No Taxation Without Administration: Wealth Assessment in the Formation of the Fiscal State

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November 9, 2018

Abstract. Rulers face serious difficulties in their efforts to extract wealth from society through taxation. Historically, taxation was often low and attempts to increase it frequently caused revolts. Over time, however, taxation has increased dramatically while violent resistance has virtually disappeared. We present a model that shows how these patterns can be understood as arising from the Crown's desire to maximize its income from taxation in a context where it is institutionally unconstrained but lacks valuable information about the wealth of its subjects. We show tax revolts to be an effective means of communication about the burden of taxation. Resistance, especially if it is likely to be futile, provides the Crown with credible evidence that it may need to reduce its demands. While the poor form the base of any violent opposition, the rich occasionally join in resistance to take advantage of tax relief and avoid a tax increase that acquiescence might entail in the future. Several surprising features of the empirical record emerge naturally from our analysis, including the rich joining the poor in resistance, tax relief being provided even after successfully quelling the opposition, and the simultaneous increase in taxation and decrease in violent resistance. The growth of the state can be understood as a direct consequence of administrative improvements in assessing wealth rather than centralization of power, monopolization of violence, or provision of public goods.

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*Corresponding author: slantchev@ucsd.edu. We gratefully acknowledge support from the National Science Foundation (Grant #SES-1153441). Previous versions of this paper were presented at the Political Economy Workshop (New York University), Civil Conflict and Rationality Workshop (Barcelona Graduate School of Economics, Spain), UC Center Sacramento, Security, Peace, and Conflict Workshop (Duke University), the 13th Jan Tinbergen European Peace Science Conference (Milan, Italy), Peace Science Society – ISA Joint Conference (Budapest, Hungary), CISS/United Nations University – CRISS Joint Conference (Bruges, Belgium), the IR Workshop (Stanford University), the Institute for Political Economy and Governance (Barcelona, Spain), the Institute for Political Economy (Waseda University, Tokyo, Japan), and the Graduate School of Decision Sciences (Universität Konstanz, Germany).

For all their fabulous wealth expressed in glittering palaces, sumptuous entertainment, lavish construction, and extravagant military spending, the princes of early modern Europe were burdened by heavy debt and often hovered on the brink of bankruptcy.[1] Their income regularly failed to keep up with their expenditures, and it was the rare prince who managed to accumulate any reserves (which were at any rate squandered by their successors). The perpetual deficits reveal that extracting resources from the populace was difficult—attempts to increase taxes often provoked both local and widespread revolts, which were costly to put down and further injured the Crown's revenue. Raising money from their subjects was a major princely concern even if it was disliked and despised. The history of the rise of the modern centralized bureaucratic state in Europe is often told as a story about the growth of the government's ability to get into the pockets of its citizens.[2]

But why was the Crown so often in financial trouble? It was certainly not because there was no money to be had: although many of its demands were met with cries about poverty and insufficiency, money invariably turned up when subsidies were granted; when the Crown was paying usurious rates on its debts, estates and urban elites could borrow at low rates in their name, and even profit from arbitrage by lending the proceeds to the Crown.[3]

The failure to secure adequate finance is even more puzzling when one accounts for the significant advantages the Crown enjoyed in the asymmetry of its coercive power, extensive legal privilege, large patronage network, and social primacy. It was very difficult in law and in practice to organize serious resistance to a determined monarch's quest for funds. And yet, there was the startling phenomenon of rich elites and broke princes.

The puzzle becomes more mystifying when one notes that the vast majority of violent resistance to taxation came from the lower strata of society—the peasants and the urban poor—that is, precisely from those who stood the best chance of being forcefully suppressed for their trouble. The poor could not organize and coordinate on more than a regional scale, did not have the proper equipment and the training to use it effectively, did not have access to networks of contacts to raise funds for supply and logistics, and found it difficult to articulate resistance messages that would transcend their local interests. And yet they engaged in an activity that was certain to get punished, sometimes quite savagely so.[4]

All of this is capped by the most perplexing trend of all: violent resistance to taxation declined while taxes were going up. In a sense, it is this observation that the scholarly literature on state formation seeks to explain. Previous authors have argued the rise of modern centralized states can be attributed to increased coercive powers of the central authority, enhanced legal capacity, expanding menus of public goods, and financial innovations enabling cheap borrowing.[5] The thing is, in the chronology of the fiscal state, pacification with higher taxation quite often preceded any of these developments and innovations.

[1] Collins (1988, 220–22); Bonney (1981, 273); Bonney (1991, 352–5). The early modern period is usually taken to be between the Middle Ages and the Industrial Revolution, or roughly the 15th through 18th centuries.

[2] Brewer (1990); Yun-Casalilla, O'Brien, and Comín (2012).

[3] Carsten (1959); Tracy (1985); Drelichman and Voth (2016).

[4] Part of the reason is that elites often managed to shift burden of taxation on peasantry and urban dwellers. This was true even when there were no formal exemptions but elites controlled assessments and apportionment of taxes (Blaufarb, 2012, 30–1).

[5] Dickson (1967); Tilly (1992); Glete (2002); Besley and Persson (2009).

In this paper, we show how the fiscal state could have come about solely through improvements in administrative capacity to assess wealth. Our explanation focuses on the interaction of two features of early modern polities: the Crown's moral hazard problem with respect to the taxpayers and the asymmetric information about taxable wealth between the Crown and its subjects.

Throughout the 13th to 18th centuries, the Crown had significant control over how it spent its revenue, including complete control over domain income and customary dues inherited from feudal rights and prerogatives or from perpetual grants. But even with other sources of income, such as extraordinary taxes and temporary grants, the Crown had leeway in how it chose to spend the funds. This was so even in places with representative assemblies and territories where the power of the purse was non-existent, developed slowly, or tended to be tentative.[6] Since there were few institutional checks on its expenditures, the Crown could not commit to spend the funds in a manner consistent with the interests of the taxpayers. This reduced the willingness of the taxpayers to contribute to Crown endeavors.[7]

In theory, the Crown could overcome some of this reluctance by making appropriate promises and then sticking to them. In practice, princes tended to adopt an alternative method: demand as much tax as traffic would bear. When representative institutions were present, this tended to produce elaborate bargaining that generally granted the substance of the demands while extracting some concessions, often regarding privileges of apportionment, collection, and administration.[8] Sometimes, however, agreement could not be reached and the Crown attempted to collect taxes on its own. This is when it encountered the coercive constraint on its demands: the tax revolt.[9]

The Crown could deploy its formidable legal and military advantages in coercive bargaining with its subjects. If the Crown knew the wealth of the taxpayers, it could in principle extract everything above what they could expect to get by revolting. In this, however, the Crown labored under a serious disadvantage: it only had a vague idea about the actual wealth of its subjects. This inability to assess taxable wealth was widespread and, although the Crown did introduce various innovations to cope with it, the taxpayers tended to retain the informational edge. Even when they were willing to comply, taxpayers had to worry whether their behavior today would reveal something that the Crown could then use against them tomorrow. Every deal the Crown tried to offer the taxpayers would be scrutinized for possible repercussions for future taxes. This further reduced the value of taxes to the taxpayers and their willingness to contribute to the treasury.

We demonstrate that in the under-institutionalized political environment where the Crown faces a moral hazard problem in spending and a credible commitment problem in its use of information, the struggle over wealth assessment manifests itself in taxpayers obfuscating the Crown's inferences about their wealth by refusing demands and revolting (sometimes despite being able to pay the tax demanded). We show how the strategic imperatives of the

[6] Attempts to audit the Crown were interpreted as interference in policy domains that were customarily princely prerogatives and could easily become dangerous cases of *lèse majesté*.

[7] Kiser and Linton (2002), for instance, find that only offensive wars—which did not benefit the local taxpayers—were associated with tax revolts in France.

[8] We explain in the next section why the elites were so interested in these.

[9] While it was sometimes possible to evade payment of taxes or flee the jurisdiction to avoid them, in practice these strategies were not readily available to taxpayers whose wealth was locked in immovable land.

Crown needing to raise funds and the populace needing to protect knowledge of their wealth tend to produce (i) taxes that were much lower than what the asymmetries in coercive and agenda-setting powers would lead one to expect; (ii) frequent tax revolts by the poorer strata of society despite the relatively low probability that the rebels would prevail; (iii) rulers granting tax relief even after suppressing tax revolts successfully; (iv) occasional, and more dangerous, revolts joined by the wealthy; and (v) rulers increasing tax demands after their previous ones have been accepted.

The struggle over control of tax assessment can help explain the puzzle of powerful yet seriously underfunded princes. The analysis shows how the Crown could dramatically increase its income from taxation without any change in its coercive capacity or its ability (or willingness) to provide public goods, and how it could do this while simultaneously reducing the resistance to its tax demands. The rise of the fiscal state could thus be traced to improvements in the administrative machinery for assessing the wealth of the taxpayers rather than to political institutions designed to constrain the Crown, military institutions designed to centralize and monopolize violence, or economic institutions designed to enable the Crown to borrow more cheaply.

This paper proceeds as follows. Section 1 provides reasons for the basic modeling setup, and some historical evidence in support. Section 2 specifies the model and its assumptions. Section 3 presents the analysis with the ruler being perfectly informed about the wealth to be taxed, and establishes the benchmark result for very high coercive taxation without resistance by the taxpayers. Section 4 introduces the problem of wealth assessment and motivates the assumption that the ruler and the taxpayer have asymmetric information about taxable wealth. Section 5 analyzes the model with the ruler being imperfectly informed about that wealth, and establishes the twin results of severely underfunded rulers and endemic tax resistance, especially by the poor. Section 6 draws out empirical implications from the analysis and provides some evidence in their support. Section 7 relates our results to several theoretical models involving a ratchet effect in production and taxation. Section 8 concludes.

1 Royal Prerogative and the Crown's Coercive and Legal Powers

Royal prerogative refers to the Crown's exclusive authority and right to decide on certain kinds of policies. The most important among them were the economy (establishing markets, charging tolls, hunting and fishing, coinage, mining, unclaimed property), the judiciary and administration (high justice, law and order), and foreign affairs (diplomacy and war). Of these, the preparation for and fighting of wars were by far the most expensive undertakings, regularly accounting for more than half of Crown spending in peacetime (and often exceeding two thirds when debt service on war-related loans was included), and easily outstripping its revenue during war. These were also the areas where the preferences of the Crown were most likely to diverge from those of elites and regular taxpayers alike. While the latter might be interested in security, defense, and mercantilism, the Crown's concerns were often dynastic, territorial, and matters of *gloire*.[10]

When policies were financed out of the Crown's "ordinary revenue" there was not much

[10] Holsti (1990); Lynn (1999).

anyone could do about this divergence except for elites to offer counsel and hope for the best. But when the policies had to be financed at least in part with additional "extraordinary" contributions, the royal prerogative created a moral hazard problem. The Crown could not credibly commit to use the resources granted by taxpayers for things they wanted. And since the exigent circumstances that necessitated the demand for these additional subsidies were almost invariably military, the taxpayers had yet another reason to worry about the consequences of providing the Crown with financial relief: the forces raised could be used to coerce further contributions.

The dual problems of potential misuse of funds and suspected intent to wield tyrannical power could lead to resistance of royal requests for contributions. The Crown had to negotiate with elites, impressing, cajoling, and threatening in turn, and all that it could hope to achieve was quasi-voluntary (and often temporary) consent, or rather, acquiescence, to taxes that tended to fall below the Crown's requests.[11]

In this bargaining in the shadow of power, the Crown enjoyed certain advantages in custom, law, and practice. It could rely on its network of patronage to mobilize support for its demands, it could point to custom that turned some grants into ritualistic expressions of consent, and it could choose to interpret and selectively enforce existing (and sometimes forgotten) laws. It could also awe opponents with the social prestige of the throne and intimidate them with threats of fines, imprisonments, confiscations, exiles, and executions (Webber and Wildavsky, 1986, 282).

The asymmetry in bargaining power made it difficult to resist demands with reciprocal agreements—the so-called "redress before relief" whereby new subsidies are granted only when the Crown has addressed the grievances of the taxpayers. To make such arrangements stick, the subjects would have to coordinate to press their demands, resist individual temptations to defect to the Crown, monitor the implementation of any promises over time, and agree to punish the Crown for prior malfeasance when a new opportunity arose. Almost none of this was possible with communications being difficult and slow, travel expensive and risky, and opportunities for assembly and duration of meetings limited to the Crown's discretion.[12] Moreover, whatever concessions the Crown agreed to make in times of financial distress could be reneged on and clawed back after the emergency passed.[13] Any sort of shared policy-making between the Crown and a collective body representing taxpayer interests could not occur while the Crown had vast independent sources of revenue (Rosenthal, 1998).

Expenses, however, kept climbing up. Some of it was driven by the profligacy of indi-

[11] Henneman (1971); Levi (1989, 25–6); (Downing, 1992, 74–78).

[12] For example, during the 1710s the Estates of Württemberg repeatedly protested that Duke Eberhard Louis kept a permanent military force that they had not authorized; that the ducal War Council was collecting an excise tax that had been granted in emergency during the War of the Spanish Succession in 1704 and had been illegally taken out of their control; that the military was levying tax arrears and impressing people into building a new ducal residence; and so on. Even though they sometimes threatened to withhold their consent to funding, they never acted on those threats and in fact proceeded to authorize that very same excise tax year after year. In some ways this was just a repeat of the earlier episode when the duke had proceeded to levy taxes without the consent of the Estates after the failed diet of 1699 (Carsten, 1959, 104–14). See Stasavage (2010) for the geographical constraint.

[13] This was among the main reasons the negotiations between Charles I and Parliament eventually broke down in 1641 even though the king had assented to all parliamentary demands (Seel and Smith, 2001, 62).

vidual princes who simply lived beyond their means. Some of it was driven by the price revolution and the resulting inflation (especially acute during the sixteenth century) that forced the Crown to increase revenue several-fold just to keep up. And some of it, of course, was driven by the military revolution and the increasing costs of warfare.[14] Even the vast revenues of the French Crown "were barely enough to pay for the great effort to keep the kingdom from falling to pieces" (Wolfe, 1972, 247). As the fiscal strain grew, the relations between Crowns and their elites became tenser because the insatiable appetite for resources, especially in a time of need, often pushed monarchs into more overtly coercive behavior (Russell, 1982, 208). The Crown began to resort to direct threats to punish recalcitrant members of the elite, revoke traditional liberties of rights of Estates, collect taxes without seeking their consent, disregard their admonitions about spending choices, and even deny them existence altogether.

Only when the Crown became unable to finance itself through its ordinary sources could representative bodies acquire the institutional permanence and independence that permitted the development of the power of the purse that could reduce the moral hazard problem, and even then the process was slow and uneven.[15] Most of the time and in most of the places, there was only one way to resist the Crown's tax demands: evade payment.

The simplest (and often successful) expedient was to refuse to pay, which could be especially effective when the Crown was financially strapped because of internal or external turmoil (Collins, 1988, 214). Evasion could also be effective when the Crown lacked the administrative capacity to enforce collection, which could be especially pronounced in customs or excise (Webber and Wildavsky, 1986, 271). Collective and overt tax evasion, however, tended to express itself in revolts.

These revolts were not meant to overturn the social order, usher in a revolution, or effect some sort of grand redistributive scheme.[16] They were local and occasionally regional, they were quite specific in that they tended to be triggered by a specific tax or new law, and they aimed merely at rectifying that particular wrong.[17] Most of these local tax revolts have either gone unrecorded (because the poor were illiterate and the wealthy had little interest in recording their doings) or unstudied by historians, who prefer to focus on resistance on the larger scale (Burg, 2004, xvi–xviii). But even looking at the more famous tax-related uprisings one is struck by their prevalence. Between 1536 and 1675, Spain experienced 3 major tax-related revolts, England 5, and France 19 (Brustein and Levi, 1987). Burg (2004) reports 73 for Europe between 1500 and 1700. This is certainly an underestimate. Kiser and Linton (2002) count 35 tax revolts between 1514 and 1675 in France alone, meaning that the Crown was dealing with some sort of violent tax resistance during 20% of

[14]Goldstone (1991, 95–98); Bonney (1991, 354); Seel and Smith (2001, 38); Körner (1995); Storrs (2009); Downing (1992).

[15]Cox (2016)

[16]Elliott (1969, 41–5, 55),Bercé (1990, 169–319).

[17]Mousnier (1979, 730–1) also relates tax revolts to the general expectation that the Crown was supposed to meet its regular expenses with revenue from the royal domain, reserving taxation only for exceptional circumstances like war. Every new tax or an increase in an existing tax was perceived as extortion, and there was little chance that people would voluntarily accept large changes of customary rules. As he notes, "the expressed motive of most popular revolts was the excessiveness, real or alleged, of taxation." Beik (1997) also notes the specific nature of urban protests.

the years.[18]

Consistent with these observations, our theoretical model incorporates several key features: (i) royal prerogative: once a tax is granted, there is no way to control how the Crown spends the money; (ii) elite privilege: the Crown's spending of public money tends to favor the wealthier elites; (iii) proposal power: the Crown has the initiative in making tax demands and could threaten that failure to accept them would result in their (involuntary) collection, which gives its demands a flavor of ultimata; and (iv) coercive advantage: whereas those who resist stand to be dispossessed, the Crown faces no comparable risk. The model abstracts away from representation and coordination for collective action among taxpayers and elites, as well as the possibility that the Crown could ally itself with some segment of society in order to increase its ability to extract resources from another. It also does not deal with different fiscal systems — a vast topic on its own — and with the mundane, but possibly important, issues of tax avoidance and evasion. The last two assumptions are meant to be broadly consistent with practice and are deliberately chosen in their more extreme variants in order to give the Crown an overall edge with respect to the taxpayers. If we find that despite these advantages the Crown still ends up under-taxing, then our results will be much more convincing than if we found under-taxation when the Crown is assumed to be in a weaker position.

2 The Model

The Crown bargains with a Subject with wealth $y \geq 0$ over the amount of taxation.[19] The Crown demands a payment of $x \geq 0$, and the Subject can either acquiesce and pay or resist. If the Subject pays, the Crown provides a good produced from x that yields the Subject the payoff $U(x; y)$. This function is continuous in both arguments and strictly increasing in y. To incorporate the notion of royal prerogative, we assume that for any amount of taxation, the Crown provides some benefit to the Subject (e.g., patronage, enforcement of property rights, defense), but the Subject has no direct control over that provision. The payoff function represents this in a simple way: the Subject's utility is strictly concave in x (so that he benefits from some taxes but finds whatever the Crown is willing to provide unattractive at higher levels of taxation). We normalize $U(0; y) = y$ (only private consumption when there is no tax), and $U(y; y) = 0$ (no utility when all wealth is taxed away). Finally, since these benefits tend to accrue disproportionately to the wealthy, we assume that

Assumption 1 (Wealth Privilege). U is supermodular: $\frac{\mathrm{d}^2 U}{\mathrm{d}x \mathrm{d}y} > 0$.

If the Subject resists, tax evasion succeeds with probability $p(y)$, which is strictly increasing in his wealth, and he avoids paying taxes, foregoing any benefit the Crown would have provided with them, so his payoff is $U(0; y)$. If the Crown prevails, it permanently expropriates the Subject's wealth, yielding him the payoff $U(y; y)$. Resistance is always risky,

[18] Not all revolts were tax-related, of course. Some were provoked by royal attacks on the church, and some were elite-led reactions to Crown attempts to centralize and dilute their local power (Brustein and Levi, 1987, 474–6). Elite support was especially important in the large-scale rebellions that could not survive without it (Barkey, 1991). In some cases, the taxes being resisted were wild and wholly imaginary, such as taxes on unborn children or hats (Le Roy Ladurie, 1974, 269).

[19] We refer to the Crown as "it" and to the Subject as "he" for ease of exposition.

$p(y) \in (0, 1)$ for all $y > 0$, so the Subject's expected payoff from resistance is:

$$R(y) = p(y)U(0; y).$$

We assume that wealthier subjects stand to lose more from resistance than poorer subjects. Let $U(0; y) - p(y)U(0; y) = (1 - p(y))y$ be the difference between the Subject's payoff when he is free from taxation and the expected value of rebelling to obtain that freedom. This expression captures the opportunity costs of revolt. We assume it is increasing in wealth:

ASSUMPTION 2 (OPPORTUNITY COSTS). $(1 - p(y))U(0; y)$ is increasing in y.

Another way of saying this is that for any tax demanded, the difference between what the Subject would obtain by paying and what he can secure by resisting is increasing in wealth. Even though they are more likely to resist successfully, wealthier subjects also derive larger benefits from peaceful private consumption than poorer subjects.[20]

The Crown's payoff when it obtains x is $V(x)$, where we assume that the utility function is strictly increasing, concave, and normalized so that $V(0) = 0$. The Crown's expected payoff when the Subject resists is

$$W(y) = (1 - p(y))V(y).$$

This assumes that the only cost of a tax revolt is foregone taxation to the Crown and foregone royal benefits for the Subject. Although this does characterize a few tax revolts, most involve fighting that cause destruction quite apart from these costs. Formally, these costs make revolts less attractive to both sides and increase the incentives to find a mutually acceptable deal. Both actors already prefer to avoid a revolt in our model (see Lemma 2 and Lemma 3 below). Since our results also hold with explicit costs of rebellion, we omit them from the specification for the sake of simplicity.

The interaction takes place over two periods that are structurally identical. In each, the Crown demands x, and the Subject decides whether to pay or to resist. If he resists and the Crown prevails, his wealth is expropriated and the game ends. If he resists successfully or acquiesces, the period ends and players receive their per-period payoffs. The total payoff is the undiscounted sum of the per-period payoffs.

The model incorporates the lack of institutional constraints on the Crown by assuming that the Crown is free to set whatever tax demands it chooses. In particular, it cannot precommit to the second-period taxation at the outset.

The solution concept is subgame perfect equilibrium.

[20] Assumption 2 can be written equivalently in terms of the probability of survival $p(y)$: $p(y) + y \cdot \frac{\mathrm{d}p}{\mathrm{d}y} < 1$. That is, we restrict attention to resistance technologies that do not admit very large changes in the probability of survival for small increases in wealth. Common contest-success functions satisfy this assumption. For example, $p(y) = e^{-\frac{1}{y}}$ and the standard ratio form $p(y) = \frac{\omega y^\alpha}{\omega y^\alpha + \psi^\alpha}$, with $\omega > 0$, $\alpha \in (0, 1]$, and $\psi > 0$, where the latter measures the Crown's coercive resources. As we shall see, Assumption 2 is sufficient for the results we obtain, but it is not necessary.

3 Acquiescence to High Coercive Taxation

Consider the second period and suppose the Subject either acquiesced in the first period or survived a revolt. By subgame perfection, for any given x, the Subject will pay if, and only if, $U(x; y) \geq R(y)$. Since the Crown's payoff is increasing in the tax, it will demand the highest tax the Subject would agree to, denoted $x_{\mathrm{k}}(y)$, which implies that the equilibrium *coercive tax* is defined by

$$U(x_{\mathrm{k}}(y); y) = R(y).$$

This equation has a unique solution because $U(0; y) > R(y)$, $R(y)$ is constant, and $U(x; y)$ concave in x together imply that $U(x; y)$ and $R(y)$ will intersect only once, at some $x_{\mathrm{k}}(y) > 0$. The equilibrium tax has an important property:

LEMMA 1. *The coercive tax $x_{\mathrm{k}}(y)$ is increasing in the Subject's wealth.* □

This result, illustrated in Figure 1, might (or should be) surprising.[21] Recall that since the probability of surviving the revolt is increasing in wealth and because winning means keeping one's wealth, the richer the Subject, the higher the expected payoff from rebellion. Since the Crown is taxing at the revolt constraint, one might expect that this should induce it to offer better deals to the rich.[22]

In this model, however, the acceptable tax demand is increasing in wealth: the richer the Subject, the more he can be induced to pay *even though* his payoff from rebellion is higher because he also stands to lose more by resisting. We now show that the Crown would never provoke resistance with its demands:

LEMMA 2. *The Crown strictly prefers to obtain the coercive tax than to cause the Subject to rebel: $V(x_{\mathrm{k}}(y)) > W(y)$.* □

This is why there is no need to assume additional costs of rebellion in order to provide sufficient incentives to the actors to avoid violence in equilibrium. Lemma 2 also implies that the Crown must demand $x_{\mathrm{k}}(y)$ in the second period, which the Subject accepts, in the unique equilibrium. This is so because any lower demand is also accepted, yielding a lower peaceful payoff, whereas any higher demand is rejected, yielding a lower expected payoff from the revolt. That $x_{\mathrm{k}}(y)$ must be accepted in equilibrium even though Subject is indifferent follows from the fact that if it were not so, the Crown would have no best response: for any positive probability of rejection, it would be strictly better off making a slightly smaller demand that is accepted with certainty. Thus, in the unique (because $x_{\mathrm{k}}(y)$ is strictly increasing) equilibrium the Crown taxes the Subject with known income all the way down to his indifference point between paying that tax and rebelling.

Consider now the first period. Define three demands: the initial one the Crown makes in the first period, x_1, and the two outcome-contingent demands it can make in the second period, one after acceptance of the first-period demand (x_{A}), and another after its rejection

[21] We used the following functional forms: $U(x; y) = y - x + \lambda\sqrt{x(y - x)}$, where $\lambda = 3$ measures the importance of royal benefits to the Subject, and $p(y) = y/(1 + y)$.

[22] This is what happens in the standard models of bargaining in the shadow of power where the types with higher expected payoffs from fighting must be offered more attractive peace terms to be induced not to fight (Powell, 2002).

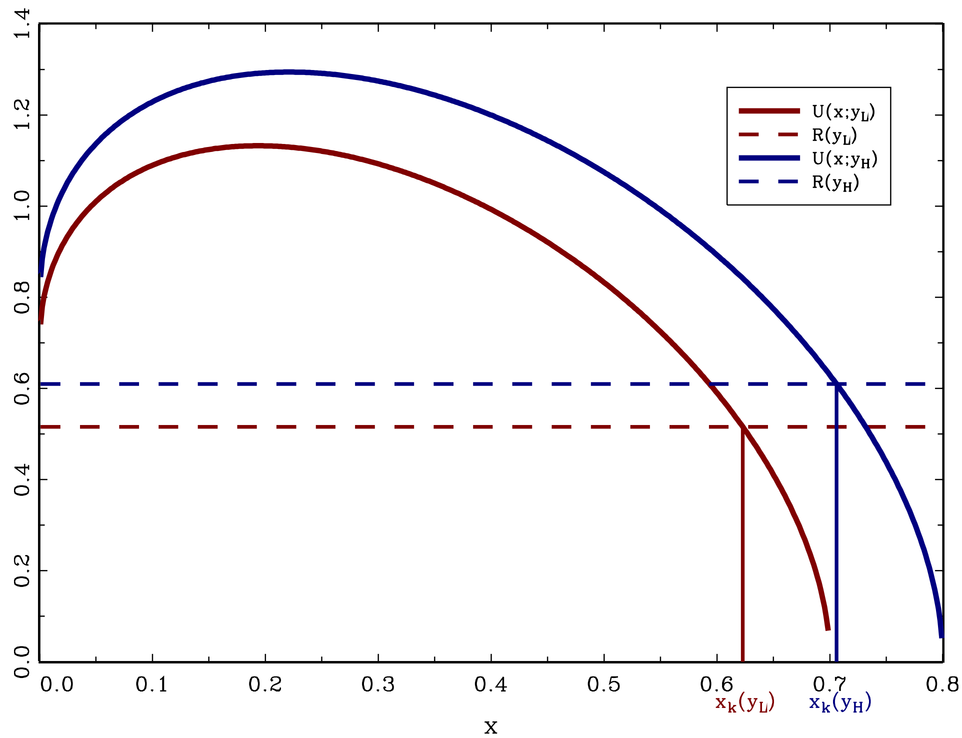

Figure 1: Coercive Taxation and Rebellion ($y_{\mathrm{L}} = 0.7$, $y_{\mathrm{H}} = 0.8$).

(x_{R}). As we have seen, if the Subject acquiesces in the first period or survives a revolt, the Crown will impose the coercive tax in the second period regardless of what happens in first:

$$x_{\mathrm{A}} = x_{\mathrm{R}} = x_{\mathrm{k}}(y),$$

which means that the Subject will always accept in the second period. Consider now an arbitrary x_1, and note that if the Subject accepts that, his payoff is:

$$U(x_1; y) + U(x_{\mathrm{k}}(y); y) = U(x_1; y) + R(y),$$

whereas if he rejects it, his payoff is

$$R(y) + p(y)U(x_{\mathrm{k}}(y); y) = R(y) + p(y)R(y).$$

Thus, the Subject accepts x_1 only if $U(x_1; y) \geq p(y)R(y)$. Since the Crown has no best response to the Subject rejecting with positive probability when indifferent, in equilibrium it must be that the Subject accepts in that case. Because the Crown's payoff is increasing in the accepted demand, it follows that the Crown must make the Subject indifferent. Let $x_{\mathrm{d}}(y)$ uniquely solve

$$U(x_{\mathrm{d}}(y); y) = p(y)R(y), \tag{1}$$

and be a first-period coercive tax.[23]

Since the Subject would accept any lower first-period tax without affecting the second-period payoffs, the Crown cannot profit by reducing taxation. The other possibility, of course, is that the Crown induces a revolt by making some unacceptable demand: the gives it a chance to expropriate the Subject if the revolt fails, while still yielding the coercive tax in the second period if the revolt succeeds. If a revolt occurs, the Crown prevails with probability $1 - p(y)$, in which case it expropriates the entire wealth, and if it loses, which happens with probability $p(y)$, it imposes the single-period violence constrained tax $x_{\mathrm{k}}(y)$. Thus, the expected second-period payoff from a revolt in the first period is

$$G(y) = (1 - p(y))V(y) + p(y)V(x_{\mathrm{k}}(y)), \tag{2}$$

and we can show that even without discounting this payoff is worse than the coercive tax the Crown could extract peacefully in the first period.

LEMMA 3. *The Crown prefers the coercive first-period tax to the expected second-period payoff from a gamble on revolt:* $V(x_{\mathrm{d}}(y)) > G(y)$. □

Since the total payoff from inducing a revolt in the first period is $W(y) + G(y)$, and the total payoff from peaceful coercive taxation is $V(x_d(y)) + V(x_k(y))$, lemmata 2 and 3 immediately imply that we can characterize the solution to the game:

PROPOSITION 1. *In the unique subgame-perfect equilibrium, the Crown demands $x_{\mathrm{d}}(y)$ in the first period and $x_{\mathrm{k}}(y)$ in the second period, and the Subject accepts both.* □

An important question now arises with respect to the two different tax demands: could it be that the Crown's inability to commit not to demand anything less than the coercive tax in the second period forces it to make concessions in the first? The answer turns out to be negative. In fact, the Crown is even *more* demanding in the first-period. The coercive second-period tax is actually an instance of tax relief.

LEMMA 4. *The first-period tax exceeds the second-period coercive tax: $x_{\mathrm{d}}(y) > x_{\mathrm{k}}(y)$, and is also increasing in the Subject's wealth.* □

The intuition behind this result is straightforward. In the second period the Crown will extract the maximum represented by the coercive tax. It cannot commit to anything less or demand anything more. Since the post-revolt and post-acceptance taxes are the same, from the Subject's perspective resistance can only yield tax relief in the current period; it cannot alter the terms the Subject would have to agree to in the second period. The attraction of a possible tax relief, however, is seriously dampened by the risk that he will be permanently expropriated. Whereas any possible benefit from resistance can only accrue in first period, the losses from defeat persist in both. In particular, this means that the Subject not only risks losing today but also risks not being around tomorrow, when paying even the onerous tax is strictly better than being expropriated with certainty. This makes resistance in the

[23]The equation has a unique solution because $p(y)R(y)$ is constant in x while $U(x; y)$ is concave in x, which together imply that $U(x; y)$ and $p(y)R(y)$ will intersect at most twice. Since $U(0; y) > R(y) > p(y)R(y) > 0 = U(y; y)$, it follows that they will intersect once, at some $x_{\mathrm{d}}(y) > 0$.

first period strictly worse than the single-period revolt payoff, and as a result the Crown can demand, and obtain, a higher tax.

We conclude that the Crown gains tremendously from its proposal power — which enables it to extract all wealth up to the Subject's reservation point — and from its coercive advantage — which enables it to demand an even larger tax in the first period than the already high tax in the second. The Crown's royal prerogative and its inability to pre-commit do not appear to be much of a problem.

RESULT 1. *When the Crown knows the wealth of the taxpayers, its superior proposal and coercive powers yield a significant advantage: taxes are high (and increasing in wealth) but no revolts occur.*

In this environment, the Crown should be funded well beyond what its subjects prefer as far as taxes go, and yet none of them would resist. Given how stacked the interaction is in favor of the Crown, this result is unsurprising. It does, however, flatly contradict the observed empirical patterns of under-taxation, tax privileges for the wealthy, and frequent tax revolts (especially by the poor). We now show that all of these can be explained if we maintain all our assumptions and merely introduce another feature of the early modern world: the asymmetric information about the wealth of the Subject.

4 The Problem of Wealth Assessment

So far, we assumed that the Crown knows the wealth of the Subject. Since wealth is also related to the opportunity costs of revolt and the probability of successful evasion, knowing it allows the Crown to estimate the Subject's willingness to pay. This in turn allows the Crown to calibrate its coercive demands and extract everything up to the revolt constraint without provoking resistance.[24] Thus, the assumption that the Crown is perfectly informed about the Subject's wealth appears critical. However, during the early modern period it was very likely violated. Obtaining reliable assessments of the taxpayers wealth was prohibitively costly, potentially disruptive politically, and often simply beyond the administrative or technological capacity of the Crown.[25]

Consider some of the problems that beset the various forms of revenue extraction. Taxing trade through tolls or customs duties was possible (especially at ports, town gates, river crossings, and mountain passes) but it was much more difficult when frontiers were ill-defined and porous. Even in England rulers quickly found that agreement of the merchants was necessary to collect them, and duties remained at nominal rates for decades.[26] Excise taxes required a relatively sophisticated administrative apparatus that remained well beyond the capacity of most polities for a long time (Brewer, 1990, 104–08). In addition, indirect

[24]Kiser and Linton (2002, 890-1) argue that incomplete information about the probability of revolt—which in our model is generated by uncertainty over the reservation price—could cause rulers to miscalculate and inadvertently provoke resistance. They do not study the implications of this uncertainty for behavior of the taxpayers.

[25]Rulers often had only vague idea about their receipts and expenditures (Guenée, 1981, Ch. 6), and obtaining information from their domains could be exceedingly costly (Yun-Casalilla, O'Brien, and Comín, 2012).

[26]Guenée (1981, 97); Goldstone (1991, 97)

taxes like excise and customs increased prices and threatened to affect adversely the scope of commercial activities and the volume of trade, which could actually decrease the tax yield.

Taxing income cannot be done until there is a way to ascertain that income, which can be quite impossible in a world where few systematic records are kept, where accounting practices are primitive (double-entry book-keeping was slow to spread from Italy, and even as late as the 18th century the French Crown did not have anything resembling a budget), and where there is no pressing need to determine wealth in order to engage in daily economic activities. It was only with the rise of state officials living on known salaries, landlords receiving income from contractual rents, and the Corporation that had to keep track of revenues to pay dividends that some income became easy to assess.

In order to tax property—which could also be used as an index of wealth—rulers had to carry out difficult, time-consuming, and expensive valuations of property.[27] In practice, this meant that they could not do so very often and as a result taxpayers were assessed on past values of their property, often as distant in the past as several centuries.[28] To make matters worse, assessments could be easily tampered with either by holding property in some less traceable forms, by colluding with the tax assessors who were often one own's neighbors (who presumably had a better idea about one's holdings than an outsider), or by the simple expedient of lying — since rulers were often reduced to relying on self-reported valuations and the practice of requiring an oath was inconsistently used.[29]

Two common ways of trying to get around the problem of assessing income is by ignoring it or by looking for some more easily measurable proxy for wealth. The old capitation taxes, which imposed a fixed amount on an individual as defined by a census, were straightforward to impose since they required no assessment of anything except the relevant population. Related variants were the hearth and window taxes that were imposed on dwellings since these were easier to count than people (although counting hearths still required entry into private dwellings). Deeply regressive, these taxes obviously failed to tap into much wealth since they had to be affordable by the poorest members of the taxpayer population. Even then, since it was the peasants who were usually the poorest and because the tax took no account of how ability to pay could change with circumstances, these impositions could become unbearable after bad harvests or in tough economic conditions, and could trigger revolts.[30]

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[27]Might be practicable in towns but much harder for countryside.

[28]Even the efficient Dutch had to rely on seventeenth-century assessments when collecting extraordinary property taxes in the eighteenth-century (Aalbers, 1977, 85–6).

[29](Braddick, 1996, 94, 163–4); (Goldstone, 1991, 97–8).

[30]A slightly improved approach to the capitation tax that still necessitated no inquiry into the wealth of the subjects was to introduce gradations according to social rank, with the latter presumably serving as an index of wealth. Thus, the poll tax introduced by Crown Louis XIV in 1695 defined twenty-two classes of society from the lowest comprising day laborers and servants, who paid 1 *livre*, all the way up to the *Dauphin* himself, who paid 2,000 *livres*. Analogous variants were also occasionally used in England since the 14th century, and came into wide use during the fiscal stresses of the tumultuous 16th century. With such crude indicators of wealth, however, these tax were simultaneously too burdensome for many in the lower ranks (who lived on a small margin) and too light for those in the top ranks. The flat impositions spurred efforts to gain exemptions and when this was not possible produced rumblings of revolt. When Württemberg saw a graduated poll tax in the wake of the French invasion and exactions in 1707–08, someone scribbled a warning on the doors of the Estates' house in Stuttgart: "if you consent to the Duke's demands, we shall revolt." The tax was promptly

We are not the first to note the importance of taxation to the development state institutions, and to the effects various fiscal systems can have on government revenue. In his brief but very perceptive essay on economic history, Hicks (1969, 81–4) argues that the reason for the "chronic deficiency of tax revenue" can be traced to the Crown's inability to tap into the wealth of much of society. Scholars have analyzed various fiscal systems precisely from the vantage point of how easy taxes were to collect (which includes assessment, administration, and enforcement). The widespread practice of tax-farming, for instance, is a rational way of dealing with the problem of asymmetric information about the tax base in such an environment, especially when the tax rights are auctioned off at fairly regular intervals to a large group of bidders. Tilly (1992) bases part of his argument about the different paths to the modern state on the presumed difficulty of collecting land taxes than taxes on commerce. Ertman (1997, 16) rightly disputes this assumption (using the evidence collected by Brewer (1990) on the excise tax in England) but then goes to the other extreme by asserting that "land taxes were not difficult to administer, because central governments could dispense with the time-consuming business of wealth or income assessments and instead simply demand fixed amounts from each local area."

This remarkably sanguine view seriously underestimates the need to figure out what was there to be taxed. It is now well-documented, for example, that royal tax figures in France "represented proposed revenues, or 'hoped-for' revenues, not in any sense money actually collected. [. . .] All the chilling tax figures from the 1640s are mere imagination; they have little foundation in reality" (Collins, 1988, 200–05).

The English Crown did not fare much better. For instance, the fifteenth, a tax on personal property that required the government to assess the current wealth of the taxpayers, developed during the reign of Edward I but "ossified [by 1336 and] remained basically unchanged until its termination in 1624 [. . .] [F]rom tax to tax and irrespective of economic growth or decline, each ward and vill made the same contribution as in 1334. Moreover, from 1336 onwards the assessment of individual wealth was placed beyond the competence of centrally appointed officials and reserved for the local community to determine" (Bush, 1991, 381–2). The under-assessment of wealth in England was so extreme in the early 17th century that subsidies dropped in yield when national wealth was going up (Braddick, 1996, 163).[31]

The persistent theme, of course, is that taxpayers knew more about their wealth than the Crown did, and that it was very expensive for the Crown to acquire the necessary information (Yun-Casalilla, O'Brien, and Comín, 2012). Consistent with the problematic nature of wealth assessments, we now assume that the Crown and the Subject are asymmetrically

abandoned and when another duke attempted to impose it in 1764, the Estates managed to obtain an injunction from the Imperial court that put a stop to it. See Carsten (1959, 110, 140–2).

[31]Even when the Crown and the elites agreed on the size of the tax base, disagreements about what was possible to tax without ruining the taxpayers could create open ruptures. In 1626, the Catalan Corts was confronted with a demand for 250,000 ducats a year for fifteen years by Olivares who was desperately trying to plug the fiscal hole in which the Spanish Crown was descending. The money was intended for military upkeep, with Olivares promising that it will all be spent in the province and collected exclusively by locals, but it exceeded the customary contribution by nearly 60 percent. Even though both sides had estimated the population of the principality to be about one million (the actual figure was closer to 400,000), there was a "sharp divergence between the king's advisers and the Catalans over the fiscal resources of the Principality," with the latter considering the demand exorbitant and ruinous (Elliott, 1984, 237–38).

informed about the wealth. We maintain all other assumptions as is; in particular, royal prerogative implies that the Crown is unconstrained in how it uses any information it acquires from the taxpayers regarding their wealth.

5 Taxation and Revolts under Asymmetric Information

Suppose now that the Subject is perfectly informed about his wealth but the Crown does not observe it. Consider two types of Subject: rich, with income y_{H}, and poor, with income $y_{\mathrm{L}} < y_{\mathrm{H}}$. Let $q \in (0, 1)$ denote the Crown's prior belief that the Subject is rich. For ease of exposition, we shall use the following short-hand notation: $x_i \equiv x_{\mathrm{k}}(y_i)$ and $p_i \equiv p(y_i)$. The solution concept is sequential equilibrium (Kreps and Wilson, 1982; Myerson, 1991).[32]

We now characterize the properties that optimal strategies must have in any sequential equilibrium. Since the Crown cannot pre-commit to any particular tax in the second period, its strategy must be sequentially rational given its updated beliefs, which we denote by q_2:

LEMMA 5. *In any sequential equilibrium, in the second period the Crown demands x_{L} if $q_2 < q_2^*$ and x_{H} if $q_2 > q_2^*$, where*

$$q_2^* = \frac{V(x_{\mathrm{L}}) - W(y_{\mathrm{L}})}{V(x_{\mathrm{H}}) - W(y_{\mathrm{L}})}.$$

If $q_2 = q_2^*$, the Crown is indifferent and can mix between these two demands. □

In the second period the Crown attempts to impose a high tax only when it is sufficiently convinced that the Subject is wealthy; otherwise, it settles for a low tax. This means that the Subject is simultaneously threatened by the possibility that the Crown concludes that he is rich and demands a high tax — the *ratchet effect*, and attracted to the possibility that the Crown concludes that he is poor and demands a low tax — *tax relief*. Since the Crown is going to attempt to infer his wealth from his behavior in the first period and from the outcome of revolt should one occur, this incentive distorts the Subject's behavior in that period. We now turn to the analysis of these strategies.

Consider the first-period strategy of the poor Subject. The Crown only ever demands either x_{L} or x_{H} in the second period (Lemma 5). The poor Subject accepts the former and rejects the latter, but in both cases his payoff is the same because $U(x_{\mathrm{L}}; y_{\mathrm{L}}) = R(y_{\mathrm{L}})$ by the definition of x_{L}. This makes the Crown's offer irrelevant to his expectations, which in turn renders the Crown's beliefs immaterial to his strategic considerations in the first period. Since the poor Subject has no incentive to manipulate the Crown's beliefs and because he gets in expectation the equivalent of the complete-information second-period payoff, it should come as no surprise that his strategy in the first period is exactly the strategy he would have had under complete information:

LEMMA 6. *In any sequential equilibrium, the poor Subject accepts any $x_1 < x_{\mathrm{d}}(y_{\mathrm{L}})$ and rejects any $x_1 > x_{\mathrm{d}}(y_{\mathrm{L}})$. Moreover, if the rich Subject accepts $x_1 = x_{\mathrm{d}}(y_{\mathrm{L}})$, then the poor one does as well.* □

[32] Sequential equilibrium has not been formalized for games with infinite pure-strategy sets like ours. However, one can restrict the space of possible tax demands to a large discrete collection without losing any key results. As Lemma 9 shows, in any equilibrium the Crown must choose from three first-period tax demands.

The strategies for the poor Subject for any $x_1 \neq x_d(y_L)$ follow from strict dominance, and therefore hold irrespective of beliefs on the path and, in a sequential equilibrium, off the path as well. That he also accepts $x_1 = x_d(y_L)$ in any equilibrium is predicated on the assumption that the rich Subject does so as well. As we shall prove in Lemma 8, this assumption is satisfied in any sequential equilibrium, so Lemma 6 offers a complete characterization of the behavior of the poor Subject. Because the Crown's lack of information does not distort the behavior of the poor type, all dynamics of any interest must come from the behavior of the rich Subject.

Since the poor Subject revolts for any demand $x_1 > x_d(y_L)$, we begin by asking whether the rich Subject would be willing to accept some such demand with certainty. In equilibrium, doing so would fully reveal his wealth so that $x_R(x_1) = x_L$ and $x_A(x_1) = x_H$. If y_H accepts x_1, his payoff is $U(x_1; y_H) + U(x_H; y_H) = U(x_1; y_H) + R(y_H)$, and if he rejects it, his payoff is $R(y_H) + p_H U(x_L; y_H)$. Therefore, he strictly prefers to accept when $U(x_1; y_H) > p_H U(x_L; y_H)$, and strictly prefers to resist if the inequality is reversed. Let x_w be the larger root of:[33]

$$U(x_w; y_H) = p_H U(x_L; y_H). \tag{3}$$

As one might expect, since the poor type must be willing to revolt when the Crown demands x_w, it is the case that this demand exceeds $x_d(y_L)$. This separating tax must be intermediate:

LEMMA 7. *The tax demands are ordered as follows:* $x_L < x_d(y_L) < x_w < x_d(y_H)$. *Moreover, if y_L is sufficiently smaller than y_H, then $x_w < x_H$ as well.* □

This shows that if the Crown is to get the rich Subject to agree to a tax that exceeds the tax that the poor Subject is willing to pay, it cannot hope to get that tax as high as it would have been able to under complete information. The fact that the tax is separating implies ratcheting after acceptance and relief after revolt, so the Crown must provide an incentive for the rich Subject not to revolt. Since it cannot commit not to offer relief after the revolt and because the rich Subject is more likely to survive that revolt, the only inducement the Crown can offer is in the form of a considerably lower tax in the first period. We can now specify the optimal behavior of the rich Subject:

LEMMA 8. *In any sequential equilibrium, the rich Subject accepts any $x_1 \leq x_w$, rejects any $x_1 \in (x_w, x_d(y_H)]$ with certainty if $q \leq q_1^*$ and with probability r^* if $q > q_1^*$, where*

$$q_1^* = \frac{q_2^* p_L}{q_2^* p_L + (1 - q_2^*) p_H} \quad \textit{and} \quad r^* = \left(\frac{p_L}{p_H}\right)\left[\frac{q_2^*(1-q)}{q(1-q_2^*)}\right],$$

and rejects any $x_1 > x_d(y_H)$ with certainty. □

The results for demands $x \in (x_w, x_d(y_H)]$ when $q > q_1^*$—that is, where the rich Subject plays a mixed strategy—deserve some clarification. Here, the rich Subject rejects the

[33] Equation (3) either has one solution that is greater than the unconstrained optimum or two, with one on each side of that optimum. We are always interested in the solution that exceeds the unconstrained optimum because anything less than it requires no coercion by the Crown.

Crown's initial demand with probability r^*, which is constant in x. This might seem puzzling at first: why does he revolt with the same probability after different demands? After all, accepting a higher tax is making him worse off. The reason, then, must be that the consequences of resistance must also be getting worse as the initial demand increases. We now show that this is indeed the case.

Since the rich Subject is willing to mix, he must be indifferent between accepting a demand that leads to a high second-period tax of x_{H} and rejecting it. Since the demands in question exceed x_{w}, it cannot be the case that he expects resistance to produce certain tax relief, $x_{\mathrm{R}} \neq x_{\mathrm{L}}$, because then he strictly prefers to revolt. Moreover, it cannot be the case that he expects resistance to lead to the high tax, $x_{\mathrm{R}} \neq x_{\mathrm{H}}$, because for any demand less than $x_{\mathrm{d}}(y_{\mathrm{H}})$ he strictly prefers not to revolt. It must, therefore, be the case that he is uncertain about the post-revolt tax; that is, the Crown must be mixing as well.

Let $h(x)$ denote the probability with which the Crown makes the high demand x_{H} after a revolt over a demand $x \in (x_{\mathrm{w}}, x_{\mathrm{d}}(y_{\mathrm{H}})]$. Since the rich Subject is indifferent, this probability must satisfy:

$$U(x; y_{\mathrm{H}}) + U(x_{\mathrm{H}}; y_{\mathrm{H}}) = R(y_{\mathrm{H}}) + p_{\mathrm{H}}\Big[h(x)U(x_{\mathrm{H}}; y_{\mathrm{H}}) + (1 - h(x))U(x_{\mathrm{L}}; y_{\mathrm{H}})\Big],$$

which tells us that in equilibrium, the rich Subject expects the high demand to be made with probability

$$h(x) = \frac{U(x_{\mathrm{w}}; y_{\mathrm{H}}) - U(x; y_{\mathrm{H}})}{U(x_{\mathrm{w}}; y_{\mathrm{H}}) - U(x_{\mathrm{d}}(y_{\mathrm{H}}); y_{\mathrm{H}})} \in (0, 1]. \tag{4}$$

It is easy to verify that $h(x)$ is increasing in x; that is, the larger the Crown's initial demand, the more likely is it to demand the high tax if the revolt fails. Thus, even though acquiescing to a larger demand makes the rich Subject worse off, the expected payoff of resistance is also decreasing because the Crown becomes much less likely to offer tax relief in response. In equilibrium, the Crown's strategy keeps the rich Subject indifferent even as x increases, which in turn explains why his probability of acceptance can be independent of the initial demand.

The Crown's response has another, seemingly puzzling, property: it causes the rich Subject to reject $x_{\mathrm{d}}(y_{\mathrm{H}})$ with positive probability even though he accepts it with certainty under complete information. To see this, note that $h(x_{\mathrm{d}}(y_{\mathrm{H}})) = 1$, which means that $x_{\mathrm{R}} = x_{\mathrm{H}}$. But since only the rich Subject accepts with positive probability, $x_{\mathrm{A}} = x_{\mathrm{H}}$ as well. Recall now that under complete information, the rich Subject always accepts $x_{\mathrm{d}}(y_{\mathrm{H}})$ when he expects x_{H} in the second period (Proposition 1). But here, the rich Subject still rejects it with probability $r^* > 0$. Why?

The answer is that the Crown's unwillingness to offer tax relief here is predicated on the post-rebellion inference that the Subject is rich with sufficiently high probability. If the rich Subject were to accept $x_{\mathrm{d}}(y_{\mathrm{H}})$ with certainty, then the only possible post-rebellion inference the Crown could make is that the Subject must be poor (recall that resisting here is strictly dominant for the poor Subject). This would cause the Crown to offer tax relief after a revolt, $x_{\mathrm{R}} = x_{\mathrm{L}}$. But in that case, the rich Subject would strictly prefer to resist given the high initial demand, a contradiction to the supposition that he accepts it. In other words, the rich Subject cannot accept the same demand he would under complete information because doing so would change the Crown's behavior in the second period in a way that would render such acceptance suboptimal.

Observe further that after a revolt, the Crown is indifferent between making the acceptable demand x_{L} and the one that only the rich Subject would accept. In principle, there is no reason why it should not choose x_{L} and avoid further resistance. Why cause the poor to revolt with ever higher probability, as $h(x)$ increasing shows?

Because in order to get the rich Subject to agree to a relatively high first-period tax demand, the Crown must threaten that rebellion would not lead to tax relief. The only way it can make this threat credible is by making the high tax demand with sufficiently high probability, which of course causes the poor Subject to revolt. Since the first-period demands here are still not as high as $x_{\mathrm{d}}(y_{\mathrm{H}})$, the Crown does not need to threaten to demand $x_{\mathrm{R}} = x_{\mathrm{H}}$ with certainty, just with a high enough probability. It is worth noting, however, that after the equilibrium demand $x_{\mathrm{d}}(y_{\mathrm{H}})$, the Crown must be certain not to offer any tax relief, so the rebellion of the poor Subject in the second period is ensured.

The remainder of this section completes the analysis. Lemmata 6 and 8 fully characterize the responses to any demand in any sequential equilibrium, while Lemma 5 and equation (4) fully specify the Crown's second-period strategy as well. This is sufficient to determine the set of possible first-period demands for the Crown. The following result establishes that the Crown will only ever make one of at most three demands:

LEMMA 9. *In any sequential equilibrium, the Crown demands some $x_1 \in \{x_{\mathrm{d}}(y_{\mathrm{L}}), x_{\mathrm{w}}\}$ if $q \leq q_1^*$, and some $x_1 \in \{x_{\mathrm{d}}(y_{\mathrm{L}}), x_{\mathrm{w}}, x_{\mathrm{d}}(y_{\mathrm{H}})\}$ otherwise.* □

The only thing that remains now is to determine how the Crown will choose among the demands listed in Lemma 9. Let $V_1(x)$ be the Crown's expected equilibrium payoff from demanding x in the first period. We can now show that these payoffs are non-decreasing in q but at different rates: the payoff from $x_{\mathrm{d}}(y_{\mathrm{H}})$ strictly increases at the fastest rate; the payoff from x_{w} strictly increases at a slower rate; and the payoff from $x_{\mathrm{d}}(y_{\mathrm{L}})$, either strictly increases at the slowest rate or is constant. This implies that any two payoff functions will have at most one intersection, as shown in the following result.

LEMMA 10. *There exist non-degenerate probabilities q_{L}, q_{M}, and q_{H} such that*

(i) $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{w}}) \Leftrightarrow q < q_{\mathrm{L}}$, *and* $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) < V_1(x_{\mathrm{w}}) \Leftrightarrow q > q_{\mathrm{L}}$;

(ii) $V_1(x_{\mathrm{w}}) > V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) \Leftrightarrow q < q_{\mathrm{H}}$, *and* $V_1(x_{\mathrm{w}}) < V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) \Leftrightarrow q > q_{\mathrm{H}}$;

(iii) $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) \Leftrightarrow q < q_{\mathrm{M}}$, *and* $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) < V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) \Leftrightarrow q > q_{\mathrm{M}}$.

Moreover, the only possible configurations are $q_{\mathrm{L}} < q_{\mathrm{M}} < q_{\mathrm{H}}$ and $q_{\mathrm{H}} < q_{\mathrm{M}} < q_{\mathrm{L}}$. □

This result means that the equilibrium choice for the Crown can be simply characterized by the configuration of the cut-points:

PROPOSITION 2. *In any sequential equilibrium, the Crown's initial demand is:*

$$x_1^* = \begin{cases} x_{\mathrm{d}}(y_{\mathrm{L}}) & \text{if } q \leq q_{\mathrm{L}} \\ x_{\mathrm{w}} & \text{if } q \in (q_{\mathrm{L}}, q_{\mathrm{H}}] \\ x_{\mathrm{d}}(y_{\mathrm{H}}) & \text{if } q > q_{\mathrm{H}} \end{cases} \qquad \text{if } q_{\mathrm{L}} < q_{\mathrm{H}},$$

and

$$x_1^* = \begin{cases} x_d(y_L) & \textit{if } q \leq q_M \\ x_d(y_H) & \textit{if } q > q_M \end{cases} \qquad \textit{otherwise.} \qquad \square$$

As this proposition makes clear, any sequential equilibrium will generate the same probability distribution over the outcomes given the prior, and in that sense the equilibrium is unique. The only multiplicity that arises concerns off-the-paths beliefs.[34]

Proposition 2 shows that the introduction of asymmetric information about wealth wipes out most of the gains the Crown's coercive and agenda-setting advantages were supposed to confer (Result 1). To see this, consider two sets of comparisons. If the Crown knows that Subject's wealth is y_L, then it will peacefully obtain $x_d(y_L)$ in the first period and x_L in the second. When it deals with the same Subject but is uncertain about his wealth,

- if $q \leq q_L$, the Crown's demands are the same as in the complete information case (same payoff);
- if $q \in (q_L, q_H)$, the Crown's demand induces a revolt in the first period, and is the same in the second (worse payoff);
- if $q \geq q_H$, the Crown's demands induce revolts in both periods (worst payoff).

If the Crown knows that Subject's wealth is y_H, then it will peacefully obtain $x_d(y_H)$ in the first period and x_H in the second. When it deals with the same Subject but is uncertain about his wealth,

- if $q \geq q_H$, the Crown's demands are the same as in the complete information case, but the first-period demand induces a revolt with positive probability (worse payoff);
- if $q \in (q_L, q_H)$, the Crown's demand is lower in the first period, and the same in the second (even worse payoff);
- if $q \leq q_L$, the Crown's demands are much lower in both periods (worst payoff).

In this way, asymmetric information about wealth proves to be a significant obstacle to the Crown in its quest for money:

RESULT 2. *Even when the Crown enjoys agenda-setting and coercive advantages, its ability to extract wealth from society is seriously hindered by its lack of information about the wealth it is trying to tax. The Crown is forced into taxation that is either low (but peaceful), moderate (but riddled with exemptions for the wealthy and still provoking the poor into resistance), or high but risky (because it not only causes the poor to revolt but also sometimes provokes the wealthy as well).*

We now turn to some general qualitative implications of our results.

[34]For example, Lemma 8 establishes that when $q \leq q_1^*$, any demand $x \in (x_w, x_d(y_H))$ causes the Subject to rebel irrespective of type. This leaves acceptance off the path of play, and the equilibrium requires that the Crown demand $x_A = x_H$ in that case. There are many beliefs that can support this as a sequentially rational strategy, and in the proof we exhibit one assessment whose beliefs converge to assigning probability 1 on the rich Subject after acceptance as the fully mixed strategies converge to the equilibrium ones. There are, of course, other sequences that can do the same thing.

6 Discussion

We now examine some empirical implications of the theoretical mechanism, with some evidence in support.

6.1 Elite Resistance to Wealth Assessment

The central feature of the mechanism is that the credible revelation of information about wealth is impeded by the strategic incentives of the rich Subject. Even when both Crown and the poor Subject strictly prefer to move to complete information, the rich Subject is better off in the asymmetric information equilibrium (except when $q \geq q_{\mathrm{H}}$, where he is indifferent) because he either gets taxed very lightly ($q \leq q_{\mathrm{L}}$) or at least gets to enjoy some exemptions ($q \in (q_{\mathrm{L}}, q_{\mathrm{H}})$). The mechanism implies that the rich would try very hard to keep information about their wealth private, and that they would resist any measures that could allow rulers a glimpse into their financial situation.

There is, in fact, substantial evidence that elites were quite aware of their informational advantage and jealously guarded it. Whenever possible—that is, whenever a representative institution existed—resistance took the form of insisting on local collection and administration of the tax.[35] In the first instance, this allowed elites to shift the tax burden onto peasants and urban dwellers (Brustein and Levi, 1987, 471–3). But since only the total amounts were negotiated, it also prevented the Crown from forming an accurate estimate of what could be taxed, especially from the elites. Moreover, using a local administrative apparatus also prevented the Crown from continuing to collect the tax without consent. Even when the wealthy were not legally exempt from taxation, as in England, tax incidence could be very uneven, mostly because of the "chronic under-assessment of the political elite" (Seel and Smith, 2001, 29). Similarly, French financiers sought to make wealth valuations "difficult, time-consuming, and expensive to the government" out of fear of "retrospective taxation" should the Crown discover that there was more to be extracted (Bonney, 1981, 278).

The Crown was not unaware of the problem, and sometimes made determined attempts to collect relevant information although its ability to do so tended to be severely constrained politically.[36] It sometimes tried to elicit that information by having assessors and taxpayers to swear about the valuations, as was done in England. The practice was abandoned after 1566, probably because it was merely creating a nation of perjurers (Braddick, 1996, 94). The perpetual hostility to wealth assessment caused William Petty to admit that the "objection against this so exact computation of the Rents and [worth] of lands, &c. is, that the Sovereign would know too exactly every mans Estate." To this he had only to offer the tepid defense that "it would be a great discommodity to the Prince to take more than he needs," which of course caused him to wonder "where is the evil of this so exact knowledge?"[37]

[35] Blaufarb (2012, 25–31); Beik (1985, 118); Swann (2003, 81). Even though bargaining dealt with total revenues to be raised, the elites were much more concerned that the mechanics of administration be left in their hands.

[36] Soll (2011); Higgs (2004).

[37] "A Treatise of Taxes & Contributions" in Petty (1899, 53–4). Available online at The Online Library of Liberty, `http://oll.libertyfund.org/?option=com_staticxt&staticfile=show.php`

Perceptive observes knew very well where this evil lay because taxpayers could not rely on the tender mercies of benevolent monarchs. For instance, Francis Bacon praised Queen Elizabeth I for raising funds "by the assent of parliament, according to the ancient customs of this realm" and then asserted that her subjects paid their taxes "with great good-will and cheerfulness" because of her spending the money exclusively for "defence and preservation of the subject, not upon excessive buildings, nor upon immoderate donatives, nor upon triumphs and pleasures: or any the like veins of dissipation of treasure, which have been familiar to many kings." But he then made plain that the actual source of this tax merriment was not to be found in self-restraint by the monarch but in the prosaic fact that the subjects had been "taxed and also assessed with a very light and gentle hand", for "the Englishman is the most master of his own valuation, and the least bitten in his purse of any nation of Europe."[38]

The English were not the only ones concerned with keeping their wealth information away from the Crown, as two examples from France and Germany illustrate. The Estates of Languedoc, one of the few *pays d'états* of ancien régime France that survived until the Revolution, met annually as a single assembly, and deliberated in secrecy. Even though they were always convoked by the king, the Estates admitted no royal representatives to these deliberations, retained no records of the substance of debates, and offered no tally of the votes to accompany their final decision (Beik, 1985, 127–30). The need to conceal the nature of deliberations was more general, however. The first order of business for the assembly after checking the credentials of the deputies, was to swear to "serve the king and the province faithfully and to reveal nothing, by speech or writing, of anything said or done in the assembly that might be harmful and prejudicial to the assembly or to the individuals composing it".[39] The estates of Burgundy excluded the royal intendant from meetings to the point that he complained that he did not even known what they were talking about, and only informed the governor about their decisions through deputations (Swann, 2003, 77–8).

The Estates of Württemberg did have to acquiesce to the presence of ducal officials—the so-called *Amtleute* who were supposed to represent rural districts, a right that the deputies from the towns claimed for themselves—but they managed to exclude them from the agenda-setting committee that was elected at the beginning of each session. This committee deliberated in secrecy, discussed ducal demands, prepared the list of grievances, and all other matters of interest. It then submitted its recommendation to the full assembly for formal approval, which was nearly always granted. In this way, the ducal officials were "excluded from the confidential deliberations of the committee which they might have tried to influence in the prince's favour, or might have divulged to him."[40]

`%3Ftitle=1677`, accessed June 14, 2013.

[38] "Certain observations upon a libel published this present year, 1592, entitled, A declaration of the true causes of the great troubles presupposed to be intended against the realm of England," in Bacon (1824, Volume III, pp. 71–2). The argument is all the more convincing when one recalls that Bacon's object here was to counter what he considered a libelous claim that England was in an impoverished state as a result of ruinous taxation due to Elizabeth I's wars with Spain. Even as he was defending the Queen's policies Bacon could not but dwell on the causes of uncommonly low taxes.

[39] Mousnier (1979, 618); Brink (1980, 438).

[40] Carsten (1959, 26–8). The *Amtleute* were never allowed to participate in the standing Small and Large Committees that made decisions between diets of the full Estates. The Small Committee in particular controlled the financial administration of the Estates and could authorize grants within certain limits without the need to

There is also evidence that taxpayers recognized how information acquired through assessments could be used against them in the future, and that the Crown explicitly sought to minimize the commitment problem it was facing. The 1487 subsidy for an army of 10,000 archers in England authorized royal commissioners to assess the wealth, and it quite explicitly denied that this could become a precedent "considering that there never was before that time any like grant made" and it even provided that the certificates of wealth these commissioners made would be "never returnable in any of the king's court of record." Despite these precautions, the subsidy was quite unpopular, led to widespread resistance, and in the end collected no more than £27,000 of the £75,000 it was supposed to. Five of the six tax rebellions in the 15th century in England were directed against changes in the system of direct taxation.[41]

6.2 Revolts as Signaling Devices

Our model conceives of revolts as a (primitive but effective) form of communication in a political environment where signaling that the burden of taxation is unacceptable is very difficult because there exist no useful channels through which such signals can be sent (e.g., limited, if any, representation), because the use of such channels is prohibitively costly (e.g., submitting petitions to the sovereign), or because the signal is too easily manipulable to be meaningful. The view of revolts as communication whose goal is to alter undesirable (in this case, tax) policy is consistent with the empirical record, which shows fairly unambiguously that revolts almost never aim at overturning the social order or even removing the ruler and that they are almost always suppressed.[42] One is then left to wonder what the point of these revolts was. Our model reveals one such role:

RESULT 3. *Even when a revolt has no chance of overthrowing the Crown or impose any limit on subsequent policy, it can nevertheless induce the Crown — through the information it reveals — to change policy in its wake. Moreover, the fact that a revolt can potentially occur influences the Crown's present policy as well.*

In other words, revolts can succeed in the sense of causing the Crown to alter its policies because they can signal to the Crown that its attempted policies are so unacceptable that the subjects are willing to revolt despite the severe handicap they face. The Crown then has an incentive to react to this new information by adjusting its policies even when it suppresses the revolt itself. This possibility of influencing future policy provides an incentive for the revolt, and because this incentive exists, the Crown will take it into account even in its current policy. By explaining how this communication, policy revision, and choice of current policy happen the model can rationalize the otherwise puzzlingly large number of revolts and the subsequent policy changes.

This can explain the phenomenon of the common European myths about a good king deceived by bad ministers into creating a new tax, or about a tax remission granted by the king but subverted by the tax collectors Bercé (1990). Having these scapegoats offered the

convoke a diet.

[41] Dowell (1884, 169–70); Bush (1991, 382–3).

[42] Bercé (1987); Le Roy Ladurie (1974).

Crown plausible deniability, which allowed it to grant some demands without appearing to have been coerced. This, in turn, made it plausible that some concessions could be forthcoming, providing the resisters with some hope of success. Mousnier (1979, 741) notes how in 17th century France, the tax revolts' "frequency, [...] their forms of organization, and the ways in which they began and developed made them almost an institution." One is struck by how often these revolts unfolded in similar ways, with the rulers suppressing them (sometimes the rebels would simply surrender without a fight), executing a few ring-leaders, and then granting some relief of the grievances. Without the expectation that policy change would follow, people who could not hope to effect such change by force would not bother to revolt.

The informational explanation can thus account for the findings from the analysis of 267 rebellions in Tokugawa Japan between 1603 and 1868. Steele, Paik, and Tanaka (2017) show that during the period when the ruling class had unquestionable dominance in coercive capability, the average post-rebellion tax tended to be significantly lower than the pre-rebellion level.

6.3 Prevalence of Revolts by the Poor

The model implies that the vast majority of revolts would tend to originate in the poorer strata of society, which will only occasionally be joined by the wealthier elites.[43] When the equilibrium is separating, it is only the poor that revolt (and only in the first period), and when it is semi-separating, the poor revolt in both periods while the rich sometimes revolt but only in the first period. Inducing the wealthy to acquiesce to the demanded tax boils down to providing them with privileges in the sense that the tax is much lower than what it would have been had the Crown been certain of their wealth ($x_{\mathrm{w}} < x_{\mathrm{d}}(y_{\mathrm{H}})$), and even the weaker incentive that sometimes provokes them into rebellion is accompanied by a tax reduction in the next period ($x_{\mathrm{H}} < x_{\mathrm{d}}(y_{\mathrm{H}})$).[44] Whenever these elites expect the Crown to ratchet its future tax demand, the present tax must offer them sufficient compensation for not joining the poor in a revolt and obtaining the tax relief they expect. In other words, the model can explain the following patterns:

RESULT 4. *Most tax revolts will involve the poorer segments of society, and only rarely the wealthier ones. Moreover, when the wealthy accept a given level of taxation, it will often be outright privilege (proportionally lower than what the poor pay) or be accompanied with the (credible) expectation of a reduction in the future. A post-acceptance ratchet for the wealthy is possible but it requires a larger present compensation.*

6.4 Income Inequality

In the low-taxation equilibrium, no information is ever revealed, whereas in the moderate taxation one all of it is. In neither does the distribution of wealth have any impact on the

[43] For number and types of revolts in 17th century France that exhibit these patterns, see Bercé (1990, 169–243). Zagorin (1982, 175–275) provides a more general comparative study.

[44] See Kwass (2000) on the politics of taxation privilege.

information the Crown has in the second period. This is not the case in the high-taxation equilibrium, where there is only partial revelation, which depends on the Subject's wealth. The information transmitted in this equilibrium is always limited by the requirement that the Crown's posterior be precisely q_2^*. What happens to this inference as income inequality increases? Fix some y_L and consider the effect of increasing y_H. It is easy to see that

$$\frac{\mathrm{d}\, q_2^*}{\mathrm{d}\, y_\mathrm{H}} < 0$$

because x_H is increasing in y_H (Lemma 1). When the payoff from getting the high tax from the rich Subject increases (because his wealth is larger), the belief the Crown has to have about the Subject being rich before it is willing to impose that tax is smaller. This suggests that the rich Subject would not have to rebel so frequently, as indeed turns out to be the case:

$$\frac{\mathrm{d}\, r^*}{\mathrm{d}\, y_\mathrm{H}} = \left(\frac{p_\mathrm{L}}{p_\mathrm{H}}\right)\left(\frac{1-q}{q}\right)\left(\frac{1}{1-q_2^*}\right)\left(\frac{1}{1-q_2^*}\cdot\frac{\mathrm{d}\, q_2^*}{\mathrm{d}\, y_\mathrm{H}} - \frac{q_2^*}{p_\mathrm{H}}\cdot\frac{\mathrm{d}\, p_\mathrm{H}}{\mathrm{d}\, y_\mathrm{H}}\right) < 0.$$

This might be surprising when we recall that the tax he is accepting with higher probability is also larger ($x_\mathrm{d}(y_\mathrm{H})$ increases in y_H by Lemma 4). In other words, as inequality increases because the rich Subject gets richer, he becomes more likely to accept the high tax demand even though this demand has also gotten larger. Why is this happening? The Crown can impose a higher initial tax on the rich Subject only if it can credibly commit to imposing a higher tax in the second period. Ironically, the growing wealth of that Subject improves that credibility because it makes the post-rebellion risk-return trade-off demand more attractive to the Crown, and so it can commit not to offer tax relief. This weakens the incentive of the rich to join the revolt by the poor, and enables the Crown to extract a much higher expected return.

RESULT 5. *As income inequality increases, the rich become less likely to join the poor in revolt. The Crown is more likely to succeed in imposing even higher taxes. Although the extent of resistance to these taxes by the poor remains unchanged, the absence of elite support for that resistance improves the consolidation of the Crown as its tax revenues increase.*

This illustrates a fundamental aspect of our mechanism that explains the interaction between commitment and informational problems: the driving force behind the result is not the threat of the ratchet but the promise of relief.

6.5 Elites and the Problem of Tax Relief

One might be tempted to think that the most relevant problem of the Crown's relative lack of constraint would manifest itself through the ratchet effect: the rich do not want to reveal their wealth by accepting a high tax demand because doing so would cause the Crown to saddle them with even more burdensome taxes; and as a result they sometimes revolt, which in turn causes the Crown to lower its demands. For this logic to work, however, there must be some benefit of rebelling — after all, it is a fairly risky activity. This benefit must come in the form of possible tax relief that would not have occurred without the violence. But this suggests that the Crown does have a strategy that would severely reduce the incentives to revolt: it has to threaten to keep the taxes high when it fails to expropriate the rebels. Ironically, this is when the Crown's lack of constraint acquires a bite for the Crown cannot commit not to provide tax relief when it concludes that its subjects are likely poorer than it initially thought.

RESULT 6. *It is the seemingly benign aspect of the Crown's behavior — reducing taxes when informed by revolt that its subjects are being taxed beyond their endurance — that is furnishing the incentive to the rich to conceal their wealth by hiding behind the same grievance and in the end leads to under-taxation.*

Since the rich are more likely to survive the rebellion, the Crown can infer that rebels it has failed to defeat are also more likely to be wealthy, which does impart credibility to its threat to keep post-rebellion taxes high. This, however, comes at a very high price: if the Crown is wrong and its subjects are in fact poor, this strategy produces endemic strife; if the Crown is right, then this strategy induces the rich to rebel with positive probability as well. Thus, the Crown would only pursue such a strategy if it is sufficiently convinced that these risks are low — in all other circumstances it opts for safer and (much) lower taxes.

A telling dynamic occurs when the Crown is sufficiently optimistic to demand the highest tax $x_{\mathrm{d}}(y_{\mathrm{H}})$, whose acceptance by the rich is predicated on the Crown deliberately triggering a rebellion by the poor even in the second period. We already explained why the equilibrium requires that the Crown not offer post-rebellion tax relief: doing so would induce the rich to reject the demand to join the revolt with certainty. The substantive implication is that the Crown sacrifices the poor in an effort to commit credibly to a high post-rebellion tax so that the rich would be coerced into accepting the very high initial demand with positive probability.

This further suggests that attempts to increase taxation beyond moderate levels must not only generate frequent tax revolts by the poor, but also involve the Crown standing firm in its demands after quashing these revolts. Thus, if the Crown prefers the high tax regime to the moderate one with perks for the rich, it will not only demand higher taxes but will be increasingly unlikely to offer tax relief after a revolt. The resulting severe treatment of the poor (who are now revolting in the second period as well) will therefore come at the same time as the Crown puts the squeeze on the rich by eliminating their privileges.

Does this result imply that the second period tax is higher than the initial demand; that is, does the ratchet effect show up? It certainly does not when the equilibrium is pooling or semi-separating because in both instances the Crown actually provides tax relief upon acceptance ($x_{\mathrm{L}} < x_{\mathrm{d}}(y_{\mathrm{L}})$ and $x_{\mathrm{H}} < x_{\mathrm{d}}(y_{\mathrm{H}})$, respectively). The second-period demand can only exceed the first-period demand when the equilibrium is separating, and then only when the difference between the wealth of the rich and the poor types is substantial ($x_{\mathrm{H}} > x_{\mathrm{w}}$ per Lemma 7). Thus, while the ratchet is indeed possible, it never causes the rich to revolt in order to hide the information and prevent it. Instead, it causes the Crown to offer a first-period tax that the rich are willing to accept. One anecdote consistent with this comes from 1772, when the *vintiémes* for the district of Tours was increased by 100,000 *livres*, which prompted a complaint from the local administrator who wrote that "It is the facility with which the 250,000 *livres* were obtained by the last increase which has doubtless suggested that cruel step."[45]

6.6 Administrative Capacity: Higher Taxes with Less Resistance

As we have now seen, the difficulty of assessing the subjects' wealth can lead to persistent under-taxation and to frequent tax revolts. It was not merely evasion that reduced the taxes but the strategic constraints of their extraction in the shadow of threats of violence. An important implication of this analysis is that as the ability to conceal taxable wealth decreases (e.g., due to increased state capacity to inquire into the wealth of the subjects or the development of actuarial techniques to estimate it more reliably), the problems caused by asymmetric information should also decrease.

Consider what happens if the true wealth is y_{L} and the Crown's information improves (q decreases). The incidence of revolts will decrease (from occurring in both periods, to occurring only in the first period, to not occurring at all), and the successful first-period tax will become $x_{\mathrm{d}}(y_{\mathrm{L}})$. This is the highest tax that the Crown can extract, and it will do so without risking rebellion.

Consider now what happens if the true wealth is y_{H} and the Crown's information improves (q increases). The first-period tax will increase to the highest possible level $x_{\mathrm{d}}(y_{\mathrm{H}}) > x_{\mathrm{w}} > x_{\mathrm{d}}(y_{\mathrm{L}})$ while the probability that it provokes resistance will decrease significantly (in the limit, as $q \to 1$, the probability of a revolt goes to 0).

Thus, as its information improves, the Crown simultaneously extracts taxes closer to the maximum possible and runs lower chances of resistance. This improvement in the Crown's finances is neither due to an increase in its coercive powers (as much of the state-formation literature would have it) nor to it providing better inducements in the form of more service or public goods provision (as much of the literature on voluntary taxation would have it) nor to building fiscal capacity that merely allows it to collect taxes more efficiently.[46]

RESULT 7. *As the Crown's administrative capacity grows, taxes would tend to increase while at the same time the incidence and severity of tax revolts would tend to decrease. This effect will occur even if the state does not develop more extensive coercive powers and even if it does not offer more goods and services to its citizens.*

7 Relation to the Literature

The vast literature on optimal taxation does not, in general, consider the combination of commitment and informational issues that we raised in this paper. We have, however, identified two articles that are relevant. Freixas, Guesnerie, and Tirole (1985) analyze central

[45] Alexis de Tocqueville, *The Old Regime and the French Revolution,* note 70, p. 287.

[46] Tilly (1992); Besley and Persson (2010). The British government, which had supposedly been a constitutional monarchy since at least the middle of the eighteenth century did not really do much in terms of public services until 1870. Its budget, which had spent in excess of 80% on the military and debt service on average during the eighteenth century, still allocated 9% on the civil service, 56% on the military, and 35% on war-related debts in the nineteenth. See Webber and Wildavsky (1986, 289), and Hoppit (2017, 95–98).

planning under asymmetric information about the productivity of a firm when the planner cannot commit to a revision procedure. Dillén and Lundholm (1996) use this framework to study the dynamic problem of a taxing authority that must set rates on two individuals whose labor income is known but whose efficiency is not. Although the taxing authority maximizes social welfare unlike our Crown, which maximizes extraction, the incentives of the taxpayers are similar in the models: the high efficiency citizen wants to conceal that fact in order to avoid paying a transfer to the low efficiency one, whereas the latter wants to reveal his type to get that transfer. However, in neither case does the central authority have the power to expropriate a player who refuses the demanded production or tax. Moreover, the latter analysis is restricted to both types of citizen being equally likely while assuming that the budget constraint varies between periods. This makes it harder to relate the results to our model, whose predictions depend on the priors, and where incomes do not vary between periods. Consequently, we focus on Freixas, Guesnerie, and Tirole (1985) even though their setup seems quite different from ours.

The ratchet effect is the situation in which the firm underproduces in order to avoid demanding schemes in the future. In order to induce revelation, the central planner must be generous in the first period. This changes nothing for the low productivity firm, whose optimal action in the first period is the same as the optimal one-period action. (This is not the case in our model where the Crown's coercive advantage allows it to extract a strictly higher tax in the first period than in one-period action.) We could think of the Crown's demand in our model as analogous to the reverse of the bonus offered by the central planner: a *large* demand is analogous to a *small* bonus. This makes the "baseline" case the one where the poor type revolts and we can then consider what happens when the Crown starts with a large demand and then lowers it. A very large demand will, of course, cause the rich type to rebel as well: although we do not obtain such a result in equilibrium, the dynamic is analogous to pooling on low production. Reducing the demand eventually provides some incentive to the rich type to separate, and so he does that by sometimes accepting it and sometimes rebelling (the semi-separating form). Reducing the demand further strengthens that incentive, and eventually causes the rich type to separate fully by accepting it with certainty. The sequencing of the forms the equilibrium can take — from pooling to semi-separating to separating — is what happens in their model, and the logic has a similar flavor: smaller demands (larger bonuses) increase the rich (productive) type's costs of pooling with the poor (unproductive) type and induce separation. We have thus recovered the analogue to their "well-behaved" case without having placed many restrictions on the payoffs. (We also find that in contrast to the central planner, who does not have an optimal first-period demand, the Crown's initial tax is uniquely determined by its prior beliefs.)

This, of course, is where their model ends because, by definition, the unproductive firm cannot mimic the productive firm, so increasing the bonus beyond achieving this separation has no effect. In our model, however, lowering the demand does have an effect: it changes the behavior of the poor type, which switches to acceptance. When this happens, the rich type pools with it on accepting as well, which yields the low-taxation equilibrium in which no information is revealed. In other words, our model suggests a non-linear effect of tax demands, which go from inducing partial revelation of information, to full revelation, to no information transmission at all.

8 Conclusion

We began with the observation that in the early modern period, the Crown often seems to have been unable to tap effectively into the wealth available to the potential taxpayers.

Our central result is that despite coercive and bargaining advantages that would enable the Crown to impose high taxes without meeting resistance when it is fully informed about the wealth of the subjects, the Crown is in a very weak position when it lacks that information. Since the Crown always acts in its best interests given what it knows, it cannot commit not to provide tax relief after a revolt when it infers that the taxpayer is likely poor. This gives a rich subject incentive to exploit his informational advantage by joining the revolt to secure that concession. The Crown is forced into lowering the taxes that the rich Subject pays to compensate for the loss that observing wealth information would lead to in the future. Since the ability to impose high taxes without conflict today depends on the credible threat not to reward refusal to pay them, the Crown's lack of institutional constraints turns into a serious liability.

This mechanism can illuminate several features of the state-formation period that we find fascinating. As we observed, concern with concealing one's private wealth from the prying eyes of the Crown seems to have been a defining issue when it came to taxation. We took this to motivate the existence of asymmetric information about wealth in the model, but the mechanism reveals why such concern was rational. All else equal, providing the Crown with better information allows it to impose higher taxes with less resistance without necessarily giving the taxpayer anything in return. Because the tax is determined by bargaining in the shadow of coercion (rather than in a contractual or legislative manner), revealing one's wealth merely enables the Crown to drive a harder bargain. In fact, it could cause rich elites to lose any tax breaks and privileges they might have acquired when the Crown had to incentivize them not to participate in tax revolts by the poor. This rationalizes resistance to any schemes that would have enabled the Crown to look more closely at the resources of the subjects.

This leads us to the second aspect of taxation during this period that the model can explain: why it was so riddled with exceptions for the rich. After all, they were the people best positioned to contribute more without worsening their own situation perceptibly. As our mechanism indicates, the Crown can volunteer to lower the taxes the rich pay as a compensation for any revelation of their wealth that would be implicit when they fail to join a tax revolt by the poor. This compensation can be quite significant because it has to prevent the rich from resistance to the tax even when they expect that doing so would cause the Crown to offer tax relief in the future.

When the Crown compensates the rich adequately, it can safely infer that any subjects that still prefer to revolt must be poor. This can explain another frequent, and perhaps puzzling, aspect of these tax revolts: the relief that the Crown often offered afterwards. When one considers the overwhelming coercive advantage that the Crown enjoyed with respect to the poor, it is clear that the Crown could have easily crushed any such revolt. In many cases the tax relief came *after* the Crown had already successfully dealt with the revolt, which means it cannot have been a bargaining concession designed to avoid conflict. If this were a strategy to prevent future conflict, then it must have been the case that the revolt somehow changed the Crown's information about how much taxation the subjects were willing to accept. The mechanism shows that such information transmission can occur when the Crown's strategy causes the rich to abstain from the revolt. In other words, tax relief after a revolt occurs because the Crown's tax policy has successfully separated the rich from the poor.

This, in turn, explains revolts by the poor, which otherwise would have been completely pointless given the Crown's vast power relative to them. The poor need not be desperate, misled, deluded, or mistaken to choose resistance. Indeed, the model shows that in many cases their actions are rationalizable because they are consequential: the Crown does respond by lowering its demands. (Of course, as the semi-separating tax regime demonstrates, this might not always work.)

The model also suggests an alternative explanation for what is sometimes called the "ratchet effect in taxation" — the finding that once taxes go up during some emergency, they seldom return to their previous levels even after the emergency has passed.[47] One reason for this could be that once they acquiesce to supposedly temporary new taxes, the subjects become accustomed to them, so it becomes easier to continue the policy. Another reason could be bureaucratic capture: every policy creates a constituency that benefits from it, and if this constituency is politically important, it could block any attempts to reduce the taxes to prior levels. None of these explanations seems to be useful for the early modern period. The primary beneficiary of new taxes was the entity imposing them — the Crown — and the bureaucracies that administered them were either not particularly large or not very important politically. If the Crown wished to rescind taxes, there would have been little effective resistance to doing so. Moreover, most emergencies did not last long enough for accustomization to take place.

Our model suggests that when subjects agree to pay for increased taxes during an emergency, they end up revealing their ability to do so, and this is bound to figure in the Crown's subsequent calculations. Of course, when the opportunity cost of not paying the taxes is very high — as it would be in an emergency — the revealed willingness to pay might not be the most useful guide to the future. On the other hand, the fact that taxpayers could deliver the requested amount could undermine claims that what the Crown was demanding was outside the realm of the possible. This might be one reason why after the Catholics withdrew from his territories at the conclusion of the Thirty Years War, the Hohenzollern Elector imposed taxation that was far higher than the pre-war levels but tracked closely what the occupying armies had been able to extract with their forced contributions from his conquered lands (Wilson, 2009).

Finally, the model suggests that the path to state-formation might not be through increased coercive powers of the central authority or its expanding menu of public services, but something far more prosaic: an improvement in the administrative capacity to assess wealth. The fiscal state could have come about merely because the Crown learned how to measure what was there to tax effectively, and with this it could increase its revenue while facing less resistance to its extractive policies. No taxation without administration.

[47] Peacock and Wiseman (1961); Higgs (1987).

References

Aalbers, J. 1977. "Holland's Financial Problems (1713–1733) and the Wars against Louis XIV." In *Britain and the Netherlands*, ed. A. C. Duke, and C. A. Tamse. The Hague: Martinus Nijhoff.

Bacon, Francis. 1824. *The Works of Francis Bacon, Baron of Verulam, Viscount St. Albans, and Lord High Chancellor of England. In Ten Volumes*. London: W. Baynes and Son.

Barkey, Karen. 1991. "Rebellious Alliances: The State and Peasant Unrest in Early Seventeenth-Century France." *American Sociological Review* 56(December): 699–715.

Beik, William. 1985. *Absolutism and Society in Seventeenth-century France: State Power and Provincial Aristocracy in Languedoc*. Cambridge: Cambridge University Press.

Beik, William. 1997. *Urban Protest in Seventeenth-Century France: The Culture of Retribution*. Cambridge: Cambridge University Press.

Bercé, Yves-Marie. 1987. *Revolt and Revolution in Early Modern Europe: An Eassy on the History of Political Violence*. New York: St. Martin's Press.

Bercé, Yves-Marie. 1990. *History of Peasant Revolts: The Social Origins of Rebellion in Early Modern France*. Ithaca: Cornell University Press.

Besley, Timothy, and Torsten Persson. 2009. "The Origins of State Capacity: Property Rights, Taxation, and Politics." *American Economic Review* 99(4): 1218–1244.

Besley, Timothy, and Torsten Persson. 2010. "State Capacity, Conflict, and Development." *Econometrica* 78(1): 1–34.

Blaufarb, Rafe. 2012. *The Politics of Fiscal Privilege in Provence, 1530s–1830s*. Washington, D.C.: The Catholic University of America Press.

Bonney, Richard. 1981. *The King's Debts: Finance and Politics in France, 1589-1661*. Oxford: Clarendon Press.

Bonney, Richard. 1991. *The European Dynastic States, 1494–1660*. Oxford: Oxford University Press.

Braddick, Michael J. 1996. *The Nerves of State: Taxation and the Financing of the English State, 1558–1714*. Manchester: Manchester University Press.

Brewer, John. 1990. *The Sinews of Power: War, Money, and the English State, 1688–1783*. Cambridge: Harvard University Press.

Brink, James Eastgate. 1980. "The Estates General of Languedoc: Struggles for Provincial Autonomy in Early 16th Century France." *Legislative Studies Quarterly* 5(August): 437–46.

Brustein, William, and Margaret Levi. 1987. "The Georgraphy of Rebellion: Rulers, Rebels, and Region, 1500 to 1700." *Theory and Society* 16(July): 467–495.

Burg, David F. 2004. *A World History of Tax Rebellions: An Encyclopedia of Tax Rebels, Revolts, and Riots from Antiquity to the Present*. New York: Routledge.

Bush, Michael. 1991. "Tax Reform and Rebellion in Early Tudor England." *History* 76(248): 379–400.

Carsten, F. L. 1959. *Princes and Parliaments in Germany from the Fifteenth to the Eighteenth Century*. Oxford: Oxford University Press.

Collins, James B. 1988. *Fiscal Limits of Absolutism: Direct Taxation in Early Seventeenth-Century France*. Berkeley: University of California Press.

Cox, Gary W. 2016. *Marketing Sovereign Promises: Monopoly Brokerage and the Growth of the English State*. Cambridge: Cambridge University Press.

Dickson, P.G.M. 1967. *The Financial Revolution in England: A Study in the Development of Public Credit*. London: Macmillan.

Dillén, Mats, and Michael Lundholm. 1996. "Dynamic income taxation, redistribution, and the ratchet effect." *Journal of Public Economics* 59: 69–93.

Dowell, Stephen. 1884. *A History of Taxation and Taxes in England from the Earliest Times to the Present Day*. London: Longmand, Green & Co.

Downing, Brian. 1992. *The Military Revolution and Political Change*. Princeton: Princeton University Press.

Drelichman, Mauricio, and Hans-Joachim Voth. 2016. *Lending to the Borrower from Hell: Debt, Taxes, and Default in the Age of Philip II*. Princeton: Princeton University Press.

Elliott, J. H. 1969. "Revolution and Continuity in Early Modern Europe." *Past & Present* 42(February): 35–56.

Elliott, J. H. 1984. *The Revolt of the Catalans: A Study in the Decline of Spain, 1598–1640*. Cambridge: Cambridge University Press.

Ertman, Thomas. 1997. *Birth of the Leviathan: Building States and Regimes in Medieval and Early Modern Europe*. Cambridge: Cambridge University Press.

Freixas, Xavier, Roger Guesnerie, and Jean Tirole. 1985. "Planning under Incomplete Information and the Ratchet Effect." *Review of Economic Studies* 52(2): 173–191.

Glete, Jan. 2002. *War and the State in Early Modern Europe: Spain, the Dutch Republic, and Sweden as Fiscal-military States*. London: Routledge.

Goldstone, Jack A. 1991. *Revolution and Rebellion in the Early Modern World*. Berkeley: The University of California Press.

Guenée, Bernard. 1981. *States and Rulers in Later Medieval Europe*. Oxford: Basil Blackwell.

Henneman, John Bell. 1971. *Royal Taxation in Fourteenth Century France: The Development of War Financing, 1322-1356*. Princeton: Princeton University Press.

Hicks, John. 1969. *A Theory of Economic History*. Oxford: Oxford University Press.

Higgs, Edward. 2004. *The Information State in England: The Central Collection of Information on Citizens since 1500*. New York: Plagrave Macmillan.

Higgs, Robert. 1987. *Crisis and Leviathan: Critical Episodes in the Growth of American Government*. Oxford: Oxford University Press.

Holsti, Kalevi J. 1990. *Peace and War: Armed Conflicts and International Order, 1648–1989*. Cambridge: Cambridge University Press.

Hoppit, Julian. 2017. *Britain's Political Economies: Parliament and Economic Life, 1660–1800*. Cambridge: Cambridge University Press.

Kiser, Edgar, and April Linton. 2002. "The Hinges of History: State-Making and Revolt in Early Modern France." *American Sociological Review* 67(6): 889–910.

Körner, Martin. 1995. "Expenditure." In *Economic Systems and State Finance*, ed. Richard Bonney. Oxford: Oxford University Press pp. 393–422.

Kreps, David, and Robert Wilson. 1982. "Sequential Equilibrium." *Econometrica* 50: 863–94.

Kwass, Michael. 2000. *Privilege and the Politics of Taxation in Eighteenth-Century France*. Cambridge: Cambridge University Press.

Le Roy Ladurie, Emmanuel. 1974. *The Peasants of Languedoc*. Urbana-Champaign: University of Illinois Press.

Levi, Margaret. 1989. *Of Rule and Revenue*. Berkeley: University of California Press.

Lynn, John A. 1999. *The Wars of Louis XIV, 1667–1714*. London: Longman.

Mousnier, Roland. 1979. *The Institutions of France under the Absolute Monarchy, 1598–1789. Volume I: Society and the State*. Chicago: The University of Chicago Press.

Myerson, Roger B.İ991. *Game Theory: Analysis of Conflict*. Cambridge: Harvard University Press.

Peacock, Alan T., and Jack Wiseman. 1961. *The Growth of Public Expenditure in the United Kingdom*. Princeton: Princeton University Press.

Petty, William. 1899. *The Economic Writings of Sir William Petty, together with The Observations upon Bills of Mortality, more probably by Captain John Graunt. Two Volumes*. London: Cambridge University Press.

Powell, Robert. 2002. "Bargaining Theory and International Conflict." *Annual Review of Political Science* 5: 1–30.

Rosenthal, Jean-Laurent. 1998. "The Political Economy of Absolutism Reconsidered." In *Analytic Narratives*, ed. Robert H. Bates, Avner Greif, Margaret Levi, Jean-Laurent Rosenthal, and Barry R. Weingast. Princeton: Princeton University Press.

Russell, Conrad S. R. 1982. "Monarchies, Wars, and Estates in England, France, and Spain, c. 1580 – c. 1640." *Legislative Studies Quarterly* 7(May): 205–220.

Seel, Graham E., and David L. Smith. 2001. *Crown and Parliaments, 1558–1689*. Cambridge: Cambridge University Press.

Soll, Jacob. 2011. *The Information Master: Jean-Baptiste Colbert's Secret State Intelligence System*. Ann Arbor: The University of Michigan Press.

Stasavage, David. 2010. "When Distance Mattered: Geographic Scale and the Development of European Representative Assemblies." *American Political Science Review* 104(November): 625–643.

Steele, Abbey, Christopher Paik, and Seiki Tanaka. 2017. "Constraining the Samurai: Rebellion and Taxation in Early Modern Japan." *International Studies Quarterly* 61(June): 352–370.

Storrs, Christopher, ed. 2009. *The Fiscal-Military State in Eighteenth-Century Europe*. Farnham: Ashgate.

Swann, Julian. 2003. *Provincial Power and Absolute Monarchy: The Estates General of Burgundy, 1661–1790*. Cambridge: Cambridge University Press.

Tilly, Charles. 1992. *Coercion, Capital, and European States, AD 990-1992*. Oxford: Blackwell.

Tracy, James D. 1985. *A Financial Revolution in the Habsburg Netherlands: Renten and Renteniers in the County of Holland, 1515–1565*. Berkeley: University of California Press.

Webber, Carolyn, and Aaron Wildavsky. 1986. *A History of Taxation and Expenditure in the Western World*. New York: Simon and Schuster.

Wilson, Peter H. 2009. "Prussia as a Fiscal-Military State, 1640–1806." *in* Storrs (2009).

Wolfe, Martin. 1972. *The Fiscal System of Renaissance France*. New Haven: Yale University Press.

Yun-Casalilla, Bartolomé, Patrick O'Brien, and Francisco Comín Comín, eds. 2012. *The Rise of Fiscal States: A Global History, 1500–1914*. Cambridge: Cambridge University Press.

Zagorin, Perez. 1982. *Rebels & Rulers, 1500–1660*. Cambridge: Cambridge University Press.

A Proofs

Proof of Lemma 1. Take some $y_L < y_H$ and let $x_L = x_{\mathrm{k}}(y_L) > 0$ and $x_H = x_{\mathrm{k}}(y_H) > 0$. We need to show that $x_L < x_H$.

Observe that $\theta(y) = U(0; y) - R(y)$ and recall that under our assumption about p, it is strictly increasing, so $\theta(y_L) < \theta(y_H)$. Define $\hat{U}(x; y) = U(x; y) - \theta(y)$ and note that it inherits the concavity in x and the supermodularity of $U(x; y)$. By supermodularity, we have:

$$\hat{U}(x_L; y_H) - \hat{U}(x_L; y_L) > \hat{U}(0; y_H) - \hat{U}(0; y_L).$$

By definition, $\hat{U}(0; y_H) = R(y_H)$, $\hat{U}(0; y_L) = R(y_L)$, and $\hat{U}(x_L; y_L) = U(x_L; y_L) - \theta(y_L)$. Moreover, since x_L is such that $U(x_L; y_L) = R(y_L)$, we can write $\hat{U}(x_L; y_L) = R(y_L) - \theta(y_L)$. Using these identities, we can write the inequality above as:

$$\hat{U}(x_L; y_H) - R(y_L) + \theta(y_L) > R(y_H) - R(y_L),$$

which simplifies to

$$\hat{U}(x_L; y_H) + \theta(y_L) > R(y_H).$$

But now we obtain:

$$U(x_L; y_H) = \hat{U}(x_L; y_H) + \theta(y_H) > \hat{U}(x_L; y_H) + \theta(y_L) > R(y_H),$$

where the first inequality follows from $\theta(y)$ increasing. But since $U(0; y_H) > R(y_H)$, the definition of x_H tells us that $U(x; y_H) > R(y_H)$ for all $x < x_H$ while $U(x; y_H) < R(y_H)$ for all $x > x_H$. This means the our finding of $U(x_L; y_H) > R(y_H)$ implies that $x_L < x_H$. ■

Proof of Lemma 2. Recall that $\theta(y) = U(0; y) - R(y)$ and note that

$$V(\theta(y)) = V(p(y)(0) + (1 - p(y))y) \geq p(y)V(0) + (1 - p(y))V(y) = W(y),$$

where the inequality follows from the concavity of V and $V(0) = 0$. It is therefore sufficient to show that $V(x_{\mathrm{k}}(y)) > V(\theta(y))$. Since V is increasing, we must prove that $x_{\mathrm{k}}(y) > \theta(y)$, which holds because:

$$\begin{aligned} U(\theta(y); y) &= U(p(y)(0) + (1 - p(y))y; y) \\ &> p(y)U(0; y) + (1 - p(y))U(y; y) = R(y) = U(x_{\mathrm{k}}(y); y), \end{aligned}$$

where the inequality follows from the strict concavity of U in x and $U(y; y) = 0$. ■

Proof of Lemma 3. By concavity, we know that $G(y) = (1-p(y))V(y)+p(y)V(x_{\mathrm{k}}(y)) \leq V((1 - p(y))y + p(y)x_{\mathrm{k}}(y))$, and since V is increasing it will be sufficient to show that $x_{\mathrm{d}}(y) > (1 - p(y))y + p(y)x_{\mathrm{k}}(y)$. Since U is decreasing in x in this region, this is equivalent to showing that $U(x_{\mathrm{d}}(y); y) < U((1 - p(y))y + p(y)x_{\mathrm{k}}(y); y)$. But now the strict concavity of U implies that

$$\begin{aligned} U\big(1 - p(y))y + p(y)x_{\mathrm{k}}(y); y\big) &> (1 - p(y))U(y; y) + p(y)U(x_{\mathrm{k}}(y); y) \\ &= p(y)U(x_{\mathrm{k}}(y); y) = p(y)R(y) = U(x_{\mathrm{d}}(y); y). \end{aligned}$$

where the first equality follows from $U(y; y) = 0$, and the rest from the definitions of $x_{\mathrm{k}}(y)$ and $x_{\mathrm{d}}(y)$. ■

Proof of Proposition 1. In the second period the Subject accepts only $x \leq x_{\mathrm{k}}(y)$, and since by Lemma 2 the Crown is always strictly better off taxing at the violence-constrained maximum than inducing rebellion, the strategies in the second period are optimal. The definition of $x_{\mathrm{d}}(y)$ is such that it is the highest tax the Subject would accept in the first period when expecting $x_{\mathrm{k}}(y)$ in the second, so it is also the optimal peaceful demand for the Crown. The payoff from making this demand is

$$V_1(x_{\mathrm{d}}(y)) = V(x_{\mathrm{d}}(y)) + V(x_{\mathrm{k}}(y)).$$

The only remaining possibility is that the Crown makes an unacceptable first-period demand and so induces a rebellion in that period. Consider some $x > x_{\mathrm{d}}(y)$ that the Subject rejects, so the Crown's payoff would be:

$$V_1(x) = W(y) + \big[(1 - p(y))V(y) + p(y)V(x_{\mathrm{k}}(y))\big] = W(y) + G(y).$$

Observe now that $V_1(x) < V_1(x_{\mathrm{d}})$ obtains because $V(x_{\mathrm{k}}(y)) > W(y)$ by Lemma 2 and $V(x_{\mathrm{d}}(y)) > G(y)$ by Lemma 3. We conclude that the Crown cannot prefer to induce rebellion in the first period, which implies that the unique peaceful equilibrium is also the unique equilibrium. ∎

Proof of Lemma 4. Since $U(x_{\mathrm{k}}(y); y) = R(y) > p(y)R(y) = U(x_{\mathrm{d}}(y); y)$ and $U(x; y)$ is decreasing at $x_{\mathrm{k}}(y)$, the fact that $x_{\mathrm{d}}(y)$ is unique implies that $x_{\mathrm{k}}(y) < x_{\mathrm{d}}(y)$. Define

$$\hat{c}(y) = U(0; y) - p(y)R(y) = \big(1 - p^2(y)\big)y = (1 + p(y))\theta(y),$$

and observe that since $\theta(y)$ and $p(y)$ are both increasing in y, so is $\hat{c}(y)$. Define now $\hat{U}(x; y) = U(x; y) - \hat{c}(y)$. The rest of the proof replicates the proof of Lemma 1. ∎

Proof of Lemma 5. By subgame perfection, the rich Subject rejects any $x > x_{\mathrm{H}}$, and accepts any $x \leq x_{\mathrm{H}}$, whereas the poor Subject rejects any $x > x_{\mathrm{L}}$, and accepts any $x \leq x_{\mathrm{L}}$. Since $x_{\mathrm{L}} < x_{\mathrm{H}}$ by Lemma 1, these imply that any $x \leq x_{\mathrm{L}}$ will be accepted with certainty, any $x > x_{\mathrm{H}}$ will be rejected with certainty, and any $x \in (x_{\mathrm{L}}, x_{\mathrm{H}}]$ will be accepted only if the Subject is rich. Since the Crown's payoff is strictly increasing, it follows that x_{L} strictly dominates any $x < x_{\mathrm{L}}$ and x_{H} strictly dominates any $x \in (x_{\mathrm{L}}, x_{\mathrm{H}})$. Since Lemma 2 also tells us that x_{H} strictly dominates any $x > x_{\mathrm{H}}$, it follows that in any equilibrium the Crown would only ever make one of two demands: x_{L}, which yields $V(x_{\mathrm{L}})$, and x_{H}, which yields $(1-q_2)W(y_{\mathrm{L}})+q_2V(x_{\mathrm{H}})$. Setting the payoffs equal to each other and solving for q_2 yields the value of q_2^*. The claim follows immediately. ∎

Proof of Lemma 6. If y_{L} accepts x_1, his payoff is $U(x_1; y_{\mathrm{L}}) + R(y_{\mathrm{L}})$, and if he rejects it his payoff is $R(y_{\mathrm{L}}) + p_{\mathrm{L}}R(y_{\mathrm{L}})$. Thus, $U(x_1; y_{\mathrm{L}}) \gtreqless p_{\mathrm{L}}R(y_{\mathrm{L}}) \Leftrightarrow x_1 \lesseqgtr x_{\mathrm{d}}(y_{\mathrm{L}})$. The first claim follows immediately.

To establish the second claim, consider $x_1 = x_{\mathrm{d}}(y_{\mathrm{L}})$ and assume that the rich type accepts it in equilibrium. The proof proceeds by supposing that in some equilibrium the poor type rejects this demand with positive probability, and then showing that the Crown could always obtain a strictly better payoff by deviating to a slightly smaller demand that is accepted with certainty. In other words, the Crown has no best response if $x_{\mathrm{d}}(y_{\mathrm{L}})$ is rejected with positive probability, implying that in equilibrium both types must be accepting it.

Observe first that since the poor type accepts any $x < x_1$, the rich type must accept it as well. To see this, consider the worst that he can expect in the second period if he accepts now: $R(y_{\mathrm{H}})$. Accepting x then yields at least $U(x; y_{\mathrm{H}}) + R(yH)$, whereas rejecting it yields at most $R(y_{\mathrm{H}}) + p_{\mathrm{H}} U(x_{\mathrm{H}}; y_{\mathrm{H}}) = R(y_{\mathrm{H}}) + U(x_{\mathrm{d}}(y_{\mathrm{H}}); y_{\mathrm{H}})$. But since $U(x; y_{\mathrm{H}}) > U(x_{\mathrm{d}}(y_{\mathrm{H}}); y_{\mathrm{H}})$ for any $x < x_{\mathrm{d}}(y_{\mathrm{H}})$, it follows that he must accept $x < x_{\mathrm{d}}(y_{\mathrm{L}})$. We thus conclude that $x < x_1$ must be accepted by both types in any equilibrium.

Suppose now that the poor type rejects x_1 in equilibrium with probability $r > 0$. Since by assumption the rich type accepts it with certainty, it is only the poor type who ever rebels with positive probability, which implies that $x_{\mathrm{R}} = x_{\mathrm{L}}$. The Crown's payoff from demanding x_1 then is

$$q\Big[V(x_1) + V(x_{\mathrm{H}})\Big] + (1-q)\Big[r\big(W(y_{\mathrm{L}}) + G(y_{\mathrm{L}})\big) + (1-r)\big(V(x_1) + \mathrm{E}\left[V(x_{\mathrm{A}}); y_{\mathrm{L}}\right]\big)\Big],$$

where $\mathrm{E}\left[V(x_{\mathrm{A}}); y_{\mathrm{L}}\right] \in \{V(x_{\mathrm{L}}), W(y_{\mathrm{L}})\}$ denotes the Crown's expected payoff from the way the poor type responds to x_{A} (the rich type always accepts it). Note also that the updated belief after acceptance is

$$q_2 = \frac{q}{1 + (1-r)(1-q)} > q.$$

We have two cases to consider.

Case I: $q \geq q_2^*$, which implies that $q_2 > q_2^*$ after acceptance, so $x_{\mathrm{A}} = x_{\mathrm{H}}$. Since the poor type rejects this, $\mathrm{E}\left[V(x_{\mathrm{A}}); y_{\mathrm{L}}\right] = W(y_{\mathrm{L}})$, so the Crown's expected payoff can be written as

$$qV(x_1) + qV(x_{\mathrm{H}}) + (1-q)W(y_{\mathrm{L}}) + (1-q)\big[rG(y_{\mathrm{L}}) + (1-r)V(x_1)\big]. \tag{5}$$

Consider now a deviation to some $x < x_1$, which is accepted by both types. In this case, $q_2 = q \geq q_2^*$, so the Crown will demand $x_{\mathrm{A}} = x_{\mathrm{H}}$ still.[48] The Crown's expected payoff from this deviation is:

$$q\Big[V(x) + V(x_{\mathrm{H}})\Big] + (1-q)\Big[V(x) + W(y_{\mathrm{L}})\Big] = V(x) + qV(x_{\mathrm{H}}) + (1-q)W(y_{\mathrm{L}}).$$

This deviation would be profitable if

$$V(x) > qV(x_1) + (1-q)\big[rG(y_{\mathrm{L}}) + (1-r)V(x_1)\big]. \tag{6}$$

By Lemma 3, $G(y_{\mathrm{L}}) < V(x_1)$, which implies that $qV(x_1) + (1-q)\big[rG(y_{\mathrm{L}}) + (1-r)V(x_1)\big] < V(x_1)$. But since $\lim_{x \to x_1^-} V(x) = V(x_1)$, it follows that there exists $\hat{x} < x_1$ such that (6) is satisfied for any $x \in (\hat{x}, x_1)$. Thus, the Crown has no best response if $q \geq q_2^*$.

Case II: $q < q_2^*$, in which case we have two possibilities to consider. Suppose first that $q_2 \geq q_2^*$, which means that $x_{\mathrm{A}} = x_{\mathrm{H}}$, so the Crown's equilibrium expected payoff is again given by (5). Consider a deviation to some $x < x_1$, which both types accept. Since

[48] Technically, if $q = q_2^*$, the Crown could demand either x_{L} or x_{H}, but since it is indifferent, we can calculate the expected payoff assuming x_{H} without loss of generality. This situation has measure zero anyway.

$q < q_2^*$, $x_{\mathrm{A}} = x_{\mathrm{L}}$, which both types accept as well. The Crown's expected payoff from this deviation is:

$$V(x) + V(x_{\mathrm{L}}) > V(x) + qV(x_{\mathrm{H}}) + (1-q)W(y_{\mathrm{L}}),$$

where the inequality follows from $q < q_2^*$. This reduces this situation to the case we examined above, so the deviation is profitable for x sufficiently close to x_1.

Suppose now that $q_2 < q_2^*$, which means that $x_{\mathrm{A}} = x_{\mathrm{L}}$. Since the poor type accepts this, $\mathrm{E}\left[V(x_{\mathrm{A}}); y_{\mathrm{L}}\right] = V(x_{\mathrm{L}})$, so the Crown's expected payoff can be written as:

$$\begin{aligned} q\Big[V(x_1) + V(x_{\mathrm{L}})\Big] + (1-q)\Big[r\big(W(y_{\mathrm{L}}) + G(y_{\mathrm{L}})\big) + (1-r)\big(V(x_1) + V(x_{\mathrm{L}})\big)\Big] \\ < q\Big[V(x_1) + V(x_{\mathrm{L}})\Big] + (1-q)\Big[V(x_1) + V(x_{\mathrm{L}})\Big] = V(x_1) + V(x_{\mathrm{L}}), \quad \blacksquare \end{aligned}$$

where the inequality follows from Lemma 2 and Lemma 3, which imply that $W(y_{\mathrm{L}}) + G(y_{\mathrm{L}}) < V(x_1) + V(x_{\mathrm{L}})$. We already know that deviating to some $x < x_1$ yields an expected payoff of $V(x) + V(x_{\mathrm{L}})$. Since taking x sufficiently close to x_1 makes this payoff arbitrarily close to $V(x_1) + V(x_{\mathrm{L}})$, it also makes the deviation profitable. Thus, the Crown has no best response if $q < q_2^*$ as well.

We conclude that in any equilibrium, the poor type must accept $x_{\mathrm{d}}(y_{\mathrm{L}})$ with certainty whenever the rich type accepts it.

Proof of Lemma 7. First, $x_{\mathrm{L}} < x_{\mathrm{d}}(y_{\mathrm{L}})$ is just $x_{\mathrm{k}}(y) < x_{\mathrm{d}}(y)$, which we established in Lemma 4. We now show that $x_{\mathrm{d}}(y_{\mathrm{L}}) < x_{\mathrm{w}}$. We show first that $U(x_{\mathrm{d}}(y_{\mathrm{L}}); y_{\mathrm{H}}) > p_{\mathrm{H}}U(x_{\mathrm{L}}; y_{\mathrm{H}}) = U(x_{\mathrm{w}}; y_{\mathrm{H}})$. Since $x_{\mathrm{d}}(y_{\mathrm{L}}) > x_{\mathrm{L}}$, supermodularity yields

$$U(x_{\mathrm{d}}(y_{\mathrm{L}}); y_{\mathrm{H}}) + U(x_{\mathrm{L}}; y_{\mathrm{L}}) > U(x_{\mathrm{L}}; y_{\mathrm{H}}) + U(x_{\mathrm{d}}(x_{\mathrm{L}}); y_{\mathrm{L}}),$$

and since $U(x_{\mathrm{d}}(y_{\mathrm{L}}); y_{\mathrm{L}}) = p_{\mathrm{L}}R(y_{\mathrm{L}})$ and $U(x_{\mathrm{L}}; y_{\mathrm{L}}) = R(y_{\mathrm{L}})$, we can write this inequality as

$$U(x_{\mathrm{d}}(y_{\mathrm{L}}); y_{\mathrm{H}}) > U(x_{\mathrm{L}}; y_{\mathrm{H}}) - (1-p_{\mathrm{L}})R(y_{\mathrm{L}}).$$

Thus, it will be sufficient to show that

$$U(x_{\mathrm{L}}; y_{\mathrm{H}}) - (1-p_{\mathrm{L}})R(y_{\mathrm{L}}) > p_{\mathrm{H}}U(x_{\mathrm{L}}; y_{\mathrm{H}}).$$

We can rewrite this as:

$$(1-p_{\mathrm{H}})U(x_{\mathrm{L}}; y_{\mathrm{H}}) > (1-p_{\mathrm{L}})R(y_{\mathrm{L}}) = p_{\mathrm{L}}\theta(y_{\mathrm{L}}).$$

Since $p_{\mathrm{H}}\theta(y_{\mathrm{H}}) > p_{\mathrm{L}}\theta(y_{\mathrm{L}})$, it will be sufficient to show that

$$(1-p_{\mathrm{H}})U(x_{\mathrm{L}}; y_{\mathrm{H}}) > p_{\mathrm{H}}\theta(y_{\mathrm{H}}) = (1-p_{\mathrm{H}})R(y_{\mathrm{H}}),$$

which holds because $x_{\mathrm{L}} < x_{\mathrm{H}}$ implies that $U(x_{\mathrm{L}}; y_{\mathrm{H}}) > U(x_{\mathrm{H}}; y_{\mathrm{H}}) = R(y_{\mathrm{H}})$. Thus, $U(x_{\mathrm{d}}(y_{\mathrm{L}}); y_{\mathrm{H}}) > U(x_{\mathrm{w}}; y_{\mathrm{H}})$. Since $U(x; y_{\mathrm{H}})$ is decreasing for all $x \geq x_{\mathrm{w}}$, this implies that $x_{\mathrm{d}}(y_{\mathrm{L}}) < x_{\mathrm{w}}$, as required. We finally need to show that $x_{\mathrm{w}} < x_{\mathrm{d}}(y_{\mathrm{H}})$. But since $x_{\mathrm{L}} < x_{\mathrm{H}}$ implies that $U(x_{\mathrm{w}}; y_{\mathrm{H}}) = p_{\mathrm{H}}U(x_{\mathrm{L}}; y_{\mathrm{H}}) > p_{\mathrm{H}}U(x_{\mathrm{H}}; y_{\mathrm{H}}) = U(x_{\mathrm{d}}(y_{\mathrm{H}}); y_{\mathrm{H}})$, the result follows.

Consider now the relationship between x_{w} and x_{H}:

$$U(x_{\mathrm{w}}; y_{\mathrm{H}}) = p_{\mathrm{H}} U(x_{\mathrm{L}}; y_{\mathrm{H}}) \gtrless p_{\mathrm{H}} U(0; y_{\mathrm{H}}) = U(x_{\mathrm{H}}; y_{\mathrm{H}})$$
$$U(x_{\mathrm{L}}; y_{\mathrm{H}}) \gtrless U(0; y_{\mathrm{H}}).$$

Recalling that $U(x; y)$ is concave in x and noting that

$$\lim_{y_{\mathrm{L}} \to 0} U(x_{\mathrm{L}}; y_{\mathrm{H}}) = U(0; y_{\mathrm{H}})$$
$$\lim_{y_{\mathrm{L}} \to y_{\mathrm{H}}} U(x_{\mathrm{L}}; y_{\mathrm{H}}) = U(x_{\mathrm{H}}; y_{\mathrm{H}}) < U(0; y_{\mathrm{H}}),$$

we conclude that there exists $\widetilde{y_{\mathrm{L}}} \in (0, y_{\mathrm{H}})$ such that $U(x_{\mathrm{k}}(\widetilde{y_{\mathrm{L}}}); y_{\mathrm{H}}) = U(0; y_{\mathrm{H}})$ with the property that $U(x_{\mathrm{k}}(y); y_{\mathrm{H}}) > U(0; y_{\mathrm{H}})$ for all $y < \widetilde{y_{\mathrm{L}}}$ and $U(x_{\mathrm{k}}(y); y_{\mathrm{H}}) < U(0; y_{\mathrm{H}})$ for all $y > \widetilde{y_{\mathrm{L}}}$.[49] In other words, for y_{L} sufficiently smaller than y_{H}, the inequality $U(x_{\mathrm{L}}; y_{\mathrm{H}}) > U(0; y_{\mathrm{H}})$ obtains, which implies that $U(x_{\mathrm{w}}; y_{\mathrm{H}}) > U(x_{\mathrm{H}}; y_{\mathrm{H}})$. Since U(x;$y_{\mathrm{H}}$) is strictly decreasing for any $x > x_{\mathrm{H}}$, it follows that $x_{\mathrm{w}} < x_{\mathrm{H}}$ must be the case. ■

Proof of Lemma 8. Consider some $x < x_{\mathrm{d}}(y_{\mathrm{L}})$. We established that the rich type must accept any such x in the proof of Lemma 6.

Consider $x = x_{\mathrm{d}}(y_{\mathrm{L}})$. Accepting yields *at least* $U(x_{\mathrm{d}}(y_{\mathrm{L}}); y_{\mathrm{H}}) + R(y_{\mathrm{H}})$, while rejection yields *at most* $R(y_{\mathrm{H}}) + p_{\mathrm{H}} U(x_{\mathrm{L}}; y_{\mathrm{H}}) = R(y_{\mathrm{H}}) + U(x_{\mathrm{w}}; y_{\mathrm{H}})$. Thus, it is sufficient to show that $U(x_{\mathrm{d}}(y_{\mathrm{L}}); y_{\mathrm{H}}) > U(x_{\mathrm{w}}; y_{\mathrm{H}})$, which we know obtains because $x_{\mathrm{d}}(y_{\mathrm{L}}) < x_{\mathrm{w}}$ by Lemma 7. Thus, the rich type must accept this demand.

Consider some $x \in (x_{\mathrm{d}}(y_{\mathrm{L}}), x_{\mathrm{w}})$. Since the poor type rejects this with certainty, if the rich type accepts this with positive probability, $x_{\mathrm{A}} = x_{\mathrm{H}}$ must obtain. Thus, the payoff from acceptance is $U(x; y_{\mathrm{H}}) + R(y_{\mathrm{H}})$. The *most* the rich type can expect after rebellion is $x_{\mathrm{R}} = x_{\mathrm{L}}$, with a payoff of $R(y_{\mathrm{H}}) + U(x_{\mathrm{L}}; y_{\mathrm{H}}) = R(y_{\mathrm{H}}) + U(x_{\mathrm{w}}; y_{\mathrm{H}})$. Thus, it is sufficient to show that $U(x; y_{\mathrm{H}}) > U(x_{\mathrm{w}}; y_{\mathrm{H}})$, which obtains because $x < x_{\mathrm{w}}$. Thus, the rich type must accept any such demand.

Consider $x = x_{\mathrm{w}}$. Since the poor type rejects it with certainty, if the rich type accepts this with positive probability, $x_{\mathrm{A}} = x_{\mathrm{H}}$ must obtain. Thus, the payoff from acceptance is $U(x_{\mathrm{w}}; y_{\mathrm{H}}) + R(y_{\mathrm{H}})$. This is also the payoff from rebellion if the Crown demands $x_{\mathrm{R}} = x_{\mathrm{L}}$, which implies that if the Crown were to mix or demand x_{H}, acceptance would be strictly preferable to rejection. Thus, acceptance is optimal for any post-rebellion belief that is at least $q_2 \geq q_2^*$. Since the Crown must demand $x_{\mathrm{R}} = x_{\mathrm{L}}$ for any $q_2 < q_2^*$, in all these situations the rich type must be indifferent. We now show that he will neither mix nor rebel with certainty whenever $q_2 < q_2^*$. We do this by showing that if he were to reject x_{w} with positive probability, the Crown does not have a best response. Let r denote the probability that y_{H} rejects x_{w} and suppose $r > 0$. The Crown's expected payoff from demanding x_{w} is *at most* (because $x_{\mathrm{A}} = x_{\mathrm{H}}$):

$$q\Big[r\big(2W(y_{\mathrm{H}}) + p_{\mathrm{H}} V(x_{\mathrm{L}})\big) + (1-r)\big(V(x_{\mathrm{w}}) + V(x_{\mathrm{H}})\big)\Big] + (1-q)\Big[W(y_{\mathrm{L}}) + G(y_{\mathrm{L}})\Big].$$

[49] Using our functional form, we can easily find this analytically. Solving $U(x; y_{\mathrm{H}}) = U(0; y_{\mathrm{H}}) = y_{\mathrm{H}}$ requires solving $y_{\mathrm{H}} - x + \lambda\sqrt{x(y_{\mathrm{H}} - x)} = y_{\mathrm{H}}$, so $x = \left(\frac{\lambda}{1+\lambda}\right) y_{\mathrm{H}}$. We then need to find y such that $x_{\mathrm{k}}(y) = x$.

Consider a deviation to some $x \in (x_d(y_L), x_w)$, which the poor type rejects but the rich type accepts. The Crown's payoff would be

$$q\Big[V(x) + V(x_H)\Big] + (1-q)\Big[W(y_L) + G(y_L)\Big].$$

To establish that this deviation is profitable, we need to show that

$$r\big(2W(y_H) + p_H V(x_L)\big) + (1-r)\big(V(x_w) + V(x_H))\big) < V(x) + V(x_H).$$

Note first that $U(x_w; y_H) = p_H U(x_L; y_H) = p_H U(x_L; y_H) + (1 - p_H)U(y_H; y_H) \le U(p_H x_L + (1 - p_H)y_H; y_H)$ by concavity of U. Since U is decreasing in this region, this implies that $x_w \ge p_H x_L + (1-p_H)y_H$. This now implies that $V(x_w) \ge V(p_H x_L + (1-p_H)y_H) \ge (1 - p_H)V(y_H) + p_H V(x_L) = W(y_H) + p_H V(x_L)$, where the first inequality follows from V increasing, and the second from V concave. We now obtain:

$$\begin{aligned} r\big(2W(y_H) + p_H V(x_L)\big) &+ (1-r)\big(V(x_w) + V(x_H))\big) \\ &\le r\big(V(x_w) + W(y_H)\big) + (1-r)\big(V(x_w) + V(x_H))\big) \\ &< V(x_w) + V(x_H), \end{aligned}$$

where the first inequality follows from the implication above, and the second from $V(x_H) > W(y_H)$ by Lemma 2. But since we can make $V(x) + V(x_H)$ arbitrarily close to $V(x_w) + V(x_H)$ by taking x sufficiently close to x_w, there always exists a profitable deviation. In other words, it cannot be that the rich type rejects x_w with positive probability in any sequential equilibrium. Thus, the rich type must accept this demand.

Consider some $x \in (x_w, x_d(y_H))$. Since y_L rejects this with certainty, it cannot be the case that y_H accepts it with certainty. If he were to do so, there would be full separation with $x_A = x_H$ and $x_R = x_L$, but we know that in that case y_H strictly prefers to reject any $x > x_w$, a contradiction. There are now two possibilities to consider: y_H rejects with certainty or mixes.

If he were to reject with certainty, the post-rejection belief is given by Bayes rule:

$$q_2 = \frac{q p_H}{q p_H + (1-q) p_L}, \tag{7}$$

where we note that although the types pooled on rejection, some information is revealed because the rich type is more likely to survive the revolt. Since the Crown's response is governed by Lemma 5, we can rewrite $q_2 \lesseqgtr q_2^*$ in terms of the prior $q \lesseqgtr q_1^*$, where the latter is specified in the lemma. After a revolt, the Crown will demand x_H if $q > q_1^*$, x_L if $q < q_1^*$, and possibly a mixture when $q = q_1^*$. Since the latter event has measure zero, we shall fold it into the first subcase. Suppose, then, that $q \le q_1^*$, in which case rebellion yields the rich type an expected payoff of $R(y_H) + p_H U(x_L; y_H) = R(y_H) + U(x_w; y_H)$. Deviating to acceptance cannot be profitable if $x_A = x_H$ because in that case his payoff would be $U(x; y_H) + R(y_H)$, which is strictly worse because $x > x_w$. To induce the Crown to make that demand, it has to be the case that its post-acceptance belief (which is off the path of play) is at least q_2^*. We can construct a sequence of beliefs that satisfies this in the limit by letting the probability that the rich type accepts x be ε, and the probability that the poor type accepts it be ε^2. The limit of the post-acceptance belief from these fully mixed strategies is

$$\lim_{\varepsilon \to 0} \frac{q\varepsilon}{q\varepsilon + (1-q)\varepsilon^2} = 1,$$

which satisfies the condition. Thus, we conclude that when $q \leq q_1^*$, the rich type would rebel with certainty and the Crown would demand $x_A = x_H$ and $x_R = x_L$ (i.e., it will offer tax relief after a rebellion).

Suppose now that $q > q_1^*$, which means that $x_R = x_H$, so the rich type's payoff from rebellion is $R(y_H) + p_H R(y_H) = R(y_H) + U(x_d(y_H); y_H)$. The *worst* it can expect after acceptance is $x_A = x_H$, which would yield his a payoff of $U(x; y_H) + R(y_H)$. He must prefer to accept whenever $U(x; y_H) > U(x_d(y_H); y_H)$, which is satisfied because $x < x_d(y_H)$. Thus, in any equilibrium, the rich type would accept such a demand, a contradiction to the supposition that he rejects it with certainty. Thus, pooling on rebellion cannot be a sequential equilibrium if $q > q_1^*$.

This leaves us a final possibility: the rich type rejects x with probability $r^*(x) \in (0, 1)$. Since he is the only one who accepts x with positive probability, it immediately follows that $x_A = x_H$, so the payoff from accepting is $U(x; y_H) + R(y_H)$. The payoff from rebelling is $R(y_H) + p_H U(x_R; y_H)$. Since he is willing to mix in equilibrium, he must be indifferent, so $U(x; y_H) = p_H U(x_R; y_H)$ must obtain. This immediately implies that $x_R \neq x_H$ because in that case $p_H U(x_H; y_H) = p_H R(y_H) = U(x_d(y_H); y_H) < U(x; y_H)$, where the last inequality follows from $x < x_d(y_H)$. It also implies that $x_R \neq x_L$ because in that case $p_H U(x_L; y_H) = U(x_w; y_H) > U(x; y_H)$, where the last inequality follows from $x > x_w$. We conclude that the Crown must be playing a mixed strategy in any equilibrium in which the rich type mixes. But then it must be the case that the post-rebellion belief specifically makes the Crown indifferent:

$$q_2 = \frac{q p_H r^*(x)}{q p_H r^*(x) + (1-q) p_L} = q_2^*.$$

Solving this yields r^* specified in the lemma, where we dropped the functional notation because the mixing probability is clearly independent of x. It is readily shown that $r^* < 1$ only if $q > q_1^*$. Thus, we conclude that when $q > q_1^*$, the rich type rebels with probability r^* and accepts the demand with probability $1 - r^*$. This completes the characterization of the rich type's strategy for any demand in this range.

Consider $x = x_d(y_H)$. Since the poor type rejects this with certainty, if the rich type would accept this with positive probability, $x_A = x_H$ in any equilibrium. If he were to accept with certainty, $x_R = x_L$, in which case $x_d(y_H) > x_w$ implies that the rich type would strictly prefer to rebel, a contradiction. Therefore, there are only two possibilities to consider: he either rebels with certainty or mixes.

Suppose he rebels with certainty, in which case the Crown's post-revolt belief is given by (7). From the argument above, we know that if $q \leq q_1^*$, the Crown demands $x_R = x_L$, and if $q > q_1^*$ it demands $x_R = x_H$. This means that if $q \leq q_1^*$, the rich type's payoff from rebellion is $R(y_H) + U(x_w; y_H)$. Deviating to acceptance cannot be profitable if $x_A = x_H$ because in that case his payoff would be $U(x_d(y_H); y_H) + R(y_H)$, which is strictly worse because $x_w < x_d(y_H)$. It is straightforward to induce the Crown to make this demand in a sequential equilibrium (we have seen that the fully mixed strategy of the rich accepting

$x_d(y_H)$ with probability ε and the poor accepting it with probability ε^2 yields a sequence of beliefs that converges on putting probability 1 on the rich type as the strategies converge to rejection, $\varepsilon \to 0$). Thus, we conclude that in a sequential equilibrium the rich type must reject $x_d(y_H)$ with certainty whenever $q \leq q_1^*$.

If, on the other hand, $q > q_1^*$, the rich type's payoff from rebellion is $R(y_H)+p_H R(y_H) = R(y_H)+U(x_d(y_H); y_H)$. Since acceptance yields *at least* $U(x_d(y_H); y_H)+R(y_H)$, the only way to sustain an equilibrium in which he rejects with positive probability is when $x_A = x_H$, which at most makes his indifferent. We now show that it cannot be the case that he rebels with certainty when indifferent. If he were to rebel list the poor type, $x_R = x_H$, and the Crown's expected payoff from demanding $x_d(y_H)$ would be

$$q\Big[W(y_H) + G(y_H)\Big] + (1-q)\Big[2W(y_L) + p_L W(y_L)\Big].$$

If the Crown were to deviate to some $x \in (x_w, x_d(y_H))$, which the poor type rejects but the rich type accepts with probability r^*, so that $x_A = x_H$ and $x_R = x_H$ (this is without loss of generality since we know the Crown would be indifferent after rebellion), its payoff would be

$$q\Big[r^*\big(W(y_H) + G(y_H)\big) + (1-r^*)\big(V(x) + V(x_H)\big)\Big] + (1-q)\Big[2W(y_L) + p_L W(y_L)\Big].$$

This deviation would be profitable whenever

$$r^*\big(W(y_H) + G(y_H)\big) + (1-r^*)\big(V(x) + V(x_H)\big) > W(y_H) + G(y_H).$$

To establish this, it is sufficient to show that $V(x) + V(x_H) > W(y_H) + G(y_H)$. We know that $V(x_d(y_H)) + V(x_H) > W(y_H) + G(y_H)$ because $V(x_H) > W(y_H)$ by Lemma 2, and $V(x_d(y_H)) > G(y_H)$ by Lemma 3. Since we can make $V(x)$ arbitrarily close to $V(x_d(y_H))$ by taking x sufficiently close to $x_d(y_H)$, a profitable deviation must always exist. Thus, if the rich type were to reject $x_d(y_H)$ with certainty when $q > q_1^*$, the Crown would have no best response. This means that in any sequential equilibrium, he must be mixing in this case. We have already seen that he would only mix if the Crown mixes after rebellion, which in turn pins down his rejection probability to r^*. Thus, we conclude that if $q > q_1^*$ the rich type must reject $x_d(y_H)$ with probability r^*.

Consider some $x > x_d(y_H)$. By Lemma 6, the poor type rejects this with certainty, which means that in any equilibrium in which the rich type accepts with positive probability, $x_A(x) = x_H$. The *least* that the rich type could expect after revolting is $x_R(x) = x_H$. But we know that when faced with x_H irrespective of what he does, he strictly prefers to revolt for any $x > x_d(y_H)$. Thus, the rich type must reject any such demand. ∎

Proof of Lemma 9. Lemmata 6 and 8 help us establish that any demand not stated in the lemma is strictly dominated by one of the demands stated there. Observe first that any demand $x < x_d(y_L)$ is strictly dominated by $x_d(y_L)$ itself because all such demands are always accepted, the post-acceptance demand is invariant (it is either x_L if $q \leq q_2^*$ and x_H otherwise), and the Crown's payoff is strictly increasing in the accepted demand. Analogously, any demand $x \in (x_d(y_L), x_w)$ is strictly dominated by x_w because all such demands are always rejected by the poor type and accepted by the poor type, which implies that the second period payoff for the Crown, $V(x_{\mathrm{L}})$ after rebellion and $V(x_{\mathrm{H}})$ after acceptance, is the same for all such demands, so the Crown is better off making the highest possible first-period demand with these properties. Thus, in any sequential equilibrium the Crown would never demand any $x < x_{\mathrm{d}}(y_{\mathrm{L}})$ or any $x \in (x_{\mathrm{d}}(y_{\mathrm{L}}), x_{\mathrm{w}})$. We now turn to the remaining possibilities.

Assume now that $q \leq q_1^*$. The only demands we need to consider are $x > x_{\mathrm{w}}$, which are rejected with certainty, and $x_{\mathrm{R}} = x_{\mathrm{L}}$. We now show that any such unacceptable demand is strictly dominated by x_{w}. The Crown's expected payoff from an unacceptable demand is

$$q\big[2W(y_{\mathrm{H}}) + p_{\mathrm{L}}V(x_{\mathrm{L}})\big] + (1-q)\big[W(y_{\mathrm{L}}) + G(y_{\mathrm{L}})\big],$$

and the payoff from x_{w} is

$$q\big[V(x_{\mathrm{w}}) + V(x_{\mathrm{H}})\big] + (1-q)\big[W(y_{\mathrm{L}}) + G(y_{\mathrm{L}})\big].$$

The separating demand is strictly preferable whenever

$$V(x_{\mathrm{w}}) + V(x_{\mathrm{H}}) > 2W(y_{\mathrm{H}}) + p_{\mathrm{L}}V(x_{\mathrm{L}}),$$

which holds because $V(x_{\mathrm{H}}) > W(y_{\mathrm{H}})$ by Lemma 2, and $V(x_{\mathrm{w}}) > W(y_{\mathrm{H}}) + p_{\mathrm{H}}V(x_{\mathrm{L}})$, which was established in the proof of Lemma 8. This exhausts the possibilities when $q \leq q_1^*$ and establishes the first part of the claim.

Assume now that $q > q_1^*$. Consider first demands $x \in (x_{\mathrm{w}}, x_{\mathrm{d}}(y_{\mathrm{H}}))$, which the poor type rejects with certainty but the rich rejects with probability $r^* \in (0,1)$. For any such demand, the Crown is indifferent after rebellion (so we can take $V(x_{\mathrm{H}})$ as the payoff in that case without loss of generality) and demands x_{H} after acceptance. Since the rejection probability is constant in the demand as are the second period payoffs, the Crown's payoff is strictly increasing in the demand accepted, which implies that $x_{\mathrm{d}}(y_{\mathrm{H}})$ is preferable to any lower demand with these properties.

The only remaining demands we need to consider are $x > x_{\mathrm{d}}(y_{\mathrm{H}})$, which are rejected with certainty, and $x_{\mathrm{R}} = x_{\mathrm{H}}$. We now show that any such unacceptable demand is strictly dominated by $x_{\mathrm{d}}(y_{\mathrm{H}})$. The Crown's expected payoff from an unacceptable demand is

$$q\big[W(y_{\mathrm{H}}) + G(y_{\mathrm{H}})\big] + (1-q)\big[2W(yL) + p_{\mathrm{L}}W(y_{\mathrm{L}})\big],$$

and the payoff from $x_{\mathrm{d}}(y_{\mathrm{H}})$ is

$$q\Big[r^*\big(W(y_{\mathrm{H}}) + G(y_{\mathrm{H}})\big) + (1-r^*)\big(V(x_{\mathrm{d}}(y_{\mathrm{H}})) + V(x_{\mathrm{H}})\big)\Big] + (1-q)\Big[2W(y_{\mathrm{L}}) + p_{\mathrm{L}}W(y_{\mathrm{L}})\Big].$$

The semi-separating demand is strictly preferable whenever

$$W(y_{\mathrm{H}}) + G(y_{\mathrm{H}}) < r^*\big[W(y_{\mathrm{H}}) + G(y_{\mathrm{H}})\big] + (1-r^*)\big[V(x_{\mathrm{d}}(y_{\mathrm{H}})) + V(x_{\mathrm{H}})\big],$$

which holds for any r^* because $V(x_{\mathrm{d}}(y_{\mathrm{H}})) > G(y_{\mathrm{H}})$ by Lemma 3, and $V(x_{\mathrm{H}}) > W(y_{\mathrm{H}})$ by Lemma 2. This exhausts the possibilities when $q > q_1^*$ and establishes the second part of the claim. ∎

Proof of Lemma 10. We begin by writing the expected payoffs to the Crown. Consider then the Crown's payoff from the semi-separating high demand $x_{\mathrm{d}}(y_{\mathrm{H}})$. Since only the rich Subject accepts this with positive probability, $x_{\mathrm{A}} = x_{\mathrm{H}}$. Since $h(x_{\mathrm{d}}(y_{\mathrm{H}})) = 1$, the Crown offers no tax relief after rebellion, so $x_{\mathrm{R}} = x_{\mathrm{H}}$ as well. Thus, the poor Subject revolts in both periods, whereas the rich Subject revolts only in the first period with probability r^*. The Crown's expected payoff is:

$$V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) = q\Big[r^*\big(W(y_{\mathrm{H}}) + G(y_{\mathrm{H}})\big) + (1-r^*)\big(V(x_{\mathrm{d}}(y_{\mathrm{H}})) + V(x_{\mathrm{H}})\big)\Big] \\ + (1-q)\Big[2W(y_{\mathrm{L}}) + p_{\mathrm{L}}W(y_{\mathrm{L}})\Big],$$

which we can rewrite, using the fact that

$$r^* = \frac{(1-q)\zeta}{q}, \quad \text{where} \quad \zeta = \left(\frac{p_{\mathrm{L}}}{p_{\mathrm{H}}}\right)\left[\frac{V(x_{\mathrm{L}}) - W(y_{\mathrm{L}})}{V(x_{\mathrm{H}}) - V(x_{\mathrm{L}})}\right] > 0,$$

as

$$V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) = q\Big[V(x_{\mathrm{d}}(y_{\mathrm{H}})) + V(x_{\mathrm{H}})\Big] + (1-q)\Big[W(y_{\mathrm{L}}) + (1+p_{\mathrm{L}})W(y_{\mathrm{L}}) - \zeta\eta\Big], \tag{8}$$

where

$$\eta = V(x_{\mathrm{d}}(y_{\mathrm{H}})) - G(y_{\mathrm{H}}) + V(x_{\mathrm{H}}) - W(y_{\mathrm{H}}) > 0,$$

where the last inequality follows from $V(x_{\mathrm{d}}(y_{\mathrm{H}})) > G(y_{\mathrm{H}})$ by Lemma 3 and $V(x_{\mathrm{H}}) > W(y_{\mathrm{H}})$ by Lemma 2. Since $\zeta\eta > 0$, this further implies that

$$\frac{\mathrm{d}\,V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))}{\mathrm{d}\,q} = V(x_{\mathrm{d}}(y_{\mathrm{H}})) - (1+p_{\mathrm{L}})W(y_{\mathrm{L}}) + V(x_{\mathrm{H}}) - W(y_{\mathrm{L}}) + \zeta\eta > 0$$

because $(1+p_{\mathrm{L}})W(y_{\mathrm{L}}) < G(y_{\mathrm{L}})$ and $V(x_{\mathrm{H}}) > V(x_{\mathrm{L}}) > W(y_{\mathrm{L}})$.

Consider now the separating demand x_{w}. Since the rich Subject accepts it with certainty and the poor Subject rejects it, $x_{\mathrm{A}} = x_{\mathrm{H}}$ and $x_{\mathrm{R}} = x_{\mathrm{L}}$. The Crown's expected payoff is:

$$V_1(x_{\mathrm{w}}) = q\Big[V(x_{\mathrm{w}}) + V(x_{\mathrm{H}})\Big] + (1-q)\Big[W(y_{\mathrm{L}}) + G(y_{\mathrm{L}})\Big]. \tag{9}$$

This is also strictly increasing:

$$\frac{\mathrm{d}\,V_1(x_{\mathrm{w}})}{\mathrm{d}\,q} = V(x_{\mathrm{w}}) - G(y_{\mathrm{L}}) + V(x_{\mathrm{H}}) - W(y_{\mathrm{L}}) > 0$$

because $V(x_{\mathrm{w}}) > V(x_{\mathrm{d}}(y_{\mathrm{L}})) > G(y_{\mathrm{L}})$, where the first inequality follows from Lemma 7 and the second from Lemma 3, and $V(x_{\mathrm{H}}) > V(x_{\mathrm{L}}) > W(y_{\mathrm{L}})$, where the first inequality follows from Lemma 1 and the second from Lemma 2. Moreover, note that

$$\frac{\mathrm{d}\,V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))}{\mathrm{d}\,q} > \frac{\mathrm{d}\,V_1(x_{\mathrm{w}})}{\mathrm{d}\,q} \tag{10}$$

because $V(x_{\mathrm{d}}(y_{\mathrm{H}})) > V(x_{\mathrm{w}})$, $G(y_{\mathrm{L}}) > (1+p_{\mathrm{L}})W(y_{\mathrm{L}})$, and $\zeta\eta > 0$. It immediately follows that $V_1(x_{\mathrm{w}}) > V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))$ if, and only if, $q < q_{\mathrm{H}}$, where

$$q_{\mathrm{H}} = \frac{G(y_{\mathrm{L}}) - (1+p_{\mathrm{L}})W(y_{\mathrm{L}}) + \zeta\eta}{V(x_{\mathrm{d}}(y_{\mathrm{H}})) - V(x_{\mathrm{w}}) + G(y_{\mathrm{L}}) - (1+p_{\mathrm{L}})W(y_{\mathrm{L}}) + \zeta\eta} \in (0,1). \tag{11}$$

We have thus established that the payoff from the semi-separating demand $x_{\mathrm{d}}(y_{\mathrm{H}})$ is increasing faster than the payoff from the separating demand x_{w}, and that the separating demand is preferable only when $q < q_{\mathrm{H}}$. Of course, the semi-separating demand is only ever relevant when $q > q_1^*$ (by Lemma 8).

The pooling demand is more involved. Consider $x_{\mathrm{d}}(y_{\mathrm{L}})$ when $q \leq q_2^*$, in which case $x_{\mathrm{A}} = x_{\mathrm{L}}$, so the Crown's payoff is

$$V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) = V(x_{\mathrm{d}}(y_{\mathrm{L}})) + V(x_{\mathrm{L}}). \tag{12}$$

It is now easy to see that $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{w}})$ if, and only if, $q < q_{\mathrm{L}}$, where

$$q_{\mathrm{L}} = \frac{V(x_{\mathrm{d}}(y_{\mathrm{L}})) - G(y_{\mathrm{L}}) + V(x_{\mathrm{L}}) - W(y_{\mathrm{L}})}{V(x_{\mathrm{w}}) - G(y_{\mathrm{L}}) + V(x_{\mathrm{H}}) - W(y_{\mathrm{L}})} \in (0,1). \tag{13}$$

Analogously, $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))$ if, and only if, $q < q_{\mathrm{M}}$, where

$$q_{\mathrm{M}} = \frac{V(x_{\mathrm{d}}(y_{\mathrm{L}})) - (1+p_{\mathrm{L}})W(y_{\mathrm{L}}) + V(x_{\mathrm{L}}) - W(y_{\mathrm{L}}) + \zeta\eta}{V(x_{\mathrm{d}}(y_{\mathrm{H}})) - (1+p_{\mathrm{L}})W(y_{\mathrm{L}}) + V(x_{\mathrm{H}}) - W(y_{\mathrm{L}}) + \zeta\eta} \in (0,1). \tag{14}$$

To determine the possible configurations, consider first $q \leq q_2^*$. By (12), $V(x_{\mathrm{d}}(y_{\mathrm{L}}))$ is constant in q. Suppose first that $q_{\mathrm{L}} < q_{\mathrm{M}}$, in which case (10) tells us that $V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))$ and $V_1(x_{\mathrm{w}})$ must intersect at $q_{\mathrm{H}} > q_{\mathrm{M}}$, which yields the first cut-point configuration: $q_{\mathrm{L}} < q_{\mathrm{M}} < q_{\mathrm{H}}$. Suppose now that $q_{\mathrm{M}} < q_{\mathrm{L}}$, in which case (10) implies that $V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))$ and $V_1(x_{\mathrm{w}})$ must intersect at $q_{\mathrm{H}} < q_{\mathrm{M}}$, which yields the second cut-point configuration: $q_{\mathrm{H}} < q_{\mathrm{M}} < q_{\mathrm{L}}$.

Consider now the pooling demand $x_{\mathrm{d}}(y_{\mathrm{L}})$ when $q > q_2^*$, in which case $x_{\mathrm{A}} = x_{\mathrm{H}}$, so the Crown's payoff is

$$V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) = V(x_{\mathrm{d}}(y_{\mathrm{L}})) + qV(x_{\mathrm{H}}) + (1-q)W(y_{\mathrm{L}}), \tag{15}$$

which is strictly increasing:

$$\frac{\mathrm{d}\, V_1(x_{\mathrm{d}}(y_{\mathrm{L}}))}{\mathrm{d}\, q} = V(x_{\mathrm{H}}) - W(y_{\mathrm{L}}) > 0.$$

Moreover, it is readily observed that

$$\frac{\mathrm{d}\, V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))}{\mathrm{d}\, q} > \frac{\mathrm{d}\, V_1(x_{\mathrm{w}})}{\mathrm{d}\, q} > \frac{\mathrm{d}\, V_1(x_{\mathrm{d}}(y_{\mathrm{L}}))}{\mathrm{d}\, q}. \tag{16}$$

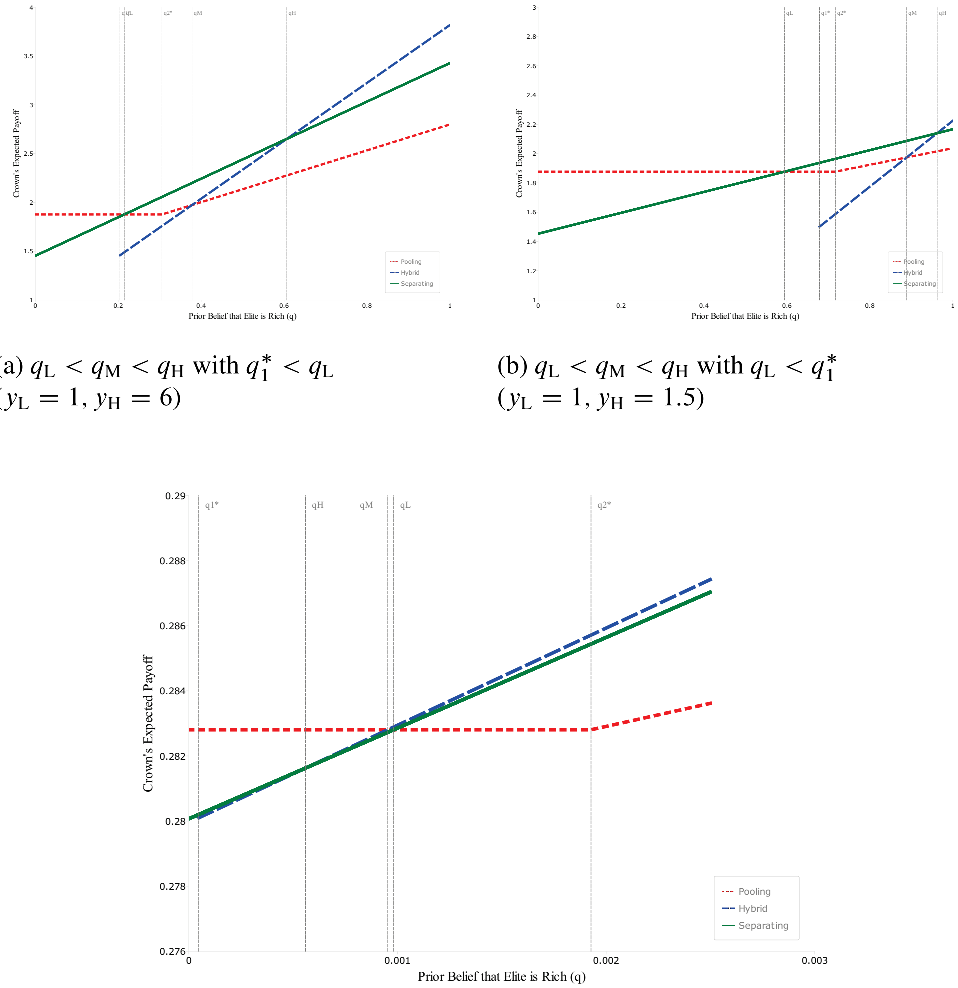
We now specify the two cut-point probabilities that depend on (15). We can see that $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{w}})$ if, and only if, $q < q_{\mathrm{L}}$, where

$$q_{\mathrm{L}} = \frac{V(x_{\mathrm{d}}(y_{\mathrm{H}})) - G(y_{\mathrm{L}})}{V(x_{\mathrm{w}}) - G(y_{\mathrm{L}})} \in (0,1). \tag{17}$$

Analogously, $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))$ if, and only if, $q < q_{\mathrm{M}}$, where

$$q_{\mathrm{M}} = \frac{V(x_{\mathrm{d}}(y_{\mathrm{L}})) - (1+p_{\mathrm{L}})W(y_{\mathrm{L}}) + \zeta\eta}{V(x_{\mathrm{d}}(y_{\mathrm{H}})) - (1+p_{\mathrm{L}})W(y_{\mathrm{L}}) + \zeta\eta} \in (0,1). \tag{18}$$

Given these cut-point probabilities, (16) implies that the analysis of optimal demands is exactly the same as the one we just carried out. Thus, define q_{H} as (11), define q_{L} as (13) if $q \leq q_2^*$ and (17) otherwise, and define q_{M} as (14) if $q \leq q_2^*$ and (18) otherwise. The claim then follows. ■



(a) $q_{\mathrm{L}} < q_{\mathrm{M}} < q_{\mathrm{H}}$ with $q_1^* < q_{\mathrm{L}}$ ($y_{\mathrm{L}} = 1, y_{\mathrm{H}} = 6$)

(b) $q_{\mathrm{L}} < q_{\mathrm{M}} < q_{\mathrm{H}}$ with $q_{\mathrm{L}} < q_1^*$ ($y_{\mathrm{L}} = 1, y_{\mathrm{H}} = 1.5$)

(c) $q_{\mathrm{H}} < q_{\mathrm{M}} < q_{\mathrm{L}}$ ($y_{\mathrm{L}} = 0.02, y_{\mathrm{H}} = 4$)

Figure 2: The Two Possible Cut-Point Configurations.

Proof of Proposition 2. The proof is easier to follow with Figure 2.

There are two cut-point configurations to consider (Lemma 10). Suppose first that the configuration is $q_{\mathrm{L}} < q_{\mathrm{M}} < q_{\mathrm{H}}$. This implies that $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{w}}) > V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))$ for any $q < q_{\mathrm{L}}$, $V_1(x_{\mathrm{w}}) > V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))$ for any $q \in (q_{\mathrm{L}}, q_{\mathrm{M}})$, $V_1(x_{\mathrm{w}}) > V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) > V_1(x_{\mathrm{d}}(y_{\mathrm{L}}))$ for any $q \in (q_{\mathrm{M}}, q_{\mathrm{H}})$, and $V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) > V_1(x_{\mathrm{w}}) > V_1(x_{\mathrm{d}}(y_{\mathrm{L}}))$ for any $q > q_{\mathrm{H}}$. Thus, in any sequential equilibrium the Crown demands $x_{\mathrm{d}}(y_{\mathrm{L}})$ if $q \leq q_{\mathrm{L}}$, demands x_{w} if $q \in (q_{\mathrm{L}}, q_{\mathrm{H}}]$, and demands $x_{\mathrm{d}}(y_{\mathrm{H}})$ if $q > q_{\mathrm{H}}$ whenever this configuration obtains.

Suppose now that the configuration is $q_{\mathrm{H}} < q_{\mathrm{M}} < q_{\mathrm{L}}$. This implies that $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{w}}) > V_1(x_{\mathrm{d}}(y_{\mathrm{H}}))$ for any $q < q_{\mathrm{H}}$, $V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) > V_1(x_{\mathrm{w}})$ for any $q \in (q_{\mathrm{H}}, q_{\mathrm{M}})$, $V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) > V_1(x_{\mathrm{d}}(y_{\mathrm{L}})) > V_1(x_{\mathrm{w}})$ for any $q \in (q_{\mathrm{M}}, q_{\mathrm{L}})$, and $V_1(x_{\mathrm{d}}(y_{\mathrm{H}})) > V_1(x_{\mathrm{w}}) > V_1(x_{\mathrm{d}}(y_{\mathrm{L}}))$ for any $q > q_{\mathrm{L}}$. Thus, in any sequential equilibrium the Crown demands $x_{\mathrm{d}}(y_{\mathrm{L}})$ if $q \leq q_{\mathrm{M}}$, and demands $x_{\mathrm{d}}(y_{\mathrm{H}})$ if $q > q_{\mathrm{M}}$ whenever this configuration obtains. ■