Voting and Trading: The Shareholder's Dilemma: *

Adam Meirowitz
David Eccles School of Business, University of Utah

Shaoting Pi
David Eccles School of Business, University of Utah

October 13, 2020

Abstract

We develop a model of governance by shareholders that can buy/sell shares. Voting for the better policy maximizes portfolio value only when pivotal; otherwise it is better to vote against one's information, distort the market, and then trade at the distorted price. In equilibrium voting informativeness balances these forces and is low. As the number of shareholders grows the probability of making the correct decision converges to a quantity less than the probability that a single agent with one signal makes the correct decision. We consider institutional features like voting blocks and transparency reforms in light of these equilibrium effects.

KEYWORDS: common values voting; shareholder voting, corporate governance, information aggregation, strategic voting

*We are grateful for comments from or discussions with Arjada Bard, Shmuel Baruch, Philip Bond, Laurent Bouton, Mike Cooper, John Hatfield, Davidson Heath, Mark Jansen, Wei Jian, Yihui Pan, Howard Rosenthal and participants at the 2020 SITE conference.

1 Introduction

The tendency of publicly traded firms to rely on shareholder voting is not only a norm but codified through regulatory requirements and, if anything, the trend is to expand the authority of shareholders to influence firm governance. Reflexively, we might expect that this is a best-case application for models that assume common-values and insights that rely on this feature. After all, shareholders are united by the concern for return on their investment in the firm they govern and this is a more likely case of common-values than any political election we might imagine. Not withstanding important work on strategic voting in common-values problems the literature provides a sense that even if they are not unique efficient equilibria do exist under reasonable conditions and thus governance by shareholders with alligned interests may turn out well. [1]

This assessment, however, is premature as most work on voting in the corporate context postulates a direct narrow objective of making the correct decision for the firm. The starting point for this paper is the observation that shareholders might more reasonably be thought to maximize the expected value of their portfolio and have to make two types of choices. They vote and they trade. We investigate whether opportunities to trade can impact incentives to vote in the firm's interest.[2] In the presence of liquidity an incentive problem surfaces and equilibrium forces must reduce the level of information aggregation in response to a potential opportunity to create informational advantages when voting and arbitrage these advantages into informational rents when trading. In particular, equilibrium forces must temper incentives for shareholders to vote against their assessment of the firm's interests in order to generate an informational advantage over the market that they can capitalize on by trading strategically. Our analysis provides an intuition for why equilibria to a model with voting and trading involve far less information aggregation and are much less likely to select the optimal decision than one would expect from a common values problem.

[1] Although investors may have different attitudes towards risk or time horizons that can lead to differences of opinion in the presence of risk and uncertainty, the fact that shareholders have opted to invest in this particular firm might naturally cause sorting which would even reduce heterogeneity on these more minor attributes. We might then reasonably expect that shareholders have closely aligned incentives when voting. At least if the voting rule is chosen to bypass the incentives problems developed in Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997) and the subsequent literature on voting in common values elections or if one appeals to mixed strategies as in McLennan (1998).

[2] One might take work that captures incentives problems when investors hold shares in firms with correlated returns as congruent with our concern but that work is motivated by a different tradeoff.

More precisely, we present a theoretical account of shareholder voting as a means to potentially aggregate private information and find the presence of a sharp incentive problem even when all investors have identical risk and time preferences as well as identical initial portfolios. Shareholders, that are assumed to care only about maximizing their returns, in fact may have perverse incentives when voting over firm policy in the presence of liquidity. The key feature in our account is that current shareholders do not need to remain shareholders; they can sell or they can increase their share holdings in the firm. The desirability of either of these actions depends on the price at which they transact and thus shareholders will care not just about influencing firm decisions, they will care about influencing market prices. But if there is any information in voting then market prices must also react to votes. This then creates the possibility that strategic voting will allow for the creation of informational advantages over the market which can be translated into informational rents by strategic voting and trading in the presence of liquidity. To flesh out the incentives faced by shareholder voters in the presence of liquidity we develop a simple model of voting and trading. We find that voting for the option that the shareholder believes to be optimal for the firm is only optimal for her if she turns out to be the pivotal voter (that is the vote is nearly a tie). In all other realizations of the vote her payoffs are maximized by casting a vote for the option she believes to be worse for the firm and capitalizing on the informational advantage she has over the market. Equilibrium then requires that voting must be sufficiently noisy relative to how strong shareholders private signals are so as to balance how the market reacts to votes with the incentive to select the policies they think are best. Thus, equilibrium will be consistent with the idea that there is not too much information contained in voting even if shareholders in aggregate possess a lot of relevant information. Interestingly, the model yields equilibria in which voting is close to random and uninformative and it also yields equilibria in which voters are very lopsided and uninformative–the latter potentially matching what we tend to see in practice. In the limit as the number of shareholders gets large the probability of making the decision that is better for the firm is bounded away from 1, that is information aggregation fails. These equilibria are driven by the fact that were voting more correlated with information then incentives to take advantage of informational rents would exist, but in equilibrium these incentives are either not present or just balanced with incentives to correctly influence policy.

Importantly, in equilibrium shareholders may not feel strong pressures to influence market prices through their vote precisely because of this balancing. Put differently market prices may not be very responsive to voting because voting is not very informative in equilibrium. It is instructive to draw an analogy with the absence of arbitrage opportunities in an equilibrium to a canonical trading model. Here, incentives to influence market prices through voting can be driving equilibrium behavior even though in equilibrium shareholders don't see desirable opportunities to manipulate prices by voting. Where these features not balance in equilibrium, a block holder would see opportunities to distort share prices up in advance of selling off by casting votes that are seen as strong votes of confidence or investors seeking to increase their holdings in the firm would have opportunities to deflate prices prior to the purchase of additional shares by voting in opposition to management.

This paper makes three contributions. The first one is about corporate governance. The takeaway is that there is room to rethink what motivates shareholders when making firm decisions and thus there are analytical insights that can be obtained by rethinking a range of questions about shareholder governance. Our primary results are negative in that they point to failures in information aggregation. But more broadly, thinking about the connections between trading and voting makes it possible to better understand how a broad range of institutional features might impact voting and governance in ways that previous work has missed. To help push future work in this direction we provide a few suggestive extensions that consider the role of liquidity in the presence of block voters, the role of voting transparency and reporting in the presence of insider voters and the incentives for information acquisition. The second contribution speaks to scholars of collective choice and policymaking more broadly. It is more theoretical and pertains to the general process of utilizing voting theory to provide traction on a broad range of organizations or institutions that use voting to make decisions. Although various organizations make choices with the same mechanism (here simple-majority rule) it is valuable for models to capture key differences: how is shareholder voting on executive compensation different from legislative voting on whether to authorize military action or a vote by VP's of a firm on a high risk corporate strategy. One such difference is the selection and retention process which dictates who has standing to vote and what their potential outside options are. For us, the key feature is that voters may distort policy in order to create opportunities to extract informational rents by buying or selling shares (and voting rights) at prices that they can influence.[3] The third contribution is technical. We present a model in which the informational environment is very nice yet

[3]In a different direction Gieczewski and Kosterina (2020) consider how attrition related to failures in experimentation can lead to a more risk tolerant electorate which causes the organization to pursue the risky protocol longer than optimal.

the limiting probability of making the correct decision is not 0 or 1; as such questions about information aggregation hinge on characterizing rates of convergence. Our limiting analysis hinges on the study of a complicated equilibrium condition. The condition, however, centers around describing the expected incremental value of one more success to a Bayesian trying to forecast the likelihood that a coin of unknown bias lands on heads from from observing n tosses. Use of Taylor approximations and a direct argument about what equilibrium values converge to allow us to obtain a closed from characterization for rates of convergence of equilibrium behavior and limiting probabilities of making the correct decision. To the extent that this kind of random variable can arise in other learning settings the approach may have additional value to applied theorists.

We close this section by providing an informal intuition for how voting and trading might interact. In particular we flesh out the possibility that shareholders will care about more than just selecting the policy they think is best for the firm before jumping into the model itself. We then discuss related work in corporate governance and voting theory. The model is then developed and results presented. To improve the flow of the paper, the more nuanced proofs appear in the appendix. We then close the analysis by presenting several extensions that involve voting blocks, agents that face prohibitive costs to trading or voting incorrectly and evaluate the effect of reforms that require extreme transparency.

1.1 An intuition

To flesh out the central intuition, we begin by walking through a stylized but related problem that a shareholder may face. Imagine a firm that has to make an important and public decision. Further consider an institutional investor that possess private information that speaks to how the available decisions the firm can make will impact the firm's profitability. Imagine that the investor can make a public announcement revealing the hard information prior to the firm's choice. Such a message has two audiences. The firm management, understanding that the investor has information may choose to rely (at least partially) on the speech in making its decision. Second, traders may let the speech inform their evaluation of the firm's decision and assessment of the firm's value after they observe its decision.

How would the investor evaluate this opportunity to reveal her private information? Now, because the institutional shareholder has a stake in the firm, she might like to see it make the best available decision. This is true if she intends to keep or increase her stake in the firm. But, the investor also has the opportunity to sell shares in the firm. What determines whether the investor wants to change the number of shares of the firm in her portfolio after the policy choice is made? The answer in the presence of liquidity is not that she will decide based on whether the firm made an optimal choice. Instead what matters in this calculus is whether she thinks the market is over or under pricing the firm based on the decision it made.

Returning to the question of whether the investor wants to reveal her information we see that there are two forces at work. By revealing her information she can potentially improve the quality of the firm's decision and thus increase the value of her shares. If the investor does not reveal her information she may lower the likelihood that the firm makes the right choice and this lowers the expected value of her shares if she keeps them. But if the investor keeps quiet she may wait to see what policy is chosen and then use her informational advantage over the market to extract expected rents: (1) if she thinks the firm chose wisely then because her information was withheld it may not be incorporated into the market price and thus the price may not be optimistic enough and she buys at a "bargain" price. (2) if she thinks the firm chose poorly then again because her information was withheld it is not incorporated into the market price and thus the market price may not be pessimistic enough and she sells at an "inflated" price. Either of these options can be more compelling than revealing her information, potentially improving the odds that the firm chooses correctly and then having the price of shares adjust to correctly capture her information. The upside of reveling is that the firm is more likely to make the choice that maximizes the value of the shares she owns. But the gains from concealing information and trading at distorted prices can be higher.

Although our focus is not on speech-making or information disclosure by institutional investors the starting point for our model is that in voting settings where there is some private information to be aggregated voting can have an element of strategic signaling. When a shareholder votes she may impact the outcome of the vote and thus the policy but she may also impact the level of support for the choice that is made and this can affect assessments of the firms decision and its valuation. Combining this insight with the observation that shareholders aren't tied to maximizing the value of the firm, but instead they are likely driven by the desire to maximize the value of their portfolio leads us to consider problems of corporate voting and trading as similar to problems of signaling and trading. Figuring out how the equilibrium forces balance out these different pressures allows us to better understand the degree of information aggregation involved in corporate decision-making and the connections between voting and trading behavior. A robust feature that was central to our informal discussion above and which turns out to hold in a much larger set of models than considered here is the fact that only when shareholders believe all of their private information is revealed to the market will they be willing to not-trade in the presence of full liquidity.

1.2 Related Literature

A large literature perhaps starting with the Marquis de Condorcet (1776) seeks to understand voting in settings where agents posses private information about the desirability of choices. A key intuition is the finding by Austen-Smith and Banks (1996) that equilibrium behavior requires that in evaluating their information, agents must also condition on the event that they are pivotal. This equilibrium phenomena has been shown to lead to interesting distortions and accounting for these distortions is central to work on institutional design, for example the choice of voting rule (Feddersen and Pesendorfer 1998, Duggan and Martinelli 2001, Meirowitz 2002). Recent work seeks to understand how seemingly fine differences in the informational environment effect the nature of voting behavior and whether information is efficiently aggregated when there are a large number of voters (Feddersen and Pesendorfer 1997, Bhatacharya 2013, Mandler 2012, Acharya and Meirowitz 2017). We maintain the most parsimonious if not canonical assumptions here, abstracting from questions of rule choice. As a result, without liquidity our model is one in which a natural pure strategy Bayesian Nash equilibrium would fully aggregate information. In otherwords, absent liquidity simple majority rule is the correct rule to use in our setting. An important caveat to work aimed primarily at the study of sincere voting is the finding in McLennan (1998) that for a common values problem there will exist optimal equilibria that aggregate information well. Our insight here is that with liquidity the game is not one of common-values and so his insight does not help to solve the problem. Kim and Fey (2007) show that if one adds a set of voters with adversarial preferences as a primitive to a common values voting model information aggregation can fail. In our model although all voters start out with the same preferences something a kin to adversarial induced preferences emerges endogenously as a result of near future opportunities to trade strategically and differences in private information.

The connections between strategic voting (in political economy) and shareholder voting are natural. Maug (1999), introduces proxy voting to this framework. Maug and Rydqvist (2008) consider natural questions about shareholder control in this setting. Levit and Malenko (2011) and Ekmekci and Lauermann (2019) explore non-binding voting and Malenko and Malenko (2019) add shareholder information acquisition from proxy advisory firms. Bar-Isaac and Shapiro (2020) study blockholder voting. Bond and Eraslan (2010) consider strategic voting over proposals that are strategically chosen (by say management requiring board approval). In these and other models of voting in finance the incentives of agents are limited to the policy choice at hand (the case of no-liquidity in our model). We think subsequent work marrying the informational and institutional features of these papers to our setting with voting and trading might be particularly illuminating. In contemporary and mostly compatible work Li, Maug and Schwartz-Ziv (2019) find that shareholder voting is too heterogeneous to be explained by informational models and argue for models of opinions, "However, these [informational] models imply that shareholder's beliefs converge after observing the meeting outcome, giving rise to lower volatility and volume after the meeting which is not inline with our evidence." (page 6). Our point of departure is that when voting does not fully reveal the shareholders' private information (as in all equilibria to our model), sufficient interim differences in beliefs persist to support active and heterogeneous trade. In this sense the logic of lemma 1 may represent a fruitful way for future theoretical work to reconcile the empirical pattern cited above. Moving away from models of information, Levit, Malenko, and Maug (2020) study the link between trading and voting when shareholders have heterogeneous preferences but there is no asymmetric information. The model develops an intuition for how shareholder support endogenously forms through trading before voting.

Our, contention, however is not that there is not additional value to adding heterogeneous preferences or distinct filters for processing information to models of shareholder behavior.[4] Rather, we think efforts to capture richer informational and preference environments need to keep track of the impact of trading opportunities on shareholder behavior and the ways that equilibrium information processing at the voting stage may affect strategic trading-and the ways that strategic trading may affect voting. Moving forward, we see promise in adapting features of the informational environment and market learning in Bannerjee and Kremer (2010) and Bollerslev et. al. (2018) to models that capture voting and trading.[5] The logic behind lemma 1 is likely to hold in a much larger set of models.

A theoretical perspective on our paper is that we situate the voting problem in a (slightly) broader strategic environment and see how this shapes incentives and equilib-

[4]See also a separate empirical argument for heterogeneous preferences in Bolton et. al (2018)

[5]To be fair, the idea of building off these approaches appears in Liv, Maug and Schwartz-Ziv.

rium behavior in the voting problem. Earlier work in political economy, Razin (2003) and Meirowitz and Shotts (2009) considered strategic voting when voting determined not only the identity election winner but also the policy implemented by the winner. In this setting the identity of the winner is determined by a discontinuous function of votes and the strength of the voting mandate has a smoother impact on a payoff relevant term. In that literature this resulted in a softening of the importance of being pivotal. But, the structure of preference over mandates in those papers is quite dissimilar to the structure of preferences over vote counts here and so the connections are weak. Moreover, the finding of limited information transmission here is in contrast to finding of strong signaling in the earlier papers. Outside the voting context, models of common value auctions often have strong connections with models of voting in common values problems. Atakin and Ekmecki (2014), consider a common values auction where the winners must decide how to use the item and show that prices will not aggregate information in monotone equilibria. In that setting the value of winning depends on being able to make the correct use decision and the fact that winning is more informative when there is rationing (a consequence of pooling at the bidding stage) drives flat bidding strategies. Thus, the possibility that information will effect the ability to make choices after voting or bidding and this effects the earlier action is common to both our paper and theirs. Importantly, Atakin and Ekmecki show that although price is not informative other statistics about the bidding behavior are. In contrast we find that in the limit all observable statistics based on voting become uninformative.

Beyond work on strategic voting in settings with imperfect information there are two other relevant sets of papers. Brav and Matthews (2011) recognize the potential for strategic portfolio choice and voting. They study the effects of empty voting by a single strategic actor that can acquire additional votes and then make call orders prior to voting. They show that there are incentives to deviate from one-share one vote and that the welfare consequences can go either way. But, because trading is not possible after the vote and informational rents cannot be created in their paper the possibility of incentives to vote against the firm's interest do not surface. It is not difficult to see that in a natural extension of Brav and Matthews in which the strategic actor could buy or sell shares after voting is observed, the incentives presented here would appear. The new margin would involve trading off the probability of getting the right choice (noisy because of the noise voters) and the price at which new shares could be purchased. A large literature starting with Grossman and Hart (1980,1988) and more recently including Iaryczower and Oliveros (2017), Dekel and Wolinsky (2011), Harris and Raviv (1988), and Blair, Golbe and Gerard (1989) explores the relevance of acquiring votes and vote buying in corporate control contests. Insight about the difference between efficiency and shareholder profits as well as some of the interesting tradeoffs associated with deviations from one share-one vote are studied. The particular tensions to vote against the firms' interest in expectation of optimal trading behavior, however do not surface in these studies but extensions to include post vote trading are possible and might prove informative.

The theoretical finance literature makes conflicting predictions about the impact of liquidity on corporate governance. Coffee (1991) and Bhide (1993) argue that stock market liquidity impairs governance. Maug (1998) argues that liquidity makes corporate governance more effective. Admati and Pfleiderer (2009), Edmans (2009), and Edmans and Manso (2011) show investors exit by liquidating their shares and this is a governance mechanism in itself. The empirical literature around this question also yields mixed findings. Edmans, Fang, and Zur (2013) demonstrates stock liquidity has a positive effect on blockholder governance. But Back, Li, and Ljungqvist (2015) shows greater liquidity negatively impact governance on average. In this work the focus is on the direct result of selling and not voting. None of these papers consider the potential for incentives created by liquidity to distort voting and thus impact governance. Our paper finds the ability of shareholders to change their portfolios shortly after voting weakens their incentives to vote informatively and thus reduces the informativeness of corporate voting. In the equilibria where voting is consensual we see a severe breakdown of shareholder oversight.

Two other areas of policy importance in the finance literature are addressed. Firstly, we extend our model by adding a blockholder. Blockholders are generally expected to have stronger incentives to sincerely votes due to their concentrated holdings and reputation concerns on strategic voting. However, we find that blockholders may persistently engage in strategic voting unless strict liquidation restrictions after voting are imposed. Secondly, we extend our model by considering the existence of certain shareholders that are assumed to vote sincerely, such as insiders or passive index funds. Perhaps surprisingly, after considering the interactions between these behavioral sincere-voting shareholders and strategic-voting shareholders, we find the existence of sincere-voting shareholders harms information aggregation rather than benefit it. To remedy this problem, we analyze the effect of a regulation requiring real-time disclosure of voting. Our analysis speaks to recent studies and debates on how to improve the current disclosure regulations on shareholder voting.[6] At the end of this extension, we also note that the value of voting transparency depends on whether shareholders have different levels of informativeness in equilibrium. In particular, we find that when shareholders have equal quality information and take symmetric voting strategies, transparency of voting does not improve governance via voting. However, when different types of shareholders use different voting strategies, strengthening transparency can improve information aggregation. Our predictions about the link between voting transparency and heterogeneity of shareholders may encourage future empirical studies and enable empirical work to better inform policy discussions. Succinctly, the model here provides traction on how an empirical question (how much heterogeneity is out there) relates to a policy question (the consequences of real-time vote reporting requirements).

2 The Model

2.1 Modeling considerations

In order to better understand how market opportunities influence the incentives of shareholders involved in corporate governance we develop a model in which shareholders interact in a voting stage and a trading stage. The voting problem is one of aggregating private information. The primitives of the informational environment are chosen so that the voting problem itself is not subject to the types of incentive problems and inefficiencies that have been well studied in the voting literature (Austen-Smith and Banks 1996 and many others that follow). The key intuition from this literature is that even in common values problems so-called sincere voting, voting one's signal, is often not consistent with equilibrium behavior. Rationality requires that one condition on being pivotal and when voting is responsive, being pivotal implies something about the information possessed by everyone else and this can influence one's posterior on the state. When the number of voters is large the information contained in this pivotal event can overwhelm one's private information. We discuss this literature below, but in order to contrast our key insight from extent work as much as possible we focus on an informational environ-

[6] In 2003, SEC received over 8,000 comment letters about their proposed rules on mutual fund voting disclosure. SEC described this as an extraordinary level of public interest and vigorous debate. However, Cremers and Romano (2011) find the results of the 2003 mutual fund voting disclosure rule do not conform to the SEC's expectation. In 2020, as the ongoing focus and efforts, SEC proposed to comprehensively modify the disclosure framework of mutual fund and exchange-traded fund for retail investors' needs.

ment in which this incentive problem is absent. Without liquidity, the problem would be trivial in the sense that an equilibrium with sincere voting would exist and this would efficiently aggregate the private information of shareholders.

In developing the trading environment we abstract away from the question of how markets aggregate information from traders. See for example Kyle (1985). We focus on a model in which prices are assumed to perfectly aggregate all publicly available information and none of the information leaked by shareholder's contemporaneous trading behavior. Shareholders face a liquidity trader or market maker that simply posts a security price at the expected value of the share given all public information. Shareholders then submit orders which the market maker executes at the posted price. We limit attention to small orders: buy or sell one share or hold. This seems most congruent with the posted price mechanism. A common justification for assuming that prices are "fair" given public information is the assumption that the market faces strong competitive pressures and thus price is subject to a 0-expected profit or no-arbitrage condition. Our justification for not capturing the effect of shareholder's contemporaneous trading strategies on price is that even small orders will represent opportunities for shareholders to benefit and thus it is sufficient to capture these local incentives in order to better understand the effect of trading opportunities on voting incentives. Extensions that allow price to aggregate information contained in shareholder orders are possible and if a martingale condition on shareholder beliefs about the effect of other shareholders' orders on price is satisfied the results here carry through for risk neutral traders. In our model then prices are based on all public information from the voting and correct conjectures about the voting strategies. We focus on the decision to buy sell or hold, thus limiting our focus to local trading incentives.

2.2 Primitives

We develop a two period model. In the first period a collection of n (odd) shareholders each endowed with one share of a stock in the firm vote on a binary policy and then in the second period the shareholders have the opportunity to buy an additional share or sell their share or hold their share of the firm.

Voting consists of making a decision $x \in \{0, 1\}$ under simple majority rule. The shareholders face uncertainty about which decision is better for the firm. Formally, denote the underlying state by $\omega \in \{0, 1\}$ with the interpretation that if $x = \omega$ each share has value 1 and if $x \neq \omega$ each share has value 0. The common prior is that $Pr(\omega = 1) = \frac{1}{2}$.

Shareholders are assumed to posses imperfect information about ω. Prior to voting each shareholder i receives a private signal $s_i \in \{0, 1\}$. Signals are conditionally independent with $Pr(s_i = \omega) = q$, with $q \in (\frac{1}{2}, 1)$. By $s = \{s_1, s_2, ..., s_n\}$ we denote a profile of the signal. After observing only their own private signals, shareholders cast ballots $v_i \in \{0, 1\}$. By $v = (v_1, v_2, ..., v_n)$ we denote a profile of votes. Whichever policy receives more votes is selected. By $t = \sum_{i=1}^{n} v_i$ we denote the publicly available vote tally. It is convenient to also describe the tally from shareholders other than i, denoted $t_{-i} = \sum_{j \in n-\{i\}} v_j$

In period 2, after observing the policy x, and the vote count $t = \sum_{i=1}^{n} v_i$, and the common price $P_x(t)$ each trader submits an order $b_i \in \{-1, 0, 1\}$ with the interpretation that $b_i = -1$ denotes selling their share, $b_i = 0$ denotes holding and $b_i = 1$ denotes buying an additional share. Trades are executed at the common price $P_x(t)$ which is assumed to satisfy a no-arbitrage condition,

$$P_x(t) = E[1_{x=\omega}|t]$$

where the expectation is taken over a version of conditional probability that is based on a correct conjecture of the joint probability $Pr(t|s)$. As long as strategies are measurable, we may conveniently write,

$$P_x(t) = Pr(\omega = x|x, t)$$

where the conditional probability satisfies Bayes rule given correct conjectures of the voting strategies. Note that because shareholders can compute the price based on the public infomation it does not matter whether we assume that the price is posted before or after orders are submitted.

Finally, the state is observed and the value of the share is realized. One interpretation is that the firm provides a one-time dividend of either 1 or 0 for each share and the game ends. Thus, at the end of the game the value of each share is given by

$$v(x, \omega) = \begin{cases} 1 \text{ if } \omega = x \\ 0 \text{ otherwise} \end{cases} \quad (1)$$

and an agent that bought a share obtains payoff $2v(x, \omega) - P_x(t)$, an agent that sold a share receives payoff $P_x(t)$ and an agent that made no trades obtains payoff $v(x, \omega)$.

As the game involves a potentially large population of agents that differ only in their private information making sequential choices with imperfect information we seek Perfect Bayesian Nash equilibria in symmetric strategies.[7] Put simply, voting strategies can be described by a number m with the interpretation that $Pr(v_i = s_i | s_i) = m$.[8]

As the price $P_x(t)$ can be inferred directly from the public vote total, t, trading strategies are functions of the triple (s_i, v_i, t). In principle these orders can be in mixed strategies with the mixtures depending on both arguments. As we will show sequential rationality pins down orders by a fair amount and it is not necessary to invest in much notation for tracking these dependencies.

Lemma 1. *In any Perfect Bayesian Equilibrium at an information set* (s_i, v_i, t) *reached with positive probability a shareholder buys (sells) if*

$$Pr(\omega = x | t, s_i, v_i) > (<) P_x(t) = Pr(\omega = x | t).$$

Proof.. This is an immediate consequence of the payoff structure. The expected utility to buying is $2Pr(\omega = x|t, s_i, v_i) - Pr(\omega = x|t)$ while the expected utility to selling is $Pr(\omega = x|t)$ and so the benefit from buying is $2Pr(\omega = x|t, s_i, v_i) - 2Pr(\omega = x|t)$ which is positive (negative) iff $Pr(\omega = x|t, s_i, v_i) > (<) Pr(\omega = x|t)$.

■

<u>Remark:</u> *Lemma 1 is not without predictive power. In any equilibrium that does not fully reveal private information, there will be a wedge between the shareholder's posterior and the market maker's posterior. Thus there will be a wedge between what the shareholder thinks the share is worth and the posted price. The lemma then predicts that traders will take heterogeneous positions in the market based on heterogeneity in the realizations of their private signals and votes.*

---

[7] Because the game treats each state symmetrically we focus on equilbria in which not only are the player strategies symmetric but also the probability that a player's vote corresponds to her signal is independent of the signal.

[8] If $m > \frac{1}{2}$ then voting is positively correlated with shareholder information and we might expect the vote count t to be an informative public signal. Naturally the closer m is to 1 the better t aggregates shareholder information and by a law of large numbers the larger is n the higher is the probability that the correct decision is made (assuming m is bounded above $\frac{1}{2}$).

3 A benchmark with no liquidity

Note that if trading where not possible (i.e. the second period market were closed) then the voting in stage 1 would be a simple problem of common values. Each shareholder would obtain a payoff of 1 if $x = \omega$ and 0 otherwise. Although a large literature has developed subtle insights into the potential for incentive problems and inefficiencies in problems of this form, our model has enough symmetry so that sincere voting and information aggregation are consistent with equilibrium. The key insight from informational models of voting without trading is that a best response in the one period voting game require that shareholder i vote for the policy supported by her signal, termed sincere voting, if $Pr(\omega = s_i | s_i, t_{-i} = \frac{n-1}{2}) > \frac{1}{2}$. Importantly the optimal choice is not prescribed by examining $Pr(\omega = s_i|s_i)$. Rather in a Bayesian Nash equilibrium best responding to one's signal and the equilibrium conjecture of how others are voting is equivalent to voting for the best policy when conditioning on private information and the event that i's vote is decisive. In other words a voting strategy must be optimal if players condition on their private signal as well as the hypothetical event that they are pivotal (and so $t_{-i} = \frac{n-1}{2}$). Given that

$$Pr(\omega = s_i | s_i, t_{-i} = \frac{n-1}{2}) = \frac{(q(1-q))^{\frac{n-1}{2}} q}{(q(1-q))^{\frac{n-1}{2}} q + (q(1-q))^{\frac{n-1}{2}} (1-q)} > \frac{1}{2}$$

sincere voting is an equilibrium in the benchmark game with no market. Note that this strategy profile maximizes the probability that the correct choice is made by the firm and it maximizes the sum of shareholder payoffs. Moreover, as n goes to infinity a strong law of large numbers implies that the correct decision is made almost surely. One might take these assessments as a strong defense of shareholder voting as an efficient form of corporate governance when shareholders possess relevant information, $q > \frac{1}{2}$.[9]

4 The Impossibility of Sincere Voting

Moving now to the two period model with liquidity in which shareholders can trade we see that the incentives are very different. At a first pass the difference can be seen as a consequence of lemma 1; shareholders will trade if they have different posteriors than the market maker after voting. Despite the fact that the underlying policy-making

[9]Several of the cited papers demonstrate how this result fails when changes to the voting rule or information structure of the one period game are made. A brief review of this literature appears below.

problem does not create any incentives for inefficient voting if shareholder care only about selecting the correct policy there is not an equilibrium in which private information is used efficiently to select the correct policy. In other words sincere voting fails. The proof involves showing that by deviating at the voting stage and fooling the market maker a shareholder can maximize the gap between her posterior beliefs and the market makers and extract maximal information rents in the market.

Define sincere voting as a strategy profile in which $Pr(v_i = 1|s_i = 1) = Pr(v_i = 0|s_i = 0) = 1$.

Theorem 1 (Sincere voting fails)**.** *In the two period model with a market there is no equilibrium with sincere voting.*

Proof. Suppose by way of a contradiction that there is an equilibrium with sincere voting. The support of t is then $\{0, 1, 2, ..., n\}$ Moreover, under sincere voting t is a sufficient statistic for the private signals. Let

$$\rho(t) = Pr(\omega = 1|t) = \frac{q^t(1-q)^{n-t}}{q^t(1-q)^{n-t} + (1-q)^t q^{n-t}}$$

which is increasing in t. If $t \geq \frac{n+1}{2}$ then

$$P_1(t) = \rho(t)$$

If $t < \frac{n+1}{2}$ then

$$P_0(t) = 1 - \rho(t)$$

Hold fixed all players other than i at the equilibrium strategy. Following sincere voting, the information t, s_i is the same as t and so by lemma 1 i will be indifferent between any order given that the market price satisfies $P_x(t) = Pr(\omega = x|s_i, t_{-i})$. Importantly under sincere voting i's payoff coincides with the probability that the correct decision is made.

Consider now player i with signal $s_i = 0$ and the deviation for i of selecting $v_i(0) = 1$ and then selecting the pure trading strategy concentrated on 1 if $x = 0$ (so $t < \frac{n+1}{2}$) and concentrated on -1 if $x = 1$ (so $t \geq \frac{n+1}{2}$). Put simply, i is betting on the firm if choice matches her signal despite her vote and betting against the firm if choice matches her vote and is contrary to her signal.

We now show that for each realization of t_{-i} this deviation yields a positive gain over the payoff to following the conjectured equilibrium strategy. There are three cases.

Case 1: $t_{-i} < \frac{n-1}{2}$ then $x = 0$ and i buys an additional share under the deviation. To i the expected value of each share is $1 - \rho(S)$ and the price is $p_0(t) = 1 - \rho(S+1)$, thus $i's$ gain over the equilibrium payoff is

$$2(1-\rho(S)) - (1-\rho(S+1)) - [2(1-\rho(S)) - (1-\rho(S))]$$

which is strictly positive since $\rho(S) < \rho(S+1)$.

Case 2: $t_{-i} \geq \frac{n+1}{2}$ then $x = 1$ and i sells her share under the deviation. Each share is worth $\rho(S)$ but it sells for $p_1(t) = \rho(S+1)$. Since the latter is larger the gain over the conjectured equilibrium payoff is

$$\rho(S+1) - \rho(S)$$

which is strictly positive.

Case 3: $S = \frac{n-1}{2}$. i is pivotal and under the deviation $x = 1$ and i sells her share. Each share is worth $\rho(S)$ but it sells for $\rho(S+1)$. If instead i does not deviate from the equilibrium strategy $x = 0$ and the payoff to i is $(1-\rho(S))$. The gain from the deviation is

$$\rho(S+1) - (1-\rho(S))$$

Under thy symmetry of the model (prior of $\frac{1}{2}$ and equal error probabilities)

$$\rho(\frac{n+1}{2}) = 1 - \rho(\frac{n-1}{2})$$

and thus the deviation is weakly profitable.

Since cases 1 and 2 occur with strictly positive probability the gain from deviation is strictly positive and our assumption of a sincere equilibrium cannot hold. ■

The proof of this result contains the two central intuitions of this paper. First, shareholders have an informational advantage over the market and protecting this might allow them to extract informational rents when trading. Revealing their information can be less profitable than fooling the market. Second the benefits of fooling the market can be seen to obtain whenever the shareholder is not pivotal (cases 1 and 2 in the proof of theorem 1). In contrast when the shareholder is pivotal fooling the market comes at a cost; from the shareholder's perspective the wrong policy is chosen. In the case of a separating/sincere strategy profile this cost when pivotal is not high because under sincere voting the market learns everything and thus the price does not leave any rents over for the shareholder. In other words because the shareholder is indifferent between buying or selling when pivotal in a sincere strategy profile there is not a cost to causing the wrong policy to be chosen. As we will see in the sequel if voting is not sincere but still partially informative ($m := Pr(v_i = s_i|s_i) \in (\frac{1}{2}, 1)$) then voting incorrectly can involve a cost when pivotal. But it still yields benefits when not pivotal. Our analysis will need to determine how m, the likelihood a shareholder votes her signal, can balance these effects.

5 Partially informative voting

We have thus seen that there cannot be equilibria with sincere voting. Since this type of equilibrium involves efficient use of the private information to maximize the chance that $x = \omega$, we are guaranteed efficiency losses. We will show below that there are pooling equilibria in which voting is unrelated to the private signals and no shareholder is ever pivotal. We now seek to find out how responsive voting (and by extension) policy-making can be in equilibrium?

We lead with a simple example that demonstrates it is possible for voting to convey some information.

5.1 Building Intuition: Three Shareholders

Assume that $n = 3$. We seek to find a mixed strategy characterized by m. Our analysis will focus on shareholder 1 holding the remaining shareholders voting strategies at m, and assuming that trading satisfied lemma 1.[10] Consider first the case where shareholder 1 receives signal $s_1 = 1$. In a mixed strategy-equilibrium, she is indifferent between voting 1 and voting 0. If she votes for the policy 1, her expected payoff is

$$\begin{aligned}
&EU[v_1 = 1|s_1 = 1] \\
&= Pr(t_{-i} = 0|s_1 = 1)Pr(\omega = 0|t = 1) \\
&+ Pr(t_{-i} = 1|s_1 = 1)(2Pr(\omega = 1|s_1 = 1, t_{-i} = 1) - Pr(\omega = 1|t = 2)) \\
&+ Pr(t_{-i} = 2|s_1 = 1)(2Pr(\omega = 1|s_1 = 1, t_{-i} = 2) - Pr(\omega = 1|t = 3)).
\end{aligned} \tag{2}$$

[10]Note that for $m < 1$ lemma 1 implies a strict preference for buying or selling and thus mixing at the trading stage is ruled out.

If she votes for the policy 0, her expected payoff is

$$
\begin{aligned}
&EU[v_1 = 0|s_1 = 1] \\
&= Pr(t_{-i} = 0|s_1 = 1)Pr(\omega = 0|t = 0) \\
&+ Pr(t_{-i} = 1|s_1 = 1)Pr(\omega = 0|t = 1) \\
&+ Pr(t_{-i} = 2|s_1 = 1)(2Pr(\omega = 1|s_1 = 1, t_{-i} = 2) - Pr(\omega = 1|t = 2)).
\end{aligned} \tag{3}
$$

Therefore, the indifference condition is

$$
\begin{aligned}
&EU[v_1 = 0|s_1 = 1] - EU[v_1 = 1|s_1 = 1] \\
&= Pr(t_{-i} = 0|s_1 = 1)(Pr(\omega = 0|t = 0) - Pr(\omega = 0|t = 1)) \\
&+ Pr(t_{-i} = 1|s_1 = 1)(Pr(\omega = 0|t = 1) - 2Pr(\omega = 1|s_1 = 1, t_{-i} = 1) + Pr(\omega = 1|t = 2)) \\
&+ Pr(t_{-i} = 2|s_1 = 1)(Pr(\omega = 1|t = 3) - Pr(\omega = 1|t = 2)) = 0
\end{aligned} \tag{4}
$$

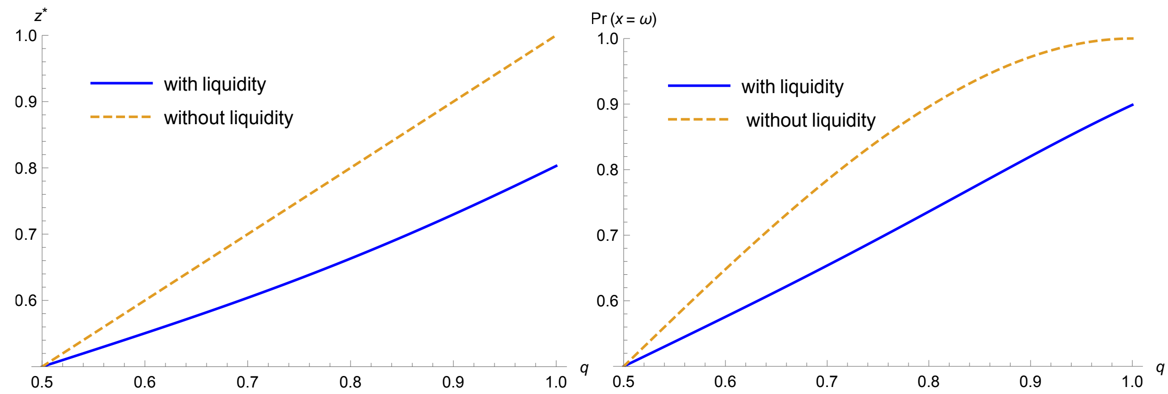
Rearranging terms yields an expression that is easier to interpret,

$$
\begin{aligned}
&Pr(t_{-i} = 0|s_1 = 1)(Pr(\omega = 1|t = 1) - Pr(\omega = 1|t = 0)) \\
&+ Pr(t_{-i} = 1|s_1 = 1)(Pr(\omega = 1|t = 2) - Pr(\omega = 1|t = 1)) \\
&+ Pr(t_{-i} = 2|s_1 = 1)(Pr(\omega = 1|t = 3) - Pr(\omega = 1|t = 2)) \\
&= Pr(t_{-i} = 1|s_1 = 1)(2Pr(\omega = 1|s_1 = 1, t_{-i} = 1) - 1)
\end{aligned} \tag{5}
$$

It is instructive to think of the LHS as the signaling effect from voting against one's signal while the RHS is the pivotal effect of shaping policy when pivotal. Given the mixed strategy profile, m, the probability that a shareholder votes for the better policy is $z := Pr(v_i = \omega|\omega) = qm + (1-q)(1-m)$. Using Bayes rule repeatedly and simplifying allows us to write the indifference condition as

$$
\frac{(2z-1)(3-8(1-z)z)}{1-3(1-z)z} = 2(2q-1)
$$

The same indifference condition obtains when $s_1 = 0$. Figure 1 plots the solutions of the above indifference condition z^* as a function of q. Although the equilibrium mixture is m^* it is more informative to view the solution as z^* since this term describes the informativeness of a vote in the equilibrium.



(a) z^* as a function of q (b) $Pr(x = \omega)$

Figure 1: The informativeness of a vote and probability of choosing right policy with or without liquidity

We see then that the presence of a market results in inefficiencies. The likelihood that each voter votes for the correct policy is q in the efficient equilibrium to the benchmark where trading is not allowed. Figure 1 shows that with a market the equilibrium probability z^* is always below this benchmark.

5.2 Large numbers of shareholders and information aggregation

The indifference condition from our three person example highlights the forces at work in a mixed strategy equilibrium. Because voting is not fully informative, each shareholder maintains an information advantage over the market in equilibrium. Even when $v_i = s_i$ the market maker only takes this vote as a signal of strength z whereas i interprets her signal as having strength q. Because of this, shareholder i believes the stock is overpriced if $x \neq s_i$ and i believes the stock is underpriced if $x = s_i$. Accordingly, the shareholder is obtaining rents. By voting incorrectly she can move the price a little and increase her rents. But this comes at a cost. If the shareholder is pivotal and votes incorrectly then she misses out on the opportunity to purchase an additional share at a "discount" and receive the benefit of increasing the value of the endowed share she owns and instead best responds after her vote by selling her share at a better than fair price. This involves a loss (in contrast to the case of sincere voting). But the loss happens only if i is pivotal. Being pivotal is very unlikely, when n is large. Moreover, being pivotal is less likely the farther z is from $\frac{1}{2}$. But the benefit from voting incorrectly which is the expected ability of one signal of strength z to move the market maker's posterior on ω gets smaller the larger is n. Equilibrium involves balancing these effects. Analysis of equilibria when n gets large involves understanding the rates at which these effect get small and how z^* has to move to balance these changes.

To generalize from the case of $n = 3$, conjecture that there is an equilibrium in which each shareholder votes her signal with probability m. We first rely on lemma 1 to observe that given this, each shareholder will buy if $x = s_i$ and sell if $x \neq s_i$. Given this, when $t_{-i} \neq \frac{n-1}{2}$ voting against one's signal moves price in the direction that improves i's payoff. But when $t_{-i} = \frac{n-1}{2}$ voting against ones signal lowers the value of the share that i owns. To derive the relevant indifference condition, consider first shareholder 1 with signal $s_1 = 1$. Assume all other shareholders vote their signal with probability m at each signal.

At each $t_{-i} < \frac{n-1}{2}$ the payoff from $v_1 = 0$ is $P_0(t_{-i})$ while the payoff from voting $v_1 = 1$ is $P_0(t_{-i} + 1)$. Accordingly the difference in expected utility for shareholder 1 is $P_0(t_{-i}) - P_0(t_{-i} + 1)$. Similarly, at each $t_{-i} > \frac{n-1}{2}$ the payoff from $v_1 = 0$ is $2Pr(\omega = 1|t_{-i}, s_1 = 1) - P_1(t_{-i})$ while the payoff from voting $v_1 = 1$ is $2Pr(\omega = 1|t_{-i}, s_1 = 1) - P_1(t_{-i} + 1)$. Accordingly the difference in expected utility for shareholder 1 is $P_1(t_{-i} + 1) - P_1(t_{-i})$.

The change in price from voting 1 as opposed to 0 corresponds to the change in the market maker's posterior given $t = t_{-i} + 1$ or $t = t_{-i}$. We can write this difference as a function of t_i the number of votes for 1 from the the voters, $n - \{i\}$

$$\begin{aligned}\Delta P(t_{-i}) = &\frac{z^{t_{-i}+1}(1-z)^{n-1-t_{-i}-1}}{z^{t_{-i}+1}(1-z)^{n-1-t_{-i}-1} + z^{n-1-t_{-i}-1}(1-z)^{t_{-i}+1}} \\ &- \frac{z^{t_{-i}}(1-z)^{n-1-t_{-i}}}{z^{t_{-i}}(1-z)^{n-1-t_{-i}} + z^{n-1-t_{-i}}(1-z)^{t_{-i}}}\end{aligned} \tag{6}$$

In the remaining case where $t_{-i} = \frac{n-1}{2}$ and 1 is pivotal the payoff from $v_1 = s_1 = 1$ is $2Pr(\omega = 1|t_{-i}, s_1 = 1) - P_1(t_{-i} + 1)$ and the payoff to voting $v_1 = 0$ is $P_0(t_{-i})$. Notice that

$$P_0(\frac{n-1}{2}) = 1 - \rho_z(\frac{n-1}{2}) = z = \rho_z(\frac{n+1}{2}) = P_1(\frac{n+1}{2})$$

where $\rho_z(t)$ is given by

$$\rho_z(t) = \frac{z^t(1-z)^{n-t}}{z^t(1-z)^{n-t} + (1-z)^t z^{n-t}}$$

and given $t_{-i} = \frac{n-1}{2}, s_1 = 1$, the expected value of each share is q, and so in this pivotal event the payoff difference is

$$\Delta U(piv) = 2q - 2P_0(\frac{n-1}{2}) = 2q - 2z$$

Combining, in order for player 1 with signal $s_1 = 1$ to be willing to randomize the following indifference condition must hold

$$\sum_{t_{-i}=0}^{\frac{n-1}{2}-1} [Pr(t_{-i}|s_1 = 1)\Delta P(t_{-i})] + \sum_{t_{-i}=\frac{n+1}{2}}^{n-1} [Pr(t_{-i}|s_1 = 1)\Delta P(t_{-i})] = Pr(t_{-i} = \frac{n-1}{2}|s_1 = 1)\Delta U(piv) \tag{7}$$

where

$$Pr(t_{-i}|s_1 = 1) = \binom{n-1}{t_{-i}}(qz^{t_{-i}}(1-z)^{n-1-t_{-i}} + (1-q)(1-z)^{t_{-i}}z^{n-1-t_{-i}})$$

In the sequel it is convenient to refer to the indifference condition as $LHS(z, q, n) = RHS(z, q, n)$.

Note that RHS of the indifference condition can be written as

$$\begin{aligned} &Pr(t_{-i}|s_1 = 1)\Delta U(piv) \\ &= Pr(t_{-i}|s_1 = 1)(2Pr(\omega = 1|t_{-i}, s_1 = 1) - P_1(t_{-i} + 1) - P_0(t_{-i})) \\ &= Pr(t_{-i}|s_1 = 1)[2Pr(\omega = 1|t_{-i}, s_1 = 1) - P_1(t_{-i} + 1) - (1 - P_1(t_{-i}))] \\ &= Pr(t_{-i}|s_1 = 1)(2Pr(\omega = 1|t_{-i}, s_1 = 1) - \Delta P(t_{-i}) - 1) \end{aligned} \tag{8}$$

where $t_i = \frac{n-1}{2}$.

By moving $-Pr(t_{-i}|s_1 = 1)\Delta P(t_{-i})$ from RHS to the LHS, we can rearrange the indifference condition to be

$$\underbrace{\sum_{t_{-i}=0}^{n-1} [Pr(t_{-i}|s_1 = 1)\Delta P(t_{-i})]}_{\textit{Signal Effect}} = \underbrace{Pr(t_{-i} = \frac{n-1}{2}|s_1 = 1)(2Pr(\omega = 1|t_{-i} = \frac{n-1}{2}, s_1 = 1) - 1)}_{\textit{Pivotal Effect}} \tag{9}$$

Since $\sum_{t_{-i}=0}^{n-1}[Pr(t_{-i}|s_1 = 1)\Delta P(t_{-i})]$ measures the effects of shareholder 1's vote on moving prices in all cases, we name it signal effect. Because $Pr(t_{-i} = \frac{n-1}{2}|s_1 = 1)(2Pr(\omega = 1|t_{-i}, s_1 = 1) - 1)$ measures the effect of shareholder 1's vote on selecting correct policy when she is pivotal, we name it pivotal effect. As a consequence, the indifference condition implies that the signal effect is equal to the pivotal effect.

We now show that there is always a symmetric equilibrium, that is the indifference condition can be solved.

Theorem 2. *For any $n \geq 3$ (odd) and $q \in (1/2, 1)$, there exists a symmetric mixed strategy equilibrium, characterized by $m^*(q,n)$ that induces a probability that $v_i = \omega$ given by $z^*(q,n) = qm^*(q,n) + (1-q)(1-m^*(q,n))$.*

Proof. Fix $n \geq 3$ (odd) and $q \in (1/2, 1)$. Although, the equilibrium is described by m, it is convenient for us to work with the value $z = qm + (1-q)m$. Define

$$h(z) = \underbrace{\sum_{t_{-i}=0}^{n-1} [Pr(t_{-i}|s_1 = 1)\Delta P(t_{-i})]}_{\text{Signal Effect}} - \underbrace{Pr(t_{-i} = \frac{n-1}{2}|s_1 = 1)(2Pr(\omega = 1|t_{-i} = \frac{n-1}{2}, s_1 = 1) - 1)}_{\text{Pivotal Effect}}$$

First, when $z = \frac{1}{2}$, $\Delta P(t_{-i}) = 0$ but $Pr(t_{-i} = \frac{n-1}{2}|s_1 = 1)(2Pr(\omega = 1|t_{-i}, s_1 = 1) - 1) > 0$. Thus, we know $h(\frac{1}{2}) < 0$.

Second, when $z = q$, $\Delta P(t_{-i} = \frac{n-1}{2}) = 2q - 1$. To see this, observe that

$$\begin{aligned} &\Delta P(t_{-i} = \frac{n-1}{2}) \\ &= \frac{q^{\frac{n+1}{2}}(1-q)^{\frac{n-1}{2}}}{q^{\frac{n+1}{2}}(1-q)^{\frac{n-1}{2}} + (1-q)^{\frac{n+1}{2}}q^{\frac{n-1}{2}}} - \frac{q^{\frac{n-1}{2}}(1-q)^{\frac{n+1}{2}}}{q^{\frac{n-1}{2}}(1-q)^{\frac{n+1}{2}} + (1-q)^{\frac{n-1}{2}}q^{\frac{n+1}{2}}} \\ &= q - (1-q) \\ &= 2q - 1 \end{aligned} \tag{10}$$

Moreover, $2Pr(\omega = 1|t_{-i} = \frac{n-1}{2}, s_1 = 1) - 1)$ is also equal to $2q - 1$. Then,

$$h(q) = \sum_{t_{-i}=0}^{\frac{n-1}{2}-1} Pr(t_{-i}|s_1 = 1)\Delta P(t_{-i}) + \sum_{t_{-i}=0}^{\frac{n-1}{2}+1} Pr(t_{-i}|s_1 = 1)\Delta P(t_{-i})$$

Since $\Delta P(t_{-i}) > 0$ when $z = q$, we know $h(q) > 0$.

By continuity of $h(z)$ and the intermediate theorem, we know a $z \in (\frac{1}{2}, q)$ satisfying $h(z) = 0$ exists.

■

From theorem 1 we know that sincere voting cannot be supported but if m is close to 1 the equilibrium will aggregate information almost as well as sincere voting. We now show, that in fact when the number of shareholder is large voting must be almost uncorrelated with private information. In particular we show that in the limit m converges to $\frac{1}{2}$. For the remainder of the section fix q and by z_n we denote the value of z that satisfies the equilibrium cutpoint at q, n. By theorem 2 at least one value exists. For convenience we suppress the dependent on q when it does not introduce any confusion.

For values of n that allow the use of simple numeric methods to solve for the equilibrium we can get a sense for how $z^*(q, n)$ depends on the parameters. In particular Figure 2 exhibits the equilibrium value of z^* for a range of parameters: n is between 9 and 99 and q is between $\frac{1}{2}$ and 1. On this range of the parameter space z increases with q and decreases with n.

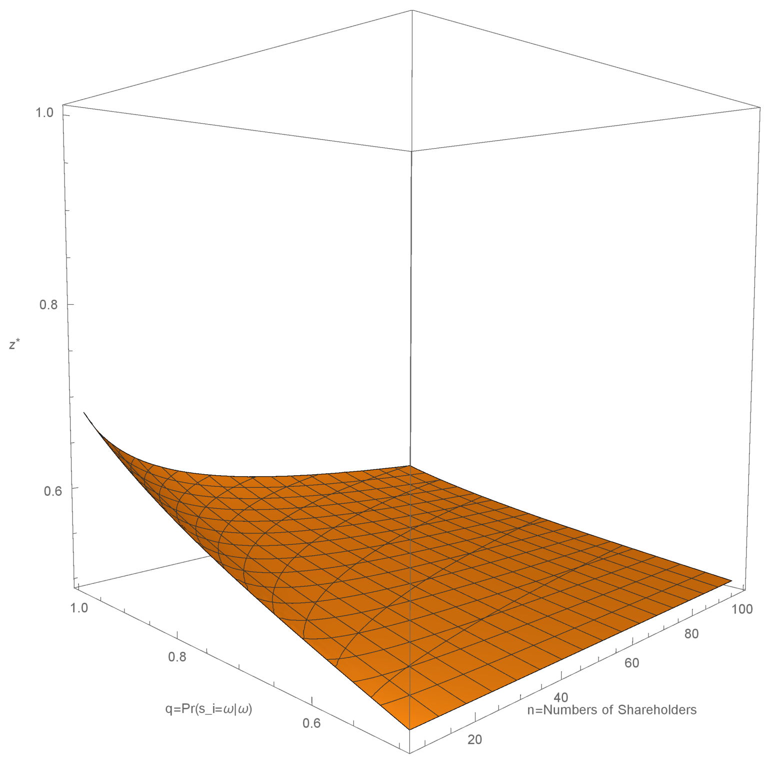


Figure 2: z^* as a function of q and n

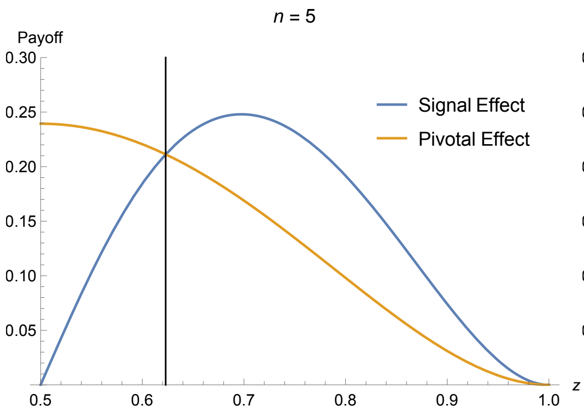
We close by using analytic methods to show that as n goes to infinity if equilibria of this form exist the probability that a voter votes correctly, $z^*(q, n)$ converges to $\frac{1}{2}$. Just as numeric analysis of the indifference condition becomes difficult as n gets big, we do not know of any methods that allow direct analysis of limits of the LHS of the indifference condition.[11] Our approach then is to convert the problem to one of two limits. For each n we first carefully select a subset $k < n$ of the possible realizations of t_{-i}, we show that if k gets large enough the only way to solve the indifference condition is for z to converge to $\frac{1}{2}$ and we then take limits in n so that it is possible to select this sequence of k's.[12] We close this section by illustrating that any sequence of symmetric equilibria in which voting strategies are not constant in type (termed responsive) must become uninformative.

Theorem 3. *Fix $q \in (\frac{1}{2}, 1)$. As n goes to infinity any sequence of responsive symmetric equilibrium mixtures, m_n^* and equilibrium probabilities of voting correctly z_n^* both converge to $\frac{1}{2}$.*

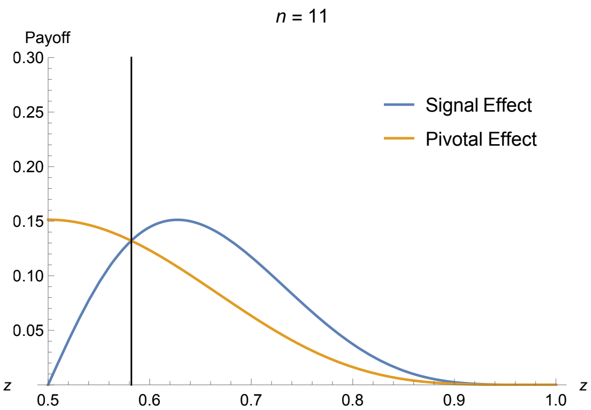
The proof is rather long and thus appears in the appendix. It is instructive to get a sense of the magnitudes through a few numeric examples. Figure 3 shows the equilibrium value of z^* under a few different values of n holding $q = \frac{4}{5}$.

[11] The challenge is that the LHS which is essentially the expected impact of one signal on a Bayesian posterior cannot be directly translated into expressions for which the binomial theorem or Stirling's formula apply.

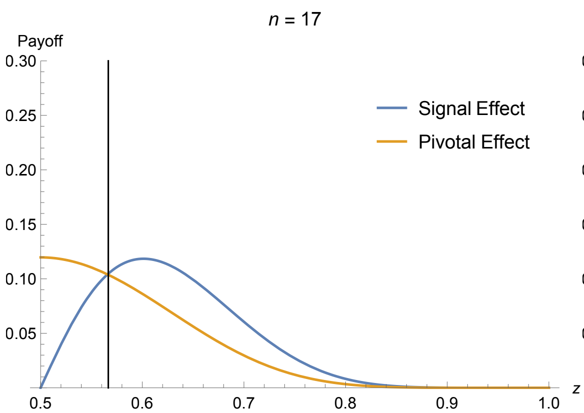
[12] This approach works because of the fact that the probability of t_{-i} taking a value a fixed number of increments larger than $\frac{n-1}{2}$ is independent of n. As n grows the effect of a signal on the posterior when t_{-i} takes a particular value vanishes but the likelihood of this value of t_{-i} is constant. Of course as n grows additional values of t_{-i} become possible.



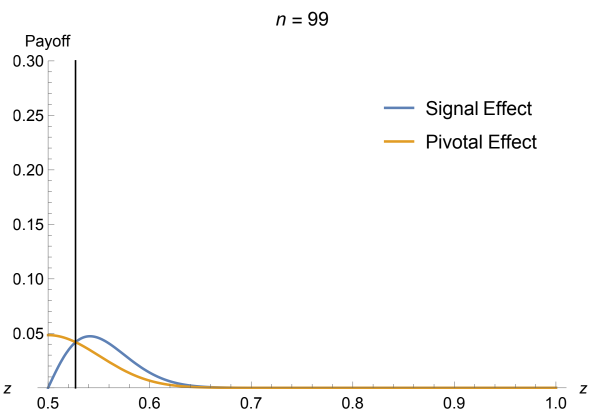
(a) when $n = 5$, z^* is about 0.64



(b) when $n = 11$, z^* is about 0.58



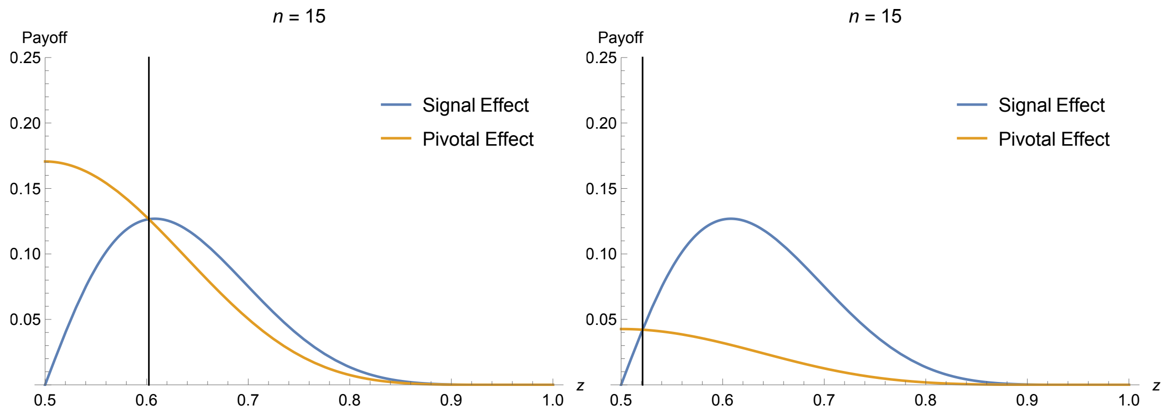
(c) when $n = 17$, z^* is about 0.56



(d) when $n = 99$, z^* is about 0.52

Figure 3: z^*, the informativeness of a vote at equilibrium, converges to $\frac{1}{2}$ *as* $n \to \infty$

In thinking about how changes in q, the underlying informativeness of private signals, affect equilibrium behavior, it is instructive to note that the signal effect does not depend on q while the pivotal effect increases with q. Therefore, the intersection between RHS and LHS moves to the left when q becomes smaller. In Figure 4 we fix $n = 15$ and plot z^* as a function of q.



(a) when $q = 0.9$, z^* is about 0.60 (b) when $q = 0.6$, z^* is about 0.52

Figure 4: z^* decreases with $q = Pr(s_i = \omega|\omega)$

Theorem 3 does not provide any insight about the limit of the probability that the majority-rule policy choice is correct (if the limit of this probability even exists). The possibilities are that even though the informativeness of each individual vote is vanishing the likelihood of making the correct decision in the aggregate still converges to 1 (strongly or weakly) or that in the limit there is uncertainty about whether the correct choice is made. Limiting uncertainty about whether the correct choice will be made in voting models with conditionally independent signals is rare. But see Ahn and Oliveros (2012) where the limiting probability is not necessarily degenerate in the different setting of voting over multiple alternatives.

More precisely, although Theorem 3 tells us that the probability that each vote is correct is converging to $\frac{1}{2}$ we must recognize that it is converging from above. If this rate of convergence is slow enough then we may still obtain information aggregation. The idea is that if the mixture converges slow enough then the fact that more and more voters are being added as n grows swamps the fact that each vote is becoming less correlated with the state. More lower quality votes are good enough in the limit. But on the other-hand if the mixture converges fast enough then this will dominate the addition of new informed voters as n grows. Figure 3 shows that the mixture tends to fall quickly but this does not resolve the question. It turns out that, here, the limiting probability is not degenerate and so full aggregation does not occur. In the limit the probability of making the correct decision is less than 1. We state and prove this as theorem 4.

Theorem 4. *In any sequence of symmetric mixed strategy equilibria as $n \to \infty$, the probability of making the correct decision converges to*

$$\Phi\left(\frac{1}{\sqrt{2}}\frac{\sqrt{16(p-1)p+\pi+4}-\sqrt{\pi}}{2p-1}\right)$$

which is strictly greater than 0 and strictly less than 1.

The proof of Theorem 4 also appears in the appendix. The result is quite useful as it allows us to evaluate the limiting probability that shareholders make the correct decision in equilibrium and see how it varies with the paramerer q. Figure 5 plots the value of this limiting probability and shows that not-surprisingly as q increases from $\frac{1}{2}$ to 1 the collective does better.

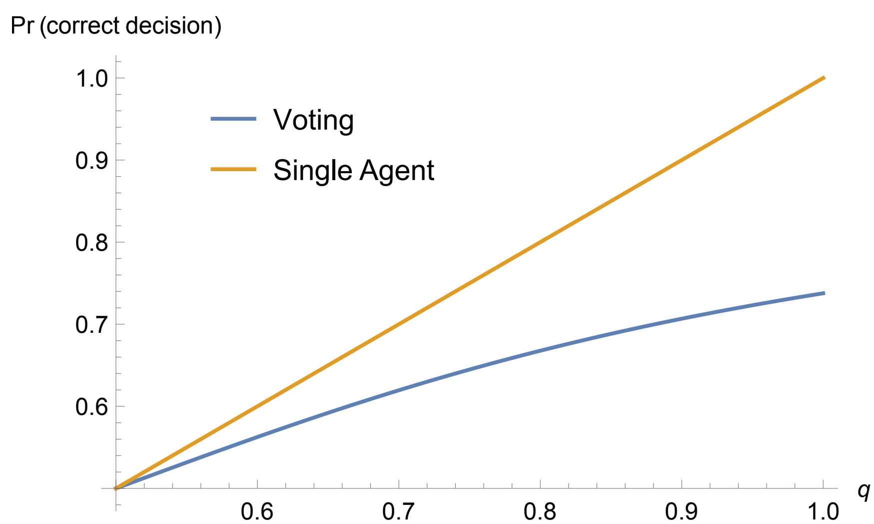


Figure 5: Pr(correct decision) as $n \to \infty$

Interestingly the figure reveals that the limiting probability from equilibrium voting lies below the identity mapping which illustrates the probability that a single agent (dictator) would make the correct decision if she had access to only one signal of quality q. Thus theorem 4 really tells us two things about information aggregation. Not only does the limiting probability of making the correct decision fail to converge to 1 even though a Bayesian with access to all of the information obtained by the shareholders would be able to make the correct decision with probability approaching 1 but in fact an even weaker standard fails. In equilibrium the group does worse than a dictator that has access to only 1 signal. The idea that majority rule would beat one agent is sometimes termed a Condorcet jury theorem of the first type and the idea that majority rule would asymptotically make the correct decision is sometimes termed a Condorcet jury theorem of the second type. Theorem 4 shows that Condorcet jury theorems of the second type fail in our environment. From the figure which uses the characterization in theorem 4 we have the following corollary.

Corollary 1. *In equilibrium a Condorcet jury theorem of the first type never obtains; regardless of how many shareholders we have the probability of making the correct decision in equilibrium is less than the probability that a single agent receiving one signal would make the correct decision.*

5.3 Shareholder Welfare and the Value of Information Quality

Without liquidity, shareholder utility is equivalent to the likelihood of selecting the correct policy (and thus obtaining share value of 1). With liquidity in equilibrium shareholders are trading with the market-maker and extracting informational rents by trading at a discount, accordingly expected utility need not coincide with the equilibrium probability of making the correct choice, $Pr(x^* = \omega)$. Let U^* denote the expected utility of a shareholder in equilibrium.

Theorem 5. *For any fixed $q \in (\frac{1}{2}, 1)$ consider any sequence of symmetric responsive mixed strategy equilibria, In the limit $U^* > Pr(x^* = \omega)$*

The proof appears in the appendix. Interestingly, it is sometimes the case that U^* is even higher than the probability a Bayesian would make the correct decision with n signals of quality q. That is the rent extraction can more than offset the loss in likelihood of selecting the correct policy. But this possibility obtains only for small values of n as when n goes to infinity, the Bayesian's probability of success converges to 1 fairly quickly.

Although a full analysis of endogenous information acquisition is beyond our focus, an interesting insight comes out of comparing how U^* changes with q and how $Pr(x^* = \omega)$ changes with q. Without liquidity (i.e. in standard voting models) the marginal value to all shareholders of information quality would correspond to $\frac{\Delta n Pr(x^*=\omega)}{\Delta q}$ or the derivative if it exists. With liquidity the marginal value of information quality to the initial shareholders is $\frac{\Delta n U^*}{\Delta q}$. As we now show the latter is larger indicating that although in equilibrium shareholders do not efficiently utilize the available information to make the correct decision for the firm with the highest possible probability they do obtain high value from information in the form of rent extraction from the rest of the market.

If we thought of a centralized or cooperative process that influenced the signal quality of shareholder's information we might expect to see more investment with liquidity than without. The argument is proven in the appendix.

Theorem 6. *In the limit the marginal value of information is higher with liquidity than without.*

5.4 Consensus equilibria

It is not difficult to see that there is one other form of symmetric equilibrium. Consider a pooling strategy profile in which all shareholders vote $v_i(s_i) = 1$ regardless of their type, and in which the market maker's off the path beliefs are that a vote for 0 is equally likely to have come from either type. Given this the probability of being pivotal is 0 and the market-maker does not adjust price based on t. Accordingly, there is no profitable deviation. Although the argument is brief it is primarily a restatement of the above paragraph and we relegate the proof of the following result to the appendix.

Theorem 7. *For any n, q there exists two symmetric pooling equilibria. In the first, $v_i(s_i) = 1$ for all s_i. The market maker's belief (and thus share price) on and off the path is $Pr(\omega = 1|t) = \frac{1}{2}$. In the second $v_i(s_i) = 0$ for all s_i. The market maker's belief (and thus share price) on and off the path is that $Pr(\omega = 1|t) = \frac{1}{2}$*

These equilibria involve no information aggregation.

6 Extension: Blockholder

6.1 Adding a Blockholder

In this section, we extend the model by adding a blockholder. Suppose the firm has n shares in total. The blockholder has b shares while each of the remaining retail shareholder has one share. The blockholder receives a signal $s_b \in \{0, 1\}$, while each retail shareholder i receives a signal $s_r^i \in \{0, 1\}$. Signals are conditionally independent with $Pr(s_b = \omega) = p > Pr(s_r^i = \omega) = q$. This ordering captures conventional wisdom that blockholders, usually professional investors or institutional investors, have better information than ordinary small investors.

In the voting stage, the blockholder can vote for either policy with her b shares but cannot split its votes and cast ballots for both policies simultaneously. Given the symmetry of the model we focus on equilibria in which the blockholder votes for a policy that is consistent with her signal with the same probability following each signal, m_b, thus the probability that her votes are the same as the state is $z_b = Pr(\omega = v_b|\omega) = m_b p + (1 - m_b)(1 - p)$. Similarly, a small shareholder i votes for the policy supported by her signal with probability, m, so her vote is aligned with the state with probability $z = Pr(\omega = v_i|\omega) = mq + (1 - m)(1 - q)$.

In the trading stage, small shareholders have full liquidity; each of them can buy or sell one share or simply hold their share. But the blockholder has limited liquidity; the blockholder can buy or sell l shares or simply hold its portfolio. Thus, liquidity is parameterized by $l \leq b$. This limitation captures the idea that blockholders may find it prohibitively costly to liquidate all of their holdings because of market frictions or contracts with clients. We will analyze how changes in the degree of liquidity l impact the incentives and equilibrium level of voting informativeness.

6.2 The Effects of Block Voting

Adding a more informed blockholder to the basic model results in three conceptual changes. First, it affects how the market perceives the voting results and thus how prices are set. The market maker would like to know which way the block voted and under some vote counts, t, this can be perfectly inferred but under others it cannot. If $t < b$, the marker maker knows that the blockholder voted for policy 0. However, if $b \leq t \leq n - b$, the market maker is unable to infer how the blockholder voted. For $b \leq t \leq n - b$, either the blockholder voted for policy 1 and $t - b$ small shareholders voted for policy 1 or the blockholder voted for policy 0 and t small shareholders voted for policy 0. If $t > n - b$, the market maker can infer that the blockholder voted for policy 1.

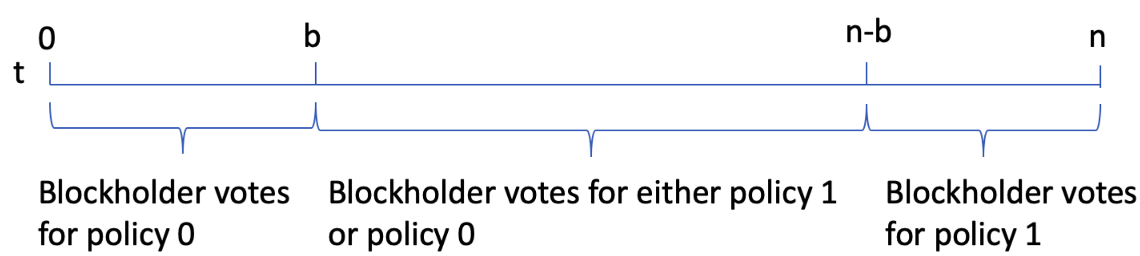


Figure 6: The Inference of the Blockholder's Votes from t

As a result, the marker maker sets P as a piece-wise function of t,

$$P = \begin{cases} \frac{z_b\binom{n-b}{t}(1-z)^t z^{n-b-t}}{z_b\binom{n-b}{t}(1-z)^t z^{n-b-t}+(1-z_b)\binom{n-b}{t}z^t(1-z)^{n-b-t}} & \text{if } t < b \\ \frac{B}{A+B} & \text{if } b \leq t < \frac{n+1}{2} \\ \frac{A}{A+B} & \text{if } \frac{n+1}{2} \leq t \leq n-b \\ \frac{z_b\binom{n-b}{t-b}(z)^{t-b}(1-z)^{n-t}}{z_b\binom{n-b}{t-b}(z)^{t-b}(1-z)^{n-t}+(1-z_b)\binom{n-b}{t-b}(1-z)^{t-b}(z)^{n-t}} & \text{if } n-b < t \end{cases}$$

where

$$\begin{aligned} A &= z_b\binom{n-b}{t-b}z^{t-b}(1-z)^{n-t} + (1-z_b)\binom{n-b}{t}z^t(1-z)^{n-b-t} \\ B &= (1-z_b)\binom{n-b}{t-b}(1-z)^{t-b}z^{n-t} + z_b\binom{n-b}{t}(1-z)^t z^{n-b-t} \end{aligned} \tag{11}$$

Second, the existence of a blockholder alters how small shareholders forecast how others vote. Consider a small shareholder and let $\bar{t}$ denote the number of votes for 1 by all other shareholders. In order for $\bar{t} < b$, to obtain the blockholder must vote for policy 0. Thus,

$$\begin{aligned} &Pr(\bar{t}|s_i) \\ &= (1-z_b)\binom{n-b-1}{\bar{t}}z^{\bar{t}}(1-z)^{n-b-1-\bar{t}}Pr(\omega = 1|s_i) \\ &+ z_b\binom{n-b-1}{\bar{t}}(1-z)^{\bar{t}}z^{n-b-1-\bar{t}}Pr(\omega = 0|s_i) \end{aligned} \tag{12}$$

if $\bar{t} < b$.

However, if $b \leq \bar{t} \leq n-b$ obtains then two configurations are possible: the case where the blockholder votes for policy 1 and the case where the blockholder votes for policy 0. So,

$$\begin{aligned} &Pr(\bar{t}|s_i) \\ &= [z_b\binom{n-b-1}{\bar{t}-b}z^{\bar{t}-b}(1-z)^{n-1-\bar{t}} + (1-z_b)\binom{n-b-1}{\bar{t}}z^{\bar{t}}(1-z)^{n-b-1-\bar{t}}]Pr(\omega = 1|s_i) \\ &+ [(1-z_b)\binom{n-b-1}{\bar{t}-b}(1-z)^{\bar{t}-b}z^{n-1-\bar{t}} + z_b\binom{n-b-1}{\bar{t}}(1-z)^{\bar{t}}z^{n-b-1-\bar{t}}]Pr(\omega = 0|s_i) \end{aligned} \tag{13}$$

if $b \leq \bar{t} \leq n-b$.

If $\bar{t} > n - b$, the blockholder must vote for policy 1. So,

$$
\begin{aligned}
&Pr(\bar{t}|s_i) \\
&= z_b \binom{n-b-1}{\bar{t}-b} z^{\bar{t}-b}(1-z)^{n-1-\bar{t}} Pr(\omega = 1|s_i) \\
&+ (1-z_b)\binom{n-b-1}{\bar{t}-b}(1-z)^{\bar{t}-b} z^{n-1-\bar{t}} Pr(\omega = 0|s_i)
\end{aligned}
\tag{14}
$$

if $\bar{t} > n - b$.

Third, because the blockholder owns b shares, she is pivotal for a range of different voting profiles. We use $t' \in [0, n-b]$ to denote the voting counts of all shareholders except the blockholder. Then, as long as $t' \in [\frac{n+1}{2} - b, \frac{n+1}{2})$, the blockholder is pivotal: the policy will coincide with how the blockholder votes.

6.3 The Effect of Liquidity

In this section, we show that the blockholder's incentive to reveal its private information through voting depends on the degree of liquidity.

The indiference conditions are derived and analyzed in the online appendix. The take-away points are: The pivotal effect is independent of the degree of liquidity, l, but the signal effect is proportional to l. In particular the equilibrium condition can be written as $\frac{P(z)}{S(z,z_b)} = l$ where $P(z)$ is the pivotal effect and $S(z, z_b)$ is the signal effect. Thus as l increases the ratio on the left hand side needs to increase, and thus the pivotal effect needs to increase and or the signaling effect decreases. In general this involves less informative voting. Rather than sort out the effects on z and z_b analytically we present a numeric example to showcase the effects and magnitudes.

A completely mixed strategy equilibrium, (z, z_b) must satisfy the indifference conditions of the blockholder and small shareholders.

The indifference condition for the blockholder is

$$
\sum_{t'=\frac{n+1}{2}-b}^{\frac{n+1}{2}-1} [Pr(t'|s_b)(2bPr(\omega = 1|s_b, t') - b)] = l \sum_{t'=0}^{n-b} [Pr(t'|s_b)(Pr(\omega = 1|t' + b) - Pr(\omega = 1|t'))]
\tag{15}
$$

The indifference condition for the small shareholders is

$$Pr(\bar{t} = \frac{n-1}{2}|s_i^r)[2Pr(\omega = 1|\bar{t} = \frac{n+1}{2} - 1, s_i^r) - 1] = \sum_{\bar{t}=0}^{n-1}[Pr(\bar{t}|s_i^r)(Pr(\omega = 1|\bar{t}+1) - Pr(\omega = 1|\bar{t}))] \tag{16}$$

We give numerical examples of the mixed-strategy equilibrium. Suppose a firm has 15 shares in total and the blockholder owns 4 shares and each of the other small shareholders has 1 share. Suppose the signals s_b and s_i^r are correct with probabilities $p = \frac{4}{5}$ and $q = \frac{2}{3}$ respectively. We draw a table to show the mixed-strategy equilibria when the blockholder can liquidate $l \in \{1, 2, 3, 4\}$ shares.[13]

$n = 15, b = 4, p = \frac{4}{5}, q = \frac{2}{3}$	z_b^*	z^*
$l = 1$	0.800	0.642
$l = 2$	0.778	0.639
$l = 3$	0.671	0.619
$l = 4$	0.623	0.603

The examples highlight our predictions. In particular, when the blockholder has 4 shares but only can liquidate 1 share, her voting can be fairly informative in equilibrium. But as liquidity increases the informational value of blockholder votes must decrease until the case of full liquidity where we see completely mixed strategy equilibrium where all votes are fairly uninformative.

Moving beyond concerns about liquidity, the inclusion of a blockholder to the basic model allows us to think about heterogeneity in voting between institutional shareholders and retail shareholders. To make this point sharp not that in this case with $n = 15$, $p = \frac{4}{5}$, $q = \frac{2}{3}$, for the case of $l = 4$ there is another equilibrium: $z_b^* = 0.572, z = \frac{2}{3}$. If the blockholder votes this way sincere voting is a best response for the small shareholders and when the shareholders vote sincerely the blockholder is indifferent.

[13]Direct calculation is demanding even for these sized examples. We, thus, use numeric approximation to compute the solutions for the tables in this section.

6.4 Reputation Costs of Voting the Wrong Way

Now we suppose the blockholder suffers a cost, C if her voting is inconsistent with trading behavior just after the vote. A natural rationale is that this type of devious behavior may involve reputational costs. We show that as long as C is not large, the blockholder still randomizes between voting with and against her signal in equilibrium.

$n = 15, b = 4, p = \frac{4}{5}, q = \frac{2}{3}, l = 4$	z_b^*	z^*
$C = 0$	0.623	0.603
$C = 0.1$	0.640	0.610
$C = 0.2$	0.661	0.620
$C = 0.3$	0.686	0.622
$C = 0.4$	0.715	0.629
$C = 0.5$	0.747	0.634

It is important to interpret the magnitudes of C correctly. Recall the blockholder holds 4 shares in this example. This means the stakes of getting the policy correct have magnitude of 4. The range of C in the figure is 0 to .5. The latter then means that the shareholders reputational cost is an eighth of the gain from selecting the correct policy and increasing the value of her holdings. In this example that largest value of C that supports a non degenerate mixed strategy is $\frac{2}{3}$.

7 Extension: Insider Voting and Passive Voting

In practice, some voters may face strong pressure to vote according to their signals or significant costs or constraints related to trading. For example, insiders of a firm, such as the board of directors, could face restrictions on liquidating their shares. Thus, they may not have the incentives to vote strategically to generate informational rents. In addition, some passive funds, such as index funds, may passively follow voting recommendations from proxy firms. In this section we illustrate how to incorporate heterogeneity in the ability to take advantage of informational rents into the model and investigate whether the existence of some shareholders that are assumed to vote sincerely necessarily improves the incentives faced by others and whether the presence of these sincere players necessarily improves the level of information aggregation. We will call agents that are assumed to mechanically vote their signal, sincere-voting shareholders.

It might seem natural to expect that that the presence of shareholders that automatically vote sincerely (voting for the option favored by their private signal) could improve the information aggregation of shareholder voting. However, we find that this expectation fails to consider the interactions between the sincere-voting shareholders and strategic-voting shareholders. In particular, we find that the existence of sincere-voting shareholders, causes the equilibrium strategies of strategic shareholders to change and in particular they become less likely to vote for the policy favored by their private signal. In particular as the number of sincere-voting shareholders increases, the votes of strategic shareholders become more noisy. In equilibrium, the level of information aggregation depends on which effect dominates. We find that which effect dominates depends on the underlying environment.

To analyze the information aggregation given the existence of sincere-voting shareholders, we extend our 3-shareholders baseline model by assuming shareholder 2 and shareholder 3 sincerely vote for their signal ($v_2 = s_2$ and $v_3 = s_3$). Shareholder 2 and shareholder 3 could be viewed as insiders or passive index funds. Then, we find the equilibrium value of $z^* = Pr(v_1 = s_1)$ for shareholder 1 who is assumed to be a fully strategic player. In the online appendix the expected utilities are derived.

Equating $EU(v_1 = 1|s_1 = 1)$ with $EU(v_1 = 0|s_1 = 1)$, we can solve z^* as a function of q. As shown in the figure below, z^* is decreasing with q.

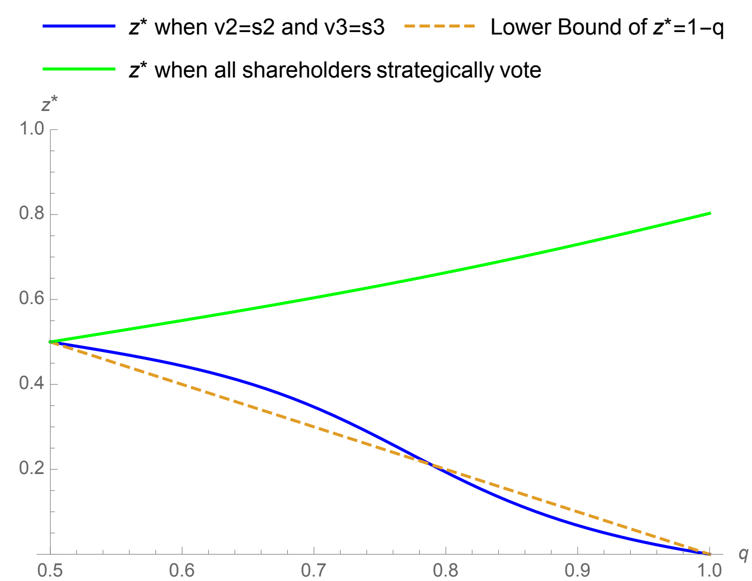


Figure 7: z^* of Shareholder 1 when Shareholder 2 and 3 Sincerely Vote

The figure plots values of z that solve the indifference condition for player 1. Recall that the lower bound of z^* is $1-q$. So, when the blue line is below the yellow line, the mixed-strategy equilibrium does not exist as it is not possible to find a mixture that yields $z < 1-q$. The blue line crosses the yellow line at $q = \frac{1}{6}\left(\sqrt{3}+3\right)$. So, when $q > \frac{1}{6}\left(\sqrt{3}+3\right)$, the mixed-strategy equilibrium cannot be sustained and the only equilibrium has shareholder 1 always votes against her signal ($z^* = 1-q$).

Now, we compare the information aggregation when shareholder 2 and shareholder 3 are sincere-voters with the information aggregation when all shareholders are strategic (the case of our first 3 player example above).

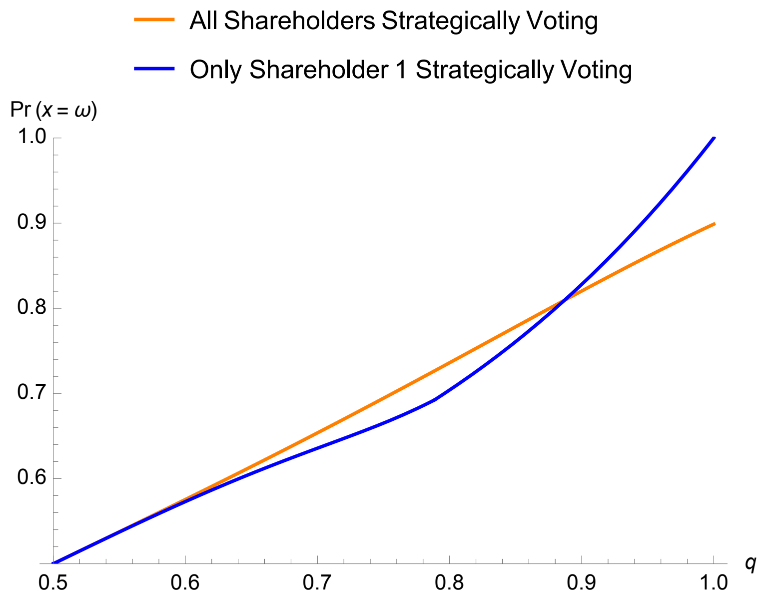


Figure 8: $Pr(x = \omega)$ Given the Existence of Sincere-voting Shareholders

As can be seen from the above figure, $Pr(x = \omega)$ is larger when all shareholders are strategic unless q is sufficiently large ($q > 0.89$). The equilibrium reaction of a strategic shareholder more than offsets the potential efficiency gains from having two shareholders mechanically voting sincerely unless the underlying signal quality is very high. Even though it would naively seem that some voters using their information helps things, equilibrium reactions can turn out to be so severe that things get worse.

Since the presence of shareholders that are voting their signals does not necessarily improve information aggregation, we examine whether other institutional changes might improve information aggregation in the presence of sincere-voting shareholders A novel regulation, that we consider involves requiring that all shareholders immediately disclose how they vote as soon as the voting finishes.[14] This regulation would improve information aggregation in our context. To see why note that when all shareholders report how they vote, the market can distinguish the votes of shareholders that are very likely to be voting sincerely from shareholders that may be acting strategically. In the presence of this kind of advanced updating by the market voting against one's signal will yield smaller informational rents and strategic voting will be less profitable. Thus

[14] Current regulations require institutional shareholders, such as mutual funds, to file Form N-PX with the SEC once a year. `https://www.sec.gov/reportspubs/investor-publications/investorpubsmfproxyvotinghtm.html` However, we find that only if a shareholder is required to disclose her voting before she can trade will we unambiguously see an increase the informativeness of voting compared with the baseline model.

in equilibrium strategic players will vote their signal with higher probability. To see how this mechanism works, we allow the market maker to observe $\{v_1, v_2, v_3\}$. So, the pricing function is

$$P_x = Pr(\omega = x|v_1, v_2, v_3)$$

The expected utilities under this variant are derived in the final section of the online appendix. Equating $EU(v_1 = 1|s_1 = 1)$ with $EU(v_1 = 0|s_1 = 1)$, we solve z^* at equilibrium.

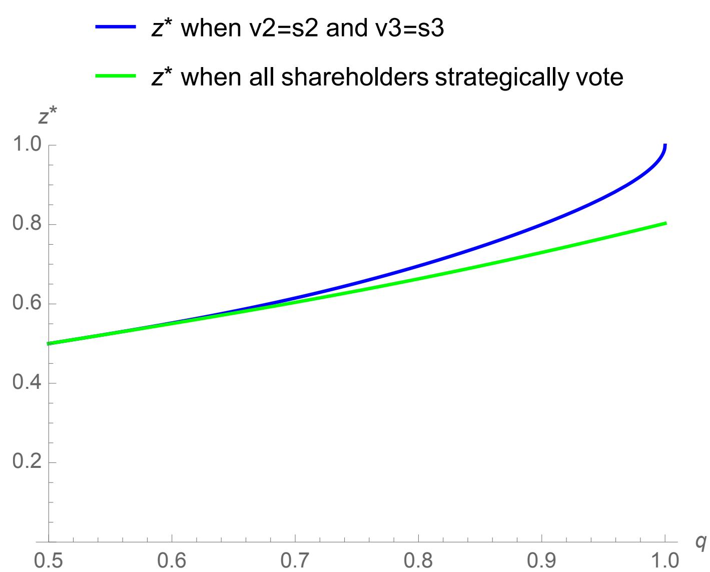


Figure 9: Under the Proposed Regulation requiring timely transparency, z^* of Shareholder 1 when Shareholder 2 and 3 Vote Sincerely

It is instructive to compare the equilibrium value of z^* from Figures 7 and 9. z^* in Figure 9 now increases with q. Moreover, z^* in Figure 9 is even greater than the z^* in the baseline model where all shareholders vote strategically. Hence, when all shareholders disclose how they vote before they trade, the existence of sincere-voting shareholders can help suppress the strategic voting of other shareholders. There are two positive effects. The shareholders that vote sincerely directly improve governance and when voting is transparent to the market the presence of these sincere types induces strategic voters to vote correctly with a higher probability.

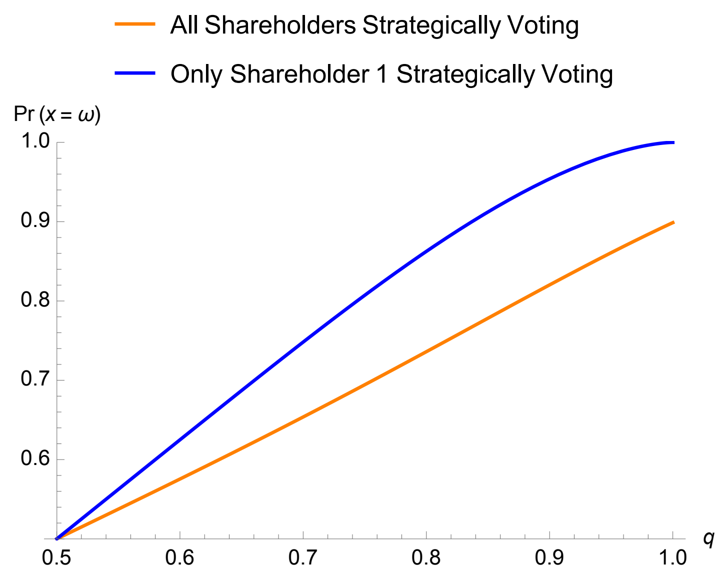


Figure 10: $Pr(x = \omega)$ Under the Proposed Transparency Regulation

Figure 10 shows that this transparency improves information aggregation. We note that transparency of this form alone does not need to improve governance. For example in the baseline model the symmetric mixed strategy equilibrium found above is not altered if each vote is observed prior to trading. With symmetry, t is a sufficient statistic. But with asymmetries t is not a sufficient statistic and transparency before trading tends to improve equilibrium levels of information aggregation in these extensions.

8 Conclusion

The starting point here is that shareholders do not have an absolute incentive to maximize the value of firms that they have a stake in. Starting instead with the primitive assumption that shareholders as investors seek to maximize their returns, the possibility of trading introduces a potential wedge. In settings where shareholders posses private information, incentives to use this information in their trading behavior and not reveal it when voting may lead to distortions in corporate policy-making. Equilibrium forces must balance out these incentives and accounting for this provides an explanation for uninformative voting even when shareholders have access to high quality information. Moreover, because differences in private information may remain present at interim stages of the model, shareholders may be seen to behave heterogeneously in the market. We note, the presence of pooling equilibria in which there is no uncertainty about the voting outcome. These equilibria may match up with received wisdom that shareholder votes typically serve as a rubber stamp on the decisions made by management. Moreover, in all the equilibria found here, correlations between market prices and individual votes should be weak and shareholders should be heterogeneous in their post vote market behavior. A casual reading of the empirical literature supports these predictions.

Importantly, the inefficiencies that stem from the shareholder's dilemma do not stem from mis-alignment between votes and shares or vote-trading–the topic of much theoretical, empirical and policy oriented research. The analysis in our paper does, however, provide some guidance for policymakers to consider. Our analysis isolates reductions of liquidity following important votes and reporting policies that reveal how individuals vote in a timely manner as possible policy levers for enhancing the efficiency of governance especially when block and insider or passive voters are present.

Accounting for the shareholder's dilemma in studies of shareholder voting in which there is any informational component may prove valuable. In work on information acquisition, rule choice, mergers and vote trading the default assumptions on shareholder objectives may be inconsistent with a broader perspective of the shareholders choice environment. To be sure, the model is sparse. In the interest of abstracting away from features that are already well understood in the classical voting literature we have worked with an informational environment that is as simple as possible. Moreover, we have abstracted away from asymmetries in the number of shares owned by voters.[15] Finally, we have ignored considerations that involve direct communication between shareholders. But preliminary work makes clear that adding these features does not make the potential for generating informational rents by strategic voting and extracting them by strategic trading go away. Instead inclusion of trading in richer voting models is likely to provide richer and more nuanced assessments than extant work provides. Moreover, the model presents some promising challenges for empirical work; it relates forms of heterogeneity to the welfare effects of real-time reporting requirements and provides context for work on post vote trading behavior as well as correlational work on voting and returns.[16]

[15] See for example Maug (1999) for work incorporating some of these features in the standard informational voting model without a trading stage.

[16] The relationship between private information, voting and trading opportunities that comes out of our work may help explain findings in work relating shareholder votes to other economic variables. See for example Brav et. al. (2018).

References

1. Acharya, Avidit and Adam Meirowitz (2017). "Sincere Voting in Large Elections," *Games and Economics Behavior*, 101 (1): 121-131.

2. Admati, Anat R., and Paul Pfleiderer (2009). "The Wall Street Walk and shareholder activism: Exit as a form of voice." *Review of Financial Studies* 22.7: 2645-2685.

3. Ahn, David, and Santiago Oliveros (2012). "Combinatorial Voting," *Econometrica*, 80 (1) :89-141.

4. Atakan, Alp, and Mehmet Ekmekci (2014). "Auctions, actions, and the failure of information aggregation," *American Economic Review* 104(7):2014-2048.

5. Austen-Smith, David and Jeffrey Banks (1996). "Information Aggregation, Rationality, and the Condorcet Jury Theorem," *American Political Science Review*, 90, 1, 34-45.

6. Back, Kerry, Tao Li, and Alexander Ljungqvist (2015). "Liquidity and governance." *ECGI-Finance Working Paper* 388 .

7. Bannerjee, Snehal, and Ilan Kramer (2010). "Disagreement and Learning: Dynamic Patterns of Trade," *Journal of Finance* 65(4):1269-1302.

8. Bar-Isaac, Heski, and Joel Shapiro (2020) "Blockholder voting." *Journal of Financial Economics* 136.3 (2020): 695-717.

9. Bhattacharya, Sourav (2013) "Preference Monotonicity and Information Aggregaton in Electons." *Econometrica* 81(3):1229-1247.

10. Bhide, Amar (1993). "The hidden costs of stock market liquidity." *Journal of financial economics* 34.1: 31-51.

11. Blair, D.H, D, Golbe, and J.M. Gerard (1989). "Unbundling the Voting Rights and Profit Claims of Common Shares" *Journal of Political Economy* 97:420-443.

12. Bollerslev, Tim, Jia Li, and Yuan Xue (2018) "Volume, Volatility, and Public News Announcements," *Review of Economic Studies* 85(4):2005-2041.

13. Bolton, Patrick, Tao Li, Enrichetta Ravina, and Howard Rosenthal (2018) "Investor Idealogy" *ECGI Finance Working Paper*

14. Bond, Phillip, and Hulya Eraslan, H., (2010). "Strategic Voting over Strategic Proposals," *Review of Economic Studies* 77(2):459-490.

15. Brav, Alon, and Richmond Matthews (2011). "Empty Voting and the Efficiencey of Corporate Governance". *Journal of Financial Economics* 99(2):289-307.

16. Brav, Alon, Wei Jiang, Tao Li, and James Pinnington (2018). "Picking friends before picking (proxy) fights: How mutual fund voting shapes proxy contests." *Columbia Business School Research Paper* 18-16 (2018).

17. Coffee, John C(1991)."Liquidity versus control: The institutional investor as corporate monitor." *Columbia law review* 91.6 (1991): 1277-1368.

18. Condorcet, Nicolas de (1785). "Essai sur l'application de l'analyse á la probabilité des décisions rendues á la pluralité de voix [Essay on the Application of Analysis to the Probability of Majority Decisions]," Paris.

19. Cremers, KJ Martijn, and Roberta Romano. "Institutional investors and proxy voting on compensation plans: The impact of the 2003 mutual fund voting disclosure rule." *American Law and Economics Review* 13.1 (2011): 220-268.

20. Dekel, Eddie, and Asher Wolinsky (2011) "Buying Shares and/or Votes for Corporate Control" *Review of Economic Studies* 79:196-226.

21. Duggan, John and Cesar Martinelli (2001) "A Bayesian Model of Voting in Juries." *Games and Economic Behavior*, 37(2): 259-294.

22. Edmans, Alex (2009)."Blockholder trading, market efficiency, and managerial myopia." *The Journal of Finance* 64.6: 2481-2513.

23. Edmans, Alex, and Gustavo Manso(2010)."Governance through trading and intervention: A theory of multiple blockholders." *Review of Financial Studies* 24.7: 2395-2428.

24. Edmans, Alex, Vivian W. Fang, and Emanuel Zur(2013)."The effect of liquidity on governance." *Review of Financial Studies* 26.6: 1443-1482.

25. Ekmekci, Mehmet, and Stephan Lauermann (2019) "Informal Elections with Dispersed Information," *CRC TR 224 Discussion Paper Series* University of Bonn and University of Mannheim, Germany.

26. Feddersen, Timothy and Wolfgang Pesendorfer (1997). "Voting Behavior and Information Aggregation in Large Elections with Private Information." *Econometrica*, 65, 5, 1029-1058.

27. Feddersen, Timothy and Wolfgang Pesendorfer (1998). "Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting." *American Political Science Review*, 92(1):, 23-35.

28. Gieczewski, German and Svetlana Kosterina (2020) "Endogenous Experimentation in Organizations," *Princeton University Typescript.*

29. Good, I.J. and Lawrence Mayer. (1975) "Estimating the Efficacy of a Vote," *Behavioral Science*, 20, 1, 25-33.

30. Grossman, Sandy. and Oliver Hart (1980). "Takeover Bids, the Free-Rider Problem, and the Theory of Corporation" *Bell Journal of Economics* 11:42-64

31. Grossman, Sandy. and Oliver Hart (1988). "One Share-One Vote and the Market for Corporate Control" *Journal of Financial Economics* 20:175-202.

32. Harris, M. and A. Raviv (1988). "Corporate Governance: Voting Rights and Majority Rules" *Journal of Financial Economics* 20:203-235.

33. Iaryczower, Matias and Santiago Oliveros (2017). "Competing for Loyalty: The Dynamics of Rallying Support" *The American Economic Review* 107:2990-3005.

34. Kim, Jaehoon and Mark Fey (2007) "The Swing Voters Curse with Adversarial Preferences," *Journal of Economic Theory* 135: 236-252.

35. Kyle, Albert (1985) "Continuous Auctions and Insider Trading," *Econometrica* 53(6):1315-1335

36. Levit, Doron and Nadya Malenko (2011) "Nonbinding Voting for Shareholder Proposals," *Journal of Finance* 66(5):1579-1614.

37. Levit, Doron, Nadya Malenko, and Ernst Maug (2020). "Trading and Shareholder Democracy," *European Corporate Governance Institute Finance Working Paper* 631/2019.

38. Li, Sophia, Ernst Maug, and Miriam Schwartz (2019). "When Shareholders Disagree: Trading AFter Shareholder Meetings," *European Corporate Governance Institute Finance Working Paper*.

39. Malenko, Andrey and Nadya Malenko (2019) "Proxy Advisory Firms: The Economics of Selling Information to Voters," *Journal of Finance*, forthcoming.

40. Mandler, Michael (2012) "The Fragility of Information Aggregation in Large Elections" *Games and Economic Behavior*, 74, 1, 257-268.

41. Maug, Ernst (1998)."Large shareholders as monitors: Is there a tradeoff between liquidity and control?." *Journal of finance*, 53.1: 65-98.

42. Maug, Ernst (1999) "How Effective Is Proxy Voting? Information Aggregation and Conflict Resolution in Corporate Voting Contests," *University of Mannheim Typescript*.

43. Maug, Ernst and Kristian Rydqvist (2008) "Do Shareholders Vote Strategically? Voting Behavior, Proposal Screening, and Majority Rules," *Review of Finance* 13(1):47-79.

44. Maug, Ernst and Bilge Yilmaz (2002) "Two-class Voting: A Mechanism for Conflict Resolutoin," *American Economic Review* 92(5):1448-1471.

45. McLennan, Andrew (1998) "Consequences of the Condorecet Jury Theorems for Beneficial Information Aggregation by Rational Agents," *American Political Science Review* 92(2):413-418.

46. Meirowitz, Adam (2002). "Informative Voting and Condorcet Jury Theorems with a Continuum of Types," *Social Choice and Welfare* 19(1);219-236.

47. Meirowitz, Adam and Kenneth Shotts (2009)."Pivots Versus Signals in Elections," *Journal of Economic Theory*144(2):744-771.

48. Razin, Ronny (2003)."Signaling and Election Motivations in a Voting Model with Common Values and Responsive Candidates," *Econometrica* 71(4):1083-1119.

.

A Appendix: Proofs Omitted From Main Text

A.1 Proof of Theorem 3

Proof.

We begin the proof from the indifference condition rewritten below

$$\underbrace{\sum_{t_{-i}=0}^{n-1}[Pr(t_{-i}|s_1=1)\Delta P(t_{-i})]}_{\textit{Signal Effect}} = \underbrace{Pr(t_{-i}=\frac{n-1}{2}|s_1=1)(2Pr(\omega=1|t_{-i},s_1=1)-1)}_{\textit{Pivotal Effect}} \tag{17}$$

First, we can rewrite the signal effect as $\sum_{k=-n}^{n-2}[Pr(t_{-i}=\frac{n+k}{2}|s_1=1)\Delta P(t_{-i})]$, where each k is an odd number. We then divide the both sides of the indifference condition by $Pr(t_{-i}=\frac{n-1}{2}|s_1=1)$ to get

$$\sum_{k=-n}^{n-2}\frac{\binom{n-1}{\frac{n+k}{2}}}{\binom{n-1}{\frac{n-1}{2}}}[q(\frac{z}{1-z})^{\frac{1}{2}+\frac{1}{2}k}+(1-q)(\frac{1-z}{z})^{\frac{1}{2}+\frac{1}{2}k}](\frac{1}{\left(\frac{z}{1-z}\right)^{-k-2}+1}-\frac{1}{\left(\frac{z}{1-z}\right)^{-k}+1})=2q-1 \tag{18}$$

Now we consider subset of the terms in the sum, namely taking the index set from $k=-1$ and to $k=\overline{k(n)}$, where $\overline{k(n)}\leq n-1$. Note that below when taking n to infinity, $k=\overline{k(n)}$ could be any arbitrary large but finite number. Because each term of the series above is strictly greater than 0, the sum over the subset of terms (we call it the sub-sum) is strictly smaller than the original sum. Thus, we have

$$\sum_{k=-1}^{\overline{k(n)}}\frac{\binom{n-1}{\frac{n+k}{2}}}{\binom{n-1}{\frac{n-1}{2}}}[q(\frac{z}{1-z})^{\frac{1}{2}+\frac{1}{2}k}+(1-q)(\frac{1-z}{z})^{\frac{1}{2}+\frac{1}{2}k}]\left(\frac{1}{\left(\frac{z}{1-z}\right)^{-k-2}+1}-\frac{1}{\left(\frac{z}{1-z}\right)^{-k}+1}\right)<2q-1 \tag{19}$$

Because $(1-q)(\frac{1-z}{z})^{\frac{1}{2}+\frac{1}{2}k}\left(\frac{1}{\left(\frac{z}{1-z}\right)^{-k-2}+1}-\frac{1}{\left(\frac{z}{1-z}\right)^{-k}+1}\right)>0$ for any k, we have

$$\sum_{k=-1}^{\overline{k(n)}}\frac{\binom{n-1}{\frac{n+k}{2}}}{\binom{n-1}{\frac{n-1}{2}}}(\frac{z}{1-z})^{\frac{1}{2}+\frac{1}{2}k}\left(\frac{1}{\left(\frac{z}{1-z}\right)^{-k-2}+1}-\frac{1}{\left(\frac{z}{1-z}\right)^{-k}+1}\right)<\frac{2q-1}{q} \tag{20}$$

Since $q \in (\frac{1}{2}, 1)$, we know the maximum of $\frac{2q-1}{q}$ is smaller than 1. Thus, the above inequality requires that the sub-sum is smaller than 1.

$$\sum_{k=-1}^{\overline{k(n)}} \frac{\binom{n-1}{\frac{n+k}{2}}}{\binom{n-1}{\frac{n-1}{2}}} (\frac{z}{1-z})^{\frac{1}{2}+\frac{1}{2}k} \left(\frac{1}{\left(\frac{z}{1-z}\right)^{-k-2}+1} - \frac{1}{\left(\frac{z}{1-z}\right)^{-k}+1} \right) < 1 \tag{21}$$

Contrary to the theorem, suppose now that as $n \to \infty$ a sequence of equilibria exist with m_n, z_n not converging to $\frac{1}{2}$. Since these probabilities live in a bounded set, a subsequence of equilibria must exist with $z_m^*(n) \to z^* > \frac{1}{2}$. We drop the tracking of subsequences and just refer to this as z_n^*. The terms in the sum are continuous in z^* and so this implies that

$$\lim_{n\to\infty} \sum_{k=-1}^{\overline{k(n)}} \frac{\binom{n-1}{\frac{n+k}{2}}}{\binom{n-1}{\frac{n-1}{2}}} (\frac{z^*}{1-z^*})^{\frac{1}{2}+\frac{1}{2}k} \left(\frac{1}{\left(\frac{z^*}{1-z^*}\right)^{-k-2}+1} - \frac{1}{\left(\frac{z^*}{1-z^*}\right)^{-k}+1} \right) < 1 \tag{22}$$

where $\frac{\binom{n-1}{\frac{n+k}{2}}}{\binom{n-1}{\frac{n-1}{2}}}$ converges to 1 as $n \to \infty$ and k is bounded.

As we are taking limits as $n \to \infty$ and the above requires only that $\overline{k(n)} < n$ we are interested in the sub-sum for large $\overline{k(n)}$. Note that sinces $z^* > \frac{1}{2}$, $\frac{z^*}{1-z^*}$ is strictly greater 1. Thus, for $k > 0$, the term $(\frac{z^*}{1-z^*})^{\frac{1}{2}+\frac{1}{2}k}$ in the above inequality is divergent as k grows. On the other hand, $2((\frac{z^*}{1-z^*})^{-k-2}+1)((\frac{z^*}{1-z^*})^{-k}+1)$ converges to 2 as k grows because

$$\begin{aligned} &((\frac{z^*}{1-z^*})^{-k-2}+1)((\frac{z^*}{1-z^*})^{-k}+1) \\ &= (\frac{z^*}{1-z^*})^{-2k-2} + (\frac{z^*}{1-z^*})^{-k-2} + (\frac{z^*}{1-z^*})^{-k} + 1 \end{aligned} \tag{23}$$

and the first three terms converge to 0 as k grows. So, the divergent $(\frac{z^*}{1-z^*})^{\frac{1}{2}+\frac{1}{2}k}$ is much greater than the convergent $2((\frac{z^*}{1-z^*})^{-k-2}+1)((\frac{z^*}{1-z^*})^{-k}+1)$ as k grows.

Taking limits as $n \to \infty$, and $z_n^* \to z^* > \frac{1}{2}$ and selecting large $\overline{k(n)} < n$, a lower bound of the sub-sum can be derived when the divergent $(\frac{z^*}{1-z^*})^{\frac{1}{2}+\frac{1}{2}k}$ is replaced by the convergent $2((\frac{z^*}{1-z^*})^{-k-2}+1)((\frac{z^*}{1-z^*})^{-k}+1)$. Because the sub-sum is smaller than 1, its lower bound must be smaller than 1.

$$2z^* - 1 + \lim_{n\to\infty} \sum_{k=1}^{\overline{k(n)}} 2((\frac{z^*}{1-z^*})^{-k-2}+1)((\frac{z^*}{1-z^*})^{-k}+1) \left(\frac{1}{\left(\frac{z^*}{1-z^*}\right)^{-k-2}+1} - \frac{1}{\left(\frac{z^*}{1-z^*}\right)^{-k}+1} \right) < 1 \tag{24}$$

$\Rightarrow$

$$2z^* - 1 + \lim_{n\to\infty} \sum_{k=1}^{\overline{k(n)}} 2[(\frac{z^*}{1-z^*})^{-k} - (\frac{z^*}{1-z^*})^{-k-2}] < 1 \tag{25}$$

$\Rightarrow$

$$2z^* - 1 + 2(1 - (\frac{z^*}{1-z^*})^{-2}) \lim_{n\to\infty} \sum_{k=1}^{\overline{k(n)}} (\frac{z^*}{1-z^*})^{-k} < 1 \tag{26}$$

As $z^* > \frac{1}{2}$, we know $(\frac{z^*}{1-z^*})^{-2} < 1$. So, the series $(\frac{z^*}{1-z^*})^{-k}$ is a convergent geometric series and the inequality requires

$$2z^* - 1 + \frac{2-2z^*}{z^*} < 1 \tag{27}$$

A necessary condition for the inequality above to be true is that the minimum of the left hand side of the inequality is smaller than 1. Define the left hand side of the inequality as a function of z^*. The necessary condition is

$$\min_{\frac{1}{2}<z^*\leq q} f(z^*) = \min_{\frac{1}{2}<z^*\leq q} 2z^* - 1 + \frac{2-2z^*}{z^*} < 1 \tag{28}$$

We show this necessary condition cannot be true. Because $\frac{df(z^*)}{dz^*} < 0$ when $\frac{1}{2} < z^* \leq q < 1$, the function $f(z^*)$ is strictly decreasing when $\frac{1}{2} < z^* \leq q < 1$. Thus, $\min_{\frac{1}{2}<z^*\leq q} f(z^*) > f(1) = 1$, which establishes the contradiction.

As a consequence, when $n \to \infty$, any sequence of equilibria must have $z_n^* \to \frac{1}{2}$. ■

A.2 Proof of Theorem 4

Proof. Consider an arbitrary n and equilibrium z_n. The probability of making the correct decision through voting is

$$\begin{aligned}
&Pr(\text{correct decision}) \\
&= Pr(x=1|\omega=1)Pr(\omega=1) + Pr(x=0|\omega=0)Pr(\omega=0) \\
&= \frac{1}{2}\left(\sum_{t=\frac{n+1}{2}}^{t=n} \binom{n}{t} z_n^t (1-z_n)^{n-t} + \sum_{t=0}^{t=\frac{n-1}{2}} \binom{n}{t} (1-z_n)^t z_n^{n-t}\right)
\end{aligned} \tag{29}$$

Since $\sum_{t=\frac{n+1}{2}}^{t=n} \binom{n}{t} z_n^t (1-z_n)^{n-t} \equiv \sum_{t=0}^{t=\frac{n-1}{2}} \binom{n}{t} (1-z_n)^t z_n^{n-t}$, we have

$$Pr(\text{correct decision}) = \sum_{t=\frac{n+1}{2}}^{t=n} \binom{n}{t} z_n^t (1-z_n)^{n-t} \tag{30}$$

Standard arguments imply that a central limit theorem applies: for n large, $Pr(\text{correct decision})$ is approximately

$$\begin{aligned} &1-\Phi\left(\frac{\frac{n}{2}-nz_n}{\sqrt{nz_n(1-z_n)}}\right) \\ &= 1-\Phi\left(\frac{\sqrt{n}(\frac{1}{2}-z_n)}{\sqrt{z_n(1-z_n)}}\right) \\ &= \Phi\left(\frac{\sqrt{n}(z_n-\frac{1}{2})}{\sqrt{z_n(1-z_n)}}\right) \end{aligned} \tag{31}$$

Accordingly, when $n \to \infty$, the probability of making the correct decision depends on $\lim_{n\to\infty} \frac{\sqrt{n}(z_n-\frac{1}{2})}{\sqrt{z_n(1-z_n)}}$.

To find this limit, recall that the indifference condition is

$$\begin{aligned} &\sum_{t=0}^{n-1} \binom{n-1}{t} [q z_n^t (1-z_n)^{n-t-1} + (1-q)(1-z_n)^t z_n^{n-t-1}] \\ &\cdot [\frac{z_n^{t+1}(1-z_n)^{n-t-1}}{z_n^{t+1}(1-z_n)^{n-t-1} + (1-z_n)^{t+1} z_n^{n-t-1}} - \frac{z_n^t (1-z_n)^{n-t}}{(1-z_n)^t z_n^{n-t} + z_n^t (1-z_n)^{n-t}}] \\ &= \binom{n-1}{\frac{n-1}{2}} z_n^{\frac{n-1}{2}} (1-z_n)^{\frac{n-1}{2}} 2q-1 \end{aligned} \tag{32}$$

For fixed n we can view the LHS of the indifference condition as a function of z_n, $LHS(z_n;n)$ and the RHS of the indifference condition as a function $RHS(z_n,n)$. It is common to use an iterative approach to approximate the solution to a non-linear sytem. This would involve finding z_n solving a Taylor expansion of the system and then iteratively improving the point where the expansion is taken by using the solution to the previous step. As we have already shown that z_n converges to $\frac{1}{2}$ it is sufficient to take the expansions once at $z_n = \frac{1}{2}$

The second degree Taylor expansion of $LHS(z_n; n)$ is

$$LHS(z_n; n) = LHS(\frac{1}{2}; n) + \frac{LHS'(\frac{1}{2}; n)}{1!}(z_n - \frac{1}{2}) + \frac{LHS''(\frac{1}{2}; n)}{2!}(z_n - \frac{1}{2})^2 + \mathcal{O}(z_n - 1/2)^3$$

where $\mathcal{O}(z_n - 1/2)^3$ vanishes an order faster than $(z_n - \frac{1}{2})^3$

We have

$$LHS(\frac{1}{2}; n) = 0 \tag{33}$$

$$LHS'(\frac{1}{2}; n) = \sum_{t=0}^{n-1} \binom{n-1}{t} 2^{2-n} = 2 \tag{34}$$

$$LHS''(\frac{1}{2}; n) = -\sum_{t=0}^{n-1} \binom{n-1}{t} 2^{4-n}(2p-1)(n-2t-1) = 0 \tag{35}$$

The last two equations use the binomial theorem that $\sum_{k=0}^{n-1} \binom{n-1}{t} = 2^{n-1}$ and the fact that $\sum_{t=0}^{n-1} \binom{n-1}{t}(n-2t-1) = 0$.

Thus, we have

$$LHS(z_n) = 2z_n - 1 + \mathcal{O}(z_n - 1/2)^3$$

Similarly, we can also view the right hand side of the indifference condition as a function of z_n, $RHS(z_n; n)$. The Taylor expansion of $RHS(z_n; n)$ at degree of 2 is

$$RHS(z_n; n) = RHS(\frac{1}{2}; n) + \frac{RHS'(\frac{1}{2}; n)}{1!}(z_n - \frac{1}{2}) + \frac{RHS''(\frac{1}{2}; n)}{2!}(z_n - \frac{1}{2})^2 + \mathcal{O}(z_n - 1/2)^3$$

We have

$$RHS(\frac{1}{2}; n) = \binom{n-1}{\frac{n-1}{2}} (\frac{1}{2})^{n-1}(2q-1) \tag{36}$$

$$RHS'(\frac{1}{2}; n) = 0 \tag{37}$$

$$RHS''(\frac{1}{2}; n) = -\binom{n-1}{\frac{n-1}{2}} 2^{3-n}(n-1)(2q-1) \tag{38}$$

Thus, we have

$$RHS(z_n) = \binom{n-1}{\frac{n-1}{2}} (\frac{1}{2})^{n-1}(2q-1) - \binom{n-1}{\frac{n-1}{2}} 2^{2-n}(n-1)(2q-1)(z_n - \frac{1}{2})^2 + \mathcal{O}(z_n - 1/2)^3$$

Since we have already proved that $z_n \to \frac{1}{2}$ when $n \to \infty$, the $\mathcal{O}$ terms vanish as $n \to \infty$. Thus, an approximation of z_n can be given by solving

$$2z_n - 1 = \binom{n-1}{\frac{n-1}{2}}(\frac{1}{2})^{n-1}(2q-1) - \binom{n-1}{\frac{n-1}{2}}2^{2-n}(n-1)(2q-1)(z_n - \frac{1}{2})^2$$

We obtain

$$z_n \to \frac{1}{2} + \frac{2^n\left(\sqrt{2^{3-2n}(n-1)(1-2q)^2C_n^2+1}-1\right)}{4(n-1)(2q-1)C_n}, \quad C_n = \binom{n-1}{\frac{n-1}{2}}.$$

Using Stirling's approximation, we have $C_n = \binom{n-1}{\frac{n-1}{2}}$ is approximately $\frac{2^{n-\frac{1}{2}}}{\sqrt{\pi}\sqrt{n-1}}$. So,

$$z_n \to \frac{1}{2} + \frac{1}{2\sqrt{2n-2}}\frac{\sqrt{16(q-1)q+\pi+4}-\sqrt{\pi}}{2q-1}.$$

Thus,

$$\lim_{n\to\infty}\sqrt{n}(z_n - \frac{1}{2}) = \frac{1}{2\sqrt{2}}\frac{\sqrt{16(q-1)q+\pi+4}-\sqrt{\pi}}{2q-1}$$

which is finite. Consequently,

$$Pr(\text{correct decision}) \to \Phi\left(\frac{1}{\sqrt{2}}\frac{\sqrt{16(q-1)q+\pi+4}-\sqrt{\pi}}{2q-1}\right) < 1. \qquad (39)$$

■

A.3 Proof of Theorem 5

Proof.. As equilibrium play is in mixed strategies we can analyze the payoff to either action.

$EU(v_i = 0|s_i = 1)$

$$= \sum_{t'=0}^{\frac{n-1}{2}} Pr(t'|s_i = 1)Pr(\omega = 0|t') + \sum_{t'=\frac{n+1}{2}}^{n-1} Pr(t'|s_i = 1)(2Pr(\omega = 1|t', s_i = 1) - Pr(\omega = 1|t'))$$

$$= \sum_{t'=0}^{\frac{n-1}{2}} Pr(t'|s_i = 1))(1 - Pr(\omega = 1|t')) + \sum_{t'=\frac{n+1}{2}}^{n-1} Pr(t'|s_i = 1)(2Pr(\omega = 1|t', s_i = 1) - Pr(\omega = 1|t')) \tag{40}$$

Note that

$$-\sum_{t'=0}^{\frac{n-1}{2}} Pr(t'|s_i = 1)Pr(\omega = 1|t') - \sum_{t'=\frac{n+1}{2}}^{n-1} Pr(t'|s_i = 1)Pr(\omega = 1|t')$$
$$= -\sum_{t'=0}^{n-1} Pr(t'|s_i = 1)Pr(\omega = 1|t') \tag{41}$$

So, we can write $EU(v_i = 0|s_i = 1)$ as

$EU(v_i = 0|s_i = 1)$

$$= \sum_{t'=0}^{\frac{n-1}{2}} Pr(t'|s_i = 1) + \sum_{t'=\frac{n+1}{2}}^{n-1} Pr(t'|s_i = 1)2Pr(\omega = 1|t', s_i = 1) - \sum_{t'=0}^{n-1} Pr(t'|s_i = 1)Pr(\omega = 1|t') \tag{42}$$

First, we study the sum of the first two terms.

$$\sum_{t'=0}^{\frac{n-1}{2}} Pr(t'|s_i=1) + \sum_{t'=\frac{n+1}{2}}^{n-1} Pr(t'|s_i=1)2Pr(\omega=1|t',s_i=1)$$

$$= \sum_{t'=0}^{\frac{n-1}{2}} \binom{n-1}{t'} [qz^{t'}(1-z)^{n-1-t'} + (1-q)(1-z)^{t'}z^{n-1-t'}]$$

$$+ \sum_{t'=\frac{n+1}{2}}^{n-1} \binom{n-1}{t'} [qz^{t'}(1-z)^{n-1-t'} + (1-q)(1-z)^{t'}z^{n-1-t'}] 2\frac{qz^{t'}(1-z)^{n-1-t'}}{qz^{t'}(1-z)^{n-1-t'} + (1-q)(1-z)^{t'}z^{n-1-t'}}$$

$$= \sum_{t'=0}^{\frac{n-1}{2}} \binom{n-1}{t'} [qz^{t'}(1-z)^{n-1-t'} + (1-q)(1-z)^{t'}z^{n-1-t'}] + \sum_{t'=\frac{n+1}{2}}^{n-1} \binom{n-1}{t'} 2qz^{t'}(1-z)^{n-1-t'} \tag{43}$$

Note that

$$\sum_{t'=0}^{\frac{n-1}{2}} \binom{n-1}{t'} qz^{t'}(1-z)^{n-1-t'} + \sum_{t'=\frac{n+1}{2}}^{n-1} \binom{n-1}{t'} qz^{t'}(1-z)^{n-1-t'}$$

$$= q\sum_{t'=0}^{n-1} \binom{n-1}{t'} z^{t'}(1-z)^{n-1-t'} \tag{44}$$

$$= q$$

Thus, we can write equation (43) as

$$= q + \sum_{t'=0}^{\frac{n-1}{2}} \binom{n-1}{t'} (1-q)(1-z)^{t'} z^{n-1-t'} + \sum_{t'=\frac{n+1}{2}}^{n-1} \binom{n-1}{t'} q z^{t'} (1-z)^{n-1-t'}$$

$$= q + \sum_{t'=0}^{\frac{n-1}{2}} \binom{n-1}{t'} (1-q)(1-z)^{t'} z^{n-1-t'} + \sum_{t'=\frac{n-1}{2}}^{n-1} \binom{n-1}{t'} q z^{t'} (1-z)^{n-1-t'} \tag{45}$$

$$- \binom{n-1}{\frac{n-1}{2}} q z^{\frac{n-1}{2}} (1-z)^{n-1-\frac{n-1}{2}}$$

Because of the symmetry of binomial distribution, we know

$$\sum_{t'=0}^{\frac{n-1}{2}} \binom{n-1}{t'} (1-z)^{t'} z^{n-1-t'} = \sum_{t'=\frac{n-1}{2}}^{n-1} \binom{n-1}{t'} z^{t'} (1-z)^{n-1-t'} \tag{46}$$

Following this, we can write equation (45) as

$$= q + \sum_{t'=\frac{n-1}{2}}^{n-1} \binom{n-1}{t'} z^{t'} (1-z)^{n-1-t'} - \binom{n-1}{\frac{n-1}{2}} q z^{\frac{n-1}{2}} (1-z)^{n-1-\frac{n-1}{2}} \tag{47}$$

Thus, we have

$EU(v_i = 0 | s_i = 1)$

$$= q + \sum_{t'=\frac{n-1}{2}}^{n-1} \binom{n-1}{t'} z^{t'} (1-z)^{n-1-t'} - \binom{n-1}{\frac{n-1}{2}} q z^{\frac{n-1}{2}} (1-z)^{n-1-\frac{n-1}{2}} - \sum_{t'=0}^{n-1} Pr(t'|s_i = 1) Pr(\omega = 1|t') \tag{48}$$

Note that the first term in the above equation

$$\sum_{t'=\frac{n-1}{2}}^{n-1} \binom{n-1}{t'} z^{t'}(1-z)^{n-1-t'} \tag{49}$$

represents the probability that at least $\frac{n-1}{2}$ voters out of $n-1$ voters votes for signal.

The second term

$$\binom{n-1}{\frac{n-1}{2}} qz^{\frac{n-1}{2}}(1-z)^{n-1-\frac{n-1}{2}} = Pr(exact\ \frac{n-1}{2}\text{out of n-1 votes vote for signal})q$$

We then do a Taylor expansion of the third term.

$$\begin{aligned}
&\sum_{t'=0}^{n-1} Pr(t'|s_i=1)Pr(\omega=1|t') \\
&= \sum_{t'=0}^{n-1} \binom{n}{t'} 2^{-n} - \sum_{t'=0}^{n-1} \binom{n}{t'} 2^{1-n} \left(z-\frac{1}{2}\right)(2np-4pt'-2p+1) + O\left(\left(z-\frac{1}{2}\right)^2\right) \\
&= 2^{-1} - (z-\frac{1}{2}) + O\left(\left(z-\frac{1}{2}\right)^2\right)
\end{aligned} \tag{50}$$

Finally, we take the limit of $EU(v_i|s_i=1)$.

$$\begin{aligned}
&\lim_{n\to\infty} EU(v_i=0|s_i=1) \\
&= q + \lim_{n\to\infty} Pr(\text{at least } \frac{n-1}{2} \text{ voters out of n-1 voters votes for signal}) \\
&- \lim_{n\to\infty} \binom{n-1}{\frac{n-1}{2}} qz^{\frac{n-1}{2}}(1-z)^{n-1-\frac{n-1}{2}} \\
&- \lim_{n\to\infty} \sum_{t'=0}^{n-1} Pr(t'|s_i=1)Pr(\omega=1|t') \\
&= q + \lim_{n\to\infty} \Phi(\frac{\sqrt{n-1}(z-\frac{1}{2})}{\sqrt{z(1-z)}}) - 0 - \frac{1}{2}
\end{aligned} \tag{51}$$

According to Theorem 4, we have

$$\lim_{n\to\infty} \Phi(\frac{\sqrt{n-1}(z-\frac{1}{2})}{\sqrt{z(1-z)}}) = \Phi\left(\frac{1}{\sqrt{2}}\frac{\sqrt{16(q-1)q+\pi+4}-\sqrt{\pi}}{2q-1}\right)$$

Therefore, we have

$$\lim_{n\to\infty} EU(v_i=0|s_i=1) - \lim_{n\to\infty} Pr(correct)$$

$$= q + \Phi\left(\frac{1}{\sqrt{2}}\frac{\sqrt{16(q-1)q+\pi+4}-\sqrt{\pi}}{2q-1}\right) - \frac{1}{2} - \Phi\left(\frac{1}{\sqrt{2}}\frac{\sqrt{16(q-1)q+\pi+4}-\sqrt{\pi}}{2q-1}\right)$$

$$= q - \frac{1}{2} > 0 \tag{52}$$

■

A.4 Proof of Theorem 6

Proof. We now consider $U^* - Pr(correct\ when\ v_i = s_i\ all\ i)$, where the latter is both the probability a Bayesian with n conditionally iid signals of quality q makes the correct decision and the eqm probability of making the correct decision when there is no liquidity. Following the proof of Theorem 5, we can express the limit as

$$\lim_{n\to\infty} EU(v_i=0|s_i=1) - \lim_{n\to\infty} Pr(correct\ when\ v_i = s_i\ all\ i)$$

$$= q + \Phi\left(\frac{1}{\sqrt{2}}\frac{\sqrt{16(q-1)q+\pi+4}-\sqrt{\pi}}{2q-1}\right) - \frac{1}{2} - 1 \tag{53}$$

This term is strictly increasing in q. To see this, we differentiate the equation over q and get

$$1 + \phi\left(\frac{1}{\sqrt{2}}\frac{\sqrt{16(q-1)q+\pi+4}-\sqrt{\pi}}{2q-1}\right)\frac{\sqrt{2}\left(\sqrt{\pi}-\frac{\pi}{\sqrt{4(1-2q)^2+\pi}}\right)}{(1-2q)^2}$$

Since

$$\sqrt{\pi} - \frac{\pi}{\sqrt{4(1-2q)^2 + \pi}} > \sqrt{\pi} - \frac{\pi}{\sqrt{\pi}} = 0$$

we know $\lim_{n\to\infty} EU(v_i = 0|s_i = 1) - \lim_{n\to\infty}$ is increasing in q. It remains to show that this implies that the derivative of U^* is larger than the derivative of the probability of making the correct decision. Consider an arbitrary function of the form $h(q) = f(q) - g(q)$ that is strictly increasing. By the chain rule $h'(q) = f'(q) - g'(q)$, and thus $h'(q) > 0$ implies $f'(q) > g'(q)$, thus the result follows.

■

A.5 Proof of Theorem 7

Proof. It is sufficient to make 2 observations. First on the path the market-maker's beliefs are consistent with Bayes' rule. Second, under either profile a single deviation in voting cannot change the policy or the price and thus payoffs are flat in any single deviation at the voting stage ■

B Web Appendix

B.1 Derivations for Extension 6

Suppose that the blockholder gets signal $s_b = 1$. If she votes for policy 1 her expected payoff is

$$\begin{aligned} &\sum_{t'=0}^{\frac{n+1}{2}-b-1} [Pr(t'|s_b)((b-l)Pr(\omega=0|s_b,t') + lPr(\omega=0|t'+b))] \\ &+ \sum_{t'=\frac{n+1}{2}-b}^{n-b} [Pr(t'|s_b)((b+l)Pr(\omega=1|s_b,t') - lPr(\omega=1|t'+b))] \end{aligned} \tag{54}$$

If she votes for policy 0 her expected payoff is

$$\begin{aligned} &\sum_{t'=0}^{\frac{n+1}{2}-1} [Pr(t'|s_b)((b-l)Pr(\omega=0|s_b,t') + lPr(\omega=0|t'))] \\ &+ \sum_{t'=\frac{n+1}{2}}^{n-b} [Pr(t'|s_b)((b+l)Pr(\omega=1|s_b,t') - lPr(\omega=1|t'))] \end{aligned} \tag{55}$$

The difference between these two expected payoffs is

$$\begin{aligned} &\sum_{t'=0}^{\frac{n+1}{2}-b-1} [Pr(t'|s_b)l(Pr(\omega=0|t'+b) - Pr(\omega=0|t'))] \\ &- \sum_{t'=\frac{n+1}{2}-b}^{\frac{n+1}{2}-1} [Pr(t'|s_b)((b-l)Pr(\omega=0|s_b,t') + lPr(\omega=0|t'))] \\ &+ \sum_{t'=\frac{n+1}{2}-b}^{\frac{n+1}{2}-1} [Pr(t'|s_b)((b+l)Pr(\omega=1|s_b,t') - lPr(\omega=1|t'+b))] \\ &+ \sum_{t'=\frac{n+1}{2}}^{n-b} [Pr(t'|s_b)l(Pr(\omega=1|t') - Pr(\omega=1|t'+b))] \end{aligned} \tag{56}$$

We can simplify the above equation to

$$\underbrace{\sum_{t'=\frac{n+1}{2}-b}^{\frac{n+1}{2}-1}[Pr(t'|s_b)(2bPr(\omega=1|s_b,t')-b)]}_{\text{pivotal effect}}-l\underbrace{\sum_{t'=0}^{n-b}[Pr(t'|s_b)(Pr(\omega=1|t'+b)-Pr(\omega=1|t'))]}_{\text{signal effect}} \tag{57}$$

B.2 Derivations for Extension 7

Suppose without the lose of generality that shareholder 1 has signal $s_1 = 1$. If she votes for policy 1, her expected payoff is

$$\begin{aligned}
&EU(v_1=1|s_1=1)\\
=&Pr(v_2=0,v_3=0|s_1=1)Pr(\omega=0|t=1)\\
+&Pr(v_2=1,v_3=0|s_1=1)(2Pr(\omega=1|s_1=1,v_2=1,v_3=0)-Pr(\omega=1|t=2))\\
+&Pr(v_2=0,v_3=1|s_1=1)(2Pr(\omega=1|s_1=1,v_2=0,v_3=1)-Pr(\omega=1|t=2))\\
+&Pr(v_2=1,v_3=1|s_1=1)(2Pr(\omega=1|s_1=1,v_2=1,v_3=1)-Pr(\omega=1|t=3))\\
=&(q(1-q)^2+(1-q)q^2)\frac{(1-z)q^2+2z(1-q)q}{(1-z)q^2+2z(1-q)q+z(1-q)^2+2(1-z)q(1-q)}\\
+&2(q^2(1-q)+(1-q)^2q)[2\frac{q^2(1-q)}{q^2(1-q)+(1-q)^2q}-\frac{2zq(1-q)+(1-z)q^2}{2zq(1-q)+(1-z)q^2+2(1-z)q(1-q)+z(1-q)^2}]\\
+&(q^3+(1-q)^3)(2\frac{q^3}{q^3+(1-q)^3}-\frac{zq^2}{zq^2+(1-z)(1-q)^2})
\end{aligned} \tag{58}$$

But if she votes for 0, her expected payoff is

$$
\begin{aligned}
&EU(v_1 = 0|s_1 = 1)\\
=&Pr(v_2 = 0, v_3 = 0|s_1 = 1)Pr(\omega = 0|t = 0)\\
+&Pr(v_2 = 1, v_3 = 0|s_1 = 1)Pr(\omega = 0|t = 1)\\
+&Pr(v_2 = 0, v_3 = 1|s_1 = 1)Pr(\omega = 0|t = 1)\\
+&Pr(v_2 = 1, v_3 = 1|s_1 = 1)(2Pr(\omega = 1|s_1 = 1, v_2 = 1, v_3 = 1) - Pr(\omega = 1|t = 2))\\
=&(q(1-q)^2 + (1-q)q^2)\frac{zq^2}{zq^2 + (1-z)(1-q)^2}\\
+&2(q^2(1-q) + (1-q)^2 q)\frac{(1-z)q^2 + 2z(1-q)q}{(1-z)q^2 + 2z(1-q)q + z(1-q)^2 + 2(1-z)q(1-q)}\\
+&(q^3 + (1-q)^3)(2\frac{q^3}{q^3 + (1-q)^3} - \frac{2zq(1-q) + (1-z)q^2}{2zq(1-q) + (1-z)q^2 + 2(1-z)q(1-q) + z(1-q)^2})
\end{aligned}
\tag{59}
$$

Now consider the transparency reform described. Suppose without the lose of generality that shareholder 1 has signal $s_1 = 1$. If she votes for policy 1, her expected payoff is

$$
\begin{aligned}
&EU(v_1 = 1|s_1 = 1)\\
=&Pr(v_2 = 0, v_3 = 0|s_1 = 1)Pr(\omega = 0|v_1 = 1, v_2 = 0, v_3 = 0)\\
+&Pr(v_2 = 1, v_3 = 0|s_1 = 1)(2Pr(\omega = 1|s_1 = 1, v_2 = 1, v_3 = 0) - Pr(\omega = 1|v_1 = 1, v_2 = 1, v_3 = 0))\\
+&Pr(v_2 = 0, v_3 = 1|s_1 = 1)(2Pr(\omega = 1|s_1 = 1, v_2 = 0, v_3 = 1) - Pr(\omega = 1|v_1 = 1, v_2 = 0, v_3 = 1))\\
+&Pr(v_2 = 1, v_3 = 1|s_1 = 1)(2Pr(\omega = 1|s_1 = 1, v_2 = 1, v_3 = 1) - Pr(\omega = 1|v_1 = 1, v_2 = 1, v_3 = 1))\\
=&(q(1-q)^2 + (1-q)q^2)\frac{(1-z)q^2}{(1-z)q^2 + z(1-q)^2}\\
+&2(q^2(1-q) + (1-q)^2q)(2\frac{q^2(1-q)}{q^2(1-q) + (1-q)^2q} - \frac{zq(1-q)}{zq(1-q) + (1-z)(1-q)q})\\
+&(q^3 + (1-q)^3)(2\frac{q^3}{q^3 + (1-q)^3} - \frac{zq^2}{zq^2 + (1-z)(1-q)^2})
\end{aligned}
\tag{60}
$$

On the other hand, if she votes for policy 0, her expected payoff is

$$
\begin{aligned}
&EU(v_1 = 0|s_1 = 1) \\
=&Pr(v_2 = 0, v_3 = 0|s_1 = 1)Pr(\omega = 0|v_1 = 0, v_2 = 0, v_3 = 0) \\
+&Pr(v_2 = 1, v_3 = 0|s_1 = 1)Pr(\omega = 0|v_1 = 0, v_2 = 1, v_3 = 0) \\
+&Pr(v_2 = 0, v_3 = 1|s_1 = 1)Pr(\omega = 0|v_1 = 0, v_2 = 0, v_3 = 1) \\
+&Pr(v_2 = 1, v_3 = 1|s_1 = 1)(2Pr(\omega = 1|s_1 = 1, v_2 = 1, v_3 = 1) - Pr(\omega = 1|v_1 = 0, v_2 = 1, v_3 = 1)) \\
=&(q(1-q)^2 + (1-q)q^2)\frac{zq^2}{zq^2 + (1-z)(1-q)^2} \\
+&2(q^2(1-q) + (1-q)^2 q)\frac{zq(1-q)}{zq(1-q) + (1-z)(1-q)q} \\
+&(q^3 + (1-q)^3)(2\frac{q^3}{q^3 + (1-q)^3} - \frac{(1-z)q^2}{(1-z)q^2 + z(1-q)^2})
\end{aligned}
\tag{61}
$$