# Voter Turnout and Preference Aggregation*


Kei Kawai†
University of California at Berkeley

Yuta Toyama‡
Northwestern University

Yasutora Watanabe§
University of Tokyo


August 2017


**Abstract**

We study how voter turnout affects the aggregation of voter preferences in elections. When voting is voluntary, election outcomes disproportionately aggregate the preferences of voters with low voting cost and high preference intensity. We show identification of the correlation structure among preferences, costs, and perceptions of voting efficacy, and explore how the correlation affects preference aggregation. Using county-level data from the 2004 U.S. presidential election, we find that young, low-income, less-educated, and minority voters are underrepresented. All of these groups tend to prefer Democrats, except for the less-educated. Democrats would have won the majority of the popular and the electoral votes if all eligible voters had turned out.



keyword: *voter turnout, preference aggregation, election*



*We thank Heski Bar-Isaac, Alessandra Casella, Hulya Eraslan, Matias Iaryczower, Karam Kang, Alessandro Lizzeri, Antonio Merlo, Eduardo Morales, Kris Ramsay, Raul Sanchez de la Sierra, Laura Silver, Francesco Trebbi, and Chamna Yoon for helpful comments. Yasutora Watanabe acknowledges financial support from the Research Grants Council of Hong Kong Government under the General Research Fund Projects 16506015 and 16528116.



†Department of Economics, University of California at Berkeley 530 Evans Hall #3880 Berkeley, CA, 94720-3880. Email: kei@berkeley.edu

‡Department of Economics, Northwestern University, 2211 Campus Drive, Evanston, IL 60208. Email: yuta-toyama@u.northwestern.edu

§Department of Economics, University of Tokyo, 7-3-1 Hongo, Tokyo, Japan. Email: yasutora.watanabe@gmail.com

# 1 Introduction

Democracies rely on elections to aggregate the preferences of their citizens. Elections, however, aggregate the preferences of only those that participate. The importance of participation for preference aggregation is documented by studies of suffrage expansion in various contexts, such as the abolition of apartheid in South Africa (Kroth et al., 2013), the passage of the Voting Rights Act of 1965 (Husted and Kenny, 1997; Cascio and Washington, 2013), and the passage of women's suffrage laws (Miller, 2008). Less dramatic measures that have reduced the voting costs of certain groups of voters have also been found to affect policy in important ways (Fujiwara, 2015).

While most democracies now enjoy universal suffrage, participation in elections is far from perfect, given the voluntary nature of voting. To the extent that the preferences of those that turn out are systematically different from those that do not, election outcomes may poorly aggregate the preferences of all citizens. Thus, how well elections aggregate the preferences of citizens and whose preferences are underrepresented are open questions, even in mature democracies.

The issue of preference aggregation and underrepresentation are also relevant from a policy perspective. The concern that the preferences of certain groups of voters are underrepresented has led some to argue for compulsory voting (see, e.g., Lijphart, 1997). More moderate policy proposals, such as introducing Internet voting, relaxing registration requirements, and making election day a holiday, are motivated by similar concerns. Understanding how voter turnout affects preference aggregation can provide a basis for more informed discussions of these policy proposals. This paper also contributes to the policy debate on partisan districting, in particular, the discussion over how to measure gerrymandering. Gerrymandering can be considered as an intentional attempt by one party to aggregate preferences disproportionately in its favor. Much of the recent discussion on gerrymandering focuses on how well actual votes are translated into seat shares, but ignores how preferences map to seat shares. Given that turnout is endogenous, however, it is possible for sophisticated planners to design redistricting plans that map vote shares to seat shares well, but map preferences to seat shares poorly. Studying how underlying preferences, rather than votes, are aggregated into election outcomes provides a coherent alternative even when turnout is endogenous.

In this study, we explore the extent to which preferences are aggregated in elections, which hinges on how the preferences, voting costs, and perceptions of voting efficacy are correlated. We show identification of the joint distribution of these three terms, and estimate it using county-level voting data from the 2004 U.S. presidential election. We find that young, low-income, less-educated, and minority voters have a high cost of voting and that all of these groups tend to prefer the Democrats, except for the less-educated. We find that voter ID requirement is associated with higher voting costs while the effect of same-day registration is not statistically significant. We then simulate the counterfactual election outcome when all voters vote. The difference between the simulated and actual outcomes allows us to quantify the degree to which preferences are aggregated. In our counterfactual, the two-party vote share of the Democrats increases by about 3.8%, and the Democrats win the plurality of the electoral votes. We also consider another counterfactual in which we elimiate endogeneity in turnout that is state-specific. This allows us to gauge the sensitivity of the efficiency gap, an influential measure of gerrymandering, to endogeneity of turnout.

The key challenge in studying the effect of turnout on preference aggregation is to identify the correlation between preferences and voting costs in the population. In particular, we need to identify how voter characteristics such as race and income simultaneously determine preferences and costs. However, this is not a straightforward task because a high level of turnout among a particular set of voters may be due to low voting cost or high preference intensity.

To illustrate, consider a plurality rule election in which voters have private values and choose to vote for candidate $A$ or candidate $B$ or not to turn out. Applying a discrete choice framework to the voter's decision, let $u_A(\mathbf{x})$ and $u_B(\mathbf{x})$ denote the utility of voting for candidates $A$ and $B$, respectively, and $c(\mathbf{x})$ denote the cost of voting (relative to not voting), where $\mathbf{x}$ is a vector of voter characteristics. Then, the voter's mean utilities are as follows:

$$
\begin{aligned}
V_A(\mathbf{x}) &= u_A(\mathbf{x}) - c(\mathbf{x}), \\
V_B(\mathbf{x}) &= u_B(\mathbf{x}) - c(\mathbf{x}), \text{ and} \\
V_0(\mathbf{x}) &= 0,
\end{aligned}
$$

where $V_0$ represents the mean utility of not turning out. While one can identify $V_A(\mathbf{x}) = u_A(\mathbf{x}) - c(\mathbf{x})$ and $V_B(\mathbf{x}) = u_B(\mathbf{x}) - c(\mathbf{x})$ by using vote share and turnout data (see Berry, 1994; Hotz and Miller, 1993), $u_A(\cdot)$, $u_B(\cdot)$, and $c(\cdot)$ are not separately identified without further restrictions. This is because making a voter care more about the election outcome (say, by adding an arbitrary function, $g(\mathbf{x})$, to both $u_A(\mathbf{x})$ and $u_B(\mathbf{x})$) is observationally equivalent to lowering the voting cost (by subtracting $g(\mathbf{x})$ from $c(\mathbf{x})$). Even if there are exogenous cost shifters $\mathbf{z}$ (e.g., rainfall), they do not help separately identify $u_A(\cdot)$, $u_B(\cdot)$, and $c(\cdot)$.[1] Thus, most existing studies impose ad-hoc exclusion restrictions on the way that $\mathbf{x}$ enters $u_A(\cdot)$, $u_B(\cdot)$, and $c(\cdot)$, assuming that $\mathbf{x}$ is excluded from either $u_k(\cdot)$ or $c(\cdot)$. Imposing such exclusion restrictions assumes away the correlation structure among these terms and precludes the possibility that the preferences of those with high voting costs are different from those with low voting costs. Note that this identification challenge exists regardless of whether the data are available at the individual level or at the aggregate level.

In this paper, we uncover the correlation structure between preferences and costs in a setting in which $\mathbf{x}$ is allowed to enter both $u_k(\cdot)$ and $c(\cdot)$. Our identification is based on the simple observation that, unlike consumer choice problems in which choosing not to buy results in the outcome of not obtaining the good, choosing not to turn out still results in either $A$ or $B$ winning the election. This observation implies that the voter's choice is determined by the utility difference between the two election outcomes rather than by the levels of utility associated with each outcome.[2] Barkume (1976) first used this observation to separately identify $u_k(\cdot)$ and $c(\cdot)$ in the context of property tax referenda for school districts.

To see how this observation leads to the identification of $u_k(\cdot)$ and $c(\cdot)$, consider the calculus of voting models of Downs (1957) and Riker and Ordeshook (1968). In these models, the utility of voting for candidate $k$ can be expressed as $u_k = pb_k$, where $p$ is the voter's beliefs that she is pivotal; $b_A$ is the utility difference between having candidate $A$ in office and candidate $B$ in office; and $b_B$ is defined similarly.[3] Hence,

[1] Suppose that the cost function is separated into two parts as $c = c_{\mathbf{x}}(\mathbf{x}) + c_{\mathbf{z}}(\mathbf{z})$, where $\mathbf{z}$ is a vector of cost shifters that is exluded from $u_A(\cdot)$ and $u_B(\cdot)$. Then, $u_A(\cdot) - c_{\mathbf{x}}(\cdot)$, $u_B(\cdot) - c_{\mathbf{x}}(\cdot)$ and $c_{\mathbf{z}}(\cdot)$ are all separately identified. However, $u_A(\cdot)$, $u_B(\cdot)$ and $c_{\mathbf{x}}(\cdot)$ are not separately identified. See the subsection titled "Exogenous Cost Shifters" towards the end of Section 4 for details.

[2] This implication holds as long as voters care about the ultimate outcome of the election. However, it may not hold for models in which voters gain utility from the act of voting for a candidate, such as models of expressive voting.

[3] More precisely, the utility of voting for candidate $k$ *relative to not turning out* can be expressed as $u_k = pb_k$, by normalizing the utility of not turning out to be zero. See Appendix A for details.

we have $b_A = -b_B$. The mean utilities can now be expressed as

$$\begin{aligned} V_A(\mathbf{x}) &= \quad pb_A(\mathbf{x}) - c(\mathbf{x}), \\ V_B(\mathbf{x}) &= -pb_A(\mathbf{x}) - c(\mathbf{x}), \text{ and} \\ V_0(\mathbf{x}) &= 0. \end{aligned}$$

The property $b_A = -b_B$ allows us to separately identify preferences and costs. By adding the first two expressions above, we have $V_A(\mathbf{x})+ V_B(\mathbf{x}) = -2c(\mathbf{x})$ because $pb_A(\mathbf{x})$ cancels out. Given that $V_A(\mathbf{x})$ and $V_B(\mathbf{x})$ are both identified from the vote share and turnout data, $c(\cdot)$ is identified. Similarly, we can identify $pb_A(\cdot)$ because we have $V_A(\mathbf{x})- V_B(\mathbf{x}) = 2pb_A(\mathbf{x})$, and $V_A(\mathbf{x})- V_B(\mathbf{x})$ is identified. Although this may appear mechanical, there is a straightforward intuition behind this result. $V_A(\mathbf{x})+ V_B(\mathbf{x})$ is identified primarily by voter turnout, and $V_A(\mathbf{x})- V_B(\mathbf{x})$ is identified primarily by the vote share margin. Hence, voter turnout pins down $c(\cdot)$, while the vote share margin pins down $pb_A(\cdot)$.[4]

In this paper, we retain the basic structure of the calculus of voting model but do not place additional restrictions on $p$, such as rational expectations, in which $p$ equals the actual pivot probability. In our model, we interpret $p$ more broadly as the voter's perception of voting efficacy, which is allowed to differ across individuals and to be correlated with the true pivot probability in a general manner. In particular, we let $p$ be a function of individual characteristics and the state in which the voter lives, as $p = p_s \times \tilde{p}(\mathbf{x})$, where $p_s$ is a state-specific coefficient and $\tilde{p}(\cdot)$ is a function of voter characteristics, $\mathbf{x}$. By letting $p$ depend on each state, we can take into account the nature of the electoral college system.[5] We show that the ratios of the state-specific components of efficacy, $p_s/p_{s'}$ $(\forall s, s')$, are identified directly from the data. Morever, we show that $\tilde{p}(\cdot)$, $b_A(\cdot)$ and $c(\cdot)$ are identified up to a scalar normalization.[6] Our identification discussion does not depend on equilibrium restrictions on $p$, such as

[4]See **?** for identification of voter preferences in a spatial voting model with full turnout.

[5]Under the electoral college system, perceptions of voting efficacy may differ significantly across states. For example, electoral outcomes in battleground states such as Ohio were predicted to be much closer than outcomes in party strongholds such as Texas. Hence, we allow for the possibility that $p$ is higher for voters in Ohio than for voters in Texas.

[6]More precisely, we can identify $p(\cdot)b_A(\cdot)$ state by state given that we have many counties within each state. Assuming that $\tilde{p}(\cdot)$ and $b_A(\cdot)$ are common across states, we can identify $p_s/p_{s'}$. We also show that $\tilde{p}(\cdot)$ and $b_A(\cdot)$ are separately identified up to a scalar multiple in our full specification with county-level shocks to preferences and costs.

rational expectations. Therefore, our identification and estimation results are agnostic about how voters formulate $p$.

Given the debate over how to model voter turnout, we briefly review the literature in order to relate our model to the various models of voter turnout.[7] The model that we estimate in this paper is based on the decision theoretic model of voter turnout introduced by Downs (1957) and Riker and Ordeshook (1968). In their models, a voter turns out and votes for the preferred candidate if $pb - c + d > 0$, where $p$ is the voter's beliefs over the pivot probability; $b$ is the utility difference from having one's preferred candidate in office relative to the other; $c$ is the physical and psychological costs of voting; and $d$ is the benefit from fulfilling one's civic duty to vote. While the original studies do not endogenize any of these terms, the decision theoretic model has provided a basic conceptual framework for much of the subsequent work on voting and turnout.

Studies subsequent to Riker and Ordeshook (1968) endogenize or micro-found each of the terms in the calculus of voting model in various ways. Ledyard (1984) and Palfrey and Rosenthal (1983, 1985) introduce the pivotal voter model, in which the pivot probability $p$ is endogenized in a rational expectations equilibrium. They show that there exists an equilibrium with positive turnout in which voters have consistent beliefs about the pivot probability. Coate et al. (2008), however, point out that the rational expectations pivotal voter model has difficulties matching the data on either the level of turnout or the winning margin.[8] Moreover, using laboratory experiments, Duffy and Tavits (2008) finds that voters' subjective pivot probabilities are much higher than the actual pivot probability, which is at odds with the rational expectations assumption.

More recently, there have been attempts at endogenizing $p$ in ways other than rational expectations. For example, Minozzi (2013) proposes a model based on cognitive dissonance in the spirit of Akerlof and Dickens (1982) and Brunnermeier and Parker (2005). In his model, voters jointly choose $p$ and whether or not to turn out in or-

[7]For a survey of the literature, see, e.g., Dhillon and Peralta (2002), Feddersen (2004), and Merlo (2006).

[8]Note, however, that with aggregate uncertainty, Myatt (2012) shows that the level of turnout can still be high with rational expectations. Levine and Palfrey (2007) also show that combining the quantal response equilibrium with the pivotal voter model can generate high turnout and finds that the results of laboratory experiments are consistent with the model prediction.

der to maximize subjective expected utility. Kanazawa (1998) introduces a model of reinforcement learning in which boundedly rational voters, who cannot compute the equilibrium pivot probabilities, form expectations about $p$ from the correlation between their own past voting behavior and past election outcomes (see, also, Bendor et al., 2003; Esponda and Pouzo, 2016, for similar approaches). While these models are based on the basic calculus of voting model, the $p$ term in them no longer carries the interpretation of the actual pivot probability.

Another strand of the literature endogenizes the $c$ and $d$ terms. Harsanyi (1980) and Feddersen and Sandroni (2006) endogenize the $d$ term by proposing a rule-utilitarian model in which voters receive a warm-glow payoff from voting ethically. Based on their approach, Coate and Conlin (2004) estimate a group-utilitarian model of turnout. Shachar and Nalebuff (1999) also endogenize the $d$ term by considering a follow-the-leader model in which elites persuade voters to turn out. In a paper studying split-ticket voting and selective abstention in multiple elections, Degan and Merlo (2011) consider a model that endogenizes $c$ to reflect the voter's psychological cost of making mistakes.

In our paper, we bring the calculus of voting model to the data without taking a particular stance on how the $p$, $b$, $c$, or $d$ terms are endogenized. Specifically, our identification and estimation do not use the restriction that $p$ is equal to the actual pivot probability, as in the rational expectations model. The $p$ term that we recover can be broadly interpreted as the voter's perception of voting efficacy. We purposely aim to be agnostic about the different ways of modeling voter turnout so that our estimates of voter preferences and costs are robust to the specific way in which the $p$, $b$, $c$, or $d$ terms are endogenized. Instead of imposing equilibrium restrictions of a particular model a priori, we let the data directly identify the $p$, $b$, and $c - d$ terms.

Relatedly, our study does not impose a priori restrictions on how the covariates enter the $p$, $b$, or $c - d$ terms, allowing, instead, the same set of covariates to affect all three terms. This is important because the way in which covariates enter the $p$, $b$, and $c - d$ terms determines the correlation structure among them, which, in turn, determines how well preferences are aggregated. In most existing studies, the sets of covariates that enter the $p$, $b$ and $c - d$ terms are disjoint, precluding the possibility that preferences and costs are correlated. For example, Coate and Conlin (2004) and Coate et al. (2008) include demographic characteristics only in the $b$ term,[9] while Shachar

[9]To be more precise, Coate and Conlin (2004) and Coate et al. (2008) use demographic characteristics as covariates for the fraction of the population supporting one side.

and Nalebuff (1999) include them in the $c - d$ term. In contrast, we let each demographic characteristic enter all three terms, allowing us to study the effects of turnout on preference aggregation.[10]

We use county-level data on voting outcomes from the 2004 U.S. presidential election to estimate the model.[11] A benefit of using actual voting data over survey data is that we can avoid serious misreporting issues often associated with survey data, such as the overreporting of turnout and reporting bias in vote choice (see, e.g., Atkeson, 1999; DellaVigna et al., 2015). Our data on turnout and vote share incorporate the number of non-citizens and felons to account for the difference between the voting-eligible population and the voting-age population (McDonald and Popkin, 2001). We construct the joint distribution of demographic characteristics within each county from the 5% Public Use Microdata Sample of the Census.

We find that young, less-educated, low-income, and religious voters have high voting costs, as do African Americans, Hispanics, and other minorities. Moreover, young voters have low perception of voting efficacy, which further depresses turnout among this group. Overall, young, less-educated, and low-income voters are particularly underrepresented. We find that voters in states with ID requirements tend to have higher voting costs while voters in states with same-day registration do not seem to have costs that are different from other voters.[12] In terms of preferences, minority, young, highly educated, low-income, and non-religious voters are more likely to prefer Democrats. These results are qualitatively in line with the findings based on survey data that document the differences in preferences between voters and non-voters (see, e.g., Citrin et al., 2003; Brunell and DiNardo, 2004; Martinez and Gill, 2005; Leighley and Nagler, 2013).

Our results show that, overall, there is a positive correlation between voting cost

[10] One possible exception is Degan and Merlo (2011). They consider a model based on the theories of regret in which the cost term is endogenized in a way that captures voters' preferences over candidates. They include the same set of covariates in the $c$ and $d$ terms. In one of their counterfactual analyses, they consider the effect of increasing voter turnout, focusing on split-ticket voting and selective abstention across presidential and congressional elections.

[11] Although we use aggregate data, we account for the issue of ecological fallacy by computing the behavior of individual voters and aggregating them at the county level.

[12] Existing research finds that ID requirements have only a small or no effect on voter turnout (see, e.g., **?**). Because our estimate of the effect ID requirement is idenitified off of cross-section variation, it may be picking up state-level unobserved heterogeneity that are correlated with the existence of ID requirements. The evidence on whether same-day registration affects turnout is mixed (see, e.g., literature review in **?**).

and preference for Democrats that can be accounted for through observable characteristics. Except for two voter characteristics–years of schooling and being religious–we find that demographic characteristics that are associated with a higher cost of voting are also associated with preferring Democrats. We also find that unobservable cost shocks are positively correlated with unobservable preference shocks for Democrats. These correlations result in fewer Democratic votes relative to the preferences of the underlying population. Our estimate of turnout is significantly lower among the electorate who prefer Democrats to Republicans, at $55.7$%, compared with turnout among those who prefer Republicans to Democrats, at $64.5$%.[13] Moreover, we find that voters who have a strong preference for one of the parties are more likely to turn out, suggesting that preference *intensity* affects preference aggregation (see Campbell, 1999; Casella, 2005; Lalley and Weyl, 2015).

Regarding our results on the perception of voting efficacy, we find substantial across-state variation in our estimates of $p_s$, the state-specific coefficient in $p$. Furthermore, the estimates are correlated with the ex-post closeness of the election: Battleground states such as Ohio and Wisconsin tend to have high estimates of $p_s$, while party strongholds such as New Jersey and California have low estimates, which is consistent with the comparative statics of the pivotal voter model with rational expectations. However, the magnitude of the estimated ratio of $p_s$ is, at most, three for any pair of states. This is in contrast to a much larger variation in the ratio implied by the pivotal voter model.[14] Our results are more consistent with models of turnout in which voters' perception of efficacy is only weakly correlated with the actual pivot probabilities.

In our first counterfactual experiment, we simulate the voting outcome when all voters vote. We find that the vote share of the Democrats increases in all states. Overall, the increase in the Democrats' two-party vote share is about $3.7$%. We also find that the increase in the Democratic vote share would overturn the election results in nine states, including key states such as Florida and Ohio, resulting in the Democrats

[13] See DeNardo (1980) and Tucker and DeNardo (1986) for studies that report negative correlation between turnout and the Democratic vote share using aggregate data. For more recent work, see Hansford and Gomez (2010) who use rainfall as an instrument for turnout.

[14] The pivotal voter model with rational expectations predicts high variation in the ratio of pivot probabilities across states, given the winner-take-all nature of the electoral college system. Voters in only a handful of swing states have a reasonable probability of being pivotal (see, e.g., Shachar and Nalebuff, 1999).

winning a plurality of the electoral votes.

**In our second counterfactual experiment, we compare the actual election outcome with that when we equalize the state-specific component of efficacy across states (set $p_s = p_{s'}$). Equalizing $p_s$ across states eliminates endogeneity in turnout that is state-specific. This counterfactual is motivated by a growing interest in measuring gerrymandering since the U.S. Supreme Court case, League of United Latin American Citizens v Perry. In that case, the justices expressed support for the standard of "partisan symmetry", introduced by King et al., which requires that districting plans treat the two parties equally with respect to the conversion of votes to seat shares. More recently, a metric of gerrymandering called the "efficiency gap", introduced by Stephanopolus and McGhee, has become influential – its applicability as a standard is one of the central issues in a current U.S. Supereme Court case, Gill v. Whitford.**[15]

**The concept of partisan symmetry and the efficiency gap both offer measures of how well actual votes are translated into seat shares. However, interpreting them as a measure of representation requires that actual vote shares reflect preferences, which we show is not the case. Moreover, given that districting determines both the turnout as well as the vote shares in each district, it is possible to design redistricting plans that map vote shares to seat shares well, and hamper representation at the same time.**[16] **The potential for gaming raises concerns about adopting these measures as part of legal doctrine. This is a point that seems to have been largely overlooked in the discussion on measuring gerrymandering. The counterfactual experiment gauges the robustness of the efficiency gap to endogenous turnout by elimiating state-specific endogeneity in turnout. We find that equalizing $p_s$ across states while keeping tunout at the actual level changes the efficiency gap by 1.3 percentage points. One natural alternative to comparing actual votes and seat shares is to compare underlying voter preferences and seat shares. If we think about elections as a way to aggregate preferences into**

[15] See, e.g., a news reporting by Wines (2017). https://www.nytimes.com/2017/10/01/us/wisconsinsupreme-court-gerrymander.htm

[16] In Table 7, we illustrate this point using a numerical example provided in **?**. The example is a hypothetical districting plan in which party A's seat share far exceeds party A's vote share, and the efficiency gap is very large in favor of party A. However, we show that by endogenizing turnout, it is possible to make the efficiency gap of the districting plan close to zero without changing party A's seat share.

**outcomes, evaluating the electoral system in terms of its ability to aggregate preferences seems most coherent.**

# 2 Model

Anticipating the empirical application of the paper, we tailor our model to the U.S. presidential election. Let $s \in \{1, ..., S\}$ denote a U.S. state and $m \in \{1, ..., M_s\}$ denote a county in state $s$.

**Preference of Voters** We consider a model of voting with two candidates, $D$ and $R$. Each voter chooses to vote for one of the two candidates or not to vote. We let $b_{nk}$ denote voter $n$'s utility from having candidate $k \in \{D, R\}$ in office, $p_n$ denote her perception of voting efficacy, and $c_n$ denote her cost of voting. Given that there are only two possible outcomes (either $D$ wins or $R$ wins the election), the utility of voting for candidate $k$, $U_{nk}$ depends only on $b_{nD} - b_{nR}$ rather on than $b_{nD}$ and $b_{nR}$ individually:

$$
\begin{aligned}
U_{nD} &= p_n(b_{nD} - b_{nR}) - c_n, && (1)\\
U_{nR} &= p_n(b_{nR} - b_{nD}) - c_n, && (2)\\
U_{n0} &= 0,
\end{aligned}
$$

where $U_{n0}$ is the utility of not turning out, which we normalize to zero (see Appendix A for a derivation).[17] When $p_n$ is the actual pivot probability, our model is the same as the equilibrium of the pivotal voter model of Palfrey and Rosenthal (1983, 1985). However, we interpret $p_n$ broadly as the voter's subjective perception of voting efficacy, as we discuss below. The cost of voting, $c_n$, includes both physical and psychological costs, as well as possible benefits of fulfilling one's civic duty. Hence, $c_n$ can be either positive or negative. When $c_n$ is negative, the voter turns out regardless of the value of $p_n$ and $b_{nD} - b_{nR}$.

We let the preferences of voter $n$ in county $m$ of state $s$ depend on her demographic

[17] Note that expressions (1) and (2) take the familiar form $pb - c$.

characteristics, $\mathbf{x}_n$, as follows:

$$b_{nk} = b_k(\mathbf{x}_n) + \lambda_{sk} + \xi_{mk} + \varepsilon_{nk}, \text{ for } k \in \{D, R\},$$

where $\lambda_{sk}$ is a state-specific preference intercept that captures state-level heterogeneity in voter preferences. $\xi_{mk}$ and $\varepsilon_{nk}$ are unobserved random preference shocks at the county level and at the individual level, respectively. $\xi_{mk}$ captures the unobserved factors that affect preferences at the county level, such as the benefits that the voters in county $m$ receive from policies supported by candidate $k$. Then, the expression for the utility difference is as follows:

$$b_{nR} - b_{nD} = b(\mathbf{x}_n) + \lambda_s + \xi_m + \varepsilon_n,$$

where $b(\mathbf{x}_n) = b_R(\mathbf{x}_n) - b_D(\mathbf{x}_n)$, $\lambda_s = \lambda_{sR} - \lambda_{sD}$, $\xi_m = \xi_{mR} - \xi_{mD}$, and $\varepsilon_n = \varepsilon_{nR} - \varepsilon_{nD}$. We assume that $\varepsilon_n$ follows the standard normal distribution.

We also let voting cost $c_n$ be a function of voter $n$'s characteristics as

$$c_n = c_s(\mathbf{x}_n) + \eta_m,$$

where $c_s(\mathbf{x}_n)$ is the cost function and $\eta_m$ is a county-level shock on the cost of voting. The cost function, $c_s(\cdot)$, depends on $s$ to allow for state-level cost shifters, such as voter ID requirements and same-day registration. Given that previous studies (e.g., Smith, 2001) find that neither the presence nor the closeness of gubernatorial and congressional elections affect turnout in presidential elections,[18] we do not incorporate other elections in our model. We assume that $\xi_m$ and $\eta_m$ are both independent of $\mathbf{x}_n$, but we allow $\xi_m$ and $\eta_m$ to be correlated with each other.

We let the voting efficacy term, $p_n$, depend on both the demographic characteristics of voter $n$ and the state in which she votes as follows:

$$p_n = p_s(\mathbf{x}_n) = p_s \times \tilde{p}(\mathbf{x}_n),$$

where $p_s$ is a state specific coefficient that we estimate. It is important to let $p_n$ depend on the state in which the voter votes because of the winner-take-all nature of the elec-

[18]The identification of the model does not depend on whether or not we include state-specific constant terms in $c_n$.

toral votes in each state.[19] For example, in the 2004 presidential election, a vote in key states such as Ohio was predicted to matter considerably more than a vote elsewhere. Our specification also allows for the possibility that $p_n$ depends on voters' characteristics, $\mathbf{x}_n$. Previous work has shown that voters' social and economic status affects her general sense of political efficacy (see, e.g., Karp and Banducci, 2008).

Note that our specification corresponds to the equilibrium of the pivotal voter model with rational expectations if we set $p_s$ equal to the actual pivot probability in state $s$ and set $\tilde{p}(\mathbf{x}_n)$ equal to 1. In this sense, our specification nests the pivotal voter model as a special case. However, instead of imposing the pivotal voter model (and, hence, placing equilibrium restrictions on $p_n$), we estimate $p_s$ and $\tilde{p}(\cdot)$ directly from the data. This approach allows us to interpret $p_n$ consistently with models of turnout that endogenize $p_n$ in various ways.

Substituting the expressions for $b_{nR} - b_{nD}$, $c_n$, and $p_n$ into equations (1) and (2), the utility from choosing each of the alternatives can be expressed as follows:

$$\begin{aligned} U_{nD}(\mathbf{x}_n) &= p_s(\mathbf{x}_n)\left[-b_s(\mathbf{x}_n) - \xi_m - \varepsilon_n\right] - c_s(\mathbf{x}_n) - \eta_m, \\ U_{nR}(\mathbf{x}_n) &= p_s(\mathbf{x}_n)\left[b_s(\mathbf{x}_n) + \xi_m + \varepsilon_n\right] - c_s(\mathbf{x}_n) - \eta_m, \\ U_{n0}(\mathbf{x}_n) &= 0, \end{aligned}$$

where $b_s(\mathbf{x}_n)$ denotes $b(\mathbf{x}_n) + \lambda_s$.

**A Voter's Decision** Voter $n$'s problem is to choose the alternative $k \in \{D, R, 0\}$ that provides her with the highest utility:

$$k = \underset{\kappa \in \{D,R,0\}}{\arg\max}\, U_{n\kappa}(\mathbf{x}_n). \tag{3}$$

[19]In U.S. presidential elections, the winner is determined by the Electoral College. Each U.S. state is allocated a number of electoral votes, roughly in proportion to the state's population. The electoral votes of each state are awarded on a winner-takes-all basis in all states, except for Maine and Nebraska. The Presidential candidate who wins the plurality of electoral votes becomes the winner of the election.

We can write the probability that voter $n$ votes for candidate $R$ as

$$\begin{aligned}
&\Pr\left(R = \arg\max_{\kappa\in\{D,R,0\}} U_{n\kappa}\right)\\
&\quad = \Pr\left(U_{nR} > U_{nD} \text{ and } U_{nR} > 0\right)\\
&\quad = \Pr\left(\varepsilon_n > -b_s(\mathbf{x}_n) - \xi_m \text{ and } \varepsilon_n > -b_s(\mathbf{x}_n) - \xi_m + \frac{c_s(\mathbf{x}_n) + \eta_m}{p_s(\mathbf{x}_n)}\right)\\
&\quad = 1 - \Phi\left(\max\left\{-b_s(\mathbf{x}_n) - \xi_m, -b_s(\mathbf{x}_n) - \xi_m + \frac{c_s(\mathbf{x}_n) + \eta_m}{p_s(\mathbf{x}_n)}\right\}\right),
\end{aligned}$$

where $\Phi$ is the CDF of the standard normal. We can derive a similar expression for candidate $D$.

Figure 1 depicts the behavior of a voter as a function of $\varepsilon_n$. There are two cases to consider: one in which the cost of voting is positive (Case 1) and the other in which the cost of voting is negative (Case 2). In Case 1, a voter with a strong preference for one of the candidates (which corresponds to a large positive realization or a large negative realization of $\varepsilon_n$) votes for her preferred candidate, while a voter who is relatively indifferent between the two candidates does not turn out. That is, a voter with high preference intensity relative to cost turns out, while a voter with low preference intensity does not. In Case 2, a voter always votes, regardless of her preference intensity, as the cost of voting is negative.

**Vote Share and Voter Turnout** We can express the vote share for candidate $k$ in county $m$, $v_{k,m}$, and the fraction of voters who do not turn out, $v_{0,m}$, as follows:

$$v_{R,m} \equiv \int 1 - \Phi\left(\max\left\{-b_s(\mathbf{x}_n) - \xi_m, -b_s(\mathbf{x}_n) - \xi_m + \frac{c_s(\mathbf{x}_n) + \eta_m}{p_s(\mathbf{x}_n)}\right\}\right) dF_{\mathbf{x},m}(\mathbf{x}_n), \tag{4}$$

$$v_{D,m} \equiv \int \Phi\left(\min\left\{-b_s(\mathbf{x}_n) - \xi_m, -b_s(\mathbf{x}_n) - \xi_m - \frac{c_s(\mathbf{x}_n) + \eta_m}{p_s(\mathbf{x}_n)}\right\}\right) dF_{\mathbf{x},m}(\mathbf{x}_n), \tag{5}$$

$$v_{0,m} \equiv 1 - v_{D,m} - v_{R,m} \tag{6}$$

where $F_{\mathbf{x},m}$ denotes the distribution of $\mathbf{x}$ in county $m$. Denoting the number of eligible voters in county $m$ by $N_m$ and the number of counties in state $s$ as $M_s$, the vote share for candidate $k$ in state $s$ can be expressed as $\sum_{m=1}^{M_s} N_m v_{k,m} \Big/ \sum_{m=1}^{M_s} N_m$. The candi-

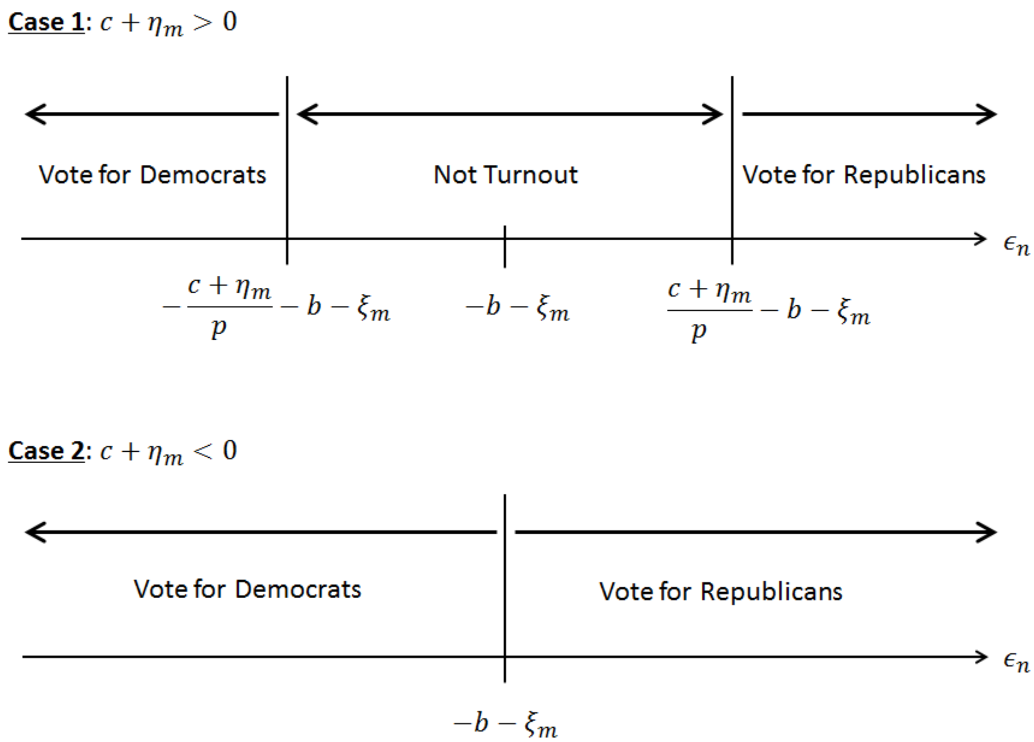


Figure 1: Voter's Decision as a Function of $\varepsilon_n$. The top panel corresponds to the case in which a voter has positive costs of voting. The bottom panel corresponds to the case in which a voter has negative costs of voting.

date with the highest vote share in state $s$ is allocated all of the electors of that state.[20] The candidate who wins the plurality of the electors becomes the overall winner of the presidential election.

**Advertising and Campaign Visits** An important feature of presidential elections not explicitly modeled thus far is the campaign activities of candidates. Candidates target key states with advertisements and campaign visits during the election. These campaign activities are endogenous and depend on the expected closeness of the race in each state (see, e.g., Strömberg, 2008; Gordon and Hartmann, 2013).

While we do not have a specific model of political campaigns, the model accounts for the effect of campaigns on voters through the state fixed effect in the voter's utility, $\lambda_s$. Because we treat $\lambda_s$ as parameters to be estimated, $\lambda_s$ may be arbitrarily correlated

[20]Maine and Nebraska use a different allocation method. Hence, we drop these two states from our sample.

with the closeness of the race in the state. Hence, our estimates of voter preferences are consistent even when campaign activities depend on the closeness. Similarly, our results based on the estimates, such as our discussion on underrepresentation and preference intensity, are not affected. We note, however, that the results of our counterfactual experiments take the level of campaigning as given.

**Discussion on Voter's Information** One factor that we do not specifically model is voter's information. One way to explicitly model information is by endogenizing the voters' information acquisition (see, e.g., **?**Degan and Merlo, 2011). In these models, the voters decide on the amount of information to acquire about the candidates—which helps them form preferences over candidates—by paying the cost of information acquisition. Our specification of voter preference can be thought of as the indirect utility of these models to the extent that information acquisition costs are functions of voter demographics. In fact, in our estimation results, we find that income and education are associated with low voting costs, which suggests that the information acquisition cost may comprise an important part of the voting cost (as the opportunity cost for these voters tend to be higher).

Another way to model information is to consider a common value environment in which voters obtain signals about the quality of the candidates (see, e.g., **??**). In these models, voters' utility consists partly of the expected quality of the candidates, which is computed by conditioning on the event that the voter is pivotal. To the extent that the prior beliefs over candidate quality and the signal distribution depend on the voter's demographic characteristics, the common value component is also a function of these characteristics. Hence, our specification of the utility can be interpreted as capturing a combination of private value and common value components in a reduced-form way.

**Discussion on $p$** The modeling in our paper is purposely agnostic about how $p$ is endogenized: We do not impose a particular model of $p$, such as rational expectations (Palfrey and Rosenthal, 1983, 1985), overconfidence (Duffy and Tavits, 2008), or cognitive dissonance (Minozzi, 2013). Similarly, our estimation approach avoids using restrictions specific to a particular way of modeling voter beliefs. The important point for our purpose is that there exists an equilibrium $p$ that corresponds to the data-generating process regardless of the way in which $p$ is endogenized. Our approach is to identify and estimate both the model primitives and the equilibrium $p$ directly from the data with as little structure as possible. This empirical strategy is similar in spirit to that in the estimation of incomplete models, in which some primitives are estimated from the data without fully specifying a model. For example, Haile and Tamer (2003) recovers bidder values without fully specifying a model of the English auction, using only the restriction that the winning bid lies between the valuations of the losers and the winner. Given that their estimation procedure also avoids using restrictions specific to a particular model of the English auction, the estimates are consistent under a variety of models.

In Section 4, we show that the key primitives of the model are identified without fully specifying how voters form $p$. We show that the equilibrium $p$ is also identified directly from the data.[21] The strength of our approach is that we impose few restrictions on beliefs, and, thus, our estimates of preferences and costs are consistent under a variety of behavioral assumptions regarding how $p$ is formed. On the other hand, this approach limits the types of counterfactual experiments that we can conduct since we do not specify a particular model regarding $p$.

# 3 Data

In this section, we describe our data and provide summary statistics. We combine county-level voting data, demographics data, and state-level data on ID requirements and same-day registration. The county-level voting data is obtained from David Leip's Atlas of U.S. Presidential Elections. This dataset is a compilation of election data from official sources such as state boards of elections. The demographics data is obtained from the U.S. Census Bureau. We construct the data on eligible voters for each county by combining the population estimates from the 2004 Annual Estimates of the Resident Population and age and citizenship information from the 2000 Census. We then adjust for the number of felons at the state level using the data from McDonald (2016). Hence, our data account for the difference between the voting age population and voting eligible population (see McDonald and Popkin, 2001). The data sources on voter ID requirements and same-day registration are from the National Conference of State Legislatures (NCSL) and **?**.

[21] More precisely, $p_s/p_{s'}$ is identified for any states $s$ and $s'$, and $\tilde{p}(\cdot)$ is identified up to a scalar normalization. See Section 4 for details.

We construct the joint distribution of voters' demographic characteristics and citizenship at the county level from the 2000 Census by combining the county-level marginal distribution of each demographic variable and the 5% Public Use Microdata Sample (see Appendix B for details). We augment the Census data with county-level information on religion using the Religious Congregations and Membership Study 2000. In particular, we define the variable Religious using adherence to either "Evangelical Denominations" or "Church of Jesus Christ of Latter-day Saints."

Our data consist of a total of 2,909 counties from forty states. Because we need a large number of counties within each state to identify the state-specific parameters, $p_s$ and $\lambda_s$, we drop states that have fewer than 15 counties. These states are Alaska, Connecticut, District of Columbia, Delaware, Hawaii, Massachusetts, New Hampshire, Rhode Island, and Vermont. In addition, we drop Maine and Nebraska because these two states do not adopt the winner-takes-all rule to allocate electors. We also drop counties with a population below 1,000 because their vote shares and turnout rates can be extreme due to small population size.[22] Table 1 presents the summary statistics of the county-level vote share, turnout, and demographic characteristics. Note that a Hispanic person may be of any race according to the definition used in the Census.

In order to illustrate the degree to which turnout and expected closeness are related, Figure 2 plots the relationship between the (ex-post) winning margin and voter turnout at the state level. The two variables are negatively correlated, although the fitted line is relatively flat. The slope of the fitted line implies that a decrease in the (ex-post) winning margin of ten percentage points is associated with an increase in turnout of only about 1.6 percentage points. While the negative correlation may be capturing some of the forces of the rational-expectations pivotal voter model, the flatness of the slope suggests that turnout is unlikely to be fully accounted for by the pivotal voter model.

# 4 Identification

In this section, we discuss the identification of our model as the number of counties *within each state* becomes large ($M_s \to \infty$, $\forall s$). Given that we have state-specific parameters for $p_s(\cdot)$ and $b_s(\cdot)$, we require the number of observations for each state

[22]In addition, we drop one county, Chattahoochee, GA, as the turnout rate is extremely low (18.8%) relative to all other counties. The turnout rate for the next lowest county is 33%.

| Voting Data | | | | | |
|---|---|---|---|---|---|
| | Obs | Mean | Std. Dev. | Min | Max |
| Vote Share: Democrat | 2,909 | 0.24 | 0.08 | 0.04 | 0.57 |
| Vote Share: Republican | 2,909 | 0.37 | 0.09 | 0.07 | 0.70 |
| Turnout Rate | 2,909 | 0.60 | 0.09 | 0.33 | 0.89 |

| County Demographics | | | | | |
|---|---|---|---|---|---|
| | Obs | Mean | Std. Dev. | Min | Max |
| % Hispanic | 2,909 | 0.05 | 0.12 | 0.00 | 0.97 |
| % Black/African American | 2,909 | 0.09 | 0.15 | 0.00 | 0.87 |
| % Neither Black nor White | 2,909 | 0.05 | 0.08 | 0.00 | 0.95 |
| Mean Age | 2,909 | 46.75 | 2.64 | 35.94 | 56.07 |
| Mean Income (USD 1,000) | 2,909 | 42.71 | 9.23 | 23.33 | 93.40 |
| Mean Years of Schooling | 2,909 | 12.86 | 0.60 | 10.84 | 15.18 |
| % Religious | 2,909 | 0.26 | 0.17 | 0.00 | 1.00 |

Table 1: Summary Statistics of Voting Outcome and Demographic Characteristics of Eligible Voters. For Age, Income, and Years of Schooling, the table reports the mean, standard deviation, minimum, and maximum of the county mean. “% Religious” is the share of the population with adherence to either “Evangelical Denomination” or “Church of Jesus Christ of Latter-day Saints.”

to be large. Our discussion in this section builds on the idea initially proposed by Barkume (1976) in the context of property tax referenda for school districts.

Recall that the observed vote shares are expressed as:

$$\begin{aligned}
v_{R,m} &\equiv \int 1-\Phi\left(\max\left\{\text{-}b_s(\mathbf{x}_n)-\xi_m, \text{-}b_s(\mathbf{x}_n)-\xi_m+\frac{c_s(\mathbf{x}_n)+\eta_m}{p_s(\mathbf{x}_n)}\right\}\right) dF_{\mathbf{x},m}(\mathbf{x}_n),\\
v_{D,m} &\equiv \int \Phi\left(\min\left\{\text{-}b_s(\mathbf{x}_n)-\xi_m, \text{-}b_s(\mathbf{x}_n)-\xi_m-\frac{c_s(\mathbf{x}_n)+\eta_m}{p_s(\mathbf{x}_n)}\right\}\right) dF_{\mathbf{x},m}(\mathbf{x}_n),\\
v_{0,m} &\equiv 1-v_{D,m}-v_{R,m}.
\end{aligned}$$

For exposition, consider the simple case in which there is no heterogeneity in voters' observable characteristics, so that $\mathbf{x}_n = \overline{\mathbf{x}}_m$ for all $n$ in county $m$.[23] In this case, the

[23]Note that we are well aware of the issues of ecological fallacy. In what follows, we consider a simplified setup with $\mathbf{x}_n = \overline{\mathbf{x}}_m$ for all $n$ in county $m$, just for expositional purposes. In our empirical exercise, we fully address the fact that each county has a distribution of $\mathbf{x}$ by integrating the vote share for each $\mathbf{x}$ with respect to $F_{\mathbf{x},m}(\cdot)$.

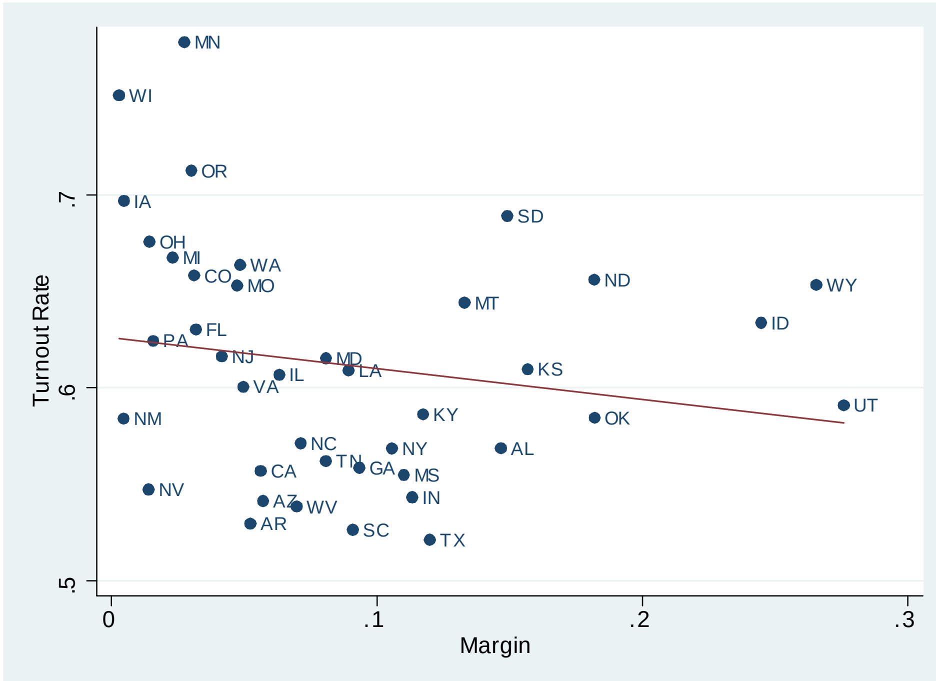


Figure 2: Relationship between the Ex-Post Winning Margin and Voter Turnout. The slope coefficient is $-0.16$ and not statistically significant.

above expressions simplify as follows:

$$v_{R,m} \equiv 1 - \Phi\left(\max\left\{-b_s(\overline{\mathbf{x}}_m) - \xi_m, -b_s(\overline{\mathbf{x}}_m) - \xi_m + \frac{c_s(\overline{\mathbf{x}}_m) + \eta_m}{p_s(\overline{\mathbf{x}}_m)}\right\}\right), \quad (7)$$

$$v_{D,m} \equiv \Phi\left(\min\left\{-b_s(\overline{\mathbf{x}}_m) - \xi_m, -b_s(\overline{\mathbf{x}}_m) - \xi_m - \frac{c_s(\overline{\mathbf{x}}_m) + \eta_m}{p_s(\overline{\mathbf{x}}_m)}\right\}\right), \quad (8)$$

$$v_{0,m} \equiv 1 - v_{D,m} - v_{R,m}. \quad (9)$$

We now show that the primitives of the model are identified from expressions (7), (8), and (9).

Using the fact that $\Phi$ is a strictly increasing function, we can rewrite expressions (7) and (8) as follows:

$$\Phi^{-1}(1 - v_{R,m}) = \max\left\{-b_s(\overline{\mathbf{x}}_m) - \xi_m, \; -b_s(\overline{\mathbf{x}}_m) - \xi_m + \frac{c_s(\overline{\mathbf{x}}_m) + \eta_m}{p_s(\overline{\mathbf{x}}_m)}\right\},$$

$$\Phi^{-1}(v_{D,m}) = \min\left\{-b_s(\overline{\mathbf{x}}_m) - \xi_m, \; -b_s(\overline{\mathbf{x}}_m) - \xi_m - \frac{c_s(\overline{\mathbf{x}}_m) + \eta_m}{p_s(\overline{\mathbf{x}}_m)}\right\}.$$

Rearranging these two equations, we obtain the following expressions:

$$\begin{aligned}
\frac{\Phi^{-1}\left(1-v_{R,m}\right)+\Phi^{-1}\left(v_{D,m}\right)}{-2} &= b_s(\overline{\mathbf{x}}_m)+\xi_m, \text{ and} && (10)\\
\frac{\Phi^{-1}\left(1-v_{R,m}\right)-\Phi^{-1}\left(v_{D,m}\right)}{2} &= \max\left\{0, \frac{c_s(\overline{\mathbf{x}}_m)}{p_s(\overline{\mathbf{x}}_m)}+\frac{\eta_m}{p_s(\overline{\mathbf{x}}_m)}\right\}. && (11)
\end{aligned}$$

Note that the left hand side of (10) closely reflects the difference in vote share, and the left hand side of (11) reflects the turnout rate. This is because, if we ignore the nonlinearity of $\Phi^{-1}(\cdot)$ and the denominator, the left hand side of (10) reduces to $1 - v_{R,m} + v_{D,m}$ and the left hand side of (11) to $1 - v_{R,m} - v_{D,m}$. The former is one minus the difference in vote share, and the latter is one minus voter turnout. The left hand side of expressions (10) and (11) can be directly computed using data on vote shares, $v_{D,m}$ and $v_{R,m}$.

We first consider the identification of $b_s(\cdot)$ and the distribution of $\xi$, $F_\xi(\cdot)$. Taking the expectation of (10) conditional on $\overline{\mathbf{x}}_m$, we have

$$\mathbf{E}\left[\left.\frac{\Phi^{-1}\left(1-v_{R,m}\right)+\Phi^{-1}\left(v_{D,m}\right)}{-2}\right|\overline{\mathbf{x}}_m\right]=b_s(\overline{\mathbf{x}}_m), \tag{12}$$

because $\mathbf{E}\left[\xi_m|\overline{\mathbf{x}}_m\right] = 0$. As the left hand side of the above expression is identified, $b_s(\cdot)$ is (nonparametrically) identified for each $s$ (note that the asymptotics is with respect to the number of counties within each state). Given that $b_s(\cdot)$ is the utility difference between Republicans and Democrats, it is intuitive that $b_s(\cdot)$ is identified by the difference in vote share.

Now, consider the identification of $F_\xi(\cdot)$. Given that $b_s(\cdot)$ is identified and the left hand side of (10) is observable, each realization of $\xi_m$ can be recovered from (10). Hence, $F_\xi(\cdot)$ is also identified. Note that if $b_s(\cdot)$ is linear in $\overline{\mathbf{x}}_m$ (i.e., $b_s(\overline{\mathbf{x}}_m) = \boldsymbol{\beta}\overline{\mathbf{x}}_m$), one can simply regress the left hand side of expression (10) on $\overline{\mathbf{x}}_m$ by OLS to obtain $\boldsymbol{\beta}$ as coefficients and $\xi_m$ as residuals.

We now discuss the identification of $p_s(\cdot)$, $c_s(\cdot)$, and $F_\eta(\cdot)$. For simplicity, consider the case in which the second term inside the max operator of expression (11) is positive with probability 1–i.e.,

$$\frac{\Phi^{-1}\left(1-v_{R,m}\right)-\Phi^{-1}\left(v_{D,m}\right)}{2}=\frac{c_s(\overline{\mathbf{x}}_m)}{p_s(\overline{\mathbf{x}}_m)}+\frac{\eta_m}{p_s(\overline{\mathbf{x}}_m)}. \tag{13}$$

This corresponds to the case in which the turnout rate is always less than 100%. We show in Appendix C that $p_s(\cdot)$, $c_s(\cdot)$, and $F_\eta(\cdot)$ are identified also for the case in which the turnout rate is allowed to be 100%.

Taking the conditional moments of (13), we have

$$\mathbf{E}\left[\left.\frac{\Phi^{-1}\left(1-v_{R,m}\right)-\Phi^{-1}\left(v_{D,m}\right)}{2}\right|\overline{\mathbf{x}}_m\right] = \frac{c_s(\overline{\mathbf{x}}_m)}{p_s(\overline{\mathbf{x}}_m)}, \text{ and} \tag{14}$$

$$\mathbf{Var}\left[\left.\frac{\Phi^{-1}\left(1-v_{R,m}\right)-\Phi^{-1}\left(v_{D,m}\right)}{2}\right|\overline{\mathbf{x}}_m\right] = \frac{\sigma_\eta^2}{(p_s(\overline{\mathbf{x}}_m))^2}, \tag{15}$$

where $\sigma_\eta^2$ is the variance of $\eta_m$. Using (15), $p_s(\cdot)$ is identified up to a scalar constant (i.e., up to $\sigma_\eta^2$) because the left hand side of (15) is identified. This implies that $c_s(\overline{\mathbf{x}}_m)$ is also identified up to $\sigma_\eta^2$ using (14). Given that $p_s(\cdot)$ and $c_s(\cdot)$ are identified, we can recover the realization of $\eta_m$ from (13), implying that $F_\eta(\cdot)$ is also identified up to $\sigma_\eta^2$. Intuitively, the left hand side of (14) and (15) closely reflect the mean and variance of the rate of abstention. Hence, the average cost of voting normalized by $p_s$ $(c_s(\cdot)/p_s(\cdot))$ is identified from (14).

Importantly, while $p_s(\cdot)$ is identified only up to a scalar constant, the ratio $p_{s'}/p_{s''}$ for any states $s'$ and $s''$ is identifed given our specification of $p_s(\cdot)$ as $p_s(\cdot) = p_s \times \tilde{p}(\cdot)$. To see this, note that the ratio of (15) for two counties with the same demographics in states $s'$ and $s''$ directly identify $p_{s'}/p_{s''}$.

The discussion has, thus far, been based on the simplified case in which all voters in county $m$ have the same demographic characteristics–i.e., $\mathbf{x}_n = \overline{\mathbf{x}}_m$ for all $n$ in $m$. As long as there is sufficient variation in $F_{\mathbf{x},m}(\mathbf{x})$, we can recover the vote shares conditional on each $\mathbf{x}$ and apply the identification discussion above.

**Correlation between Unobserved Cost and Preference Shocks** Our identification makes no assumptions regarding the correlation between the unobservables $\xi_m$ and $\eta_m$. As $\xi_m$ and $\eta_m$ enter separately in (10) and (11), $\xi_m \perp \overline{\mathbf{x}}_m$ and $\eta_m \perp \overline{\mathbf{x}}_m$ are sufficient to identify the unknown primitives on the right hand side in each equation. Hence, we do not require any restrictions on the joint distribution of $\xi_m$ and $\eta_m$. In fact, we can nonparametrically identify the joint distribution of $\xi_m$ and $\eta_m$ from the joint distribution of the residuals in each equation. In our estimation, we specify the joint distribution of $\xi_m$ and $\eta_m$ as a bivariate Normal with correlation coefficient $\rho$.

**Exogenous Cost Shifters** Lastly, we discuss identification when there exist instruments (e.g., rainfall) that shift the cost of voting but not the preferences of the voters. The point we wish to make is that the existence of exogenous cost shifters are neither necessary nor sufficient for identification.

To illustrate this point, consider the following discrete choice setup with instruments $\mathbf{z}_n$,

$$\begin{aligned} V_A &= u_A(\mathbf{x}_n) - c_{\mathbf{x}}(\mathbf{x}_n) - c_{\mathbf{z}}(\mathbf{z}_n) \\ V_B &= u_B(\mathbf{x}_n) - c_{\mathbf{x}}(\mathbf{x}_n) - c_{\mathbf{z}}(\mathbf{z}_n) \\ V_0 &= 0, \end{aligned}$$

where $V_k$ denotes the mean utility of choosing $k \in \{A, B, 0\}$. Here, $u_A(\mathbf{x}_n)$ is not necessarily equal to $-u_B(\mathbf{x}_n)$, and the cost function is separated into two components, $c_{\mathbf{x}}(\mathbf{x}_n)$ and $c_{\mathbf{z}}(\mathbf{z}_n)$, where $\mathbf{z}_n$ is a vector of cost shifters excluded from $u_k(\mathbf{x}_n)$. For any arbitrary function $g(\mathbf{x}_n)$, consider an alternative model with $\tilde{u}_k(\mathbf{x}_n) = u_k(\mathbf{x}_n) + g(\mathbf{x}_n)$ ($k \in \{A, B\}$) and $\tilde{c}_{\mathbf{x}}(\mathbf{x}_n) = c_{\mathbf{x}}(\mathbf{x}_n) + g(\mathbf{x}_n)$, as follows:

$$\begin{aligned} V_A &= \tilde{u}_A(\mathbf{x}_n) - \tilde{c}_{\mathbf{x}}(\mathbf{x}_n) - c_{\mathbf{z}}(\mathbf{z}_n) \\ V_B &= \tilde{u}_B(\mathbf{x}_n) - \tilde{c}_{\mathbf{x}}(\mathbf{x}_n) - c_{\mathbf{z}}(\mathbf{z}_n) \\ V_0 &= 0. \end{aligned}$$

Because $\tilde{u}_k(\mathbf{x}_n) - \tilde{c}_{\mathbf{x}}(\mathbf{x}_n) = u_k(\mathbf{x}_n) - c_{\mathbf{x}}(\mathbf{x}_n)$, the two models are observationally equivalent, and thus, $u_k(\cdot)$ and $c_{\mathbf{x}}(\cdot)$ are not separately identified. In particular, the correlation between preferences and costs cannot be identified because this model cannot rule out $c_{\mathbf{x}}(\mathbf{x}_n) = 0$, $\forall \mathbf{x}_n$. This is true even if $\mathbf{z}_n$ has a rich support. Hence, it is *not* the availability of instruments, but rather the observation that we can express $u_A(\mathbf{x}_n) = -u_B(\mathbf{x}_n)$ that identifies the primitives of the model.

# 5 Specification and Estimation

## 5.1 Specification

We now specify $b_s(\cdot)$, $c_s(\cdot)$, $p_s(\cdot)$ and the joint distribution of $\xi_m$ and $\eta_m$ for our estimation. We specify $b_s(\cdot)$, which is the utility difference from having candidates $R$ and $D$ in office, as a function of a state-level preference shock, $\lambda_s$, and demographic characteristics, $\mathbf{x}_n$, consisting of age, race, income, religion, and years of schooling:

$$b_s(\mathbf{x}_n) = \lambda_s + \boldsymbol{\beta}_b'\mathbf{x}_n.$$

The intercept, $\lambda_s$, is a parameter that we estimate for each state. It captures the state-level net preference shock for the Republicans that demographic characteristics do not account for. Note that the linear specification can be derived from a spatial voting model in which a voter's bliss point is linear in $\mathbf{x}_n$.[24]

Voting cost, $c_s(\cdot)$, is also specified as a linear function of $\mathbf{x}_n$ as

$$c_s(\mathbf{x}_n) = \boldsymbol{\beta}_c[1, \mathbf{x}_n']' + \boldsymbol{\beta}_c^{ID} ID_s + \boldsymbol{\beta}_c^{REG} REG_s,$$

where $ID_s$ is a dummy variable that indicates whether voter ID is required in state $s$ and $REG_s$ corresponds to whether the state allows same-day registration. We do not specifically model the presence of other elections, such as gubernatorial and senatorial elections, because previous studies (e.g., Smith, 2001) find that neither the presence nor the closeness of other elections affects turnout in presidential elections. We also do not include weather-related variables in $c(\cdot)$ because there was insufficient variation in precipitation or temperature on the day of the 2004 presidential election to have affected turnout significantly.[25]

We specify the voter's perception of efficacy as $p_s \times \tilde{p}(\mathbf{x}_n)$, where $\tilde{p}(\cdot)$ is a function

[24]To illustrate this point, consider a unidimensional spatial voting model in which candidate $D$'s ideological position is 0; candidate $R$'s position is 1; and a voter's bliss point is $\alpha_n = \beta^{bliss}\mathbf{x}_n$. Under the quadratic loss function, the utility from electing candidates $D$ and $R$ are $-\alpha_n^2$ and $-(1-\alpha_n)^2$, respectively, and the utility difference, $b_s(\mathbf{x}_n)$, is written as $-(1-\alpha_n)^2 + \alpha_n^2 = 2\beta^{bliss}\mathbf{x}_n - 1$. Thus, $b_s(\mathbf{x}_n)$ is linear in $\mathbf{x}_n$ in such a model.

[25]We included weather variables in the simple model that assumes $\mathbf{x}_n = \overline{\mathbf{x}}_m$ (i.e., the demographic characteristics of voters in each county are assumed to be the same within county) and found the coefficients on the weather variables to be small and insignificant.

of her age, income, and years of schooling, as follows:[26]

$$\tilde{p}(\mathbf{x}_n) = \exp(\boldsymbol{\beta}_p' \mathbf{x}_n).$$

We normalize $p_s = 1$ for Alabama and normalize $\tilde{p}(\cdot)$ such that $\tilde{p}(\bar{\mathbf{x}}) = 1,$ where $\bar{\mathbf{x}}$ is the national average of $\mathbf{x}_n$.[27]

We specify the joint distribution of county-level preference shock $\xi$ and cost shock $\eta$ as a bivariate normal, $N(0, \Sigma)$, where $\Sigma$ is the variance-covariance matrix with diagonal elements equal to $\sigma_\xi^2$, $\sigma_\eta^2$ and off-diagonal elements $\rho\sigma_\xi\sigma_\eta$.

## 5.2 Estimation

We use the method of moments to estimate the model parameters.[28] Recall that the vote shares (as a fraction of eligible voters) and turnout in county $m$ are given by expressions (4), (5) and (6), where $F_{\mathbf{x},m}$ is the distribution of $\mathbf{x}_n$ in county $m$. For a fixed vector of the model parameters, $\boldsymbol{\theta}$ =($\boldsymbol{\beta}_b$,$\{\lambda_s\}$,$\boldsymbol{\beta}_c$,$\{p_s\}$,$\boldsymbol{\beta}_p$,$\sigma_\xi$,$\sigma_\eta$,$\rho$), we can compute the moments of expressions (4), (5) and (6) by integrating over $\xi$ and $\eta$. Our estimation is based on matching the moments generated by the model with the corresponding sample moments.

Specifically, we define the first and second order moments implied by the model as follows:

$$\begin{aligned}
\hat{v}_{k,m}(\boldsymbol{\theta}) &= \mathbf{E}_{\xi,\eta}[v_{k,m}(\xi,\eta;\boldsymbol{\theta})], \forall k \in \{D,R\}, \\
\hat{v}_{k,m}^{squared}(\boldsymbol{\theta}) &= \mathbf{E}_{\xi,\eta}[v_{k,m}(\xi,\eta;\boldsymbol{\theta})^2], \forall k \in \{D,R\}, \\
\hat{v}_{m}^{cross}(\boldsymbol{\theta}) &= \mathbf{E}_{\xi,\eta}[v_{D,m}(\xi,\eta;\boldsymbol{\theta}) v_{R,m}(\xi,\eta;\boldsymbol{\theta})],
\end{aligned}$$

[26] The set of variables included in $p_s(\mathbf{x}_n)$ is a subset of $\mathbf{x}_n$ that takes continuous values. Here, we do not include dummy variables such as race, and religion. The reason is as follows. The variation in $c(\cdot)$ changes the utility level additively, while the variation in $p_s(\cdot)$ changes it multiplicatively as $p_s \times b_s$. As dummy variables take only 0 and 1, it is difficult, in practice, to distinguish whether the effects of those variables are additive or multiplicative. Thus, estimating the model with dummy variables in both cost and efficacy is difficult, and we include only continuous variables in $p_s(\cdot)$.

[27] Note that we need two normalizations. Because we express $p_s(\mathbf{x}_n)$ as $p_s \times \tilde{p}(\mathbf{x}_n)$, we need a scalar normalization on either $p_s$ or $\tilde{p}(\mathbf{x}_n)$. We normalize $p_s = 1$ for Alabama. We also need an additional normalization because $p_s(\cdot)$ is identified only up to the variance of $\eta$– i.e., the level of $p_s$ is not identified in our model. Assuming that $\tilde{p}(\bar{\mathbf{x}}) = 1$ eliminates this degree of freedom.

[28] In contrast to maximum likelihood estimation, which requires us to solve for $(\xi, \eta)$ that rationalizes the observed vote share for each parameter value, the method of moments only requires integration with respect to $\xi$ and $\eta$ by simulation. The latter is substantially less costly in terms of computation.

where $v_{k,m}(\xi,\eta;\boldsymbol{\theta})$ is the vote share of candidate $k$ given a realization of $(\xi,\eta)$ and parameter $\boldsymbol{\theta}$.[29,30] Denoting the observed vote share of candidate $k$ in county $m$ as $\mathsf{v}_{k,m}$, our objective function, $J(\theta)$, is given by

$$J(\boldsymbol{\theta}) = \sum_{k=\{D,R\}} \left( \frac{J_{1,k}(\boldsymbol{\theta})}{\widehat{\mathbf{Var}}(\mathsf{v}_{k,m})} + \frac{J_{2,k}(\boldsymbol{\theta})}{\widehat{\mathbf{Var}}(\mathsf{v}_{k,m}^2)} \right) + \frac{J_3(\boldsymbol{\theta})}{\widehat{\mathbf{Var}}(\mathsf{v}_{D,m}\mathsf{v}_{R,m})},$$

where

$$\begin{aligned} J_{1,k}(\boldsymbol{\theta}) &= \frac{1}{M}\sum_{s=1}^{S}\sum_{m=1}^{M_s}(\hat{v}_{k,m}(\boldsymbol{\theta}) - \mathsf{v}_{k,m})^2, \forall k \in \{D,R\}, \\ J_{2,k}(\boldsymbol{\theta}) &= \frac{1}{M}\sum_{s=1}^{S}\sum_{m=1}^{M_s}(\hat{v}_{k,m}^{squared}(\boldsymbol{\theta}) - \mathsf{v}_{k,m}^2)^2, \forall k \in \{D,R\}, \\ J_3(\boldsymbol{\theta}) &= \frac{1}{M}\sum_{s=1}^{S}\sum_{m=1}^{M_s}\left(\hat{v}_{m}^{cross}(\boldsymbol{\theta}) - \mathsf{v}_{D,m}\mathsf{v}_{R,m}\right)^2. \end{aligned}$$

$J_{1,k}$ is the sum of the squared differences between the expectation of the predicted vote share ($\hat{v}_{k,m}(\boldsymbol{\theta})$) and the actual vote share ($\mathsf{v}_{k,m}$). $J_{2,k}$ is the sum of the squared differences between $\hat{v}_{k,m}^{squared}(\boldsymbol{\theta})$ and the squared vote share, $\mathsf{v}_{k,m}^2$. $J_3$ is the sum of the squared differences between the predicted and actual cross terms. $M$ is the total number of counties across all states, $\sum_{s=1}^{S} M_s$, and $\widehat{\mathbf{Var}}(z)$ denotes the sample variance of $z$.

The construction of our objective function follows our identification argument closely. The first moment, $J_{1,k}$, matches the conditional expectation of the vote shares from the model with that from the data. Intuitively, $J_{1,k}$ corresponds to (12) and (14), and pins down $\boldsymbol{\beta}_b$, $\{\lambda_s\}$, $\boldsymbol{\beta}_c/p_s$, $\boldsymbol{\beta}_c/\boldsymbol{\beta}_p$ and $p_s/p_{s'}$. The second and third moments,

[29] Computing $\hat{v}_{k,m}(\boldsymbol{\theta})$, $\hat{v}_{k,m}^{squared}(\boldsymbol{\theta})$, and $\hat{v}_{m}^{cross}(\boldsymbol{\theta})$ requires integration over ($\xi$,$\eta$). For integration, we use a quadrature with $[5 \times 5]$ nodes and pruning (see Jäckel, 2005) with a total of 21 nodes.

[30] Note that we do not need to know the value of $\rho$ for computing $v_{k,m}(\xi,\eta;\boldsymbol{\theta})$. Thus, for each $\boldsymbol{\theta}\backslash\{\rho\}$ (i.e., the parameters except for $\rho$), we can compute $v_{k,m}(\xi,\eta;\boldsymbol{\theta})$ and solve for the implied realizations $(\xi_m,\eta_m)$ that give the observed vote shares in county $m$. Berry (1994) guarantees that there exists a unique pair of $(\xi_m,\eta_m)$ that equates $v_{k,m}(\xi,\eta;\boldsymbol{\theta})$ to the observed shares. By computing the correlation between the implied realizations of $\xi_m$ and $\eta_m$, we can obtain the value of $\rho$ that is consistent with the observed data. We then impose this value of $\rho$ to compute $\hat{v}_{k,m}(\boldsymbol{\theta}), \hat{v}_{k,m}^{squared}(\boldsymbol{\theta})$, and $\hat{v}_{m}^{cross}(\boldsymbol{\theta})$. Our estmation procedure can be thought of as minimizing the objective function with respect to $\boldsymbol{\theta}\backslash\{\rho\}$, and the estimate of $\rho$ is obtained by computing the correlation between the implied realizations of $\xi_m$ and $\eta_m$ given the estimate of $\boldsymbol{\theta}\backslash\{\rho\}$.

$J_{2,k}$ and $J_3$, correspond to (15). These moments pin down $\boldsymbol{\beta}_p$,$\sigma_\xi$,$\sigma_\eta$ and $\rho$.

# 6 Results

The set of parameters that we estimate include those that are common across all states ($\boldsymbol{\beta}_b$,$\sigma_\xi$, $\boldsymbol{\beta}_c$,$\sigma_\eta$,$\boldsymbol{\beta}_p$,$\rho$) and those that are specific to each state ($\{\lambda_s\}$,$\{p_s\}$). Table 2 reports the estimates of the former set, while Figures 4 and 5 plot the parameter estimates of the latter set.

**Estimates of $\boldsymbol{\beta}_b$, $\sigma_\xi$, $\boldsymbol{\beta}_c$, $\sigma_\eta$, $\boldsymbol{\beta}_p$, and $\rho$** The first column of Table 2 reports the estimates of the preference parameters. We find that Age and Income enter the utility difference, $b_R - b_D$, positively, implying that old and high-income voters are more likely to prefer Republicans. Years of Schooling enters the utility difference negatively, thus more-educated voters are more likely to prefer Democrats. We also find that Hispanics, African Americans, and Other Races prefer Democrats. The Religion variable carries a positive coefficient, implying that religious voters prefer Republicans. The estimate of the constant term corresponds to the preference of the voter who has $\mathbf{x}_n$ equal to the national average and has $\lambda_s$ equal to Alabama.

In the second column of Table 2, we report the estimates of the cost parameters. The estimated costs are net of any benefits from fulfilling civic duty. Moreover, our cost estimates include not only physical and opportunity costs but also psychological costs, such as information acquisition costs. We find that Age, Years of Schooling, and Income enter voting cost negatively. This implies that older, more-educated, and higher-income voters have a lower cost of voting.[31] Hispanics, African Americans, and Other Races have a higher cost of voting relative to non-Hispanics and Whites. Religious voters also have a higher cost of voting compared to non-religious voters. Our estimates of voter ID requirements and same day registration on voting costs are both statistically and economically insignificant.[32] The estimate of the constant term

[31] Given that high-income and more-educated voters tend to have high opportunity cost, the negative coefficients on income and education might suggest that information acquisition cost can be an important part of the voting cost.

[32] Cantoni (2017) finds that voter ID requirements have only a small or no effect on voter turnout using panel data. The evidence on the effect of same-day registration is mixed (see, e.g., Braconnier, et al., 2017).

| | Preference | | Cost | | Efficacy | |
|---|---|---|---|---|---|---|
| | Estimate | SE | Estimate | SE | Estimate | SE |
| Age | 0.0185 | (0.0035) | -0.0104 | (0.0090) | 0.0615 | (0.0158) |
| Years of Schooling | -0.0797 | (0.0156) | -0.2323 | (0.0646) | -0.1688 | (0.0648) |
| log(income) | 0.3748 | (0.0493) | -0.1709 | (0.0672) | -0.0112 | (0.1597) |
| Hispanic | -0.2530 | (0.0842) | 0.0563 | (0.0631) | | |
| African American | -1.3204 | (0.0583) | 0.2370 | (0.0617) | | |
| Other Races | -0.7368 | (0.0760) | 0.2367 | (0.0798) | | |
| Religious | 0.6619 | (0.0566) | 0.1175 | (0.0393) | | |
| Constant | 0.0970 | (0.0382) | 0.3592 | (0.0423) | | |
| Sigma | 0.1992 | (0.0080) | 0.0154 | (0.2702) | | |
| ID requirement | | | 0.1351 | (0.0471) | | |
| Same-day-registration | | | -0.0032 | (0.0874) | | |
| Rho | | -0.0875 (0.0504) | | | | |

Table 2: Parameter Estimates. The table reports the parameter estimates of voters' preferences, costs, and perception of voting efficacy. The estimate of the constant terms in the first and second columns corresponds to the preference and costs of the voter who has $\mathbf{x_n}$ equal to the national average and has $\lambda_s$ equal to Alabama. The variable log(income) is the log of income divided by 1000. Excluded categories are non-Hispanic, White, and non-Religious. Standard errors are computed by analytically deriving the asymptotic variance covariance matrix. The standard errors are reported in parentheses.

in the second column corresponds to the cost of a voter whose characteristics are set to the national mean.

The third column of Table 2 reports the estimates of the efficacy parameters. We find that Age enters the perception of efficacy positively, while Years of Schooling and Income enter negatively. This implies that older, less-educated, and lower-income citizens tend to have a higher perception of efficacy. Given that older voters have lower voting costs as well, they are more likely to be overrepresented than young voters. Regarding Years of Schooling and Income, the overall effect on participation depends on the relative magnitudes of the cost and efficacy coefficients. We discuss the net effect in the next subsection.

The last row of Table 2 reports the estimate of $\rho$, which is the correlation between unobservable shocks $\xi$ and $\eta$. The estimate is negative (–0.084), implying that the correlation in the unobservable shocks tends to suppress turnout among voters who prefer Democrats.

**Representation and Preference Aggregation** To better understand what our estimates imply about voter preferences and representation across demographic groups, Figure 3 plots the estimated share of voters who prefer Democrats over Republicans (right axis) and an estimated measure of voter representation (left axis) by demographic groups. The share of voters who prefer Democrats over Republicans is simply the two-party vote share of the Democrats unconditional on turnout, computed using the preference estimates. The representation measure that we use is based on **?**, and it is defined as follows:

$$WR(\mathbf{x}) = \frac{\text{share of group } \mathbf{x} \text{ among those who turn out}}{\text{share of group } \mathbf{x} \text{ among the overall electorate}},$$

where $\mathbf{x}$ is a demographic group. Overrepresented demographic groups have representation measures larger than one, while underrepresented groups have measures less than one.

Figure 3 shows that there are significant differences in the representation and preference measures among demographic groups. For example, panel (a) shows that the representation measure of 75-year-old voters is 1.65, while that of 25-year-old voters is 0.37. The fraction of 75-year-old voters who prefer Democrats is 36.4%, while that of 25-year-old voters is 59.9%.

Figure 3 also illustrates how demographic characteristics are related to preference aggregation. In particular, if the two curves in each panel have the same (opposite) slope, the voters who prefer Republicans (Democrats) are underrepresented. For example, in panels (a), (c), and (d), the overrepresented groups tend to prefer Republicans, while the underrepresented groups tend to prefer Democrats.

Panel (a) of Figure 3 shows that old voters are overrepresented *and* prefer Republicans, while young voters are underrepresented *and* tend to prefer Democrats. Similarly, low-income voters and those who are Hispanic, African American, and of Other Races are underrepresented and tend to prefer Democrats. On the other hand, panel (f) shows that religious voters are underrepresented and prefer Republicans. These results show that there is a systematic selection in the preferences of those who turn out.

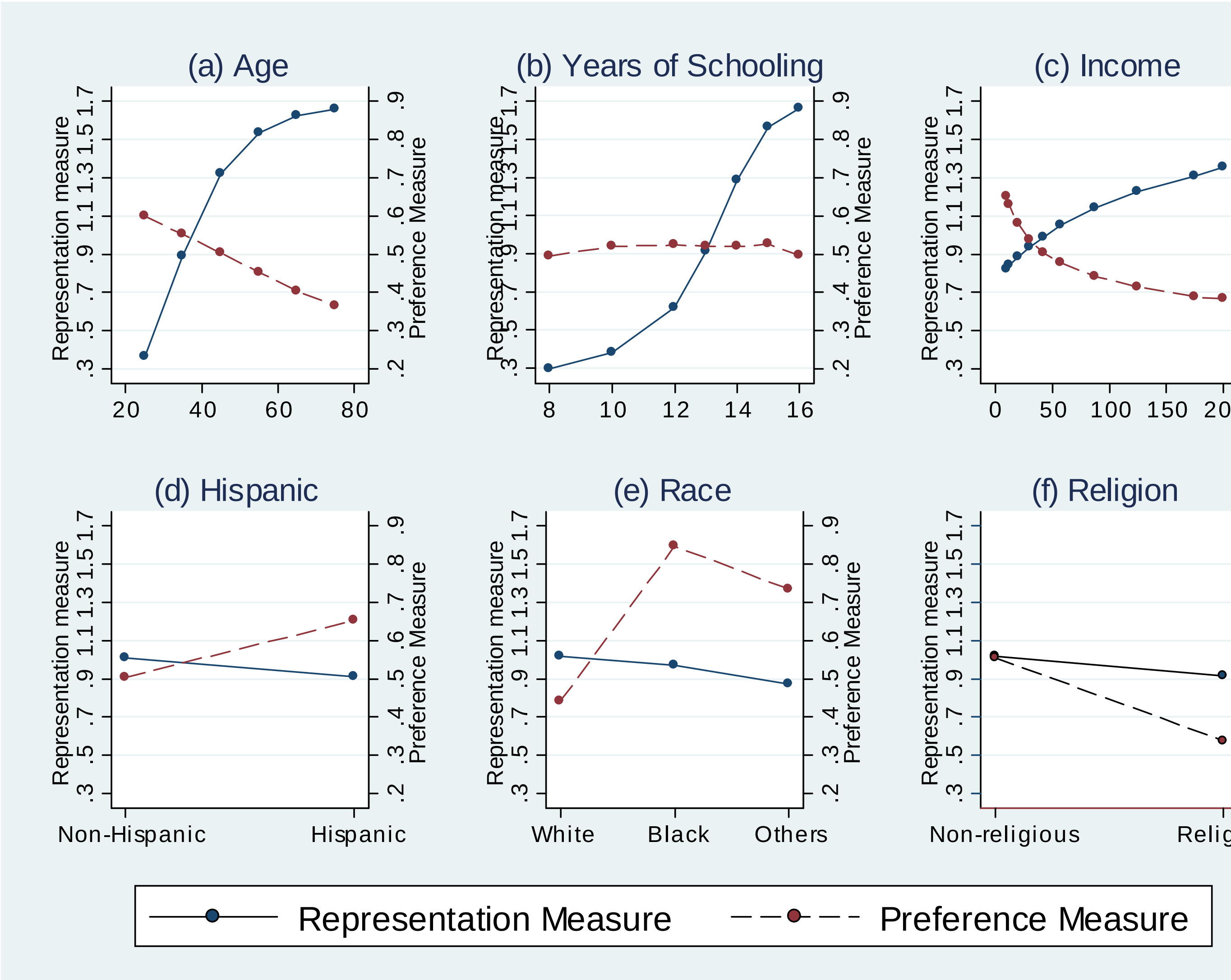


Figure 3: Representation and Preference by Demographic Characteristics. The horizontal axis represents the level/category of the demographic variable. The left vertical axis corresponds to the representation measure, and the right vertical axis corresponds to preference of the group in terms of the two-party vote share. The horizontal axis in Panel (c) is income in 1,000 USD.

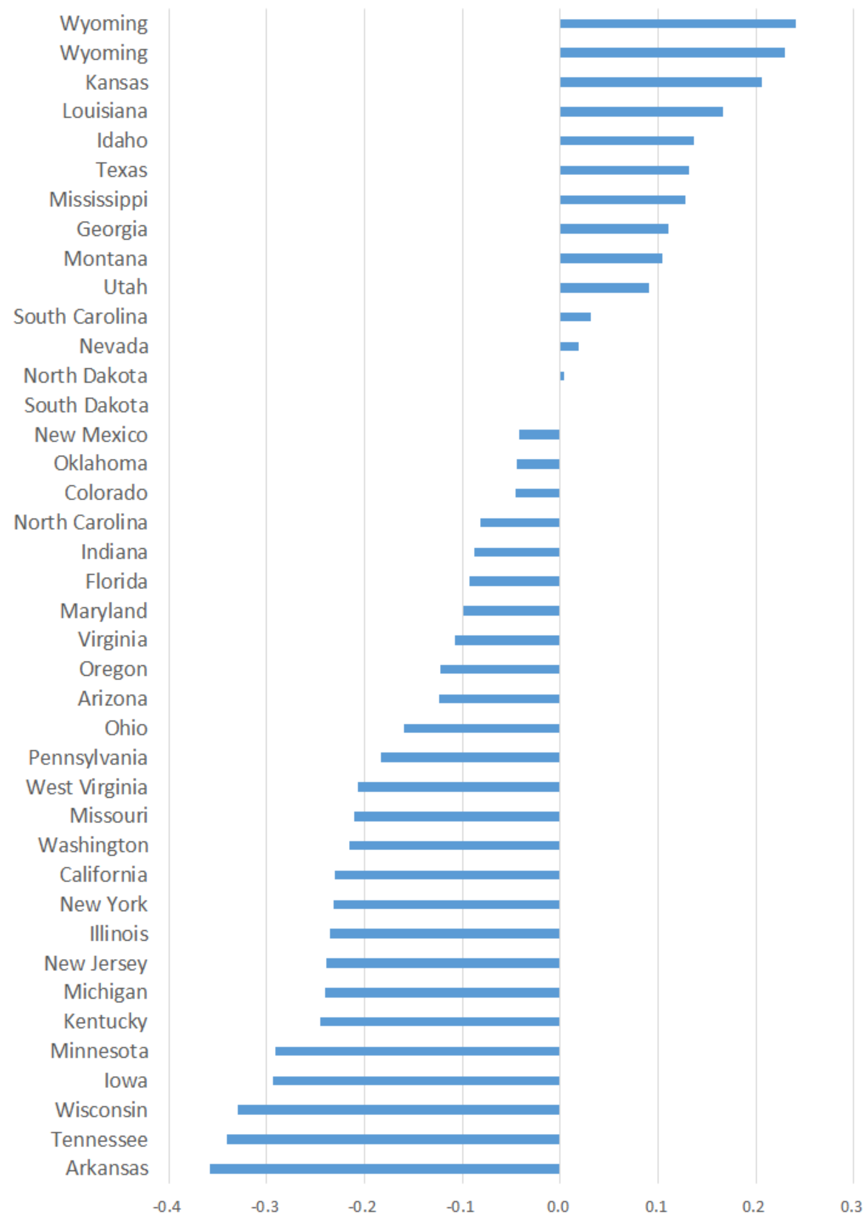


Figure 4: Estimates of the State-level Preference Intercept, $\lambda_s$. $\lambda_{Alabama}$ is normalized to zero.

**Estimates of State-Specific Effects, $\lambda_s$ and $p_s$** Figure 4 plots our estimates of the state specific intercepts in the voter's utility relative to that of Alabama, which is normalized to 0 (Table 9 in the Online Appendix reports the point estimates and the standard errors). Larger values of $\lambda_s$ imply that the voters in the corresponding state prefer Republicans, *net of the effect of demographic characteristics*. These state fixed effects may include the inherent preferences of voters and/or the effect of campaign activities by candidates. The figure show that states such as Wyoming, Kansas, and Louisiana have the strongest preference for Republicans, while states such as Arkansas and Wisconsin have the strongest preference for Democrats. Overall, Democratic strongholds such as New York and California tend to have low estimated values of $\lambda_s$, while Republican strongholds such as Georgia and Texas tend to have high estimates.

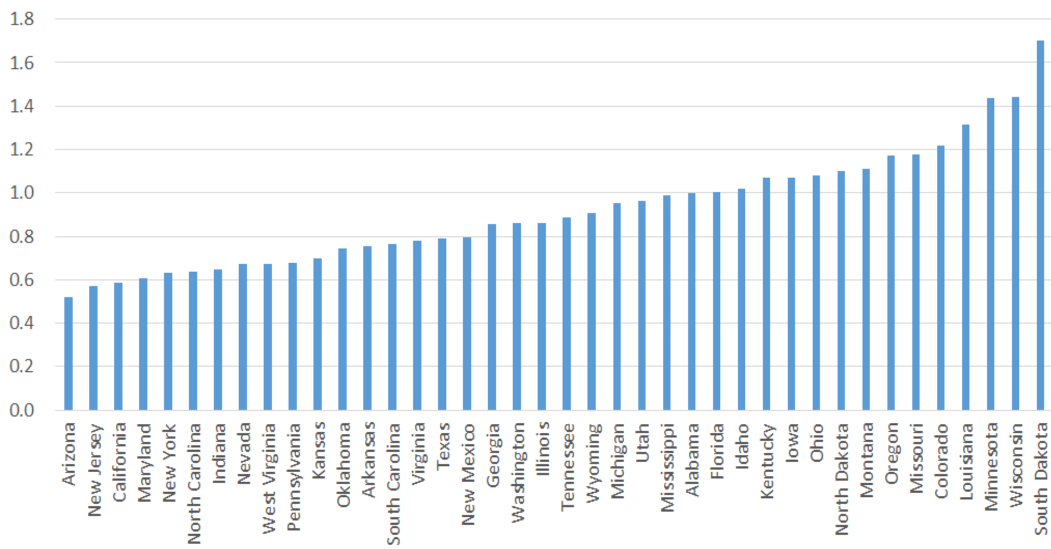


Figure 5: Estimates of the State-level Efficacy Coefficient, $p_s$. Our estimates of $p_s$ are relative to $p_{Alabama}$ where $p_{Alabama}$ is normalized to 1.

Figure 5 plots the estimates of the state-specific component of the perception of voting efficacy, $p_s$, with normalization $p_{Alabama} = 1$ (Table 10 in the Online Appendix reports the point estimates and standard errors). High values of $p_s$ correspond to high perception of voting efficacy, after controlling for demographics. The perception of voting efficacy varies across states, which may partly reflect the fact that the electors are determined at the state level. We find that battleground states such as Minnesota, Wisconsin, Ohio and Iowa have some of the highest estimated values of

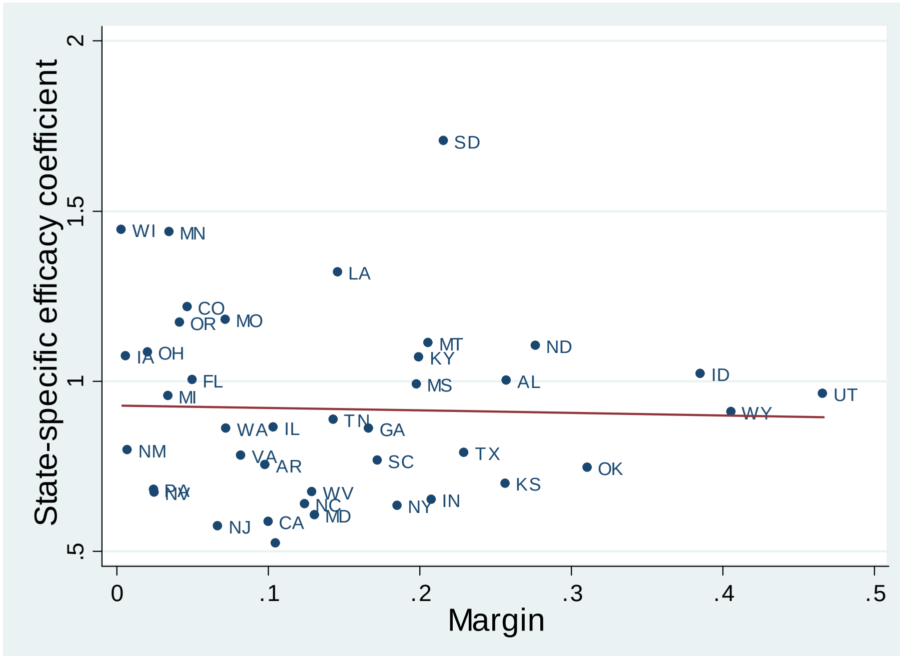


Figure 6: Margin and State-specific Efficacy. The horizontal axis is the actual winning margin and the vertical axis is the estimate of the state-specific component in voting efficacy. The fitted line has a slope of $-0.15$.

voting efficacy. We also find that states considered party strongholds, such as California and Texas, have low estimated values. These results suggest a positive relationship between perception of voting efficacy and pivot probability. To illustrate this relationship, Figure 6 plots the estimates of the state-specific coefficient of efficacy, $p_s$, and the winning margin. The estimated perception of voting efficacy and margin have a negative relationship, with a slope of $-0.15$.

While some of the forces of the pivotal voter model seem to be at play, the estimated values of $p_s$ suggest that the rational expectations pivotal voter model is unlikely to explain the overall voting pattern very well. Models of voting based on rational expectations would require $p_s$ in battleground states to be orders of magnitude greater than those in party strongholds (see, e.g., Shachar and Nalebuff, 1999). However, our estimates of $p_s$ fall within a narrow range: the ratio of the estimated state-level efficacy parameters, $p_s/p_{s'}$, is, at most, three. Our results highlight the importance of relaxing the assumption of rational expectations on the pivot probability.

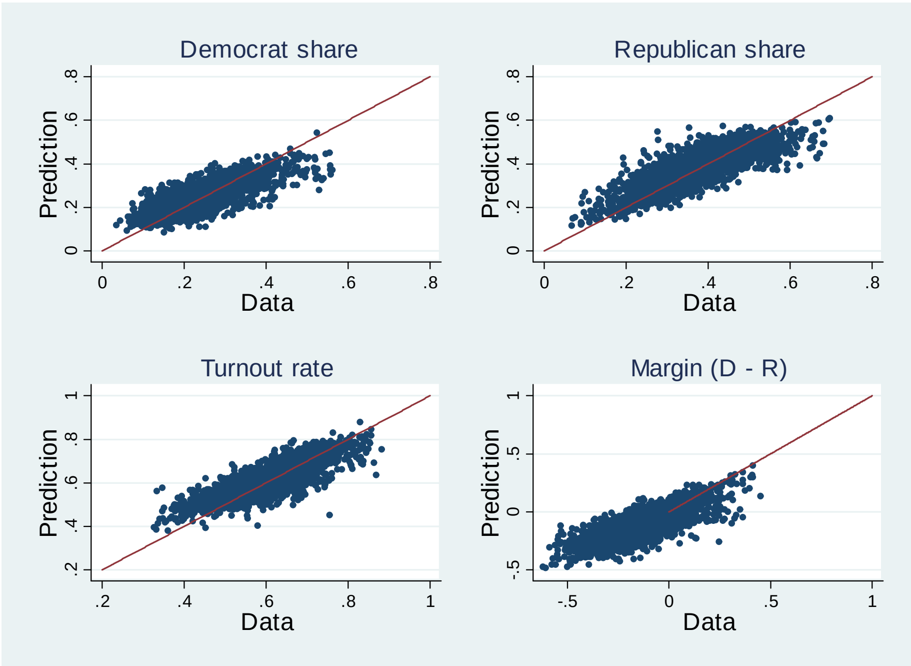


Figure 7: Fit. The figure plots the predicted vote share, turnout, and vote share margin against the data for each county.

**Fit** To assess the fit of our model, Figure 7 plots the county-level vote share, voter turnout, and vote share margin predicted from the model against the data. The predicted vote share, turnout, and vote share margin are computed by evaluating expressions (4), (5), and (6) at the estiamted parameter and integrating out $\xi$ and $\eta$. The plots line up around the 45-degree line, which suggests that the model fits the data well. In Online Appendix B, we provide further discussion of fit.

In previous work, Coate et al. (2008) discusses the difficulty of fitting the winning margin using the rational expectations pivotal voter model. In our paper, we do not impose the rational expectations assumption, and the model fits the winning margin in the data well.

**Turnout and Preference Intensity** We now discuss how preference intensity is related to turnout and how this affects preference aggregation. Our discussion is closely related to the theoretical work of Campbell (1999) that shows that minorities with intense preferences can win elections with costly voting (see, also, Casella, 2005; Lalley and Weyl, 2015). Note that our discussion of intensity in this subsection depends on the distributional assumption of idiosyncratic preference error $\varepsilon_n$.

Figure 8 plots the histogram of $b(\mathbf{x}_n) \equiv b_s(\mathbf{x}_n) + \varepsilon_n$ for all eligible voters in the forty states included in our sample (top panel), and the proportion of those who turn out for given levels of $b(\mathbf{x}_n)$ (bottom panel). The top panel shows that the distribution of the utility difference is roughly centered around zero, and has a slightly fatter tail on the Democrats' side. The bottom panel shows that turnout is higher among Republican supporters than Democratic supporters at the same level of preference intensity. For example, voters with preferences in the bin [–2.5, –2) turns out with 70.3%, while voters with preference in the bin [2, 2.5) turns out with 87.6%. Overall, we estimate that turnout is significantly lower among the electorate who prefer Democrats over Republicans, at $55.7$%, than turnout among those who prefer Republicans over Democrats, at $64.5$%.

The panel also shows that there is high turnout among voters with high preference intensity for either party. For example, voters with preferences in the bin [–2.5, –2) are almost twice as likely to turn out as those with preferences in the bin [–0.5, 0) (70.3% compared to 36.5%). This implies that voters with intense preferences effectively have "more votes" than those who are indifferent, as pointed out by Campbell (1999) (see, also, Casella, 2005; Lalley and Weyl, 2015).

# 7 Counterfactual Experiment

We conduct two counterfactual experiments to quantify the degree to which the correlation among preferences, voting cost, and efficacy affects preference aggregation.

## 7.1 Preference Aggregation under Compulsory Voting

In our first counterfactual experiment, we consider what the election outcome would be if the preferences of all eligible voters were aggregated. The counterfactual result can be thought of as the election outcome under compulsory voting. The election outcome under this counterfactual can be computed by setting voting cost to zero. That is, individuals vote for Democrats or Republicans depending on the sign of $\widehat{b}_s(\mathbf{x}_n) + \widehat{\xi}_m + \varepsilon_n$, where $\widehat{b}_s(\cdot)$ and $\widehat{\xi}_m$ are the estimates of the net utility difference and county-level

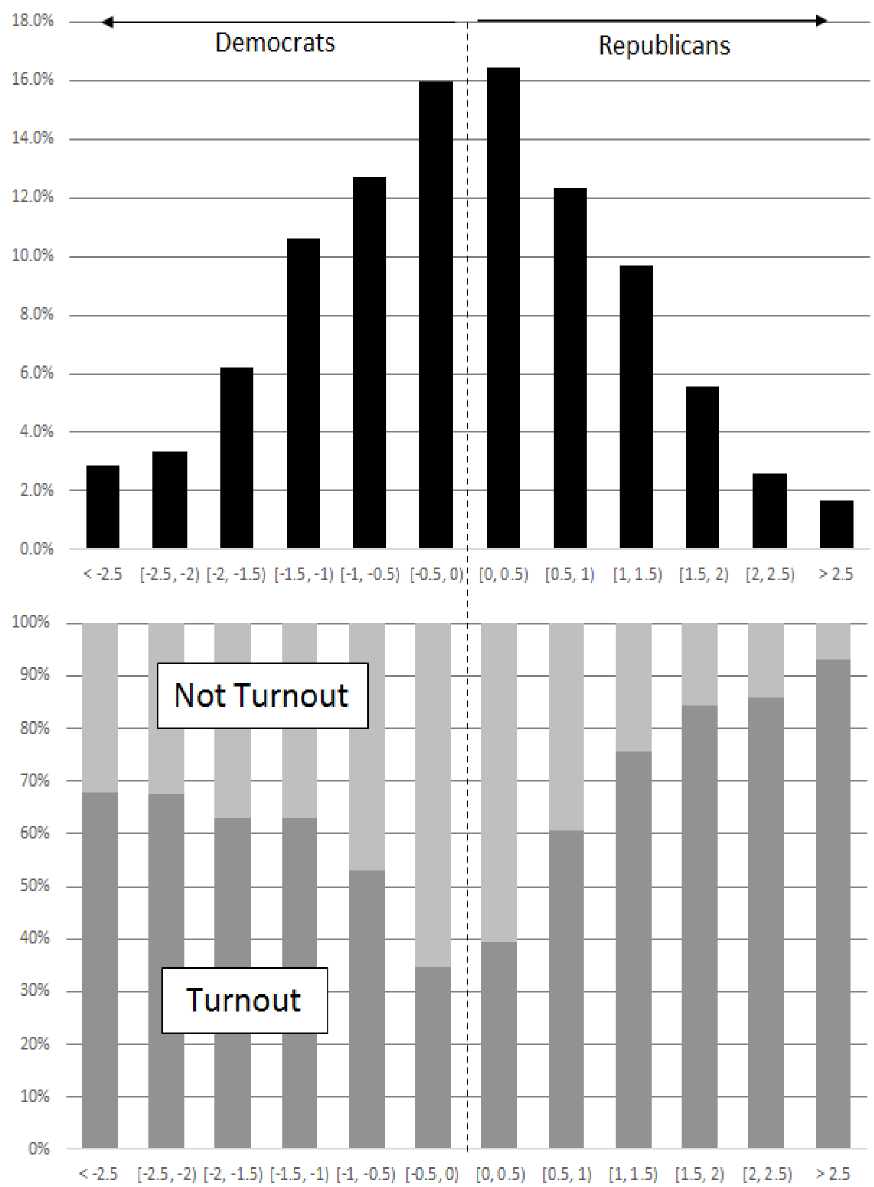


Figure 8: Histogram of Preference Intensity and Turnout Rate by Preference Intensity. The top panel plots the histogram of the utility difference, $b(\mathbf{x}_n)$. The bottom panel plots the proportion of those who turn out for given levels of preference intensity.

preference shock.[33] Hence, we can express the counterfactual county-level vote shares, $\tilde{v}_{D,m}$ and $\tilde{v}_{R,m}$, as

$$\begin{aligned}\tilde{v}_{D,m} &= \int \Phi\left(-\widehat{b}_s(\mathbf{x}_n) - \widehat{\xi}_m\right) dF_{\mathbf{x},m}(\mathbf{x}_n),\\ \tilde{v}_{R,m} &= 1 - \tilde{v}_{D,m}.\end{aligned}$$

Note that our counterfactual results are robust to equilibrium adjustments to voters' perception of efficacy because a voter's decision depends only on the sign of the utility difference irrespective of the perception of efficacy.

| | Two-Party Vote Share | | | # of Electors | |
|---|---|---|---|---|---|
| | Democrats | Republicans | Turnout Rate | Democrats | Republicans |
| Actual | 48.2% | 51.8% | 60.1% | 208 | 278 |
| Counterfactual | 51.9% | 48.1% | 100.0% | 310 | 176 |
| | (1.1%) | (1.1%) | n.a. | (30.1) | (30.1) |

Table 3: Counterfactual Outcome Under Full Turnout. The table compares the actual outcome with the counterfactual outcome in which all voters turn out. The reported outcomes do not include the results for the eleven states that we drop from the sample. Standard errors are reported in parentheses.

Table 3 compares the actual outcomes with the counterfactual outcomes for the forty states in our sample. The first row of Table 3 reports the actual vote share, turnout rate, and the number of electors for the two parties. We report our counterfactual results in the second row. We find that the Democratic two-party vote share increases from 48.2% to 51.9% in the counterfactual, reflecting our earlier finding that the preference for Democrats and voting costs are positively correlated.

In terms of electoral votes, we find that the Democrats increase the number of electoral votes by 102, from 208 to 310. Although ten states and the District of Columbia (D.C.) are not included in our sample, the number of 310 electoral votes is larger than that needed to win the election (270) even if those excluded states and D.C. all vote for the Republican electors.[34] Hence, our estimates suggest that the Democrats would

[33] Note that there is a unique value of $(\hat{\xi}_m, \hat{\eta}_m)$ that rationalizes the actual vote outcome given our estimates of preference, cost and perception of efficacy, as discussed in footnote 30. We use these values of $(\hat{\xi}_m, \hat{\eta}_m)$ to compute our counterfactual outcome.

[34] There are a total of 538 electoral votes, including the states that are excluded from our sample. A

| | Two-Party Vote Share of Democrats | | | Turnout Rate | # of Electors for Democrats | | | Prob(Democrats win in counterfactual) |
|---|---|---|---|---|---|---|---|---|
| | Actual | Counterfactual | | Actual | Actual | Counterfactual | | |
| Alabama | 37.1% | 43.0% | (1.2%) | 56.9% | 0 | 0 | (0.0) | 0.0% |
| Arizona | 44.7% | 50.0% | (1.4%) | 54.1% | 0 | 10 | (5.0) | 49.5% |
| Arkansas | 45.1% | 50.2% | (1.3%) | 53.0% | 0 | 6 | (3.0) | 51.5% |
| California | 55.0% | 58.1% | (1.4%) | 55.7% | 55 | 55 | (0.0) | 100.0% |
| Colorado | 47.6% | 50.5% | (0.9%) | 65.8% | 0 | 9 | (4.4) | 62.6% |
| Florida | 47.5% | 51.2% | (1.0%) | 63.0% | 0 | 27 | (8.8) | 87.9% |
| Georgia | 41.6% | 46.7% | (1.1%) | 55.9% | 0 | 0 | (0.0) | 0.0% |
| Idaho | 30.7% | 36.0% | (1.1%) | 63.4% | 0 | 0 | (0.0) | 0.0% |
| Illinois | 55.2% | 58.0% | (1.2%) | 60.7% | 21 | 21 | (0.0) | 100.0% |
| Indiana | 39.6% | 45.5% | (1.3%) | 54.3% | 0 | 0 | (0.0) | 0.0% |
| Iowa | 49.7% | 52.0% | (0.9%) | 69.7% | 0 | 7 | (0.0) | 100.0% |
| Kansas | 37.1% | 42.4% | (1.1%) | 61.0% | 0 | 0 | (0.0) | 0.0% |
| Kentucky | 40.0% | 44.9% | (1.1%) | 58.6% | 0 | 0 | (0.0) | 0.0% |
| Louisiana | 42.7% | 47.2% | (1.0%) | 60.9% | 0 | 0 | (0.0) | 0.0% |
| Maryland | 56.6% | 59.7% | (1.1%) | 61.5% | 10 | 10 | (0.0) | 100.0% |
| Michigan | 51.7% | 54.5% | (1.0%) | 66.7% | 17 | 17 | (0.0) | 100.0% |
| Minnesota | 51.8% | 52.8% | (0.7%) | 77.9% | 10 | 10 | (0.0) | 100.0% |
| Mississippi | 40.1% | 45.9% | (1.2%) | 55.5% | 0 | 0 | (0.0) | 0.0% |
| Missouri | 46.4% | 49.5% | (0.9%) | 65.3% | 0 | 0 | (4.7) | 24.6% |
| Montana | 39.7% | 44.3% | (0.9%) | 64.4% | 0 | 0 | (0.0) | 0.0% |
| Nevada | 48.7% | 53.3% | (1.3%) | 54.7% | 0 | 5 | (0.0) | 100.0% |
| New Jersey | 53.4% | 56.5% | (1.2%) | 61.6% | 15 | 15 | (0.0) | 100.0% |
| New Mexico | 49.6% | 53.2% | (1.2%) | 58.4% | 0 | 5 | (0.3) | 99.7% |
| New York | 59.3% | 62.6% | (1.3%) | 56.9% | 31 | 31 | (0.0) | 100.0% |
| North Carolina | 43.8% | 48.2% | (1.2%) | 57.1% | 0 | 0 | (3.1) | 4.4% |
| North Dakota | 36.1% | 41.1% | (0.9%) | 65.6% | 0 | 0 | (0.0) | 0.0% |
| Ohio | 48.9% | 51.6% | (0.9%) | 67.6% | 0 | 20 | (4.1) | 95.6% |
| Oklahoma | 34.4% | 40.6% | (1.2%) | 58.4% | 0 | 0 | (0.0) | 0.0% |
| Oregon | 52.1% | 53.7% | (0.9%) | 71.3% | 7 | 7 | (0.0) | 100.0% |
| Pennsylvania | 51.3% | 54.1% | (1.1%) | 62.4% | 21 | 21 | (0.0) | 100.0% |
| South Carolina | 41.4% | 47.2% | (1.3%) | 52.6% | 0 | 0 | (0.8) | 1.0% |
| South Dakota | 39.2% | 43.2% | (0.7%) | 68.9% | 0 | 0 | (0.0) | 0.0% |
| Tennessee | 42.8% | 47.7% | (1.2%) | 56.2% | 0 | 0 | (1.3) | 1.3% |
| Texas | 38.5% | 45.1% | (1.4%) | 52.1% | 0 | 0 | (0.0) | 0.0% |
| Utah | 26.7% | 32.5% | (1.5%) | 59.1% | 0 | 0 | (0.0) | 0.0% |
| Virginia | 45.9% | 50.1% | (1.0%) | 60.1% | 0 | 13 | (6.5) | 45.1% |
| Washington | 53.6% | 55.7% | (1.1%) | 66.4% | 11 | 11 | (0.0) | 100.0% |
| West Virginia | 43.5% | 49.2% | (1.3%) | 53.8% | 0 | 0 | (2.2) | 26.3% |
| Wisconsin | 50.2% | 51.8% | (0.7%) | 75.2% | 10 | 10 | (0.0) | 100.0% |
| Wyoming | 29.7% | 35.1% | (1.1%) | 65.3% | 0 | 0 | (0.0) | 0.0% |

Table 4: State-level Simulation Results under Full Turnout. The shaded rows correspond to the states in which the winning party in the counterfactual differs from that of the actual data. The total number of electors is 538 (the number of electors for the states included in our data is 486), and 270 electors are needed to win the election. Standard errors are reported in parenthesis.

likely have won the 2004 presidential election if the preferences of all voters had been aggregated. The standard errors in our parameter estimates translate to an 87.7% confidence level that the number of electors for the Democrats exceeds 270. Note that this number is a lower bound on the confidence level that the Democrats win the election because it assumes that all states excluded from our sample vote for the Republicans. If we assume, instead, that all states excluded from our sample vote in the same way as they did in the actual election, the confidence level that the Democrats win is 96.0%.

Table 4 presents the state-level breakdown of the counterfactual results for the forty states in our sample. We find that the two-party vote share of the Democrats increases in the counterfactual in all states, and that the results are overturned in nine states (shaded in the table) in the counterfactual. The table also shows that there is considerable heterogeneity in the magnitude of the change across states. For example, in Texas, we find that the change in the two-party vote share for the Democrats is more than five percentage points (from 38.5% to 45.1%), while, in Minnesota, the change is only 1.0 percentage point. An important variable that explains the heterogeneity is the actual turnout. Figure 9 plots the state-level change in the two-party vote share against turnout, and shows that the change tends to be small in states with high voter turnout, while it tends to be large in states with low voter turnout. We explain this pattern below in Section 7.1.2.

#### 7.1.1 Decomposition of Change by Demographics

We now examine the contribution of each demographic group to the overall change in the two-party vote share between the actual and the counterfactual. We do so by decomposing the change in the two-party vote share by demographic groups. Denoting the actual aggregate vote share as $v_k$, $k \in \{D, R\}$, and the counterfactual aggregate vote share as $\tilde{v}_k$, $k \in \{D, R\}$, the change is written as

$$\begin{aligned}\frac{\tilde{v}_D}{\tilde{v}_D + \tilde{v}_R} - \frac{v_D}{v_D + v_R} &= \int \left( \tilde{v}_D(\mathbf{x}) - \frac{v_D}{v_D + v_R} \right) f(\mathbf{x}) d\mathbf{x}, \\ &= \int \Delta(\mathbf{x}) f(\mathbf{x}) d\mathbf{x},\end{aligned}$$

candidate needs 270 electoral votes to win.

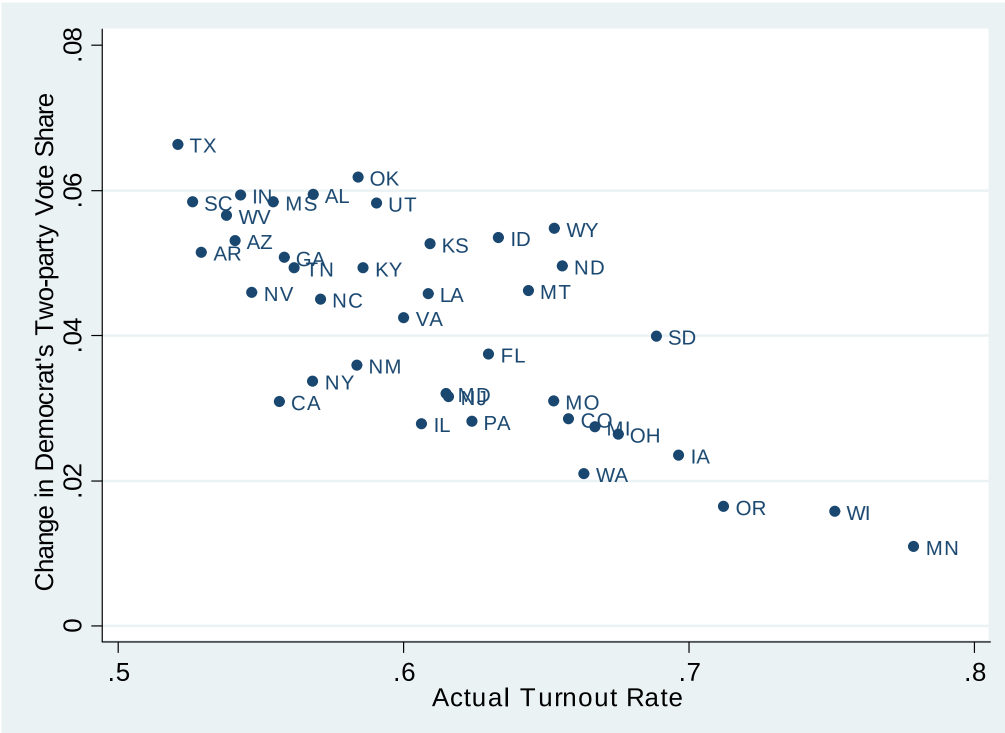


Figure 9: Changes in the Democrat's Vote Share by State. The figure plots the changes in the Democrat's two-party vote share between the actual ant the counterfactual against the level of actual turnout rate.

where $\tilde{v}_D(\mathbf{x})$ denotes the counterfactual vote share of demographic group $\mathbf{x}$, $\Delta(\mathbf{x}) \equiv \tilde{v}_D(\mathbf{x}) - \frac{v_D}{v_D+v_R}$, and $f(\cdot)$ denotes the distribution of demographic characteristics across the forty states included in our sample. Note that $\tilde{v}_D(\mathbf{x}) + \tilde{v}_R(\mathbf{x}) = 1$. We can decompose the change in the two-party vote share as follows.

$$\int \Delta(\mathbf{x}) f(\mathbf{x}) d\mathbf{x} = \int_{\mathbf{x}\in X_1} \Delta(\mathbf{x}) f(\mathbf{x}) d\mathbf{x} + \cdots + \int_{\mathbf{x}\in X_H} \Delta(\mathbf{x}) f(\mathbf{x}) d\mathbf{x},$$

where $\{X_1, ..., X_H\}$ is any partition of the set of demographic characteristics. An example would be a partition of population into three age groups; the groups of 18- to 34-year-olds, 35- to 59-year-olds, and 60-year-olds and above. Thus, $\int_{\mathbf{x}\in X_h} \Delta(\mathbf{x}) f(\mathbf{x}) d\mathbf{x}$ can be interpreted as the contribution of $X_h$ to the change in the aggregate two-party vote share.

Table 5 reports the results of the decomposition. We consider decomposition by age, years of schooling, income, race, Hispanic and non-Hispanic, and Religious and non-Religious. The first column reports the contribution of a particular subgroup to the change, the second column reports the share of the subgroup in the population, and the third column reports the contribution from the subgroup per unit of population, which is simply the number in the first column divided by the number in the second column. For example, age group 18 to 34 contributes 4.15 percentage points to the change in the aggregate two-party vote share. Also, the contribution of a unit of population of this group is 0.13 percentage points. The table shows that age group 18 to 34 and the income group that makes less than $25,000 have large effects on the Democratic two-party vote share. Similarly, African Americans and Other races have a significant impact on the counterfactual vote share. These are the groups that tend to have strong preferences for the Democrats and have low turnout.

| Age | [a] Contribution | [b] Share (%) | [a] / [b] | Income | [a] Contribution | [b] Share (%) | [a] / [b] |
|---|---|---|---|---|---|---|---|
| 18 to 34 | 4.15 pp | 32.1 | 0.13 pp | Less than $25,000 | 4.48 pp | 29.8 | 0.15 pp |
| 35 to 59 | 1.62 pp | 46.0 | 0.04 pp | $25,000 to $49,999 | 1.00 pp | 29.7 | 0.03 pp |
| 60 and above | -1.91 pp | 21.9 | -0.09 pp | $50,000 to $75,000 | -0.30 pp | 19.2 | -0.02 pp |
| | | | | $75,000 to $99,999 | -0.41 pp | 9.9 | -0.04 pp |
| | | | | $10,0000 to $149,999 | -0.50 pp | 7.3 | -0.07 pp |
| | | | | $150,000 and above | -0.41 pp | 4.1 | -0.10 pp |

| Education | [a] Contribution | [b] Share (%) | [a] / [b] | Race | [a] Contribution | [b] Share (%) | [a] / [b] |
|---|---|---|---|---|---|---|---|
| Less than 9 years | 0.05 pp | 3.6 | 0.01 pp | White | -2.88 pp | 78.6 | -0.04 pp |
| 9 years to 11 years | 0.30 pp | 8.6 | 0.03 pp | African-American | 2.03 pp | 12.9 | 0.16 pp |
| High School Graduate | 0.99 pp | 26.8 | 0.04 pp | Others | 4.72 pp | 8.5 | 0.55 pp |
| Associate Degree | 1.12 pp | 31.0 | 0.04 pp | | | | |
| College Graduate | 1.42 pp | 30.0 | 0.05 pp | | | | |

| Hispanic | [a] Contribution | [b] Share (%) | [a] / [b] | Religious | [a] Contribution | [b] Share (%) | [a] / [b] |
|---|---|---|---|---|---|---|---|
| Hispanic | 1.53 pp | 9.8 | 0.16 pp | Religious | -2.15 pp | 16.7 | -0.13 pp |
| Non-Hispanic | 2.34 pp | 90.2 | 0.03 pp | Non-Religious | 6.02 pp | 83.3 | 0.07 pp |

Table 5: Decomposition by Demographics. The numbers show how each demographic group contributes to the change in the two-party vote share between counterfactual and actual data. Note that the changes in each column add up to the same number.

### 7.1.2 Decomposition of State-level Deviation

In this subsection, we explore the relationship between (i) the state-level turnout; and (ii) the state-level change in the Democratic two-party vote share between the actual and the counterfactual. Recall that Figure 9 shows a negative correlation between these two. This correlation can be explained either by the differences in the demographic composition across states or by the differences in the way voters (of a given demographic characteristic) vote across states.

To disentangle these effects, consider how much the change in the two-party vote share in a given state deviates from the national mean, that is

$$\underbrace{\int \Delta^s(\mathbf{x}) f^s(\mathbf{x}) d\mathbf{x}}_{\text{state-level change for state } s} - \underbrace{\int \Delta(\mathbf{x}) f(\mathbf{x}) d\mathbf{x}}_{\text{aggregate change}},$$

where $\Delta^s(\mathbf{x})$ denotes the change in the two-party vote share of voters with demographics $\mathbf{x}$ in state $s$, and $f^s(\mathbf{x})$ the distribution of type $\mathbf{x}$ voter in state $s$. This can be decomposed as follows

$$\begin{aligned}\int \Delta^s(\mathbf{x}) f^s(\mathbf{x}) d\mathbf{x} - \int \Delta(\mathbf{x}) f(\mathbf{x}) d\mathbf{x} = &\int \left(\Delta^s(\mathbf{x}) - \Delta(\mathbf{x})\right) f(\mathbf{x}) d\mathbf{x} \\ &+ \int \Delta(\mathbf{x}) \left(f^s(\mathbf{x}) - f(\mathbf{x})\right) d\mathbf{x} \\ &+ \int \left(\Delta^s(\mathbf{x}) - \Delta(\mathbf{x})\right) \left(f^s(\mathbf{x}) - f(\mathbf{x})\right) d\mathbf{x}. \quad (16)\end{aligned}$$

The first term on the right hand side corresponds to the deviation due to the way that voters vote in state $s$ holding the demographic composition fixed; the second term to the demographic composition in state $s$; and the third term to the cross term between the two.

The first five columns of Table 6 present the summary statistics of each of the three terms, and the last column reports their correlation with the state-level voter turnout. The number of observations is 40, which is the number of states in our sample. The table shows that only the first two terms are important; the third term has much smaller mean and standard deviation compared to the first two terms. Of the first two terms, only the second term, $\int \Delta(\mathbf{x}) \left(f^s(\mathbf{x}) - f(\mathbf{x})\right) d\mathbf{x}$, correlates with the state-level voter turnout. This implies that the negative correlation between the turnout and the change

| | Obs | Mean | Std. Dev. | Min | Max | Correlation with Turnout Rate |
|---|---|---|---|---|---|---|
| First Term | 40 | 1.81% | 5.02% | -6.92% | 19.12% | 0.04 |
| Second Term | 40 | -1.56% | 4.05% | -13.67% | 6.52% | -0.29 |
| Third Term | 40 | 0.03% | 0.75% | -3.52% | 1.10% | -0.12 |

Table 6: Decomposition of State-Level Deviation. Correlation in the table refers to correlation coeffcieint between each term in in expression (16) and the state-level voter turnout .

in the two-party vote share is driven mostly by variation in demographics rather than by the change in the way voters vote.

To further explore which demographics drive the correlation, Figure 10 plots the relationship between the state-level turnout and the fraction of young (18-34 years old), lowest income (below $25,000), and African American voters. These are the demographic groups that make a large contribution to the change in vote share as discussed in Section 7.1.1 on Table 5. The figure shows that states with a larger fraction of young, low-income, and African American voters tend to be associated with lower turnout. Hence, combined with the fact that $\Delta(\mathbf{x})$ is higher for these groups, as shown in the previous subsection, the deviation in the size of these groups, $f^s(\mathbf{x}) - f(\mathbf{x})$, contributes to the negative correlation between turnout and change in the Democratic two-party vote share.

## 7.2 Efficiency Gap and Endogenous Turnout

In our second counterfactual experiment, we study the implications of endogenous turnout for using the efficiency gap as a measure of gerrymandering. Our counterfactual is motivated by a recent U.S. Supreme Court case involving districting for Wisconsin's state legislature (*Gill v. Whitford*) in which the plaintiffs introduces a metric called the "efficiency gap" to measure the extent of gerrymandering.

The efficiency gap is a measure of how well vote shares map into seat shares, and it is defined as the difference in the wasted votes between the parties divided by the total votes. The top half of Table 7 is a numerical example from **?** that illustrates how the efficiency gap is computed. The wasted vote for party $k$ in district $d$ is either $v_{kd} - (1/2)(v_{kd} + v_{-kd})$ or $v_{kd}$, depending on whether $v_{kd}$ is greater than $v_{-kd}$, where

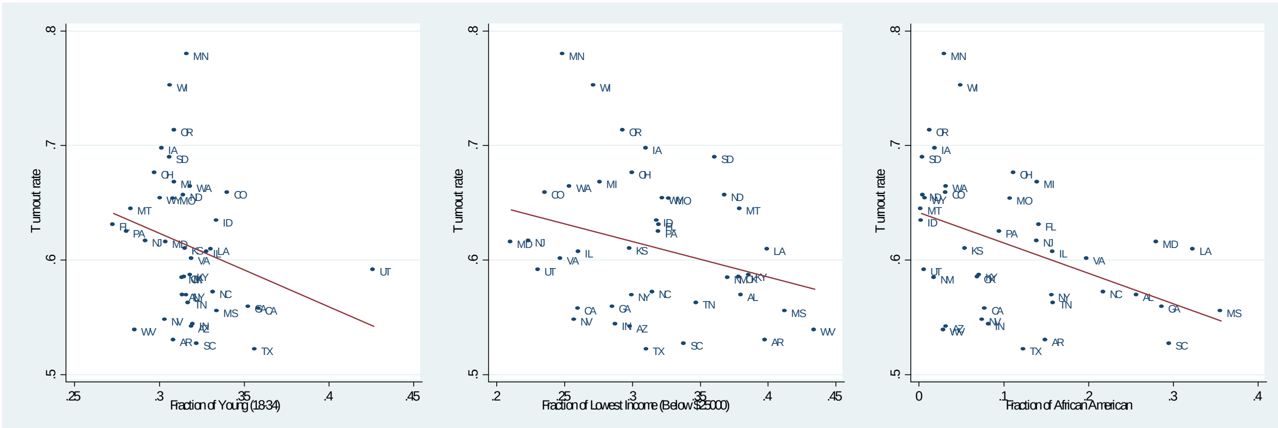


Figure 10: Demographic Composition and Turnout. The left panel plots the state-level voter turnout against the fraction of eligible voters in 18-34 age group; the center panel the fraction of those with lowest income bracket; and the right panel the fraction of African Americans.

$v_{kd}$ is the votes obtained by party $k$ in district $d$ and $v_{-kd}$ is the votes obtained by the other party. The efficiency gap is simply the sum of the difference in the wasted votes across districts, divided by the total votes. Stephanopolous and McGhee argue that an efficiency gap exceeding 8% should be presumptively unlawful. Table 7 is an example of a districting plan that favors party A (party A wins districts 1 through 8 with a state-wide vote share of 55%), and indeed, this is captured by the fact that this has a high efficiency gap of 20%.

While the efficiency gap captures the inbalance in the way votes are translated to seat shares, the validity of the efficiency gap as a measure of gerrymandering requires that turnout is exogenous. To see this, consider again the example in Table 7, but suppose now that there are low cost voters (who always vote) and high cost voters (who vote only when the partisanship of the district is balanced) in equal proportion. The bottom part of Table 7 illustrates a redistricting plan in which the shares of A and B supporters are kept the same as in the original Stephanopolous and McGhee's example, but voters now have heterogeneous costs. In this plan, the high cost voters turn out only in districts 4 to 8, which are the districts with a balanced partisanship. This example lowers the efficiency gap from 20% to 4.87% while keeping the seat shares unchanged.

This example illustrates the potential issue with using the efficiency gap as a measure of gerrymandering. The original example of Stephanopolous and McGhee and

Table 7: Example: Stephanopolous

| District | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Exogenous Turnout: Stephanopolous and McGhee's example | | | | | | | | |
| Total Population | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 1000 | |
| A supporter | 70 | 70 | 70 | 54 | 54 | 54 | 54 | 54 | 35 | 35 | 550 | |
| B supporter | 30 | 30 | 30 | 46 | 46 | 46 | 46 | 46 | 65 | 65 | 450 | |
| A waste vote | 20 | 20 | 20 | 4 | 4 | 4 | 4 | 4 | 35 | 35 | 150 | Efficiency Gap |
| B waste vote | 30 | 30 | 30 | 46 | 46 | 46 | 46 | 46 | 15 | 15 | 350 | 20.00% |
| | | | | | Endogenous Turnout | | | | | | | |
| Low Cost A Supporter | 70 | 70 | 70 | 0 | 0 | 0 | 0 | 0 | 33 | 32 | 275 | |
| High Cost A Supporter | 0 | 0 | 0 | 54 | 54 | 54 | 54 | 54 | 2 | 3 | 275 | |
| Low Cost B Supproter | 0 | 0 | 0 | 46 | 43 | 23 | 23 | 23 | 34 | 33 | 225 | |
| High Cost B Supporter | 30 | 30 | 30 | 0 | 3 | 23 | 23 | 23 | 31 | 32 | 225 | |
| A vote | 70 | 70 | 70 | 54 | 54 | 54 | 54 | 54 | 33 | 32 | 545 | |
| B vote | 0 | 0 | 0 | 46 | 46 | 46 | 46 | 46 | 34 | 33 | 297 | |
| Turnout | 70 | 70 | 70 | 100 | 100 | 100 | 100 | 100 | 67 | 65 | 842 | |
| A waste vote | 35 | 35 | 35 | 4 | 4 | 4 | 4 | 4 | 33 | 32 | 190 | Efficiency Gap |
| B waste vote | 0 | 0 | 0 | 46 | 46 | 46 | 46 | 46 | 0.5 | 0.5 | 231 | 4.87% |

the modified example are just as biased for party A in terms of mapping the overall preferences of the voters to seat shares, and yet they result in a very different efficiency gap measures. Moreover, districting planners can take advantage of the heterogeneity in voting costs to draw plans that give one party a disproportionate advantage while keeping the efficiency gap low. Whether the efficiency gap is a good measure of gerrymandering depends on the extent to which turnout levels can be manipulated through districting plans.

In order to empirically evaluate the robustness of the efficiency gap to endogenous turnout, we compute the efficiency gap for the 2004 U.S. Presidential election when we equalize the state-specific component of efficacy across states (set $p_s = p_{s'}$). Equalizing $p_s$ across states eliminates endogeneity in turnout that is district specific. Although the intended use of the efficiency gap measure is mainly for Congressional and state legislative elections, it is possible to compute the efficiency gap for presidential elections as well. In our context, we compute the wasted vote in each state for the two parties and then sum the difference across all of the states.

Table 8 reports the efficiency gap when we equalize $p$ across states. The table considers five different levels of $p$ to target aggregate turnout levels between 40% and 80% in 10% increments. The results are reported in rows (2) to (6). The first row corresponds to the efficency gap computed using the actual data. Comparing the first

| Turnout | Vote share | | Two-party Vote Share | # of electors | | # of winning states | | Efficiency gap |
|---|---|---|---|---|---|---|---|---|
| | D | R | for D | D | R | D | R | |
| 60.1% (Data) | 29.0% | 31.1% | 48.2% | 208 | 278 | 11 | 29 | -0.5% |
| 40% | 18.3% | 21.7% | 40.0% | 143 | 343 | 6 | 34 | -8.8% |
| 50% | 23.5% | 26.5% | 50.0% | 171 | 315 | 8 | 32 | -4.8% |
| 60% | 29.0% | 31.0% | 60.0% | 198 | 288 | 10 | 30 | -1.8% |
| 70% | 34.9% | 35.1% | 70.0% | 225 | 261 | 14 | 26 | -0.4% |
| 80% | 40.9% | 39.1% | 80.0% | 272 | 214 | 16 | 24 | 7.3% |

Table 8: Efficiency gap

row to the (x) row, we find that the efficiency gap changes by 1.7%. This implies that equalizing $p_s$ across states in a way that does not change the overall turnout still affects the efficiency gap by 1.7%. The table also shows that the efficiency gap changes by about 16% when turnout is exogenously increased from 40% to 80%. Relative to the proposed threshold value of 8% above which districting plans should be deemed presumptively unlawful, these changes in the efficiency measure seem significant.

The sensitivity of the efficiency gap to endogeneiy of turnout points to a more general problem with comparing actual votes and seat shares as a measure of gerrymandering. While our discussion has so far focused on the efficiency gap, any measure based on a comparison beween actual votes and seat shares, is subject to the same concerns. This includes the concept of partisan symmetry, which a majority of the Supreme court justices expressed support in the case *League of United Latin American Citizens v Perry*. The extant arguments seem to take turnout as fixed and exogenous. Our results illustrate the importance of considering the implications of endogenous turnout when thinking abou how to measure gerrymandering. One natural alternative is to compare the difference between the actual underlying voter preferences and seat shares. If we think about elections as a way to aggregate preferences into outcomes, evaluating the electoral system in terms of its ability to aggregate preferences seems most coherent.

# References

**Akerlof, George A and William T Dickens**, "The Economic Consequences of Cognitive Dissonance," *American Economic Review*, 1982, pp. 307–319.

**Atkeson, Lonna Rae**, “"Sure, I Voted for the Winner!" Overreport of the Primary Vote for the Party Nominee in the National Election Studies,” *Political Behavior*, 1999, *21* (3), 197–215.

**Barkume, Anthony J**, “Identification of Preference for Election Outcomes from Aggregate Voting Data,” *Public Choice*, 1976, *27* (1), 41–58.

**Bendor, Jonathan, Daniel Diermeier, and Michael Ting**, “A Behavioral Model of Turnout,” *American Political Science Review*, 2003, *97* (02), 261–280.

**Berry, Steven T**, “Estimating Discrete-Choice Models of Product Differentiation,” *RAND Journal of Economics*, 1994, *25* (2), 242–262.

**Brunell, Thomas L and John DiNardo**, “A Propensity Score Reweighting Approach to Estimating the Partisan Effects of Full Turnout in American Presidential Elections,” *Political Analysis*, 2004, *12* (1), 28–45.

**Brunnermeier, Markus K and Jonathan A Parker**, “Optimal Expectations,” *American Economic Review*, 2005, *95*, 4.

**Campbell, Colin M**, “Large Electorates and Decisive Minorities,” *Journal of Political Economy*, 1999, *107* (6), 1199–1217.

**Cascio, Elizabeth U and Ebonya Washington**, “Valuing the Vote: The Redistribution of Voting Rights and State Funds Following the Voting Rights Act of 1965,” *Quarterly Journal of Economics*, 2013.

**Casella, Alessandra**, “Storable Votes,” *Games and Economic Behavior*, 2005, *51* (2), 391–419.

**Citrin, Jack, Eric Schickler, and John Sides**, “What if Everyone Voted? Simulating the Impact of Increased Turnout in Senate Elections,” *American Journal of Political Science*, 2003, *47* (1), 75–90.

**Coate, Stephen and Michael Conlin**, “A Group Rule-Utilitarian Approach to Voter Turnout: Theory and Evidence,” *American Economic Review*, 2004, pp. 1476–1504.

**_ , _ , and Andrea Moro**, “The Performance of Pivotal-Voter Models in Small-Scale Elections: Evidence from Texas Liquor Referenda,” *Journal of Public Economics*, 2008, *92* (3), 582–596.

**Degan, Arianna and Antonio Merlo**, "A Structural Model of Turnout and Voting in Multiple Elections," *Journal of the European Economic Association*, 2011, *9* (2), 209–245.

**DellaVigna, Stefano, John A List, Ulrike Malmendier, and Gautam Rao**, "Voting to Tell Others," Technical Report 2015.

**DeNardo, James**, "Turnout and the Vote: The Joke's on the Democrats," *American Political Science Review*, 1980, *74* (02), 406–420.

**Dhillon, Amrita and Susana Peralta**, "Economic Theories of Voter Turnout," *Economic Journal*, 2002, *112* (480), F332–F352.

**Downs, Anthony**, *An Economic Theory of Democracy*, New York, 1957.

**Duffy, John and Margit Tavits**, "Beliefs and Voting Decisions: A Test of the Pivotal Voter Model," *American Journal of Political Science*, 2008, *52* (3), 603–618.

**Esponda, Ignacio and Demian Pouzo**, "Conditional Retrospective Voting in Large Elections," *American Economic Journal: Microeconomics, Forthcoming*, 2016.

**Feddersen, Timothy and Alvaro Sandroni**, "A Theory of Participation in Elections," *American Economic Review*, 2006, pp. 1271–1282.

**Feddersen, Timothy J**, "Rational Choice Theory and the Paradox of Not Voting," *Journal of Economic Perspectives*, 2004, pp. 99–112.

**Fujiwara, Thomas**, "Voting Technology, Political Responsiveness, and Infant Health: Evidence from Brazil," *Econometrica*, 2015, *83* (2), 423–464.

**Gordon, Brett R and Wesley R Hartmann**, "Advertising Effects in Presidential Elections," *Marketing Science*, 2013, *32* (1), 19–35.

**Haile, Philip A and Elie Tamer**, "Inference with an Incomplete Model of English Auctions," *Journal of Political Economy*, 2003, *111* (1), 1–51.

**Hansford, Thomas G and Brad T Gomez**, "Estimating the Electoral Effects of Voter Turnout," *American Political Science Review*, 2010, *104* (02), 268–288.

**Harsanyi, John C**, “Rule Utilitarianism, Rights, Obligations and the Theory of Rational Behavior,” *Theory and Decision*, 1980, *12* (2), 115–133.

**Hotz, V Joseph and Robert A Miller**, “Conditional Choice Probabilities and the Estimation of Dynamic Models,” *Review of Economic Studies*, 1993, *60* (3), 497–529.

**Husted, Thomas A and Lawrence W Kenny**, “The Effect of the Expansion of the Voting Franchise on the Size of Government,” *Journal of Political Economy*, 1997, pp. 54–82.

**Jäckel, Peter**, “A Note on Multivariate Gauss-Hermite Quadrature,” Technical Report 2005. Sourced from: www.pjaeckel.webspace. virginmedia.com/ANoteOnMultivariateGaussHermiteQuadrature.pdf.

**Kanazawa, Satoshi**, “A Possible Solution to the Paradox of Voter Turnout,” *Journal of Politics*, 1998, *60* (04), 974–995.

**Karp, Jeffrey A and Susan A Banducci**, “Political Efficacy and Participation in Twenty-Seven Democracies: How Electoral Systems Shape Political Behaviour,” *British Journal of Political Science*, 2008, *38* (02), 311–334.

**Kroth, Verena, Valentino Larcinese, and Joachim Wehner**, “A Better Life For All? Democratization and Electrification in Post-Apartheid South Africa,” Technical Report 2013.

**Lalley, P Steven and E Glen Weyl**, “Quadratic Voting,” Technical Report 2015.

**Ledyard, John O**, “The Pure Theory of Large Two-Candidate Elections,” *Public Choice*, 1984, *44* (1), 7–41.

**Leighley, Jan E and Jonathan Nagler**, *Who Votes Now?: Demographics, Issues, Inequality, and Turnout in the United States*, Princeton University Press, 2013.

**Levine, David K and Thomas R Palfrey**, “The Paradox of Voter Participation? A Laboratory Study,” *American Political Science Review*, 2007, *101* (01), 143–158.

**Lijphart, Arend**, “Unequal Participation: Democracy’s Unresolved Dilemma Presidential Address,” *American Political Science Review*, 1997, *91* (01), 1–14.

**Martinez, Michael D and Jeff Gill**, "The Effects of Turnout on Partisan Outcomes in US Presidential Elections 1960–2000," *Journal of Politics*, 2005, *67* (4), 1248–1274.

**McDonald, Michael P.**, "1980-2014 November General Election," 2016. accessed May 26th, 2016.

**McDonald, Michael P and Samuel L Popkin**, "The Myth of the Vanishing Voter," *American Political Science Review*, 2001, *95* (4), 963–974.

**Merlo, Antonio**, "Whither Political Economy? Theories, Facts and Issues," in Whitney Newey Richard Blundell and Torsten Persson, eds., *Advances in Economics and Econometrics, Theory and Applications: Ninth World Congress of the Econometric Society*, Vol. I, Cambridge: Cambridge University Press, 2006, pp. 381–421.

**Miller, Grant**, "Women's Suffrage, Political Responsiveness, and Child Survival in American History," *The Quarterly Journal of Economics*, 2008, *123* (3), 1287.

**Minozzi, William**, "Endogenous Beliefs in Models of Politics," *American Journal of Political Science*, 2013, *57* (3), 566–581.

**Myatt, David**, "A Rational Choice Theory of Voter Turnout," Technical Report 2012.

**Palfrey, Thomas R and Howard Rosenthal**, "A Strategic Calculus of Voting," *Public Choice*, 1983, *41* (1), 7–53.

— **and** — , "Voter Participation and Strategic Uncertainty," *American Political Science Review*, 1985, *79* (01), 62–78.

**Riker, William H and Peter C Ordeshook**, "A Theory of the Calculus of Voting," *American Political Science Review*, 1968, *62* (01), 25–42.

**Shachar, Ron and Barry Nalebuff**, "Follow the Leader: Theory and Evidence on Political Participation," *American Economic Review*, 1999, pp. 525–547.

**Smith, Mark A**, "The Contingent Effects of Ballot Initiatives and Candidate Races on Turnout," *American Journal of Political Science*, 2001, pp. 700–706.

**Strömberg, David**, "How the Electoral College Influences Campaigns and Policy: the Probability of Being Florida," *American Economic Review*, 2008, *98* (3), 769–807.

**Tucker, Harvey J and James DeNardo**, "Does Heavy Turnout Help Democrats in Presidential Elections?," *American Political Science Review*, 1986, *80* (04), 1291–1304.

# 8 Appendix

## 8.1 Appendix A: Derivation of the Calculus of Voting Model

In this Appendix, we follow Riker and Ordeshook (1968) and present a derivation of expressions (1) and (2). We classify the situation of a voter into the following five mutually exclusive events:

$E_0$: votes for $D$ and $R$ are tied without her vote;

$E_{D1}$: $D$ has exactly one more vote than $R$ without her vote;

$E_{R1}$: $R$ has exactly one more vote than $D$ without her vote;

$E_{D2}$: $D$ has two or more votes than $R$ without her vote;

$E_{R2}$: $R$ has two or more votes than $D$ without her vote.

Let $q_l$ denote the probability of $E_l$ for $l \in \{0, D1, R1, D2, R2\}$. Let $\pi$ be the probability that $D$ wins the election in case of a tie. Then, the utility of the voter for voting for candidates $D$ and $R$, as well as not voting, are written as

$$\begin{aligned} U_D &= q_0 b_D + q_{D1} b_D + q_{R1}(\pi b_D + (1-\pi) b_R) + q_{D2} b_D + q_{R2} b_R - c, \\ U_R &= q_0 b_R + q_{D1}(\pi b_D + (1-\pi) b_R) + q_{R1} b_R + q_{D2} b_D + q_{R2} b_R - c, \\ U_0 &= q_0(\pi b_D + (1-\pi) b_R) + (q_{D1} + q_{D2}) b_D + (q_{R1} + q_{R2}) b_R. \end{aligned}$$

By taking the difference between voting for $D$ and not voting, we have

$$\begin{aligned} U_D - U_0 &= q_0 b_D + q_{D1} b_D + q_{R1}(\pi b_D + (1-\pi) b_R) + q_{D2} b_D + q_{R2} b_R - c \\ &\quad - q_0(\pi b_D + (1-\pi) b_R) - (q_{D1} + q_{D2}) b_D - (q_{R1} + q_{R2}) b_R \\ &= \left(q_0(1-\pi) + q_{R1}\pi\right) b_D - \left(q_0(1-\pi) + q_{R1}\pi\right) b_R - c \\ &= \left(q_0(1-\pi) + q_{R1}\pi\right)(b_D - b_R) - c. \end{aligned}$$

Similarly, we have

$$\begin{aligned} U_R - U_0 &= q_0 b_R + q_{R1} b_R + q_{D1}(\pi b_D + (1-\pi) b_R) + q_{R2} b_R + q_{D2} b_D - c \\ &\quad - q_0(\pi b_D + (1-\pi) b_R) - (q_{R1} + q_{R2}) b_R - (q_{D1} + q_{D2}) b_D \\ &= (q_{D1}(1-\pi) + q_0 \pi)\, b_R + (-q_{D1}(1-\pi) - q_0 \pi)\, b_D \\ &= (q_{D1}(1-\pi) + q_0 \pi)\,(b_R - b_D) - c. \end{aligned}$$

Assuming that $q_0 = q_{D1} = q_{R1} \equiv p$ in a large election (see page 103 of **?**, for a justification), we can rewrite $U_D$, $U_R$, and $U_0$ as

$$\begin{aligned} U_D &= p(b_D - b_R) - c \\ U_R &= p(b_R - b_D) - c \\ U_0 &= 0. \end{aligned}$$

## 8.2 Appendix B: Data Construction

In this Appendix, we explain how we construct the joint distribution of demographic characteristics and citizenship status at the county level. We first use the 5% Public Use Microdata Sample of the 2000 U.S. Census (hereafter PUMS), which is an individual-level dataset, to estimate the covariance matrix between the demographic variables and citizenship information within each public use microdata area (PUMA). In particular, we estimate the joint distribution of the discrete demographic characteristics (Race, Hispanic, Citizenship) by counting the frequency of occurrence. We then estimate a covariance matrix for the continuous demographic variables (Age, Income, Years of Schooling) for each bin. Because the PUMA and counties do not necessarily coincide, we estimate covariance matrices for each PUMA and then use the correspondence chart provided in the PUMS website to obtain estimates at the county level.

In the second step, we construct the joint distribution of demographic characteristics by combining the covariance matrix estimated in the first step and the marginal distributions of each of the demographic variables at the county level obtained from Census Summary File 1 through File 3. We discretize continuous variables into coarse bins. We discretize age into three bins: (1)18-34 years old; (2) 35-59 years old; and (3) above 60 years old; income into 6 bins: (1) $0-$25,000; (2) $25,000-$50,000; (3) $50,000-$75,000; (4) $75,000-$100,000; (5) $100,000-$150,000; and (6) above

\$150,000; and years of schooling into 5 bins: (1) Less than 9th grade; (2) 9th-12th grade with no diploma; (3) high school graduate; (4) some college with no degree or associate degree; and (5) bachelor's degree or higher. Thus, there are 540 bins in total. The joint distribution of demographic characteristics that we create gives us a probability mass over each of the 540 bins for each county.

Finally, we augment the census data with religion data obtained from Religious Congregations and Membership Study 2000. These data contain information on the share of the population with adherence to either "Evangelical Denominations" or "Church of Jesus Christ of Latter-day Saints" at the county level. Because the Census does not collect information on religion, we do not know the correlation between the religion variable and the demographic characteristics in the Census. Thus, we assume independence of the religion variable and other demographic variables. As a result, there are 1,080 bins in our demographics distribution.

## 8.3 Appendix C: Identification of $c(\cdot)$, $p(\cdot)$, and $F_\eta$ in the general case

In this Appendix, we show that $c(\cdot)$, $p(\cdot)$, and $F_\eta$ are identified even when the max operator in equation (11) binds with positive probability. Note that our argument in the main text considered only the case in which the max operator never binds. Recall that

$$\underbrace{\frac{\Phi^{-1}\left(1-v_{R,m}\right)-\Phi^{-1}\left(v_{D,m}\right)}{2}}_{\equiv Y_m}=\max\left\{0,\frac{c(\bar{\mathbf{x}}_m)}{p(\bar{\mathbf{x}}_m)}+\frac{\eta_m}{p(\bar{\mathbf{x}}_m)}\right\},\eta_m\perp\bar{\mathbf{x}}_m. \tag{17}$$

In this Appendix, we work with the normalization that the value of $p(\cdot)$ at some $\bar{\mathbf{x}}_m = \mathbf{x}_0$ as $p(\mathbf{x}_0) = 1$. This amounts to a particular normalization of variance of $\eta$. Note that the distribution of $Y_m$ (the left hand side of equation (17)) conditional on $\bar{\mathbf{x}}_m = \mathbf{x}_0$ is a truncated distribution with mass at zero. Figure 11 illustrates this when the mass at zero is less than $50\%$, and $F_\eta$ is symmetric and single-peaked at zero.

First, we present our identification discussion for the case that $F_\eta$ is symmetric and single-peaked at zero. As Figure 11 illustrates, the median of $Y_m$ conditional on $\bar{\mathbf{x}}_m = \mathbf{x}_0$ directly identifies $c(\mathbf{x}_0)$ under these assumptions. Also, the density of $\eta$, $f_\eta$, is identified above the point of truncation. Formally, $f_\eta(F_\eta^{-1}(t))$ is identified for any

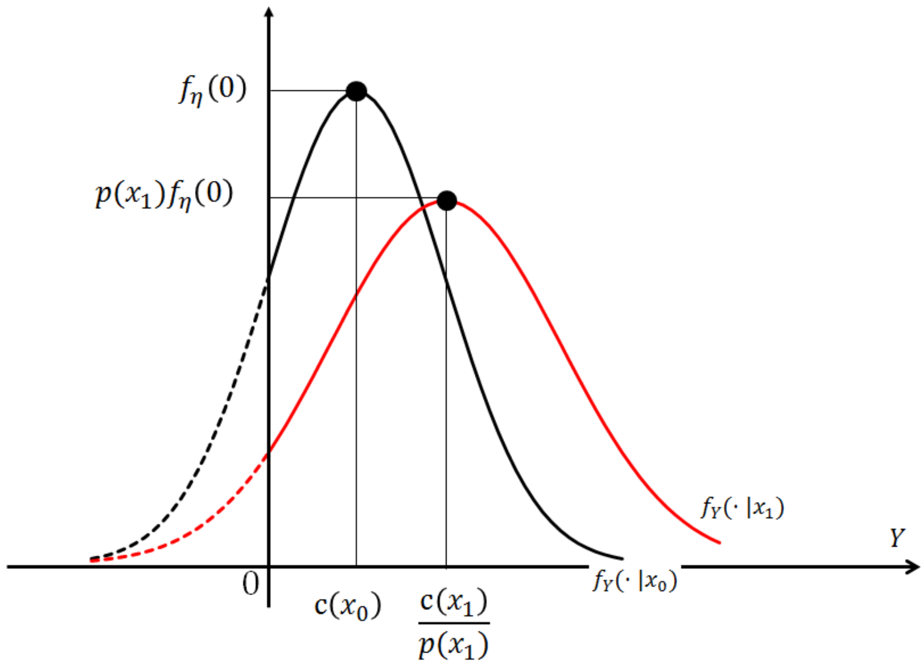


Figure 11: The distribution of $Y_m$ conditional on $\mathbf{x} = \mathbf{x}_0$ and $\mathbf{x} = \mathbf{x}_1$ when the distribution of $\eta$ is symmetric and single-peaked, and $t(\mathbf{x}_0)$, $t(\mathbf{x}_1) < 0.5$, where $t(\mathbf{x})$ is the probability that $Y_m$ is equal to zero conditional on $\mathbf{x}$. Note that the distribution of $Y_m$ is truncated at zero. The conditional median of $Y_m$ identifies $c(\mathbf{x}_0)$ and $c(\mathbf{x}_1)/p(\mathbf{x}_1)$, and the height of the density at the conditional median identifies $f_\eta(0)$ and $p(\mathbf{x}_1)f_\eta(0)$.

$t > t(\mathbf{x}_0)$, where

$$t(\mathbf{x}_0) = \Pr\left(Y_m = 0 | \, \mathbf{x}_0\right).$$

Hence, $f_\eta(0)$ is identified from the height of the density of $Y_m$ at the median.

Now, consider $\mathbf{x}_1 \neq \mathbf{x}_0$. Assume, again, that $t(\mathbf{x}_1) < 0.5$. Then, $c(\mathbf{x}_1)/p(\mathbf{x}_1)$ is identified from the conditional median of $Y_m$, and $p(\mathbf{x}_1)f_\eta(0)$ is identified by the height of the conditional density of $Y_m$ at the median. Given that $f_\eta(0)$ is identified, $c(\mathbf{x}_1)$ and $p(\mathbf{x}_1)$ are both identified. Moreover, $F_\eta$ is identified over its full support if there exists sufficient variation in $\mathbf{x}$, i.e., $\inf_{\mathbf{x}} t(\mathbf{x}) = 0$.

We now consider the case in which $F_\eta$ is not restricted to being symmetric and single-peaked and $t(\mathbf{x}_0)$ may be less than 0.5. The distribution of $Y_m$ is identified above $t(\mathbf{x}_0)$, as before. Now, consider $\mathbf{x}_1 \neq \mathbf{x}_0$. Similar to before, we identify $p(\mathbf{x}_1)f_\eta(F_\eta^{-1}(\tau))$ for $\tau$ above $t(\mathbf{x}_1)$.[35] If we let $\tau$ be any number larger than $\max\{t(\mathbf{x}_0), t(\mathbf{x}_1)\}$,

[35] Note that we identify $f_{\eta/p(\mathbf{x}_1)}\left(F^{-1}_{\eta/p(\mathbf{x}_1)}(t)\right)$, where $f_{\eta/p(\mathbf{x}_1)}(\cdot)$ and $F^{-1}_{\eta/p(\mathbf{x}_1)}(\cdot)$ are the density

both $f_\eta(F_\eta^{-1}(\tau))$ and $p(\mathbf{x}_1)f_\eta(F_\eta^{-1}(\tau))$ are identified. Hence, $p(\mathbf{x}_1)$ is identified. Similarly, $p(\cdot)$ is identified for all $\mathbf{x}$.

We now consider identification of $c(\cdot)$. We present two alternative assumptions on $F_\eta$ and show that $c(\cdot)$ can be identified under either assumption. First, assume that the median of $\eta$ is zero, $Med(\eta) = 0$, and that there exists $\mathbf{x} = \mathbf{x}_2$ such that $t(\mathbf{x}_2) < 1/2$. The latter assumption means that more than half of the counties have turnout less than $100\%$ when $\mathbf{x} = \mathbf{x}_2$. Then, the median of $Y_m$ conditional on $\mathbf{x}_2$ identifies $c(\mathbf{x}_2)/p(\mathbf{x}_2)$. Now, consider any $\mathbf{x}_1 \neq \mathbf{x}_2$ and let $\tau$ be any number larger than $\max\{t(\mathbf{x}_2), t(\mathbf{x}_1)\}$. Let $z_1$ and $z_2$ be the $\tau$ quantile of $Y_m$ conditional on $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. $z_1$ and $z_2$ are clearly identified. Then,

$$
\begin{aligned}
& p(\mathbf{x}_1)\left[F^{-1}_{\eta/p(\mathbf{x}_1)}(\tau) - F^{-1}_{\eta/p(\mathbf{x}_1)}(1/2)\right] = p(\mathbf{x}_2)\left[F^{-1}_{\eta/p(\mathbf{x}_2)}(\tau) - F^{-1}_{\eta/p(\mathbf{x}_2)}(1/2)\right] \\
& \Leftrightarrow p(\mathbf{x}_1)\left[z_1 - \frac{c(\mathbf{x}_1)}{p(\mathbf{x}_1)}\right] = p(\mathbf{x}_2)\left[z_2 - \frac{c(\mathbf{x}_2)}{p(\mathbf{x}_2)}\right] \\
& \Leftrightarrow \frac{c(\mathbf{x}_1)}{p(\mathbf{x}_1)} = z_1 - \frac{p(\mathbf{x}_1)}{p(\mathbf{x}_2)}\left(z_2 - \frac{c(\mathbf{x}_2)}{p(\mathbf{x}_2)}\right). \qquad (18)
\end{aligned}
$$

Given that all of the terms on the right hand side of (18) are identified, $c(\mathbf{x}_1)/p(\mathbf{x}_1)$ is identified.

Alternatively, assume that $\mathbf{E}(\eta) = 0$ and $\inf_{\mathbf{x}} t(\mathbf{x}) = 0$. We now show that $c(\cdot)$ is identified under these alternative assumptions. Intuitively, this latter assumption means that there exist values of $\mathbf{x}$ for which the max operator is never binding. In this case, we can fully recover the distribution of $F_\eta(\cdot)$. Then, we can identify the distribution of $c(\mathbf{x})/p(\mathbf{x}) + \eta_m/p(\mathbf{x})$ for any $\mathbf{x}$. Hence, we identify $c(\cdot)/p(\cdot)$.

of $\eta/p(\mathbf{x}_1)$ and the inverse distribution of $\eta/p(\mathbf{x}_1)$, respectively. Note that $f_{\eta/p(\mathbf{x}_1)}\left(F^{-1}_{\eta/p(\mathbf{x}_1)}(t)\right) = p(\mathbf{x}_1)f_\eta(F_\eta^{-1}(t))$.

## 8.4 Online Appendix A: Additional Tables

We report the estimates of state-specific effects on preference and efficacy in Tables 9 and 10, which we use to plot Figures 4 and 5.

| | Estimate | SE | | Estimate | SE |
|---|---|---|---|---|---|
| Alabama | 0 (Normalized) | | Nevada | 0.019 | (0.061) |
| Arizona | -0.124 | (0.051) | New Jersey | -0.239 | (0.043) |
| Arkansas | -0.358 | (0.035) | New Mexico | -0.042 | (0.070) |
| California | -0.231 | (0.062) | New York | -0.232 | (0.038) |
| Colorado | -0.046 | (0.055) | North Carolina | -0.081 | (0.034) |
| Florida | -0.093 | (0.040) | North Dakota | 0.004 | (0.041) |
| Georgia | 0.110 | (0.029) | Ohio | -0.160 | (0.036) |
| Idaho | 0.137 | (0.044) | Oklahoma | -0.045 | (0.042) |
| Illinois | -0.235 | (0.032) | Oregon | -0.123 | (0.055) |
| Indiana | -0.088 | (0.033) | Pennsylvania | -0.183 | (0.038) |
| Iowa | -0.294 | (0.034) | South Carolina | 0.031 | (0.031) |
| Kansas | 0.206 | (0.038) | South Dakota | 0.000 | (0.040) |
| Kentucky | -0.246 | (0.034) | Tennessee | -0.340 | (0.036) |
| Louisiana | 0.167 | (0.031) | Texas | 0.131 | (0.035) |
| Maryland | -0.099 | (0.054) | Utah | 0.091 | (0.062) |
| Michigan | -0.240 | (0.033) | Virginia | -0.108 | (0.035) |
| Minnesota | -0.291 | (0.034) | Washington | -0.216 | (0.057) |
| Mississippi | 0.128 | (0.033) | West Virginia | -0.207 | (0.038) |
| Missouri | -0.211 | (0.031) | Wisconsin | -0.330 | (0.035) |
| Montana | 0.105 | (0.044) | Wyoming | 0.241 | (0.057) |

Table 9: Estimates of State Preference Fixed Effects Relative to $\lambda_{Alabama}$. Standard errors are reported in parentheses. Higher values imply a stronger preference for Democrats.

## 8.5 Online Appendix B: Fit

In this Appendix, we report further on the fit of the model. Figure 12 plots the distribution of Democratic and Republican vote shares in the data and in the model prediction. The figure shows that the model fits the data well for all ranges of the Democratic and Republican vote shares. Lastly, we compute the $\chi^2$ statistic for the goodness-of-fit test. The $\chi^2$ test statistics for Democratic and Republican vote shares are 14.07 and 45.29,

|  | Estimate | SE |  | Estimate | SE |
|---|---|---|---|---|---|
| Alabama | 1 (Normalized) |  | Nevada | 0.671 | (0.074) |
| Arizona | 0.522 | (0.097) | New Jersey | 0.571 | (0.056) |
| Arkansas | 0.752 | (0.045) | New Mexico | 0.795 | (0.085) |
| California | 0.584 | (0.056) | New York | 0.632 | (0.058) |
| Colorado | 1.216 | (0.084) | North Carolina | 0.636 | (0.047) |
| Florida | 1.002 | (0.057) | North Dakota | 1.101 | (0.062) |
| Georgia | 0.859 | (0.041) | Ohio | 1.082 | (0.067) |
| Idaho | 1.019 | (0.163) | Oklahoma | 0.743 | (0.052) |
| Illinois | 0.861 | (0.056) | Oregon | 1.171 | (0.091) |
| Indiana | 0.649 | (0.050) | Pennsylvania | 0.679 | (0.052) |
| Iowa | 1.072 | (0.069) | South Carolina | 0.765 | (0.047) |
| Kansas | 0.697 | (0.054) | South Dakota | 1.704 | (0.118) |
| Kentucky | 1.069 | (0.044) | Tennessee | 0.885 | (0.046) |
| Louisiana | 1.317 | (0.083) | Texas | 0.788 | (0.043) |
| Maryland | 0.604 | (0.055) | Utah | 0.961 | (0.091) |
| Michigan | 0.955 | (0.062) | Virginia | 0.780 | (0.045) |
| Minnesota | 1.436 | (0.211) | Washington | 0.859 | (0.065) |
| Mississippi | 0.989 | (0.067) | West Virginia | 0.673 | (0.047) |
| Missouri | 1.179 | (0.050) | Wisconsin | 1.443 | (0.209) |
| Montana | 1.110 | (0.062) | Wyoming | 0.908 | (0.157) |

Table 10: Estimates of State-level Fixed Effects of Voting Efficacy. Standard errors are reported in parentheses. Alabama is set to 1 for normalization.

respectively. The former do not reject the null that these two distributions are the same at 5% level, while the latter rejects it.

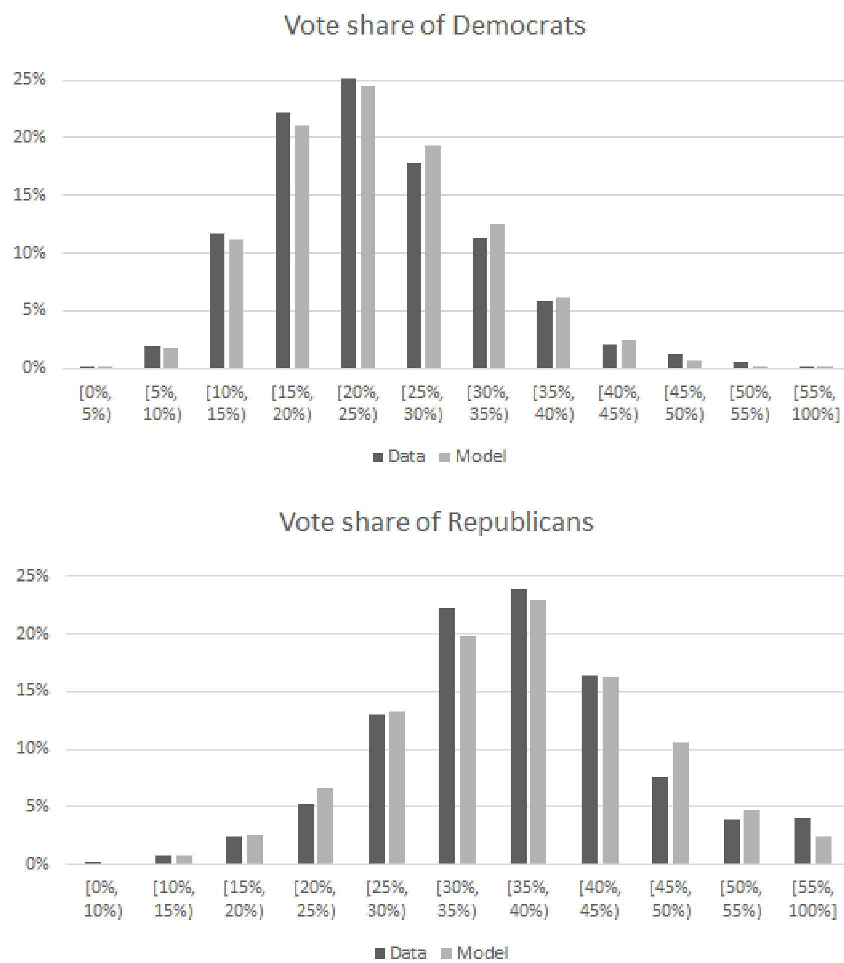


Figure 12: Model Fit. The top panel plots the distributions of Democratic vote share in the data and in the model prediction. The bottom panel plots the distributions for the Republican vote share.