Biased Experts, Majority Rule, and the Optimal Composition of Committee*

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Abstract

An uninformed principal or planner appoints a committee of experts to vote on a multi-attribute alternative, such as an interdisciplinary project. Each expert can evaluate only one attribute and is biased toward it (specialty bias). The principal values all attributes of the alternative equally but has a status quo bias, reflecting the organizational cost of a change. We study whether the principal would compose the committee of more or less specialty-biased experts. We show that her optimal composition is nonmonotonic in the majority rule, with the most biased experts appointed under intermediate rules. We then show that the principal would be less concerned about the committee composition if its members can be uninformed, as they induce the informed to vote less strategically. Surprisingly, although uninformed members lower the quality of the committee’s decision, the principal may prefer to have some when its composition is suboptimal and the majority rule is sufficiently extreme, such as the unanimity. By the same logic, the principal may exclude some informed experts from the committee.

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1 Introduction

Decision-making by committee is commonplace. Its advantage is, perhaps, most apparent when the alternative in question has multiple attributes, each of which requires evaluation by

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an expert. Examples include interdisciplinary projects, company decisions involving various business units, and legislative bills affecting several states. To the extent that expert evaluations are aggregated, the committee will make a better decision. There are, however, two potential problems: first, an expert, such as a scientist, often knows his evaluation privately, and second, he is likely to be “biased” or “partisan” toward his field of specialty and put more weight on his evaluation.¹

In this paper, we investigate the optimal partisanship in the committee, given the voting procedure.² Specifically, given the majority rule for the alternative’s approval, we ask whether a status quo biased principal would appoint more or less partisan experts to the committee. We then ask how the optimal committee composition would change when some members might be uninformed, as they may have lacked the time or access for a thorough evaluation of the alternative before the vote. Last, we explore whether or not the principal prefers a fully informed committee.

To address these questions, we draw upon the recent literature (discussed below) and employ a model of collective decision with payoff interdependencies. An uninformed principal, e.g., a university administrator, forms a panel of experts who vote between a multi-attribute alternative, e.g., an interdisciplinary project, and status quo according to a fixed majority rule. Prior to the vote, each expert receives private information about the “quality” of one attribute (his specialty), but he may also value others. The expert’s weight on his information captures his specialty bias or partisanship. We study how this bias affects equilibrium voting, and, in turn, the expected payoff of the principal who cares about the alternative’s average quality. Unlike experts, the principal is assumed to possess a status quo bias, reflecting the organizational cost of a change that is mostly ignored by the committee.³

It is intuitive that a status quo biased principal would, in general, appoint partisan experts. Our first contribution is to show that their optimal partisanship depends crucially on the majority rule and does so nonmonotonically. If the majority requirement for the alternative is weak, the principal wants the least partisan experts in the committee. In contrast, if the

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¹Such partisanship may also be due to having different cultural and cognitive backgrounds that distort one’s processing of others’ information. This argument is frequently used for explaining why agents may “agree to disagree”; e.g., Morris (1995).

²In the United States, the meetings and voting procedures of organizations, clubs, legislative bodies, and other deliberative assemblies are generally governed by Robert’s Rules of Order introduced in 1876 (for the latest edition, see Robert et al. 2011).

³Such a preference conflict between a decision-maker and informed agents toward the status quo is commonly assumed in the literature on cheap talk and voting games, e.g., Che and Kartik (2009), Battaglini (2017), and Gradwohl and Feddersen (2018).
majority requirement is moderate, she wants the most partisan. For the remaining majority
rules, those close to unanimity, the principal prefers modestly partisan experts.

To understand why, note that the principal would not care about the committee compo-
sition if she could control how its members vote, i.e., if she could contract on their approval
standards. In practice, though, such control is infeasible since members hold private informa-
tion. Instead, the principal may try to influence their equilibrium voting by their composition.
We show that less partisan experts, those who place a greater weight on others’ information,
vote more strategically: conditional on the information gleaned from the pivotal event, they
are more willing to ignore their own. When the alternative requires few affirmative votes for
approval, the pivotal event carries negative news, leading a less partisan expert to raise his
approval standard and do so toward the principal’s desired level. In contrast, when the alter-
native requires an intermediate number of affirmative votes, the pivotal event carries positive
news, leading a less partisan expert to lower his approval standard. To prevent such diver-
gence from her desired standard, the principal appoints more partisan experts who place less
weight on the pivotal event. Finally, when the alternative requires a strong consensus, includ-
ing the unanimity, the principal strikes a balance between the two cases and appoints mod-
erately partisan experts. Overall, given the majority rule, the principal manages the amount
of strategic voting in the committee by its composition and uses it to her advantage to bring
members’ approval standards closer to her preferred level.

Next, we extend our model to include uninformed experts in the committee. We discover
that by making their votes the least pivotal, the uninformed members delegate the decision to
the informed in equilibrium since their payoffs are interdependent. In particular, the unin-
formed reject the alternative when the majority requirement for its approval is weak, accept
when it is strong, and strictly mix in-between. The strict mixing by the uninformed occurs
because the pivotal event conveys no news in equilibrium, which also induces the informed
to vote sincerely or nonstrategically independent of their partisanship. Hence, the presence
of uninformed members reduces the principal’s ability to influence informed voters by their
composition. Put differently, the committee’s composition is more instrumental for the prin-
cipal when experts are more likely to be informed.

Does the last observation, however, mean that the principal wants all experts to be in-
formed? Interestingly, not always. We show that when the principal cannot pick optimally
biased experts to the committee, because they may be unavailable in the population, she may
prefer some to be uninformed. Although uninformed experts decrease the quality of the de-
cision, they help bring the alternative’s approval rate closer to the optimum owing to their
delegation incentive mentioned above. This finding implies that the principal may some-
times raise – not lower – the cost of information for committee members, perhaps, by limiting
their access to information or by rushing the vote on the alternative. We also show that if
the principal cannot prevent experts from being informed, she may exclude some from the
committee; i.e., the principal may not consult an informed expert for each attribute of the
alternative.

Related Literature. As alluded to above, our paper builds on the recent literature on vot-
ing as a means of preference aggregation in committees. Committee members are assumed to
have conflicting interests, formalized as interdependent valuations, in that each is biased or
partisan toward his information.\footnote{Hence, even if their information were public, partisan members would not necessarily agree on the alternative. There is, of course, an extensive literature on voting as a means of information aggregation in committees; e.g., Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996). This line of research typically assumes common interest among committee members.} Using this specification, Yildirim (2012) identifies credible
majority rules if a social planner cannot commit to one ex ante. Focusing on ad-hoc commit-
tees, we fix the voting rule here as an institution but examine the optimal partisanship and
informed voting in the committee. Roesler (2016) introduces privately known partisanship
and shows that under the unanimity rule, each member relaxes his equilibrium approval stan-
dard as the rest of the group becomes stochastically more partisan. Unlike her (and keeping
up with most other studies in this literature), we assume symmetric and commonly known
partisanship to explore the planner’s preference for it depending on the majority rule. In
doing so, we also establish the important link between approval standards and the major-
ity rule.\footnote{It is worth noting that Roesler’s result for the unanimity rule is not at odds with our Proposition 1(c) below, because, under symmetry, we find the total effect of partisanship on a player’s equilibrium strategy.} In a dynamic model, Moldovanu and Shi (2013) study collective search under a
unanimous agreement and compare equilibrium acceptance standards across specialist and
“generalist” members.\footnote{Previous papers by Albrecht et al. (2010) and Compte and Jehiel (2010) allow for more general majority rules in a collective search model but restrict attention to pure private values, i.e., to the most partisan members. Aside from their different focus, none of these dynamic models reduces to our static analysis of equilibrium voting.} Using different specifications with single-peaked preferences and
interdependent bliss points, Gruner and Kiel (2004) and Rosar (2015) compare social perfor-
mannces of mean and median aggregation rules for (continuous) reports. They find that the
mean aggregation rule dominates if members are not too partisan or the committee is large
enough. Here, we restrict attention to binary reports, i.e., votes, but consider more general
aggregation rules.
In highlighting the need for biased experts to improve decision-making, our paper is also related to those that emphasize their role in acquiring costly information; e.g., Dewatripont and Tirole (1999), Cai (2009), Prendergast (2007), Gerardi and Yariv (2008), Che and Kartik (2009), and Kartik et al. (2017). Unlike theirs, our model features costless information. As such, our paper is more related to those that emphasize the role of biased experts in increasing information transmission in persuasion and cheap-talk games with exogenous information; e.g., Shin (1998), Krishna and Morgan (2001), and Battacharya and Mukherjee (2013). These papers allow for more general communication between experts and the decision-maker than simple votes, but they restrict attention to two-expert committees, in which majority rule is not crucial. An exception in this regard is Li and Suen (2004), who consider simple votes between two biased experts but do not explore the issue of the optimal composition, which would result in appointing unbiased experts in their setting.\(^7\)

Similar to ours, several papers also point to the optimality of having uninformed committee members. Among them, Caillaud and Tirole (2007) and Bardhi and Guo (2018) argue that the sponsor of a proposal may best persuade a group by selectively informing some members and relying on rubber-stamping by the uninformed. Although the uninformed also rubber-stamp in equilibrium in our extended model, their presence is desired by a social-minded principal who values every attribute of the alternative – not by a self-interested sponsor who simply wants the alternative approved. Gershkov and Szentes (2009) show that a utilitarian planner may optimally decide under imprecise information as it becomes harder to motivate an additional expert to acquire costly information. Without the latter concern, however, the planner would prefer to have all informed experts in their model. Battacharya and Mukherjee (2013) examine a persuasion game without commitment. They find that when experts are moderately biased toward a policy, increasing their likelihood of being informed may lead to worse outcomes as it affects decision-maker’s default policy and experts’ disclosure strategies in equilibrium. In our model, the majority rule is fixed, so the lack of commitment is not the source of having uninformed members. Last but not least, Name-Correa and Yildirim (2018) conclude that when the committee is susceptible to outside influence, increasing its size may help deter capture even though new members are expected to remain uninformed and rubber-stamp.

The rest of the paper is organized as follows. In the next section, we lay out the model,
followed by the characterization of equilibrium voting in Section 3. In Section 4, we study the principal’s preference for the committee composition. In Section 5, we extend the analysis to committees with uninformed members and study the principal’s preference for a fully informed committee. Section 6 concludes. All proofs and auxiliary results are relegated to an appendix.

2 The Model

An uninformed principal must decide between the status quo and a multi-attribute alternative by appointing an ad-hoc committee of $n > 1$ interested experts. For instance, a university administrator must decide whether or not to implement an interdisciplinary project by selecting a researcher from each discipline. Based on his specialty, expert $i$ can evaluate the quality of attribute $i$, which yields a private signal $\theta_i$ independently drawn from a twice continuously differentiable distribution $F$, with mean 0 and a positive density $f$ on the support $\theta < 0 < \bar{\theta}$. Besides his own, however, expert $i$ may also value others’ signals about the alternative as we formalize by the following interdependent payoff:

$$v_i^A = \left( \beta + \frac{1 - \beta}{n} \right) \theta_i + (1 - \beta) \frac{\sum_{j \neq i} \theta_j}{n}. \tag{1}$$

The interdependence parameter $\beta \in [0, 1]$ is commonly known and captures the degree of specialty bias or partisanship: the higher $\beta$ is, the more an expert cares about his dimension of the alternative, with $\beta = 0$ and $\beta = 1$ referring to the committees with the least and the most partisan members, respectively. The status quo is assumed to yield a normalized payoff of 0 to each expert. So, other than the specialty bias, expert $i$ has no status quo bias: $E[v_i^A] = 0$, where $E[.]$ denotes the expectation operator.

Upon receiving their private signals, committee members simultaneously vote Yes/No, and the alternative is accepted if it garners at least $k \in \{1, ..., n\}$ affirmative votes, i.e., $k$-majority. The principal’s payoff from the alternative is:

$$w^A = \frac{\sum_i \theta_i}{n}. \tag{2}$$

The payoff specification in (1) can be equally interpreted as other-regarding preferences: expert $i$’s material payoff is his signal $\theta_i$, but he cares about others’ payoffs, too (see Cooper and Kagel, 2016 for a recent survey). Such (linearly) interdependent payoffs are also exploited in auction theory (see Krishna, 2009 for a review), where $\beta = 0$ and $\beta = 1$ refer to pure common- and pure private-value auctions, respectively.
The principal, representing the organization, cares about the average quality of the alternative, weighing its attributes equally. If the alternative is rejected, the principal obtains a fixed payoff \( w^0 > 0 \) from the status quo. Thus, unlike experts, the principal has a status quo bias; perhaps, implementing the alternative involves an organizational cost that experts ignore. Such a preference conflict between a decision-maker and informed agents toward the status quo is commonly assumed in the literature on cheap talk and voting games, e.g., Che and Kartik (2009), Battaglini (2017), and Gradwohl and Feddersen (2018). Moreover, our results are robust to a negative status quo bias, \( w^0 < 0 \), as established in Corollary 3 below.

As is standard in the literature, given ex ante symmetric players, we solve for symmetric (Bayesian-Nash) equilibria of the voting game. To eliminate trivial equilibria associated with nonunanimous rules, we also require them to be responsive or interior. We begin our analysis by characterizing equilibrium voting and then proceed to the principal’s preference for the committee’s composition, namely, for the parameter \( \beta \).

### 3 Equilibrium voting

Since signals are independent and \( v^A_i \) is strictly increasing in \( \theta_i \), it is readily verified that expert \( i \) follows a cutoff strategy: approve the project if \( \theta_i > \theta^*_i \), and disapprove if \( \theta_i < \theta^*_i \). In a symmetric equilibrium, suppose that all but expert \( i \) adopt a cutoff \( \theta^*_i \). In determining his, expert \( i \) needs to consider only the pivotal event in which there are \( k \) approval and \( n - k \) disapproval votes except for his. Conditional on this event, (1) implies that expert \( i \)'s expected payoff is:

\[
v^A(\theta_i; \theta^*, k, \beta) = (\beta + \frac{1 - \beta}{n})\theta_i + \frac{1 - \beta}{n} \left[ ((k - 1)E^+[\theta^*] + (n - k)E^-[\theta^*]) \right],
\]

where \( E^+[x] = E[\theta|\theta > x] \) and \( E^-[x] = E[\theta|\theta < x] \). The symmetric cutoff \( \theta^* = \theta^*(k, \beta) \) constitutes an (interior) equilibrium if and only if it satisfies the following indifference condition:

\[
v^A(\theta^*; \theta^*, k, \beta) = 0.
\]

**Lemma 1** There is a unique symmetric equilibrium.

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9 This is equivalent to assuming a utilitarian principal for the alternative, \( w^A = \frac{\sum_i v^A_i}{n} \), since, from (1), \( \sum_i v^A_i = \sum_i \theta_i \), which is independent of \( \beta \).

10 For instance, under a nonunanimity rule, a symmetric equilibrium with all members’ voting Yes independent of their private information always exists since each vote then becomes nonpivotal.

11 His decision when indifferent is immaterial as it is a zero probability event.
The equilibrium existence is by continuity. Its uniqueness follows from the fact that both conditional means, \( E^+[x] \) and \( E^-[x] \), are strictly increasing in the cutoff, \( x \).

To understand equilibrium strategies, we introduce the notion of sincere or nonstrategic voting. Expert \( i \) is said to vote sincerely if he conditions his vote only on his private information (Austen-Smith and Banks, 1996). Absent any status quo bias for the experts, sincere voting corresponds to adopting a cutoff of 0 in our model.\(^{12}\) It is immediate from (4) that voting is always sincere for the most partisan (pure private-value) experts, i.e., \( \theta^*(k, 1) = 0 \) for all \( k \). For the rest, Proposition 1 shows that equilibrium voting is generically strategic and depends on the degree of partisanship.

**Proposition 1** Let \( \beta < 1 \) and \( \kappa^* = 1 + (n - 1)[1 - F(0)] \). In equilibrium,

(a) \( E^-[\theta^*(\cdot)] < 0 < E^+[\theta^*(\cdot)] \),

(b) \( \theta^*(k, \beta) \) is strictly decreasing in \( k \),

(c) \( \text{sgn} \left[ \frac{\partial \theta^*(k, \beta)}{\partial \beta} \right] = \text{sgn} \left[ -\theta^*(k, \beta) \right] = \text{sgn} \left[ k - \kappa^* \right] \).

Refer to Figure 1. The threshold rule \( \kappa^* \in (1, n) \) is the real solution to \( \theta^*(k, \beta) = 0 \). If an integer, it induces sincere voting for all \( \beta \). Generically though, \( \kappa^* \) is noninteger except for special cases; for instance, \( \kappa^* = \frac{n+1}{2} \) (the simple majority) when \( n \) is odd, and the signal distribution is symmetric so that \( F(0) = \frac{1}{2} \). Part (a) confirms that a positive (resp. negative) vote, on average, carries positive (resp. negative) news about the alternative. Given this, part (b) says that when the majority rule requires more approval votes for the alternative, experts relax their approval standards as they hold a more favorable view of others’ opinions in the event of being pivotal. Part (c) reveals that for any \( k \), an increase in \( \beta \) moves the equilibrium cutoff closer to 0, sincere voting, because experts put less weight on the pivotal event. That is,

\(^{12}\)Formally, since \( E[\theta_i] = 0 \), we have from (1) that \( \text{sgn} \left( E[v^A_i | \theta_i] \right) = \text{sgn} \left( \theta_i \right) \).
for any \( k \), an increase in \( \beta \) causes a counterclockwise rotation in the equilibrium cutoff.

![Equilibrium Voting: \(\beta_1 < \beta_2\)](image)

Part (c) of Proposition 1 can also inform us of how the approval probability of the alternative changes with partisanship. Note that for an arbitrary cutoff \( x \in [\theta, \bar{\theta}] \), the alternative is approved with the probability:

\[
P(x; k) = \sum_{m=k}^{n} p(x; m),
\]

where \( p(x; m) = \binom{n}{m} [1 - F(x)]^m [F(x)]^{n-m} \).

**Corollary 1** Let \( \beta < 1 \). Then, \( \text{sgn} \left[ \frac{\partial P(x; k, \beta)}{\partial \beta} \right] = \text{sgn} [\kappa^* - k]. \)

Corollary 1 follows because \( \frac{\partial P(x; k)}{\partial x} < 0 \), as expected. It says that the chances of the alternative’s acceptance decrease (resp. increase) with partisanship if a sufficiently strong (resp. weak) consensus is required.

Armed with experts’ voting behavior, we next examine the principal’s preference for their composition, i.e., \( \beta \).

## 4 Committee composition

Note that conditional on an arbitrary cutoff \( x \) and the vote profile with \( m \) Yes votes, the principal’s ex post payoff from implementing the project is:

\[
w^A(x; m) = \frac{mE^+[x] + (n-m)E^-[x]}{n},
\]

9
and, in turn, using (5), her ex ante payoff before a committee vote is:

$$\bar{w}(x; k, w^0) = \sum_{m=k}^{n} p(x; m) w^A(x; m) + \sum_{m=0}^{k-1} p(x; m) w^0,$$

where the first term on the right-hand side of (7) is her expected payoff from the alternative’s acceptance and the second term is her expected payoff from its rejection.

To characterize the principal’s payoff under equilibrium voting, we first establish a benchmark of optimal voting. Suppose that the principal could dictate the cutoff $x$ such that expert $i$ accepts the alternative whenever $\theta_i > x$. Then, the principal would solve the following program:

$$\max_{x \in \theta} \bar{w}(x; k, w^0).$$

**Lemma 2** There is a unique solution, $\theta^o(k, w^0)$, to (8). Moreover, $\theta^o(.) \in (\theta, \bar{\theta})$ if $w^0 < \frac{k}{n} \bar{\theta}$, and $\theta^o(.) = \bar{\theta}$ if $w^0 \geq \frac{k}{n} \bar{\theta}$.

The existence of a maximum is immediate since $\bar{w}(x; .)$ is continuous. Its uniqueness is due to the latter’s single-peakedness in $x$, as established in Lemma A2 and depicted in Figure 2.

![Figure 2. Single-Peakedness of Expected Payoff](image)

Intuitively, a higher cutoff raises the average quality of the accepted alternative but lowers the probability of acceptance. An interior cutoff means that the principal meaningfully delegates the decision to the committee. And Lemma 2 shows that she will delegate if her status quo bias is not too severe, which is more likely to be the case for a voting rule closer to unanimity. Otherwise, it would be optimal for the principal to set the highest standard for a Yes vote,
i.e., $\theta^o = \theta$, ensuring the rejection of the alternative. Building on Lemma 2, the next result characterizes the optimal voting strategy, which is depicted in Figure 3.

**Proposition 2 (Optimal voting)** Suppose $w^0 < \frac{\theta}{n}$. Then,

(a) $\theta^o(k, w^0)$ is strictly decreasing in $k$ and strictly increasing in $w^0$,

(b) $\text{sgn} [\theta^o(.)] = \text{sgn} [\kappa^o(w^0) - k]$, where

$$\kappa^o(w^0) = \kappa^* + \frac{nw^0}{E^+[0] - E^-[0]}.$$

Part (a) of Proposition 2 indicates that the principal with a stronger status quo bias would ask experts to raise their approval standards for the alternative. She would also ask them for higher approval standards if the majority rule is less demanding for the alternative. Refining these observations, part (b) identifies the critical rule which, if integer, would render sincere voting optimal for the principal. Since, unlike the experts, the principal has a status quo bias, her critical rule for sincere voting requires more approval votes than the experts’, i.e., $\kappa^o(w^0) \geq \kappa^*$. It is readily checked that $\kappa^o(w^0)$ is strictly increasing in $w^0$.$^{13}$

![Figure 3. Optimal vs. Equilibrium Voting](image)

$^{13}$Though not our focus here, we note that the voting rule, $\kappa^o(w^0)$, that induces sincere voting would, generally, not be optimal if the principal could *jointly* choose the cutoff and the majority rule. That is, having a status quo bias, the principal would desire some strategic voting in the committee.
Note that the optimal voting strategy, \( \theta^o(k, w^0) \), is independent of the specialty bias, \( \beta \). That is, the principal would not be concerned about the committee’s composition if she could control how its members vote. In practice, though, the principal lacks such control as expert evaluations are private. Instead, the principal may try to influence the equilibrium voting by appointing a committee with more or less partisan specialists. Combining the previous two, the next proposition offers a sharp characterization of the principal’s preference for committee composition.

**Proposition 3 (Committee composition)** Let \( \varpi(\theta^o(k, \beta); k, w^0) \) be the principal’s ex ante payoff in equilibrium. Then, her payoff is strictly decreasing in \( \beta \) for \( k \in [1, \kappa^* \) ), and strictly increasing for \( k \in (\kappa^*, \kappa^o(w^0)] \). Finally, for \( k \in (\kappa^o(w^0), n] \), the principal’s payoff is strictly increasing in \( \beta \) for \( \beta < \bar{\beta}(k, w^0) \), and strictly decreasing for \( \beta > \bar{\beta}(k, w^0) \), where the cutoff bias is given by

\[
\bar{\beta}(k, w^0) = \frac{w^0}{w^0 - \theta^o(k, w^0)} \in (0, 1).
\]

Moreover, \( \bar{\beta}(k, w^0) \) is strictly decreasing in \( k \) and strictly increasing in \( w^0 \).

Proposition 3 reveals that the principal’s preference for committee composition depends critically on the majority rule. If the majority rule is relatively lenient toward the alternative, i.e., \( k < \kappa^* \), the principal wants less partisan specialists in the committee. Graphically, since her expected payoff in (8) is single-peaked in the cutoff, \( x \), the principal wants a clockwise rotation of the equilibrium cutoff in Figure 3, which means lower partisanship, \( \beta \), by Figure 1.\(^{14}\) In words, with only a few affirmative votes required for the approval of the alternative, the principal desires a high approval standard, \( \theta^o > 0 \). And by Proposition 1, less partisan experts, gleaning negative news from the pivotal event and placing more weight on it, serve this purpose by raising their equilibrium voting cutoffs toward the desired level. If, on the other hand, the majority requirement is moderate, i.e., \( \kappa^* < k \leq \kappa^o \), then the principal wants more partisan specialists – a counterclockwise rotation of the equilibrium cutoff in Figure 3. Although the principal still aims for a high approval standard in this case, less partisan experts, now gleaning positive news from the pivotal event, would lower their cutoffs, diverging from the optimal level. Finally, if the majority requirement is sufficiently stringent for the alternative, i.e., \( k > \kappa^o(w^0) \), both the principal and members would apply low approval standards, i.e., \( \theta^o < 0 \) and \( \theta^* < 0 \), and the two would coincide for some moderately partisan

\(^{14}\)As shown in the proof of Proposition 3, \( \theta^o(k, \beta) \leq \theta^o(k, w^0) \) for \( k \in [1, \kappa^* \) ) and all \( \beta \in [0, 1] \).
experts. The monotonicity of the principal’s payoff is, again, due to the single-peakedness identified in Lemma 2.

Overall, Proposition 3 reveals that by forming a committee with more or less partisan members, the principal controls the amount of strategic voting, and in doing so, she brings the equilibrium voting standard closer to her optimum. In fact, if, for each attribute of the alternative, the principal has access to experts with any degree of partisanship, the following result, which is immediate from Proposition 3 and depicted in Figure 4, shows her optimal choice.

**Corollary 2 (Optimal composition)** Let \( \beta^o(k, w^0) = \arg \max_{\beta \in [0,1]} \pi(\theta^*(k, \beta); k, w^0) \). Then,

\[
\beta^o(k, w^0) = \begin{cases} 
0 & \text{if } k < \kappa^* \\
[0,1] & \text{if } k = \kappa^* \\
1 & \text{if } \kappa^* < k \leq \kappa^o(w^0) \\
= \overline{\beta}(k, w^0) \in [0,1] & \text{if } \kappa^o(w^0) < k \leq n.
\end{cases}
\]

Three observations are in order. First, the optimal committee composition is nonmonotonic in the majority rule, \( k \). While, for a low \( k \), the principal would appoint a committee with the least partisan and hence the most strategic specialists, she would do the opposite for an intermediate \( k \): appoint a committee with the most partisan specialists who would vote sincerely. For a high \( k \), she would strike a balance between the two cases, with the optimal bias, \( \overline{\beta}(\cdot) \), decreasing in \( k \).
Second, given that both $\kappa^o(w^0)$ and $\bar{\beta}(k, w^0)$ are increasing in $w^0$, the optimal committee would involve (weakly) more partisan experts as the principal becomes more status quo biased. Recall from Figure 3 that a principal with a stronger bias wants experts to apply a higher approval standard so that their chances of approving a mediocre alternative is lower. The observation then follows from Figure 1: equilibrium approval standard rotates counterclockwise as partisanship increases.

Third, the ability to compose the committee is most useful to the principal when the majority requirement is sufficiently strong; i.e., when $k \geq \kappa^o(w^0)$, so that $\theta^*(k, \bar{\beta}) = \theta^o(k, w^0)$. That is, the principal can implement optimal voting by the committee composition when $k \geq \kappa^o(w^0)$, which includes the unanimity rule. Otherwise, the equilibrium approval standard remains too low for the principal.\footnote{Formally, $\theta^*(k, 0) = \theta^o(k, 0) < \theta^o(k, w^0)$ for $k \leq \kappa^*$, and $\theta^*(k, 1) = 0 < \theta^o(k, w^0)$ for $\kappa^* < k < \kappa^o(w^0)$.}

Example 1 illustrates Corollary 2.

Example 1 Let $n = 11$, $w^0 = 3/11$, and $\theta_i \sim U[-1,1]$. Then, $\kappa^* = 6$ and $\kappa^o = 9$. Hence, $\beta^o = 0$ for $k < 6$; $\beta^o = 1$ for $6 < k \leq 9$; and $\beta^o = 18/(11k - 81)$ for $9 < k \leq 11$.

We close this section by noting that isomorphic results would obtain if the principal had a negative status quo bias, i.e., $w^0 < 0$. In particular, noticing from Proposition 2 that $\kappa^o(w^0) < \kappa^*$ in this case, the following corollary to Proposition B1 in the appendix is immediate for the optimal committee composition.

Corollary 3 (Negative status quo bias) Suppose $w^0 < 0$. Then,

$$
\begin{align*}
\beta^o(k, w^0) &\begin{cases} 
= \bar{\beta}(k, w^0) \in [0,1) & \text{if } k < \kappa^o(w^0) \\
= 1 & \text{if } \kappa^o(w^0) \leq k < \kappa^* \\
\in [0,1] & \text{if } k = \kappa^* \\
= 0 & \text{if } \kappa^* < k \leq n.
\end{cases}
\end{align*}
$$

5 Imperfectly informed committees

Up to now, we have assumed that each expert learns about his dimension of the alternative before the vote. In reality, though, some experts may remain uninformed; perhaps, they lacked the time or access for a thorough investigation. The presence of uninformed members is likely to affect the equilibrium voting for all, and, in turn, the principal’s preference for
biased experts. In the next subsection, we characterize equilibrium voting for both types of experts and determine the optimal composition of an imperfectly informed committee. We then investigate whether or not the principal wants a fully informed committee.

5.1 Committee composition with uninformed members

Let expert \( i \) privately learn his signal \( \theta_i \) with an exogenous and independent probability \( \lambda \in (0, 1) \), and remain uninformed with probability \( 1 - \lambda \). To determine the symmetric equilibrium, also let an uninformed committee member approve the alternative with probability \( \pi \in [0, 1] \) while an informed member continues to follow a cutoff strategy \( x \in [\theta, \bar{\theta}] \) as in the baseline. Then, the ex ante probability that a member approves the alternative is:

\[
\phi(x, \pi; \lambda) = \lambda[1 - F(x)] + (1 - \lambda)\pi. \tag{9}
\]

From (9) and the fact that \( E[\theta] = 0 \), the expected signal values of Yes and No votes are, respectively,

\[
E^+[x, \pi; \lambda] = \frac{\lambda \int_{\theta}^{\bar{\theta}} \theta dF(\theta)}{\phi(x, \pi; \lambda)} \quad \text{and} \quad E^-[x, \pi; \lambda] = \frac{\lambda \int_{\theta}^{\bar{\theta}} \theta dF(\theta)}{1 - \phi(x, \pi; \lambda)}. \tag{10}
\]

As a result, the total expected value of others’ signals in the pivotal event can be written:

\[
h(x, \pi; k, \lambda) = (k - 1)E^+[x, \pi; \lambda] + (n - k)E^-[x, \pi; \lambda]. \tag{11}
\]

Let \( x = \theta^* \) and \( \pi = \pi^* \) denote the equilibrium voting strategies for the informed and uninformed experts, respectively. As in (4), given \( \pi^* \), the (interior) cutoff signal \( \theta^* \) must leave an informed expert indifferent between accepting and rejecting the alternative in the pivotal event, namely

\[
(\beta + \frac{1 - \beta}{n})\theta^* + \frac{1 - \beta}{n}h(\theta^*, \pi^*; k, \lambda) = 0. \tag{12}
\]

As for the uninformed, they also condition their strategies on the pivotal event. Since \( E[\theta] = 0 \), an uninformed expert’s payoff from the alternative in the pivotal event is:

\[
\frac{1 - \beta}{n}h(\theta^*, \pi^*; k, \lambda). \tag{13}
\]

Let \( \beta < 1 \). Then, \( \pi^* \), the equilibrium probability of a Yes vote for an uninformed expert, must satisfy the following best response to \( \theta^* \):

\[
\pi^* = \begin{cases} 
1 & \text{if } h(\theta^*, 1; k, \lambda) > 0 \\
\frac{k - 1 - \lambda(1 - F(0))}{1 - \lambda} & \text{if } h(\theta^*, \pi^*; k, \lambda) = 0 \\
0 & \text{if } h(\theta^*, 0; k, \lambda) < 0.
\end{cases} \tag{14}
\]
In words, an uninformed member approves the alternative if the pivotal event carries positive news, and disapproves if it carries negative news. It is, however, possible that the pivotal event carries no news in equilibrium, i.e., \( h(\cdot) = 0 \). In this case, (12) implies sincere voting by the informed experts, and solving \( h(0, \pi^*; k, \lambda) = 0 \) for \( \pi^* \), we find the randomized strategy for the uninformed. It is readily verified that the randomized strategy is feasible, i.e., \( \pi^* \in [0, 1] \), if and only if \( \kappa'(\lambda) \leq k \leq \kappa^h(\lambda) \) where

\[
\kappa'(\lambda) = 1 + (n - 1)\lambda[1 - F(0)] \quad \text{and} \quad \kappa^h(\lambda) = 1 + (n - 1)[1 - \lambda F(0)].
\]

Lemma 3 \( \kappa'(\lambda) \) is increasing and \( \kappa^h(\lambda) \) is decreasing in \( \lambda \), with \( \kappa'(1) = \kappa^h(1) = \kappa^*, \kappa'(0) = 1 \), and \( \kappa^h(0) = n \).

The characterization of the equilibrium voting is then obtained from (12), (14), and (15).

**Proposition 4** (Equilibrium voting) Let \( \beta < 1 \). There is a unique symmetric equilibrium, and it has these properties:

\[
\begin{align*}
\theta^* > 0 \text{ and } \pi^* &= 0 \quad \text{if } k < \kappa'(\lambda) \\
\theta^* &= 0 \text{ and } \pi^* = \frac{k - 1 - \lambda(1 - F(0))}{1 - \lambda} \quad \text{if } \kappa'(\lambda) \leq k \leq \kappa^h(\lambda) \\
\theta^* < 0 \text{ and } \pi^* &= 1 \quad \text{if } \kappa^h(\lambda) < k.
\end{align*}
\]

Moreover,

(a) \( \pi^* \) is independent of \( \beta \),

(b) \( \text{sgn} \left[ \partial \theta^* / \partial \beta \right] = \text{sgn} \left[ -\theta^* \right] \).

The existence of a symmetric equilibrium follows from the fact that \( h(x, \pi; k, \lambda) \) is continuous in \( x \). Unlike in the base model (\( \lambda = 1 \)), however, its uniqueness is not immediate because for \( \lambda < 1 \), the conditional expectations \( E^+[x, \pi; \lambda] \) and \( E^-[x, \pi; \lambda] \), and, in turn, \( h(x, \pi; k, \lambda) \) can be nonmonotonic in \( x \). Nevertheless, we establish their monotonicity on the equilibrium path, leading us to the equilibrium uniqueness for \( \beta < 1 \). For \( \beta = 1 \), there is a trivial

---

\(^{16}\) In solving \( h(0, \pi^*; k, \lambda) = 0 \), note that \( \int_0^\beta dF(\theta) = -\int_0^\beta dF(\theta) \) in (10) given \( E[\theta] = 0 \).

\(^{17}\) Lemma C1 in the appendix shows that \( \frac{\partial}{\partial \lambda} E^+[x, \pi; \lambda] = \frac{1}{\lambda(1 - \lambda)} \frac{\partial \lambda F(x)}{\partial \lambda} (E^+[x, \pi; \lambda] - x) \). Thus, \( \lim_{\lambda \to 0} \frac{\partial}{\partial \lambda} E^+[x, \pi; \lambda] = 0 \) and \( \lim_{\lambda \to 1} \frac{\partial}{\partial \lambda} E^+[x, \pi; \lambda] < 0 \) for \( \lambda \in (0, 1) \) and \( \pi > 0 \). Intuitively, all else equal, an informed expert’s raising his already high approval standard reduces the expected value of a Yes vote. The reason is that the Yes vote is more likely to have come from an uninformed expert in this case.
multiplicity of equilibrium since, from (13), uninformed members receive a zero payoff in the pivotal event regardless of its news content. By continuity, we assume that the equilibrium for $\beta - 1^-$ applies to $\beta = 1$.

Inspecting Proposition 4, it is interesting to observe that uninformed members delegate the decision to the informed in equilibrium. To understand, consider $k = 1$ and $k = n$, under which the alternative is either accepted by one Yes vote or rejected by one No vote, respectively. In each case, the uninformed make their votes nonpivotal by always rejecting the alternative for $k = 1$ and always accepting it for $k = n$. This equilibrium strategy is also consistent for an intermediate $k$ since $\pi^*$ is strictly increasing in $k$; that is, the uninformed are more likely to cast Yes votes as the majority requirement becomes stronger.\textsuperscript{18,19}

Next, we observe that unlike in the base model, the presence of uninformed members induces sincere voting by the informed for a nontrivial region of majority rules, i.e., $\theta^* = 0$ for $\kappa^l(\lambda) \leq k \leq \kappa^h(\lambda)$. In fact, given the properties of $\kappa^l(\lambda)$ and $\kappa^h(\lambda)$ in Lemma 3, sincere voting by the informed becomes more prevalent as the fraction of the uninformed grows. As mentioned above, the reason is that with a significant fraction of the uninformed, the pivotal event carries no significant news in equilibrium to compel an informed expert to ignore his signal. Last but not least, while the committee composition, $\beta$, does not affect the voting behavior of the uninformed experts, it affects that of the informed as in the base model whenever they vote strategically, i.e., whenever $\theta^* \neq 0$. Hence, when forming the committee, the principal targets the informed members.

To characterize the optimal committee composition, we modify the baseline analysis, and note that conditional on the vote profile with $m$ Yes votes, the principal’s ex post welfare is given by:

$$w^A(x, \pi; m, \lambda) = \frac{mE^+[x, \pi; \lambda] + (n - m)E^-[x, \pi; \lambda]}{n}.$$

\textsuperscript{18}The fact that the uninformed members vote to delegate the decision to the informed suggests that the former could simply abstain, which we do not allow here. Our restriction is, however, without loss of generality, because in our model, the affirmative vote of at least $k$ members in the committee is required for the alternative, implying that abstentions would count as No votes. Such treatment of abstentions is consistent with Robert’s Rules of Order for committee conduct alluded to in the Introduction and widely adopted by organizations, clubs, legislative bodies, and other deliberative assemblies in the United States. For more on this point, see <http://mrsc.org/Home/Stay-Informed/MRSC-Insight/April-2013/How-Are-Abstentions-Handled-When-Counting-Votes.aspx>.

\textsuperscript{19}Though our model is very different, the uninformed members’ incentive to delegate the decision to the informed is reminiscent of Feddersen and Pesendorfer’s (1996) swing voter. In elections, the uninformed achieve such delegation by abstaining, since only those who vote at the polls decide the outcome. As mentioned in the previous footnote, however, small assemblies may treat abstentions as No votes or even disallow them.
and, in turn, her ex ante welfare before the vote is:

\[ w(x, \pi; w^0, \lambda) = \sum_{m=k}^{n} p(x, \pi; m, \lambda)w^A(x, \pi; m, \lambda) + \sum_{m=0}^{k-1} p(x, \pi; m, \lambda)w^0, \]

where \( p(x, \pi; m, \lambda) = \binom{n}{m} \phi(\cdot)^m[1 - \phi(\cdot)]^{n-m}. \)

As a result, the principal’s ex ante payoff under the equilibrium strategies is:

\[
\bar{w}^*(k, \beta, w^0, \lambda) = \bar{w}(\theta^*(k, \beta, \lambda), \pi^*(k, \lambda); w^0, \lambda). \tag{16}
\]

Clearly, \( \bar{w}^*(\cdot) \) is independent of \( \beta \) for \( k^i(\lambda) \leq k \leq k^h(\lambda) \). That is, the principal is neutral to the committee composition for moderately strong majority rules, since gleaning no significant news from the pivotal event, the informed members always vote sincerely in this region. To determine how \( \bar{w}^*(\cdot) \) changes with \( \beta \) for the remaining, more extreme majority rules, we consider an auxiliary problem, much like (8), in which given \( \pi^*(\cdot) \), the principal optimally sets the cutoff for the informed experts: \(^{20}\)

\[
\bar{\theta}^0(k, w^0, \lambda) = \arg \max_{x} \bar{w}(x, \pi^*(k, \lambda); w^0, \lambda). \tag{17}
\]

**Lemma 4** Let \( \bar{\kappa}^0(w^0, \lambda) = k^h(\lambda) + \frac{m w^0}{E^+[\lambda] - E^-[\lambda]}. \) Then,

\[
\text{sgn} \left[ \bar{\theta}^0(k, w^0, \lambda) \right] = \text{sgn} \left[ \bar{\kappa}^0(w^0, \lambda) - k \right].
\]

Moreover,

(a) \( \bar{\kappa}^0(\cdot) \) is strictly increasing in \( w^0 \) and strictly decreasing in \( \lambda \),

(b) \( \bar{\kappa}^0(w^0, \lambda) \geq k^h(\lambda) \),

(c) \( \bar{\kappa}^0(w^0, 1) = \kappa^0(w^0) \).

Lemma 4 mimics Lemma 2. Unlike the optimal cutoff \( \theta^0 \) for an all informed committee, however, \( \bar{\theta}^0 \) need not be unique since, as alluded to above, \( E^+[x, \pi; \lambda] \) and \( E^-[x, \pi; \lambda] \) need not be monotonic in \( x \). Nevertheless, from Proposition 4 and Lemma 4, we prove Proposition C1 in the appendix. The following characterization of the optimal composition, which is depicted in Figure 5, is then immediate.

\(^{20}\)Note that in (17), the principal does not choose \( \pi \) in the auxiliary problem, but this is not needed to establish the optimal composition given that \( \pi^* \) does not depend on \( \beta \).
Proposition 5 (Optimal composition) Let $\beta^0(k, w^0, \lambda) = \arg\max_{\beta \in [0, 1]} \overline{v}(k, \beta, w^0, \lambda)$. Then,

$$\beta^0(k, w^0, \lambda) = \begin{cases} 
0 & \text{if } k < \kappa^l(\lambda) \\
\in [0, 1] & \text{if } \kappa^l(\lambda) \leq k \leq \kappa^h(\lambda) \\
1 & \text{if } \kappa^h(\lambda) < k \leq \overline{w}^0(\lambda) \\
\beta(k, w^0, \lambda) \in [0, 1] & \text{if } \overline{w}^0(\lambda) < k \leq \kappa^o \\
\end{cases}$$

where $\beta(.) = \frac{w^0}{w^0 - \theta^0(k, w^0, \lambda)}$ uniquely solves: $\theta^0(k, w^0, \lambda) = \theta^*(k, \beta, \lambda)$. Moreover, $\beta(.)$ is strictly decreasing in $k$ and $\lambda$, and strictly increasing in $w^0$.

Proposition 5 extends Corollary 1. The key difference is that when the committee is imperfectly informed, the principal is neutral to its composition for a nontrivial set of majority rules: $\kappa^l(\lambda) \leq k \leq \kappa^h(\lambda)$. In other words, the composition of an imperfectly informed committee is important to the principal only for extreme voting rules such as the unanimity.

5.2 Optimally informed committee

In light of Lemma 3, Proposition 5 further implies that the principal has less influence over a less informed committee through its composition: as $\lambda$ decreases, the dark region between $\kappa^l(\lambda)$ and $\kappa^h(\lambda)$ in Figure 5 grows. This observation raises the following question: does the principal prefer a fully informed committee, i.e., $\lambda = 1$? Interestingly, not always. The next proposition shows that the principal may prefer some uninformed members in the committee when its composition is suboptimal and the majority requirement is extreme.
Proposition 6 (Optimal information) Let $\beta = 1$ and $\lambda^o \in \arg \max_{\lambda \in [0,1]} \pi^*(k, 1, w^0, \lambda)$. Also, let $\kappa^*$ be an integer. Then,

$$
\lambda^o = \begin{cases} 
1, & \text{if } \kappa^* \leq k \leq \kappa^o(w^0) + 1 - F(0) \\
\neq 1, & \text{otherwise.}
\end{cases}
$$

In particular, $\lambda^o \neq 1$ for $k = n$ if and only if $w^0 < \left(1 - \frac{1}{nF(0)}\right) E^+[0]$.

To understand Proposition 6, recall that informed experts vote sincerely, $\theta^* = 0$, when they are the most partisan, $\beta = 1$. Such nonstrategic voting is suboptimal for the principal except for the knife-edge voting rule $k = \kappa^o(w^0)$. Refer to Figure 3 (with a horizontal Equilibrium line). Note that under the unanimity rule, $k = n$, informed experts over-reject the alternative, $\theta^o < \theta^*$. Intuitively, under the unanimous agreement, a single informed expert with a slightly negative signal can vote down an otherwise high quality alternative. To mitigate the problem of over-rejection, the principal optimally introduces uninformed members, $\lambda^o \neq 1$, since, by Proposition 4, she expects them to accept the alternative and delegate the decision to the informed. The principal can achieve her objective by limiting the committee’s access to information or the time to process it before the vote. While lowering the quality of the committee’s decision, the uninformed members increase the alternative’s probability of approval. A similar argument also explains why the principal prefers an imperfectly informed committee when the voting rule is a dictatorship, $k = 1$. Unlike with the unanimity, though, informed experts over-accept the alternative under this rule, $\theta^o > \theta^*$, which the uninformed members help alleviate by rejecting it in equilibrium. Proposition 6 shows that the intuition from the two extreme voting rules carries over to less extreme ones depending on the parameter values. For intermediate voting rules, however, the principal wants all committee members to be informed, i.e., $\lambda^o = 1$. Although, as is clear from Figure 3, the problem of over-rejection or over-acceptance is also present under intermediate voting rules, it is not severe enough for the principal to lower the quality of the decision.

Proposition 6 suggests that when the principal cannot introduce uninformed members to the committee (perhaps, limiting information about the alternative is infeasible), she may optimally exclude some informed experts from the decision. That is, the principal may consult fewer experts than the alternative requires for a fully informed decision. Our last result demonstrates this possibility.
Proposition 7 (Optimal committee size) Let $\beta = \lambda = 1$. Then, for $k = 1$ (dictatorship) and $k = m$ (unanimity), there is a threshold $n_k^* < \infty$ such that the optimal committee excludes some experts, i.e., $m^o < n$, if and only if $n > n_k^*$. Moreover, if $k = \left\lfloor \frac{m + 1}{2} \right\rfloor$ (simple majority), $n$ is odd, and $F(0) = \frac{1}{2}$, then the optimal committee includes all experts, i.e., $m^o = n$.

The intuition behind Proposition 7 is similar to Proposition 6. Consider, for instance, the unanimity rule. Proposition 7 shows that when the number of informed experts is sufficiently large, the principal excludes some from the committee to attenuate the over-rejection problem. The principal may also appoint a smaller committee of the informed experts under a dictatorship rule to alleviate the over-acceptance problem. Under the simple majority rule, however, the optimal committee includes all informed experts. Under this rule, the problem of over-acceptance is not severe enough for the principal to risk a less informed decision. It is readily verified that in Example 1 above, the optimal committee size would be $m^o = 1$ for a dictatorship, $m^o = n = 11$ for the simple majority, and $m^o \in \{7, 8\}$ for the unanimity.

Together, Propositions 6 and 7 reveal that information design for the committee can be as valuable a tool for the principal as its composition. As such, it relates to several studies discussed in the Introduction, especially Caillaud and Tirole (2007) and Bardhi and Guo (2018). In different settings, these authors also discover that a committee may be best persuaded to approve an alternative by selectively informing some of its members and relying on rubber-stamping of the uninformed. We add to their insights by highlighting the importance of the majority rule as well as the committee design.

6 Conclusion

Decision-making by committee is a staple of democratic organizations. While organizations often pre-commit to the voting rule, be it simple majority or unanimity, they are flexible in appointing the members of a specific committee. Such flexibility appears especially relevant when committee members are experts with specialty bias: each can evaluate one attribute of the alternative but is also biased toward it. In this paper, building on the recent models of collective decision-making with interdependent payoffs, we investigate how a status quo biased principal or planner chooses the optimal composition of a committee.

While it is intuitive that a status quo biased principal would appoint biased experts, we show that the optimal specialty bias would depend crucially on the majority rule. If a weak majority is required for the alternative’s approval, the principal prefers the least biased mem-
bers, whereas if the majority requirement is moderate, she prefers the most biased. And for the remaining majority rules, those closer to unanimity, the principal prefers modestly biased members. The intuition behind such nonmonotonicity comes from the fact that the majority rule determines how much a committee member learns from the pivotal event in equilibrium, and his specialty bias (or the degree of payoff interdependence) determines how much he cares about this event to ignore his evaluation. Hence, by choosing its composition, the principal chooses the amount of strategic voting in the committee to implement her optimal approval standard.

By extending the analysis, we have found that the committee composition is less important when some members can be uninformed. The reason is that learning little from the pivotal event in this case, informed members vote less strategically, regardless of their specialty bias. Nevertheless, when the committee composition is not optimal, we show that the principal may prefer some members to be uninformed, perhaps by limiting the committee’s access to information or rushing the vote on the alternative. While lowering the quality of the decision, the uninformed members help correct for the alternative’s probability of approval since, in equilibrium, they tend to delegate the decision to the informed. We have also found that when she cannot restrict the experts’ access to information, the principal may optimally exclude some from the committee. That is, the principal may use fewer experts than required for a fully informed decision on the alternative.
Appendix A: Proofs for Section 3

In the appendix, the subscripts of functions refer to partial derivatives.

Proof of Lemma 1. Using (3), define \( V(x; k, \beta) = v^A(x; x, k, \beta) \). Note that \( V(x; k, 1) = x \), which, from (4), implies that \( \theta^* = 0 \) is the unique equilibrium for \( \beta = 1 \).

Next, suppose \( \beta < 1 \). Since \( E[\theta] < E[\theta] + \frac{1 - \beta}{n} \theta \), it follows that

\[
V(\theta; k, \beta) = V(\theta; n, \beta) = [1 - (n - 1) \frac{1 - \beta}{n}] \theta < 0
\]

and

\[
V(\bar{\theta}; k, \beta) = V(\bar{\theta}; 1, \beta) = [1 - (n - 1) \frac{1 - \beta}{n}] \bar{\theta} > 0.
\]

In addition, \( V(x; k, \beta) > 0 \) since, with appropriate limit arguments for \( x = \theta \) and \( \bar{\theta} \),

\[
E^+[x] = \frac{f(x)}{1 - F(x)} (E^+ - x) > 0 \quad \text{and} \quad E^-[x] = \frac{f(x)}{F(x)} (x - E^-) > 0.
\] (A-1)

From these three facts, there exists a unique (and interior) solution, \( \theta^*(k, \beta) \in (\theta, \bar{\theta}) \), to \( V(x; k, \beta) = 0 \) which also constitutes an equilibrium by (4).

Proof of Proposition 1. Let \( \beta < 1 \). Then, by Lemma 1, \( \theta^*(k, \beta) \in (\theta, \bar{\theta}) \) for all \( \beta \) and \( k \). To prove part (a), note that

\[
F(\theta^*)E^-[\theta^*] + (1 - F(\theta^*))E^+[\theta^*] = E[\theta] = 0
\]

by the law of iterated expectations. Since \( F(\theta^*) \in (0, 1) \) and \( E^-[\theta^*] < E^+[\theta^*] \), it follows that \( E^-[\theta^*] < 0 < E^+[\theta^*] \).

Next, recall from the previous proof that \( V(x; k, \beta) \) is strictly increasing in \( x \) and in \( k \). Moreover,

\[
V(\theta^*(k, \beta); k, \beta) = 0.
\] (A-2)

Thus, \( \theta^*(k, \beta) \) is strictly decreasing in \( k \), as claimed in part (b).

To prove part (c), we implicitly differentiate (A-2) with respect to \( \beta \) and find

\[
\theta^*_\beta(.) = - \frac{V_\beta(\theta^*(.), \cdot)}{V_x(.)}.
\]

Simple algebra shows \( V_\beta(\theta^*(.), \cdot) = \frac{\theta^*(.) - \theta(\cdot)}{1 - \beta} \). Since \( V_x(.) > 0 \) and \( \beta < 1 \), we conclude that

\[
\text{sgn} \left[ \theta^*_\beta(k, \beta) \right] = \text{sgn} \left[ -\theta^*(k, \beta) \right].
\]
Finally, treating \( k \) as a continuous variable and solving \( V(0; k, \beta) = 0 \) for \( k \), we find
\[
\kappa^* = \frac{E^+[0] - nE^-[0]}{E^+[0] - E^-[0]},
\]
(A-3)
Note that
\[
F(0) = \frac{E^+[0]}{E^+[0] - E^-[0]},
\]
since \( F(0)E^-[0] + (1 - F(0))E^+[0] = E[\theta] = 0 \) by the law of iterated expectations. Substituting from (A-4), (A-3) reduces to:
\[
\kappa^* = 1 + (n - 1)[1 - F(0)],
\]
as defined in the proposition. Clearly, \( \theta^*(\kappa^*, \beta) = 0 \). Hence, by part (b), \( \theta^*(k, \beta) > 0 \) for \( k < \kappa^* \), and \( \theta^*(k, \beta) < 0 \) for \( k > \kappa^* \). Equivalently stated,
\[
\text{sgn} [-\theta^*(k, \beta)] = \text{sgn}[k - \kappa^*],
\]
proving part (c).

Before proving Corollary 1 and Lemma 2, we record two auxiliary lemmas.

**Lemma A1.** Let \( \overline{w}^A(x, k) = \sum_{m=k}^n p(x; m, n)w^A(x, m) \) and \( P(x, k) = \sum_{m=k}^n p(x; m, n) \) denote the principal’s expected payoff from the alternative, and the probability of its acceptance, respectively, where \( p(x; m, n) = \binom{n}{m}[1 - F(x)]^m[F(x)]^{n-m} \) is the binomial probability. Then,

(a) \( P_x(x, k) = -nf(x)p(x; k - 1, n - 1) \)

(b) \( \overline{w}^A(x, k) = p(x; k - 1, n - 1) \int_0^\pi \theta dF(\theta) \)

(c) \( \overline{w}_x^A(x, k) = \frac{P_x(x, k)}{n} \{ x + (k - 1)E^+[x] + (n - k)E^-[x] \} \).

**Proof.** For conciseness, let \( \phi = 1 - F(x) \) in this proof. Part (a) follows because
\[
\frac{\partial}{\partial \phi} \left( \sum_{i=k}^n \binom{n}{i} \phi^i (1-\phi)^{n-i} \right) = n \binom{n-1}{k-1} \phi^{k-1} (1-\phi)^{n-k}.
\]
For part (b), we re-write
\[
\overline{w}^A(x; k, n) = \Lambda(\phi, x, k, n)
\]
\[
= \sum_{i=k}^n \binom{n}{i} \phi^i (1-\phi)^{n-i} \left[ \frac{iE^+(x) + (n-i)E^-(x)}{n} \right]
\]
\[
= \sum_{i=k}^n \binom{n}{i} \phi^i (1-\phi)^{n-i} \left[ \frac{i \int_0^\pi \theta dF(\theta)}{n \phi} + \frac{n - i \int_0^\pi \theta dF(\theta)}{n (1-\phi)} \right].
\]
Given that 
\[
\frac{i}{n} \binom{n}{i} = \binom{n-1}{i-1}, \quad \frac{n-i}{n} \binom{n}{i} = \binom{n-1}{i}
\]
and 
\[
\sum_{i=k}^{n} \binom{n-1}{i-1} \phi^{i-1} (1-\phi)^{n-i} = \sum_{i=k}^{n-1} \binom{n-1}{i} \phi^{i} (1-\phi)^{n-1-i},
\]
we have
\[
\Lambda(\phi, x, k, n) = \int_{x}^{\overline{\theta}} \theta dF(\theta) \left[ \sum_{i=k}^{n-1} \binom{n-1}{i} \phi^{i} (1-\phi)^{n-1-i} \right] + \int_{x}^{\overline{\theta}} \theta dF(\theta) \left[ \sum_{i=k}^{n-1} \binom{n-1}{i} \phi^{i} (1-\phi)^{n-1-i} \right]
\]
\[
= \left( \int_{x}^{\overline{\theta}} \theta dF(\theta) + \int_{x}^{\overline{\theta}} \theta dF(\theta) \right) \left[ \sum_{i=k}^{n-1} \binom{n-1}{i} \phi^{i} (1-\phi)^{n-1-i} \right]
\]
\[
+ \int_{x}^{\overline{\theta}} \theta dF(\theta) \left[ \binom{n-1}{k-1} \phi^{k-1} (1-\phi)^{n-k} \right]
\]
\[
= E[\theta] \sum_{i=k}^{n-1} \binom{n-1}{i} \phi^{i} (1-\phi)^{n-1-i} + \int_{x}^{\overline{\theta}} \theta dF(\theta) \left[ \binom{n-1}{k-1} \phi^{k-1} (1-\phi)^{n-k} \right].
\]

Given that \(E[\theta] = 0\), we obtain the desired expression:
\[
\Lambda(\phi, x, k, n) = \binom{n-1}{k-1} \phi^{k-1} (1-\phi)^{n-k} \int_{x}^{\overline{\theta}} \theta dF(\theta) \quad \text{(A-6)}
\]
or equivalently,
\[
\overline{w}^{\Lambda}(x, k) = p(x; k-1, n-1) \int_{x}^{\overline{\theta}} \theta dF(\theta). \quad \text{(A-7)}
\]

To prove part (c), recall \(\phi = 1 - F(x)\) and differentiate (A-6) with respect to \(x\):
\[
\overline{w}^{\Lambda}_{x}(x, k) = \Lambda_{\phi}(\cdot) \frac{\partial \phi}{\partial x} + \Lambda_{x}(\cdot)
\]
\[
= -f(x) \left[ \binom{n-1}{k-1} \left[ (k-1) \phi^{k-2} (1-\phi)^{n-k} - (n-k) \phi^{k-1} (1-\phi)^{n-1-k} \right] \int_{x}^{\overline{\theta}} \theta dF(\theta) \right]
\]
\[
+ \binom{n-1}{k-1} \phi^{k-1} (1-\phi)^{n-k} (-xf(x)).
\]

Since \(\int_{x}^{\overline{\theta}} \theta dF(\theta) = \phi E^{+}(x)\), and \(\phi E^{+}(x) + (1-\phi) E^{-}(x) = E[\theta] = 0\), we further have
\[
\Lambda_{\phi}(\cdot) \frac{\partial \phi}{\partial x} + \Lambda_{x}(\cdot) = -f(x) \left\{ \binom{n-1}{k-1} \phi^{k-1} (1-\phi)^{n-k} \left[ (k-1) E^{+}(x) + (n-k) E^{-}(x) \right] \right. \]
\[
\left. + \binom{n-1}{k-1} \phi^{k-1} (1-\phi)^{n-k} x \right\}
\]
or equivalently,
\[ w_A^x(x; k) = -f(x) \left( \frac{n-1}{k-1} \right) \phi^{k-1}(1-\phi)^{n-k} \{ (k-1)E^+(x) + (n-k)E^-(x) + x \}, \]
which, after substituting for \( \phi = 1 - F(x) \), produces the expression for \( w_A^x(x; k) \) in part (c).

**Lemma A2.** \( w(x; k, w^0) \) is strictly quasi-concave or single-peaked in \( x \).

**Proof.** By Lemma A1, the ex ante payoff in (7) can be re-stated:
\[
\bar{w}(x; k, w^0) = \bar{w}^A(x, k) + [1 - P(x, k)]w^0. \tag{A-8}
\]
Differentiating with respect to \( x \) and substituting from Lemma A1, we find
\[
\bar{w}_x(x; k, w^0) = \frac{P_x(x, k)}{n} \{ x + (k-1)E^+[x] + (n-k)E^-[x] - nw^0 \}
= \frac{P_x(x, k)}{n} \left( H(x, k) - nw^0 \right) \tag{A-9}
\]
where
\[
H(x, k) = x + (k-1)E^+[x] + (n-k)E^-[x]. \tag{A-10}
\]
Clearly, \( H_x(x, k) > 0 \) since \( E^+[x] > 0 \) and \( E^-[x] > 0 \) by (A-1). Moreover, since \( P_x(x, k) < 0 \) by Lemma A1(a), further differentiation yields
\[
\operatorname{sgn} \left[ w_{xx}(x; k, w^0) \right|_{w_x(x; k) = 0} = \operatorname{sgn} [-H_x(x, k)] < 0.
\]
Hence, \( \bar{w}(x; k, w^0) \) is strictly quasi-concave or single-peaked in \( x \).

**Proof of Corollary 1.** Let \( \beta < 1 \). Simple differentiation shows
\[
\frac{\partial P(\theta^*(k, \beta), k)}{\partial \beta} = P_x(\theta^*(k, \beta), k) \times \theta^*_\beta(k, \beta).
\]
Since \( P_x(.) < 0 \) by Lemma A1(a), Proposition 1(c) implies
\[
\operatorname{sgn} \left[ \frac{\partial P(\theta^*(k, \beta), k)}{\partial \beta} \right] = \operatorname{sgn} \left[ -\theta^*_\beta(k, \beta) \right]
= \operatorname{sgn} [\kappa^* - k].
\]

■
Appendix B: Proofs for Section 4

Proof of Lemma 2. As mentioned in the text, the existence of a solution to (8) follows because \( w(x; \cdot) \) is continuous and the feasible set, \([\theta, \bar{\theta}]\), is compact. Its uniqueness follows from Lemma A2. To establish when the solution is interior, i.e., \( \theta^o \in (\theta, \bar{\theta}) \), note from (A-9) that

\[
\text{sgn} \left[ \bar{w}_x(x; k, w^0) \right] = \text{sgn} \left[ nw^0 - H(x, k) \right], \tag{B-1}
\]

since \( P_x(x, k) < 0 \) by Lemma A1(a). Moreover, since \( \underline{\theta} < 0 < \overline{\theta} \),

\[
H(\theta, k) = (n - k + 1)\theta < 0 \quad \text{and} \quad H(\bar{\theta}, k) = k\bar{\theta} > 0.
\]

Thus,

\[
\bar{w}_x(\theta, k, w^0) > 0 \quad \text{and} \quad \text{sgn} \left[ \bar{w}_x(\theta, k, w^0) \right] = \text{sgn} \left[ nw^0 - k\theta \right].
\]

From here (and the single-peakedness by Lemma A2), \( \theta^o \in (\theta, \bar{\theta}) \) if and only if \( \bar{w}_x(\theta, k, w^0) < 0 \), or equivalently \( w^0 < \frac{k}{n}\theta \). As such, \( \theta^o = \bar{\theta} \) whenever \( w^0 \geq \frac{k}{n}\theta \). \( \blacksquare \)

Proof of Proposition 2. Suppose \( w^0 < \frac{\overline{\theta}}{n} \). Then, \( w^0 < \frac{k}{n}\theta \) for all \( k \). By Lemma 2, the unique solution \( \theta^o(k, w^0) \) to (8) is interior, and by (B-1), it satisfies the following first-order condition:

\[
nw^0 - H(x, k) = 0, \tag{B-2}
\]

where \( H(x, k) \) is defined in (A-10).

Note that \( H_x(x, k) > 0 \), as established in the proof of Lemma A2, and that \( H(x, k) \) is strictly increasing in \( k \) given that \( E^-[x] < E^+[x] \). Hence, (B-2) implies that \( \theta^o(k, w^0) \) is strictly decreasing in \( k \) and strictly increasing in \( w^0 \), as claimed in part (a).

To show part (b), first observe that

\[
H(0, k) = (k - 1)E^+[0] + (n - k)E^-[0] \\
= k(E^+[0] - E^-[0]) - E^+[0] + nE^-[0] \\
= (E^+[0] - E^-[0]) \left( k - \frac{E^+[0]}{E^+[0] - E^-[0]} + n \frac{E^-[0]}{E^+[0] - E^-[0]} \right). \tag{B-3}
\]

Inserting (A-4) and (A-5) into (B-3), we find

\[
H(0, k) = (E^+[0] - E^-[0]) (k - F(0) - n(1 - F(0))) \\
= (E^+[0] - E^-[0])(k - \kappa^*).
\]
Setting $x = 0$ in (B-2) and solving $H(0,k) = nw^0$ for $k$, we obtain

$$\kappa^0(w^0) = \kappa^* + \frac{nw^0}{E^+[0] - E^-[0]}.$$  

Finally, since $\theta^0(\kappa^0(w^0), w^0) = 0$ and the fact that $\theta^0(k, w^0)$ is strictly decreasing in $k$ by part (a), we conclude that $\text{sgn} \left[ \theta^0(k, w^0) \right] = \text{sgn} \left[ \kappa^0(w^0) - k \right].$  

**Proof of Proposition 3.** Let $\overline{w}(\theta^*(k, \beta); k, w^0)$ be the principal’s expected equilibrium payoff. Recall from Lemma A2 above, which is depicted in Figure 2, that $\overline{w}(x; k, w^0)$ is single-peaked in $x$. In particular, $\overline{w}_x(x; k, w^0) > 0$ for $x < \theta^0(k, w^0)$, and $\overline{w}_x(x; k, w^0) < 0$ for $x > \theta^0(k, w^0)$.

Suppose $k \in [1, \kappa^*)$. Then, by Propositions 1(c) and 2(b), we have

$$0 < \theta^*(k, \beta) \leq \theta^*(k, 0) = \theta^0(k, 0) < \theta^0(k, w^0),$$

where the equality is due to (B-2) and the fact that (4) can be re-stated as: $H(\theta^*, k) = -n \frac{\beta}{1-\beta} \theta^*$. Since $\theta^* < \theta^0$, and $\theta^*_\beta < 0$ by Proposition 1(c), we have

$$\frac{d}{d\beta} \overline{w}(\theta^*; k, w^0) = \overline{w}_x(\theta^*; k, w^0) \times \theta^*_\beta < 0.$$

Now suppose $k \in (\kappa^*, \kappa^0(w^0)]$. Then, $\overline{w}(\theta^*; k, w^0)$ is strictly increasing in $\beta$ since, in this case,

$$\theta^*(k, \beta) < 0 \leq \theta^0(k, w^0),$$

but $\theta^*_\beta > 0$.

Last, suppose $k \in (\kappa^0(w^0), n]$. In this case, the optimal voting can be implemented in equilibrium for some $\beta$. To find it, we set $\theta^*(k, \beta) = \theta^0(k, w^0)$ and employ (B-2), simplifying (4) to:

$$\beta \theta^0(\cdot) + \frac{1-\beta}{n} nw^0 = 0$$

and yielding

$$\overline{\beta}(k, w^0) = \frac{w^0}{w^0 - \theta^0(k, w^0)}.$$  

Since $\theta^0(\cdot) < 0$ and it is strictly decreasing in $k$, we have that $\overline{\beta}(k, w^0) \in (0, 1)$ and it is strictly decreasing in $k$. Moreover, since $\theta^*(k, \overline{\beta}(\cdot)) = \theta^0(k, w^0)$ (by definition); $\theta^*_\overline{\beta}(k, \beta) > 0$ (by Proposition 1); and $\theta^*_{\overline{\beta}}(k, w^0) > 0$ (by Proposition 2), $\overline{\beta}(k, w^0)$ is strictly increasing in $w^0$. Finally, since $\theta^*_\beta(k, \beta) > 0$,

$$\text{sgn} \left[ \theta^*(k, \beta) - \theta^0(k, w^0) \right] = \text{sgn} \left[ \beta - \overline{\beta}(k, w^0) \right].$$
Hence, by Lemma A2, \( w(\theta^*(k, \beta); \cdot) \) is strictly increasing in \( \beta \) for \( \beta < \overline{\beta}(k, w^0) \), and strictly decreasing in \( \beta \) for \( \beta > \overline{\beta}(k, w^0) \), as claimed. ■

**Proof of Corollary 2.** Let \( \beta^0(k, w^0) = \arg \max_{\beta \in [0,1]} w(\theta^*(k, \beta); k, w^0) \). Suppose \( k \in [1, \kappa^*] \). Then, by Proposition 3, \( w(\theta^*(k, \beta); \cdot) \) is strictly decreasing in \( \beta \), and thus \( \beta^0(k, w^0) = 0 \). For \( k > \kappa^* \), the corollary is also immediate from Proposition 3. For \( k = \kappa^* \), we know from Proposition 1(c) that \( \theta^*(\kappa^*, \beta) = 0 \) for all \( \beta \), so the principal is indifferent to the committee composition, i.e., \( \beta^0(\kappa^*, w^0) \in [0,1] \).

Before proving Corollary 3, we report the following result similar to Proposition 3.

**Proposition B1. (negative status quo bias)** Suppose \( w^0 < 0 \). The principal’s ex ante equilibrium payoff \( w(\theta^*(k, \beta); k, w^0) \) is strictly decreasing in \( \beta \) for \( k \in (\kappa^*, n] \) but strictly increasing for \( k \in [\kappa^0(w^0), \kappa^*) \). Finally, for \( k \in [1, \kappa^0(w^0)) \), there is a unique cutoff \( \overline{\beta}(k, w^0) \in (0, 1) \) such that her equilibrium payoff is strictly increasing in \( \beta \) for \( \beta < \overline{\beta}(k, w^0) \), and strictly decreasing in \( \beta \) for \( \beta > \overline{\beta}(k, w^0) \).

**Proof.** By replacing \( w^0 \) with \( -w^0 \), the proof mimics that of Proposition 3. In particular, it is evident from Lemma A2 that \( w(x; \cdot) \) continues to be single-peaked except that \( \kappa^0(w^0) < \kappa^* \).

■

**Proof of Corollary 3.** As in Corollary 2, the result directly follows from Proposition B1. ■

**Appendix C: Proofs for Section 5**

**Proof of Lemma 3.** From (15), it is clear that \( \kappa^l(\lambda) \) is increasing, and \( \kappa^h(\lambda) \) is decreasing in \( \lambda \). Moreover, \( \kappa^l(1) = \kappa^h(1) = 1 + (n - 1)[1 - F(0)] \), which is equal to \( \kappa^* \) by (A-5). Finally, \( \kappa^l(0) = 1 \) and \( \kappa^h(0) = n \), as claimed. ■

The following lemma is useful in the proof of Proposition 4.

**Lemma C1.** \( E^+[x, \pi; \lambda] \) and \( E^-[x, \pi; \lambda] \) are both increasing in \( x \) if one of the following holds: (1) \( x > 0 \) and \( \pi = 0 \), (2) \( x < 0 \) and \( \pi = 1 \), or (3) \( x = 0 \).

**Proof.** Using (9) and (10), we derive

\[
\frac{\partial}{\partial x} E^+[x, \pi; \lambda] = \frac{\lambda f(x)}{\phi(x, \pi; \lambda)} (E^+[x, \pi; \lambda] - x) \quad \text{and} \quad \frac{\partial}{\partial x} E^-[x, \pi; \lambda] = \frac{\lambda f(x)}{1 - \phi(x)} (x - E^-[x, \pi; \lambda]).
\]

Suppose \( x > 0 \) and \( \pi = 0 \). Then, clearly \( E^+[x, 0; \lambda] > x \) and \( x > 0 > E^-[x, 0; \lambda] \). Next, if \( x < 0 \) and \( \pi = 1 \), then \( E^+[x, 1; \lambda] > 0 > x \) and \( x > E^-[x, 1; \lambda] \). In both cases, \( \frac{\partial}{\partial x} E^+[x, \pi; \lambda] > 0 \) and \( \frac{\partial}{\partial x} E^-[x, \pi; \lambda] > 0 \). Finally, if \( x = 0 \), the claim trivially follows since \( E^+[0, \pi; \lambda] > 0 > E^-[0, \pi; \lambda] \). ■
In what follows, let
\[ V(x, \pi; k, \beta, \lambda) = (\beta + \frac{1-\beta}{n})x + \frac{1-\beta}{n} h(x, \pi; k, \lambda) \] (C-1)

where
\[ h(x, \pi; k, \lambda) = (k - 1)E^+[x, \pi; \lambda] + (n - k)E^-[x, \pi; \lambda], \]
as defined in (11).

**Proof of Proposition 4.** Let \( \beta < 1 \). The equilibrium characterization is by construction. First, suppose that \( h(\theta^*, \pi^*; k, \lambda) = 0 \). Then, \( \theta^* = 0 \) by (12), and solving \( h(0, \pi^*; k, \lambda) = 0 \) for \( \pi^* \), we find \( \pi^* = \frac{k-1}{1-\lambda(1-F(0))} \). Since \( \pi^* \) is strictly increasing in \( k \), \( \pi^* \in [0,1] \) if and only if \( \kappa^l(\lambda) \leq k \leq \kappa^h(\lambda) \) where \( \kappa^l(\lambda) \) and \( \kappa^h(\lambda) \) are as defined in (15).

Next, take \( k < \kappa^l(\lambda) \). Then, \( \pi^* = 0 \) or 1. Suppose \( \pi^* = 1 \). Then, \( h(\theta^*, 1; k, \lambda) > 0 \), which, from (12), implies \( \theta^* < 0 \). Now note the following two properties: (1) \( h(x, \pi; k, \lambda) \) is strictly increasing in \( k \) since \( E^+[x, \pi; \lambda] > E^-[x, \pi; \lambda] \) for all \( x \), and (2) by Lemma C1, \( h(x, 1; k, \lambda) \) is strictly increasing in \( x \) for \( x \leq 0 \). Therefore,
\[ h(\theta^*, 1; k, \lambda) < h(0, 1; \kappa^l(\lambda), \lambda) < h(0, 0; \kappa^l(\lambda), \lambda) = 0, \]
where the equality follows from the definition of \( \kappa^l(\lambda) \). Hence, \( h(\theta^*, 1; k, \lambda) < 0 \), a contradiction. This means that \( \pi^* = 0 \) for \( k < \kappa^l(\lambda) \). Then, \( h(\theta^*, 0; .) < 0 \) and, in turn, \( \theta^* > 0 \). To show that for \( k < \kappa^l(\lambda) \), such an equilibrium exists and it is unique, note from (C-1) that
\[ V(0, 0; k, \beta, \lambda) = \frac{1-\beta}{n} h(0, 0; k, \lambda) = \frac{1-\beta}{n} \left\{ (k-1) \int_0^\theta \vartheta dF(\vartheta) + (n-k) \frac{\lambda}{1-\lambda(1-F(0))} \right\}. \]

Since \( E[\theta] = 0 \), we have \( \int_0^\theta \vartheta dF(\vartheta) = -\int_0^\theta \vartheta dF(\vartheta) \). As a result,
\[ sgn \left[ V(0, 0; k, \beta, \lambda) \right] = sgn \left[ \frac{k-1}{1-F(0)} - (n-k) \frac{\lambda}{1-\lambda(1-F(0))} \right] = sgn \left[ k - \kappa^l(\lambda) \right] < 0. \]

On the other hand,
\[ V(\theta, 0; k, \beta, \lambda) = (\beta + \frac{1-\beta}{n})\theta + \frac{1-\beta}{n} h(\theta, 0; k, \lambda) > 0 \]

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because \( \overline{\theta} > 0 \), \( E^+ [\overline{\theta}, 0; \lambda] > 0 \), and \( E^- [\overline{\theta}, 0; \lambda] = 0 \), implying \( h(\overline{\theta}, 0; k, \lambda) > 0 \). Hence, there is \( \theta^* \in (0, \overline{\theta}) \) that solves: \( V(\theta^*, 0; k, \beta, \lambda) = 0 \). The solution is also unique since \( V_x(x, 0; k, \beta, \lambda) > 0 \) due to the fact that \( h_x(x, 0; k, \lambda) \) for \( x > 0 \) by Lemma C1.

Finally, implicitly differentiating \( V(\theta^*, 0; k, \beta, \lambda) = 0 \) with respect to \( \beta \), we find that for \( k < \kappa^l(\lambda) \),

\[
\theta^*_\beta = -\frac{V_\beta(\theta^*, 0; k, \beta, \lambda)}{V_x(\theta^*, 0; k, \beta, \lambda)} < 0,
\]

where \( V_\beta(\cdot) > 0 \) since \( \theta^* > 0 \) and \( h(\theta^*, 0; k, \lambda) < 0 \). Equivalently, \( \text{sgn} \left[ \theta^*_\beta \right] = \text{sgn} [-\theta^*] \) for \( k < \kappa^l(\lambda) \), as claimed in part (b). Together with the fact that \( \pi^* = 0 \) is independent of \( \beta \), the proof is complete for \( k < \kappa^l(\lambda) \).

A similar line of argument also proves the claims in the proposition for \( k > \kappa^h(\lambda) \). The conclusions in parts (a) and (b) trivially hold for \( \kappa^l(\lambda) \leq k \leq \kappa^h(\lambda) \) since both \( \pi^* \) and \( \theta^* \) are independent of \( \beta \) in this region of \( k \).

**Proof of Lemma 4.** We first show that \( \overline{\kappa}^o(w^0, \lambda) = \kappa^h(\lambda) + E^+ [0, 1; \lambda] + \frac{nw^0}{E^+ [0, 1; \lambda] - E^- [0, 1; \lambda]} \), as defined in Lemma 4, uniquely solves

\[
h(0, \pi^*(k, \lambda); k, \lambda) = nw^0.
\]

To do so, let \( \overline{\kappa} \) be a solution to (C-2). We prove that \( \overline{\kappa} > \kappa^h \). Suppose, to the contrary, that \( \overline{\kappa} \leq \kappa^h \). In particular, suppose \( \overline{\kappa} \in [\kappa^l, \kappa^h] \). Then, \( \theta^* = 0 \) by Proposition 4, and, in turn, \( h(0, \pi^*(\overline{\kappa}, \lambda); \overline{\kappa}, \lambda) = 0 \) by (12), a contradiction. Next, suppose \( \overline{\kappa} < \kappa^l \). Then, \( \pi^*(\overline{\kappa}, \lambda) = \pi^*(\kappa^l, \lambda) = 0 \) by Proposition 4, and because \( h(0, 0; k, \lambda) \) is strictly increasing in \( k \), \( h(0, 0; \overline{\kappa}, \lambda) < h(0, 0; \kappa^l, \lambda) = 0 < nw^0 \), a contradiction. Hence, \( \overline{\kappa} > \kappa^h \), which implies \( \pi^*(\overline{\kappa}, \lambda) = 1 \). Given that \( h(0, 1; k, \lambda) \) is also strictly increasing in \( k \), there is a unique \( k \) that solves \( h(0, 1; k, \lambda) = nw^0 \). Recalling, by definition, that

\[
h(0, 1; k, \lambda) = (k - 1)E^+ [0, 1; \lambda] + (n - k)E^- [0, 1; \lambda],
\]

we find

\[
\overline{\kappa}^o(w^0, \lambda) = \frac{E^+ [0, 1; \lambda] - nE^- [0, 1; \lambda]}{E^+ [0, 1; \lambda] - E^- [0, 1; \lambda]} + \frac{nw^0}{E^+ [0, 1; \lambda] - E^- [0, 1; \lambda]}.\]

Though not obvious from the definition of \( \kappa^h(\lambda) \) in (15), it can also be written as:

\[
\kappa^h(\lambda) = \frac{E^+ [0, 1; \lambda] - nE^- [0, 1; \lambda]}{E^+ [0, 1; \lambda] - E^- [0, 1; \lambda]}
\]

since it solves: \( h(0, 1; \kappa^h, \lambda) = 0 \). Thus, \( \overline{\kappa}^o(w^0, \lambda) = \kappa^h(\lambda) + \frac{nw^0}{E^+ [0, 1; \lambda] - E^- [0, 1; \lambda]} \), as claimed. From here, parts (a), (b), and (c) follow.

Finally, we prove that \( \text{sgn} \left[ \theta^o(k, w^0, \lambda) \right] = \text{sgn} \left[ \overline{\kappa}^o(w^0, \lambda) - k \right] \). To this end, recall the following definition from the text:

\[
\overline{w}(x, \pi^*(k, \lambda); w^0, \lambda) = \overline{w}^A(x, \pi^*(k, \lambda); k, \lambda) + (1 - P(x, \pi^*(k, \lambda); k, \lambda)) w^0,
\]
where
\[ \overline{w}^A(x, \pi^*(k, \lambda); k, \lambda) = \sum_{m=k}^{n} p(x, \pi^*(k, \lambda); m, \lambda)w^A(x, \pi^*(k, \lambda); m, \lambda) \]
and
\[ P(x, \pi^*(k, \lambda); k, \lambda) = \sum_{m=k}^{n} p(x, \pi^*(k, \lambda); m, \lambda). \]

By mimicking the proof of Lemma A1(b), it is readily established that
\[ \overline{w}^A(x, \pi^*(k, \lambda); k, \lambda) = \lambda p(x, \pi^*(k, \lambda); k - 1, n - 1, \lambda) \int_{x}^{\bar{\theta}} \theta dF(\theta). \] (C-3)

Moreover, by mimicking the proof of Lemma A1(c), we find that
\[ \overline{w}^A_0(x, \pi^*(k, \lambda); k, \lambda) = \lambda \frac{P(x, \pi^*(k, \lambda); k, \lambda)}{n} \times \{ x + (k - 1)E^+ [x, \pi^*(k, \lambda); \lambda] + (n - k)E^- [x, \pi^*(k, \lambda); \lambda] \}. \]

Differentiating \( \overline{w}(x, \pi^*(k, \lambda); w^0, \lambda) \) with respect to \( x \) as in the proof of Lemma A2, we, therefore, obtain
\[ \text{sgn} [\overline{w}(x; .)] = \text{sgn} [nw^0 - x - h(x, \pi^*(k, \lambda); k, \lambda)]. \] (C-4)

Suppose that for a given \( k, \bar{\theta}^0(.) \) solves \( \overline{w}_x(x; .) = 0 \), or, equivalently,
\[ \bar{\theta}^0(.) = nw^0 - h(\bar{\theta}^0(.), \pi^*; k, \lambda). \] (C-5)

To establish that \( \bar{\theta}^0(.) > 0 \) for \( k < \bar{\kappa}^o \), suppose, to the contrary, that \( \bar{\theta}^0(.) < 0 \) for some \( k' < \bar{\kappa}^o \) (since \( \bar{\theta}^0(.) = 0 \) if and only if \( k = \bar{\kappa}^o \)). Then, \( \bar{\theta}^0(k, w^0, \lambda) < 0 \) for any \( k < \bar{\kappa}^o \).

Thus, by (C-5), \( h(\bar{\theta}^0(k, w^0, \lambda), 1; k, \lambda) > nw^0 \) for any \( k \in (k^0, \bar{\kappa}^o) \). Now note the following two properties for \( h(x, \pi; k, \lambda) \) for \( k \) in \( (k^0, \bar{\kappa}^o) \): (1) \( h(x, \pi; k, \lambda) \) is strictly increasing in \( k \); and (2) by Lemma C1, \( h(x, 1; k, \lambda) \) is strictly increasing in \( x \) for \( x \leq 0 \). These facts imply that \( h(\bar{\theta}^0(k, w^0, \lambda), 1; k, \lambda) < h(0, 1; \bar{\kappa}^o, \lambda) = nw^0 \), a contradiction. As a result, \( \bar{\theta}^0(.) > 0 \) for every \( k < \bar{\kappa}^o \).

Next consider \( k > \bar{\kappa}^o \). Given that \( h_x(0, 1; \bar{\kappa}^o, \lambda) > 0 \), and \( h(x, \pi; k, \lambda) \) is strictly increasing in \( k \), we have \( \bar{\theta}^0(k, w^0, \lambda) < 0 \) for \( k \) close enough to \( \bar{\kappa}^o \). Moreover, since, by Lemma C1, \( h_x(x, 1; k, \lambda) > 0 \) for \( x \leq 0 \), we have \( \bar{\theta}^0(.) > 0 \), as desired.

To prove Proposition 5, we state and prove Proposition C1, which generalizes Proposition 3. The following lemma is useful in doing so.

**Lemma C2.** \( \overline{w}(x, \pi^*(k, \lambda); w^0, \lambda) \) is single-peaked in \( x \) if \( k \leq k^0(\lambda) \) or \( k \geq \bar{\kappa}^o(w^0, \lambda) \).
Proof. Suppose \( k \leq \kappa^i(\lambda) \) or \( k \geq \bar{\kappa}(w^0,\lambda) \). Then, by Proposition 4, \( \pi^* = 0 \) and \( \pi^* = 1 \), respectively. By Lemma C1, \( h_\pi(x,\pi^*;k,\lambda) > 0 \) for: (1) \( k \leq \kappa^i(\lambda) \) and \( x \geq 0 \), or (2) \( k \geq \bar{\kappa}(w^0,\lambda) \) and \( x \leq 0 \). Using (C-4), we, therefore, have

\[
\text{sgn} \left[ \frac{\partial^2 x}{\partial x^2} \bigg|_{x=0} \right] = \text{sgn} \left[ -(h_x(\cdot)+1) \right] < 0,
\]

proving the single-peakedness. \( \blacksquare \)

**Proposition C1.** For \( k \in [1,\kappa^i(\lambda)) \), \( \bar{\omega}^*(\cdot) \) is strictly decreasing in \( \beta \) whereas for \( k \in (\kappa^h(\lambda),\bar{\kappa}(w^0,\lambda)) \], \( \bar{\omega}^*(\cdot) \) is strictly increasing in \( \beta \). Moreover, for \( k \in (\bar{\kappa}(w^0,\lambda),n] \), there is a unique cutoff \( \bar{\theta}(k,w^0,\lambda) \in (0,1) \) such that \( \bar{\omega}^*(\cdot) \) is strictly increasing in \( \beta \) for \( \beta < \bar{\theta}(\cdot) \) and strictly decreasing in \( \beta \) for \( \beta > \bar{\theta}(\cdot) \), where \( \bar{\theta}(\cdot) \) is strictly increasing in \( w^0 \). Finally, for \( k \in [\kappa^i(\lambda),\kappa^h(\lambda)] \) we have that \( \bar{\omega}^*(\cdot) \) is independent of \( \beta \).

**Proof.** Recall from (16) that \( \bar{\omega}^*(k,\beta,w^0,\lambda) = \bar{\omega}(\theta^*(k,\beta,\lambda),\pi^*(k,\lambda);w^0,\lambda) \). Suppose \( k \in [1,\kappa^i(\lambda)) \). Then, by Proposition 4 and Lemma 4, we have

\[
0 < \theta^*(k,\beta,\lambda) \leq \theta^*(k,0,\lambda) = \bar{\theta}^0(k,0,\lambda) < \bar{\theta}^0(k,w^0,\lambda).
\]

Since \( \theta^0_\beta(\cdot) < 0 \) in this region of \( k \), the single-peakedness identified in Lemma C2 implies that \( \bar{\omega}^*(\cdot) \) is strictly decreasing in \( \beta \). Now suppose \( k \in (\kappa^h(\lambda),\bar{\kappa}(w^0,\lambda)] \). Then,

\[
\theta^*(k,\beta,\lambda) < 0 \leq \bar{\theta}^0(k,w^0,\lambda).
\]

Given that (i) by Lemma C1, \( h_\pi(x,1;k,\lambda) > 0 \) whenever \( x \leq 0 \), and (ii) \( h(x,1;k,\lambda) \) is increasing in \( k \), we have that \( \bar{\omega}_\pi(x,\cdot) \) is strictly increasing in \( x \). Thus, since \( \theta^*(k,\beta,\lambda) < 0 \) and \( \theta^0_\beta(k,w^0,\lambda) > 0 \), we conclude that \( \bar{\omega}^*(\cdot) \) is strictly increasing in \( \beta \).

Next, suppose \( k \in (\bar{\kappa}(w^0,\lambda),n] \). Then, by Proposition 4, \( \pi^*(k,\lambda) = 1 \), and from (C-4), \( \bar{\theta}^0(k,w^0,\lambda) \) solves

\[
x + h(x,1;k,\lambda) = nu^0.
\]

By Lemma C1, the left-hand side of (C-6) is strictly increasing in \( x, k, \) and \( \lambda \). Hence, \( \bar{\theta}^0(\cdot) \) is strictly decreasing in \( k \) and \( \lambda \), and strictly increasing in \( w^0 \). Now set \( \theta^*(k,\beta,\lambda) = \bar{\theta}^0(k,w^0,\lambda) \) in (12) and use (C-6) to find

\[
\bar{\theta}(k,w^0,\lambda) = \frac{w^0}{w^0 - \bar{\theta}^0(k,w^0,\lambda)}.
\]

Clearly, since \( \bar{\theta}^0(\cdot) < 0 \), so \( \bar{\theta}(\cdot) \in (0,1) \). Moreover, \( \bar{\theta}(\cdot) \) is strictly decreasing in \( k \) and \( \lambda \). To see that it is strictly increasing in \( w^0 \), simply recall that \( \theta^*(k,\beta,\lambda) = \bar{\theta}^0(k,w^0,\lambda) \) and the fact that
where \( \theta^*(k, w^0, \lambda) > 0 \) in this region. In sum, we find

\[
\text{sgn} \left[ \theta^*(k, \beta, \lambda) - \tilde{\theta}^*(k, w^0, \lambda) \right] = \text{sgn} \left[ \beta - \tilde{\beta}(k, w^0, \lambda) \right].
\]

Hence, by the single-peakedness identified in Lemma C2, \( \overline{\omega}^*(.) \) is strictly increasing in \( \beta \) for \( \beta < \tilde{\beta}(.) \) and strictly decreasing in \( \beta \) for \( \beta > \tilde{\beta}(.) \), as claimed. Finally, for \( k \in [\kappa^l(\lambda), \kappa^h(\lambda)] \), we have from Proposition 4 that \( \theta^*(k, \beta, \lambda) = 0 \) for all \( \beta \). Therefore, \( \overline{\omega}^*(.) \) is independent of \( \beta \).

**Proof of Proposition 5.** Immediate from Proposition C1. □

**Proof of Proposition 6.** Let \( \beta = 1 \). Then, by (12), \( \theta^*(k, 1, \lambda) = 0 \) for all \( \lambda \). From (16) and (C-3), the principal’s ex ante payoff can be written:

\[
\overline{\omega}^*(k, 1, w^0, \lambda) = \overline{\omega}(0, \pi^*; w^0, \lambda) = \lambda p(\phi^*; k - 1, n - 1)S(0) + (1 - P(\phi^*, k))w^0 = \omega(\lambda)
\]

where \( \pi^* = \pi^*(k, \lambda) \), \( \phi^* = \lambda [1 - F(0)] + (1 - \lambda) \pi^* \), \( p(\phi; k - 1, n - 1) = \binom{n-1}{k-1} \phi^{k-1}(1 - \phi)^{n-k} \), \( S(0) = \int_0^\pi dF(\theta) \), and \( P(\phi, k) = \sum_{m=k}^n p(\phi; m, n) \).

Differentiating with respect to \( \lambda \) and using the fact that \( p(\phi; .) = np(.) \) from Lemma A1, we find

\[
\omega'(\lambda) = \frac{p(.)S(0) + \lambda p(\phi^*)S(0) - p(\phi^*)w^0}{p(.)} = \frac{p(.)[S(0) - n\phi^*w^0] + \lambda p(\phi^*)S(0)}{p(.)} = \frac{p(.)[S(0) - n\phi^*\omega^0 + \lambda \frac{p(\phi)}{p(.)} \phi^*S(0)]}{p(.)},
\]

where

\[
\frac{p(\phi)}{p(.)} = \frac{k - 1}{\phi^*} - \frac{n - k}{1 - \phi^*}.
\]

To establish the result, we prove three claims about the sign of \( \omega'(\lambda) \) across different values of \( k \).

**Claim 1.** (a) \( \omega'(\lambda) < 0 \) if \( k > \overline{\kappa}^0(w^0, \lambda) + 1 - \lambda F(0) \), and (b) \( \omega'(\lambda) > 0 \) if \( \kappa^h(\lambda) < k < \overline{\kappa}^0(w^0, \lambda) + 1 - \lambda F(0) \).

**Proof.** First, recall from Lemma 4 that

\[
\overline{\kappa}^0(w^0, \lambda) = \kappa^h(\lambda) + \frac{n\omega^0}{E^+[0, 1; \lambda] - E^-[0, 1; \lambda]},
\]

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where \( E^+[0, 1; \lambda] - E^-[0, 1; \lambda] = \lambda S(0) \left( \frac{1}{1 - \lambda F(0)} + \frac{1}{\lambda F(0)} \right) \) and \( \kappa^h(\lambda) = 1 + (n - 1)[1 - \lambda F(0)] \) from (10) and (15), respectively.

Next, suppose \( k > \kappa^h(\lambda) \). Proposition 4 implies that \( \pi^* = 1 \) and, in turn, \( \phi^* = 1 - \lambda F(0) \) and \( \phi^*_\lambda = -F(0) \). Then, (C-7) implies that \( \omega'(\lambda) < 0 \) if and only if
\[
\frac{k - 1}{1 - \lambda F(0)} - \frac{n - k}{\lambda F(0)} > \frac{1}{\lambda} \left[ \frac{1}{F(0)} + \frac{n w^0}{S(0)} \right]
\]
or, after some algebra, if and only if
\[
k > \tilde{\kappa}^*(w^0, \lambda) + 1 - \lambda F(0).
\]

Part (b) similarly follows. □

Claim 2. \( \omega'(\lambda) > 0 \) if \( \kappa^l(\lambda) \leq k \leq \kappa^h(\lambda) \).

Proof. For \( \kappa^l(\lambda) \leq k \leq \kappa^h(\lambda) \), Proposition 4 reveals that \( \phi^* = \frac{k - 1}{n - 1} \), which implies \( \phi^*_\lambda = 0 \). Hence, from (C-7),
\[
\omega'(\lambda) = p(.) S(0) > 0. \quad \Box
\]

Claim 3. Suppose \( k < \kappa^* \). Then, \( \omega'(1) < 0 \).

Proof. Suppose \( k < \kappa^* \), and recall that \( \kappa^* = \kappa^l(1) \). From Proposition 4, \( \pi^* = 0 \) and, in turn, \( \phi^* = \lambda[1 - F(0)] \) and \( \phi^*_\lambda = 1 - F(0) \). Then, (C-7) implies that \( \omega'(1) < 0 \) if and only if
\[
\frac{n - k}{F(0)} - \frac{k - 1}{1 - F(0)} > \frac{1}{1 - F(0)} - \frac{n w^0}{S(0)}
\]
or, equivalently,
\[
k < \kappa^* - F(0) \left( 1 - \frac{n w^0}{1 - F(0)} \right).
\]

Note that \( \kappa^* \geq \kappa^* - F(0) \), implying \( \kappa^* > \kappa^* - 1 \). Since \( \kappa^* \) is assumed to be integer, \( \omega'(1) < 0 \) for \( k < \kappa^* \). □

Armed with Claims 1-3, the conditions for \( \lambda^o = 1 \) and \( \lambda^o \neq 1 \) follow from Lemmas 3 and 4. Then, for \( k = n \), we have that \( \lambda^o \neq 1 \) if and only if \( \kappa^o(w^0) + 1 - F(0) < n \). Substituting for \( \kappa^o(w^0) \) from Proposition 2, straightforward algebra reveals that \( \lambda^o \neq 1 \) for \( k = n \) if and only if \( w^0 < \left( 1 - \frac{1}{\lambda F(0)} \right) E^+[0] \). ■

Proof of Proposition 7. Let \( \lambda = 1 \) so that all experts are informed. Also let the principal appoint \( m \in \{1, \ldots, n\} \) experts to the committee. Extending Lemma A1 and abusing the notation to introduce \( m \), her expected payoff is found to be
\[
\bar{w}(x; k, m) = \frac{m}{n} p(x; k - 1, m - 1) \int_x^\infty \theta dF(\theta) + (1 - P(x; k, m)) w^o. \quad \text{(C-8)}
\]
Now suppose $\beta = 1$. Then, $\theta^* = 0$. For $k = 1$ (dictatorship), the optimal committee size solves

$$m^o = \arg \max_{1 \leq m \leq n} \bar{w}(0; 1, m).$$

Note that $p(0; 0, m - 1) = F^{m-1}(0)$ and $P(0; 1, m) = 1 - F^m(0)$. Thus, since $\int_0^{\theta} \theta dF(\theta) = (1 - F(0))E^+[0]$,

$$\bar{w}(0; 1, m) = \frac{m}{n} F^{m-1}(0)(1 - F(0))E^+[0] + F^m(0)w^0.$$

Treating $m$ as a continuous variable and differentiating with respect to $m$, we obtain

$$\frac{\partial \bar{w}(0; 1, m)}{\partial m} = \ln(F(0))F^{m-1}(0)(1 - F(0)) \left( m + \frac{1}{\ln(F(0))} + \frac{F(0)nw^0}{(1 - F(0))E^+[0]} \right)$$

$$= \ln(F(0))F^{m-1}(0)(1 - F(0)) \frac{E^+[0]}{n} (m - \bar{m}^o)$$

where

$$\bar{m}^o = -\frac{1}{\ln(F(0))} + \frac{F(0)}{(1 - F(0))E^+[0]} nw^0.$$

Moreover,

$$\frac{\partial^2 \bar{w}(0; 1, m)}{\partial m^2} \bigg|_{m=\bar{m}^o} = \ln(F(0))F^{m^o-1}(0)(1 - F(0)) \frac{E^+[0]}{n} < 0.$$

Hence, the optimal committee size is

$$m^o = \min\{\bar{m}^o, n\}.$$

Straightforward algebra reveals that $m^o < n$ if and only if $\bar{m}^o < n$, or

$$n > n_1^* = \frac{\frac{1}{\ln(F(0))}}{1 + \frac{F(0)nw^0}{(1 - F(0))E^+[0]}}.$$

Clearly, $0 < n_1^* < \infty$.

Next, for $k = n$ (unanimity),

$$\bar{w}(0; m, n) = \frac{m}{n} (1 - F(0))^m E^+[0] + [1 - (1 - F(0))^m] w^0.$$

In this case, we find

$$\bar{m}^o = -\frac{1}{\ln(1 - F(0))} + \frac{w^0}{E^+[0]} n.$$

Hence, $m^o < n$ if and only if $\bar{m}^o < n$, or

$$n > n_n^* = \frac{\frac{1}{\ln(1 - F(0))}}{1 - \frac{w^0}{E^+[0]}}.$$
Note that $0 < n^*_n < \infty$ if $w^0 < E^+[0]$. The latter condition is satisfied; otherwise, the principal would not appoint a committee for $w^0 \geq E^+[0]$.

To prove the last part of the proposition, suppose that $k = \lceil \frac{m+1}{2} \rceil$ (the simple majority), $n$ is odd, and $F(0) = \frac{1}{2}$. To establish $m^o = n$, we first show that $m^o$ cannot be an even integer. To see this, pick an even integer $m_1 \leq n - 1$, and let $m_2 = m_1 + 1$. Then,

$$k_1 = k_2 - 1 = \frac{m_1}{2},$$

and thus,

$$\frac{p(0; k_1 - 1, m_1 - 1) \frac{m_1}{n}}{p(0; k_2 - 1, m_2 - 1) \frac{m_2}{n}} = \frac{m_1}{m_1 + 1} < 1. \quad (C-9)$$

Moreover, since $m_1$ is even and $F(0) = \frac{1}{2}$,

$$P(0; k_2, m_2) = \frac{1}{2} < P(0; k_1, m_1). \quad (C-10)$$

From (C-9) and (C-10), (C-8) implies that

$$\bar{w}(0; k_1, m_1) < \bar{w}(0; k_2, m_2).$$

Thus, the optimal committee size $m^o$ must be odd.

Next suppose that $m_1 \leq n - 2$ is odd and $m_2 = m_1 + 2$. Clearly, $k_1 = \lceil \frac{m_1 + 1}{2} \rceil = \frac{m_1 + 1}{2}$ and $k_2 = k_1 + 1$. Thus, after some algebra,

$$\frac{p(0; k_1 - 1; m_1 - 1) \frac{m_1}{n}}{p(0; k_2 - 1; m_2 - 1) \frac{m_2}{n}} = \frac{\frac{m_1 - 1}{2}}{\frac{m_1 + 1}{2}} \frac{\frac{m_1 - 1}{m_1 + 2}}{\frac{m_1 + 1}{m_1 + 2}} \frac{m_1}{m_1 + 2}$$

$$= \frac{m_1 + 1}{m_1 + 2} < 1.$$  

Moreover, $P(0; k_1, m_1) = P(0; k_2, m_2) = \frac{1}{2}$, implying that

$$\bar{w}(0; k_1, m_1) < \bar{w}(0; k_2, m_2).$$

Hence, $m^o$ must be odd and $n - 1 \leq m^o \leq n$. Given that $n$ is odd, $m^o = n$, as claimed. \qed
References


