A Model of Focusing in Political Choice

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Abstract

This paper develops a theoretical model of voters’ and politicians’ behavior based on the notion that voters focus disproportionately on, and hence overweight, certain attributes of policies. We assume that policies have two attributes and that voters focus more on the attribute in which their options differ more. First, we consider exogenous policies and show that voters’ selective attention polarizes the electorate. Second, we consider the endogenous supply of policies by office-motivated politicians who take voters’ distorted focus into account. We show that voters’ selective attention leads to inefficient policies, which cater excessively to a subset of voters: social groups that are larger, have more distorted focus, are more moderate, and are more sensitive to changes in a single attribute are more influential. Finally, we show that augmenting the classical models of voting and electoral competition with selective attention can contribute to explain puzzling stylized facts as the inverse correlation between income inequality and redistribution or the backlash effect of extreme policies.

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1 Introduction

Evaluating policy alternatives is a difficult task. In fact, many important political decisions involve multiple consequences, even for the same voter. On June 23, 2016, UK citizens were asked to choose between two different levels of integration with their neighboring countries: remaining a member the European Union (EU) or leaving the EU. This choice will have had multiple consequences: given a voter’s preferences and beliefs, each option had some relative advantages or benefits (for example, leaving the EU can be associated with fewer immigrants from other EU countries, no contributions to the EU budget, and regulatory independence) and some relative disadvantages or costs (for example, leaving the EU can be associated with higher prices of imported goods, less competitive exports and reduced international influence). Beyond this example, many public policies have multiple consequences and involve a trade-off between benefits and costs, not only for society as a whole but also from the perspective of the single citizen. A prominent example is the size of government: higher revenues give governments the ability to provide more public goods (infrastructure, mandatory spending programs, etc.) but require higher taxation. Other examples are the degree of government surveillance (more surveillance means a lower chance of terrorist attacks but also less privacy and more limitations to personal freedom); the degree of industry regulation (more intervention means higher consumer protection and lower risk of systemic crises but also less competition and product innovation); immigration policy (more openness means a larger working age population and more sustainable social security programs but also higher heterogeneity of preferences and potential social turmoil); and the degree of environmental regulation (stricter regulation means higher quality of life and lower chances of environmental catastrophes but also higher costs of production and private investments). In all these domains, how the different consequences are weighted is crucial for the resolution of the trade-off and the formation of voters’ preferences.

People often focus disproportionately on certain consequences, or attributes, of their available options, overweighting these features in estimating the overall value of an option. The social sciences literature offers many hints of this judgement bias, in various choice domains: American retirees focus on weather when comparing perspective life in California versus Ohio but differences in weather do not account for their satisfaction after the choice (Schkade and Kahneman, 1998); harmful consumption is overly responsive to price (Gruber and Köszegi, 2001; Abaluck, 2011), or taste (Hare, Camerer and Rangel, 2009); voters are prone to systematic misperceptions of the tax burden (e.g., fiscal illusion, or the relative “invisibility” of indirect taxes as compared to more “visible” direct

\[\text{See The Economist’s “Brexit” Backgrounder, published on February 24, 2016, \url{http://www.economist.com/blogs/graphicdetail/2016/02/graphics-britain-s-referendum-eu-membership}.}\]
taxes; see Mill, 1848; Buchanan, 1967; Sausgruber and Tyran, 2005; Chetty, Looney and Kroft, 2009) which lead them to overweight the benefits of government size relative to its costs; and voters overweight the economic costs of immigration (Brader, Valentino and Suhay, 2008) or the cultural threat posed by European integration (McLaren, 2002) relative to their benefits.

Building on this evidence, economists have recently developed models where the choice set can distort the relative weights a consumer attaches to the attributes of an alternative. The theoretical implications of this selective attention (or focusing or salient-thinking) for political behavior are largely unexplored and unclear. In fact, most theories of voting are based on the classic model of choice where the subjective value each option gives to a decision-maker is independent of the other available options.

In this paper, we develop a model of voters’ and politicians’ behavior based on the idea that voters focus more on attributes in which their available policies differ more. This assumption is based on the notion that our limited cognitive resources are attracted by a subset of the available sensory data (Taylor and Thompson, 1982) and, in particular, that “our mind has a useful capability to focus on whatever is odd, different or unusual” (Kahneman, 2011). Section 3 presents our framework. We consider a continuum of voters in different social groups who choose the location of a unidimensional policy (e.g., the size of government). Each policy has two attributes: it gives to all voters in the same social groups benefits and costs. For voter in a given group, the consumption utility from a policy equals the difference between its benefits and its costs. However, when evaluating policies, voters use focus-weighted utility instead of consumption utility. We assume that voters focus more on the attribute in which options differ more, that is, on the attribute which delivers the greater range of consumption utility.

In Section 4, we analyze the consequences of voters’ selective attention for their preferences over an exogenous pair of policies. We show that voters focus on the relative advantage—that is, the larger benefits or the smaller costs—of the policy which gives them the higher consumption utility. As a consequence, focusing does not affect what policy a voter prefers but it strengthens the intensity of preferences between this policy and the alternative (that is, it polarizes the electorate). We then consider the effect of focus on the endogenous formation of voters’ choice set. In Section 5, we introduce focusing voters into a model of electoral competition between two office-motivated parties. In the unique equilibrium of this game, the two parties offer the same policy and, thus, voters have undistorted focus. Nonetheless, any deviation from the equilibrium policies triggers voters’ distorted attention (potentially in different direction for different voters) and, thus, the electorate’s selective attention affects the politicians’ electoral calculus. We show that the equilibrium policies are generically different than the ones emerging

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2We discuss this literature in Section 2.
with rational voters and do not maximize utilitarian welfare; and that politicians are more likely to inefficiently cater to larger groups, to groups with more distorted focus, to groups that are more sensitive to changes in the attribute they focus on (in equilibrium), and to groups that are more moderate.

In Section 6, we explore the relevance of voters’ distorted attention in one important application—fiscal policy. In particular, we consider a stylized Meltzer and Richard (1981) model where parties offer a public good funded by a proportional tax rate and show the model helps explain facts that are puzzling from the perspective of existing political economy theories—the negative correlation between income inequality and both the support for redistribution (Ashok, Kuziemko and Washington, 2015) and the top marginal tax rates (Piketty, Saez and Stantcheva, 2014). Following a marginal deviation from the convergent equilibrium policies, poor voters (who prefer more redistribution) focus on the public good’s benefits, while rich voters (who prefer less redistribution) focus on the public good’s costs. If increased income inequality affects costs more than it affects benefits, selective attention amplifies rich voters’ marginal sensitivity to policies more than poor voters’, as the latter group focuses on benefits and underweights costs, and makes rich voters more responsive to electoral platforms. This leads rich voters to become more influential in the politicians’ calculus and, thus, to obtain less redistribution than before even if most of the electorate would benefit from more redistribution.

Finally, in Section 7, we consider more general choice sets, with a finite number of policies. We show that, when the choice set includes more than two policies, focusing not only affects the intensity of preferences but it can also affect its ranking. We discuss how the introduction of extreme policies in the voters’ choice set or consideration set (for example, a policy enacted in a neighboring country; a policy measure suggested or required by an external body, like the EU Commission; a novel policy introduced in the public debate by the media or an extreme party) can generate a backlash effect and change voters’ preferences, making them perceive more favorably the policies at the other end of the spectrum. We claim that this can explain the growing support for EU integration (and pro-EU parties) in European countries (including the UK) after Brexit.

2 Related Literature

Our work is primarily related to a recent, yet rapidly growing, research program in behavioral political economy, which studies electoral competition or political agency models when voters employ decision heuristics or are prone to cognitive biases. This literature considers voters who are subject to negativity bias or loss aversion (Alesina and Pasarelli, 2015; Lockwood and Rockey, 2015), correlation neglect (Levy and Razin, 2015), overconfidence (Ortoleva and Snowberg, 2015), time-inconsistency (Bisin, Lizzeri and
Yariv, 2015), reluctance to explicitly consider trade-offs (Patty, 2007), self-serving bias in moral judgement (Passarelli and Tabellini, Forthcoming). More closely related to this paper, Callander and Wilson (2006, 2008) introduce a theory of Downsian competition with context-dependent voting where the propensity to turn out and vote for the preferred candidate is greater when the other candidate is more extreme, and apply it to the puzzle of why politicians are ambiguous in their campaigns.

This paper also contributes to the theoretical literature on selective attention in economic choice. K˝ oszegi and Szeidl (2012), Bordalo, Gennaioli and Shleifer (2012, 2013a,b, 2015a,b), Cunningham (2013), and Bushong, Rabin and Schwartzstein (2015) introduce models where the choice set distorts the relative weights a consumer attaches to the attributes of an alternative. These models do not consider choices over options with correlated attributes and agents with heterogeneous preferences. More importantly, these models do not study the aggregation of the agents’ conflicting preferences in a collective choice, and the endogenous formation of the choice set by political candidates.

Less closely related to this paper is the theoretical literature on poorly informed voters (Glaeser, Ponzetto and Shapiro, 2005; Gavazza and Lizzeri, 2009; Gul and Pesendorfer, 2009; Ponzetto, 2011; Glaeser and Ponzetto, 2014; Prato and Wolton, 2016; Ogden, 2016; Matˇ ejka and Tabellini, 2016). Contrary to our model, where voters have complete information on policies, these works consider voters who are uncertain about candidates’ policies and receive or acquire information prior to casting their vote. The most closely related contributions are Prato and Wolton (2016), Ogden (2016) and Matˇ ejka and Tabellini (2016) who consider politicians’ incentives when voters have limited cognitive resources (or attention) and allocate them endogenously to improve the available information on their policy options. The selective attention we study is inherently different from this rational inattention: while the former concerns stimulus-driven and ex-post allocation of attention, the latter concerns goal-driven and ex-ante allocation of attention. The (unconscious) bottom-up process we introduce and the (conscious) top-down process studied by the existing literature have both been shown to be important channels contributing simultaneously and independently to a decision-maker’s overall allocation of attention in performing a task (Connor, Egeth and Yantis, 2004; Ciaramelli, Grady, Levine, Ween and Moscovitch, 2010; Pinto, van der Leij, Sligte, Lamme and Scholte, 2013).

3 Model

Consider a continuum of voters who belong to $n \geq 2$ social groups. The fraction of voters in group $i \in N = \{1, \ldots, n\}$ is $m_i > 0$, with $\sum_{i \in N} m_i = 1$. All voters from the same social group have the same policy preferences. In particular, each policy $p \in \mathbb{R}_+$
has two attributes: it provides voters in group $i$ with benefits, $B_i(p)$, and with costs, $C_i(p)$. Therefore, a voter in group $i$ derives consumption utility from policy $p$ equal to:

$$V_i(p) = B_i(p) - C_i(p).$$  \hspace{1cm} (1)

The same policy can yield different benefits and costs to voters in different social groups.

As we discussed in the Introduction, there are many examples of policies with multiple consequences for voters and involving a trade-off between benefits and costs.

We make the following assumptions on the benefit and cost functions:

**Assumption 1.** (A1) For all $i \in N$ and all $p \in \mathbb{R}_+$, (a) benefits are increasing and concave in $p$: $B_i(p) \geq 0$, $B_i'(p) > 0$, $B_i''(p) \leq 0$; (b) costs are increasing and convex in $p$: $C_i(p) \geq 0$, $C_i'(p) > 0$, $C_i''(p) \geq 0$; (c) at least one inequality between $B_i''(p) \geq 0$ and $C_i''(p) \leq 0$ is strict.

**Assumption 2.** (A2) For all $i \in N$, $V_i$ admits an interior maximum at $p_i$ (group $i$’s “consumption bliss point”): there exists $p_i > 0$ such that $B_i'(p_i) - C_i'(p_i) = 0$.

**Assumption 3.** (A3) For all $i \in N$ and all $p \in \mathbb{R}_+$, if $i < n$, $B_i'(p) \leq B_{i+1}'(p)$ and $C_i'(p) \geq C_{i+1}'(p)$, with at least one strict inequality.

Assumptions A1 and A2 imply that $V_i(p)$ is strictly concave in $p$ and single-peaked around $p_i$, group $i$’s consumption bliss point. Since $B_i'(p_i) - C_i'(p_i) = 0$, Assumption A3 implies that $B_{i+1}'(p_i) - C_{i+1}'(p_i) > 0$ and, thus, $p_i < p_{i+1}$ for all $i < n$: social groups with a lower index have a lower consumption bliss point. When we assume A1 and A2 but not A3, we index social groups so that $p_i < p_{i+1}$ for all $i < n$.

Our key assumption and main departure from the classical political economy models is that, when evaluating policies, voters use their focus-weighted utility rather than their consumption utility. Consider a choice set composed of two policies: $\mathcal{P} = \{p_A, p_B\}$.\footnote{In Section 7, we consider a finite choice set, $\mathcal{P} = \{p_A, p_B, \ldots\}$, with $|\mathcal{P}| \geq 2$.}

Let $\Delta_i^B(\mathcal{P})$ be the range of benefits in $\mathcal{P}$ for voters in group $i$:

$$\Delta_i^B(\mathcal{P}) = |B_i(p_A) - B_i(p_B)|.$$  \hspace{1cm} (2)

Let $\Delta_i^C(\mathcal{P})$ be the range of costs in $\mathcal{P}$ for voters in group $i$:

$$\Delta_i^C(\mathcal{P}) = |C_i(p_A) - C_i(p_B)|.$$  \hspace{1cm} (3)

We assume that voters focus more on the attribute in which their available options differ more, that is, on the attribute which generates a greater range of consumption utility. This assumption is compatible with the neuroscience of human cognition and similar
versions of it have already been explored in a number of economic contexts (Loomes and Sugden, 1982; Rubinstein, 1988; Köszegi and Szeidl, 2012; Bordalo et al., 2013b, 2015a). The core tenet of this assumption is that focus is driven by the salience of an attribute. The neuroscience literature suggests that the detection of salient features of the environment is a key mechanism driving the allocation of cognitive resources and that salience typically stems from contrast (Baumeister and Vohs, 2007; Nothdurft, 2005).

Using this language, we assume that larger differences are more salient and, thus, that voters focus on the attribute with a larger range on the utility space.

Formally, we assume that voters in group $i$ focus on benefits if $\Delta^B_i(P) > \Delta^C_i(P)$, focus on costs if $\Delta^B_i(P) < \Delta^C_i(P)$ and have undistorted focus if $\Delta^B_i(P) = \Delta^C_i(P)$.

**Assumption 4. (A4)** For a voter in group $i$, the focus-weighted utility from $p \in P$ is:

$$
\tilde{V}_i(p|P) = \begin{cases} 
\frac{2}{1+\delta_i} B_i(p) - \frac{2\delta_i}{1+\delta_i} C_i(p) & \text{if } \Delta^B_i(P) > \Delta^C_i(P) \\
\frac{2\delta_i}{1+\delta_i} B_i(p) - \frac{2}{1+\delta_i} C_i(p) & \text{if } \Delta^B_i(P) < \Delta^C_i(P) \\
B_i(p) - C_i(p) & \text{if } \Delta^B_i(P) = \Delta^C_i(P)
\end{cases}
$$

where $\delta_i \in (0, 1]$ decreases in the severity of focusing.

When voters in group $i$ focus on benefits (costs), the relative weight of benefits (costs) increases, $\frac{2}{1+\delta_i} \in [1, 2)$, and the relative weight of costs (benefits) decreases, $\frac{2\delta_i}{1+\delta_i} \in (0, 1]$, as compared to rational voters’ evaluation. The weights on benefits and costs change discontinuously when the object of focus changes but remain constant when focus remains on a given attribute. As $\delta_i$ goes to 1, focusing voters in group $i$ converge to rational voters. As $\delta_i$ goes to 0, focusing voters in group $i$ consider only the attribute that attracts their attention and completely neglect the other. Voters in group $i$ focus on the same attribute for both policies in a given choice set. On the other hand, the weighing distortion is allowed to be heterogeneous among social groups. Finally, the normalization of the utility weights ensures that the sum of the weights on benefits and costs is independent of $\delta_i$ and of the attribute voters focus on. In other words, the

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4Bordalo et al. (2012, 2013a,b, 2015a,b) assume that the salience of different attributes and, thus, the decision-maker’s focus is driven by contrast, what they call ordering, and that contrast is perceived with diminishing sensitivity. We study the consequences of adding diminishing sensitivity to our model in Appendix A2 and show that this implies voters focusing on costs for any choice set. Bordalo et al. (2013b, 2015a) consider similar focus weights while Köszegi and Szeidl (2012) use weights that change continuously with focus. The advantage of the latter formulation is the absence of the discontinuity. Its disadvantage is an additional term in the derivative of voters’ utility arising from the marginal change in the weights.

5Köszegi and Szeidl (2012) make a similar assumption. In Bordalo et al. (2012, 2013a,b, 2015a,b), in principle, the salient attribute of different options can be different. However, with binary choice sets and homogeneity of degree zero, as assumed in Bordalo et al. (2013b, 2015a), the same attribute is salient for both options. Moreover, the existing evidence on human perception and cognition does not favor either specification (see Köszegi and Szeidl, 2012, for further discussion of this point).
normalization ensures that the model is not biased towards focus on any single attribute by construction.

4 Consequences of Focus on Voters’ Preferences

Consider an exogenous choice set given by $\mathcal{P} = \{p_A, p_B\}$. When $p_A = p_B$ all the voters have undistorted focus. Consider $p_A \neq p_B$ and, without loss of generality, $p_A > p_B$. By Assumption A1, $p_A$ gives, to all voters, larger benefits and larger costs than $p_B$. In this sense, $p_A$’s relative advantage lies in its larger benefits, while $p_B$’s relative advantage lies in its lower costs. Proposition 1 shows that voters focus on the relative advantage of the policy which delivers the higher consumption utility.7

**Proposition 1.** Assume A1, A4 and $\mathcal{P} = \{p_A, p_B\}$, $p_A \geq p_B$. Voters in group $i \in \mathcal{N}$, (a) focus on benefits if and only if $V_i(p_A) > V_i(p_B)$; (b) focus on costs if and only if $V_i(p_A) < V_i(p_B)$; (c) have undistorted focus if and only if $V_i(p_A) = V_i(p_B)$.

Consider a social group $i \in \mathcal{N}$ that receives higher consumption utility from $p_A$, the larger policy in the choice set. This means that, for voters in this social group, the larger benefits from $p_A$ more than compensate its larger costs. This happens if and only if the range of benefits—which measures the advantage of $p_A$ in the consumption utility space—is larger than the range of costs—which measures the disadvantage of $p_A$ in the same space. Given our assumption on the determinants of voters’ attention, this leads voters in group $i$ to focus on benefits.

Proposition 2, which uses the order-restricted preferences implied by A3,8 says that focusing separates the electorate into two contiguous subsets of social groups, or factions: a faction composed of voters with relatively high consumption bliss points—who focus on benefits—and a faction composed of voters with relatively low consumption bliss points—who focus on costs.

**Proposition 2.** Assume A1, A3, A4 and $\mathcal{P} = \{p_A, p_B\}$. For any $i \in \mathcal{N}$, (a) if voters in group $i$ focus on benefits, then voters in groups $j > i$ focus on benefits; (b) if voters in group $i$ focus on costs, then voters in group $j < i$ focus on costs; (c) if voters in group $i$ have undistorted focus and $p_A \neq p_B$, then voters in group $j < i$ focus on costs and voters in group $j > i$ focus on benefits.

Proposition 3 shows that the members of these two factions maintain the same ranking between the two policies in their choice set for any degree of focusing but that their

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7 We present all proofs in Appendix A1.
8 Order-restricted preferences satisfy the following: if $p > p'$ and $i < i'$ or if $p < p'$ and $i > i'$, then $V_i(p) > V_i(p') \Rightarrow V_{i'}(p) > V_{i'}(p')$ (Persson and Tabellini, 2000, Definition 3). When $p > p'$, we have $V_i(p) - V_i(p') = \int_p^{p'} [B_i'(x) - C_i'(x)] dx$ non-decreasing in $i$ by Assumption A3. Similarly for $p < p'$.
intensity of preferences—that is, how much each voter cares about his preferred policy and, thus, the conflict of preferences between members of the two factions—is growing in the degree of focusing (that is, decreasing in $\delta_i$).

**Proposition 3.** Assume $A1$, $A4$ and $\mathcal{P} = \{p_A, p_B\}$. For all social groups $i \in N$, (a) focusing does not change the ranking of policies in voters’ preferences, that is, the signs of $V_i(p_A) - V_i(p_B)$ and $\tilde{V_i}(p_A|\mathcal{P}) - \tilde{V_i}(p_B|\mathcal{P})$ coincide; (b) focusing increases the intensity of preferences between policies, that is, the signs of $-\left[\tilde{V_i}(p_A|\mathcal{P}) - \tilde{V_i}(p_B|\mathcal{P})\right]$ and $\frac{\partial}{\partial \delta_i} \left[\tilde{V_i}(p_A|\mathcal{P}) - \tilde{V_i}(p_B|\mathcal{P})\right]$ coincide.

To understand the intuition behind Proposition 3, consider a group $i \in N$ that receives higher consumption utility from $p_A$, the larger policy. By Proposition 1, these voters overweight the relative advantage of $p_A$ with respect to $p_B$ and underweight its relative disadvantage. As a consequence, the difference in perceived or focus-weighted utility between the two options is larger than the difference in consumption utility, that is, $\tilde{V_i}(p_A|\mathcal{P}) - \tilde{V_i}(p_B|\mathcal{P}) > V_i(p_A) - V_i(p_B)$.

The second part of Proposition 3 implies that distorted focus does not affect social choice when society votes over binary agendas and no abstention is allowed. However, as we hope to show in the rest of this paper, this does not mean that selective attention is not important in politics or collective decision making. In particular, as Proposition 3(b) suggests, focusing matters whenever the intensity of preferences affects the likelihood of casting a vote (for example, with costly voting) or the likelihood of voting for a particular candidate (for example, with stochastic choice, or whenever other considerations enter the voters’ decision). Moreover, selective attention can affect not only the intensity of preferences but also the ranking over options when the choice set is larger and includes more than two policies. We explore these last two possibilities in Sections 5, where we introduce a model of electoral competition with citizens who vote probabilistically, and in Section 7, where we show results for a finite choice set, possibly including more than two (exogenous or endogenous) options.

## 5 Electoral Competition with Focusing Voters

### 5.1 Modeling Electoral Competition

In the previous section, we considered the effect of focus on voters’ preferences over an *exogenous* choice set. In this section, we consider the effect of voters’ focus on the *endogenous* supply of policies by political parties or candidates.

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9Note that, for the same reason, selective attention will also affect any other form of costly collective action (campaign contribution; declaration of support; volunteering or canvassing; active political participation).
In particular, we introduce focusing voters into a classical model of electoral competition, the probabilistic voting model à la Lindbeck and Weibull (1987). Two identical parties, \( j \in \{A, B\} \), simultaneously announce a binding policy, \( p_j \in \mathbb{R}_+ \).\(^{10}\) Voters observe parties’ policies, evaluate them with their focus-weighted utility (rather than their consumption utility) and vote as if they are pivotal (or derive expressive utility from voting). The indirect utility voter \( v \) in group \( i \) receives when voting for each candidate is:

\[
\begin{align*}
  u_{v,i}(A) &= \tilde{V}_i(p_A|P) \\
  u_{v,i}(B) &= \tilde{V}_i(p_B|P) + \epsilon_v
\end{align*}
\]

(4)

where \( P = \{p_A, p_B\} \) is voters’ endogenous choice set and \( \epsilon_v \sim U[\frac{-1}{2\phi}, \frac{1}{2\phi}] \) is an individual-level shock to the relative popularity of party \( B \), which is realized after policies are announced but before the election. Given these assumptions, voter \( v \) in group \( i \) votes for \( A \) if and only if \( \tilde{V}_i(p_A|P) > \tilde{V}_i(p_B|P) + \epsilon_v \).

Parties are purely office-motivated and maximize their vote shares.\(^{11}\) From the parties’ perspective, the expected share of voters in group \( i \) who vote for \( A \) is:

\[
\frac{1}{2} + \phi \left[ \tilde{V}_i(p_A|P) - \tilde{V}_i(p_B|P) \right].
\]

(5)

The two parties objective functions are:

\[
\begin{align*}
  \pi_A(p_A|P) &= \frac{1}{2} + \phi \sum_{i \in N} m_i \left[ \tilde{V}_i(p_A|P) - \tilde{V}_i(p_B|P) \right] \\
  \pi_B(p_B|P) &= 1 - \pi_A(p_A|P).
\end{align*}
\]

(6)

5.2 Benchmark: Endogenous Policies with Rational Voters

In this electoral game, parties simultaneously announce their policies. For all \( j \in \{A, B\} \), a pure strategy of party \( j \) is a policy in \( \mathbb{R}_+ \) and a mixed strategy for party \( j \) is a distribution over \( \mathbb{R}_+ \). The solution concept we adopt is Nash equilibrium. As a benchmark, we first consider fully rational voters, that is, \( \delta_i = 1 \) for all \( i \in N \). In this case, \( \tilde{V}_i(p_A|P) = B_i(p_A) - C_i(p_A) \) only depends on \( p_A \), not on the entire choice set \( P \). Similarly, \( \tilde{V}_i(p_B|P) = B_i(p_B) - C_i(p_B) \) only depends on \( p_B \).

**Proposition 4.** Assume \( A1, A2 \) and \( \delta_i = 1 \) for all \( i \in N \). A Nash equilibrium in mixed strategies exists and is unique. The equilibrium policies are \( (p^*_A, p^*_B) \), where \( p^*_i \) is

\(^{10}\) Analogously, parties can announce feasible pairs \( (B_i(p_j), C_i(p_j)) \) to each group \( i \in N \).

\(^{11}\) All results we present below are robust to parties maximizing the probability of winning.

\(^{12}\) As is usual, we assume \( \phi \) is large enough such that neither party receives the vote of the entire electorate.
Figure 1: $\tilde{V}_i(p_A|\mathcal{P}) - \tilde{V}_i(p_B|\mathcal{P})$ given $\mathcal{P} = \{p_A, p_B\}$

$B_i(p) = 2\sqrt{p}, C_i(p) = \frac{p^2}{2}, \delta_i = \frac{2}{3}$

(a) $p_i < p_B$  (b) $p_B = p_i$  (c) $p_B < p_i$

the unique solution to:

$$\sum_{i \in N} m_i \left[ B_i'(p) - C_i'(p) \right] = 0. \quad (O_r)$$

Moreover, $p_i^* \in (p_1, p_n)$.

Proposition 4 shows that, when voters do not suffer from distorted focus, equilibrium policies maximize a social consumption utility function where the weight on each social group is determined by its population share, $m_i$. This means that electoral competition leads to policies that are optimal in an utilitarian sense, that is, policies that maximize the sum of voters’ utilities.\(^\dagger\)

5.3 Endogenous Policies with Focusing Voters

We now introduce focusing voters. We first consider a society composed of two groups, where $p_1 < p_2$, and then move to the more general case. In the rational benchmark, the equilibrium platforms are $(p_1^*, p_2^*)$, where $p_r^* \in (p_1, p_2)$ is the unique solution to $m_1 V_1'(p) + m_2 V_2'(p) = 0$: a marginal deviation by either party results in a gain of votes from one group which is exactly offset by a loss of votes from the other group.

Focusing changes the parties’ calculus. Consider a marginal deviation from $(p_r^*, p_r^*)$ to $(p, p_r^*)$. A first, important, implication of our assumptions is that a deviation by a single party changes voters’ evaluation of the policies offered by both parties. Formally, a deviation to $p$ changes both terms in $\tilde{V}_i(p|\{p, p_r^*\}) - \tilde{V}_i(p_r^*|\{p, p_r^*\})$.

Consider first voters in group 1, that is, voters with a lower consumption’s bliss point. Figure 1a shows that a marginal deviation from $p_r^* > p_1$ to $p$ implies that voters

\(^\dagger\)Note that this is not a feature of any electoral competition with probabilistic voting: suboptimal equilibrium policies arise if the precision of the popularity shock, $\phi$, is heterogeneous across social groups. We deliberately shut down this source of inefficiency to avoid a confounding factor and to highlight the inefficiencies that are solely due to selective attention.
in group 1, who are now choosing from the set \( \{ p, p_r^* \} \), prefer the lower policy and, thus, focus on costs. As lemma A2 formally shows, this means that the derivative of \( \tilde{V}_1(p|\{ p, p_r^* \}) - \tilde{V}_1(p_r^*|\{ p, p_r^* \}) \) with respect to \( p \) evaluated at \( p_r^* \) equals:

\[
\frac{2\delta_1}{1+\delta_1} B'_1(p_r^*) - \frac{2}{1+\delta_1} C'_1(p_r^*). \tag{7}
\]

At the margin, voters in group 1 overweight costs and underweight benefits relative to their rational counterparts. This gives parties an incentive to run on lower platforms.

At the same time, this incentive is counter-balanced by an incentive to run on larger platforms, which results from the focus of voters in group 2. As Figure 1c shows, a marginal deviation from \( p_r^* < p_2 \) to \( p \) implies that voters in group 2, who are now choosing from the set \( \{ p, p_r^* \} \), prefer the larger policy and, thus, focus on benefits. This implies that the derivative of \( \tilde{V}_2(p|\{ p, p_r^* \}) - \tilde{V}_2(p_r^*|\{ p, p_r^* \}) \) with respect to \( p \) evaluated at \( p_r^* \) equals:

\[
\frac{2}{1+\delta_2} B'_2(p_r^*) - \frac{2\delta_2}{1+\delta_2} C'_2(p_r^*). \tag{8}
\]

At the margin, voters in group 2 overweight benefits and underweight costs, creating an incentive for parties to propose larger policies. The equilibrium platforms balance these two incentives, as characterized in equation \((O_{f,2})\) in Proposition 5.

**Proposition 5.** Assume A1, A2, A4. Consider \( n = 2 \) with \( p_1 < p_2 \). A Nash equilibrium in pure strategies exists and is unique. Let:

\[
O_{f,2}(p) = \frac{2m_1}{1+\delta_1} \left[ \delta_1 B'_1(p) - C'_1(p) \right] + \frac{2m_2}{1+\delta_2} \left[ B'_2(p) - \delta_2 C'_2(p) \right]. \tag{O_{f,2}}
\]

The equilibrium platforms of the two parties are \((p_f^*, p_f^*)\), where:

(a) if \( O_{f,2}(p_1) > 0 > O_{f,2}(p_2) \), \( p_f^* = (p_1, p_2) \) is the unique solution to \( O_{f,2}(p) = 0 \);

(b) if \( O_{f,2}(p_1) \leq 0 \), \( p_f^* = p_1 \);

(c) if \( O_{f,2}(p_2) \geq 0 \), \( p_f^* = p_2 \).

Proposition 5 implies that groups that are larger and have more distorted focus are more influential in the electoral calculus. Larger groups, that is, groups with larger \( m_i \), receive larger weight in the parties’ objective function and, hence, have larger impact on the equilibrium policy. Groups with more distorted focus, that is, groups with lower \( \delta_i \), have a stronger intensity of preferences between platforms, as noted in Proposition 3, and, thus, are more sensitive to electoral announcements.

**Corollary 1.** Consider the unique equilibrium policy of the electoral competition game with focusing voters and two groups, \( p_f^* \). If \( p_f^* \in (p_1, p_2) \), \( p_f^* \) approaches \( p_i \) when \( m_i \) increases or \( \delta_i \) decreases for any \( i \in \{1, 2\} \).

\[\text{Proposition 5 follows from the more general existence and uniqueness result we prove below.}\]
It is interesting to compare the equilibrium policy, $p_f^*$, to the utilitarianly efficient policy, $p_r^*$, that emerges from competition with rational voters. In general, we can have both $p_f^* > p_r^*$ and $p_f^* < p_r^*$. In fact, with two groups and an homogeneous degree of focusing, we can characterize the direction of the inefficiency generated by selective attention.\(^\text{15}\)

**Corollary 2.** Assume $n = 2$ and $\delta_1 = \delta_2$. $p_f^* \geq p_r$ if and only if $m_2 B_2'(p_r^*) \geq m_1 C_1'(p_r^*)$.

Corollary 2 implies that equilibrium policies are generically inefficient. Politicians inefficiently cater to larger groups and to groups that are more sensitive to changes on the attribute they focus on.

Proposition 5 allows the equilibrium policy to coincide with the consumption bliss point of one of the groups, something that cannot happen with rational voters. The intuition behind this result lies in the polarization of preferences induced by focusing. Denote by $p_c^i$ the cost-focus bliss point of voters in group $i$—that is, the unique maximizer of $\tilde{V}_i$ when voters in group $i$ focus on costs. Similarly, denote by $p_b^i$ the benefit-focus bliss point—that is, the unique maximizer of $\tilde{V}_i$ when voters in group $i$ focus on benefits.\(^\text{16}\) When $\delta_i \in (0, 1)$, we have $p_c^i < p_i^b < p_i^b$, where $p_i^b$ increases and $p_i^c$ decreases with the degree of focusing. As discussed above, a marginal deviation from a pair of identical policies makes voters in group 1 focus on costs and voters in group 2 focus on benefits. Therefore, the electoral calculus of parties facing focusing voters is similar to the electoral calculus of parties facing two rational but more strongly opposed groups of voters, one with ideal policy $p_1^c < p_1$ and one with ideal policy $p_2^b > p_2$. For this reason, focusing might lead to extreme policies.

Moreover, because the equilibrium policy can coincide with the consumption bliss point of one of the groups, it can be locally unresponsive to the model parameters, that is, remain constant in some regions of the parameter space. This is another feature of equilibrium policies with focusing voters which is not shared with the case of rational voters.

**Corollary 3.** Electoral competition with focusing voters polarizes the electorate. The equilibrium policy might coincides with the consumption bliss point of a group of voters and, thus, be locally unresponsive to parameter changes.

Figure 2 shows an example of the equilibrium policy for specific functional forms of the benefits and costs functions. The two panels illustrate the comparative statics with respect to $m_1$ and $\delta_1$ (Corollary 1). In both panels, for some parameter values, the equilibrium policy coincides with $p_1$ or $p_2$, and, in these cases, it is unresponsive to

\(^{15}\)We omit the formal argument, which subtracts $(O_f)$ evaluated at $p_r$ from $(O_f, 2)$ and uses the fact that $O_f, 2(p)$ is strictly decreasing in $p$ by Assumption A2.

\(^{16}\)If $p_c^i$ does not exist set $p_c^i = 0$. Similarly, if $p_b^i$ does not exist set $p_b^i = \infty$. 

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Figure 2: Equilibrium with focusing voters

\[ B_1(p) = \sqrt{p}, \quad C_1(p) = \frac{p^2}{4}, \quad p_1 = 1, \quad B_2(p) = 4\sqrt{2p}, \quad C_2(p) = \frac{p^2}{2}, \quad p_2 = 2 \]

(a) Effect of \( m_1, \delta_1 = \delta_2 = \frac{3}{4}, m_2 = 1 - m_1 \)

(b) Effect of \( \delta_1, \delta_2 = \frac{3}{4}, m_1 = \frac{2}{5} = 1 - m_2 \)

the model parameters (Corollary 3). In both panels, \( p_f^* \) can be both above or below \( p_r^* \) (Corollary 2).

We next characterize the equilibrium of the electoral game for an arbitrary number of social groups, \( n \geq 2 \). In this more general case, the equilibrium policy is determined by a condition on the left and right derivatives of \( \tilde{D}_i(p'\{p,p\}) = \tilde{V}_i(p'\{p',p\}) - \tilde{V}_i(p\{p',p\}) \) with respect to \( p' \), evaluated at \( p' = p \). These elements, which are denoted, respectively, by \( \tilde{D}_i^- (p|\mathcal{P}) \) and \( \tilde{D}_i^+ (p|\mathcal{P}) \), capture the effect of a marginal deviation from a convergent pair of policies, \( (p,p) \), to \( (p',p) \) with \( p' < p \), for the left derivative, and \( p' > p \), for the right derivative.

**Proposition 6.** Assume \( A1, A2, A4 \) and let \( p_f^* \) be the unique solution to

\[
\sum_{i \in \mathcal{N}} m_i \tilde{D}_i^- (p|\mathcal{P}) \geq 0 \quad \sum_{i \in \mathcal{N}} m_i \tilde{D}_i^+ (p|\mathcal{P}) \leq 0. \quad (O_f)
\]

A Nash equilibrium in pure strategies exists and is unique. The equilibrium platforms of the two parties are \( (p_f^*, p_r^*) \). Moreover, \( p_f^* \in [p_1, p_n] \).

Proposition 6 shows that there exists a unique equilibrium characterized by a convergent equilibrium policy \( p_f^* \). Selective attention requires as to use this more general approach, with the left and right derivatives, to characterize \( p_f^* \). Consider Figure 1b and an electorate with three social groups. Assume that, with rational voters, the equilibrium policy coincides with \( p_2 \), the consumption bliss point of the middle group. In this case, a marginal deviation from \( (p_2, p_2) \) by either party has no effect on the votes from group 2 since \( V_2'(p_2) = 0 \). Consider now focusing voters. A marginal deviation to \( (p, p_2) \) with \( p < p_2 \) implies a faster decrease in benefits than in costs. This induces voters in group 2 to focus on benefits—that is, on the attribute decreasing at a faster rate—and, thus,
to react more strongly than rational voters to a deviation. Formally, \( \tilde{D}_2^-(p_2) > 0 \). Similarly, a marginal deviation to \((p, p_2)\) with \( p > p_2 \) implies a faster increase in costs than in benefits. This induces voters in group 2 to focus on costs and, thus, to react more strongly than rational voters to a deviation. Formally, \( \tilde{D}_2^+(p_2) < 0 \). Since \( \tilde{D}_2^-(p_2) \neq \tilde{D}_2^+(p_2) \), the objective function of party \( A \) is not differentiable in \( p_A \) when \( p_A = p_B = p_2 \) and we cannot use the derivative to characterize the equilibrium policy.\(^{17}\) Despite this, \( p^*_i \) can be characterized using the left and right derivatives of the parties’ objective functions.

In the discussion above, we assumed that \( p^*_i \) coincides with the consumption bliss point of some group. When \( p^*_i \neq p_i \) for any \( i \in N \), we do not need to use the left and right derivatives since, as shown in Lemma A2, \( \tilde{D}_i(p) \) exists when \( p \neq p_i \) for any \( i \in N \). In this case, \( p^*_i \in (p_k, p_{k+1}) \) for some \( k \in \{1, \ldots, n-1\} \) is implicitly defined by a generalized version of \((O_{f,2})\):

\[
\sum_{i=1}^{k} m_i \left[ \frac{2\delta_i}{1+\delta_i} B_i'(p^*_i) - \frac{2}{1+\delta_i} C_i'(p^*_i) \right] + \sum_{i=k+1}^{n} m_i \left[ \frac{2\delta_i}{1+\delta_i} B_i'(p^*_i) - \frac{2\delta_i}{1+\delta_i} C_i'(p^*_i) \right] = 0. \tag{9}
\]

It is immediate that the comparative statics stated in Corollary 1 for \( n = 2 \) as well as the local unresponsiveness of \( p^*_i \) to the model parameters stated in Corollary 3 carry over to the model with arbitrary number of groups.\(^{18}\)

It is interesting to determine what groups are more influential in the politicians’ calculus among those that (marginally) focus on costs in equilibrium—that is, among groups 1 through \( k \)—as well as among those that (marginally) focus on benefits in equilibrium—that is, among groups \( k + 1 \) through \( n \). Investigating what groups are more influential requires an explicit measure of influence. Our measure of influence is the weight a group receives in the expression that implicitly defines \( p^*_i \), that is, the parties’ first order condition. We can rewrite (9) as:

\[
\sum_{i=1}^{k} \frac{2m_i}{1+\delta_i} \left[ V_i'(p^*_i) - B_i'(p^*_i)(1-\delta_i) \right] + \sum_{i=k+1}^{n} \frac{2m_i}{1+\delta_i} \left[ V_i'(p^*_i) + C_i'(p^*_i)(1-\delta_i) \right] = 0. \tag{10}
\]

If, for the sake of the argument, we assume that all the groups are homogeneous in terms of their size and degree of focus, equation 10 shows that, among groups 1 through \( k \), the most influential group is the group with the largest \( B_i \). Similarly, among groups \( k + 1 \) through \( n \), the most influential group is the group with the largest \( C_i \). Under the order restricted preferences implied by A3, this means that, among the first \( k \) groups, the \( k \)-th

\(^{17}\) \( \tilde{D}_2^-(p_2) > 0 \) and \( \tilde{D}_2^+(p_2) < 0 \) imply that \( \tilde{D}_i \) has a kink at \( p_2 \) that constitutes a local maximum. \((O_{f,1})\) therefore requires the objective function of the parties to have a kink at the equilibrium policy.

\(^{18}\) When \( m_i \) increases for some \( i \), \( m_j \) has to decrease for some \( j \neq i \). We assume that \( i \) and \( j \) focus on different attributes conditional on a marginal deviation from the equilibrium policy.
Figure 3: Top 1% income share and top marginal tax rate

(a) International (2005-2009 or most recent)  

(b) US

Note: Data courtesy of Piketty et al. (2014) (see their paper for original sources). Income excludes government transfers and is before individual taxes.

6 Application: Fiscal Policy

In the last 30 years, the US (as well as other developed economics) have experienced a rapid and sustained increase in the degree of income inequality (see Figure 3, Panel a). Contrary to the predictions of the standard political economy models, this trend has not been accompanied by increased preferences for redistribution in the population (see Figure 4) or by more redistributive policies (see Figure 3, Panel b). If anything, the data points to an inverse correlation between these time series.

What is the impact of distorted focus on voters’ preferences and parties’ political offer regarding taxation and public goods provision? Can selective attention help us to explain the empirical patterns from Figures 3 and 4, that are widely regarded as puzzling?

In order to answer these questions, we introduce a basic model of fiscal policy à la Meltzer and Richard (1981) (see also Weingast, Shepsle and Johnsen, 1981). A public good, \( p \in \mathbb{R}_+ \), is financed by a proportional income tax, \( \tau \geq 0 \). Society is composed of two groups of voters, \( R \) for Rich and \( P \) for Poor, with different income: \( y_R > y_P \geq 0 \). The measure of voters in group \( i \in \{R, P\} \) is \( m_i \in (0, 1) \). The average income in society is \( \bar{y} = m_R y_R + m_P y_P \). Given public good \( p \) and tax \( \tau \), the consumption utility of voters
in group $i$ is:

$$u_i(p, \tau) = (1 - \tau) y_i + B(p). \quad (11)$$

The government budget is balanced—that is, $p = \tau \bar{y}$—and, thus, the indirect consumption utility of voters in group $i$ from public good level $p$ is:

$$V_i(p) = y_i + B(p) - \frac{y_i}{\bar{y}} p. \quad (12)$$

With respect to the general model we introduced above, the policy gives homogeneous benefits to all groups, $B_i(p) = B(p)$, but the costs are heterogeneous and proportional to the relative income, $C_i(p) = \frac{y_i}{\bar{y}} p$. The latter implies that a group’s consumption bliss point depends negatively on relative income:

$$p_i = B'^{-1}\left(\frac{y_i}{\bar{y}}\right)$$

so that $p_i$ decreases in a group’s own income and increases in the other group’s income.

As a benchmark, consider electoral competition between two office-motivated parties facing rational voters.

**Proposition 7.** Assume $\delta_R = \delta_P = 1$. A Nash equilibrium in mixed strategies exists and is unique. The equilibrium policies are $(p^*_r, p^*_r)$, where $p^*_r$ is the unique solution to:

$$B'(p^*_r) = \frac{m_R y_R + m_P y_P}{\bar{y}} = 1.$$
shares) and, hence, is efficient. Moreover, since the average marginal costs are invariant to income distribution as well as to population shares, these two variables have no impact on the equilibrium level of public good.\footnote{Note that the stylized facts from Figures 3 and 4 are also inconsistent with another workhorse model of electoral competition, the median voter model (Downs, 1957). The median voter model obtains as a special case of the probabilistic voting model when $\epsilon_v = 0$ for all voters. In this case, the equilibrium policy is the consumption bliss point of the larger group. If we assume that $P$ voters are the majority and $R$ voters are an elite, that is, $m_R < 1/2$, the equilibrium policy coincides with $p_P$, which is increasing with income inequality, that is, with larger $y_R$ or smaller $y_P$. In short, in the median voter model, larger income inequality leads to larger redistribution.} The comparative statics, however, are different if we introduce focusing voters.

**Proposition 8.** Assume $\delta_i < 1$ for any $i \in \{P, R\}$. A Nash equilibrium in pure strategies exists and is unique. The equilibrium policies are $(p^*_f, p^*_r)$, where, if $p^*_f \in (p_R, p_P)$, then $p^*_f$ is the unique solution to:

$$
\frac{2m_R}{1+\delta_R} \left[ \delta_R B'(p^*_f) - \frac{y_R}{\delta} \right] + \frac{2m_P}{1+\delta_P} \left[ B'(p^*_f) - \delta_P \frac{y_P}{\delta} \right] = 0.
$$

Moreover, (a) when $\delta_R = \delta_P$, then $p^*_f \geq p^*_r$ if and only if $\frac{m_P}{m_R} > \frac{y_R}{y_P}$; (b) when $p^*_f \in (p_R, p_P)$, then $p^*_f$ decreases with income inequality, that is, with higher $y_R$ or lower $y_P$.

The equilibrium characterization and its uniqueness are a direct consequence of Proposition 5 for the general case. The condition that defines the equilibrium policy, $p^*_f$, is the same as in the statement of Proposition 5, adapted to the application at hand. At the margin, voters in group $R$—who prefer less redistribution than $p^*_f$—focus on costs; and voters in group $P$—who prefer more redistribution than $p^*_f$—focus on benefits. The proposition shows that the equilibrium level of public goods is inefficiently high when voters in group $P$ constitute a large fraction of the population or when the level of income inequality is small. In both cases, parties inefficiently cater to voters in $P$ group.

Intuitively, parties are more likely to cater to $P$ voters when they are a larger fraction of the population because they are a larger basin of votes. More interestingly, parties are more likely to cater to $P$ voters when income inequality is small and the equilibrium level of public goods, and, hence, of redistribution, is decreasing with income inequality. To see the intuition behind this result, consider the condition that defines $p^*_f$ in Proposition 8: in this expression, income inequality only affects marginal costs. As an example, consider an increase in $y_R$. With rational voters, an increase in $y_R$ by $dy_R$ increases the marginal costs of the $R$ voters by $\frac{y_R m_R}{\delta} dy_R$ and decreases the marginal costs of the $P$ voters by $\frac{y_P m_P}{\delta} dy_R$. The former increase and the latter decrease are, respectively, for an $m_R$ and an $m_P$ fraction of the population and, thus, the two effect perfectly offset each other, making $p^*_f$ invariant to the income distribution. With focusing voters, a
higher $y_R$ still increases the marginal costs of the $R$ voters. However, the decrease in the marginal costs of the $P$ voters is under-weighted by $\delta_P$. An increase in $y_R$, thus, leads to an increase in the average population-and-focus-weighted, marginal costs and in the decrease in the demand for redistribution. Figure 5 shows how the equilibrium level of public good provision (or redistribution) changes in the Meltzer and Richard (1981) model with income inequality.

Voters’ selective attention can, thus, explain why increased income inequality is associated with constant or decreasing demand for redistribution and hence with constant or decreasing observed levels of redistribution. A natural question is whether this prediction is limited to the very simple version of the Meltzer and Richard (1981) model we presented in this Section or more general. In what follows we argue that similar comparative statics obtain in a richer version of the model.

To see this, consider a general version of the Meltzer and Richard (1981) model with two groups, $R$ and $P$. Group $P$ receives benefits $B_P(p)$ and suffers costs $C_P(p)$ from public good level $p$. Group $R$ receives benefits $B_R(p)$ and suffers costs $C_R(p)$ from public good level $p$. The equilibrium level of public goods and hence of redistribution, $p^*_f$, is implicitly defined by \((O_{f,2})\), adapted to this more general setup:

\[
\frac{2m_P}{1+\delta_P} \left[ \delta_R B'_R(p_f^*) - C'_R(p_f^*) \right] + \frac{2m_R}{1+\delta_R} \left[ B'_P(p_f^*) - \delta_P C'_P(p_f^*) \right] = 0
\]  

(13)

Instead of modeling income inequality explicitly by specifying income levels for the two groups, suppose the degree of income inequality is $\Delta \in \mathbb{R}$, where higher $\Delta$ means
higher income inequality. For \( i \in \{ P, R \} \), denote the derivative of \( B_i' \) and \( C_i' \) with respect to \( \Delta \) by \( B_i^{\Delta} \) and \( C_i^{\Delta} \) respectively. Using the implicit function theorem, higher income inequality decreases \( p_f^* \) if the following expression is negative:

\[
\frac{2m_p}{1+\delta_R} \left[ \delta_R B_i^{\Delta} (p_f^*) - C_i^{\Delta} (p_f^*) \right] + \frac{2m_p}{1+\delta_p} \left[ B_i^{\Delta} (p_f^*) - \delta_P C_i^{\Delta} (p_f^*) \right] .
\] 

(14)

To understand this condition, consider first the simple version of the Meltzer and Richard (1981) model discussed above and suppose \( \Delta = y_R \). In our simpler model, \( \Delta \) has no effect on benefits, \( C_i^{\Delta} (p_f^*) = \frac{m_{y_R}y_p}{\delta_p} \) and \( C_i^{\Delta} (p_f^*) = -\frac{m_{y_R}y_p}{\delta_p} \). Substituting these expressions into (14) shows that (14) is negative if \( \frac{-1}{1+\delta_R} + \frac{\delta_p}{1+\delta_p} < 0 \), which holds when \( \delta_p \delta_R < 1 \).

Consider now a more general model where income inequality potentially affects not only the marginal costs but also the marginal benefits of the two groups. When the effect of \( \Delta \) on the marginal cost is as in the previous paragraph, the prediction that \( p_f^* \) is decreasing in \( \Delta \) is likely to hold when \( \delta_p \) is sufficiently close to zero. In this case, the condition on (14) negative, focusing only on the effect of \( \Delta \) on the marginal costs, becomes \( \frac{-1}{1+\delta_R} < 0 \), which holds strictly. In other words, (14) remains negative even for moderate effect of \( \Delta \) on marginal benefits. More generally, the result that \( p_f^* \) is decreasing in \( \Delta \), which requires that (14) is negative, (a) is reinforced when \( B_i^{\Delta} (p_f^*) \) and \( B_i^{\Delta} (p_f^*) \) are negative but is possibly reversed otherwise; (b) holds for almost all values of \( B_i^{\Delta} (p_f^*) \) when \( \delta_R \) is sufficiently close to zero; (c) holds when \( -m_{R}C_i^{\Delta} (p_f^*) + m_{P}B_i^{\Delta} (p_f^*) < 0 \) if \( \delta_p \) and \( \delta_R \) are sufficiently low, that is, it depends only on how income inequality impacts the marginal cost of the \( R \) group and the marginal benefits of the \( P \) group.

7 Larger Choice Sets and Decoy Effects

In this section, we extend the basic framework to more than two policies. Denote by \( \mathcal{P} = \{ p_A, p_B, \ldots \} \) the voters’ choice set and assume it is finite, \( p \in \mathbb{R}_+ \) for any \( p \in \mathcal{P} \) and \( |\mathcal{P}| \geq 2 \). Let \( \mathcal{P} \) and \( \mathcal{P}_r \) be, respectively, the smallest and the largest policy in \( \mathcal{P} \). Let \( \Delta_i^B (\mathcal{P}) \) be the range of benefits in \( \mathcal{P} \) for voters in group \( i \):

\[
\Delta_i^B (\mathcal{P}) = \max_{p \in \mathcal{P}} B_i(p) - \min_{p \in \mathcal{P}} B_i(p) = B_i (\mathcal{P}_r) - B_i (\mathcal{P}_l) .
\] 

(15)

Similarly, let \( \Delta_i^C (\mathcal{P}) \) be the range of costs in \( \mathcal{P} \) for voters in group \( i \):

\[
\Delta_i^C (\mathcal{P}) = \max_{p \in \mathcal{P}} C_i(p) - \min_{p \in \mathcal{P}} C_i(p) = C_i (\mathcal{P}_r) - C_i (\mathcal{P}_l) .
\] 

(16)

The second equality in the equations above follows by Assumption A1. The focus-weighted utility of voters in group \( i \) is still defined by Assumption A4. However, with
this more general, larger, choice set, the range of benefits and costs is defined by (15) and (16) rather than by (2) and (3).

First, we consider how focusing affects voters’ preferences with a more general choice set; and how adding a policy to voters’ choice set changes their preferences over the original policies. Second, we consider what policies are endogenously offered in an electoral campaign by two office-motivated politicians, when we allow for other, exogenous policies, to belong to voters’ choice set and, thus, potentially affect voters’ focus.

Proposition 9 (analogous to Proposition 1) shows that the attribute voters focus on is determined by the comparison between the consumption utilities granted by the smallest and the largest policy in the choice set.

Proposition 9. Assume $A1$, $A4$ and $\mathcal{P} = \{\mathcal{P}_1, \ldots, \mathcal{P}_n\}$. The focus of any group is determined exclusively by the extreme policies, $\mathcal{P}_-$ and $\mathcal{P}_+$, with voters focusing on the relative advantage of the extreme policy with the higher consumption utility. Voters in group $i \in N$, (a) focus on benefits if and only if $V_i(\mathcal{P}_-) > V_i(\mathcal{P}_+)$; (b) focus on costs if and only if $V_i(\mathcal{P}_-) < V_i(\mathcal{P}_+)$; have undistorted focus if and only if $V_i(\mathcal{P}_-) = V_i(\mathcal{P}_+)$. 

7.1 The Decoy Effect on Voters’ Preferences

Given a policy $p \in \mathbb{R}_+$, define $\check{p}_i^p$ as the policy other than $p$ which gives voters in group $i$ the same consumption utility as $p$. Proposition 10 shows how expanding voters’ choice set to include an additional policy affects their focus.

Proposition 10. Assume $A1$, $A2$, $A4$. Consider two choice sets, $\mathcal{P}$ and $\mathcal{P}' = \mathcal{P} \cup \{p'\}$. For any $i \in N$, (a) if under $\mathcal{P}$ voters in group $i$ focus on benefits, after adding $p'$, they: focus on benefits if $p' < \check{p}_i^p$; have undistorted focus if $p' = \check{p}_i^p$; focus on costs if $p' > \check{p}_i^p$; (b) if under $\mathcal{P}$ voters in group $i$ focus on costs, after adding $p'$, they: focus on benefits if $p' < \check{p}_i^p$; have undistorted focus if $p' = \check{p}_i^p$; focus on costs if $p' > \check{p}_i^p$; (c) if under $\mathcal{P}$ voters in group $i$ have undistorted focus and $\mathcal{P} \neq \mathcal{P}_i$, after adding $p'$, they: focus on benefits if $p' < \mathcal{P}_i$; have undistorted focus if $p' \in [\mathcal{P}_-, \mathcal{P}_+]$; focus on costs if $p' > \mathcal{P}_i$.

The effect of expanding the choice set on voters’ focus depends on the original focus and on the location of the additional policy. When voters are focusing on benefits, adding a sufficiently large policy induces voters to focus on costs. Conversely, if voters are focusing on costs, adding a sufficiently small policy induces voters to focus on benefits. Notice that voters who are focusing on benefits can always be induced to focus on costs with a proper addition to their choice set. Formally, there always exists $p'$ such that, if voters focus on benefits under $\mathcal{P}$, then the same voters focus on costs under $\mathcal{P} \cup \{p'\}$. However, since policies are bounded below at zero, it might be impossible to induce

\footnote{If $p' \in \mathbb{R}_+$ such that $V_i(p) = V_i(p')$ and $p' \neq p$ does not exist, set $\check{p}$ to an arbitrary negative constant.}
voters who are currently focusing on costs to focus on benefits. This is the case when \( \tilde{\delta}_i < 0 \), that is, when \( \tilde{\delta}_i \) is sufficiently large.

In Proposition 11 we address the question of how adding an exogenous policy \( p_C \) to the voters’ choice set changes the evaluation of the policies in the original choice set. We say that expanding the choice set changes the focus of group \( i \) towards costs (benefits) whenever voters in group \( i \) focus on benefits (costs) or have undistorted focus under the original choice set but instead focus on costs (benefits) on the expanded choice set.

**Proposition 11.** Assume \( A1, A2, A4 \). Consider two choice sets, \( \mathcal{P} \) and \( \mathcal{P}' \), such that \( p_A \in \mathcal{P}, p_B \in \mathcal{P}, p_A > p_B \) and \( \mathcal{P}' = \mathcal{P} \cup \{p_C\} \). For any \( i \in N \), if voters in group \( i \) focus on different attributes in \( \mathcal{P} \) and \( \mathcal{P}' \) and \( \delta_i < 1 \), then, (a) if adding \( p_C \) changes focus towards costs, then \( \tilde{V}_i(p_A|\mathcal{P}) - \tilde{V}_i(p_B|\mathcal{P}) > \tilde{V}_i(p_A|\mathcal{P}') - \tilde{V}_i(p_B|\mathcal{P}') \); (b) if adding \( p_C \) changes focus towards benefits, then \( \tilde{V}_i(p_A|\mathcal{P}) - \tilde{V}_i(p_B|\mathcal{P}) < \tilde{V}_i(p_A|\mathcal{P}') - \tilde{V}_i(p_B|\mathcal{P}') \); (c) if voters have distorted focus both in \( \mathcal{P} \) and \( \mathcal{P}' \), then there exists \( \delta_i \in (0,1) \) such that for any \( \delta_i < \tilde{\delta}_i \), \( \tilde{V}_i(p_A|\mathcal{P}) - \tilde{V}_i(p_B|\mathcal{P}) \) and \( \tilde{V}_i(p_A|\mathcal{P}') - \tilde{V}_i(p_B|\mathcal{P}') \) have different (strict) signs; (d) \( p_C \in C \in \arg\min_{p \in \mathcal{P}} \tilde{V}_i(p|\mathcal{P}') \).

Proposition 11 first shows that larger policies are hurt, in terms of how their evaluation by voters in group \( i \), when focus switches towards costs or away from benefits (part a) and gain when focus switches towards benefits or away from costs (part b). In these cases, not only voters’ intensity of preferences changes, but, according to part (c), for sufficiently strong focusing, also their ranking is affected. Finally, part (d) implies that options that change the attribute voters in group \( i \) focus on are bound to lose if their fate is determined by voters in the same group. The intuition behind the result is simple. Suppose voters in group \( i \) focus on benefits under \( \mathcal{P} \). By Proposition 10, an option \( p' \) that changes the focus towards costs under \( \mathcal{P}' = \mathcal{P} \cup \{p'\} \) has to be large. But large policies are not evaluated favorably when voters focus on costs.

Proposition 11 does not rule out the possibility that the change of focus changes how voters evaluate policies that are not favored and, hence, that it only affects those policies that would never be chosen. While possible, this is unlikely. Consider part (a) of the proposition. A change of focus towards costs implies that voters in group \( i \) focus on benefits in \( \mathcal{P} \). The proposition implies that the change in focus hurts the large policy, \( p_A \), which is likely preferred over \( p_B \) when voters focus on benefits.

In fact, when the smaller choice set consider in Proposition 11, \( \mathcal{P} \), is composed of only two policies, this proposition implies that it is the policy preferred under \( \mathcal{P} \) that is hurt by the change of focus. To see this, consider \( \mathcal{P} = \{p_A, p_B\} \) with \( p_A > p_B \) and suppose voters in group \( i \) are not indifferent between \( p_A \) and \( p_B \) given \( \mathcal{P} \). If \( \tilde{V}_i(p_A|\mathcal{P}) > \tilde{V}_i(p_B|\mathcal{P}) \), by Propositions 3 and 1, voters in group \( i \) focus on benefits under \( \mathcal{P} \). Therefore, any change of focus brought about by a third policy has to be towards costs, and, by
Proposition 11(a), \( p_A \), the policy preferred in \( \mathcal{P} \), is hurt by the change of focus. If 
\[
\tilde{V}_i(p_A|\mathcal{P}) < \tilde{V}_i(p_B|\mathcal{P}),
\]
a similar argument implies that voters in group \( i \) focus on costs under \( \mathcal{P} \) and, thus, any change of focus has to be towards benefits, which, in turn, hurts \( p_B \), the policy preferred in \( \mathcal{P} \).

Corollary 5. Assume A1, A2, A4. Consider \( \mathcal{P} = \{p_A, p_B\} \) and \( \mathcal{P}' = \mathcal{P} \cup \{p_C\} \) such that 
\[
\tilde{V}_i(p_A|\mathcal{P}) > \tilde{V}_i(p_B|\mathcal{P}).
\]
For any \( i \in \mathbb{N} \), if voters in group \( i \) focus on different attributes in \( \mathcal{P} \) and \( \mathcal{P}' \) and \( \delta_i < 1 \), then, (a) \( \tilde{V}_i(p_A|\mathcal{P}) - \tilde{V}_i(p_B|\mathcal{P}) > \tilde{V}_i(p_A|\mathcal{P}') - \tilde{V}_i(p_B|\mathcal{P}') \); (b) if voters have distorted focus in \( \mathcal{P}' \), then there exists \( \tilde{\delta}_i \in (0,1) \) such that for any \( \delta_i < \tilde{\delta}_i \), 
\[
\tilde{V}_i(p_A|\mathcal{P}') < \tilde{V}_i(p_B|\mathcal{P}').
\]

Propositions 10 and 11 imply that focusing and its changes generate a backlash effect. Consider a choice set \( \mathcal{P} \) composed of two policies \( p_B \) and \( p_A > p_B \). Suppose voters in group \( i \) focus on benefits in \( \mathcal{P} \), which, in light of the discussion leading to Corollary 5, is equivalent to assuming that voters in group \( i \) prefer \( p_A \) to \( p_B \) given \( \mathcal{P} \), 
\[
\tilde{V}_i(p_A|\mathcal{P}) > \tilde{V}_i(p_B|\mathcal{P}).
\]
Now consider a third policy, \( p_C \), is added to the voter’s choice set. There are many potential channels through which an additional policy can enter the voter’s choice set or consideration set: \( p_C \) can be the policy suggested by a media outlet, a think tank, or an international organization; a policy adopted in a neighboring country; or the status-quo policy, with \( p_A \) and \( p_B \) representing two alternative reforms. Suppose the addition of \( p_C \) changes the attribute voters in group \( i \) focus on and that they now focus on costs. Since voters used to focus on benefits, this implies, by Proposition 10, that \( p_C \) has to be sufficiently large. Given a sufficiently large degree of focusing, Proposition 11 implies that the addition of \( p_C \) leads to a reversal of preferences of voters in group \( i \) who, given \( \mathcal{P}' = \mathcal{P} \cup \{p_C\} \), prefer \( p_B \) to \( p_A \) and \( p_A \) to \( p_C \). In short, the addition of a large policy leads to a preference shift towards smaller policies. The mirror version of this effect is the addition of a small policy that leads to a preference shift towards larger policies.

In order to give empirical content to the theoretical results in this Section, let policies be different degrees of integration with the European Union. If we interpret the decision of UK citizens to leave the EU as the addition of an extreme policy to the choice set of voters in other European countries, the backlash effect discussed above can potentially explain why “support for the EU has risen in Europe in the wake of Brexit” (Financial Times, November 21, 2016, see also Figure 6). Similarly, it can explain why in the Spanish parliamentary elections that were held two days after Britain’s vote to leave the European Union, “Spanish voters turned away from anti-establishment parties and endorsed the perceived safety and security of ruling conservatives” (LA Times, June 27, 2016).\textsuperscript{22}

\textsuperscript{22}See also the Financial Times, June 28, 2016: “Unidos Podemos was the big loser of Spain’s general
Figure 6: Support for EU Integration in EU Member States

Note: Data from Bertelsmann Stiftung eupinions survey (see de Vries and Hoffmann, 2017, for details).

7.2 The Decoy Effect on Electoral Competition

Suppose an additional party, party $C$ enters the election with platform $p_C \in \mathbb{R}_+$. In order to isolate the effect of $C$ on voters’ focus, we assume that voters in neither group are willing to vote for $C$. We also assume that $p_C \notin [p_1, p_n]$.

**Proposition 12.** Suppose $A1, A2, A4$. Consider an electoral competition between parties $A$ and $B$ in the presence of an additional party $C$ with policy $p_C \in \mathbb{R}_+$. There exists at most one pure strategy Nash equilibrium. If $(p_A^*, p_B^*)$ constitute a Nash equilibrium, then $p_A^* = p_B^* = p_d^*$ where $p_d^* \geq p_f^*$ if $p_f^* \geq p_C$ while $p_d^* \leq p_f^*$ if $p_f^* \leq p_C$.

Proposition 12 shows that the additional party does not create asymmetric or multiple equilibria. At the same time, despite the fact that no voters vote for it, its presence potentially changes the equilibrium policies proposed by the two mainstream or viable parties, $A$ and $B$. Namely, the policy of the additional party pushes the equilibrium away from the equilibrium that would prevail in its absence. In other words, Proposition 12 provides an electoral, endogenous policy, version of the backlash effect discussed above for exogenous policies. The intuition lies behind the effect of $p_C$ in determining the attribute voters focus on. If $p_C$ is sufficiently low and parties $A$ and $B$ locate their policies in $[p_1, p_n]$, all voters focus on benefits under the resulting choice set. This leads to larger equilibrium policies.

The characterization of the electoral equilibria with a third extreme or non-viable election, shedding more than 1m votes since the last ballot in December. [...] Unidos Podemos leaders [...] pointed to Britain’s shock decision to leave the EU just two days before the election. [...] some leftwing voters may have decided at the last minute to back more conservative options, or to stay at home.”

23Given the assumption that voters are unwilling to vote for $C$, this is perhaps a natural assumption.
party $C$ is highly complex, but becomes feasible when $p_C$ is sufficiently large. In this case, for any pair of policies announced by parties $A$ and $B$, all voters focus on costs and, hence, the electoral competition between parties $A$ and $B$ facing focusing voters is isomorphic to the electoral competition between parties $A$ and $B$ focusing rational voters who put large a weight on the costs of policies (but this weight is not affected by a marginal deviation by either party).

**Proposition 13.** Assume $A1, A2, A4$. Consider an electoral competition between parties $A$ and $B$ in the presence of an additional party $C$ with policy $p_C > \max_{i \in N} \tilde{\theta}$. A Nash equilibrium in pure strategies exists and is unique. The equilibrium policies are $(p^*_d, p^*_d)$, where $p^*_d$ is the unique solution to:

$$\max_{p \in \mathbb{R}_+} \sum_{i \in N} \frac{2m_i}{1+\delta_i} \left[ \delta_i B_i(p) - C_i(p) \right].$$

As a result of focusing, the equilibrium characterized in Proposition has several properties that would not emerge with rational voters as well as with focusing voters but only two parties. First, in this equilibrium, voters in all groups focus on costs. This is driven by the large policy of the additional party. Second, it is possible for the equilibrium policy $p^*_d$ to lie outside the interval of the consumption bliss points of the electorate; in particular, we can have $(p^*_d < p_1)$. When the electorate has a fixed focus on costs, it is no longer true that a party moving its policy below $p_1$ loses votes from all social groups. With sufficiently strong focusing by all groups, the equilibrium policy can even equal 0.

8 Conclusions

How voters (and politicians) allocate their attention is fundamental for understanding political preferences and public policies. Cognitive psychology has pointed to two complementary mechanisms: top-down, goal-driven and ex-ante allocation of attention (or rational inattention) and bottom-up, stimulus-driven and ex-post allocation of attention (or selective attention). While the existing literature in political economy has focused on the former, this is the first paper to explore the latter.

We introduce selective attention in a formal model of electoral competition by assuming that, in forming their perception of policies’ value, voters focus disproportionately on the attribute in which their options differ more. We show that selective attention leads to a polarized electorate; that politicians facing focusing voters offer policies which do not achieve utilitarian welfare; that social groups that are larger, have more distorted focus, and are more sensitive to changes on a single attribute are more influential; and
that selective attention can contribute to explain puzzling empirical patterns, as the inverse correlation between income inequality and redistribution.

Our simple framework can deliver many other interesting results that we have not explored in this paper: for example, voters with distorted focus have stronger preferences and this makes them are more likely to turn out to vote, make financial contributions, actively participate to a candidate’s campaign or engage in other forms of collective action. We believe that there are many possible directions for the next steps in this research. Regarding the model we introduced, it would be interesting to introduce heterogeneous parties (for example, policy motivated parties) or allow policies to have uncorrelated attributes (for example, electoral platforms which offer a position on many different issues or candidates who have different personal characteristics). More generally, there are many exciting open questions, as what exact features of the political environment trigger voters’ selective attention and how selective attention interacts with the conscious research for information by poorly informed voters.

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A1 Proofs

A1.1 Preliminaries

Following notation and lemmas facilitate the proofs of the propositions below. First, for any $i \in N$ and $p \in \mathbb{R}_+$, let $\tilde{p}$ be the solution to $V_i(p) = V_i(\tilde{p})$ such that $p \neq \tilde{p}$ if the solution exists and let it be an arbitrary negative constant when the solution does not exists. Notice that for $p < p_i$, $\tilde{p} > p_i$ and for $p > p_i$, $\tilde{p} < p_i$. 
Second, \( \forall i \in N, \forall p \in \mathbb{R}_+, \forall p' \in \mathbb{R}_+ \) and any choice set \( \mathcal{P} \) with \( p \in \mathcal{P} \) and \( p' \in \mathcal{P} \), let \( \tilde{D}_i(p|\mathcal{P}) = \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) \). Derivative of \( \tilde{D}_i \) with respect to \( p \) is \( \tilde{D}_i'(p|\mathcal{P}) = \frac{\partial}{\partial p} \tilde{D}_i(p|\mathcal{P}) \) and includes the effect of \( p \) directly on \( \tilde{V}_i(p|\mathcal{P}) \) as well as indirectly on both \( \tilde{V}_i(p|\mathcal{P}) \) and \( \tilde{V}_i(p'|\mathcal{P}) \) through \( \mathcal{P} \) that contains \( p \).

Third, for a real valued function \( f \), denote by \( f'^- \) and \( f'^+ \) the left and right derivative of \( f \) respectively. Fourth, let \( \mathcal{P}_1 \) and \( \mathcal{P}_2 \) be the largest and smallest elements, respectively, of \( \mathcal{P} \). Finally, let

\[
v_{b,i}(p) = \frac{2}{1+\delta_1} B_i(p) - \frac{2\delta_1}{1+\delta_1} C_i(p)
\]

\[
v_{n,i}(p) = B_i(p) - C_i(p)
\]

\[
v_{c,i}(p) = \frac{2\delta_1}{1+\delta_1} B_i(p) - \frac{2}{1+\delta_1} C_i(p)
\]

and note that, \( \forall p \in \mathbb{R}_+, \forall i \in N \) and \( \forall a \in \{b, n, c\} \), \( v''_{a,i}(p) < 0 \) by Assumption A1.

Furthermore, we have, \( \forall p \in \mathbb{R}_+ \) and \( \forall i \in N \), \( v'_{b,i}(p) \geq v'_{n,i}(p) \geq v'_{c,i}(p) \) since

\[
v'_{b,i}(p) - v'_{n,i}(p) = v'_{n,i}(p) - v'_{c,i}(p) = \frac{1-\delta_1}{1+\delta_1} [B_i'(p) + C_i'(p)] .
\]

Throughout, we use that, \( \forall i \in N \) and \( \forall p \in \mathbb{R}_+ \), \( \tilde{V}_i(p|\{p, p\}) - \tilde{V}_i(p|\{p, p'\}) = 0 \) and \( \tilde{V}_i(p|\{p, \tilde{p}\}) - \tilde{V}_i(p|\{p, \tilde{p}'\}) = 0 \) whenever \( \tilde{p} \geq 0 \). The former is immediate. The latter follows since \( \tilde{p} \geq 0 \) implies that \( V_i(p) = V_i(\tilde{p}) \), so that voters in group \( i \) have undistorted focus given choice set \( \mathcal{P} = \{p, \tilde{p}\} \).

**Lemma A1.** Assume A1. For all \( i \in N \), \( \forall p \in \mathbb{R}_+ \) and \( \forall p' \in \mathbb{R}_+ \), if \( \delta_i = 1 \), then

\[
\tilde{D}_i(p|\{p, p'\}) = \tilde{V}_i(p|\{p, p'\}) - \tilde{V}_i(p'|\{p, p'\}) \text{ is continuous in } p, \quad \tilde{D}_i'(p|\{p, p'\}) \text{ exists and } \tilde{D}_i''(p|\{p, p'\}) < 0.
\]

**Proof.** The lemma follows immediately from Assumption A1 as \( \delta_i = 1 \) implies that, \( \forall i \in N \) and \( \forall p \in \mathbb{R}_+ \), \( \tilde{D}_i(p|\{p, p'\}) = V_i(p) - V_i(p') \). \( \square \)

**Lemma A2.** Assume A1, A2, A4. For all \( i \in N \), \( \forall p \in \mathbb{R}_+ \) and \( \forall p' \in \mathbb{R}_+ \), given \( \mathcal{P} = \{p, p'\} \), if \( \delta_i < 1 \), then

1. if \( p' = p_i \), voters in group \( i \) focus on benefits when \( p < p_i \) and focus on costs when \( p > p_i \); \( \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) \) is continuous in \( p \) and is differentiable in \( p \) except at \( p = p_i \);

2. if \( p' < p_i \), voters in group \( i \) focus on benefits when \( p \in [0, p') \cup (p', \tilde{p}') \) and focus on costs when \( p > p' \); \( \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) \) is continuous and differentiable in \( p \) except at \( p = p' \) and

\[
\lim_{p \to (\tilde{p}')^-} \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) > 0
\]

\[
\lim_{p \to (\tilde{p}')^+} \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) < 0;
\]

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3. if \( p' > p_i \), voters in group \( i \) focus on benefits when \( p < \tilde{p}' \) and focus on costs when \( p \in (\tilde{p}', p') \cup (p', \infty) \); \( \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) \) is continuous and differentiable in \( p \) except at \( p = \tilde{p}' \) and
\[
\lim_{p \to (\tilde{p}')^-} \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) < 0 \quad \text{when } \tilde{p}'' > 0
\]
\[
\lim_{p \to (\tilde{p}')^+} \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) > 0 \quad \text{when } \tilde{p}'' \geq 0;
\]

4. \( \tilde{D}_i'(p|\mathcal{P}) = \frac{\partial}{\partial p} \left[ \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) \right] \) equals
\[
\frac{2}{1 + \delta_i} B_i'(p) - \frac{2 \delta_i}{1 + \delta_i} C_i'(p) \quad \text{if } p < x
\]
\[
\frac{2 \delta_i}{1 + \delta_i} B_i'(p) - \frac{2}{1 + \delta_i} C_i'(p) \quad \text{if } p > x;
\]
where \( x = p_i \) if \( p' = p_i \) and \( x = \tilde{p}' \) if \( p' \neq p_i \);

5. if \( p' = p_i \), then
\[
\tilde{D}_i^-(p_i|\mathcal{P}) = \frac{2}{1 + \delta_i} B_i'(p_i) - \frac{2 \delta_i}{1 + \delta_i} C_i'(p_i)
\]
\[
\tilde{D}_i^+(p_i|\mathcal{P}) = \frac{2 \delta_i}{1 + \delta_i} B_i'(p_i) - \frac{2}{1 + \delta_i} C_i'(p_i).
\]

**Proof.** Throughout, fix \( i \in \mathbb{N}, p \in \mathbb{R}_+, \) and \( p' \in \mathbb{R}_+ \) and let \( \mathcal{P} = \{p, p'\} \) and \( \delta_i < 1. \)

Consider part 1. Since \( p' = p_i, V_i(p) < V_i(p') \) if \( p \neq p' \) and hence, by Proposition 1, voters in group \( i \) focus on costs when \( p > p' \) and focus on benefits when \( p < p' \). Voters in group \( i \) have undistorted focus when \( p = p_i \). Hence \( \tilde{D}_i(p|\mathcal{P}) \) equals

\[
\frac{2}{1 + \delta_i} [B_i(p) - B_i(p_i)] - \frac{2 \delta_i}{1 + \delta_i} [C_i(p) - C_i(p_i)] \quad \text{if } p < p_i
\]
\[
\frac{2 \delta_i}{1 + \delta_i} [B_i(p) - B_i(p_i)] - \frac{2}{1 + \delta_i} [C_i(p) - C_i(p_i)] \quad \text{if } p > p_i
\]
\[
[B_i(p) - B_i(p_i)] - [C_i(p) - C_i(p_i)] \quad \text{if } p = p_i.
\]

\( \tilde{D}_i(p|\mathcal{P}) \) is continuous in \( p \) at any \( p \neq p_i \) since \( B_i \) and \( C_i \) are continuous. At \( p = p_i \),
\[
\lim_{p \to p_i^-} \tilde{D}_i(p|\mathcal{P}) = 0, \quad \tilde{D}_i(p_i|\mathcal{P}) = 0 \quad \text{and} \quad \lim_{p \to p_i^+} \tilde{D}_i(p|\mathcal{P}) = 0.
\]
\( \tilde{D}_i(p|\mathcal{P}) \) is differentiable in \( p \) at any \( p \neq p_i \) since \( B_i \) and \( C_i \) are differentiable.

Consider part 2. Since \( p' < p_i \), we have \( p' < p_i < \tilde{p}' \). When \( p < p' \), we have \( V_i(p) < V_i(p') \) so that, by Proposition 1, voters in group \( i \) focus on benefits. When \( p > p' \), by Proposition 1, voters in group \( i \) focus on benefits when \( V_i(p) > V_i(p') \), or, equivalently, when \( p \in (p', p') \), and focus on costs when \( V_i(p) < V_i(p') \), or, equivalently, when \( p > p' \). Voters in group \( i \) have undistorted focus when \( p \in \{p', \tilde{p}'\} \). Hence, \( \tilde{D}_i(p|\mathcal{P}) \)
\[ \frac{2}{1+\delta_i} [B_i(p) - B_i(p')] - \frac{2\delta_i}{1+\delta_i} [C_i(p) - C_i(p')] \text{ if } p \in [0, \bar{p}') \]  
\[ \frac{2\delta_i}{1+\delta_i} [B_i(p) - B_i(p')] - \frac{2}{1+\delta_i} [C_i(p) - C_i(p')] \text{ if } p > \bar{p}' \]  
(A4)

\[ \tilde{D}_i(p|\mathcal{P}) \] is continuous in \( p \) at any \( p \notin \{p', \bar{p}'\} \) since \( B_i \) and \( C_i \) are continuous. At \( p = p' \), \( \lim_{p \to (p')} \tilde{D}_i(p|\mathcal{P}) = 0 \) if \( p' > 0 \), \( \tilde{D}_i(p'|\mathcal{P}) = 0 \) and \( \lim_{p \to (p')} \tilde{D}_i(p|\mathcal{P}) = 0 \). At \( p = \bar{p}' \), \( \lim_{p \to (\bar{p}')} \tilde{D}_i(p|\mathcal{P}) \) equals

\[ \frac{2}{1+\delta_i} [B_i(\bar{p}') - B_i(p')] - \frac{2\delta_i}{1+\delta_i} [C_i(p') - C_i(p')] \]
\[ = [B_i(\bar{p}') - B_i(p')] \left( \frac{2}{1+\delta_i} - \frac{2\delta_i}{1+\delta_i} \right) > 0 \]  
(A5)

where the equality follows from \( V_i(\bar{p}') = V_i(p') \) \( \iff \) \( B_i(\bar{p}') - B_i(p') = C_i(\bar{p}') - C_i(p') \) and the inequality follows by \( \bar{p}' > p' \) and \( \delta_i < 1 \), and \( \lim_{p \to (\bar{p}')} \tilde{D}_i(p|\mathcal{P}) \) equals

\[ \frac{2\delta_i}{1+\delta_i} [B_i(\bar{p}') - B_i(p')] - \frac{2}{1+\delta_i} [C_i(p') - C_i(p')] \]
\[ = [B_i(\bar{p}') - B_i(p')] \left( \frac{2\delta_i}{1+\delta_i} - \frac{2}{1+\delta_i} \right) < 0. \]  
(A6)

\( \tilde{D}_i(p|\mathcal{P}) \) is differentiable in \( p \) at any \( p \notin \{p', \bar{p}'\} \) since \( B_i \) and \( C_i \) are differentiable. At \( p = p' \), using definition of derivative in (A4), \( \tilde{D}_i'(p'|\mathcal{P}) = \frac{2}{1+\delta_i} B'_i(p') - \frac{2\delta_i}{1+\delta_i} C'_i(p') \).

Consider part 3. Since \( p' > p_i \), \( \bar{p}' < p_i < p' \). When \( p < p' \), by Proposition 1, voters in group \( i \) focus on benefits when \( V_i(p) < V_i(p') \), or, equivalently, when \( p < \bar{p}' \), and focus on costs when \( V_i(p) > V_i(p') \), or, equivalently, when \( p \in (\bar{p}', p') \). When \( p > p' \), we have \( V_i(p) < V_i(p') \) so that, by Proposition 1, voters in group \( i \) focus on costs. Voters in group \( i \) have undistorted focus when \( p \in \{\bar{p}', p'\} \). Hence \( \tilde{D}_i(p|\mathcal{P}) \) equals

\[ \frac{2\delta_i}{1+\delta_i} [B_i(p) - B_i(p')] - \frac{2}{1+\delta_i} [C_i(p) - C_i(p')] \text{ if } p \in (\bar{p}', \infty) \]  
\[ \frac{2}{1+\delta_i} [B_i(p) - B_i(p')] - \frac{2\delta_i}{1+\delta_i} [C_i(p) - C_i(p')] \text{ if } p < \bar{p}' \]  
(A7)

\( \tilde{D}_i(p|\mathcal{P}) \) is continuous in \( p \) at any \( p \notin \{\bar{p}', p'\} \) since \( B_i \) and \( C_i \) are continuous. At \( p = p' \), \( \lim_{p \to (p')} \tilde{D}_i(p|\mathcal{P}) = 0 \), \( \tilde{D}_i(p'|\mathcal{P}) = 0 \) and \( \lim_{p \to (p')} \tilde{D}_i(p|\mathcal{P}) = 0 \). At \( p = \bar{p}' \), \( \lim_{p \to (\bar{p}')} \tilde{D}_i(p|\mathcal{P}) \) when \( \bar{p}' > 0 \) equals

\[ \frac{2}{1+\delta_i} [B_i(\bar{p}') - B_i(p')] - \frac{2\delta_i}{1+\delta_i} [C_i(\bar{p}') - C_i(p')] \]
\[ = [B_i(\bar{p}') - B_i(p')] \left( \frac{2}{1+\delta_i} - \frac{2\delta_i}{1+\delta_i} \right) < 0 \]  
(A8)
where the equality follows from \( V_i(\tilde{p}') = V_i(p') \) \( \iff \) \( B_i(\tilde{p}') - B_i(p') = C_i(\tilde{p}') - C_i(p') \) and the inequality follows by \( \tilde{p}' < p' \) and \( \delta_i < 1 \), and \( \lim_{p \to (\tilde{p}')^+} \widehat{D}_i(p|\mathcal{P}) \) when \( \tilde{p}' \geq 0 \) equals

\[
\frac{2\delta_i}{1+\delta_i} |B_i(\tilde{p}') - B_i(p')| - \frac{2}{1+\delta_i} |C_i(\tilde{p}') - C_i(p')| = |B_i(\tilde{p}') - B_i(p')| \left( \frac{2\delta_i}{1+\delta_i} - \frac{2}{1+\delta_i} \right) > 0. \tag{A9}
\]

\( \widehat{D}_i(p|\mathcal{P}) \) is differentiable in \( p \) at any \( p \notin \{\tilde{p}', p'\} \) since \( B_i \) and \( C_i \) are differentiable. At \( p = p' \), using definition of derivative in (A7), \( \widehat{D}_i(p'|\mathcal{P}) = \frac{2\delta_i}{1+\delta_i} B_i'(p') - \frac{2}{1+\delta_i} C_i'(p') \).

Part 4 for \( p = p_i \) follows from (A3), for \( p' < p_i \) follows from (A4) and for \( p' > p_i \) follows from (A7). Part 5 follows from (A3). \( \square \)

### A1.2 Proof of Proposition 1

Fix \( i \in N, p_A \in \mathbb{R}_+ \) and \( p_B \in \mathbb{R}_+ \) such that \( p_A \geq p_B \). Since \( p_A \geq p_B \), by Assumption A1, we have \( |B_i(p_A) - B_i(p_B)| = B_i(p_A) - B_i(p_B) \) and \( |C_i(p_A) - C_i(p_B)| = C_i(p_A) - C_i(p_B) \).

Part (a) follows since \( B_i(p_A) - B_i(p_B) > C_i(p_A) - C_i(p_B) \iff V_i(p_A) > V_i(p_B) \). Part (b) follows since \( B_i(p_A) - B_i(p_B) < C_i(p_A) - C_i(p_B) \iff V_i(p_A) < V_i(p_B) \). Part (c) follows since \( B_i(p_A) - B_i(p_B) = C_i(p_A) - C_i(p_B) \iff V_i(p_A) = V_i(p_B) \). \( \square \)

### A1.3 Proof of Proposition 2

We first claim that, by Assumption A3, for any \( k \in N \) and \( l \in N \) such that \( k < l \) and any \( p \in \mathbb{R}_+ \) and \( p' \in \mathbb{R}_+ \) such that \( p > p' \), \( V_k(p) - V_k(p') < V_l(p) - V_l(p') \). To see this, by A3, we have,

\[
V_k(p) - V_k(p') = \int_{p'}^p [B_k'(x) - C_k'(x)] \, dx < \int_{p'}^p [B_l'(x) - C_l'(x)] \, dx = V_l(p) - V_l(p'). \tag{A10}
\]

Now fix \( p_A \in \mathbb{R}_+ \) and \( p_B \in \mathbb{R}_+ \). It suffices to consider \( p_A \neq p_B \). When \( p_A = p_B \), then voters in all groups have undistorted focus so that parts (a) and (b) do not apply and part (c) assumes \( p_A \neq p_B \). Without loss of generality, assume \( p_A > p_B \).

To see part (a), when voters in group \( i \in N \) focus on benefits, \( V_i(p_A) > V_i(p_B) \) by Proposition 1 and it suffices to prove \( V_j(p_A) > V_j(p_B) \) when \( j > i \), which follows by the opening claim.

To see part (b), when voters in group \( i \in N \) focus on costs, \( V_i(p_A) < V_i(p_B) \) by Proposition 1 and it suffices to prove \( V_j(p_A) < V_j(p_B) \) when \( j < i \), which follows by the opening claim.

To see part (c), when voters in group \( i \in N \) have undistorted focus, \( V_i(p_A) = V_i(p_B) \) by Proposition 1. By the opening claim, \( V_j(p_A) > V_j(p_B) \) when \( j > i \), in which case
voters in group $j$ focus on benefits by Proposition 1, and $V_j(p_A) < V_j(p_B)$ when $j < i$, in which case voters in group $j$ focus on costs by Proposition 1.

\[ \square \]

A1.4 Proof of Proposition 3

Throughout, fix $i \in N$, $p_j \in \mathbb{R}_+$ for $j \in \{A, B\}$ and $j \in \{A, B\}$ and let $\mathcal{P} = \{p_A, p_B\}$. To prove part (a), we consider three cases depending on the sign of $V_i(p_j) - V_i(p_{-j})$.

**Case 1:** $V_i(p_j) = V_i(p_{-j})$: By Proposition 1, $V_i(p_j) = V_i(p_{-j})$ implies that voters in group $i$ have undistorted focus and hence $\tilde{V}_i(p_j|\mathcal{P}) = \tilde{V}_i(p_{-j}|\mathcal{P})$.

**Case 2:** $V_i(p_j) > V_i(p_{-j})$: Since $V_i(p_j) > V_i(p_{-j})$, $p_j \neq p_{-j}$. Suppose first that $p_j > p_{-j}$. Then $V_i(p_j) > V_i(p_{-j})$ implies, by Proposition 1, that voters in group $i$ focus on benefits. $\tilde{V}_i(p_j|\mathcal{P}) - \tilde{V}_i(p_{-j}|\mathcal{P})$ thus equals

\[
\frac{2}{1+\delta_i} [B_i(p_j) - B_i(p_{-j})] - \frac{2\delta_i}{1+\delta_i} [C_i(p_j) - C_i(p_{-j})] = \frac{2}{1+\delta_i} [V_i(p_j) - V_i(p_{-j})] + \frac{2(1-\delta_i)}{1+\delta_i} [C_i(p_j) - C_i(p_{-j})] > 0 \tag{A11}
\]

where the inequality follows by $V_i(p_j) > V_i(p_{-j})$ and $C_i(p_j) - C_i(p_{-j}) > 0$. Suppose now that $p_j < p_{-j}$. Then $V_i(p_j) > V_i(p_{-j})$ implies, by Proposition 1, that voters in group $i$ focus on costs. $\tilde{V}_i(p_j|\mathcal{P}) - \tilde{V}_i(p_{-j}|\mathcal{P})$ thus equals

\[
\frac{2\delta_i}{1+\delta_i} [B_i(p_j) - B_i(p_{-j})] - \frac{2}{1+\delta_i} [C_i(p_j) - C_i(p_{-j})] = -\frac{2(1-\delta_i)}{1+\delta_i} [B_i(p_j) - B_i(p_{-j})] + \frac{2}{1+\delta_i} [V_i(p_j) - V_i(p_{-j})] > 0 \tag{A12}
\]

where the inequality follows by $V_i(p_j) > V_i(p_{-j})$ and $B_i(p_j) - B_i(p_{-j}) < 0$.

**Case 3:** $V_i(p_j) < V_i(p_{-j})$: Since $V_i(p_j) < V_i(p_{-j})$, $p_j \neq p_{-j}$. Suppose first that $p_j > p_{-j}$. Then $V_i(p_j) < V_i(p_{-j})$ implies, by Proposition 1, that voters in group $i$ focus on costs. $\tilde{V}_i(p_j|\mathcal{P}) - \tilde{V}_i(p_{-j}|\mathcal{P})$ thus equals

\[
\frac{2\delta_i}{1+\delta_i} [B_i(p_j) - B_i(p_{-j})] - \frac{2}{1+\delta_i} [C_i(p_j) - C_i(p_{-j})] = -\frac{2(1-\delta_i)}{1+\delta_i} [B_i(p_j) - B_i(p_{-j})] + \frac{2}{1+\delta_i} [V_i(p_j) - V_i(p_{-j})] < 0 \tag{A13}
\]

where the inequality follows by $V_i(p_j) < V_i(p_{-j})$ and $B_i(p_j) - B_i(p_{-j}) > 0$. Suppose now that $p_j < p_{-j}$. Then $V_i(p_j) < V_i(p_{-j})$ implies, by Proposition 1, that voters in group $i$ focus on benefits. $\tilde{V}_i(p_j|\mathcal{P}) - \tilde{V}_i(p_{-j}|\mathcal{P})$ thus equals

\[
\frac{2}{1+\delta_i} [B_i(p_j) - B_i(p_{-j})] - \frac{2\delta_i}{1+\delta_i} [C_i(p_j) - C_i(p_{-j})] = \frac{2}{1+\delta_i} [V_i(p_j) - V_i(p_{-j})] + \frac{2(1-\delta_i)}{1+\delta_i} [C_i(p_j) - C_i(p_{-j})] < 0 \tag{A14}
\]

where the inequality follows by $V_i(p_j) < V_i(p_{-j})$ and $C_i(p_j) - C_i(p_{-j}) < 0$.

To prove part (b), $\tilde{V}_i(p_j|\mathcal{P}) = \tilde{V}_i(p_{-j}|\mathcal{P})$ only in Case 1 above, in which case

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\[ \tilde{V}_i(p_j | \mathcal{P}) - \tilde{V}_i(p_{-j} | \mathcal{P}) = 0 \text{ for any } \delta_i. \]  
\[ \tilde{V}_i(p_j | \mathcal{P}) > \tilde{V}_i(p_{-j} | \mathcal{P}) \text{ only in Case 2 above, in which case } \tilde{V}_i(p_j | \mathcal{P}) - \tilde{V}_i(p_{-j} | \mathcal{P}) \text{ equals} \]
\[ \frac{2}{1 + \delta_i} [B_i(p_j) - B_i(p_{-j})] - \frac{2\delta_i}{1 + \delta_i} [C_i(p_j) - C_i(p_{-j})] \text{ if } p_j > p_{-j} \]
\[ \frac{2\delta_i}{1 + \delta_i} [B_i(p_j) - B_i(p_{-j})] - \frac{2}{1 + \delta_i} [C_i(p_j) - C_i(p_{-j})] \text{ if } p_j < p_{-j} \]  
(A15)  
which is decreasing in \( \delta_i \) since \( \frac{\partial}{\partial \delta_i} \frac{2}{1 + \delta_i} < 0 \) and \( \frac{\partial}{\partial \delta_i} \frac{2\delta_i}{1 + \delta_i} > 0 \).  
\[ \tilde{V}_i(p_j | \mathcal{P}) < \tilde{V}_i(p_{-j} | \mathcal{P}) \text{ only in Case 3 above, in which case } \tilde{V}_i(p_j | \mathcal{P}) - \tilde{V}_i(p_{-j} | \mathcal{P}) \text{ equals} \]
\[ \frac{2\delta_i}{1 + \delta_i} [B_i(p_j) - B_i(p_{-j})] - \frac{2}{1 + \delta_i} [C_i(p_j) - C_i(p_{-j})] \text{ if } p_j > p_{-j} \]
\[ \frac{2}{1 + \delta_i} [B_i(p_j) - B_i(p_{-j})] - \frac{2\delta_i}{1 + \delta_i} [C_i(p_j) - C_i(p_{-j})] \text{ if } p_j < p_{-j} \]  
(A16)  
which is increasing in \( \delta_i \).

\[ \square \]

**A1.5 Proof of Proposition 4**

Since \( \delta_i = 1 \forall i \in N \), we have, \( \forall j \in \{A, B\}, \forall (p_j, p_{-j}) \in \mathbb{R}^2_+ \) and \( \forall i \in N \), \( \tilde{V}_i(p_j | \mathcal{P}) - \tilde{V}_i(p_{-j} | \mathcal{P}) = B_i(p_j) - C_i(p_j) - [B_i(p_{-j}) - C_i(p_{-j})] \). Thus, \( \forall j \in \{A, B\}, \forall (p_j, p_{-j}) \in \mathbb{R}^2_+ \) and \( \forall i \in N \), \( \tilde{V}_i(p_j | \mathcal{P}) - \tilde{V}_i(p_{-j} | \mathcal{P}) \) is strictly concave in \( p_j \) and, hence, \( \pi_j(p_j | \mathcal{P}) \) is strictly concave in \( p_j \). Therefore, \( \forall j \in \{A, B\} \) and \( \forall p_{-j} \in \mathbb{R}_+ \), the unique maximizer of \( \pi_j(p_j | \mathcal{P}) \) is \( p_j^* \), the unique solution to \( \sum_{i \in N} m_i [B_i'(p) - C_i'(p)] = 0 \). To see that \( p_j^* \) exists and is unique, note that \( \sum_{i \in N} m_i [B_i'(p) - C_i'(p)] \) is continuous and decreasing in \( p \) since its derivative \( \sum_{i \in N} m_i [B_i''(p) - C_i''(p)] < 0 \) by Assumption A1. Moreover, \( \sum_{i \in N} m_i [B_i'(p_1) - C_i'(p_1)] > 0 \) and \( \sum_{i \in N} m_i [B_i'(p_n) - C_i'(p_n)] < 0 \) by Assumption A2, which also shows that \( p_j^* \in (p_1, p_n) \).

We now argue that if a NE exists, then the parties’ equilibrium platforms are \( (p_A^*, p_B^*) \). Suppose that \( (\mu_A^*, \mu_B^*) \) constitutes a NE, where \( \mu_A^* \) is a mixed strategy, a Borel probability measure, of party \( j \in \{A, B\} \). Since \( (\mu_A^*, \mu_B^*) \) constitutes a NE in a constant-sum game, the equilibrium expected vote share equals \( \frac{1}{2} \) for both parties. Suppose, for some \( j \in \{A, B\} \), that party \( j \) contests the election with policy \( p_j = p_j^* \). Then its deviation payoff equals

\[ \pi_j(p_j^* | \{p_A^*, p_B^*\}) = \frac{1}{2} + \phi \int_{\mathbb{R}^+} \sum_{i \in N} m_i \left[ \tilde{V}_i(p_j^* | \{p_A^*, p_B^*\}) - \tilde{V}_i(p_j^* | \{p_A^*, p_B^*\}) \right] \mu_{-j}(dp). \]  
(A17)  
Since, \( \forall p_{-j} \in \mathbb{R}_+ \), \( \sum_{i \in N} m_i \left[ \tilde{V}_i(p_j^* | \{p_A^*, p_B^*\}) - \tilde{V}_i(p_{-j} | \{p_A^*, p_B^*\}) \right] \geq 0 \), with strict inequality when \( p_{-j} \neq p_j^* \), we have \( \pi_j(p_j^* | \{p_A^*, p_B^*\}) > \frac{1}{2} \) unless \( \mu_{-j}(p_j^*) = 1 \).

To see that \( (p_A^*, p_B^*) \) constitutes a NE, we have \( \pi_j(p_j^* | \{p_A^*, p_B^*\}) = \frac{1}{2} \forall j \in \{A, B\} \). If, for some \( j \in \{A, B\} \), party \( j \) deviates to \( \mu_j \) with \( \mu_j(p_j^*) < 1 \), then its deviation payoff \( \pi_j(\mu_j | \{p_A^*, p_B^*\}) < \frac{1}{2} \) by an argument similar to the one above. Therefore, neither party
Lemma A4 proves several properties of $T$ costs and groups
1 sum game, the equilibrium vote share equals
Suppose ($p_k$)
Then there exists $p < p_k$ exists (the left derivative at $p$)
by Assumption A1 and proving Proposition 6, we establish
Lemma A5. We state and prove all the lemmas first.
Lemma A4.

1. $\forall p \in \mathbb{R}^+ \text{ and } \forall k \in \{0, \ldots, n - 1\}, \, T(p, k) \geq T(p, k + 1)$;
2. $\forall p \in \mathbb{R}^+ \text{ and } \forall k \in \{0, \ldots, n\}, \, T'(p, k) < 0$;
3. $T(p, 0) > 0 \, \forall p \leq p_1 \text{ and } T(p, n) < 0 \, \forall p \geq p_n$.

Proof. For part 1, $\forall p \in \mathbb{R}^+$ and $\forall k \in \{0, \ldots, n - 1\}$:

$$T(p, k) - T(p, k + 1) = \frac{2m_{k+1}(1-\delta_{k+1})}{1+\theta_{k+1}} [B'_{k+1}(p) + C'_{k+1}(p)] \geq 0 \quad \text{(A19)}$$

where the inequality follows by Assumption A1.

Part 2 is immediate since $B''_i \leq 0$ and $C''_i \geq 0$ with at least one strict inequality $\forall i \in N$ by Assumption A1.

For part 3,

$$T(p, 0) = \sum_{i \in N} \frac{2m_i}{1+\delta_i} [B'_i(p) - C'_i(p)] + \frac{2(1-\delta_i)m_i}{1+\delta_i} C'_i(p) > 0 \quad \text{(A20)}$$

where the inequality follows from $p \leq p_1$, and

$$T(p, n) = \sum_{i \in N} \frac{2(1-\delta_i)m_i}{1+\delta_i} B'_i(p) + \frac{2m_i}{1+\delta_i} [B'_i(p) - C'_i(p)] < 0 \quad \text{(A21)}$$

where the inequality follows from $p \geq p_n$. \hfill \Box

Lemma A5. A solution, $p^*_j$, to $(O_j)$ exists, is unique and satisfies $p^*_j \in [p_1, p_n]$.

Proof. Denote by $p_0 = 0$ and $p_{n+1} = \infty$. Since $0 < p_i < p_{i+1} < \infty \, \forall i \in \{1, \ldots, n - 1\}$, we have $p_i < p_{i+1} \forall i \in \{0, \ldots, n\}$. Notice that, $\forall k \in \{0, \ldots, n\}, \, \sum_{i \in N} m_i \tilde{D}'_i(p\{p, p\}) = T(p, k)$ if $p \in (p_k, p_{k+1})$ and, $\forall k \in \{1, \ldots, n\}, \, \sum_{i \in N} m_i \tilde{D}'_i(p\{p, p\}) = T(p, k - 1)$ and $\sum_{i \in N} m_i \tilde{D}'_i(p\{p, p\}) = T(p, k)$ if $p = p_k$. The former by Lemma A2 part 4 and the latter by Lemma A2 parts 4 and 5. Therefore, if $p^*_j$ solves $(O_j)$, then either $T(p^*_j, k) = 0$ and $p^*_j \in (p_k, p_{k+1})$ for some $k \in \{0, \ldots, n\}$ or $T(p^*_j, k - 1) \geq 0, \, T(p^*_j, k) \leq 0$ and $p^*_j = p_k$ for some $k \in \{1, \ldots, n\}$. Conversely, any $p' \in \mathbb{R}^+$ such that either $T(p', k) = 0$ and $p' \in (p_k, p_{k+1})$ for some $k \in \{0, \ldots, n\}$ or $T(p', k - 1) \geq 0, \, T(p', k) \leq 0$ and $p' = p_k$ for some $k \in \{1, \ldots, n\}$ solves $(O_j)$. To prove the lemma, it thus suffices to show that $p'$ exists, is unique and $p' \in [p_1, p_n]$.

For existence, we will show that if $p' \in \mathbb{R}^+$ such that $T(p', k) = 0$ and $p' \in (p_k, p_{k+1})$ for some $k \in \{0, \ldots, n\}$ does not exist, then there exists $p' \in \mathbb{R}^+$ such that $T(p', k - 1) \geq 0, \, T(p', k) \leq 0$ and $p' = p_k$ for some $k \in \{1, \ldots, n\}$. Since $p'$ such that $T(p', k) = 0$ and $p' \in (p_k, p_{k+1})$ for some $k \in \{0, \ldots, n\}$ does not exist and since $T(p, k)$ is continuous
in $p \forall k \in \{0, \ldots, n\}$, we have, $\forall k \in \{0, \ldots, n\}$, either $T(p, k) > 0 \forall p \in (p_k, p_{k+1})$ or $T(p, k) < 0 \forall p \in (p_{k-1}, p_k)$. By Lemma A4 part 3, $T(p, 0) > 0 \forall p \in (p_0, p_1)$ and $T(p, n) < 0 \forall p \in (p_n, p_{n+1})$. Since $T(p, k) > T(p', k + 1) \forall p \in \mathbb{R}_+, \forall k \in \{0, \ldots, n - 1\}$ and $\forall p'' > p$ by Lemma A4 parts 1 and 2, there exist $k' \in \{1, \ldots, n\}$ such that, $\forall k'' \leq k'$, $T(p, k'' - 1) > 0 \forall p \in (p_{k''-1}, p_{k''})$ and, $\forall k'' \geq k'$, $T(p, k'') < 0 \forall p \in (p_{k''}, p_{k''+1})$. By continuity of $T(p, k)$ in $p \forall k \in \{0, \ldots, n\}$, we thus have $T(p_k', k' - 1) \geq 0$ and $T(p_k', k') \leq 0$.

For uniqueness, suppose either $T(p', k') = 0$ and $p' \in (p_k', p_{k'+1})$ for some $k' \in \{0, \ldots, n\}$ or $T(p', k' - 1) \geq 0$, $T(p', k') \leq 0$ and $p' = p_k'$ for some $k' \in \{0, \ldots, n\}$.

If $p' \in (p_k', p_{k'+1})$ so that $T(p', k') = 0$, then $T(p'', k'') < 0 \forall p'' > p'$ and $\forall k'' \geq k'$ by Lemma A4 parts 1 and 2 and hence $T(p'', k'') < 0 \forall p'' \in (p', p_{k'+1})$, $T(p'', k'') < 0 \forall p'' \in (p_{k'+1}, p_{k''+1})$ and $\forall k'' > k'$, and $T(p_{k'+1}, k'') < 0$ and $T(p_{k'+1}, k'' + 1) < 0 \forall k'' \geq k'$. Similarly, $T(p', k'') > 0 \forall p'' < p'$ and $\forall k'' \leq k'$ by Lemma A4 parts 1 and 2 and hence $T(p'', k') > 0 \forall p'' \in (p_{k'}, p')$, $T(p'', k'') > 0 \forall p'' \in (p_{k'}, p_{k'+1})$ and $\forall k'' < k'$, and $T(p_{k'}, k'' - 1) > 0$ and $T(p_{k'}, k'') > 0 \forall k'' \leq k'$.

If $p' = p_k'$ so that $T(p', k' - 1) \geq 0$ and $T(p', k') \leq 0$, then, by Lemma A4 parts 1 and 2, $T(p'', k'') < 0 \forall p'' \in (p_k', p_{k'+1})$ and $\forall k'' \geq k'$, and $T(p_{k'+1}, k'') < 0$ and $T(p_{k'+1}, k'' + 1) < 0 \forall k'' \geq k'$. Similarly, by Lemma A4 parts 1 and 2, $T(p'', k'' - 1) > 0 \forall p'' \in (p_{k''-1}, p_{k''})$ and $\forall k'' \leq k'$, and $T(p_{k''-1}, k'' - 2) > 0$ and $T(p_{k''-1}, k'' - 1) > 0 \forall k'' \leq k'$.

That $p' \in [p_1, p_n]$ if $T(p', k) = 0$ and $p' \in (p_k, p_{k+1})$ for some $k \in \{0, \ldots, n\}$ or $T(p', k - 1) \geq 0$, $T(p', k) \leq 0$ and $p' = p_k$ for some $k \in \{0, \ldots, n\}$ follows from Lemma A4 part 3, from $T(p, 0) > 0 \forall p < p_1$ and $T(p, n) < 0 \forall p > p_n$. \hfill \square

**Lemma A6.** Platforms $(p^*_i, p^*_f)$ constitute a Nash equilibrium.

**Proof.** It suffices to prove that $\sum_{i \in N} m_i \tilde{D}_i(p\{p, p^*_f\})$ has a (unique) maximum at $p^*_f$ as a function of $p$. Suppose first that there exists $k' \in N$ such that $p^*_f \in (p_{k'}, p_{k'+1})$. In order to recall the coming argument below, let $k'' = k' + 1$. By Lemma A2 parts 2 and 3, $\tilde{D}_i(p^*_i\{p^*_f, p^*_f\})$ exists $\forall i \in N$, and hence, since $p^*_f$ solves $(O_f)$, $\sum_{i \in N} m_i \tilde{D}_i(p^*_i\{p, p^*_f\}) = 0$. By Lemma A2 part 4, $\forall i \in N$ and $\forall p \in \mathbb{R}_+ \setminus \{p^*_f\}$, $\tilde{D}_i(p\{p, p^*_f\})$ exists and $\tilde{D}_i(p\{p, p^*_f\}) < 0$. Moreover, $\forall i \in N$, whenever $p^*_f > 0$,

$$\lim_{p \to (\mathbf{p}^*_f^-)} \tilde{D}_i(p\{p, p^*_f\}) = \frac{2}{1+\delta} B_i'(p^*_f) - \frac{2\delta}{1+\delta} C_i'(p^*_f)$$

$$\lim_{p \to (\mathbf{p}^*_f^+)} \tilde{D}_i(p\{p, p^*_f\}) = \frac{2\delta}{1+\delta} B_i'(p^*_f) - \frac{2}{1+\delta} C_i'(p^*_f)$$

(A22)

so that $\lim_{p \to (\mathbf{p}^*_f^-)} \tilde{D}_i(p\{p, p^*_f\}) > \lim_{p \to (\mathbf{p}^*_f^+)} \tilde{D}_i(p\{p, p^*_f\})$, since the difference of the limits is equal to $\frac{2(1-\delta)}{1+\delta} \left[B_i'(p^*_f) + C_i'(p^*_f)\right] > 0$. Therefore, $\forall i \in N$ and $\forall p \in \mathbb{R}_+ \setminus \{p^*_f\}$,
$\tilde{D}_i(p\{p, p_f^*\}) > \tilde{D}_i(p_f^*\{p, p_f^*\})$ when $p < p_f^*$ and $\tilde{D}_i(p\{p, p_f^*\}) < \tilde{D}_i(p_f^*\{p_f^*, p_f^*\})$ when $p > p_f^*$. Hence, $\forall p \in \mathbb{R}_+ \setminus \{\tilde{p}_f^*|i \in N\}$, $\sum_{i \in N} m_i \tilde{D}_i(p\{p, p_f^*\}) > 0$ when $p < p_f^*$ and $\sum_{i \in N} m_i \tilde{D}_i(p\{p, p_f^*\}) < 0$ when $p > p_f^*$. Now consider $\tilde{p}_f^*$. If $i \geq k''$, so that $p_i > p_f^*$, then $\tilde{p}_f^* > p_f^*$ and, by Lemma A2 part 2, $\lim_{p \rightarrow (\tilde{p}_f^*)^-} \tilde{D}_i(p\{p, p_f^*\}) > 0 = \tilde{D}_i(p_f^*\{\tilde{p}_f^*, p_f^*\}) > \lim_{p \rightarrow (\tilde{p}_f^*)^+} \tilde{D}_i(p\{p, p_f^*\})$. If $i < k'$, so that $p_i < p_f^*$, then $\tilde{p}_f^* < p_f^*$ and, by Lemma A2 part 3, $\lim_{p \rightarrow (\tilde{p}_f^*)^-} \tilde{D}_i(p\{p, p_f^*\}) < 0 = \tilde{D}_i(p_f^*\{\tilde{p}_f^*, p_f^*\}) < \lim_{p \rightarrow (\tilde{p}_f^*)^+} \tilde{D}_i(p\{p, p_f^*\})$. $\sum_{i \in N} m_i \tilde{D}_i(p\{p, p_f^*\})$ is increasing in $p$ on $[0, p_f^*)$ and decreasing on $(p_f^*, \infty)$.

Suppose now that $p_f^* = p_{k^*}$ for some $k^* \in N$. Since $p_f^*$ solves $(O_f)$, we have $\sum_{i \in N} m_i \tilde{D}_i^-(p_f^*\{p_f^*, p_f^*\}) \geq 0$ and $\sum_{i \in N} m_i \tilde{D}_i^+(p_f^*\{p_f^*, p_f^*\}) \leq 0$. The argument in the preceding paragraph applies to all $i \in N \setminus \{k^*\}$ using $k' = k^* - 1$ and $k'' = k^* + 1$. For group $k^*$, by Lemma A2 part 1, $\tilde{D}_{k^*}(p\{p, p_f^*\})$ is continuous and is differentiable except at $p_f^*$, and, by part 4, $\tilde{D}_{k^*}(p\{p, p_f^*\})$ equals

$$\frac{2}{1 + \delta_k} B_{k^*}'(p) - \frac{2\delta_k}{1 + \delta_k} C_{k^*}'(p) > \frac{2}{1 + \delta_k} [B_{k^*}'(p) - C_{k^*}'(p)] > 0 \text{ if } p < p_f^*$$

$$\frac{2\delta_k}{1 + \delta_k} B_{k^*}'(p) - \frac{2}{1 + \delta_k} C_{k^*}'(p) < \frac{2}{1 + \delta_k} [B_{k^*}'(p) - C_{k^*}'(p)] < 0 \text{ if } p > p_f^*$$  \hspace{1cm} (A23)

where the inequalities come from $p_f^* = p_{k^*}$. Therefore, $\sum_{i \in N} m_i \tilde{D}_i(p\{p, p_f^*\})$ is increasing in $p$ on $[0, p_f^*)$ and decreasing on $(p_f^*, \infty)$. \hfill $\square$

By Lemmas A3 and A5, any pair of platforms different than $(p_f^*, p_f^*)$ cannot constitute a Nash equilibrium. By Lemma A6, $(p_f^*, p_f^*)$ constitutes a Nash equilibrium. Lemma A5 shows that $p_f^* \in [p_1, p_n]$. \hfill $\square$

### A1.8 Proof of Proposition 7

From Proposition 4, with rational voters, there exists unique Nash equilibrium in mixed strategies with equilibrium platforms $(p_f^*, p^*_f)$, where $p_f^*$ is the unique solution to $(O_f)$, to $\sum_{i \in N} m_i [B_i'(p) - C_i'(p)] = 0$. Within the application, $B_i'(p) = B(p)' \forall i \in \{P, R\}$, $C_i'(p) = \frac{w_i}{p}$ and $C_{P}'(p) = \frac{w_{P}}{p}$. $B_i'(p)$ = 1 now follows after straightforward algebra. \hfill $\square$

### A1.9 Proof of Proposition 8

From Proposition 5, with focusing voters, there exists unique Nash equilibrium in pure strategies with equilibrium platforms $(p_f^*, p_f^*)$, where $p_f^*$, if $p_f^* \in (p_R, p_P)$, is the unique solution to $(O_{f, 2})$, to $\frac{2m_i}{1 + \delta_1} [\delta_1 B_1'(p) - C_1'(p)] + \frac{2m_i}{1 + \delta_2} [B_2'(p) - \delta_2 C_2'(p)] = 0$. Within the application, group $R$ corresponds to group 1, group $P$ corresponds to group 2, $B_i'(p) = B(p)' \forall i \in \{P, R\}$, $C_{R}'(p) = \frac{w_R}{p}$ and $C_{P}'(p) = \frac{w_P}{p}$. Hence the equation that implicitly
defines \( p_f^i \) reads

\[
\frac{2m_p}{1+\delta_R} \left[ \delta_R B'(p_f^i) - \frac{\eta_R}{\eta} \right] + \frac{2m_p}{1+\delta_P} \left[ B'(p_f^i) - \delta_P \frac{\eta_P}{\eta} \right] = 0. \tag{A24}
\]

Below we use the same equation that, equivalently, reads

\[
B'(p_f^i) = \frac{1 + \delta_P - \frac{m_P\eta_P}{\eta} (1 - \delta_P \delta_R)}{1 + \delta_P - m_R (1 - \delta_P \delta_R)}. \tag{A25}
\]

The \( p_f^i > p_R^i \) condition in part (a) is the condition given in Corollary 2, \( m_2 B'_2(p^*_i) \geq m_1 C'_1(p^*_i) \), adapted to the notation of the application, after using \( B'(p^*_i) = 1 \).

The comparative statics with respect to \( y_R \) and \( y_P \) in part (b) follows from \( \frac{\partial}{\partial y_R} \frac{m_P\eta_P}{\eta} = \frac{-m_P\eta P m R}{\eta^2} < 0 \) and \( \frac{\partial}{\partial y_P} \frac{m_P\eta_P}{\eta} = \frac{m_P - m_1^2 \eta P}{\eta^2} = \frac{m_P - m R \eta P}{\eta^2} > 0 \) used in (A25). When \( y_R \) increases, the numerator on the right hand side of (A25) increases. When \( y_P \) decreases, the numerator on the right hand side of (A25) increases. In both cases, \( p_f^i \) decreases. \( \square \)

### A1.10 Proof of Proposition 9

Fix \( i \in N \) and \( P \). When \( P = P_\ast \), \( \Delta_i^B(P) = \Delta_i^C(P) = 0 \) and voters in group \( i \) have undistorted focus. When \( P \neq P_\ast \), we have \( \Delta_i^B(P) - \Delta_i^C(P) = V_i(P) - V_i(P) \) by Assumption A1. Hence part (a) follows since \( \Delta_i^B(P) > \Delta_i^C(P) \leftrightarrow V_i(P) > V_i(P) \), part (b) follows since \( \Delta_i^B(P) < \Delta_i^C(P) \leftrightarrow V_i(P) < V_i(P) \) and part (c) follows since \( \Delta_i^B(P) = \Delta_i^C(P) \leftrightarrow V_i(P) = V_i(P) \). \( \square \)

### A1.11 Proof of Proposition 10

Fix \( i \in N \), \( P \) and \( P' \) such that \( P' = P \cup \{p'_i\} \).

Consider part (a). Since voters in group \( i \) focus on benefits in \( P \), \( P < P_\ast \) and, by Proposition 9, \( V_i(P) < V_i(P) \) and \( V_i(P) < V_i(P) \) jointly imply \( P < p_i \) and thus \( P > p_i > 0 \), we have \( V_i(P) = V_i(P) \). Moreover, \( V_i(P) = V_i(P) < V_i(P) \) and \( P > p_i \) imply \( P < P \). In summary, \( p_i \in (P, P) \) and \( P \in (P, P) \).

Under \( P' \), voters in group \( i \) focus as follows. When \( p'_i \in P \), then \( P' = P \), \( P' = P_\ast \) and \( V_i(p'_i) < V_i(P) < V_i(P) \) so that voters in group \( i \), by Proposition 9, focus on benefits. When \( p'_i \notin P \), then \( P' = P \) and \( P' = P_\ast \) so that voters in group \( i \) focus on benefits. When \( p'_i \in (P, P) \), then \( P' = P \), \( P' \in (P, P') \) and \( V_i(P') < V_i(P') \) so that voters in group \( i \), by Proposition 9, focus on benefits. When \( p'_i > P \), then \( P' = P \), \( P' = p' \) and \( V_i(P) = V_i(p') \) so that voters in group \( i \), by Proposition 9, have undistorted focus. When \( p'_i > P \), then \( P' = P \), \( P' = p' \) and \( V_i(P) > V_i(p') \) so that voters in group \( i \), by Proposition 9, focus on costs.

Consider part (b). Since voters in group \( i \) focus on costs in \( P \), \( P < P_\ast \) and, by
Proposition 9, \( V_i(P) > V_i(\tilde{P}) \). \( V_i(\mathcal{P}) > V_i(P) \) and \( \mathcal{P} < P \) jointly imply \( \mathcal{P} > p_i \) and thus \( \tilde{P}_i < p_i \). When \( \tilde{P}_i < 0 \), clearly \( \mathcal{P} > \tilde{P}_i \). When \( \tilde{P}_i \geq 0 \), \( V_i(\mathcal{P}) = V_i(\tilde{P}) \) and thus \( V_i(\mathcal{P}) = V_i(\tilde{P}) < V_i(P) \), which together with \( \tilde{P}_i < p_i \) implies \( \mathcal{P} > \tilde{P}_i \). In summary, \( p_i \in (\tilde{P}_i, \mathcal{P}) \) and \( \mathcal{P} \in (\tilde{P}_i, \mathcal{P}) \).

Under \( \mathcal{P}' \), voters in group \( i \) focus as follows. When \( p' > \mathcal{P} \), then \( \mathcal{P}' = \mathcal{P} \), \( \mathcal{P}' = p' \) and \( V_i(p') < V_i(\mathcal{P}) < V_i(P) \) so that voters in group \( i \), by Proposition 9, focus on costs. When \( p' = \mathcal{P} \), then \( \mathcal{P}' = \mathcal{P} \), \( \mathcal{P}' = p' \) so that voters in group \( i \) focus on costs. When \( p' \in (\tilde{P}_i, \mathcal{P}) \), then \( \mathcal{P}' \in \{\mathcal{P}, p'\} \), \( \mathcal{P}' = \mathcal{P} \) and \( V_i(\mathcal{P}) < V_i(\mathcal{P}') \in \{V_i(\mathcal{P}), V_i(p')\} \) so that voters in group \( i \), by Proposition 9, focus on costs. When \( p' = \tilde{P}_i \), so that \( \tilde{P}_i \geq 0 \), then \( \mathcal{P}' = p' \), \( \mathcal{P}' = \mathcal{P} \) and \( V_i(\mathcal{P}) = V_i(p') \) so that voters in group \( i \), by Proposition 9, have undistorted focus. When \( p' < \tilde{P}_i \), so that \( \tilde{P}_i > 0 \), then \( \mathcal{P}' = p' \), \( \mathcal{P}' = \mathcal{P} \) and \( V_i(\mathcal{P}) > V_i(p') \) so that voters in group \( i \), by Proposition 9, focus on benefits.

Consider part (c). Since voters in group \( i \) have undistorted focus in \( \mathcal{P} \), then, by Proposition 9, \( V_i(\mathcal{P}) = V_i(P) \). Since \( \mathcal{P} < P \) and \( V_i(\mathcal{P}) < V_i(P) \), we have \( p_i \in (\mathcal{P}, P) \).

Under \( \mathcal{P}' \), voters in group \( i \) focus as follows. When \( p' < \mathcal{P} \), then \( \mathcal{P}' = p' \), \( \mathcal{P}' = \mathcal{P} \) and \( V_i(p') < V_i(\mathcal{P}) < V_i(\mathcal{P}') \) so that voters in group \( i \), by Proposition 9, focus on benefits. When \( p' \in [\mathcal{P}, P] \), then \( \mathcal{P}' = \mathcal{P} \) and \( \mathcal{P}' = p' \) so that voters in group \( i \) have undistorted focus. When \( p' > \mathcal{P} \), then \( \mathcal{P}' = \mathcal{P} \), \( \mathcal{P}' = p' \) and \( V_i(p') < V_i(\mathcal{P}) = V_i(\mathcal{P}) \) so that voters in group \( i \), by Proposition 9, focus on costs.

\[ \square \]

A1.12 Proof of Proposition 11

Fix \( i \in N, \mathcal{P} \) and \( \mathcal{P}' \) such that \( \delta_i < 1, p_A \in \mathcal{P}, p_B \in \mathcal{P}, p_A > p_B \) and \( \mathcal{P} \cup \{p_C\} = \mathcal{P}' \).

To prove parts (a) and (b), we have

\[
\tilde{V}_i(p_A|\mathcal{P}) - \tilde{V}_i(p_B|\mathcal{P}) - [\tilde{V}_i(p_A|\mathcal{P}') - \tilde{V}_i(p_B|\mathcal{P}')] = c[B_i(p_A) + C_i(p_A) - [B_i(p_B) + C_i(p_B)]]
\]

(A26)

where \( c = \frac{2(1-\delta_i)}{1+\delta_i} \) when voters in group \( i \) focus on benefits in \( \mathcal{P} \) and on costs in \( \mathcal{P}' \), \( c = \frac{1-\delta_i}{1+\delta_i} \) either when voters in group \( i \) focus on benefits in \( \mathcal{P} \) and have undistorted focus in \( \mathcal{P}' \) or when voters in group \( i \) have undistorted focus in \( \mathcal{P} \) and focus on costs in \( \mathcal{P}' \), \( c = -\frac{1-\delta_i}{1+\delta_i} \) either when voters in group \( i \) focus on costs in \( \mathcal{P} \) and have undistorted focus in \( \mathcal{P}' \) or when voters in group \( i \) have undistorted focus in \( \mathcal{P} \) and focus on benefits in \( \mathcal{P}' \), and \( c = -\frac{2(1-\delta_i)}{1+\delta_i} \) when voters in group \( i \) focus on costs in \( \mathcal{P} \) and on benefits in \( \mathcal{P}' \).

Since \( p_A > p_B \), by Assumption A1, \( B_i(p_A) + C_i(p_A) - [B_i(p_B) + C_i(p_B)] > 0 \). Hence, the sign of \( \tilde{V}_i(p_A|\mathcal{P}) - \tilde{V}_i(p_B|\mathcal{P}) - [\tilde{V}_i(p_A|\mathcal{P}') - \tilde{V}_i(p_B|\mathcal{P}')] \) coincides with the sign of \( c \).
To prove part (c), consider choice set \( R \) with \( p_A \in R \) and \( p_B \in R \). Then

\[
\lim_{\delta_i \to 0} \tilde{V}_i(p_A|R) - \tilde{V}_i(p_B|R) = 2 [B_i(p_A) - B_i(p_B)] > 0
\]

\[
\lim_{\delta_i \to 0} \tilde{V}_i(p_A|R) - \tilde{V}_i(p_B|R) = -2 [C_i(p_A) - C_i(p_B)] < 0
\]

(A27)

when voters in group \( i \) focus on benefits and costs respectively, where the inequalities follow from \( p_A > p_B \) by Assumption A1. Since voters in group \( i \) focus on different attributes and have distorted focus both in \( P \) and \( P' \), they either focus on benefits in \( P \) and on cost in \( P' \), or vice versa. Thus, there has to exist \( \delta_i \in (0, 1) \) such that for any \( \delta_i < \delta_i \), \( \tilde{V}_i(p_A|P) - \tilde{V}_i(p_B|P) > 0 \) and \( \tilde{V}_i(p_A|P') - \tilde{V}_i(p_B|P') < 0 \), or vice versa.

To prove part (d), we consider three cases depending on which attribute voters in group \( i \) focus on in \( P \).

Case 1: voters in group \( i \) focus on benefits in \( P \): It suffices to show that, \( \forall p \in P \), \( V_i(p) \geq V_i(pc) \) and \( pc > p \). To see this, if voters in group \( i \) have undistorted focus in \( P' \), then \( \tilde{V}_i(p|P') - \tilde{V}_i(pc|P') = V_i(p) - V_i(pc) \), so that \( \tilde{V}_i(p|P') \geq \tilde{V}_i(pc|P') \) \( \forall p \in P' \) if \( V_i(p) \geq V_i(pc) \) \( \forall p \in P \), and if voters in group \( i \) focus on costs in \( P' \), then

\[
\tilde{V}_i(p|P') - \tilde{V}_i(pc|P')
\]

\[
= \frac{26}{1+\delta_i} B_i(p) - \frac{2}{1+\delta_i} C_i(p) - \left[ \frac{26}{1+\delta_i} B_i(pc) - \frac{2}{1+\delta_i} C_i(pc) \right]
\]

\[
= \frac{2(1-\delta_i)}{1+\delta_i} B_i(p) + \frac{2}{1+\delta_i} V_i(p) - \left[ -\frac{2(1-\delta_i)}{1+\delta_i} B_i(pc) + \frac{2}{1+\delta_i} V_i(pc) \right]
\]

\[
= \frac{2}{1+\delta_i} [V_i(p) - V_i(pc)] + \frac{2(1-\delta_i)}{1+\delta_i} [B_i(pc) - B_i(p)]
\]

(A28)

so that \( \tilde{V}_i(p|P') \geq \tilde{V}_i(pc|P') \) \( \forall p \in P' \) if \( V_i(p) \geq V_i(pc) \) and \( pc > p \) \( \forall p \in P \).

We now show that, \( \forall p \in P \), \( V_i(p) \geq V_i(pc) \) and \( pc > p \), when voters in group \( i \) focus on benefits in \( P \) and do not focus on benefits in \( P' = P \cup \{pc\} \). Since voters in group \( i \) focus on benefits, \( P < P \), so that \( V_i(P) < V_i(P) \) by Proposition 9, and hence, \( P < p_i \). \( P < p_i \) implies \( \tilde{P} > p_i \) so that, since \( V_i(\tilde{P}) = V_i(P) < V_i(P) \), \( P < \tilde{P} \). In summary, \( p_i \in (P, \tilde{P}) \) and \( P_i \in (P, \tilde{P}) \). Moreover, \( V_i(P) = \min_{p \in P} V_i(p) \). To see this, if there exists \( p \in P \) such that \( V_i(P) > V_i(p) \), then \( p > \tilde{P} \) since \( P < p_i \), but then \( p > \tilde{P} \). Therefore, it suffices to show that \( V_i(P) \geq V_i(pc) \) and \( pc > \tilde{P} \). Since voters in group \( i \) do not focus on benefits in \( P' \), by Proposition 10, \( pc \geq \tilde{P} \). Combining \( pc \geq \tilde{P} \) with \( p_i \in (P, \tilde{P}) \) and \( P_i \in (P, \tilde{P}) \), we have \( V_i(P) \geq V_i(pc) \) and \( pc > \tilde{P} \).

Case 2: voters in group \( i \) focus on costs in \( P \): It suffices to show that, \( \forall p \in P \), \( V_i(p) \geq V_i(pc) \) and \( pc < p \). To see this, if voters in group \( i \) have undistorted focus in \( P' \), then \( \tilde{V}_i(p|P') - \tilde{V}_i(pc|P') = V_i(p) - V_i(pc) \), so that \( \tilde{V}_i(p|P') \geq \tilde{V}_i(pc|P') \) \( \forall p \in P' \) if
For derivatives.

\[ V_i(p) = 2 \frac{\delta}{1 + \delta_i} B_i(p) - 2 \frac{\delta}{1 + \delta_i} C_i(p) - \left[ 2 \frac{\delta}{1 + \delta_i} B_i(p) - 2 \frac{\delta}{1 + \delta_i} C_i(p) \right] \]

(A29)

so that \( \tilde{V}_i(p|\mathcal{P}') \geq \tilde{V}_i(p|\mathcal{P}') \) \( \forall p \in \mathcal{P}' \) if \( V_i(p) \geq V_i(p_C) \) and \( p_C < p \) \( \forall p \in \mathcal{P} \).

We now show that, \( \forall p \in \mathcal{P} \), \( V_i(p) \geq V_i(p_C) \) and \( p_C < p \), when voters in group \( i \) focus on costs in \( \mathcal{P} \) and do not focus on costs in \( \mathcal{P}' = \mathcal{P} \cup \{p_C\} \). First note that focus on costs in \( \mathcal{P} \) and not in \( \mathcal{P}' \) implies, by Proposition 10, that \( p_C \leq \tilde{p}_i \) and hence \( \tilde{p}_i \geq 0 \). Since voters in group \( i \) focus on costs, \( \mathcal{P}_i < \mathcal{P} \), so that \( V_i(\mathcal{P}_i) > V_i(\mathcal{P}) \) by Proposition 9, and, hence, \( \tilde{p}_i > p_i \). \( \mathcal{P}_i > \mathcal{P} \) implies \( \tilde{p}_i < p_i \) so that, since \( V_i(\tilde{p}_i) = V_i(\mathcal{P}_i) < V_i(\mathcal{P}) \), \( \mathcal{P}_i > \tilde{p} \). In summary, \( p_i \in (\tilde{p}_i, \mathcal{P}_i) \) and \( p_i \in (\tilde{p}_i, \mathcal{P}_i) \). Moreover, \( V_i(\mathcal{P}) = \min_{p \in \mathcal{P}} V_i(p) \). To see this, if there exists \( p \in \mathcal{P} \) such that \( V_i(\mathcal{P}_i) > V_i(p) \), then \( p < \tilde{p}_i \) since \( \mathcal{P}_i > p_i \), but then \( p < \mathcal{P}_i \). Therefore, it suffices to show that \( V_i(\mathcal{P}) \geq V_i(p_C) \) and \( p_C < p \). We already have \( p_C \leq \tilde{p}_i \). Combining \( p_C \leq \tilde{p}_i \) with \( p_i \in (\tilde{p}_i, \mathcal{P}_i) \) and \( p_i \in (\tilde{p}_i, \mathcal{P}_i) \), we have \( V_i(\mathcal{P}) \geq V_i(p_C) \) and \( p_C < p \).

Case 3: voters in group \( i \) have undistorted focus in \( \mathcal{P} \): If voters in group \( i \) focus on costs in \( \mathcal{P}' \), by (A28), it suffices to show that, \( \forall p \in \mathcal{P} \), \( V_i(p) \geq V_i(p_C) \) and \( p_C > p \). If voters in group \( i \) focus on benefits in \( \mathcal{P}' \), by (A29), it suffices to show that, \( \forall p \in \mathcal{P} \), \( V_i(p) \geq V_i(p_C) \) and \( p_C < p \). Since voters in group \( i \) have undistorted focus in \( \mathcal{P} \), by Proposition 9, \( V_i(\mathcal{P}) = V_i(\mathcal{P}_i) \). Moreover, we have \( p_A > p_B \) and hence \( \mathcal{P}_i < \mathcal{P} \) so that, since \( V_i(\mathcal{P}_i) = V_i(\mathcal{P}_i) \), \( p_i \in (\mathcal{P}_i, \mathcal{P}_i) \). Thus \( V_i(\mathcal{P}_i) = V_i(\mathcal{P}_i) = \min_{p \in \mathcal{P}} V_i(p) \). If voters in group \( i \) focus on costs in \( \mathcal{P}' \), by Proposition 10, \( p_C > \mathcal{P}_i \). Combining \( p_C > \mathcal{P}_i \) with \( p_i \in (\mathcal{P}_i, \mathcal{P}_i) \) implies \( V_i(\mathcal{P}_i) > V_i(p_C) \). If voters in group \( i \) focus on benefits in \( \mathcal{P}' \), by Proposition 10, \( p_C < \mathcal{P}_i \). Combining \( p_C < \mathcal{P}_i \) with \( p_i \in (\mathcal{P}_i, \mathcal{P}_i) \) implies \( V_i(\mathcal{P}_i) > V_i(p_C) \).

\( \square \)

A1.13 Proof of Proposition 12

The proof is complicated by the fact that we need to establish properties of the parties' objective functions given presence of an additional policy. We start the proof by proving four technical Lemmas A7, A8, A9, A10. Recall that \( \hat{D}_i(p|\mathcal{P}) = \frac{\partial}{\partial p} \left[ \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) \right] \), given choice set \( \mathcal{P} \) and policies \( p \in \mathcal{P} \) and \( p' \in \mathcal{P} \). Below, when listing the policies in \( \mathcal{P} \) explicitly, we use the convention to list \( p \) and \( p' \) in the first two positions. That is, given \( \mathcal{P} = \{p, p', p''\} \), \( \hat{D}_i(p|\mathcal{P}) = \left[ \tilde{V}_i(p|\mathcal{P}) - \tilde{V}_i(p'|\mathcal{P}) \right] \), and analogously for derivatives.
**Lemma A7.** Assume $A1, A2, A4$. For all $i \in N$, $\forall p \in \mathbb{R}_+$ and $\forall p' \in \mathbb{R}_+$,

\[
\tilde{D}'_i(p|\{p,p,p'\}) = \begin{cases} 
  v'_{b,i}(p) & \text{if } p < \tilde{p}' \\
  v'_{c,i}(p) & \text{if } p > \tilde{p}' 
\end{cases}
\]

\[
\tilde{D}'_i^-(\tilde{p}'|\{\tilde{p}',\tilde{p}'',p'\}) = \begin{cases} 
  v'_{n,i}(\tilde{p}'') & \text{if } p' < p_i \\
  v'_{b,i}(\tilde{p}'') & \text{if } p' \geq p_i 
\end{cases}
\]

\[
\tilde{D}'_i^+(\tilde{p}'|\{\tilde{p}',\tilde{p}'',p'\}) = \begin{cases} 
  v'_{c,i}(\tilde{p}'') & \text{if } p' \leq p_i \\
  v'_{n,i}(\tilde{p}'') & \text{if } p' > p_i 
\end{cases}
\]

**Proof.** Fix $i \in N$, $p \in \mathbb{R}_+$ and $p' \in \mathbb{R}_+$. Note that

\[
\begin{align*}
\tilde{D}'_i^-(p|\{p,p,p'\}) &= \lim_{h \to 0^-} \tilde{D}_i(p|h\{p,p,p'\}) - \tilde{D}_i(p|\{p,p,p'\}) \\
\tilde{D}'_i^+(p|\{p,p,p'\}) &= \lim_{h \to 0^+} \tilde{D}_i(p|h\{p,p,p'\}) - \tilde{D}_i(p|\{p,p,p'\})
\end{align*}
\]  

(A30)

where $\tilde{D}_i(p|\{p,p,p'\}) = 0$. By Proposition 9, direct verification shows that there exists $h > 0$ such that, $\forall h \in (-\tilde{h}, 0)$, \(\tilde{D}_i(p + h|\{p,p,p'\}) = v_{z,i}(p + h) - v_{z,i}(p)\), where

\[
z = \begin{cases} 
  b & \text{if } (p' < p_i \land p < \tilde{p}'') \text{ or } (p' \geq p_i \land p \leq \tilde{p}'') \\
  n & \text{if } p' < p_i \land p = \tilde{p}' \\
  c & \text{if } (p' < p_i \land p > \tilde{p}'') \text{ or } (p' \geq p_i \land p > \tilde{p}'')
\end{cases}
\]  

(A31)

and such that, $\forall h \in (0, \tilde{h})$, \(\tilde{D}_i(p + h|\{p,p,p'\}) = v_{z,i}(p + h) - v_{z,i}(p)\), where

\[
z = \begin{cases} 
  b & \text{if } (p' > p_i \land p < \tilde{p}'') \text{ or } (p' \leq p_i \land p < \tilde{p}'') \\
  n & \text{if } (p' > p_i \land p = \tilde{p}'') \\
  c & \text{if } (p' > p_i \land p > \tilde{p}'') \text{ or } (p' \leq p_i \land p \geq \tilde{p}'')
\end{cases}
\]  

(A32)

which proves the lemma. \(\square\)

**Lemma A8.** Assume $A1, A2, A4$. For all $i \in N$ and $\forall p \in \mathbb{R}_+$,

\[
\tilde{D}'_i(p|\{p,p\}) = \begin{cases} 
  v'_{b,i}(p) & \text{if } p < p_i \\
  v'_{c,i}(p) & \text{if } p > p_i 
\end{cases}
\]

\[
\tilde{D}'_i^-(p_i|\{p_i,p_i\}) = v'_{b,i}(p_i) \\
\tilde{D}'_i^+(p_i|\{p_i,p_i\}) = v'_{c,i}(p_i)
\]

**Proof.** The first equality follows from Lemma A2 part 4 and the last two equalities follow
Lemma A9. Assume A1, A2, A4. For any $i \in N$, $\forall p \in \mathbb{R}_+$, $\forall p' \in \mathbb{R}_+$ and $\forall p'' \in \mathbb{R}_+$, when $p < p'$,

1. $\tilde{D}_i^{-}(p|\{p, p\}) > \tilde{D}_i^{-}(p'|\{p', p'\})$ and $\tilde{D}_i^{+}(p|\{p, p\}) > \tilde{D}_i^{+}(p'|\{p', p'\})$;

2. $\tilde{D}_i^{-}(p|\{p, p, p''\}) > \tilde{D}_i^{-}(p'|\{p', p', p''\})$ and $\tilde{D}_i^{+}(p|\{p, p, p''\}) > \tilde{D}_i^{+}(p'|\{p', p', p''\})$.

Proof. Fix $i \in N$, $p'' \in \mathbb{R}_+$, $p \in \mathbb{R}_+$ and $p' \in \mathbb{R}_+$ such that $p < p'$. To see part 1, by Lemma A8 we have

$$\tilde{D}_i^{-}(p|\{p, p\}) = \begin{cases} v_{b,i}'(p) & \text{if } p \leq p_i \\ v_{c,i}'(p) & \text{if } p > p_i \end{cases} \quad (A33)$$

and part 1 follows by $v_{b,i}'(p) < 0$, $v_{c,i}'(p) < 0$ and $v_{b,i}'(p) \geq v_{c,i}'(p)$. To see part 2, by Lemma A7 we have

$$\tilde{D}_i^{+}(p|\{p, p, p''\}) = \begin{cases} v_{b,i}'(p) & \text{if } (p'' < p_i \land p < \bar{p}'') \text{ or } (p'' \geq p_i \land p \leq \bar{p}'') \\ v_{n,i}'(p) & \text{if } (p'' < p_i \land p = \bar{p}'') \\ v_{c,i}'(p) & \text{if } (p'' < p_i \land p > \bar{p}'') \text{ or } (p'' \geq p_i \land p > \bar{p}'') \end{cases} \quad (A34)$$

so that part 2 follows by $v_{b,i}'(p) < 0$, $v_{n,i}'(p) < 0$ and $v_{b,i}'(p) \geq v_{n,i}'(p) \geq v_{c,i}'(p)$. \qed

Lemma A10. Assume A1, A2, A4. For all $i \in N$, $\forall p \in \mathbb{R}_+$ and $\forall p' \in \mathbb{R}_+$,

1. $\tilde{D}_i^{-}(p|\{p, p, p\}) = \tilde{D}_i^{-}(p|\{p, p\})$ and $\tilde{D}_i^{+}(p|\{p, p, p\}) = \tilde{D}_i^{+}(p|\{p, p\})$ if $p' = p_i$;

2. if $p' < p_i$, then

$$\tilde{D}_i^{-}(p|\{p, p, p\}) - \tilde{D}_i^{-}(p|\{p, p\}) = \begin{cases} 0 & \text{if } p \notin (p_i, \bar{p}') \\ \geq 0 & \text{if } p \in (p_i, \bar{p}') \end{cases}$$

$$\tilde{D}_i^{+}(p|\{p, p, p\}) - \tilde{D}_i^{+}(p|\{p, p\}) = \begin{cases} 0 & \text{if } p \notin [p_i, \bar{p}]' \\ \geq 0 & \text{if } p \in [p_i, \bar{p}']$$
3. if \( p' > p_i \), then

\[
\tilde{D}_i^-(p\{p, p, p'\}) - \tilde{D}_i^-(p\{p, p\}) = 0 \quad \text{if } p \notin \{p', p_i\} \\
\leq 0 \quad \text{if } p \in \{p', p_i\}
\]

\[
\tilde{D}_i^+(p\{p, p, p'\}) - \tilde{D}_i^+(p\{p, p\}) = 0 \quad \text{if } p \notin \{p', p_i\} \\
\leq 0 \quad \text{if } p \in \{p', p_i\}.
\]

**Proof.** Fix \( i \in N \), \( p \in \mathbb{R}_+ \) and \( p' \in \mathbb{R}_+ \). Part 1 follows from (A33) and (A34) and the fact that \( p' = p_i \) implies \( p_i = \tilde{p} \). To see part 2, the equality when \( p \notin \{p_i, \tilde{p}\} \) and \( p \notin \{p_i, \tilde{p}\} \) respectively follows from (A33) and (A34). The inequality when \( p \in (p_i, \tilde{p}) \) and \( p \in [p_i, \tilde{p}] \) respectively follows from \( \tilde{D}_i^-(p\{p, p\}) = v_{c,i}(p) \) when \( p > p_i \) and \( \tilde{D}_i^+(p\{p, p\}) = v_{b,i}(p) \) when \( p \geq p_i \). To see part 3, the equality when \( p \notin \{\tilde{p}, p_i\} \) and \( p \notin \{p', p_i\} \) respectively follows directly from (A33) and (A34). The inequality when \( p \in (\tilde{p}, p_i) \) and \( p \in [\tilde{p}, p_i) \) respectively follows from \( \tilde{D}_i^-(p\{p, p\}) = v_{b,i}(p) \) when \( p \leq p_i \) and \( \tilde{D}_i^+(p\{p, p\}) = v_{c,i}(p) \) when \( p < p_i \). \( \square \)

We first prove that the electoral competition game has at most one pure strategy Nash equilibrium and that, in any Nash equilibrium in pure strategies, the two competing parties offer the same policy.

Fix \( p_C \in \mathbb{R}_+ \). Suppose profile \((p_A^*, p_B^*)\) constitutes a pure strategy Nash equilibrium in the electoral competition game between parties \( A \) and \( B \) in the presence of an additional party with policy \( p_C \in \mathbb{R}_+ \). Then, by an argument similar to the one used in the proof of Proposition 6, \( \forall p^* \in \{p_A^*, p_B^*\}, \)

\[
\sum_{i \in N} m_i \tilde{D}_i^-(p^*\{p^*, p_C\}) \geq 0 \quad \sum_{i \in N} m_i \tilde{D}_i^+(p^*\{p^*, p_C\}) \leq 0. \quad (O_d)
\]

We now argue that there exists at most one \( p^* \) such that \((O_d)\) holds. Fix \( p^* \) such that \((O_d)\) holds at \( p^* \). First, we claim that \((O_d)\) fails at any \( p \in (p^*, \infty) \). To see this, since \((O_d)\) holds at \( p^* \), by Lemma A9 part 2, \( \forall p \in (p^*, \infty), 0 \geq \sum_{i \in N} m_i \tilde{D}_i^+(p^*\{p^*, p, p_C\}) \)

\[
\geq \sum_{i \in N} m_i \tilde{D}_i^+(p\{p, p, p_C\}) = \sum_{i \in N} \tilde{D}_i^+(p\{p, p, p_C\}) < 0.
\]

Hence, there exists \( p > p^* \) such that, \( \forall p \in (p^*, p) \), we have \( \sum_{i \in N} m_i \tilde{D}_i^+(p\{p, p, p_C\}) = \sum_{i \in N} \tilde{D}_i^+(p\{p, p, p_C\}) < 0 \).

Furthermore, from Lemma A7, \( \forall i \in N \) and \( \forall p \in \mathbb{R}_+ \setminus \{p_C\} \), \( \tilde{D}_i^-(p\{p, p, p_C\}) = \tilde{D}_i^+(p\{p, p, p_C\}) \).

Hence, there exists \( p > p^* \) such that, \( \forall p \in (p^*, p) \), we have \( \sum_{i \in N} m_i \tilde{D}_i^-(p\{p, p, p_C\}) = \sum_{i \in N} \tilde{D}_i^-(p\{p, p, p_C\}) < 0 \).

We now claim that \((O_d)\) fails at any \( p \in [0, p^*) \). To see this, since \((O_d)\) holds at \( p^* \), by Lemma A9 part 2, \( \forall p \in [0, p^*), \sum_{i \in N} m_i \tilde{D}_i^+(p\{p, p, p_C\}) > \sum_{i \in N} m_i \tilde{D}_i^+(p^*\{p^*, p, p_C\}) \geq 0 \).

By Lemma A7 again, there exists \( p < p^* \) such that, \( \forall p \in (p^*, p) \), we have \( \sum_{i \in N} m_i \tilde{D}_i^+(p\{p, p, p_C\}) = \sum_{i \in N} m_i \tilde{D}_i^+(p\{p, p, p_C\}) > 0 \).

Thus, by Lemma A9 part 2, \( \forall p \in [0, p^*), \sum_{i \in N} m_i \tilde{D}_i^+(p\{p, p, p_C\}) > 0 \) so that \((O_d)\) fails
at any \( p \in [0, p^*]\).

We now prove that \( p_d^* \geq p_f^* \) if \( p_f^* \geq p_C \). Recall that \( p_d^* \) is the unique solution to

\[
\sum_{i \in N} m_i \tilde{D}_i^- (p_i(p, p)) \geq 0 \quad \sum_{i \in N} m_i \tilde{D}_i^+ (p_i(p, p)) \leq 0 \quad (A35)
\]

and satisfies \( p_d^* \in [p_1, p_n] \). Suppose first that there exists \( k \in N \setminus \{n\} \) such that \( p_f^* \in (p_k, p_{k+1}) \). Then, from Lemma A8, \( \forall i \in N \) and \( \forall p \in \mathbb{R}_+ \setminus \{p_i\}, \tilde{D}_i^- (p_i(p, p)) = \tilde{D}_i^+ (p_i(p, p)) \) and hence \( \sum_{i \in N} m_i \tilde{D}_i^+ (p_f^*(p_f^*, p_f^*)) = 0 \). Thus, there exists \( \mathbb{R} < p_f^* \) such that, \( \forall p \in (\mathbb{R}, p_f^*) \),

\[
0 < \sum_{i \in N} m_i \tilde{D}_i^+ (p_i(p, p)) = \sum_{i=1}^k m_i v_{c,i}^*(p) + \sum_{i=k+1}^n m_i \tilde{D}_i^+(p_i(p, p)) \quad (A36)
\]

where the first inequality follows by Lemma A9 part 1, the first equality follows by Lemma A8 and the second inequality follows, for the first sum, since Lemma A7 implies \( \tilde{D}_i^+ (p_i(p, p, p_C)) \in \{v_{a,i}^*(p), v_{a,i}^*(p), v_{c,i}^*(p)\} \forall i \in N \) and \( \forall p \in \mathbb{R}_+ \) and we have \( v_{a,i}^*(p) \geq v_{a,i}^*(p) \forall p \in \mathbb{R}_+ \), and, for the second sum, since Lemma A10 part 2 implies \( \tilde{D}_i^+ (p_i(p, p, p_C)) = \tilde{D}_i^+ (p_i(p, p)) \forall i \in N \) such that \( p_C < p_i \) and \( \forall p \in \mathbb{R}_+ \) such that \( p < p_i \). Thus, \( \forall p \in (\mathbb{R}, p_f^*), \sum_{i \in N} m_i \tilde{D}_i^+ (p_i(p, p, p_C)) > 0 \) and hence, by Lemma A9 part 2, \( \sum_{i \in N} m_i \tilde{D}_i^+ (p_i(p, p, p_C)) > 0 \forall p \in [0, p_f^*] \).

Suppose now that there exists \( k \in N \) such that \( p_f^* = p_k \). Then we have that \( \sum_{i \in N} m_i \tilde{D}_i^- (p_f^*(p_f^*, p_f^*)) \geq 0 \) and, by Lemma A9 part 1, there exists \( \mathbb{R} < p_f^* \) such that, \( \forall p \in (\mathbb{R}, p_f^*) \), \( \sum_{i \in N} m_i \tilde{D}_i^- (p_i(p, p)) > 0 \), so that, by Lemma A8 again, we have \( \sum_{i \in N} m_i \tilde{D}_i^+ (p_i(p, p)) > 0 \). Therefore, \( \forall p \in (\mathbb{R}, p_f^*) \),

\[
0 < \sum_{i=1}^{k-1} m_i v_{c,i}^*(p) + \sum_{i=k}^n m_i \tilde{D}_i^+ (p_i(p, p)) \quad (A37)
\]

where the first equality follows by Lemma A8 and the second inequality follows, for the first sum, by similar argument as in the previous paragraph and, for the second sum, since \( \tilde{D}_i^+ (p_i(p, p, p_C)) = \tilde{D}_i^+ (p_i(p, p)) \) either \( \forall i \in N \) such that \( p_C = p_i \) and \( \forall p \in \mathbb{R}_+ \), by Lemma A10 part 1, or \( \forall i \in N \) such that \( p_C < p_i \) and \( \forall p \in \mathbb{R}_+ \) such that \( p < p_i \), by Lemma A10 part 2. Thus, \( \forall p \in (\mathbb{R}, p_f^*), \sum_{i \in N} m_i \tilde{D}_i^+ (p_i(p, p, p_C)) > 0 \) and hence, by Lemma A9 part 2, \( \sum_{i \in N} m_i \tilde{D}_i^+ (p_i(p, p, p_C)) > 0 \forall p \in [0, p_f^*] \). The proof that \( p_d^* \leq p_f^* \)
if \( p_f^* \leq p_C \) is analogous and omitted. \( \square \)

A1.14 Proof of Proposition 7.2

Suppose the policy of the additional party \( p_C > \max_{i \in N} \tilde{0} \). This implies, \( \forall i \in N, p_C > p_i \) and \( V_i(p_C) < V_i(p) \ \forall p \in [0, p_C) \). Consider any pair of policies of parties \( A \) and \( B \), \( (p_A, p_B) \) and choice set \( \mathcal{P} = \{p_A, p_B, p_C\} \). If \( \mathcal{P} = \mathcal{P}_i, p_A = p_B = p_C \), so that voters in any group \( i \) have undistorted focus. If \( \mathcal{P} < \mathcal{P}_i \), we have \( \mathcal{P}_i \leq p_C \) and \( p_C \leq \mathcal{P}_i \). The former implies that, \( \forall i \in N, V_i(\mathcal{P}_i) \geq V_i(p_C) \), and the latter implies that, \( \forall i \in N, V_i(p_C) \geq V_i(\mathcal{P}_i) \). Since \( \mathcal{P} < \mathcal{P}_i, V_i(\mathcal{P}_i) > V_i(\mathcal{P}_i) \ \forall i \in N \), so that voters in all groups focus on costs by Proposition 9.

We now argue that profile of profile \( (p_C, p_C) \) does not constitute a Nash equilibrium in the electoral game. To see this, the payoff of party \( A \) from \( (p_C, p_C) \) is \( \frac{1}{2} \). Consider deviation by party \( A \) to 0. We know that, \( \forall i \in N, V_i(0) > V_i(p_C) \). Moreover, \( \forall i \in N \), the attribute voters in group \( i \) focus on in \( \{0, p_C, p_C\} \) is equal to the attribute voters in group \( i \) focus on in \( \{0, p_C\} \). Therefore, by Proposition 3, \( \forall i \in N, \tilde{V}_i(0, \{0, p_C, p_C\}) > \tilde{V}_i(p_C, \{0, p_C, p_C\}) \) and thus payoff of party \( A \) from the deviation is (strictly) profitable.

Given \( j \in \{A, B\} \), consider policy party \( j \) contests the election with, \( p_j \), and suppose \( p_j \neq p_C \). We argue that the best response of party \( j \) to \( \{A, B\} \setminus \{j\} \) is \( p^*_A \), where \( p^*_A \) is the unique solution to

\[
\max_{p \in \mathbb{R}_+} \sum_{i \in N} \frac{2m_i}{1+m_i} [\delta_i B_i(p) - C_i(p)]. \tag{A38}
\]

This follows from the fact that given \( p_j \neq p_C \) and any \( p_{-j} \), voters in all groups focus on costs. Given this, any solution to \((A38)\) is the best response of party \( j \) to \( p_j \). By Assumption A1, the objective function in \((A38)\) is strictly concave and thus \((A38)\) has unique solution.

Since the best response of any party to the policy of its opposition that differs from \( p_C \) is \( p^*_A \), and given that \((p_C, p_C)\) does not constitute a Nash equilibrium, the electoral competition game admits unique pure strategy Nash equilibrium \((p^*_A, p^*_B)\). \( \square \)

A2 Diminishing Sensitivity

Our focus-weighted utility captures one key feature of sensory perception: human beings' perceptive apparatus is attuned to detect changes in stimuli. This is captured by *ordering*, whereby individuals focus on an attribute when it varies more than other attributes in the choice set. In addition to ordering, Bordalo et al. (2012, 2013a,b, 2015a,b) also assume that individuals perceive stimuli with *diminishing sensitivity*.

In order to consider both ordering and diminishing sensitivity in our basic framework, we can replace Assumption A4 with the following one.
Assumption 5. (\textit{A5}) For a voter in group \(i\), the focus-weighted utility from \(p \in \mathcal{P} = \{p_A, p_B\}\) with \(p_A \neq p_B\) is:

\[
\tilde{V}_i(p | \mathcal{P}) = \begin{cases} 
\frac{2\delta_i}{1+\delta_i} B_i(p) - 2\delta_i C_i(p) & \text{if } \frac{|b_i(p_A) - b_i(p_B)|}{B_i(p_A) + B_i(p_B)} > \frac{|c_i(p_A) - c_i(p_B)|}{C_i(p_A) + C_i(p_B)} \\
\frac{2\delta_i}{1+\delta_i} B_i(p) - 2\delta_i C_i(p) & \text{if } \frac{|b_i(p_A) - b_i(p_B)|}{B_i(p_A) + B_i(p_B)} < \frac{|c_i(p_A) - c_i(p_B)|}{C_i(p_A) + C_i(p_B)} \\
B_i(p) - C_i(p) & \text{if } \frac{|b_i(p_A) - b_i(p_B)|}{B_i(p_A) + B_i(p_B)} = \frac{|c_i(p_A) - c_i(p_B)|}{C_i(p_A) + C_i(p_B)}
\end{cases}
\]

where \(\delta_i \in (0, 1]\) decreases in the severity of focusing.

Consider \(\mathcal{P} = \{p_A, p_B\}\) with \(p_A > p_B > 0\). Assumption A5 implies that voters in group \(i \in N\) focus on benefits if and only if

\[
\frac{B_i(p_A) - B_i(p_B)}{B_i(p_A) + B_i(p_B)} > \frac{C_i(p_A) - C_i(p_B)}{C_i(p_A) + C_i(p_B)}. \tag{A39}
\]

After some algebra, this condition rewrites as

\[
\frac{B_i(p_A)}{C_i(p_A)} > \frac{B_i(p_B)}{C_i(p_B)}. \tag{A40}
\]

It is immediate that this condition is unlikely to hold for \(p_A > p_B\): since \(B_i\) is concave and \(C_i\) is convex, \(B_i'(p)\) is non-increasing while \(C_i'(p)\) is non-decreasing and, hence, \(B_i(p)\) is likely to eventually grow at a lower rate than \(C_i(p)\). More formally, Proposition A1 shows that, under a mild sufficient condition on \(B_i\) and \(C_i\), incorporating diminishing sensitivity à la Bordalo et al. (2012, 2013a,b, 2015a,b) into our salience function means that voters in group \(i\) focus on costs for any pair of (distinct) policies.\textsuperscript{24} Note that, for example, if \(C_i(0) = 0\), that is, when policies have no fixed cost, the condition in the statement of Proposition A1 is satisfied.

Proposition A1. Assume \(A1, A5\) and \(\mathcal{P} = \{p_A, p_B\}, p_A > p_B > 0\). For any \(i \in N\), if \(B_i(0)C_i'(0) \geq B_i'(0)C_i(0)\), then voters in group \(i\) focus on costs.

Proof. Fix \(i \in N, p_A \in \mathbb{R}^+\) and \(p_B \in \mathbb{R}^+\) such that \(p_A > p_B > 0\). By A5, voters in group \(i\) focus on costs if and only if \(\frac{B_i(p_A)}{C_i'(p_A)} < \frac{B_i(p_B)}{C_i'(p_B)}\). Since \(p_A > p_B > 0\), it suffices to show that \(\frac{B_i(p)}{C_i'(p)}\) is decreasing in \(p\) for any \(p \in \mathbb{R}^+\). We have

\[
\frac{\partial}{\partial p} \frac{B_i(p)}{C_i'(p)} = \frac{B_i'(p)C_i(p) - B_i(p)C_i'(p)}{C_i(p)^2} \tag{A41}
\]

so that we need to prove that, \(\forall p \in \mathbb{R}^+, B_i'(p)C_i(p) - B_i(p)C_i'(p) < 0\). We have,

\[
\forall p \in \mathbb{R}^+, \frac{\partial}{\partial p} \left[ B_i'(p)V_i(p) - B_i(p)C_i'(p) \right] = B_i''(p)C_i(p) - B_i(p)C_i''(p) < 0,
\]

where the

\textsuperscript{24}Some formulations of diminishing returns require the denominators in A5 to read \(xB_i(p_A) + yB_i(p_B)\) and \(xC_i(p_A) + yC_i(p_B)\), where \(x\) and \(y\) are positive constants. Proposition A1 continues to hold with this version of Assumption A5 as well since it leaves (A40) unchanged.
inequality follows from Assumption A1. Hence $B_i'(p)C_i(p) - B_i(p)C_i'(p) < 0$ for any $p \in \mathbb{R}_{++}$ if $B_i'(0)C_i(0) - B_i(0)C_i'(0) \leq 0$. □