Suppressing Dissent with Limited Information.

Gaétan Nandong

New York University

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Abstract

Technological advancement has increased governments' reliance on technology for identification purposes while also raising citizens' desire for identity protection during dissent activities. This interaction has often resulted in inadequate information in the application of repression. This paper demonstrates how, in this context, there are two distinct ways in which high punishments that enhance responsive repression can lead to increased dissent. First, if the defender learns about individual participation at the macro level—inferring based on the aggregate level of dissent—then in hostile societies, citizens can be inclined to dissent even though they anticipate repression. Second, if inference occurs at the micro level, a defender observes citizens' generated signals; the prospect of repression with inadequate information is more likely to increase dissent. In contrast to a large body of research that underscores psychological motives, these results demonstrate the role of factors such as inconsistency and the erratic nature of state reprisal. Repression with inadequate intelligence can be indiscriminate, but participation is always riskier. The inconsistency and erratic nature of the state response are viewed as signs of weakness, boosting political dissent despite the risks. The analysis further highlights the limits of humanitarian interventions, showing how these measures can inadvertently backfire.

Keywords— Repression structures, Punishment intensity, Inadequate intelligence, Marginal Risks.

The advent of new technologies has increased governments' use of technology to monitor dissidents. In response, citizens have developed novel tactics for identity protection while participating in anti-government activities. This has created a dynamic in which repression is often conducted under imperfect information. For example, university campuses in the United States have recently been the sites of public protests against the war in Gaza, where participants chose to wear face masks. Given the sensitivity of the issue regarding the conflict between Israel and Palestine, university students opted to protect their identities (using religious hijabs and medical masks) out of fear of repercussions and online harassment (Robins-Early (2024), Sammon (2024), Stanley (2024)). Anonymous protests led the Ohio attorney general to warn that students could face felony charges under anti-KKK law.[1]

Furthermore, armed conflicts like rebellions and civil wars often feature governments seeking to retaliate with limited knowledge of insurgents. Information about the rebels and their whereabouts can be limited (Kalyvas 2006; Lewis 2020). The time and location of future attacks are sometimes crucial pieces of information that are unavailable to government forces. This identification problem may also arise because combatants often hide within the civilian population. For example, during the insurgency in Iraq, a soldier from the American troops clearly pointed this out, referring to the civilians: "it's hard to separate the good from the bad" (Filkins (2005)).[2]

Underlying these different observations is the question of how deterrence operates in contentious politics when repression is implemented with insufficient information. In a seminal book on Iraq under Saddam Hussein, Blaydes 2018 argues that this form of repression reinforces group identities, ultimately increasing political dissent. In this view, inadequate information compels the regime to rely on the *minimum legible group* (e.g., ethno-religious identity) to enhance its knowledge of dissidents (Blaydes 2018, pg. 46). Given this mode

1. https://apnews.com/article/campus-protests-mask-law-ohio-55216c2b84d098edf9de69a679f83340

2. The tendency to conceal identity was also adopted by the Zapatitas rebel movement in southern Mexico (Tuckman (2002); Melgaço and Jeffrey (2021)).

of inference, expectations of repression increase political dissent by reinforcing group ties: people tend to believe that they have a 'linked-fate' and are willing to dissent despite the high risk of repression (Blaydes 2018, pg 47).

By focusing on ethnicity as the primary mode of inductive inference, this argument overlooks the myriad ways in which a government can update its knowledge of dissidents. When faced with limited information, a government can act as a rational Bayesian, building inferences using information generated by the dissident group or by individuals under surveillance. For example, a suspect may have a dissident tattoo on their knuckles, wear a suspicious shirt, or follow an online account of a dissident organization. Similarly, the number of dissident attacks may provide important clues about the organization's scope. In each of these examples, an individual's ethnic background is hardly the key to updating information about their participation. If ethnicity is irrelevant in the information-updating stage, Blayde's argument is no longer persuasive, as identities may not be salient when individuals expect repression. Because of this limitation, the patterns of political dissent and repression under conditions of limited information remain poorly understood.

The present paper takes up the challenge of analyzing the role of deterrence when repression is carried out with insufficient information. The focus is on the following questions: How does deterrence in social movements work when the government is imperfectly informed about whether a citizen is a dissident or a non-dissident? Regarding the protests on US campuses and the response of the Ohio Attorney General, what relationship exists between repressive measures and dissent that leads an imperfectly informed government to rely more on harsh measures? How should we view the efficacy of humanitarian interventions when repression is carried out with imperfect monitoring? There is the inevitable question of how the government learns about dissidents when ethnicity is hardly a factor. I focus on two modes of inductive inference that might be available to a government, depending on geographic accessibility or access to new technology. The government may rely on the number of dissident attacks to improve its information about dissidents. Alternatively, fine-grained information can be available if the government observes an individual-generated signal.

I develop a model in which citizens decide to dissent when they anticipate that a strategic and imperfectly informed government will respond with repression. There is an exogenous punishment (the repressive measures) that is common knowledge and affects both the government and citizens' payoffs. To study how citizens behave when they anticipate repression, I assume that harsh punishments increase the frequency of their application. In other words, the government never lacks credibility in enforcing repressive laws. My results show that if the punishment increases, dissidents may become more inclined to dissent even in the face of repression.

There is a lack of consensus on the impact of repression on dissent. Political process theories state that citizens consider the costs, benefits, and likelihood of success when deciding whether to participate (Tilly (1978), McAdam 1982, McCarthy and Zald 1977, Tarrow 1994). As a result, the threat of repression undermines the incentive to join a movement. In contrast, grievance-based approaches view protest as a response to an increase in grievances (Gurr 1970, Lichbach and Gurr 1981, Weede 1987). As grievances increase, people are more inclined to rebel, which in turn raises the likelihood of repression. My model is built around these two theories. A discrete number of citizens with private and independent types decide whether to join a movement while considering the risks and benefits of that decision. An individual type is the private cost they incur from state repression. Individuals also care about the movement success, the likelihood of which increases with the number of participants.

There is a defender (such as the police or minister of the interior) who responds to dissent with repression. The defender prefers to target dissidents and incurs a cost if non-dissidents are repressed. Targeting dissidents provides a private benefit, such as the satisfaction of eliminating a rebel, killing an insurgent, or the private benefit of engaging in torture, kidnapping, or sexual violence. The defender does not observe individual citizen choices. Instead, it can rely on the scale of the attack (defined as the total number of dissidents) or individual-generated signals to update its knowledge of dissidents.

The model highlights novel empirical implications for the repression-dissent nexus. The defender prefers to target dissidents while sparing non-dissidents. Under perfect information, targeting dissidents is a dominant strategy. Due to limited information, the belief about a community member's participation status is crucial for accurate retaliation. The analysis first calculates the defender's posterior belief that an individual is a dissident. An examination of the posterior belief shows that large-scale attacks make the government more likely to believe that any individual living in the community collaborated with or participated in the attack. Since attack size affects retaliation only through belief updating, a large-scale attack raises the probability that an individual is punished by increasing the government's belief that the attack could have implicated the individual as a participant. Consequently, the likelihood of government retaliation increases with both higher punishment and larger attack sizes. A similar argument holds when the government observes individual-specific signals.

In addition, responsive repression with inadequate information can inadvertently target non-dissidents. However, compared to non-dissidents, dissidents always face higher risks of retaliation. Consequently, the strategic decision to join the movement depends on the marginal risk of facing retaliation. The marginal risk of retaliation is the additional risk that an individual expects from participation. The properties of the marginal risk of facing retaliation indicate that it can decline with high punishment. If the government has access to coarse-grained information, this holds under two conditions: (i) *the community is very hostile to the government*; and (ii) *repression structures are convex*. In other words, if there are aggressive types who are committed to challenging the government, they will increase the chances that the government observes a large-scale attack, raising the probability that other members of the community are subjected to retaliation, but at a lower rate if the distribution of the benefit of retaliation is convex. Furthermore, I find that when the government relies on individual-generated signals, the marginal risk of facing retaliation always declines with high punishment, under the convexity assumption.

To spell out the main mechanism, consider both high and low levels of punishment. Joining the movement is always riskier at these two levels of punishment. However, compared to the low level, the additional risk of repression is lower at the high level of punishment under the convexity assumption. Hence, although higher punishment increases the frequency of repression, the marginal risk a dissident expects can decrease. Therefore, people are more inclined to dissent, even though they anticipate that repression will occur.

The convexity of the distribution followed by the private benefit of retaliation captures the inconsistency and erratic nature of state reprisal. The randomness of the benefit of retaliation implies that such decision can be erratic. Moreover, a convex distribution implies that every additional attacks are likely to be met with harsher response. This for example corresponds to the enforcement rule in which the Germans eliminated 100 Serbs for each German killed (Kedward 1993, 181). During the French occupation in Algeria, a similar rule implied 15,000 people killed in Setif for every 100 Europeans killed (Leites and Wolf 1970). The manner in which the state responds after observing a signal is inconsistent and erratic. Historians have argued that this inconsistency is often viewed as a sign of weakness that causes a positive correlation between repression and dissent (Lichbach 1987, Kedward 1993, Leites and Wolf 1970). The model shows that if the state's repressive response is known to be inconsistent and erratic, an announcement that the punishment of dissent will severely increase makes citizens expect the marginal cost of retaliation to wane and wilt. They expect that the government is showing signs of weakness which increase political dissent.

A recent literature argues that joining a rebellion can provide protection when repression is indiscriminate (Kalyvas 2006, Lyall 2009). Participants turn to the rebel organization seeking safety because neutrality can be more costly than rebellion. My model fails to replicate this mechanism. Although repression is also indiscriminate, participation increases the risks of facing retaliation. That is, joining the movement does not necessarily protect participants. Rather than receiving protection from rebels, in my model, citizens are more inclined to dissent as they anticipate that reprisal will be inconsistent and erratic.

The next section is a review of the literature. The model and analysis are presented next. I then examine different implications of the model, analyzing the distinction between fine-grained and coarse-grained information, the role of humanitarian interventions and the optimal level of punishment. The last section concludes.

Related Literature

Groups involved in a movement against a government often strategically self-censor when they expect a repressive response (Ritter and Conrad (2016)). However, empirical results often find repression to increase dissent (Lichbach and Gurr 1981, Fransisco 1996). In fact, the empirical literature is inconclusive on the effect of repression, as it can sometimes be negative (Hibbs 1973).

The literature on backlash also finds a positive relationship between repression and dissent. A backlash effect arises when an increase in repression is followed by an increase in dissent. The mechanism through which dissent responds to repression has often been linked to anger, frustration, emotions, feelings of unfairness, or identity activation (Rasler 1996; Lichbach 1987; Opp and Roehl 1990; Gibilisco 2021; Lebas and Young 2024). For example, repression can deepen grievances by making individuals angry about state actions. Members of a community might view state repression as deeply unfair, leading to a greater willingness to take risky actions. My model suggests that repression and dissent can be positively correlated when responsive repression is employed with inadequate information. Although a punishment raises expected repression, the inconsistency and erratic nature of its application implies a positive effect on political dissent.

The ways in which high punishment affects the frequency of its application as well as the probability of dissent has been previously studied in a game-theoretical framework (e.g. Shadmehr and Bernhardt 2011, Bueno De Mesquita et al. 2023). Both accounts assume (implicitly or not) that repression is conducted under perfect information. First, Bueno De Mesquita et al. 2023 analyzes a framework in which harsh punishment may not be credible, since such punishment can be difficult to implement. They find that this creates a channel where a harsh punishment leads to more resistance as dissidents anticipate that repression will not materialize. Second, Shadmehr and Bernhardt 2011 analyzes a model of collective action in which players are uncertain about the value of successful protest, but only have a noisy observation of it. They find a positive correlation between the likelihood of repression and the probability of dissent. Higher punishment reduces both the likelihood of repression and the incentive to dissent. In contrast to these two accounts, I consider a model in which the punishment is assumed to increase its application in an environment where the repressive government has insufficient information about dissidents.

The model provides a rational explanation for the positive effect of repression on dissent, a mechanism mediated by grievances. This result is echoed in Shadmehr 2014, who finds that income inequality (a measure of grievances) has a U-shaped effect on repression. In contrast, I study the influence of repressive measures on the willingness to dissent.

A similar framework in which deterrence is carried out with limited information is studied by Lagunoff 2024 and Baliga, Bueno De Mesquita, and Wolitzky 2020. Lagunoff 2024 analyzes a dynamic model in which at the beginning of every period a government commits to a compliance rule, and imperfectly observes citizens' act of resistance. In Lagunoff's framework, the population level of resistance is not a statistic the government relies on. Furthermore, citizens have both no private information and no coordination motives. My model also shares some feature with Baliga, Bueno De Mesquita, and Wolitzky 2020. They also study a model of deterrence with imperfect attribution. They analyze a framework where the government faces one attack at a time, while I assume that multiple attackers can attack the government at once. Moreover, in my model, the defender refines its information by observing the scale of the attack. Baliga, Bueno De Mesquita, and Wolitzky (2020) finds that when an individual becomes more aggressive, they become more suspect. As a result, others become less suspicious, which increases their aggression. This strategic complementarity is absent in the paper. In my model, when an individual becomes more aggressive, everyone else becomes more suspicious.

Model

Setup and actions

Figure 1: Citizens' Payoff Matrix.

		Success	Failure
Citizen i	*dissent*	$1 - c_i$	$-c_i$
	Abstain	0	0

The theoretical framework is an extension of the standard model of collective action described in Figure 1. The community has a discrete number $n \geq 2$ of citizens (she). Each citizen has a choice between participating in dissent activities ("dissent") or not ("abstain"). Dissent can take the form of a rebellion, an insurgency, or a protest. A citizen benefits from a successful movement if only if she participates. In this case, she receives a payoff of 1. Participation is also costly. Each citizen i has a private cost c_i, independently distributed on $[\underline{c}, \overline{c}]$. The cost of participation is distributed according to a cumulative distribution function $G(.)$ and a probability density function $g(.)$, continuous on $[\underline{c}, \overline{c}]$.

I interpret the game in Figure 1 as a reduced form of the following three-stage extensive form game. In the first stage, citizens decide whether to dissent; in the second stage, Defender (it) decides whether to retaliate; and in the third stage, the outcomes and payoffs are realized. Defender can be the police or the minister of interior.[3] The standard setup assumes that Defender has perfect information and prefers to retaliate only against dissidents. The second and third stages are therefore straightforward, as the government retaliates against a dissident with a probability of 1. As a result, only dissidents are repressed, and non-dissidents are never targeted. Therefore, by applying backward induction, dissidents face a cost c_i from a retaliation carried out in stage 2. Standard models focus primarily on analyzing the payoff matrix in Figure 1, as stage 2 is straightforward under the assumption of perfect information.

I analyze the extensive form version of the standard model by keeping the same timing but introducing imperfect monitoring. I assume that after stage 1 and before stage 2, certain characteristics of the environment undermine the ability to observe citizens' decisions. That is, Defender moves after the citizens and imperfectly observes the history of the game.

payoffs

Defender prefers to retaliate against dissidents. Retaliation against a dissident i yields $U(s, \tau, y_i)$, while a retaliation against a non-dissident yields $U(s, \tau, y_i) - 1$; where s is the level of dissent in society, or the number of attacks; s is also a measure of the amount of damages caused by dissidents. In the case of a rebellion, s is the size of a rebellion. The private benefit y_i is received from targeting individual i. The parameter τ represents the punishment. Defender incurs a cost of 1 if it makes a mistake and represses a non-dissident.

Important in the model is the punishment intensity (or repressive laws), captured by the parameter $\tau \in [\underline{\tau}, \overline{\tau}] \subset (0, \infty)$, which is common knowledge. With high punishment, more men and more guns are deployed to fight suspected dissidents. Since the goal is to analyze political dissent when dissidents anticipate that repression will materialize, I assume that

3. I use "Defender" and "Government" interchangeably. This should not cause confusion since these two actors represent the same player.

an increase in τ increases Defender's utility from retaliation $U(.)$: $\frac{\partial U}{\partial \tau} > 0$. Moreover, an increase in the size of the attack decreases Defender's payoff: $\frac{\partial U}{\partial s} < 0$.

The component y_i represents Defender's private benefit of targeted retaliation against an individual i. From the citizens' perspective, y_i is distributed on $[0, 1]$ according to a common knowledge CDF $H(.)$, with a positive density $h(.)$. To simplify the analysis and streamline calculations, I work with the uniform distribution on $[0, 1]$. The analysis will also derive the conditions on $H(.)$ that ensure that the results hold.

The shape of the distribution $H(.)$ captures *the manner in which the state responds given an observed signal about dissidents*. Its derivative $h(.)$ measures the structure of state response to an additional attack: how would the marginal response look like if, for example, the number of attacks increases from s to $s+1$? If $H(.)$ is convex, its derivative is a positive and increasing function. Thus, for a convex CDF H, an additional attack (or higher signal) is likely to be confronted with a harsher response. This defines the inconsistency in the structure of state response. For example, this is the case when the ratio of reprisals is 100 Serbs for each German killed. Another example is found during the Algerian rebellion in 1945, where the French settlers killed 15,000 people in Setif as a response to 100 Europeans killed (Leites and Wolf 1970, pg. 112). Therefore, a convex CDF $H(.)$ captures the inconsistency and erratic structure of state response. When citizens have complete information about y, the erratic nature of the structure of state response disappears. As a result, $H(.)$ is a degenerate distribution with all the mass concentrated at one point. Proposition 11 in the Appendix shows that the main result hardly holds if y is common knowledge, and the erratic nature of state response is absent.

Assuming that participation affects the decision to retaliate only through belief updating, I adopt the following specification for the government's payoff: $U(s, \tau, y_i) = v(s) + \frac{\tau}{\overline{\tau}} \times y_i$. Thus, the government's decision yields the following payoffs

$$\begin{array}{lc} \text{Retaliation} & v(s) + \frac{\tau}{\overline{\tau}} \times y_i - 1 \times \mathbb{1}\{\text{i is not a dissident}\} \\ \text{No Retaliation} & v(s). \end{array}$$

Where $v(s)$ is strictly decreasing in s. Defender relies on two modes of inference to update its belief about an individual participation. Defender can use the number of attacks (Section 2). Defender can also monitor citizens (although imperfectly) and as a result, observes an individual-generated signal (Section 3). The posterior belief is captured by the indicator function $\mathbb{1}\{\text{i is not a dissident}\}$. The interaction term $\frac{\tau}{\overline{\tau}} \times y_i$ implies that the government's ability to punish increases with τ. The normalization using $\tau/\overline{\tau}$ ensures that the benefit of retaliation does not exceed the cost. I further assume that $\frac{\overline{\tau}}{\underline{\tau}} \left(\frac{n-1}{n}\right) < 1$, so that Defender has an incentive to retaliate when someone in the community is a dissident. Finally, if Defender does not retaliate, it receives $v(s)$. An important assumption is that Defender's payoff from targeted retaliation is separable from the cost incurred after an attack. Once again, this assumption is guided by the willingness to study the effect of dissent on belief updating.

The payoff for participating in dissent depends on the number of people who participate and the intensity of punishment. The movement succeeds with probability $p(s, \tau)$ and fails with probability $1 - p(s, \tau)$. I assume that $p(,)$ is strictly increasing in s, and strictly decreasing in τ. A high punishment consists of sending more men to establish a curfew or to cordon off the streets. A harsh punishment can undermine the planning of dissenting action. The prospects of movement success then decline. Thus,

$$p(s, \tau) = \frac{\pi(s)}{d(\tau)},$$

where s is the size of attacks, $\pi(s) \in [0, 1]$, and $\pi'(.) > 0$. The function $d(\tau)$ represents the extent to which the punishment inhibits the success of the movement. I make the following simplifying assumptions. I assume that the function $d(.) > 0$ and $d'(.) \geq 0$, with $d'(\underline{\tau}) = 0$ and $d'(\overline{\tau}) = \infty$. I further assume that d'/d is strictly increasing; i.e $d(.)$ is log-convex.[4] Finally, I assume that the type space $[\underline{c}, \overline{c}]$ is rich enough: $G(0) > 0$ and $G(1) < 1$.

Timing

The game has three stages.

1. Citizens observe their private cost. They (simultaneously) decide whether to dissent or abstain.

2. Defender cannot observe the dissent decision, but instead only observes a noisy signal of it. The signal is either the aggregate level of dissent (Section 2), or is individual-specific (Section 3). Defender decides whether to retaliate and against whom to retaliate.

3. Rebellion succeeds with a probability of $p(s, \tau)$, and payoffs are realized. The equilibrium concept is the Perfect Bayesian equilibrium. I look for a symmetric equilibrium.

Model Discussion

To study how inadequate information affects the repression and dissent nexus, the framework departs from the tradition. This is the consequence of tensions between the need for tractability and the need for a model that matches reality.

The level of dissent only affects the decision to retaliate through belief updating. This assumption contrasts with standard models in which the size of the attack directly determines if the government will concede or repress. Because I analyze the impact of having a government with limited information about dissidents, I am only interested in how the information channel (through belief updating) alters decisions.

Moreover, standards model consider the decision to repress or to concede, treating these choices as substitute. However, in so many instances, successful movements are often re-

4. The following function satisfies these conditions: $d(\tau) = exp\left[\frac{(\tau-\underline{\tau})^2}{(\overline{\tau}-\tau)^4}\right]$ for $\tau \in [\underline{\tau}, \overline{\tau}]$.

pressed. People might be injured or even die on the journey towards a successful resistance campaign. According to reporters, the successful Egyptian revolution claimed the lives of at least 846 people. In addition, a political concession may follow weeks of political repression. Authoritarian governments often initially rely on repression to defend the status quo, and sometimes without anticipating a possibility of concession. Because of this observation, I assume that the probability the movement succeeds depends on the functional form, $p(.,.)$, that is arbitrary enough.

There is an important agency problem in the implementation of repression. The private benefit y_i is what the authority in charge of repression receives when following directives. Furthermore, carrying out the order of repression happens weeks before the outcome of mobilized dissent is realized. In addition, the literature often assumes that the attack size renders responsive repression difficult. This is usually termed as "the power in number". In this model, I abstract away from this assumption. Furthermore, I show that it is not needed for the result. What I mean is the following. If the authority's benefit from retaliation is in the form of

$$\frac{\tau}{\overline{\tau}} \times \frac{y_i}{s+1},$$

it strengthens the results. This is because now individuals anticipate that if many of them participate, the authority will be less inclined to retaliate because the payoff from retaliation becomes small. Although this is not assumed, I find a mechanism by which citizens are willing to turnout when they anticipate that repression will materialize.

The payoff matrix in Figure 1 shares a notion of 'pleasure in agency' which captures the selective psychological benefit one enjoys from participating in a successful movement (Wood 2003). Moreover, all types $c_i < 0$ receive an expressive benefit from participation. These are the aggressive types who join the movement to pursue a cathartic release of their grievances

(Correa, Nandong, and Shadmehr 2025). They derive a benefit from participation whether it succeeds or not. In contrast, all types $c_i > 0$ take into consideration the cost, the benefit, and the likelihood the movement succeeds (Tilly 1978).

1 Preliminaries

Perfect Information

I first analyze the case where Defender carries out responsive repression with perfect information. In this setup, it is impossible for dissidents to remain anonymous or conceal their identity. In this respect, Defender's decision to retaliate is contingent on citizens' individual action. The main point in this section is that the punishment τ is negatively associated with dissent. This result is consistent with standard deterrence theories regarding the repression-dissent nexus.

Under perfect information, the probability of retaliation can either be 1 or 0. This is because retaliation against a non-dissident is costly for Defender. The highest benefit cannot exceed the cost of 1. Thus, it is optimal to retaliate only against dissidents. Consequently, absent information friction, the government only applies repression on dissidents. This is stated in the next lemma.

Lemma 1 *Under perfect information, Defender retaliates against an individual if and only if the individual is a dissident.*

The Lemma states that in equilibrium, the probability of retaliation against individual i only depends on the individual's decision, and is given by

$$R_i = \begin{cases} 1 & \text{if individual } i \text{ dissents} \\ 0 & \text{if individual } i \text{ does not.} \end{cases}$$

Citizens' payoff matrix (Figure 1) in the first stage is a standard model of collective action with private and independent cost.

The game has multiple equilibria. However, if the pdf $g(.)$ is flat enough, the first stage has a unique equilibrium described by the threshold c^{BM}. An individual with a private signal c_i prefers to dissent if and only if $c_i \leq c^{BM}$.

Proposition 1 *There exists $\xi_0 > 0$ such that if $g(x) < \xi_0$, the game with perfect information has a unique equilibrium.*

Proof. See the Appendix.

If others participate in dissent activities with probability α, individual i increases the probability that the movement succeeds by 1 if she participates. There exists a threshold c^{BM} at which individual i is indifferent between dissent and abstain. The threshold c^{BM} solves the equation

$$\frac{\mathbb{E}[\pi(s+1)]}{d(\tau)} = c^{BM}.$$

Given that s other individuals participate, individual i's participation makes the movement succeed with probability $\pi(s+1)$. The expected probability that the movement succeeds, when i is a dissident, is given by $P(\alpha) = \mathbb{E}[\pi(s+1)]$. In other words,

$$P(\alpha) = \mathbb{E}[\pi(s+1)] = \sum_{s=0}^{n-1} \pi(s+1) \binom{n-1}{s} \alpha^s (1-\alpha)^{n-1-s}.$$

The number of players other than i who participate is a random variable that follows a binomial distribution due to independence of types and simultaneous decision-making. Thus, s players other than i participate with probability α^s, while the remaining players abstain with probability $(1-\alpha)^{n-1-s}$. Moreover, the binomial distribution $\mathcal{B}(\alpha, n-1)$ dominates the binomial distribution $\mathcal{B}(\alpha', n-1)$ in terms of first-order stochastic dominance if $\alpha > \alpha'$ (see Lemma 7 in the Appendix). Given the assumption that $\pi(.)$ is strictly increasing, the function $P(\alpha)$ is also an increasing function. Therefore, as more individuals participate (i.e., as α increases), the expected probability that the movement succeeds rises, leading to a larger net expected payoff from participation. This implies that there is strategic complementarity in the decision to dissent.

Under the condition $g(x) < \xi_0$, the probability that an individual participates exists and is unique. Moreover, as the punishment increases, individuals become less inclined to dissent. This is stated in Proposition 2.

Proposition 2 *Suppose $g(x) < \xi_0$. The probability of participation in dissent activities is decreasing in the punishment τ: $\frac{d\alpha^{BM}}{d\tau} \leq 0$.*

The proposition demonstrates that political dissent declines with the threat of coercion, measured by the punishment τ. In fact, increasing exogenous repressive measures (τ) reduces an individual's expectation that the movement will succeed. This result follows from the increase in $d(.)$. Moreover, because there is strategic complementarity in the rebel decision, an individual also believes others are less inclined to participate. These two effects reduce the overall probability of participation.

Notations

The central concern in this paper is the question about the relationship between dissent and repression when the government has limited knowledge of dissidents. I assume that Defender relies on a signal x to update his posterior belief. Defender's strategy is now a mapping from a signal space X, to the choice of retaliation. In Section 2, $X = S$ is the set of the observed number of participants $\{0, 1, 2, ...\}$. In Section 3, X is the set of individually generated signals. Let $q(x)$ be the government's posterior belief that individual i is a dissident,

$$q(x) = \Pr[i \textit{ is a dissident} \mid x].$$

Finally, a citizen's strategy is a mapping from their private cost c_i to the participation decision. I look for an equilibrium in symmetric strategies where a citizen with private type c_i dissents if $c_i < c^*$. The equilibrium analysis focuses on deriving the probability of an individual participation, $\alpha = \Pr[c_i \leq c^*]$, the government's posterior belief $q(x)$, and the probability that the government retaliates against individual i, $R(x)$. A Perfect Bayesian Equilibrium is determined by the assessment $(\alpha, R(.), q(.))$. Given the probability α, one can find the symmetric equilibrium threshold adopted by citizens, $G^{-1}(\alpha)$. Similarly, one can reverse engineer the probability of retaliation to derive the government's threshold for retaliation.

2 Imperfect Information: Macro-level Inference.

State repression

I begin by analyzing Defender's decision to retaliate against individual i. Consider the signal space $X = S$, with its elements $s = 0, 1, 2, ...$; where s is the number of attacks. Upon observing s, what is the Defender's posterior probability that i is a dissident? Suppose that the probability of participation in dissent activities is α. The government does not observe the decision to dissent, but will eventually learn α in equilibrium.

When $s = 0$, it is straightforward that $q(0) = 0$. If there is no attack, it is clear that a rational Bayesian will understand that there is no dissident. Similarly, if $s = n$, then $q(n) = 1$. If the size of the attack amounts to the whole population, the government rationally updates that a given citizen is a dissident with probability 1.

Now suppose $s = 1$. The government observes one attack, but cannot identify the person. By Bayesian updating, the government believes that

$$q(1) = \frac{\Pr[s = 1 \mid i \ is \ a \ dissident] \times \Pr[i \ is \ a \ dissident]}{\sum_{k=1}^{n} \Pr[s = 1 \mid k \ is \ a \ dissident] \times \Pr[k \ is \ a \ dissident]} = \frac{\binom{n-1}{0}\alpha(1-\alpha)^{n-1}}{\binom{n}{1}\alpha(1-\alpha)^{n-1}} = \frac{1}{n}.$$

Since a given citizen participates with probability α, any other citizen does not with probability $1 - \alpha$. Thus, the conditional probability of having one dissident given that i is already a dissident in a community of n inhabitants is the probability that none of the remaining $n - 1$ people is a dissident: $\Pr[s = 1 \mid i \ is \ a \ dissident] = (1 - \alpha)^{n-1}$. Moreover, since the events $\{s = 1 \ , \ i \ is \ a \ dissident\}_{\{i \in N\}}$, are independent with equal probability, $\sum_{k=1}^{n} \Pr[s = 1 \mid \ k \ is \ a \ dissident]. \Pr[k \ is \ a \ dissident] = \binom{n}{1}\alpha(1-\alpha)^{n-1}$. Also note that $Card(\{s = 1 \ , \ i \ is \ a \ dissident\})$ is the number of partitions of the set N, with one element.

Now suppose the government observes $s = 2$. One can deduce that

$$q(2) = \frac{\binom{n-1}{1}\alpha^2(1-\alpha)^{n-2}}{\binom{n}{2}\alpha^2(1-\alpha)^{n-2}} = \frac{2}{n}.$$

Citizen i participates with either $j \neq i$, or any other $k \neq i$. Moreover, I use the fact that the events $\{(i,j), s = 2\}_{\substack{\{i,j \in N\} \\ i \neq j}}$ form a partition of N, to apply the law of total probabilities. There are a total of $\binom{n}{2}$ such events and they have equal probability.

Finally, if the Government observes a level $s \geq 3$,

$$q(s) = \frac{\binom{n-1}{s-1}\alpha^s(1-\alpha)^{n-s}}{\binom{n}{s}\alpha^s(1-\alpha)^{n-s}} = \frac{s}{n}.$$

Therefore, in any symmetric equilibrium, the government's posterior belief is a well-behaved function of the size of attack. An important result is that increasing the number of attacks raises the government's belief that an individual living in the community is a dissident. A government with limited knowledge of dissidents always updates positively that a given individual is a dissident when the number of attacks increases. Indeed, in an effort to resolve its Bayesian inference problem, the government may mistakenly attribute the "dissident" label to a non-dissident.

The model highlights a novel logic of indiscriminate repression. Existing accounts often argue that a government that conducts repression under imperfect information will target individuals based on their ethnic background, the language they speak, or their skin color (see Kalyvas (2006), Chap. 6 for a review). The ethnic marker is often highlighted as the variable that helps the government solve its information problem. However, the functional form of the government's posterior belief shows that an alternative mechanism of responsive repression under inadequate information can be based on Bayesian updating. In this mechanism, government forces respond to the size of attacks they face by updating that many individuals in the community must have collaborated. The government always updates upward as the size of the attack increases.[5]

I now analyze the decision to retaliate. Suppose the government observes s attacks. If it retaliates against citizen i, it receives

$$v(s) + \frac{\tau}{\overline{\tau}} \times y_i$$

if citizen i is a dissident and

$$v(s) + \frac{\tau}{\overline{\tau}} \times y_i - 1$$

if citizen i is not; the government receives $v(s)$ if it does not retaliate. Hence, after observing s attacks, it is optimal to retaliate against citizen i if and only if

$$y_i \geq \frac{\overline{\tau}}{\tau}\,(1 - q(s)) = \frac{\overline{\tau}}{\tau}\left(1 - \frac{s}{n}\right),$$

5. See also Rozenas (2020) and Brandsch (2020), which highlight rationality as an explanation for the prevalence of indiscriminate repression in social movements.

for $s \geq 1$. Given the assumption that $\frac{\overline{\tau}}{\underline{\tau}}\left(\frac{n-1}{n}\right) < 1$, it turns out that for any $\tau \in [\underline{\tau}, \overline{\tau}]$ and any s such that $1 \leq s \leq n$, $\frac{\overline{\tau}}{\tau}\left(1 - \frac{s}{n}\right) \in [0, 1]$. The threshold then belongs to the support of y_i.

The government retaliates if the private benefit of targeted retaliation exceeds the probability of making a mistake. If y_i is distributed according to the uniform distribution on $[0, 1]$, Defender is expected to retaliate against citizen i with probability

$$\Pr\left[y_i \geq \frac{\overline{\tau}}{\tau}\left(1 - \frac{s}{n}\right)\right] = 1 - \frac{\overline{\tau}}{\tau}\left(1 - \frac{s}{n}\right) \equiv R(s),$$

for $s \geq 1$. I adopt the convention that $R(0) = 0$, since $q(0) = 0$ and $\frac{\tau}{\overline{\tau}}y_i - 1 \leq 0$.

The next Lemma states that the number of attacks increases the probability of retaliation against citizen i.

Lemma 2 *When a number s of people choose to attack, the probability of retaliation against citizen i is given by $R(s)$. Moreover, the number of attacks strictly increases the probability of retaliation against a given individual: $R(s) < R(s+1)$ for $s < n$.*

An increase in the size of attacks leads to a upward updating that a given individual is a dissident. This upward updating raises the government's incentive to target retaliation. As a result, the probability that it retaliates against a given individual increases. The proposition explains a puzzle that we often observe in conflict zones (rebellion or insurgencies): individuals living in a community suspected of hosting rebels are often exposed to the danger of violence from the government. The probability that individuals face such risks often depends on the frequency with which the government is confronted with rebel attacks. The analysis suggests that an increase in the number of attacks induces the government to update positively about the likelihood that an individual in the community is a rebel.

Finally, it is also worth noting that retaliation increases with the punishment τ. As τ

increases, the threshold $\frac{\overline{\tau}}{\tau}\left(1-\frac{s}{n}\right)$ declines. This implies that the probability of government retaliation increases. This follows from the credibility of the punishment. The probability of government retaliation then increases as a function of two variables: the number of attacks s, and the punishment τ. This is stated in the next lemma.

Lemma 3 *As a function of τ, the probability that the government retaliates against an individual, $R(s,\tau)$, rises as τ increases. That is, an increase in the punishment raises the likelihood that the government inflicts it.*

Political Dissent

I now analyze an individual's decision to dissent. This decision depends on whether the movement succeeds, and on the cost the individual expects to face from a retaliation. Thus, given that others participate with probability α, and under the assumption of independence, a type c_i who participates expects that the probability the movement succeeds is given by

$$\frac{P(\alpha)}{d(\tau)} = \frac{1}{d(\tau)}\left[\sum_{s=0}^{n-1}\pi(s+1)\binom{n-1}{s}\alpha^s(1-\alpha)^{n-1-s}\right].$$

Moreover, the expected cost of facing retaliation also depends on the aggregate number of participants. An individual incurs this cost only if she is targeted. Regardless of participation, the citizen knows she is targeted with some probability, as the government imperfectly observes whether the citizen is a dissident. This probability, on the other hand, depends on the number of participants, because the government's ability to retaliate is influenced by its posterior belief from observing the level of dissent.

A citizen with private cost c_i who participates receives

$$\frac{P(\alpha)}{d(r)} - c_i\sum_{s=0}^{n-1}\Pr[S^i=s]\times R(s+1) \tag{1}$$

while abstaining yields

$$-c_i \sum_{s=0}^{n-1} \Pr[S^i = s] \times R(s), \tag{2}$$

where S^i is the random variable equals to the number of citizens who participate in dissent activities, excluding i; and $R(s)$ is the probability the government retaliates. It is important to note that a dissident expects a higher probability of retaliation, compared to a non-dissident. Thus, even if repression is indiscriminate, its magnitude depends on participation.

An individual receives $P(\alpha)/d(r)$ when the movement succeeds. But participation brings an expected cost that depends on the expected probability of retaliation. If s is the size of attack excluding individual i, with probability $\Pr[S^i = s] \times R_i(s+1)$ a dissident faces retaliation (Equation (1)). Moreover, a non-dissident can pay a cost c_i if they face retaliation by mistake. This occurs depending on the size of attacks, excluding i (Equation (2)). A natural implication is that a citizen can become a suspect just because dissidents happen to live few blocks away.

A dissident has a net expected payoff given as

$$\frac{P(\alpha)}{d(r)} - c_i \sum_{s=0}^{n-1} \Pr[S^i = s] \left(R(s+1) - R(s)\right) \tag{3}$$

The term $R(s+1) - R(s)$ is the marginal probability of facing retaliation with respect to the size of attack. This measures the additional increase in the probability of government retaliation as the number of attacks increases by one. The marginal probability of government retaliation (with respect to the number of attacks) is given by

$$\begin{cases} R(s+1) - R(s) = \frac{\overline{\tau}}{n\tau} & \text{if} \quad s \geq 1 \\ R(1) - R(0) = 1 - \frac{\overline{\tau}}{\tau}\left(\frac{n-1}{n}\right) & \text{if} \quad s = 0. \end{cases} \tag{4}$$

Expression (4) states that the marginal probability of inflicting a punishment declines with the punishment when the size of the attack is not too small ($s \geq 1$). However, if the size of the attack is too small ($s = 0$), the marginal probability of applying a given punishment τ rises with the punishment. That is, although an increase in the punishment raises the probability that the government punishes (Lemma 3), its marginal infliction depends on the level of dissent in the community. If the number of attacks is very small, a harsh punishment not only increases the probability that the government applies such punishment but also raises its marginal application. However, if many people participate, the likelihood of applying the punishment increases with the punishment, but at a decreasing rate.

This observation is equivalent to the notion of ”strength in numbers” that appears in almost every classical model of collective action. In the standard tradition, a large size of attack undermines reliance on repression because such size directly incapacitates the authority. It becomes difficult for the police to suppress protests when hundreds of thousands of people occupy the streets. I have deliberately ruled out this effect by assuming that the size of the attack affects repression only through belief updating. However, I still obtain a result with a similar flavor. A large size of attack makes retaliation difficult, but only in marginal terms. Such large-scale attack reduces the rate at which the government retaliates. With a large size of attack, an increase in punishment reduces its marginal use.

And the marginal probability of retaliation (with respect to the number of attacks) is key to citizens' strategic decisions. To understand why, note that for a citizen, the marginal expected probability of facing retaliation is given by

$$
\begin{aligned}
\sum_{s=0}^{n-1} \Pr[S^i = s]\,(R(s+1) - R(s)) &= \sum_{s=1}^{n-1} \Pr[S^i = s]\,(R(s+1) - R(s)) + \Pr[S^i = 0](R(1) - R(0)) \\
&= \left(1 - (1-\alpha)^{n-1}\right) \frac{\overline{\tau}}{n\tau} + (1-\alpha)^{n-1} \left(1 - \frac{\overline{\tau}}{\tau}\left(\frac{n-1}{n}\right)\right) \\
&= \frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-\alpha)^{n-1}\right) + (1-\alpha)^{n-1}.
\end{aligned}
$$

The last equality represents a participant's net probability of facing retaliation. Thus, the net cost a participant with private type c_i anticipates is given by

$$
c_i \times \left[\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-\alpha)^{n-1}\right) + (1-\alpha)^{n-1}\right]. \tag{5}
$$

How the net probability of facing retaliation responds to the punishment τ depends on the number of people who participate. The number of people expected to participate is measured by α. Such dependence can be observed by the term $\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-\alpha)^{n-1}\right)$. As the next lemma shows, if a good fraction of aggressive types, players with a private cost $c_i \leq 0$, live in the community, any individual expects the net probability of facing retaliation to be a decreasing function of the punishment τ. This is stated in the next lemma.

Lemma 4 *Suppose $G(0) > 1 - \left(\frac{1}{n}\right)^{\frac{1}{n-1}}$. That is, a good fraction of aggressive types live in the community. Then, the net probability of facing a punishment is a decreasing function of τ:*

$$
\frac{\partial}{\partial \tau}\left[\sum_{s=0}^{n-1} \Pr[S^i = s]\,(R(s+1) - R(s))\right] < 0.
$$

If aggressive types are members of the community, the government is less likely to observe a size of attack that is too small (i.e $(1-\alpha)$ is small). Hence, by expression (4), the likelihood of applying a punishment increases, but at a lower rate. Because an increase in punishment reduces its marginal use, any citizen expects the net probability of facing a retaliation to decline with the punishment. That is, as the punishment increases the marginal cost of participation declines even though the probability the government applies the punishment increases.

Ironically, the presence of aggressive types in the community somehow mitigates the amount of (marginal) retaliation faced by participants as the punishment rises. It is important to note that aggressive types who are committed to fighting the government turn any citizen in the community into a suspect. Because the government has imperfect information about dissidents, the presence of aggressive types increases the probability an innocent is targeted. However, aggressive types also minimize the probability the government observes a size of attack that is too small. Since with these dominant types in the model, the government becomes likely to observe a size of attack greater than 1, the entire community starts expecting that an increase in the punishment reduces its marginal application (expression (4)). As I show next, this effect transfers a force that boosts participation even though individuals anticipate harsh repression to materialize.

A dissident has a net expected payoff given by

$$\frac{P(\alpha)}{d(r)} - c_i \left(\frac{\overline{\tau}}{\tau} \left(\frac{1}{n} - (1-\alpha)^{n-1} \right) + (1-\alpha)^{n-1} \right).$$

An individual with a private cost c_i participates if the private cost is smaller than

$$\frac{P(\alpha)}{d(\tau) \left(\frac{\overline{\tau}}{\tau} \left(\frac{1}{n} - (1-\alpha)^{n-1} \right) + (1-\alpha)^{n-1} \right)} \equiv c(\alpha).$$

The threshold $c(\alpha)$ represents an individual's best response function to other individuals' decision to rebel, captured by the probability α. A symmetric equilibrium in which individuals participate with probability α must satisfy the indifference condition

$$\alpha = G\left(\frac{P(\alpha)}{d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha)^{n-1}\right)+(1-\alpha)^{n-1}\right)}\right),$$

where the equation states that the equilibrium probability with which an individual participates (α) should be equal to the probability that the individual has a private cost lower than the marginal type $c(\alpha)$.

Under the condition that $g(.)$ is flat enough there exists a unique probability of participation $\alpha^*(\tau)$. In fact there exists $\xi > 0$ such that if $g(x) < \xi$ then the game has a unique equilibrium.

I state the next proposition.

Proposition 3 *There exists $\xi > 0$ such that if $g(z) < \xi$ for all $z \in [\underline{c}, \overline{c}]$, then the game has a unique perfect Bayesian equilibrium.*

In this model, equilibrium uniqueness is entirely driven by the fact that individuals coordinate on the unique equilibrium in the first stage. The direct consequence is that the relationship between the state punishment and the level of participation is robust. The next proposition investigates the effect of high punishment on incentives to participate.

Proposition 4 *Suppose $g(z) < \xi$ and that aggressive types live in the community. Then, the probability an individual participates in dissent activities is a non-monotonic function of τ. In order words, there exists a unique $\tau^* \in (\underline{\tau}, \overline{\tau})$ such that for any $\tau \leq \tau^*$ the equilibrium probability $\alpha^*(\tau)$ is increasing. Furthermore, for $\tau > \tau^*$, the equilibrium probability decreases.*

Proof: In the Appendix.

The consequence is that citizens support for the movement has a non-monotonic relationship with the punishment. $\alpha^*(\tau)$ first increases as τ increases while remaining below τ^*. The equilibrium level of rebellion then declines as τ increases above τ^*. Therefore, only harsh punishment can help to recover the standard deterrence logic.

The punishment, measured by the parameter τ, affects an individual's expected payoff from participation in two opposing ways: first, through the probability movement succeeds, $p(s, \tau)$, and second, through the net probability of facing retaliation, expression (5). The effect on the likelihood of success is negative as described above. Hence, an increase in such punishment reduces the expected probability that the movement succeeds. Moreover, the effect on the net probability of facing retaliation is also negative, if there are aggressive types. As explained extensively in the previous paragraphs, an increase in τ reduces its marginal use for large-scale attacks.

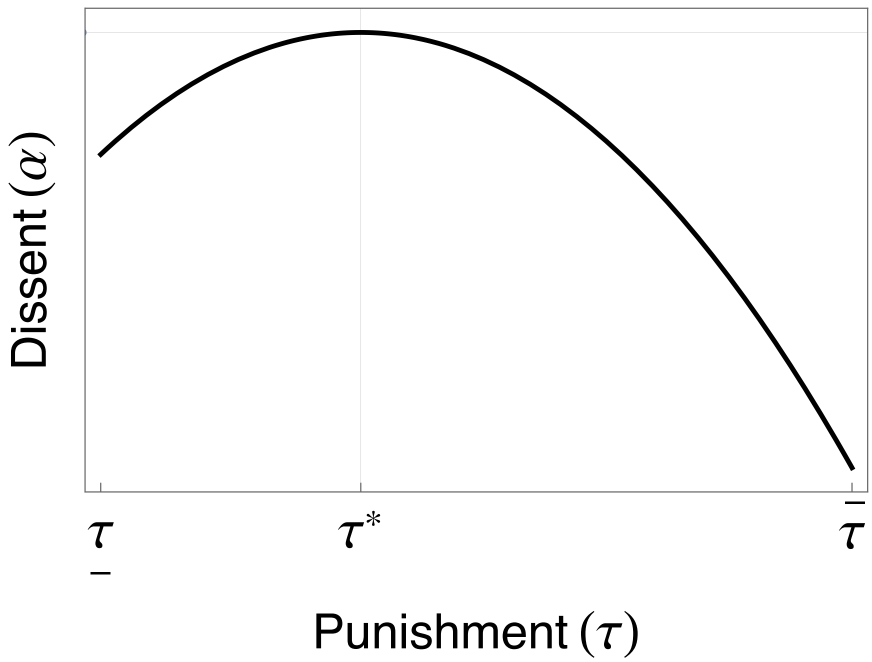

Figure 2: Punishment with Imperfect Information

When the punishment is small (below τ^*) the second effect dominates, and increasing τ has a strong effects on the net probability of facing a retaliation. As the net probability of facing a retaliation declines, individuals become more inclined to join the movement, when the movement still has good chance to succeed. However, when τ is large ($\tau \geq \tau^*$) the first effect dominates. The effect of the punishment works through its effect on the likelihood of success, as it becomes difficult for the movement to achieve success. This undermines individuals' incentives to participate in dissent activities. In this respect, credible punishments must be extremely harsh to negate participation in the movement when the government has imperfect information. This dynamic is presented in Figure 2.

I now demonstrate that in this framework there is a positive relationship between dissent and responsive repression even if dissidents anticipate repression to materialize. I first derive a measure of anticipated repression. Since citizens decide simultaneously with private information, it is difficult to know for sure the size of attack the government will observe. As a result, citizens only form an expectation of the anticipated level of repression. For each punishment τ, denote $\overline{R}(\tau, \alpha^*)$ the anticipated level of repression in equilibrium. The anticipated probability of facing repression is given by

$$\overline{R}(\tau, \alpha^*) = \sum_{k=0}^{n} R(k, \tau) \Pr[s = k \mid \alpha^*]. \tag{6}$$

The term $\Pr[s = k \mid \alpha^*]$ is the probability that the size of attack is k when the equilibrium probability of participation is α^*. It turns out

$$\begin{aligned}
\overline{R}(\tau, \alpha^*) &= \sum_{k=0}^{n} \left[1 - \frac{\overline{\tau}}{\tau}\left(1 - \frac{k}{n}\right)\right] \Pr[s = k \mid \alpha^*] \\
&= 1 - \frac{\overline{\tau}}{\tau}\left(1 - \frac{1}{n}\sum_{k=0}^{n} k \Pr[s = k \mid \alpha^*]\right) \\
&= 1 - \frac{\overline{\tau}}{\tau}(1 - \alpha^*).
\end{aligned}$$

Where the last equality uses the fact that the binomial distribution with parameter (α^*, n) has an expected value given by $n\alpha^*$. Suppose that all the conditions of Proposition 4 are satisfied. Then we have the next proposition.

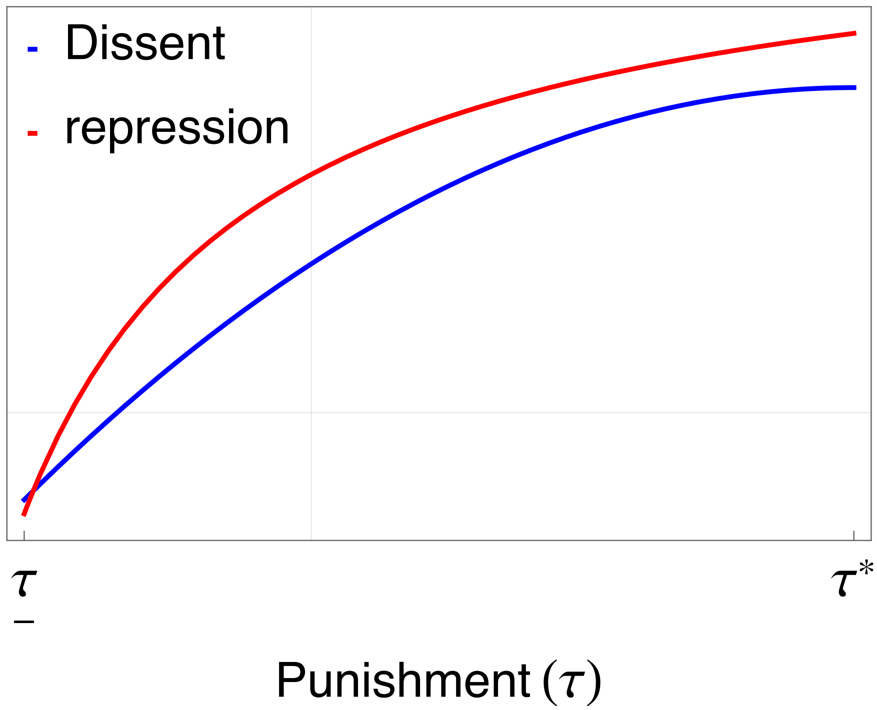

Figure 3: Repression and Dissent

Proposition 5 *Suppose the punishment τ is less than τ^*. Then, the anticipated level of repression increases with dissent. In other words, $\frac{d\alpha^*(\tau)}{d\tau} > 0$ and $\frac{d\overline{R}(\tau,\alpha^*)}{d\tau} > 0$ as well.*

It has already been demonstrated that when $\tau \leq \tau^*$, $\frac{d\alpha^*(\tau)}{d\tau}$ is positive. The proof of Proposition 5 rests on showing that it is also the case for $\overline{R}(\tau, \alpha^*)$. By inspection, it turns out that anticipated repression increases with punishment because a derivation with respect to τ yields

$$\frac{\overline{\tau}}{\tau^2}(1-\alpha^*) + \frac{\overline{\tau}}{\tau}\frac{d\alpha^*}{d\tau} > 0 \quad \text{for} \quad \tau \leq \tau^*.$$

What is the intuition behind this result? The paper now highlights the role of repression structures. I show that this result holds as long as the structure of repression measured by the CDF H is (weakly) convex.[6] Suppose the benefit from retaliation follows the distribution $H(.)$. Therefore, if $s \geq 1$, the probability the government retaliates is

6. The influence of repression structures is also analyzed in Morris and Shadmehr 2024. They study how repression structures affects the range of repertoire of collective actions.

$$R(s) = \Pr\left[y_i \geq \frac{\overline{\tau}}{\tau}\left(1 - \frac{s}{n}\right)\right] = 1 - H\left(\frac{\overline{\tau}}{\tau}\left(1 - \frac{s}{n}\right)\right).$$

The equivalent of expression (4) becomes

$$\begin{cases} R(s+1) - R(s) = H\left(\frac{\overline{\tau}}{\tau}\left(1 - \frac{s}{n}\right)\right) - H\left(\frac{\overline{\tau}}{\tau}\left(1 - \frac{s+1}{n}\right)\right) & \text{if } s \geq 1 \\ R(1) - R(0) = 1 - H\left(\frac{\overline{\tau}}{\tau}\left(\frac{n-1}{n}\right)\right) & \text{if } s = 0 \end{cases} \tag{7}$$

The next lemma shows that if $H(.)$ is (weakly) convex, then the marginal risk of retaliation declines with the high punishment, as long as $s \geq 1$.

Lemma 5 *Suppose repression structures are convex. In other words, $H(.)$ is (weakly) convex. Then for large-scale attacks ($s \geq 1$), the marginal risk of retaliation decreases with high punishment. However, for small-scale attacks ($s = 0$) the marginal risk of retaliation increases with high punishment.*

Proof:

For small-scale attacks ($s = 0$), it is clear that the marginal risk of retaliation increases with high punishment. Let's assume that $s \geq 1$. If H is weakly convex, then $h(.)$ is weakly increasing.

$$\begin{aligned} \frac{\partial}{\partial \tau}\left[R(s+1) - R(s)\right] &= \frac{\partial}{\partial \tau}\left[H\left(\frac{\overline{\tau}}{\tau}\left(1 - \frac{s}{n}\right)\right) - H\left(\frac{\overline{\tau}}{\tau}\left(1 - \frac{s+1}{n}\right)\right)\right] \\ &= \frac{\overline{\tau}}{\tau^2}\left[\left(1 - \frac{s+1}{n}\right) h\left(\frac{\overline{\tau}}{\tau}\left(1 - \frac{s+1}{n}\right)\right) - \left(1 - \frac{s}{n}\right) h\left(\frac{\overline{\tau}}{\tau}\left(1 - \frac{s}{n}\right)\right)\right] \\ &< 0 \quad \text{because } h \text{ is a (weakly) increasing function.} \end{aligned}$$

Q.E.D

Therefore, an individual expected marginal risk of retaliation becomes

$$\sum_{s=0}^{n-1}\left[H\left(\frac{\overline{\tau}}{\tau}\left(1-\frac{s}{n}\right)\right)-H\left(\frac{\overline{\tau}}{\tau}\left(1-\frac{s+1}{n}\right)\right)\right]\binom{n-1}{s}\alpha^s(1-\alpha)^{n-1-s}.$$

By Lemma 5, the marginal risk of facing retaliation decreases with large-scale attacks. The uniform distribution is then a special case. If repression structures are strictly concave, then the results remain ambiguous. It becomes difficult for a large-scale attack to reduce the marginal use of repression.

In the next proposition, I derive a condition on aggressive types that ensures that even with an arbitrary distribution $H(.)$, an increase in credible punishment can increase dissent. For the proposition, denote $\underline{h} = \inf_{u\in[0,1]} h(u)$ and $\overline{h} = \sup_{u\in[0,1]} h(u)$. Hence, with a uniform distribution on $[0,1]$, $\underline{h} = \overline{h} = 1$.

Proposition 6 *Suppose* $G(0) > 1 - \left(\frac{\underline{h}}{\underline{h}+(n-1)\overline{h}}\right)^{\frac{1}{n-1}}$. *Further assume that* $g(c) < \xi$ *for* $c \in [\underline{c}, \overline{c}]$. *There exists* τ^*, *such that for* $\tau \leq \tau^*$, $\frac{d\alpha^*}{d\tau} > 0$.

A convex cdf H measures the inconsistency and erratic nature of the state response. Lichbach 1987 argues for this inconsistency in the government's response to lead to a positive relationship between repression and dissent. According to Leites and Wolf 1970 (p.g. 108), the less complete an enforcement rule, the lower the compliance. This is mainly because of the impression of weakness such repression structure generates. Kedward 1993 (pg 181) explains that "there was no consistency in the German response to acts of armed resistance which allows for a meaningful correlation between different kinds of maquis action and incidence of reprisals". In this respect, political dissent and repression with limited information can be positively associated. This is may not be because this form of repression is often indiscriminate and targets individuals based on ethnicity (Blaydes 2018), or because of the ambiguous structure of incentives in which neutrality is more costly than participation (Kalyvas 2006, Lyall 2009).

Humanitarian Intervention and International Pressure

In the initial analysis, the only force preventing Defender from retaliating is the disutility derived from targeting innocents. However, governments often face additional restrictions that protect citizens from state atrocities. It is common for international institutions to implement measures that limit the use of repression against dissidents. Membership in international institutions can also exert pressure on repressive governments. In this context, I consider the role of third-party intervention that diminishes the benefits of targeting repression. Specifically, I analyze the effect of increasing $\overline{\tau}$ on dissent and retaliation. Since the government receives $\frac{\tau}{\overline{\tau}} y_i$ from targeting retaliation, increasing $\overline{\tau}$ limits the use of repression.

The literature is divided on the net benefits of these policies (Shadmehr and Boleslavsky 2022; Evans 2011; Kydd and Straus 2013; Grigoryan 2010; Hafner-Burton and Tsutsui 2005). Membership in a human rights treaty or a humanitarian intervention, can impel a country from using coercion less often. However, these policies are sometimes associated with increased state violence (Hafner-Burton and Tsutsui 2005; Shadmehr and Boleslavsky 2022; Grigoryan 2010). Policy measures that restrict retaliation against dissidents can also strengthen rebel organizations, undermining the intended effect of these policies (Kydd and Straus 2013). Supporters of policy intervention argue for a negative relationship between state violence and third-party intervention (Evans 2011). The model can be used to engage with this debate.

The current paper highlights a novel mechanism by which a humanitarian intervention might generate the opposite of its intended effects. As described in the next proposition, an increase in $\overline{\tau}$ may not necessarily decrease repression.

Proposition 7 *Suppose the community is hardly hostile to the government. Further assume that $G(1) < 1 - \left(\frac{1}{n}\right)^{\frac{1}{n-1}}$. Then, the relationship between $\overline{\tau}$ and $\overline{R}(\alpha^*)$ is ambiguous.*

The model highlights an informative effect through which third-party intervention can increase violence. With an increase in restrictions, the marginal risk of facing retaliation declines if the community is hardly hostile to the government:

$$\frac{\partial}{\partial \overline{\tau}}\left[\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha)^{n-1}\right)+(1-\alpha)^{n-1}\right]<0.$$

As the number of aggressive types is small, the expected probability of participation is also small, making $1-\alpha^*$ likely higher than $1/n$. Therefore, higher restrictions empower dissidents, which increases participation in dissent activities. Furthermore, since higher levels participation improve the Defender's knowledge of dissidents, retaliation can either increase or decrease depending on other factors.

Optimal Punishment

This section examines the punishment policy a government is inclined to invest in. I assume that such an investment is made early in the game, before the dissent stage. The non-monotonic association between punishment and dissent raises an important question: what's the punishment policy that minimizes the number of attacks? The government selects a punishment intensity τ to minimize the function $\alpha^*(\tau)$ at some cost $C(\tau)$. The government's problem is summarized as

$$\min_{\tau\in[\underline{\tau},\overline{\tau}]} \alpha^*(\tau)+C(\tau), \tag{8}$$

where the function $C(.)$ is such that $C'(\underline{\tau})=C(\underline{\tau})=0$ and $C''(.)>0$.

The next proposition shows that there exists an optimal punishment policy $\tau^{opt}\in(\underline{\tau},\overline{\tau})$ that solves the government's problem (8).

Proposition 8 *Consider the conditions that ensure a unique equilibrium. Denote* $\widetilde{r}$ *such that* $\frac{d\alpha^*(\widetilde{\tau})}{d\tau}+C'(\widetilde{\tau})=0$. *There exists* κ *such that if* $C(\overline{\tau})<\kappa$, *then problem* (8) *has a solution*

$r^{opt} \in (\widetilde{r}, \overline{r}]$. *Otherwise,* $r^{opt} = \underline{\tau}$.

It is important to note that although the size of the attack is a nonmonotonic function of τ, it is always the case that individuals participate more when the punishment is at the minimum ($\tau = \underline{\tau}$), compared to the maximum level of punishment $\tau = \overline{\tau}$. Once the punishment increases from $\underline{\tau}$ to τ^*, the government's objective function increases as well. This is partly due to the positive correlation between repression and dissent. Since $\lim_{\tau \to \overline{\tau}} \frac{d\alpha^*(r)}{d\tau} = -\infty$, there exists $\widetilde{\tau} \geq \tau^*$ such that the government's objective function reaches the maximum. At the point $\tau = \widetilde{\tau}$, one obtains $\frac{d\alpha^*(r)}{d\tau} + C'(\tau) = 0$. Above $\widetilde{r}$, the objective function can have multiple extrema moving towards $\alpha^*(\overline{\tau}) + C(\overline{\tau})$. Unless the maximum punishment cost is high ($C(\overline{\tau}) \geq \kappa$), a punishment level $r^{opt} \leq \overline{\tau}$ and $r^{opt} > \widetilde{r}$ is a solution to the problem 8. Therefore, harsh repressive measures are optimal.

3 Imperfect Information: Individual-Specific Signals

The analysis so far has relied on the assumption that the government learns about individual participation at the macro level. This assumption is now relaxed in a setup where signals are individual-specific. With the reliance on advanced technology, the police often monitors citizens, and as a result, can observe a noisy, and more precise signal about the individual's behavior (Nandong 2025). In light of this fact, I assume that Defender has fine-grained information about individual players.

I show that the result is stronger when Defender observes individual-specific signals. In this setup, a player's risk of repression is independent of the group's aggregate behavior. The risk of repression is mainly a function of the individual's behavior itself. This point is surprising, as there are reasons to believe that the main result can be overturned if Defender's information were more precise. First, Defender is more informed about individual behavior compared to the previous setup. Second, since the risk of repression depends more on individual behavior and less on group behavior, we can no longer rely on aggressive types to stimulate high-level participation. I find that even though the government has fine-grained information, high punishment that increases repression always reduces the marginal risk of retaliation, under the convexity assumption.

The setup assumes that when an agent chooses action $a_i \in \{0,1\}$, Defender observes a private signal $x_i = a_i + \epsilon$; where ϵ is an integer, has mean 0, and variance $\sigma \in (0,1)$, and is distributed according to

$$f(x_i \mid a_i) = \begin{cases} \frac{\sigma}{2} & \text{if } x_i = a_i + 1 \\ \frac{\sigma}{2} & \text{if } x_i = a_i - 1 \\ 1-\sigma & \text{if } x_i = a_i. \end{cases}$$

Therefore, $x \in \{-1,0,1,2\}$. I further assume that σ is such that $\frac{f(x|1)}{f(x|0)}$ is strictly increasing in x. It is easy to show that $\frac{f(x|1)}{f(x|0)}$ satisfies the Monotone Likelihood Ratio if and only if $\sigma \in (0, \frac{2}{3})$.[7] Intuitively, the two variables a and x are affiliated implies that higher value of x are likely to be associated with higher values of a. Finally, I assume that the benefit of retaliation is distributed according to a distribution $H(.)$ on $[0,1]$.

If we fix the equilibrium probability of participation α, Defender's posterior belief is given by

$$q(x) = \begin{cases} 0 & \text{if } x = -1 \\ \frac{\alpha\sigma}{\alpha\sigma + 2(1-\alpha)(1-\sigma)} & \text{if } x = 0 \\ \frac{2\alpha(1-\sigma)}{2\alpha(1-\sigma)+\sigma(1-\alpha)} & \text{if } x = 1 \\ 1 & \text{if } x = 2. \end{cases}$$

For different reasons, repression remains indiscriminate in this model. A non-dissident

7. We only need to check if $\frac{f(0|1)}{f(0|0)} < \frac{f(1|1)}{f(1|0)}$. That is, if $\frac{\sigma}{2(1-\sigma)} < \frac{2(1-\sigma)}{\sigma}$.

can be targeted mainly due to the noise in the government-observed signal. Therefore, regardless of how many people are involved in a recent attack a nondissident is targeted if $\epsilon \in \{0, 1\}$. Thus, in contrast to the previous setting, an individual needs not worry about the aggregate size of the attack when estimating the risks of repression.

It turns out that when Defender observes a signal $x = -1$, it never retaliates. Thus, $R(-1) = 0$. This is because a signal $x = -1$ can only be generated by a nondissident. Defender retaliates after observing a signal of $x \in \{0, 1\}$ with probability

$R(x) = \Pr\left[y_i \geq \frac{\overline{\tau}}{\tau}(1 - q(x))\right] = 1 - H\left(\frac{\overline{\tau}}{\tau}(1 - q(x))\right)$. Moreover, when $x = 2$, Defender always retaliates; $R(2) = 1$. A direct implication is that as x increases, the repression function $R(x)$ increases as well. Furthermore, the risk of retaliation for an individual who abstains is given by

$$\sum_x R(x)f(x \mid a = 0) \quad = \quad (1 - \sigma)R(0) + \left(\frac{\sigma}{2}\right) R(1).$$

For a dissident, it is given by

$$\sum_x R(x)f(x \mid a = 1) \quad = \quad \left(\frac{\sigma}{2}\right) R(0) + (1 - \sigma)R(1) + \left(\frac{\sigma}{2}\right) R(2)$$

The marginal risk of retaliation with respect to participation is the difference between the risk of retaliation for a participant and a non-participant.

$$\begin{aligned}
\sum_x R(x)\left(f(x \mid a=1)-f(x \mid a=0)\right) &= \left(\frac{3}{2}\sigma-1\right)\left[1-H\left(\frac{\overline{\tau}}{\tau}\left(1-q(0)\right)\right)\right] \\
&+ \left(1-\frac{3}{2}\sigma\right)\left[1-H\left(\frac{\overline{\tau}}{\tau}\left(1-q(1)\right)\right)\right]+\frac{\sigma}{2} \\
&= \frac{\sigma}{2}+\left(1-\frac{3}{2}\sigma\right)\left[H\left(\frac{\overline{\tau}}{\tau}\left(1-q(0)\right)\right)-H\left(\frac{\overline{\tau}}{\tau}\left(1-q(1)\right)\right)\right].
\end{aligned}$$

Lemma 6 *Suppose repression structures are convex. When Defender observes individual-specific signals, for each citizen, the marginal risk of repression is always decreasing with high punishment.*

Proof:

$$\begin{aligned}
&\tfrac{\partial}{\partial\tau}\left[\sum_x R(x)\left(f(x \mid a=1)-f(x \mid a=0)\right)\right] \\
&= \tfrac{\overline{\tau}}{\tau^2}\left(1-\tfrac{3}{2}\sigma\right)\times\left[(1-q(1))h\left(\tfrac{\overline{\tau}}{\tau}\left(1-q(1)\right)\right)-(1-q(0))h\left(\tfrac{\overline{\tau}}{\tau}\left(1-q(0)\right)\right)\right]
\end{aligned}$$

Since $q(1)>q(0)$, if h is (weakly) increasing,

$$\frac{\partial}{\partial\tau}\left[\sum_x R(x)\left(f(x \mid a=1)-f(x \mid a=0)\right)\right]<0.$$

Q.E.D

Thus, if state repressive response is inconsistent and erratic, a harsh punishment that increases repression also reduces the individual expected marginal risk of dissenting. I show that there exists τ_* such that participation increases with high punishment if $\tau<\tau_*$ and decreases if $\tau\geq\tau_*$.

When $H(.)$ is the uniform distribution, the marginal risk of retaliation is

$$\sum_x R(x)\left(f(x \mid a=1)-f(x \mid a=0)\right)=\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\left(\frac{\left(1-\frac{3}{2}\sigma\right)\left(2\left(\frac{1-\sigma}{\sigma}\right)-\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)}{\left(\frac{\alpha}{1-\alpha}+\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)\left(\frac{\alpha}{1-\alpha}+2\left(\frac{1-\sigma}{\sigma}\right)\right)}\right)$$
$$=\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha,\sigma)$$

where $\mathbf{Q}(\alpha,\sigma)=\left(\frac{\left(1-\frac{3}{2}\sigma\right)\left(2\left(\frac{1-\sigma}{\sigma}\right)-\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)}{\left(\frac{\alpha}{1-\alpha}+\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)\left(\frac{\alpha}{1-\alpha}+2\left(\frac{1-\sigma}{\sigma}\right)\right)}\right)$.

Given that other individuals participate with probability α, an individual player has a net expected payoff from participation written as

$$\frac{P(\alpha)}{d(\tau)}-\left(\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha,\sigma)\right)\times c_i.$$

Then an equilibrium α_* must be solution to

$$\alpha=G\left(\frac{P(\alpha)}{d(\tau)\left(\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha,\sigma)\right)}\right).$$

I enunciate the next two propositions.

Proposition 9 *There exists $\xi_1>0$ such that if $g(z)<\xi$ for $z\in[\underline{c},\overline{c}]$, the game has a unique equilibrium.*

Proposition 10 *Suppose $g(z)<\xi_1$ for $z\in[\underline{c},\overline{c}]$. There exists $\tau_*\in(\underline{\tau},\overline{\tau})$ such that the equilibrium probability of participation α_* increases if $\tau<\tau_*$, and decreases if $\tau\geq\tau_*$.*

Thus, the main result of this paper is preserved even if the government receives fine-grained information about dissidents. An increase in punishment has a positive effect on participation under the assumption of convexity. This is true even absent credibility concerns in the implementation of repression. As punishment increases, repression becomes more likely. The fact that state response is erratic and inconsistent makes individuals expect the marginal risk of retaliation to decrease with high punishment.

Conclusion

This paper examines a political environment in which a government carries out repression to suppress dissent, but has limited knowledge of dissidents. I investigate the role of two methods of inductive inference: micro and macro level inference. Each of these setup shows that political dissent has the potential to increase with repression. A sufficient condition to ensure this result holds is that state repression must be erratic and inconsistent.

The model then provides a rational explanation for the often observed positive association between repression and dissent. If the aggregate level of dissent is the source of information about dissidents, a punishment that increases repression can increase dissent in a community that is very hostile to the government. Moreover, if the government observes individual-specific signals, repression and dissent are positively associated. The analysis also highlights the role of the government repression structure, as these results are more likely when repression structures are convex. I argue that convex repression structures capture the inconsistency and erratic nature of state reprisal. Furthermore, in analyzing the role of third-party intervention in limiting state violence, I find that it can be ineffective, since political dissent can increase as a result. Suppressing dissent with limited information also requires a government to rely more on harsh measures relative to soft ones.

Appendix

Lemma 7 *Consider two binomial distributions* $\mathcal{B}(\alpha, m)$ *and* $\mathcal{B}(\alpha', m)$*. Let* $b(k, \alpha)$ *and* $b(k, \alpha')$ *be their respective pdf, where* $k \leq m$*. If* $\alpha' > \alpha$*, then* $\frac{b(k,\alpha')}{b(k,\alpha)}$ *increases in* k*.*

Proof.

$$\begin{aligned}\frac{b(k,\alpha')}{b(k,\alpha)} &= \frac{\binom{m}{k}\alpha'^k(1-\alpha')^{m-k}}{\binom{m}{k}\alpha^k(1-\alpha)^{m-k}} \\ &= \left[\left(\frac{\alpha'}{\alpha}\right)\left(\frac{1-\alpha}{1-\alpha'}\right)\right]^k\left(\frac{1-\alpha'}{1-\alpha}\right)^m\end{aligned}$$

which is increasing in k because $\left(\frac{\alpha'}{\alpha}\right)\left(\frac{1-\alpha}{1-\alpha'}\right) > 1$. *Q.E.D*

Proof Lemma 1

It is clear that the government's belief is either 1 or 0. If individual i is not active, the government's receives $v(s) + \frac{\tau}{\overline{\tau}}y_i - 1$ if it retaliates against individual i. If the government does not retaliate it receives $v(s)$. Since $\frac{\tau}{\overline{\tau}}y_i - 1 < 0$, the government prefers not to retaliate against an individual known to be a non-dissident. However, if individual i is a dissident, the government receives a net payoff of $\frac{\tau}{\overline{\tau}}y_i > 0$. Hence, the government is better off targeting only dissidents.

Q.E.D

Proof Proposition 1

A citizen with private cost c_i has a net expected payoff $\frac{\mathbb{E}[p(s+1)]}{d(\tau)} - c_i$ from dissenting. A non-dissident receives 0. It is clear that the best response to a strategy α^{BM} adopted by others is a monotone strategy described by the cut-point c^{BM}. An individual with private cost c_i is a dissident if $c_i \leq c^{BM}$.

Given that others adopt a threshold c^{BM}, the probability of dissent is given by $\alpha^{BM} = \Pr[c_i \leq c^{BM}]$. α^{BM} solves the equation

$$\alpha^{BM} = G\left(\frac{P(\alpha^{BM})}{d(\tau)}\right). \tag{9}$$

When $\alpha^{BM} = 0$, one obtains $0 < G(P(0)/d(\tau))$; when $\alpha^{BM} = 1$, $1 > G(P(1)/d(\tau))$. Thus, a solution to Equation (9) exists. It is unique if $g(c) < \xi_0 = \min_t[d(r)/P'(x)]$. The

Best-response functions must have a slope less than 1 in absolute value.

Consider the function $x - G\left(\frac{P(x)}{d(\tau)}\right)$. The derivative with respect to x yields $1 - \frac{P'(x)}{d(\tau)} g\left(\frac{P(x)}{d(\tau)}\right)$. The equation $x = G\left(\frac{P(x)}{d(\tau)}\right)$ has a unique solution if for all $c \in [\underline{c}, \overline{c}]$, $g(c) < \min_t \left[\frac{d(\tau)}{P'(t)}\right] \equiv \xi_0$.

Q.E.D

Proof Proposition 2. By the implicit function theorem

$$\begin{aligned}
\frac{d\alpha^{BM}}{d\tau} &= \frac{\partial}{\partial\tau}\left[G\left(\frac{P(\alpha^{BM})}{d(\tau)}\right)\right] + \frac{\partial}{\partial\alpha}\left[G\left(\frac{P(\alpha^{BM})}{d(\tau)}\right)\right]\frac{d\alpha^{BM}}{d\tau} \\
&= -P(\alpha^{BM})\left(\frac{d'(\tau)}{(d(\tau))^2}\right) g\left(\frac{P(\alpha^{BM})}{d(\tau)}\right) + \frac{P'}{d(\tau)} g\left(\frac{P(\alpha^{BM})}{d(\tau)}\right)\frac{d\alpha^{BM}}{d\tau}
\end{aligned}$$

$$\frac{d\alpha^{BM}}{d\tau} = -\frac{P(\alpha^{BM})\left(\frac{d'(\tau)}{(d(\tau))^2}\right) g\left(\frac{P(\alpha^{BM})}{d(\tau)}\right)}{1 - \frac{P'}{d(\tau)} g\left(\frac{P(\alpha^{BM})}{d(\tau)}\right)} \leq 0.$$

This is because $1 - \frac{P'}{d(\tau)} g\left(\frac{P(\alpha^{BM})}{d(\tau)}\right) > 0$ (if $g(c) < \xi_0$) and $d' \geq 0$. *Q.E.D*

Proof of Lemma 4.

By assumption, $G(0) > 1 - \left(\frac{1}{n}\right)^{\frac{1}{n-1}}$. Furthermore, in any symmetric equilibrium, the equilibrium threshold satisfies $c^* \geq 0$. An individual participates in dissent if the private cost c_i is less than c^*. Therefore, the equilibrium probability that an individual participates is such that $\alpha = G(c^*) \geq G(0) > 1 - \left(\frac{1}{n}\right)^{\frac{1}{n-1}}$. This means that $\frac{1}{n} - (1-\alpha)^{n-1} > 0$. Hence, $\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-\alpha)^{n-1}\right)$ is strictly decreasing.

Proof of Proposition 3. The indifference equation is given by

$$\alpha = G\left(\frac{P(\alpha)}{d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-\alpha)^{n-1}\right) + (1-\alpha)^{n-1}\right)}\right).$$

When $\alpha = 0$, $0 < G\left(\frac{P(0)}{d(\tau)\left(1 - \frac{\overline{\tau}}{\tau}\left(\frac{n-1}{n}\right)\right)}\right)$; moreover, when $\alpha = 1$, $1 > G\left(\frac{n\tau}{\overline{\tau}}\frac{P(1)}{d(\tau)}\right)$, since $\overline{c}$ is large enough. Hence, there exists a α^* solution to the indifference equation.

Under the condition that $g(.)$ is flat enough there exists a unique equilibrium $\alpha^*(\tau)$. In fact there exists $\xi > 0$ such that if $g(c) < \xi$ then the game has a unique equilibrium.

Consider the function

$$\mathbf{H}(x;\tau) \equiv \mathbf{H}(x) = x - G\left(\frac{P(x)}{d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-x)^{n-1}\right) + (1-x)^{n-1}\right)}\right)$$

defined on $[0,1]$. The derivative of this function with respect to x is given by

$$\mathbf{H}'(x) = 1 - \left(\frac{P(x)}{d(\tau)}\right)\frac{\frac{P'(x)}{P(x)}\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-x)^{n-1}\right) + (1-x)^{n-1}\right) - (n-1)(1-x)^{n-2}\left(\frac{\overline{\tau}}{\tau} - 1\right)}{\left[\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-x)^{n-1}\right) + (1-x)^{n-1}\right]^2}$$
$$\times g\left(\frac{P(x)}{d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-x)^{n-1}\right) + (1-x)^{n-1}\right)}\right).$$

A unique equilibrium exists if the derivative is positive. The condition holds if for any $c \in [\underline{c}, \overline{c}]$,

$$1 > \sup_{x\in[0,1]}\left[\frac{P(x)}{d(\tau)}\left|\frac{\frac{P'(x)}{P(x)}\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-x)^{n-1}\right) + (1-x)^{n-1}\right) - (n-1)(1-x)^{n-2}\left(\frac{\overline{\tau}}{\tau} - 1\right)}{\left[\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-x)^{n-1}\right) + (1-x)^{n-1}\right]^2}\right|\right] g(c),$$

where $|.|$ is the absolute value function.

Since the function

$$x \mapsto \frac{P(x)}{d(\tau)}\left|\frac{\frac{P'(x)}{P(x)}\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-x)^{n-1}\right) + (1-x)^{n-1} - (n-1)(1-x)^{n-2}\left(\frac{\overline{\tau}}{\tau} - 1\right)\right)}{\left[\frac{\overline{\tau}}{\tau}\left(\frac{1}{n} - (1-x)^{n-1}\right) + (1-x)^{n-1}\right]^2}\right|$$

is continuous and positive on the compact $[0,1]$, a supremium exists and is positive. Let $1/\xi$ be the supremium where $\xi > 0$.

Suppose $g(c) < \xi$ for any $c \in [\underline{c}, \overline{c}]$. Then one obtains

$$
\begin{aligned}
\mathbf{H}'(x) &= 1-\left(\frac{P(x)}{d(\tau)}\right)\frac{\frac{P'(x)}{P(x)}\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-x)^{n-1}\right)+(1-x)^{n-1}\right)-(n-1)(1-x)^{n-2}\left(\frac{\overline{\tau}}{\tau}-1\right)}{\left[\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-x)^{n-1}\right)+(1-x)^{n-1}\right]^2} \\
&\times\ g\left(\frac{P(x)}{d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-x)^{n-1}\right)+(1-x)^{n-1}\right)}\right) \\
&\geq\ 1-\frac{1}{\xi}\times g\left(\frac{P(x)}{d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-x)^{n-1}\right)+(1-x)^{n-1}\right)}\right)>0.
\end{aligned}
$$

Since the function $x\mapsto H(x)$ is strictly increasing, with $H(0)<0$ and $H(1)>0$, the equation $H(x)=0$ has a unique solution. *Q.E.D*

Proof of Proposition 4.

Suppose $g(c)<\xi$ for any $c\in[\underline{c},\overline{c}]$. Using the implicit function theorem,

$$
\frac{d\alpha^*}{d\tau}=-\frac{\frac{\partial\mathbf{H}(\alpha^*;\tau)}{\partial\tau}}{\left.\frac{\partial\mathbf{H}(x;\tau)}{\partial x}\right|_{x=\alpha^*}}. \tag{10}
$$

Given that $\left.\frac{\partial\mathbf{H}(x;\tau)}{\partial x}\right|_{x=\alpha^*}>0$ since $g(c)<\xi$, the way in which τ affects α^* depends on the sign of $\frac{\partial\mathbf{H}(\alpha^*;\tau)}{\partial\tau}$. That is,

$$
sign\left(\frac{d\alpha^*}{d\tau}\right)=-sign\left(\frac{\partial\mathbf{H}(\alpha^*;\tau)}{\partial\tau}\right)=sign\left(\frac{\partial}{\partial\tau}\left[G\left(\frac{P(\alpha^*)}{d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)}\right)\right]\right).
$$

Denote $\Delta\equiv\Delta(\alpha^*;\tau)=\frac{P(\alpha^*)}{d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)}$. I obtain

$$
\begin{aligned}
\frac{\partial}{\partial \tau}\left[G\left(\Delta(\alpha^*;\tau)\right)\right] &= -\frac{P(\alpha^*)}{\left[d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)\right]^2} \times g\left(\Delta(\alpha^*;\tau)\right) \\
&\times \left[d'(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)-\frac{\overline{\tau}}{\tau^2}d(\tau)\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)\right] \\
&= \frac{P(\alpha^*)}{\left[d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)\right]} \times g\left(\Delta(\alpha^*;\tau)\right) \\
&\times \frac{1}{\left[d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)\right]} \\
&\times \left[-d'(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)+\frac{\overline{\tau}}{\tau^2}d(\tau)\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)\right] \\
&= \frac{P(\alpha^*)}{\left[d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)\right]} \times g(\Delta(\alpha^*,\tau)) \times \left(\frac{1}{\tau}\right) \\
&\times \left[-\tau\frac{d'(\tau)}{d(\tau)}+\frac{\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)}{\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}}\right] \\
&= \Delta(\alpha^*;\tau) \times g\left(\Delta(\alpha^*;\tau)\right)\left(\frac{1}{\tau}\right)\Sigma(\alpha^*;\tau) \qquad (11)
\end{aligned}
$$

where

$$
\begin{aligned}
\Sigma(\alpha^*;\tau) &= -\tau\frac{d'(\tau)}{d(\tau)}+\frac{\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)}{\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}} \\
&= -V(\tau)+\frac{L_1(\alpha^*,\tau)}{L_1(\alpha^*,\tau)+L_2(\alpha^*,\tau)}.
\end{aligned}
$$

I denote $V(\tau)=\tau\frac{d'(\tau)}{d(\tau)}$, $L_1(\alpha^*,\tau)=\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)$ and $L_2(\alpha^*,\tau)=(1-\alpha^*)^{n-1}$. It is important to note that $L_i>0$ for $i\in\{1,2\}$.

Since all the terms in Equation (11) are positive except for $\Sigma(\alpha^*;\tau)$ which may be negative, it is clear that

$$
sign\left(\frac{d\alpha^*}{d\tau}\right)=sign\left(\frac{\partial}{\partial\tau}\left[G\left(\Delta(\alpha^*;\tau)\right)\right]\right)=sign\left(\Sigma(\alpha^*;\tau)\right).
$$

It is then sufficient and necessary to study the function Σ. As a function of τ, Σ is continuous and differentiable on the interval $[\underline{\tau}, \overline{\tau}]$. Moreover, $\Sigma(\tau = \underline{\tau}, \alpha^*) = \frac{L_1(\alpha^*, \tau=\underline{\tau})}{L_1(\alpha^*, \tau=\underline{\tau}) + L_2(\alpha^*, \tau=\underline{\tau})} > 0$, since $d'(\underline{\tau}) = 0$. In addition, at $\tau = \overline{\tau}$, since $d'(\overline{\tau})$ is extremely large, $\Sigma(\tau = \overline{\tau}, \alpha^*) < 0$. Therefore, there exists a τ^* such that $\Sigma(\tau = \tau^*, \alpha^*) = 0$.

I show that τ^* is unique. I prove this by showing that $\Sigma(\tau, \alpha^*)$ has a single-crossing. In fact, the single-crossing happens from above.

Given that α^* is also a function of τ, one obtains

$$\begin{aligned}
\frac{d\Sigma(\tau, \alpha^*)}{d\tau} &= -V'(\tau) + \frac{\left(\frac{\partial L_1}{\partial \tau} + \frac{\partial L_1}{\partial \alpha}\frac{d\alpha^*}{d\tau}\right)(L_1 + L_2) - L_1\left(\frac{\partial L_1}{\partial \tau} + \frac{\partial L_1}{\partial \alpha}\frac{d\alpha^*}{d\tau} + \frac{\partial L_2}{\partial \tau} + \frac{\partial L_2}{\partial \alpha}\frac{d\alpha^*}{d\tau}\right)}{[L_1 + L_2]^2} \\
&= -V'(\tau) + \frac{L_2\left(\frac{\partial L_1}{\partial \tau} + \frac{\partial L_1}{\partial \alpha}\frac{d\alpha^*}{d\tau}\right) - L_1\left(\frac{\partial L_2}{\partial \tau} + \frac{\partial L_2}{\partial \alpha}\frac{d\alpha^*}{d\tau}\right)}{[L_1 + L_2]^2} \\
&= -V'(\tau) + \frac{L_2\frac{\partial L_1}{\partial \tau}}{[L_1 + L_2]^2} + \frac{d\alpha^*}{d\tau}\left(\frac{L_2\frac{\partial L_1}{\partial \alpha} - L_1\frac{\partial L_2}{\partial \alpha}}{[L_1 + L_2]^2}\right) \quad \text{since } \frac{\partial L_2}{\partial \tau} = 0 \\
&= -V'(\tau) - \frac{L_2\left(\frac{\overline{\tau}}{\tau^2}\right)\left(\frac{1}{n} - (1-\alpha^*)^{n-1}\right)}{[L_1 + L_2]^2} \\
&+ \frac{d\alpha^*}{d\tau}\left(\frac{(n-1)(1-\alpha^*)^{n-2}\left(\frac{\overline{\tau}}{\tau}L_2 + L_1\right)}{[L_1 + L_2]^2}\right) \qquad (12)
\end{aligned}$$

It is clear that the two terms $-V'(\tau)$ and $-\frac{L_2\left(\frac{\overline{\tau}}{\tau^2}\right)\left(\frac{1}{n} - (1-\alpha^*)^{n-1}\right)}{[L_1+L_2]^2}$ are negative. This is because $d'(\tau)/d(\tau)$ is increasing, by log-convexity, and $\frac{1}{n} - (1-\alpha^*)^{n-1} > 0$ since $G(0) > 1 - \left(\frac{1}{n}\right)^{\frac{1}{n-1}}$. Furthermore, $(n-1)(1-\alpha^*)^{n-2}\left(\frac{\overline{\tau}}{\tau}L_2 + L_1\right)$ is positive because L_1 and L_2 are positive. Using the expression of $\frac{d\alpha^*}{d\tau}$ from Equation (10), one obtains

$$
\begin{aligned}
\frac{d\alpha^*}{d\tau} &= -\frac{\frac{\partial \mathbf{H}(\alpha^*;\tau)}{\partial \tau}}{\left.\frac{\partial \mathbf{H}(x;\tau)}{\partial x}\right|_{x=\alpha^*}} \\
&= \frac{\frac{\partial}{\partial \tau}\left[G\left(\Delta(\alpha^*;\tau)\right)\right]}{\left.\frac{\partial \mathbf{H}(x;\tau)}{\partial x}\right|_{x=\alpha^*}} \\
&= \frac{\frac{P(\alpha^*)g(\Delta(\alpha^*;\tau))\left(\frac{1}{\tau}\right)\Sigma(\alpha^*;\tau)}{\left[d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)\right]}}{\left.\frac{\partial \mathbf{H}(x;\tau)}{\partial x}\right|_{x=\alpha^*}} \text{ (using equation (11))} \\
&= \frac{P(\alpha^*)g\left(\Delta(\alpha^*;\tau)\right)\left(\frac{1}{\tau}\right)}{\left[d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)\right]\left(\left.\frac{\partial \mathbf{H}(x;\tau)}{\partial x}\right|_{x=\alpha^*}\right)}\Sigma(\alpha^*;\tau) \qquad (13)
\end{aligned}
$$

Plugging the expression (13) into (12) implies

$$
\begin{aligned}
\frac{d\Sigma(\tau,\alpha^*)}{d\tau} &= \frac{P(\alpha^*)g\left(\Delta(\alpha^*;\tau)\right)\left(\frac{1}{\tau}\right)}{\left[d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)\right]\left(\left.\frac{\partial \mathbf{H}(x;\tau)}{\partial x}\right|_{x=\alpha^*}\right)} \\
&\times \frac{(n-1)(1-\alpha^*)^{n-2}\left(\frac{\overline{\tau}}{\tau}L_2+L_1\right)}{[L_1+L_2]^2} \times \Sigma(\alpha^*;\tau) \\
&- V'(\tau) - \frac{L_2\left(\frac{\overline{\tau}}{\tau^2}\right)\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)}{[L_1+L_2]^2}
\end{aligned}
$$

Given that both $\left.\frac{\partial \mathbf{H}(x;\tau)}{\partial x}\right|_{x=\alpha^*}$ and

$$
\frac{P(\alpha^*)g\left(\Delta(\alpha^*;\tau)\right)\left(\frac{1}{\tau}\right)}{\left[d(\tau)\left(\frac{\overline{\tau}}{\tau}\left(\frac{1}{n}-(1-\alpha^*)^{n-1}\right)+(1-\alpha^*)^{n-1}\right)\right]}
$$

are positive,

It turns out that $\frac{d\Sigma(\tau,\alpha^*)}{d\tau} \leq 0$ if $\Sigma \leq 0$. Hence, Σ is a decreasing function when negative. Consequently, $\Sigma(\tau;\alpha^*)$ has a single-crossing from above.

To conclude, there exists a unique τ^* such that $\Sigma(\tau^*, \alpha^*) = 0$. Moreover, if $\tau \leq \tau^*$ then $\Sigma(\tau^*, \alpha^*) > 0$ which implies that $\frac{d\alpha^*}{d\alpha} > 0$. Furthermore, if $\tau > \tau^*$, $\Sigma(\tau, \alpha^*) < 0$ which implies that $\frac{d\alpha^*}{d\alpha} < 0$. *Q.E.D*

Proposition 11 (Common Knowledge Benefit.) *Suppose Defender's benefit of retaliation, y, is common knowledge. Then, the marginal probability of facing retaliation is always non-monotonic in the punishment. Moreover, if y is small, a high punishment always negates dissent.*

Proof of Proposition 11. I assume that y_i is common knowledge. I further consider the situation where for all i, $y_i = y$ to avoid asymmetries in the decision to dissent. It is clear that Defender retaliates if the number of attacks s exceeds $n\left(1 - \frac{\tau}{\overline{\tau}} y\right)$. Let S^i be the number of dissidents except player i. A dissident receives

$$\frac{P(\alpha)}{d(\tau)} - c_i \Pr\left[S^i \geq k(\tau) - 1\right],$$

while a player who abstains receives

$$-c_i \Pr\left[S^i \geq k(\tau)\right].$$

Where $k(\tau) = \lceil n\left(1 - \frac{\tau}{\overline{\tau}} y\right) \rceil$ is the ceiling of the number $n\left(1 - \frac{\tau}{\overline{\tau}} y\right)$. The net expected payoff then reads as

$$\frac{P(\alpha)}{d(\tau)} - c_i \Pr\left[S^i = k(\tau) - 1\right].$$

It is important to see that the marginal probability of facing retaliation $\Pr\left[S^i = k(\tau) - 1\right]$ is a non-monotonic function of τ. This is because $\frac{\partial}{\partial \tau}\left(\Pr\left[S^i = k(\tau) - 1\right]\right) = \frac{dk(\tau)}{d\tau} \times \frac{\partial}{\partial k}\left(\Pr\left[S^i = k - 1\right]\right)$. Since $S^i \sim \mathcal{B}(n-1, \alpha)$, $\frac{\partial}{\partial k}\left(\Pr\left[S^i = k - 1\right]\right)$ is positive and then negative (due to log-concavity of the Binomial). Moreover, $\frac{dk(\tau)}{d\tau} < 0$. Hence, the marginal probability of facing retaliation first decreases and then increases. Finally, if $y = 0$, the net expected payoff depends on τ only through $d(\tau)$. This implies that as τ increases participation decreases. *Q.E.D*

For the proof of Lemma 8 and Lemma 9 below, denote

$$M(\alpha;\tau) = \sum_{s=0}^{n-1} \left[H\left(\frac{\overline{\tau}}{\tau}\left(1-\frac{s}{n}\right)\right) - H\left(\frac{\overline{\tau}}{\tau}\left(1-\frac{s+1}{n}\right)\right)\right] \binom{n-1}{s} \alpha^s (1-\alpha)^{n-1-s}.$$

The next lemma shows that when the benefit of retaliation is distributed according to $H(.)$ on $[0,1]$, as long as $g(c) < \xi$ for $\xi > 0$, there is a unique equilibrium.

Lemma 8 *There exists $\xi > 0$ such that if $g(c) < \xi$ for $c \in [\underline{c}, \overline{c}]$, the game has a unique equilibrium. Moreover, for any $\tau \in [\underline{\tau}, \overline{\tau}]$, the function*

$$x \mapsto x - G\left(\frac{P(x)}{d(\tau)M(x;\tau)}\right)$$

is strictly increasing on $[0,1]$.

Proof: The proof is straightforward, and is an adaptation of the proof of proposition 4.

Lemma 9 *If $G(0) > 1 - \left(\frac{\underline{h}}{\underline{h}+(n-1)\overline{h}}\right)^{\frac{1}{n-1}}$, then for any $\tau \in [\underline{\tau}, \overline{\tau}]$, $\frac{\partial M(\alpha;\tau)}{\partial \tau} < 0$.*

Proof: Observe that $\frac{\partial M(\alpha;\tau)}{\partial \tau}$

$$\begin{aligned} = \ & \frac{\overline{\tau}}{\tau^2} \sum_{s=1}^{n-1} \left[\left(1-\frac{s+1}{n}\right) h\left(\frac{\overline{\tau}}{\tau}\left(1-\frac{s+1}{n}\right)\right) - \left(1-\frac{s}{n}\right) h\left(\frac{\overline{\tau}}{\tau}\left(1-\frac{s}{n}\right)\right)\right] \binom{n-1}{s} \alpha^s (1-\alpha)^{n-1-s} \\ + \ & (1-\alpha)^{n-1} \frac{\overline{\tau}}{\tau^2} \left(\frac{n-1}{n}\right) h\left(\frac{\overline{\tau}}{\tau}\left(\frac{n-1}{n}\right)\right). \end{aligned}$$

In the first term of the previous equality, we have $h\left(\frac{\overline{\tau}}{\tau}\left(1-\frac{s+1}{n}\right)\right) \le h\left(\frac{\overline{\tau}}{\tau}\left(1-\frac{s}{n}\right)\right)$, for H to be convex. In this respect, noting that $\underline{h} = \min_{u \in [0,1]} h(u)$,

$$\sum_{s=1}^{n-1}\left[\left(1-\tfrac{s+1}{n}\right)h\left(\tfrac{\overline{\tau}}{\tau}\left(1-\tfrac{s+1}{n}\right)\right)-\left(1-\tfrac{s}{n}\right)h\left(\tfrac{\overline{\tau}}{\tau}\left(1-\tfrac{s}{n}\right)\right)\right]\tbinom{n-1}{s}\alpha^s(1-\alpha)^{n-1-s}$$
$$\leq -\tfrac{1}{n}\underline{h}\sum_{s=1}^{n-1}\tbinom{n-1}{s}\alpha^s(1-\alpha)^{n-1-s}$$
$$= -\tfrac{1}{n}\underline{h}(1-(1-\alpha)^{n-1}).$$

In addition, since $\overline{h} = \max_{u\in[0,1]} h(u)$,

$$(1-\alpha)^{n-1}\frac{\overline{\tau}}{\tau^2}\left(\frac{n-1}{n}\right)h\left(\frac{\overline{\tau}}{\tau}\left(\frac{n-1}{n}\right)\right) \leq (1-\alpha)^{n-1}\frac{\overline{\tau}}{\tau^2}\left(\frac{n-1}{n}\right)\overline{h}.$$

I then obtain

$$\frac{\partial M(\alpha;\tau)}{\partial\tau} \leq \frac{\overline{\tau}}{n\tau^2}\left(-\underline{h}+\underline{h}(1-\alpha)^{n-1}+(n-1)\overline{h}(1-\alpha)^{n-1}\right).$$

If $(1-\alpha)^{n-1} < \frac{\underline{h}}{\underline{h}+(n-1)\overline{h}}$, or $G(0) > 1-\left(\frac{\underline{h}}{\underline{h}+(n-1)\overline{h}}\right)^{\frac{1}{n-1}}$, then $\frac{\partial M(\alpha;\tau)}{\partial\tau} < 0$. *Q.E.D*

Proof Proposition 6:

Following Lemma 8, and by the Implicit Function Theorem, the derivative of the equilibrium probability of participation α^* must have the same sign as $\frac{\partial}{\partial\tau}\left[G\left(\frac{P(\alpha^*)}{d(\tau)M(\alpha^*;\tau)}\right)\right]$.

$$\frac{\partial}{\partial\tau}\left[G\left(\frac{P(\alpha^*)}{d(\tau)M(\alpha^*;\tau)}\right)\right] = \frac{P(\alpha^*)}{d(\tau)M(\alpha^*;\tau)}\times g\left(\frac{P(\alpha^*)}{d(\tau)M(\alpha^*;\tau)}\right)\times\left[-\frac{d'(\tau)}{d(\tau)}-\frac{\frac{\partial M(\alpha^*;\tau)}{\partial\tau}}{M(\alpha^*;\tau)}\right].$$

Therefore, denote $\Sigma_2(\alpha^*;\tau) = -\frac{d'(\tau)}{d(\tau)}-\frac{\frac{\partial M(\alpha^*;\tau)}{\partial\tau}}{M(\alpha^*;\tau)}$. Since $d'(\underline{\tau})=0$, by Lemma 9, $\Sigma_2(\alpha^*;\tau=\underline{\tau})>0$. In addition, since $d'(\overline{\tau})$ is extremely large, $\Sigma_2(\alpha^*;\tau=\overline{\tau})<0$.

The analysis shows that there $\tau^\circ\in[\underline{\tau},\overline{\tau}]$, not necessarily unique, such that $\Sigma_2(\alpha^*;\tau=\tau^\circ)=0$. Denote τ^* the smallest τ° such that $\Sigma_2(\alpha^*;\tau=\tau^\circ)=0$. By continuity of Σ_2, τ^* exists. *Q.E.D*

Using Lemma 8 and the proof of Proposition 6 one can show equilibrium uniqueness, and that τ_* exists (but not necessarily unique), when $H(.)$ is an arbitrary distribution on $[0,1]$. In the next two proofs, I assume that $H(.)$ is the uniform distribution on $[0,1]$.

Proof of Proposition 9:

Consider the indifference equation

$$\alpha = G\left(\frac{P(\alpha)}{d(\tau)\left(\frac{\sigma}{2} + \frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha,\sigma)\right)}\right).$$

At $\alpha = 0$, no one participates. We have $0 < G\left(\frac{P(0)}{d(\tau)\left(\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\mathbf{Q}(0,\sigma)\right)}\right)$. Moreover, if $\alpha = 1$, then $1 > G\left(\frac{2P(1)}{\sigma d(\tau)}\right)$. Hence, there exists α_* solution to the indifference condition. Define

$$\mathbf{J}(x;\tau) = x - G\left(\frac{P(x)}{d(\tau)\left(\frac{\sigma}{2} + \frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha,\sigma)\right)}\right).$$

Similarly to the proof of Proposition 3 we show that there exists $\xi_1 > 0$ such that if $g(c) < \xi_1$ for $c \in [\underline{c}, \overline{c}]$, then $\mathbf{J}(x;\tau)$ is strictly increasing as a function of x. Moreover, we have $\mathbf{J}(0;\tau) < 0$ and $\mathbf{J}(1;\tau) > 0$. The equilibrium level of participation α_* exists and is unique. *Q.E.D*

Proof of Proposition 10

Suppose $g(c) < \xi_1$ for any $c \in [\underline{c}, \overline{c}]$. By the implicit function theorem,

$$\frac{d\alpha_*}{d\tau} = -\frac{\frac{\partial \mathbf{J}(\alpha_*;\tau)}{\partial\tau}}{\left.\frac{\partial \mathbf{J}(x;\tau)}{\partial x}\right|_{x=\alpha_*}} = \frac{\frac{\partial\left[G\left(\frac{P(\alpha_*)}{d(\tau)\left(\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha_*,\sigma)\right)}\right)\right]}{\partial\tau}}{\left.\frac{\partial \mathbf{J}(x;\tau)}{\partial x}\right|_{x=\alpha_*}}. \tag{14}$$

Since $\mathbf{J}(x;\tau)$ is strictly increasing as a function of x, $sign\left(\frac{d\alpha_*}{d\tau}\right) = sign\left(\frac{\partial\left[G\left(\frac{P(\alpha_*)}{d(\tau)\left(\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha_*,\sigma)\right)}\right)\right]}{\partial\tau}\right)$.

Denote $\Gamma \equiv \Gamma(\alpha_*;\tau) = \frac{P(\alpha_*)}{d(\tau)\left(\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha_*,\sigma)\right)}$; where $\mathbf{Q}(\alpha;\sigma) = \left(\frac{\left(1-\frac{3}{2}\sigma\right)\left(2\left(\frac{1-\sigma}{\sigma}\right)-\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)}{\left(\frac{\alpha}{1-\alpha}+\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)\left(\frac{\alpha}{1-\alpha}+2\left(\frac{1-\sigma}{\sigma}\right)\right)}\right)$.

$$
\begin{aligned}
\frac{\partial G(\Gamma(\alpha_*,\tau))}{\partial\tau} &= -\frac{P(\alpha_*)}{\left[d(\tau)\left(\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha_*,\sigma)\right)\right]^2} g\left(\Gamma(\alpha_*,\tau)\right)\left[d'(\tau)\left(\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha_*,\sigma)\right)-\frac{\overline{\tau}}{\tau^2}\mathbf{Q}(\alpha_*,\sigma)d(\tau)\right] \\
&= \Gamma(\alpha_*;\tau)\left(\frac{1}{\tau}\right)g(\Gamma(\alpha_*,\tau))\left[-\tau\frac{d'(\tau)}{d(\tau)}+\frac{\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)}{\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)+\tau\frac{\sigma}{2}}\right] \\
&= \Gamma(\alpha_*;\tau)\left(\frac{1}{\tau}\right)g(\Gamma(\alpha_*,\tau))\times\Sigma_1(\alpha_*,\tau). \qquad (15)
\end{aligned}
$$

Where

$$
\Sigma_1(\alpha_*,\tau) = -\tau\frac{d'(\tau)}{d(\tau)}+\frac{\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)}{\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)+\tau\frac{\sigma}{2}} = -V(\tau)+\frac{L_3(\alpha_*,\tau)}{L_3(\alpha_*,\tau)+L_4(\alpha_*,\tau)},
$$

$V(\tau)=\tau\frac{d'(\tau)}{d(\tau)}$, $L_3=\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)$ and $L_4(\alpha_*,\tau)=\tau\frac{\sigma}{2}$. It turns out $sign\left(\frac{\partial G(\Gamma(\alpha_*,\tau))}{\partial\tau}\right) = sign\left(\Sigma_1(\alpha_*,\tau)\right)$. Moreover, $\Sigma_1(\alpha_*,\tau=\underline{\tau}) = \frac{L_3(\alpha_*,\tau)}{L_3(\alpha_*,\tau)+L_4(\alpha_*,\tau)} > 0$; $\Sigma_1(\alpha_*,\tau=\overline{\tau}) < 0$, since $d'(\overline{\tau})$ is extremely large. Therefore, there exists τ_* such that for $\tau\le\tau_*$, $\Sigma_1(\alpha_*,\tau)>0$, and for $\tau>\tau_*$, $\Sigma_1(\alpha_*,\tau)<0$.

I show that τ_* is unique. I derive the derivative of Σ_1 with respect to τ, taking into account the fact that α_* is also a function of τ. Since $\frac{\partial L_3}{\partial\tau}=\frac{\partial[\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)]}{\partial\tau}=0$, observe that

$$
\begin{aligned}
\frac{d\Sigma_1(\alpha_*,\tau)}{d\tau} &= -V'(\tau)+\frac{\frac{\partial L_3}{\partial\alpha}\frac{d\alpha_*}{d\tau}\left[L_3+L_4\right]-\left[\frac{\partial L_4}{\partial\tau}+\frac{\partial L_3}{\partial\alpha}\frac{d\alpha_*}{d\tau}\right]L_3}{[L_3+L_4]^2} \\
&= -V'(\tau)+\frac{L_4\left(\frac{\partial L_3}{\partial\alpha}\right)\frac{d\alpha_*}{\partial\tau}-L_3\frac{\partial L_4}{\partial\tau}}{[L_3+L_4]^2} \\
&= -V'(\tau)-\frac{\overline{\tau}\left(\frac{\sigma}{2}\right)\mathbf{Q}(\alpha_*,\sigma)}{[\tau\left(\frac{\sigma}{2}\right)+\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)]^2}+\frac{\tau\left(\frac{\sigma}{2}\right)\frac{\partial(\overline{\tau}\mathbf{Q}(\alpha_*,\sigma))}{\partial\alpha}}{[\tau\left(\frac{\sigma}{2}\right)+\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)]^2}\frac{d\alpha_*}{d\tau} \qquad (16)
\end{aligned}
$$

Now consider the term $\mathbf{Q}(\alpha_*,\sigma)$.

$$\mathbf{Q}(\alpha_*,\sigma)=\left(\frac{\left(1-\frac{3}{2}\sigma\right)\left(2\left(\frac{1-\sigma}{\sigma}\right)-\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)}{\left(\frac{\alpha}{1-\alpha}+\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)\left(\frac{\alpha}{1-\alpha}+2\left(\frac{1-\sigma}{\sigma}\right)\right)}\right)=\frac{\left(1-\frac{3}{2}\sigma\right)\left(2\left(\frac{1-\sigma}{\sigma}\right)-\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)}{\left(\mathbf{A}(\alpha)+\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)\left(\mathbf{A}(\alpha)+2\left(\frac{\sigma}{1-\sigma}\right)\right)},$$

with $\mathbf{A}(\alpha)=\frac{\alpha}{1-\alpha}$. Since $\mathbf{A}'(\alpha)>0$, observe that

$$\frac{\partial(\mathbf{Q}(\alpha_*,\sigma))}{\partial\alpha}=-\frac{\left(1-\frac{3}{2}\sigma\right)\left(2\left(\frac{1-\sigma}{\sigma}\right)-\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)}{\left[\left(\mathbf{A}+\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)\left(\mathbf{A}+2\left(\frac{\sigma}{1-\sigma}\right)\right)\right]^2}\times\left[\mathbf{A}'\left(\mathbf{A}+\frac{1}{2}\left(\frac{\sigma}{1-\sigma}\right)\right)+\mathbf{A}'\left(\mathbf{A}+2\left(\frac{\sigma}{1-\sigma}\right)\right)\right]$$

is negative. Therefore, in expression (16), $\frac{\tau\left(\frac{\sigma}{2}\right)\frac{\partial(\overline{\tau}\mathbf{Q}(\alpha_*,\sigma))}{\partial\alpha}}{[\tau\left(\frac{\sigma}{2}\right)+\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)]^2}$ is negative, as well as $-V'(\tau)$ and $-\frac{\overline{\tau}\left(\frac{\sigma}{2}\right)\mathbf{Q}(\alpha_*,\sigma)}{[\tau\left(\frac{\sigma}{2}\right)+\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)]^2}$.

Furthermore, from expression (14) and (15),

$$\frac{d\alpha_*}{d\tau}=\frac{\frac{\partial\left[G\left(\frac{P(\alpha_*)}{d(\tau)\left(\frac{\sigma}{2}+\frac{\overline{\tau}}{\tau}\mathbf{Q}(\alpha_*,\sigma)\right)}\right)\right]}{\partial\tau}}{\left.\frac{\partial\mathbf{J}(x;\tau)}{\partial x}\right|_{x=\alpha_*}}=\frac{\Gamma(\alpha_*;\tau)\left(\frac{1}{\tau}\right)g(\Gamma(\alpha_*,\tau))\times\Sigma_1(\alpha_*,\tau)}{\left.\frac{\partial\mathbf{J}(x;\tau)}{\partial x}\right|_{x=\alpha_*}}. \tag{17}$$

Replacing expression (17) into equation (16), I obtain

$$\begin{aligned}\frac{d\Sigma_1(\alpha_*,\tau)}{d\tau} &= -V'(\tau)-\frac{\overline{\tau}\left(\frac{\sigma}{2}\right)\mathbf{Q}(\alpha_*,\sigma)}{[\tau\left(\frac{\sigma}{2}\right)+\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)]^2}\\ &+ \frac{\tau\left(\frac{\sigma}{2}\right)\frac{\partial(\overline{\tau}\mathbf{Q}(\alpha_*,\sigma))}{\partial\alpha}}{[\tau\left(\frac{\sigma}{2}\right)+\overline{\tau}\mathbf{Q}(\alpha_*,\sigma)]^2}\left(\frac{\Gamma(\alpha_*;\tau)\left(\frac{1}{\tau}\right)g(\Gamma(\alpha_*,\tau))}{\left.\frac{\partial\mathbf{J}(x;\tau)}{\partial x}\right|_{x=\alpha_*}}\right)\times\Sigma_1(\alpha_*,\tau).\end{aligned}$$

Based on the analysis above, since $\frac{\Gamma(\alpha_*;\tau)\left(\frac{1}{\tau}\right)g(\Gamma(\alpha_*,\tau))}{\left.\frac{\partial\mathbf{J}(x;\tau)}{\partial x}\right|_{x=\alpha_*}}>0$, $\frac{d\Sigma_1(\alpha_*,\tau)}{d\tau}\leq 0$ when $\Sigma_1>0$. Therefore, Σ_1 has a single-crossing from above. Hence, τ_* exists and is unique. *Q.E.D*

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