t^s\right) x_t^s / \left(y_t^s\right)^2 &= w^s \left(\delta^s\right)^{T-1-t'} q'\left(x_{t'}^s / y_{t'}^s\right) x_{t'}^s / \left(y_{t'}^s\right)^2 \text{ for all } t, t'. \end{aligned}$$

Taking the ratio of the first order conditions and exploiting the budget constraint like we did in the proof of Proposition 2, we get $\frac{x_t^s}{y_t^s} = \frac{x_{t'}^s}{y_{t'}^s} = \frac{X_0 - X_{-s}}{Y_0 - Y_{-s}}$. Substituting this expression in the first order condition, we obtain

$$\frac{x_t^s}{x_{t'}^s} = \frac{y_t^s}{y_{t'}^s} = \left(\delta^s\right)^{t'-t}.$$

Proposition 5(ii) follows by setting $t = t' + 1$.

For part (iii) of the proposition we can proceed in a similar way. Fix a period t, and let $X_{-t} = \sum_{s=1}^S \sum_{t' \neq t} x_{t'}^s$ and $Y_{-t} = \sum_{s=1}^S \sum_{t' \neq t} y_{t'}^s$. Candidate 1 then chooses $\left(x_t^1, ..., x_t^S\right)$ to maximize $\sum_{s=1}^S w^s \left(\delta^s\right)^{T-1-t} p\left(x_t^s, y_t^s\right)$ subject to $\sum_{s=1}^S x_t^s = X_0 - X_{-t}$, while candidate 2 chooses $\left(y_t^1, ..., y_t^S\right)$ to minimize the same expression subject to $\sum_{s=1}^S y_t^s = Y_0 - Y_{-t}$. Given $p\left(x_t^s, y_t^s\right) = q\left(x_t^s / y_t^s\right)$, the first order condition implies

$$\begin{aligned} \left(\delta^s\right)^{T-1-t} w^s q'\left(x_t^s / y_t^s\right) / y_t^s &= \left(\delta^{s'}\right)^{T-1-t} w^{s'} q'\left(x_t^{s'} / y_t^{s'}\right) / y_t^{s'} \text{ for all } s, s', \\ \left(\delta^s\right)^{T-1-t} w^s q'\left(x_t^s / y_t^s\right) x_t^s / y_t^s &= \left(\delta^{s'}\right)^{T-1-t} w^{s'} q'\left(x_t^{s'} / y_t^{s'}\right) x_t^{s'} / \left(y_t^{s'}\right)^2 \text{ for all } s, s'. \end{aligned}$$

Taking the ratio of these first order conditions and exploiting the budget constraint, we get $\frac{x_t^s}{y_t^s} = \frac{x_t^{s'}}{y_t^{s'}} = \frac{X_0 - X_{-t}}{Y_0 - Y_{-t}}$. Substituting this back in the first order condition, we obtain Proposition 5(iii):

$$\frac{x_t^s}{x_{t'}^s} = \frac{y_t^s}{y_{t'}^s} = \frac{w^s}{w^{s'}} \left(\frac{\delta^s}{\delta^{s'}}\right)^{T-1-t}.$$

Senate Elections in our Baseline Sample

Year	State
2000	DE, FL, IN, ME, MI, MN, MO, NE, NV, NY, PA, RI, VA, WA
2002	AL, AR, CO, GA, IA, LA, ME, NC, NH, NJ, OK, OR, SC, TN, TX
2004	CO, FL, GA, KY, LA, NC, OK, PA, SC, WA
2006	AZ, MD, MI, MO, NE, OH, PA, RI, TN, VA, WA, WV
2008	AK, CO, GA, ID, KS, KY, LA, ME, MS, NC, NE, NH, NM, OK, OR, SD
2010	AL, AR, CA, CO, CT, IA, IL, IN, KY, LA, MD, MO, NH, NV, NY, OR, PA, VT, WA
2012	AZ, CT, FL, HI, IN, MA, MO, MT, ND, NE, NM, NV, OH, PA, RI, VA, WI, WV
2014	AK, AR, CO, GA, IA, IL, KY, LA, ME, MI, MT, NC, NH, NM, OR, SD, VA, WV

Gubernatorial Elections in our Baseline Sample

Year	State
2000	IN, MO, NC, NH, WA, WV
2002	AL, AR, AZ, CA, CT, FL, GA, HI, IA, IL, KS, MA, MD, ME, MI, NM, NY, OK, OR, PA, RI, SC, TN, TX, WI
2004	IN, MO, NC, NH, UT, VT, WA
2006	AL, AR, AZ, CO, CT, FL, GA, IA, IL, KS, MD, ME, MI, MN, NH, NV, NY, OH, OR, PA, RI, TN, VT, WI
2008	IN, MO, NC, WA
2010	AK, AL, AR, AZ, CA, CT, FL, GA, HI, IA, ID, IL, MA, MD, MI, MN, NH, NM, NV, NY, OH, OK, OR, PA, SC, SD, TN, TX, UT, VT, WI
2012	IN, MO, MT, NC, ND, NH, WA, WV
2014	AL, AR, AZ, CO, CT, FL, GA, HI, IA, ID, IL, KS, MA, MD, ME, MI, MN, NE, NH, NM, NY, OH, OK, OR, PA, SC, TX, WI

House Elections in our Baseline Sample

Year	State-District
2000	AL-4, AR-4, CA-20, CA-49, CO-6, CT-5, FL-12, FL-22, FL-8, GA-7, KS-3, KY-3, KY-6, MI-8, MN-6, MO-2, MO-3, MO-6, NC-11, NC-8, NH-1, NH-2, NM-1, NV-1, OH-1, OH-12, OK-2, PA-10, PA-13, PA-4, TX-25, UT-2, VA-2, WA-1, WA-5, WV-2
2002	AL-1, AL-3, AR-4, CT-5, FL-22, IA-1, IA-2, IA-3, IA-4, IL-19, IN-2, KS-3, KS-4, KY-3, ME-2, MI-9, MS-3, NH-1, NH-2, NM-1, NM-2, OK-4, PA-11, PA-17, SC-3, TX-11, UT-2, WV-2
2004	CA-20, CO-3, CT-2, CT-4, FL-13, GA-12, IA-3, IN-8, KS-3, KY-3, MO-5, MO-6, NC-11, NE-2, NM-1, NM-2, NV-3, NY-27, OK-2, OR-1, TX-17, WA-5, WV-2
2006	AZ-5, AZ-8, CO-4, CO-7, CT-2, CT-4, CT-5, FL-13, FL-22, GA-12, HI-2, IA-1, IA-3, ID-1, IL-6, IN-2, IN-8, IN-9, KY-2, KY-3, KY-4, MN-6, NC-11, NH-2, NM-1, NV-3, NY-20, NY-24, NY-25, NY-29, OH-1, OH-12, OH-15, OH-18, OR-5, PA-10, SC-5, TX-17, VA-2, VA-5, VT-1, WA-5, WI-8
2008	AK-1, AL-2, AL-3, AL-5, AZ-3, AZ-5, AZ-8, CA-11, CA-4, CO-4, CT-4, CT-5, FL-16, FL-24, FL-8, GA-8, ID-1, IL-10, IN-3, KY-2, KY-3, LA-4, LA-6, MD-1, MI-7, MO-6, NC-8, NH-1, NH-2, NM-1, NM-2, NV-2, NV-3, NY-20, NY-24, NY-25, NY-26, NY-29, OH-1, OH-15, PA-10, PA-11, SC-1, VA-2, VA-5, WI-8, WV-2
2010	AL-2, AL-5, AR-2, AZ-1, AZ-5, AZ-8, CA-20, CA-45, CO-3, CO-4, CT-4, CT-5, FL-2, FL-22, FL-24, FL-8, GA-12, GA-8, HI-1, IA-1, IA-2, IA-3, IN-2, IN-8, KS-4, KY-6, MA-1, MD-1, MD-2, MI-1, MI-3, MI-7, MI-9, MN-6, MO-3, MO-4, MO-8, MS-1, NC-2, NC-5, NC-8, NE-2, NH-1, NH-2, NM-1, NM-2, NV-3, NY-20, NY-23, NY-24, NY-25, OH-1, OH-12, OH-13, OH-15, OH-16, OH-9, OK-5, OR-3, OR-5, PA-10, PA-11, PA-4, SC-2, SC-5, SD-1, TN-1, TN-4, TN-8, TN-9, TX-17, VA-2, VA-5, VA-9, WA-2, WI-8, WV-3
2012	AZ-2, CA-10, CA-24, CA-3, CA-36, CA-52, CA-7, CA-9, CO-3, CO-6, CO-7, CT-5, FL-18, GA-12, HI-1, IA-1, IA-2, IA-3, IA-4, IL-12, IL-13, IL-17, IL-8, IN-2, IN-8, KY-6, MA-6, ME-2, MI-6, MN-6, MN-8, MT-1, NC-7, ND-1, NH-1, NH-2, NM-1, NV-3, NY-19, NY-21, NY-24, NY-25, NY-27, OH-16, OH-6, PA-12, RI-1, SD-1, TX-23, UT-4, VA-2, VA-5, WI-8, WV-3
2014	AR-2, AZ-1, AZ-2, CA-21, CA-36, CA-52, CA-7, CO-6, CT-5, FL-18, FL-2, FL-26, GA-12, HI-1, IA-1, IA-2, IA-3, IL-10, IL-12, IL-13, IL-17, IN-2, ME-2, MI-7, MN-7, MN-8, MT-1, ND-1, NE-2, NH-2, NM-2, NV-3, NY-19, NY-21, NY-23, NY-24, VA-10, VA-2