Efficient Mechanisms for Public Goods with Use Exclusions*

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Abstract

This paper studies "voluntary bargaining agreements" in an environment where preferences over an excludable public good are private information. Unlike the case with a non-excludable public good, there are non-trivial conditions when there is significant provision in a large economy. The provision level converges in probability to a constant, which makes it possible to approximate the optimal solution by a simple fixed fee mechanism, which involves second degree price discrimination if identities are informative about the distribution of preferences. Truth-telling is a dominant strategy in the fixed fee mechanism.

Being able to limit a public goods' consumption does not make it a turn-blue private good. For what, after all, are the true marginal costs of having one extra family tune in on the program. They are literally zero. Why then limit any family which would receive positive pleasure from tuning in on the program from doing so? [Samuelson [24], pp 335]

1 Introduction

Virtually all theory on collective goods considers a "pure" public good, which is non-excludable and non-rival in consumption. Obviously, these properties need not go hand in hand. In fact, it

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is easier to find realistic examples of *excludable public goods*, by which I mean non-rival goods for which it is feasible to exclude consumers from usage. To the extent that copying can be prevented, a recording of a song or anything else that can be stored in digital format is an almost perfect example. Other examples include cable TV, public facilities with controlled access and excess capacity (parks, gyms, zoos, swimming pools, trains), innovations, services by the police and the fire department, and access to databases (see Brito and Oakland [3] for further examples).

Until recently, use exclusions were usually considered irrelevant. Excluding consumers is Pareto inefficient and first best can be implemented with, say, the Lindahl equilibrium mechanism. The few papers studying use exclusions in complete information models therefore had to restrict the ability to price discriminate. This is most explicit in Drèze [10], who argues that individualized Lindahl prices are never observed, and that there are good reasons for this, such as the lack of incentives for truthful revelation of preferences. Imposing a uniform price as a constraint, Drèze shows that some consumers are excluded from usage if the planner faces a binding budget constraint. The reason is that exclusions serve as an imperfect substitute for price discrimination.

Asymmetric information in itself is not sufficient to rationalize uniform prices. Versions of "pivot mechanisms" (Clarke [4], Groves [11], and d'Asperemont and Gerard-Varet [7]) can implement first best for a pure public good. Excluding consumers is then again a pure waste of resources.

This paper considers an environment where first best cannot be implemented. Mechanisms that are considered implementable must, besides incentive compatibility, also be consistent with voluntary participation and be self-financing, constraints that make it natural to think of the setup as a "voluntary bargaining problem". Alone, neither voluntary participation or budget balance suffices to create a role for exclusions: first best can be implemented with pivot mechanisms satisfying either constraint. However, if both constraints are required, first best is unattainable for reasons familiar from Myerson and Satterthwaite [18]. This creates a role for use exclusions. The basic reason is that excluding low types ameliorates the free riding problem by making it less appealing for high types to mimic lower types (Moulin [17] and Dearden [8]).

The main focus of this paper is the provision problem for a large economy. It is then well-known that it is "asymptotically impossible" to provide a non-excludable public good in a voluntary bargaining agreement (Mailath and Postlewaite [16], Rob [22], and Güth and Hellwig [12]), whereas many natural trading institutions for private goods are asymptotically first best efficient.

For an excludable public good the provision level is strictly positive with probability near one in a large economy under non-trivial parametric conditions. Hence, exclusions not only improve efficiency, but changes qualitative results significantly. Excluding customers is the *only* way to extract more than the lowest possible valuation from higher types in a large economy. This suggests that private markets for public goods should be able to operate only if consumers can be excluded, which seems roughly consistent with the array of collective goods actually provided privately.

I consider both a binary and a continuous public good. Interestingly, there is almost no qualitative difference between the two variants of the model. The reason is that the provision level is asymptotically constant in the model with a quantity choice. Adjusting the quantity based on reports makes it possible to provide more when the surplus from provision is high. Providing at higher levels when many agents have high valuations also discourages high types from misreporting. But, both these considerations vanish in a large economy. The average type conditional on being included converges in probability and the gain from using the provision rule to extract more revenues from higher types becomes negligible, since agents correctly perceive to have little influence on the quantity provided (see Al-Najjar and Smorodinsky [1] and Ledyard and Palfrey [14] for discussions of influence). Moreover, agents prefer the expectation for sure to a lottery, an effect which dominates in a large economy. The provision level therefore converges in probability.

A simple fixed fee mechanism is asymptotically optimal. This mechanism sets a user fee for each agent (depending on identity if this provides information about the distribution of the valuation), provides the good if and only if the revenues cover the costs, and allows a consumer to enjoy the good if and only if she is willing to pay the fee. This may be thought of as the obvious arrangement, but solutions to design problems are usually not this simple. Moreover, this is not optimal if the mechanism designer can force participation or if resources from outside the model are available.

The intuition is similar to the intuition for convergence in probability of the level of the public good. The essence of the incentive problem is to discourage high types to mimic low types. Efficient inclusion rules therefore only include agents with valuations above a certain threshold. Types who values the good more than the threshold type are willing to spend more only if it increases the expected consumption. But the average agent has little influence on quantity provided, and transfers from agents that are included are thus almost independent of type in large economies. The efficiency loss of fixed fees is therefore negligible with many consumers.

The fixed fee mechanism is not only simple and almost optimal. Truth-telling is a dominant strategy, budget balance holds ex post, and participation is ex post individually rational, so almost all conceivable desiderata are satisfied. The analysis thus provides some justification for the approach in Drèze [10], Brito and Oakland [3] and others, and generates a limiting model that can be used for more applied questions.

The asymptotic optimality of fixed fees is somewhat akin to the asymptotic efficiency of simple voting schemes in Ledyard and Palfrey [14],[15]. The difference is that they assume equal cost shares. Voluntary participation is then not an issue, so there is no difficulty to raise sufficient revenues. Their mechanism therefore outperforms the constrained optimal mechanism of this paper.

An important example of an excludable public good is a fixed cost in production of a private good. Cornelli [5] studies such fixed costs for a profit maximizing monopolist. Her focus is the opposite, the main insight being that, with a small number of potential customers, it is optimal to sell the good before producing it in order to use the threat of non-production to discriminate between high and low valuation customers. However, she studies a few large economy examples where similar differences between the excludable and the non-excludable case occur as in this paper.

It should be emphasized that results are sensitive to how costs of provision are treated as the economy grows. I assume that costs increase with the number of participants. Literally, the limiting results are therefore for joint changes in the number of participants and the cost of provision. However, as discussed in Section 2.1, this can be viewed as a normalization to guarantee that significant per capita contributions are needed to supply the public good.

2 The Model

2.1 Preferences and Costs

Consider an economy with a set of agents $I = \{1, ..., n\}$ bargaining over the provision of an excludable public good. Agents differ in their valuations for the public good and preferences are private information to the agents. I model this by assuming that the utility of an agent $i \in I$ is given by

$$\theta_i v(y) \Leftrightarrow t_i, \tag{1}$$

where y is the quantity of the public good, $v(\cdot)$ is a continuous, strictly increasing and concave function satisfying $v(0) = 0$, t_i is a transfer of "money" paid by agent i, and θ_i is the type of the agent, representing unobservable taste differences. If the agent is excluded from usage her utility is $\Leftrightarrow t_i$. Preferences over lotteries are of expected utility form.

The type θ_i is distributed over $\Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$ in accordance with distribution F_i, which is equipped with the continuous and strictly positive density f_i. Types are stochastically independent, so F_i is the prior about θ_i for all other agents as well as for the mechanism designer.[1] For brevity Θ is used to denote $\times_i \Theta_i$ and to avoid trivialities I assume that $\overline{\theta}_i > 0$ for all $i \in I$.

I assume that the level of the public good is bounded by $\overline{y}$ and that the *per unit* cost of providing the public good is $C(n)$. Quantity $y \in [0, \overline{y}]$ thus costs $yC(n)$ to provide. Note here that n is the number of *agents* and not the number of *users.* The good is thus fully non-rival.

The rationale for indexing costs of provision by n is that I consider sequences of economies where the number of participants approach infinity. I then assume that $C(n)/n$ has limit $c^* > 0$. The simplest example of such a sequence of cost functions is if $C(n) = c^* n$ for each n, implying that the cost of providing quantity y is $yc^* n$. Obviously, this means that the cost of providing even the tiniest amount tends to infinity as $n \to \infty$. However, all results in the paper are of the form that for a given $\epsilon > 0$ there is a finite N such that the characterization of a finite economy of size $n \geq N$ is within an ϵ distance (usually in terms of per capita surplus) from the limiting economy. For each finite economy the costs of provision goes to zero as the provision level is approaching zero. As long as limiting results are interpreted as an approximation of a finite economy the assumption that $\lim_{n\to\infty} C(n)/n = c^* > 0$ is thus merely a normalization of per capita costs to keep the provision problem "significant" for a large economy (also see Roberts [23]).[2]

Hellwig [13] studies more or less the same model, except that costs are constant as $n \to \infty$. This will in some cases make exclusions irrelevant: if the level of the public good is bounded by $\overline{y}$ and costs are constant, first best can asymptotically be implemented under the (standard) additional assumption that $\underline{\theta}_i \geq 0$ for all i. The reason is that an arbitrarily small per capita tax is sufficient for maximal provision.[3]

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[1] Independence is important. With correlated values there are circumstances where the ex post efficient rule can be implemented in the pure public goods case (Pesendorfer [21]), which eliminates any role for exclusions.

[2] At the cost of some additional complexity it is possible to handle convex cost functions $C(y, n)$. The limiting economy would then be equipped with a convex per capita cost function $c^*(y)$ such that $\lim_{n\to\infty} C(y,n)/n = c^*(y)$. Since the provision level is almost constant in a large economy even in the linear case this generalization is not very interesting: it only adds an additional force in favor of a constant provision level.

[3] If $\lim_{n\to\infty} \sum_i \underline{\theta}_i > C$ this is obvious and first best can be implemented exactly. If $\underline{\theta}_i = 0$ for all agents, the

The model becomes more specific, but the public good can be reinterpreted as a fixed cost for the production of a private good. In this case y is the quality level and $yC(n)$ the fixed cost of setting up a plant that produces quality y. For this setup to map exactly into the specification above the marginal cost of production must be zero and consumption be binary. Positive marginal costs can be "netted out", but the binary demand is a tight restriction since non-linear pricing can be used otherwise, a margin that is impossible to utilize with a public good

2.2 The Design Problem

No matter what mechanism or bargaining institution is set up in the economy, the outcome of this process should determine:

1. the level of the public good,
2. which agents should be allowed to use the public good,
3. how the costs of the public good should be shared.

By appeal to the revelation principle I restrict attention to direct mechanisms for which truth-telling is a Bayesian Nash equilibrium. Allowing randomizations in the inclusion/exclusion decision a direct mechanism can be represented as a triple (y, η, ξ), where $y : \Theta \rightarrow [0, \overline{y}]$ is the *provision rule*, $\eta : \Theta \rightarrow [0,1]^n$ is the *inclusion rule,* and $\xi : \Theta \rightarrow R_+^n$ is the *cost sharing rule.* I adopt the convention that $\eta_i(\theta)$ is the probability that agent i is allowed to consume the public good given the announcement θ. Payoffs are independent of $\eta_i(\theta)$ if $y(\theta) = 0$, so I will for convenience allow agents to be included (to consume nothing) even if the good is not produced. In principle it is also allowed to provide a positive quantity and exclude everybody, but this is obviously suboptimal. The exposition below also assumes that agent i contributes $\xi_i(\theta)$ when θ is announced no matter whether she consumes the public good or not, which is purely for notational convenience.[4]

The expected utility for agent i of type θ_i is $\eta_i(\widehat{\theta})\theta_i v(y(\widehat{\theta})) \Leftrightarrow \xi_i(\widehat{\theta})$, where $\widehat{\theta}$ denotes the vector of reported types. For truth-telling to be an equilibrium in the revelation game it must be *incentive*

result is more subtle. Then the idea is that the necessary per capita contribution approaches zero faster than the probability of being pivotal (see Hellwig [13]).

[4]The alternative is to make transfers conditional on inclusion. Due to risk-neutrality in "money" transfers this leads to the same characterization of incentive feasible provision and inclusion rules.

compatible to announce the true type for every agent i and any possible type realization. I let E_{-i} denote the expectation operator with respect to $\theta_{-i} = (\theta_1, ..., \theta_{i-1}, \theta_{i+1}, .., \theta_n)$ and express this as

$$E_{-i}\left[\eta_i(\theta)\theta_i v(y(\theta)) \Leftrightarrow \xi_i(\theta)\right] \geq E_{-i}\left[\eta_i(\widehat{\theta}_i, \theta_{-i})\theta_i v(y(\widehat{\theta}_i, \theta_{-i})) \Leftrightarrow \xi_i(\widehat{\theta}_i, \theta_{-i})\right] \ \ \forall i \in I, \theta_i, \widehat{\theta}_i \in \Theta_i. \quad (2)$$

Allocations must also be feasible in the sense that the contributions collected cover the costs of provision. It is not a priori obvious whether to impose this *ex post* or *ex ante,* but the seemingly weaker form is the *ex ante feasibility (budget balance) constraint,*

$$E\left(\sum_{i=1}^{n} \xi_i(\theta) \Leftrightarrow y(\theta) C(n)\right) \geq 0, \quad (3)$$

while the *ex post* version requires the argument of (3) to be positive for all θ. I use (3) in my analysis, but by adapting an argument from Cramton, Gibbons and Klemperer [6] one shows that the *ex ante* and *ex post* constraints are equivalent in the sense that if (3) holds, then there is a mechanism satisfying the *ex post* constraints with the same provision and exclusion rules.

Finally, I assume that *voluntary participation* or *individual rationality* must be respected. I assume that agents know their type when they decide whether to participate in the mechanism. Hence, individual rationality is imposed at the interim stage as,

$$E_{-i}\left[\eta_i(\theta)\theta_i v(y(\theta)) \Leftrightarrow \xi_i(\theta)\right] \geq 0 \qquad \forall i \in I, \theta_i \in \Theta_i. \quad (4)$$

Mechanisms that satisfy (2),(3) and (4) are called *incentive feasible.* Note here that the constraints (3) and (4) makes it natural to think of the problem as a "voluntary bargaining problem". The option to walk away from an agreement is in (4), thus capturing the notion of voluntary participation. The constraint (3) means that private consumption must be sacrificed to enjoy the public good, thus making it a bargaining setup rather than a "pure" problem of preference revelation.

2.3 Preliminaries: Characterization of Constrained Optimal Mechanisms

In this section I characterize the incentive feasible mechanisms by combining (2),(3) and (4) into a single integral constraint, which eliminates transfers from the problem. This is a routine adaptation of techniques from Baron and Myerson [2], Myerson [19], Myerson and Satterthwaite [18] and others. The purpose of the exposition is therefore to introduce notation and formal proofs are omitted.[5]

[5] Details are available in Norman [20].

Fix an arbitrary mechanism (y, η, ξ) and let $U_i(\theta_i)$ be the indirect expected utility for an agent of type θ_i. Define $t_i(\theta_i) \equiv E_{-i}\xi_i(\theta)$, which is the expected transfer for agent i of type θ_i given truthful revelation. Similarly, define $v_i(\theta_i) = E_{-i}\eta_i(\theta) v(y(\theta))$, which may be thought of as the "expected consumption benefit" (reduces to a scaling of the expected consumption when v is linear). In a truth-telling equilibrium it must be the case that

$$\begin{aligned} U_i(\theta_i) &= \max_{\widehat{\theta}_i \in \Theta_i} \theta_i E_{-i}\left(\eta_i(\widehat{\theta}_i, \theta_{-i}) v(y(\widehat{\theta}_i, \theta_{-i}))\right) \Leftrightarrow E_{-i}\left(\xi_i(\widehat{\theta}_i, \theta)\right) \\ &= \max_{\widehat{\theta}_i \in \Theta_i} \theta_i v_i(\widehat{\theta}_i) \Leftrightarrow v_i(\widehat{\theta}_i) = \theta_i v_i(\theta_i) \Leftrightarrow t_i(\theta_i) \qquad \forall\, i \in I, \theta_i \in \Theta_i, \end{aligned} \tag{5}$$

where the last equality is a consequence of the revelation principle. Using routine arguments one first shows that.

Lemma 1 *(y, η, ξ) is incentive compatible if and only if $v_i(\theta_i)$ is increasing in θ_i for all i and*

$$U_i(\theta_i) = U_i(\widehat{\theta}_i) + \int_{\widehat{\theta}_i}^{\theta_i} v_i(\theta)\, d\theta \qquad \forall i \in I, \theta_i, \widehat{\theta}_i \in \Theta_i \tag{6}$$

This is a standard result which has nothing to do with the collective nature of the good. Equally routine procedures using the characterization of incentive compatibility in Lemma 1 shows that:

Lemma 2 *Suppose (y, η, ξ) is incentive compatible. Then,*

$$E\xi_i(\theta) = \int_{\theta_1} \dots \int_{\theta_n} \left(\theta_i \Leftrightarrow \frac{(1 \Leftrightarrow F_i(\theta_i))}{f_i(\theta_i)}\right) \eta_i(\theta) v(y(\theta)) \Pi_k f_k(\theta_k)\, d\theta_k \Leftrightarrow U_i(\underline{\theta}_i) \quad \forall i \tag{7}$$

Moreover, if $v_i(\theta_i)$ is increasing in θ_i for all i (7) holds, then (y, η, ξ) is incentive compatible.

Lemma 2 fully characterizes the set of incentive compatible provision and exclusion rules. The final step is to combine incentive compatibility with the participation constraints (4) and the feasibility requirement (3). Expressed in terms of the indirect utility function, the participation constraints (4) are that $U_i(\theta_i) \geq 0$ for all i and θ_i. Since $v_i(\theta_i) = E_{-i}\eta_i(\theta) v(y(\theta)) \geq 0$ we observe from (6) that $U_i(\theta_i)$ is increasing in θ_i, so all participation constraints are fulfilled if and only if $U_i(\underline{\theta}_i) \geq 0$. Feasibility requires that

$$E \sum_i \xi_i(\theta) \geq Ey(\theta) C(n) = \int_{\theta_1} \dots \int_{\theta_n} y(\theta) C(n) \Pi_k f_k(\theta_k)\, d\theta_k, \tag{8}$$

and combining (8) with (7) and the fact that all participation constraints hold if and only if $U_i(\underline{\theta}_i) \geq 0$ for all i one concludes that the set of implementable provision and inclusion rules can be characterized without direct reference to the transfers as all (y, η) that satisfies the condition

$$\int_{\theta_1} \cdots \int_{\theta_n} \left(\sum_i \left(\theta_i \Leftrightarrow \frac{(1 \Leftrightarrow F_i(\theta_i))}{f_i(\theta_i)} \right) \eta_i(\theta) v(y(\theta)) \Leftrightarrow y(\theta) C(n) \right) \Pi_k f_k(\theta_k) d\theta_k \geq 0. \tag{9}$$

The whole discussion in this section can thus be summed up as:

Proposition 1 *There exists a contribution scheme ξ such that (y, η, ξ) satisfies (2),(3) and (4) if and only if v_i is increasing for each i and (9) holds.*

A constrained efficient mechanism is a mechanism designed to maximize social surplus as possible subject to being incentive feasible. Applying Proposition 1 that means that a constrained efficient (y, η) solves

$$\max_{\{y(\cdot), \{\eta(\cdot)\}_{i=1}^n\}} \int_{\theta_1} \cdots \int_{\theta_n} \left(\sum\nolimits_i \eta_i(\theta) v(y(\theta)) \theta_i \Leftrightarrow y(\theta) C(n) \right) \Pi_k f_k(\theta_k) d\theta_k \tag{10}$$

$$\text{s.t } \int_{\theta_1} \cdots \int_{\theta_n} \left(\sum\nolimits_i \eta_i(\theta) v(y(\theta)) \left(\theta_i \Leftrightarrow \tfrac{(1-F_i(\theta_i))}{f_i(\theta_i)} \right) \Leftrightarrow y(\theta) C(n) \right) \Pi_k f_k(\theta_k) d\theta_k \geq 0$$

$$v_i(\theta_i) = E_{-i} v(y(\theta)) \eta_i(\theta) \text{ increasing in } \theta_i \ \forall i \in I \qquad \text{(monotonicity)}$$

$$0 \leq y(\theta) \leq \overline{y} \text{ and } 0 \leq \eta_i(\theta) \leq 1 \ \forall i \in I \qquad \text{(boundary)}$$

For intuition it is useful to note that the only difference between the function in the constraint and the objective function of (10) is that the term $\theta_i \Leftrightarrow (1 \Leftrightarrow F_i(\theta_i)) / f_i(\theta_i)$ replaces θ_i in the constraint. This captures that higher types must have no incentives to mimic types with lower valuations for the public good. That is, there are informational rents for higher types that limit the mechanism designer to extract "the virtual surplus" from each agent and type rather the actual surplus.

The solution to problem (10) can be characterized by standard Lagrangian techniques which gives some useful information for the proofs in the sections to follow. Define

$$S(y, \eta) \equiv \int_{\theta_1} \cdots \int_{\theta_n} \left(\sum\nolimits_i \eta_i(\theta) \theta_i v(y(\theta)) \Leftrightarrow y(\theta) C(n) \right) \Pi_k f_k(\theta_k) d\theta_k \tag{11}$$

$$G(y, \eta) \equiv \int_{\theta_1} \cdots \int_{\theta_n} \left(\sum\nolimits_i \eta_i(\theta) v(y(\theta)) x_i(\theta_i) \Leftrightarrow y(\theta) C(n) \right) \Pi_k f_k(\theta_k) d\theta_k, \tag{12}$$

where

$$x_i(\theta_i) \equiv \theta_i \Leftrightarrow \frac{(1 \Leftrightarrow F_i(\theta_i))}{f_i(\theta_i)}. \tag{13}$$

Ignoring the monotonicity constraint, the Lagrangian for (10) is $L(y, \eta, \lambda) \equiv S(y, \eta) + \lambda G(y, \eta)$. If

$$(y^n, \eta^n) \in \arg\max L(y, \eta, \lambda^n) \tag{14}$$

and $\lambda^n G(y^n, \eta^n) = 0$, then (y^n, η^n) solves (10), given that the solution to this relaxed problem is monotonic.[6] This makes the problem very tractable since (14) is solved by pointwise optimization. The non-obvious part of Lemma 3 below is that it establishes *existence* of a value of the multiplier such that it together with associated maximizer of (14) is a saddle point of the Lagrangian, thereby proving existence of solutions to (10) as well as giving a useful characterization.

Lemma 3 *Suppose that $x_i(\theta_i)$ is weakly increasing. Then, there exists $\lambda^n \geq 0$ such that (y^n, η^n) is an optimal solution to (10) if and only if for almost all $\theta \in \Theta^n$*

$$y^n(\theta) = \arg\max_{y \in [0, \overline{y}]} \sum_{i=1}^{n} v(y) \max[0, \theta_i + \lambda^n x_i(\theta_i)] \Leftrightarrow (1 + \lambda^n) y C(n), \tag{15}$$

and where for all $i \in I$ and all θ such that $y(\theta) > 0$[7]

$$\eta_i^n(\theta) = \begin{cases} 1 & \text{if } \theta_i + \lambda^n x_i(\theta_i) \geq 0 \\ 0 & \text{otherwise} \end{cases}. \tag{16}$$

The proof follows an argument in Hellwig [13] closely and is omitted (details on how to modify the proof with use exclusions available are in Norman [20]). As usual, $x_i(\theta_i)$ is assumed to be increasing to guarantee that the unrestricted solutions are monotonic.

Certain deviations on sets of measure zero from (15) and (16) may result in an optimal solution (monotonicity requirements rule out some but not all deviations from (16)). Such deviations are irrelevant in the sense that neither the expected provision level or the probabilities of inclusion change. I will therefore ignore to add qualifiers about negligible sets in the remainder of the paper.

3 A Binary Public Good

In this section I consider the case when the public good comes as a single indivisible unit. This is a direct extension of the model in Mailath and Postlewaite [16], the only change being that use exclusions are allowed. While a single indivisible unit may seem special, one of the main lessons from the analysis of the general model is that, as the size of the economy increases, the design

[6] To see this, suppose (y', η') is such that $S(y', \eta') > S(y^n, \eta^n)$ and $G(y', \eta') \geq 0$. Then $L(y', \eta', \lambda^n) = S(y', \eta') + \lambda^n G(y', \eta') \dot{>} S(y^n, \eta^n) = L(y^n, \eta^n, \lambda^n)$, contradicting that (y^n, η^n) maximizes the Lagrangian.

[7] The inclusion rule is irrelevant when $y(\theta) = 0$, but it is without loss to assume that inclusions are always in accordance to (16).

problem reduces to a binary problem. The material of this section is therefore of interest also for the understanding of the general model.

Propositions 2 and 3 establish that the probability of provision in the surplus maximizing mechanism converges to either zero or one depending on whether $\lim_{n\to\infty}\sum_i \theta_i^*(1 \Leftrightarrow F_i(\theta_i^*))/n$ is smaller or greater than $\lim_{n\to\infty} C(n)/n$. I then show in Proposition 4 that a "fixed fee mechanism" is asymptotically optimal.

The cost $C(n)$ is now simply the cost of the project, so the feasibility constraint (3) simplifies to $E[\sum_{i=1}^n \xi_i(\theta) \Leftrightarrow \rho(\theta) C(n)] \geq 0$. I write $\rho : \Theta \to [0,1]$ for a generic (random) provision rule, where $\rho(\theta)$ is the probability of providing the public good given announcements θ. The utility of an agent of type θ_i is now $\theta_i \Leftrightarrow t_i$ if the public good is consumed and $\Leftrightarrow t_i$ otherwise, and the expected utility of agent i of type θ_i given announcements $\widehat{\theta}$ is $E_{-i}\rho(\widehat{\theta})\eta_i(\widehat{\theta})\theta_i \Leftrightarrow E_{-i}\xi_i(\widehat{\theta})$, where $\eta_i(\widehat{\theta})$ is the probability of inclusion conditional on provision. The only change in preferences is thus that $\rho(\theta)$ replaces $v(y(\theta))$, so the model is equivalent to a model with a quantity decision where $v(y) = y$ and $\overline{y} = 1$, a case obviously covered by the characterization in Section 2.3. We conclude that a surplus maximizing provision-inclusion rule must solve

$$\max_{\{\rho(\cdot),\{\eta(\cdot)\}_{i=1}^n\}} \int_{\theta_1} \cdots \int_{\theta_n} \left(\sum_i \eta_i(\theta)\theta_i \Leftrightarrow C(n)\right) \rho(\theta)\Pi_k f_k(\theta_k)\, d\theta_k \tag{17}$$

$$\text{s.t } \int_{\theta_1} \cdots \int_{\theta_n} \left(\sum_i \eta_i(\theta) x_i(\theta_i) \Leftrightarrow C(n)\right) \rho(\theta)\Pi_k f_k(\theta_k)\, d\theta_k \geq 0$$

$$\rho_i(\theta_i) = E_{-i}\rho(\theta)\eta_i(\theta) \text{ increasing in } \theta_i \ \forall\, i \in I \qquad \text{(monotonicity)}$$

$$0 \leq \rho(\theta) \leq 1 \text{ and } 0 \leq \eta_i(\theta) \leq 1 \ \forall\, i \in I \qquad \text{(boundary)}$$

and by adapting Lemma 3 we have that associated with the problem (17) there is some $\lambda^n \geq 0$ such that (ρ^n, η^n) is an optimal solution if and only if η_i^n is given by (16) for every i and θ, and

$$\rho^n(\theta) = \begin{cases} 1 & \forall\, \theta \text{ such that } \sum_i \max[0, \theta_i + \lambda^n x_i(\theta_i)] \Leftrightarrow (1+\lambda^n)\rho C(n) \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

3.1 The Surplus Maximizing Mechanism in a Large Economy

I now consider sequences of economies, where agents are added one at a time. The nth economy of a sequence consists the cost of provision $C(n)$ and agents $\{1, ..., n\}$. I let $F^n = (F_1, ..., F_n)$ denote the vector of (independent) distributions over types. An economy of size n is thus given by the pair $(C(n), F^n)$. For ease of exposition I will in the remainder of the paper assume that $x_i(\theta_i)$ is *strictly* increasing for each agent i (weak monotonicity is sufficient). I define

$$\theta_i^* = \arg\max_{\theta_i \in \Theta_i} \theta_i \left(1 \Leftrightarrow F_i\left(\theta_i\right)\right), \tag{19}$$

which may be interpreted as the "monopoly price", the price that a profit maximizing monopolist would charge if restricted to take-it-or-leave-it offers and costs of provision are sunk. Strict monotonicity of x_i implies that θ_i^* is uniquely defined, which is the reason for the assumption.

The first result provides a condition for when it is "asymptotically impossible" to provide the public good

Proposition 2 *Let $\{C(n), F^n\}_{n=1}^{\infty}$ be a sequence of economies, where, for each i, distribution F_i has a density f_i such that $x_i(\theta_i)$ is strictly increasing, where $f_i(\theta_i) > k$ for some $k > 0$, and where $\underline{\theta} \leq \underline{\theta}_i < \overline{\theta}_i \leq \overline{\theta}$ for some uniform bounds $\Leftrightarrow\infty < \underline{\theta} < \overline{\theta} < \infty$. Moreover, suppose there exists c^* such that $\lim_{n\to\infty} C(n)/n = c^*$. Finally, suppose that $\lim_{n\to\infty} \sum_i [1 \Leftrightarrow F_i(\theta_i^*)]\theta_i^*/n < c^*$, for θ_i^* defined in (19). Then, $\lim_{n\to\infty} E\rho^n(\theta) = 0$ for any sequence $\{\rho^n, \eta^n\}_{n=1}^{\infty}$ of feasible solutions to (17).*

A proof is in the appendix, but an informal sketch is instructive. An upper bound on the provision probability is found by maximizing the ex ante probability of provision subject to the constraints in (17). Since no welfare considerations enter this problem the inclusion rule maximizes the expected transfer from each agent, which is achieved by the rule

$$\eta_i^n(\theta) = \begin{cases} 1 & \text{if } \theta_i \geq \theta_i^* \\ 0 & \text{if } \theta_i < \theta_i^* \end{cases}. \tag{20}$$

A crucial implication of (20) is that $\eta_i^n(\theta)\, x_i(\theta_i)$ is stochastically independent of $\eta_j^n(\theta)\, x_j(\theta_j)$ for all i, j, and an application of Chebeshevs inequality shows that

$$\lim_{n\to\infty} \Pr\left[\left|\sum_{i=1}^{n} \frac{\eta_i^n(\theta)\, x_i(\theta_i)}{n} \Leftrightarrow \sum_{i=1}^{n} \frac{[1 \Leftrightarrow F_i(\theta_i^*)]\theta_i^*}{n}\right| \geq \epsilon\right] = 0 \tag{21}$$

for every $\epsilon > 0$. The interpretation of (21) is that the maximal revenue conditional on the project being implemented for sure converges in probability. The rule that maximizes the probability of provision involves a threshold k_n such that the public good is provided if and only if $\sum_i \eta_i^n(\theta)\, x_i(\theta_i)/n \geq k_n$. The main argument in the proof is to show that k_n must be set so that the provision probability goes to zero to satisfy the integral constraint in (17). This involves some work, but the rough idea is if the provision probability stays bounded away from zero, then the expectation of $\sum_i \eta_i^n(\theta) x_i(\theta_i)/n$ conditional on provision approaches the unconditional expectation of $\sum_i \eta_i^n(\theta) x_i(\theta_i)/n$. Expected per capita revenues conditional on provision are thus near $\sum_i [1 \Leftrightarrow F_i(\theta_i^*)]\theta_i^*/n$, which is less than $C(n)/n$ in a large economy.

Next, I turn to the case when $\lim_{n\to\infty} \sum_{i=1}^n [1 \Leftrightarrow F_i(\theta_i^*)]\theta_i^*/n > c^*$, which makes it is necessary to consider problem (17) in more detail. The first observation is that Lemma 3 implies if (η^n, ξ^n) solves (17), then there exists some θ_i^n for every $i \in I$ such that

$$\eta_i^n(\theta) = \begin{cases} 1 & \text{if } \theta_i \geq \theta_i^n \\ 0 & \text{if } \theta_i < \theta_i^n \end{cases}. \tag{22}$$

Hence, the optimal inclusion rule for agent i is again independent of announcements by other agents, which is crucial for the analysis because it makes it possible to conclude that,

$$\lim_{n\to\infty} \Pr\left[\left|\sum_{i=1}^n \frac{\eta_i^n(\theta) x_i(\theta_i)}{n} \Leftrightarrow \sum_{i=1}^n \frac{[1 \Leftrightarrow F_i(\theta_i^n)]\theta_i^n}{n}\right| \geq \epsilon\right] = 0. \tag{23}$$

The interpretation of (23) is that the total revenue converges in probability to $\sum_i [1 \Leftrightarrow F_i(\theta_i^n)]\theta_i^n/n$ if the project is undertaken for sure. Since it is feasible to set the threshold such that $\sum_i [1 \Leftrightarrow F_i(\theta_i^n)]\theta_i^n/n > C(n)/n$ in a large economy (for example by $\theta_i^n = \theta_i^*$ for all i and n) the limiting result switches from asymptotic impossibility to provision with probability 1 in this case.

Proposition 3 *Assume* $\lim_{n\to\infty} \sum_i [1 \Leftrightarrow F_i(\theta_i^*)]\theta_i^*/n > c^*$, *but that all other conditions in Proposition 2 hold. Then,* $\lim_{n\to\infty} E\rho^n(\theta) = 1$ *for any sequence* $\{\rho^n, \eta^n\}_{n=1}^\infty$ *of optimal solutions to (17).*

It cannot be optimal to provide with probability zero since it is possible to use threshold θ_i^* for each i and charge θ_i^* from each type $\theta_i \geq \theta_i^*$. The transfers collected from this mechanism exceeds the provision cost with probability converging to one, so this inclusion rule together with the rule "always provide" is incentive feasible in a large economy and generates a strictly positive surplus. Hence, if the probability of provision does not converge to 1, there must at least be some strictly positive probability of provision in the limit (of any subsequence). This requires that $\lim_{n\to\infty} \sum_i [1 \Leftrightarrow F_i(\theta_i^n)]\theta_i^n/n = c^*$. I then construct an inclusion rule near the hypothetical solution that generates an expected per capita revenue strictly larger than c^*, implying that it is feasible to provide for sure with a very small change in the inclusion rule. Increased provision is good for almost all realizations of θ and the increase in the per capita transfer is negligible, so the deviation generates a strictly larger surplus.

The main difficulty in the proof is that a *strict* budget surplus in the limit for inclusion rules close to the original is needed to guarantee feasibility for a large finite economy. This requires a convexification of the achievable expected revenues, which is achieved by randomizing over thresholds θ_i^n and θ_i^*.

3.2 A Fixed Fee Mechanism is Almost Optimal

Taken together, Proposition 2 and 3 give a sharp characterization of the efficient provision rule in a large economy. However, one may worry that unreasonably complicated transfer schemes are required. To address this, I now consider a very simple mechanisms which is almost optimal.

Definition 1 *The mechanism* (ρ, η, ξ) *is a fixed fee mechanism if the inclusion rule satisfies* $\eta_i(\theta) = 1$ *if and only if* $\theta_i \geq \widetilde{\theta}_i$ *for some* $\widetilde{\theta}_i \in \Theta_i$ *and each* i *and* $\xi_i(\theta) = \eta_i(\theta)\widetilde{\theta}_i$ *for each* i *and each* $\theta \in \Theta$

Thresholds will in general differ across agents, so price discrimination based on observables is allowed and "fixed" is relative type. Not surprisingly however, there are no gains from discriminating agents with the same distributions. That is, (16) shows that $\theta_i^n = \theta_j^n$ whenever $F_i = F_j$.

I now need an additional regularity assumption. I assume that for each i, F_i belongs to a finite set $\mathcal{F}$. This covers among other things the replica-case, in which case there is some k such that $F_i = F_{i+k}$ for all $i \geq 1$.[8] The result is:

Proposition 4 *Suppose that* $\lim_{n\to\infty} \sum_i [1 \Leftrightarrow F_i(\theta_i^*)]\theta_i^*/n \neq c^*$ *and that* $F_i \in \mathcal{F}$ *for every* i, *where* $\mathcal{F}$ *is finite. Then for each* $\epsilon > 0$ *there exists some finite* N *such that for every* $n \geq N$ *there is an incentive feasible fixed fee mechanism satisfying ex post budget balance such that the difference*

[8] The role of the assumption is to assure that the limit of the average expected revenue is strictly in between the limit average expected revenue in the optimal and revenue maximizing mechanisms if thresholds are chosen in between θ_i^n and θ_i^*. This was not an issue for Proposition 3 since randomizations were used, whereas randomizations now are ruled out by definition of a fixed fee mechanism. An alternative sufficient condition is to assume that $\theta_i(1 - F_i(\theta_i))$ is weakly concave. I conjecture that the necessary convexification can be achieved generally by an alternative strategy of proof where agents pay a price that is *either* equal to the surplus maximizing inclusion threshold *or* the revenue maximizing threshold. However, this approach introduces some other difficulties since agents in the two "groups" can not be picked arbitrarily.

in per capita surplus between this mechanism and a surplus maximizing mechanism is less than ϵ. *Moreover, truth-telling is a dominant strategy in the fixed fee mechanism.*

Hence, a fixed fee mechanism can approximate the per capita surplus of an efficient mechanism arbitrarily well. The basic intuition is that in the large economy, the difference between the per capita transfer that the planner can extract using any conceivable mechanism and by using a fixed user fee becomes negligible since the perceived influence for the average agent becomes negligible. Besides being dominant strategy implementable and simple the fixed fee mechanism also satisfies ex post voluntary participation.

Transfer schemes are indeterminate for the surplus maximizing mechanisms, but for large n, the set of θ where the provision rules differ and the set of θ_i where the inclusion rule for i differ is negligible. It therefore makes some sense also to say that an efficient mechanism is close to a fixed fee mechanism.

4 Results for the More General Model

I now return to the more general model where the public good may be provided in any quantity between 0 and $\overline{y}$. In Proposition 5 I establish that the quantity provided converges in probability to its expectation and Proposition 6 shows convergence in probability to a constant under the additional assumption that the sequence of economies is generated by replicating a finite economy. Due to convergence in probability, there is no significant loss to provide the good at a constant level if providing at all. Hence, all results from the binary model can be extended also to this case. Proposition 7 gives conditions similar to Proposition 2 and Proposition 3 for when the provision level is asymptotically zero versus strictly positive and Proposition 8 generalizes the asymptotic optimality of a fixed fee mechanism. Finally, Proposition 9 generalizes the asymptotic impossibility result for a non-excludable public good from Mailath and Postlewaite [16].

An economy is now defined by the primitives $(v, [0, \overline{y}], C(n), F^n)$. The willingness to pay for the public good is now $v(y(\theta))\theta_i$ rather than θ_i, but, like in the binary case, the perceived influence on the provision decision is (on average) negligible in a large economy. The efficient inclusion rule in (16) is of the same form as in the binary case, a threshold rule that is independent of the realized values of θ_{-i} for each agent i. Hence, (23) holds true also in this case, that is $\sum_i \eta_i^n(\theta)\, x_i(\theta_i)/n$

converges in probability to $\sum_i (1 \Leftrightarrow F_i(\theta_i^n))\theta_i^n/n$, suggesting that types $\theta_i \geq \theta_i^n$ should be taxed roughly $\theta_i^n Ev(y(\theta))$ for the right to consume the public good.

The advantages of varying the level of the public good depending on the realization of θ are that higher type realizations generate a higher social surplus for a given level of the public good and that making the level of the public good increasing in the announcements is a way to improve incentives. However, the average type conditional on being above the threshold converges in probability and incentives approach those of a fixed fee mechanism, so both these rationales for variability in y are negligible in large economies. Moreover, agents dislike variation in y if v is strictly concave, which I assume. This suggests that the level of provision in an optimal mechanism converges in probability to the expected provision level. Indeed:

Proposition 5 *Suppose $\{v, [0, \overline{y}], C(n), F^n\}_{n=1}^{\infty}$ is a sequence of economies, where every distribution F_i satisfies the regularity conditions of Proposition 2, v is strictly concave and that there exists c^* such that $C(n)/n = c^*$. Then, $\lim_{n\to\infty} \Pr(|y^n(\theta) \Leftrightarrow E(y^n(\theta))| \geq \epsilon) = 0$ for all $\epsilon > 0$ and any sequence of constrained optimal mechanisms $\{y^n, \eta^n\}$.*

The basic structure of the proof is straightforward. Assuming that the provision level does not converge in probability I consider a sequence of mechanisms which provide the expected level of the public good from the hypothetical optimal mechanism for sure and uses the same inclusion rules. Using convergence in probability of the average actual and virtual surpluses and strict concavity of v, the alternative sequence is shown to be feasible and more desirable if n is large.

Hence, while the level of the public good is endogenous, it is asymptotically a constant. We may thus think of the design problem as a binary one. For any given y the same arguments as in Section 3 tells us that y can be provided for sure (and is desirable to provide for sure) if

$$v(y) \lim_{n\to\infty} \frac{\sum_i [1 \Leftrightarrow F_i(\theta_i^*)]\theta_i^*}{n} > yc^* \tag{24}$$

holds. Clearly, (24) is harder to satisfy the higher is y, so the dividing line between the case where $y^n(\theta)$ converges to zero in probability and where it remains strictly positive can be obtained by taking the limit of (24) as y approaches zero.

There is no sense in which the constrained optimal solutions approach first best efficiency. Except for in trivial cases where the first best efficient level of provision approach zero, $\sum_i (1 \Leftrightarrow$

$F_i(\theta_i^n))\theta_i^n/n$ must converge to zero in order for the surplus to approach first best efficiency: But, then the per capita revenue, which is roughly $v(Ey^n(\theta))\sum_i(1 \Leftrightarrow F_i(\theta_i^n))\theta_i^n/n$ in a large economy, also approaches zero, whereas the per capita costs are bounded away from zero, violating feasibility.

4.1 The Replica Case

It is natural to ask whether the provision level converges in probability to a constant, that is, whether $Ey^n(\theta)$ has a well-defined limit. This analysis introduces some new technical issues. In particular, I must now establish that problems with large n are "near each other" in the sense that if average surplus S is feasible in a particular large economy, then something near S is feasible for all sufficiently large economies. To deal with this I will now restrict attention to sequences of economies that are generated by replicating a given finite economy.

Definition 2 *$\{v,[0,\overline{y}],C(n),F^n\}_{n=1}^{\infty}$ is said to be a sequence of replicas (of an economy with r agents) if there exists r such that $F_{i+r}=F_i$ for all i.*[9]

The analytical advantage of making the restriction to replications of a finite economy is that there is a well-defined "limiting economy". For sequences of replicas one shows:

Proposition 6 *Suppose that $\{v,[0,\overline{y}],C(n),F^n\}_{n=1}^{\infty}$ is a sequence of replicas. Then there exists some $y^* \in [0,\overline{y}]$ such that $\lim_{n\to\infty}\Pr[|y^n(\theta)\Leftrightarrow y^*|\geq\epsilon]=0$ for any $\epsilon>0$ and any sequence of optimal solutions to (10).*

To get a sense of how the proof works, define

$$\widetilde{\eta}_i(\theta_i,\beta)\equiv\begin{cases}1 & \text{if } (1\Leftrightarrow\beta)\theta_i+\beta x_i(\theta_i)\geq 0\\ 0 & \text{otherwise}\end{cases}. \tag{25}$$

for every i and observe that if $\beta^n=\lambda^n/1+\lambda^n$ and λ^n is the multiplier associated with the optimal mechanism, then $\widetilde{\eta}_i(\cdot,\beta^n)$ coincides with the inclusion rule for agent i in the optimal mechanism. Let r be the size of the economy being replicated and define,

$$\Phi(\beta) \equiv \int_{\theta_1}\cdots\int_{\theta_r}\frac{\sum_{i=1}^{r}\widetilde{\eta}_i(\theta_i,\beta)\theta_i}{r}\Pi_k f_k(\theta_k)\,d\theta_k. \tag{26}$$

[9]To avoid introducing additional notation I continue to add agents one at a time in rather than adding r agents at a time.

$$\Psi(\beta) \equiv \int_{\theta_1} \cdots \int_{\theta_r} \frac{\sum_{i=1}^{k} \tilde{\eta}_i(\theta_i,\beta) x_i(\theta_i)}{r} \Pi_k f_k(\theta_k)\, d\theta_k \tag{27}$$

$$Q(\beta) \equiv \arg\max_{y\in[0,\overline{y}]} v(y)\left[(1 \Leftrightarrow \beta)\Phi(\beta) + \beta\Psi(\beta)\right] \Leftrightarrow yc^*. \tag{28}$$

The basic idea is that $v(Q(\beta))\Phi(\beta) \Leftrightarrow Q(\beta)c^*$ is a good approximation of the per capita surplus in a large economy and that the constraint to the programming problem is well approximated with $v(Q(\beta))\Psi(\beta) \Leftrightarrow Q(\beta)c^*$, where β is the limiting value of $\beta^n = \lambda^n/(1+\lambda^n)$ and $\{\lambda^n\}_{n=1}^{\infty}$ is the sequence of Lagrange multipliers associated with a sequence of optimal solutions to the problem.

Using Proposition 6 and doing a Taylor approximation of the constraint we get a characterization of when provision is zero in the limit and when provision is strictly positive.

Proposition 7 *For each i, let θ_i^* be defined by (19). Moreover, suppose that:*

1. $\lim_{n\to\infty} \sum_{i=1}^{n} v'(0)\,\theta_i^*(1 \Leftrightarrow F_i(\theta_i^*))/n < c^*$. *Then* $\lim_{n\to\infty} \Pr[y^n(\theta) \geq \epsilon] = 0$ *for any* $\epsilon > 0$ *and any sequence of feasible solutions to (10)*

2. $\lim_{\epsilon\to 0} \frac{v'(\epsilon)}{\epsilon} = \infty$ *or if* $\sum_{i=1}^{r} v'(0)\,\theta_i^*(1 \Leftrightarrow F_i(\theta_i^*))/n > c^*$, *and* $\{v, [0,\overline{y}], C(n), F^n\}$ *is a sequence of replicas. Then* $y^n(\theta)$ *converges in probability to some* $y^* > 0$ *for any sequence of optimal solutions to (10).*

The result is proved by translating the problem to a binary problem by making a linear approximation of the constraint. This relationship between the binary model and the setup with a quantity decision also allows us to extend the approximate efficiency of fixed fee mechanisms to the setup with a quantity dimension. The arguments are very similar to previous proofs, so I have omitted the formal proof. All there is to verify is that there exists a provision rule that takes on two values, 0 and something close to y^*, that is both feasible and generates a surplus that can be made arbitrary close to that of the optimal mechanism for n large. This is straightforward since a small decrease of y from y^* generates a strict budget surplus by strict concavity of v. One can then appeal directly to the result for the binary model and conclude:

Proposition 8 *Suppose* $\{v, [0,\overline{y}], C(n), F^n\}$ *is a sequence of replicas. Then, for each* $\epsilon > 0$ *there exists a sequence of fixed fee mechanisms and some* N *such that the difference in per capita surplus between the fixed fee mechanism and a constrained optimal mechanism is less than* ϵ *for every* $n \geq N$. *Moreover, truth-telling is a dominant strategy in the fixed fee mechanism.*

4.2 Comparison with the Profit Maximizing Mechanism

Consider first the case with a binary public good. A profit maximizing monopolist then faces the same constraints as in (17). The objective function is expected revenues net of costs of provision, which is the left hand side of the integral constraint of the problem. Hence the profit maximizing provider would maximize the value of this integral of the sum of virtual valuations net of the costs, subject only to the boundary constraints. It is intuitive that the inclusion threshold will be set to θ_i^* (defined in (19)) for each i. Average expected virtual valuations conditional on provision still converge to $\lim_{n\to\infty}\sum_{i=1}^{n}\theta_i^*(1 \Leftrightarrow F_i(\theta_i^*))/n$ and arguments along the same lines as for the surplus maximizing case establish that the ex ante probability of provision converges to zero or one depending on whether $\lim_{n\to\infty}\sum_{i=1}^{n}\theta_i^*(1 \Leftrightarrow F_i(\theta_i^*))/n$ is greater than or smaller than c^*.

Hence, for a large economy the provision rule is almost identical to that of a benevolent planner. However the inclusion threshold θ_i^n in a constrained efficient mechanism is strictly less than θ_i^* for each i. This can be seen from observing that $\theta_i(1 \Leftrightarrow F_i(\theta_i))$ is single-peaked with maximum at θ_i^* and that the optimal mechanism satisfies the feasibility constraint with equality. Budget balance will be achieved by lowering the threshold relative θ_i^* since the social surplus is decreasing in the inclusion thresholds. We conclude that the profit maximizing mechanism is inefficient due to charging too high a price, which excludes too many potential customers from usage, but that the provision decision is not distorted in the binary case.

In the case with a quantity choice, over-exclusions occur for the same reasons as in the binary case: it is always beneficial for a profit maximizer to raise the inclusion threshold from θ_i^n to θ_i^* for any fixed provision rule $y^n(\theta)$. In terms of the provision decision this is like switching from a weighted average of per capita surplus and virtual surplus to virtual surplus alone in the determination of the level of provision in (28). The expected virtual valuation is always below the expected valuation, so in general this leads to under-provision as well as over-exclusions compared with the constrained efficient mechanism. The property that the only inefficiency is that a too high price discourages too many agents to participate is thus an artefact of the binary model and the usual monopoly result (Güth and Hellwig [12]) applies in the more general model.

4.3 Comparison with a Non-Excludable Public Good

It is of course an easy matter to remove the use exclusions from the model. In the binary case this reduces to exactly the model studied in Mailath and Postlewaite [16], but for the case with a quantity dimension to the problem this actually fills a gap in the literature. Without exclusion possibilities the set of feasible provision rules consists of all $y : \Theta \rightarrow R_+$ for which $E_{-i}v(y(\theta))$ is weakly increasing and

$$\int_{\theta_1} \ldots \int_{\theta_n} \left(\sum_i v\left(y\left(\theta\right)\right) \left(\theta_i \Leftrightarrow \frac{\left(1 \Leftrightarrow F_i\left(\theta_i\right)\right)}{f_i\left(\theta_i\right)} \right) \Leftrightarrow y\left(\theta\right) C\left(n\right) \right) \Pi_k f_k\left(\theta_k\right) d\theta_k \geq 0. \tag{29}$$

The following generalization of the asymptotic impossibility result in Mailath and Postlewaite result is then very easy to prove:

Proposition 9 *Suppose* $\lim_{n\rightarrow\infty} \sum_{i=1}^{n} v'\left(0\right) \underline{\theta}/n \Leftrightarrow \lim_{n\rightarrow\infty} C\left(n\right)/n < 0$ *and that* $\{y^n\}$ *is a sequence of incentive feasible provision rules. Then* $Ey^n\left(\theta\right) \rightarrow 0$ *as* $n \rightarrow \infty$.

Making the typical assumption that $\underline{\theta}_i = 0$ is thus sufficient for the expected provision level to be near zero in a large economy. This demonstrates that the asymptotic impossibility for voluntary agreements in a large group to realize large potential gains is not an artefact of the collective decision being a binary choice.

If costs are kept constant as n goes out of bounds, Hellwig [13] shows that the asymptotic properties depend critically on whether the level of the public good is bounded or not. Ex post efficiency can be achieved in the limit if the quantity is bounded. This is because the necessary per capita contribution is of order $1/n$ whereas the pivot probability in a mechanism that provides (the efficient level) if and only if at least m agents announce that they have a valuation above ϵ is of order $1/\sqrt{n}$. Hence it is possible to induce payments of order $1/\sqrt{n}$ from each agent (with valuation above some ϵ), so the aggregate transfers are of order $\sqrt{n}$, which is sufficient to cover cost for a large economy.

For the same reasons, the constrained efficient level of the public good approaches infinity if the efficient level is unbounded. However, the ratio of the social surplus for the constrained efficient outcome and that of the ex post efficient rule converges to zero, thus providing an analogue Proposition 9. In a sense, the assumption that the efficient level of the public good is unbounded is similar to the assumption that $\lim_{n\rightarrow\infty} C\left(n\right)/n = c^* > 0$. Both assumptions ensure that transfers of order $\sqrt{n}$ are insufficient to generate anything close to efficiency.

5 Discussion

5.1 Related Literature

The mechanism design literature on public goods provision is enormous, but the literature on excludable public goods is rather limited. The most obvious exceptions are Cornelli [5], Dearden [8], Hellwig [13], and Moulin [17]. I have already discussed the work of Cornelli and Hellwig elsewhere in the paper, so I will here focus on the other two papers.

Dearden [8] considers a slightly more general model than the binary version of the model considered here. The model allows for crowding, but even without crowding he finds that use exclusions will help overcome the free riding problem. The paper also contains some results for a large economy, but costs are held constant and exclusions are irrelevant for large economies.

Moulin [17] studies exclusions in the context of strategy-proof implementation where a class of "serial cost sharing" mechanisms fully characterizes the set of mechanisms that satisfy voluntary participation, anonymity and strategy proofness (see also Dearden [9]). The mechanism works as follows. Agents are ordered according to their announced demands, the cost to provide the lowest announced demand is divided equally among all agents, the incremental cost between the lowest and the second lowest announced demand are split among the remaining agents (i.e., all agents except the lowest demand), the incremental cost between the second and third lowest are split among the remaining agents and so on. Within this class, the possibility of exclusion from the public good helps in alleviating the free-rider problem for similar reasons as in this paper. It is also notable the mechanism has a similar flavor with the mechanisms considered in this paper in that the lowest type among those included determines the level of the public good.

5.2 Copyright Protection

Taking for granted that the collective good must be handled by a private market arrangement, the sharp contrast between an excludable and a non-excludable public good provides an efficiency rationale for establishing "rights to exclude". Clearly, this logic can be applied to copyright protection, patents, discussions about whether software companies should be forced to publish their code in an intelligible way and many other issues. In particular this seems relevant for discussions about intellectual property rights relating to copyright protection, an issue which seems critical for the music industry due to recent innovations in computer technology.

5.3 Time Consistency

There is a time inconsistency problem with the surplus maximizing mechanism, similar to that of a durable good monopolist. Once contributions are collected a benevolent planner has incentives to make the public good available to everyone, and use exclusions would therefore be non-credible. The conclusion of this would be that, unless the planner can commit, one is back in the dismal outcome of the pure public goods case.

This issue deserves more attention and I will only point out that commitment need not be a crazy assumption. In many cases transfers actually occur *ex post* (think about tollways). Requiring the mechanism to balance the budget presumably means that something bad happens if costs are not covered, such as a budget deficit or even a default on loans to pay for the construction.

6 Concluding Remarks

The basic conclusions from this paper are that it is possible for a "private market" to provide nontrivial amounts of public goods, as long as it is possible to exclude consumers, and that there are essentially no gains to exploit beyond second degree price discrimination.

However, to the extent one thinks that governments are able to compel participation one could argue that the normative recommendation still would be to let governments handle goods of collective nature and avoid exclusions completely by using pivot mechanisms that makes some agents worse off, but are beneficial for the collective. The cheap way to dismiss this argument would of course be to argue that governments seem to have little to do with welfare maximization. A more interesting approach would be to build a model that generates a disadvantage for the government for some more fundamental reason. I believe that one potentially fruitful way to think about this would be to assume that there is an additional informational problem, where some "innovator" has a better idea about the potential value of an excludable public good.

A Appendix: Proofs

To conserve space I often write $\int_{\theta\in\Theta}$ rather than $\int_{\theta_1}\cdots\int_{\theta_n}$ in the integral expressions in the proofs that follow, except when this may create confusion. I also use $dF^n(\theta)$ as shorthand notation for $\Pi_{k=1}^n f_k(\theta)\,d\theta_k$.

A.1 Proposition 2

Proof. Let $\delta = \frac{c^* - \lim_{n\to\infty}\sum_i \theta_i^*[1-F_i(\theta_i^*)]}{2} > 0$. An upper bound on the *ex ante* probability that the good is provided is found by maximizing $\int_{\theta\in\Theta}\rho^n(\theta)\,dF^n(\theta)$ subject to the constraints in (17). It is straightforward to adapt the argument in Lemma 3 to conclude that there exists λ^n such that (ρ^n,η^n) solves the problem if and only if

$$\eta_i^n(\theta) = \begin{cases} 1 & \text{if } \theta_i \geq \theta_i^* \\ 0 & \theta_i < \theta_i^* \end{cases} \qquad \rho^n(\theta) = \begin{cases} 1 & \text{if } 1 + \lambda^n\left(\sum_i \eta_i^n(\theta)\,x_i(\theta_i) \Leftrightarrow C(n)\right) \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{A1}$$

Dividing by $\lambda^n n$ and defining $k_n \equiv C(n)/n \Leftrightarrow 1/n\lambda^n$ we can express the *ex ante* probability of provision in economy n as $E\rho^n(\theta) = \Pr\left[\sum_i \eta_i^n(\theta)\,x_i(\theta_i)/n \geq k_n\right]$.

CASE 1: Suppose $k_n \geq \sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n + \delta$. We note that $E\eta_i^n(\theta)\,x_i(\theta_i) = \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]$ and that $\{\eta_i^n(\theta)\,x_i(\theta_i)\}_{i=1}^n$ is a sequence of n independent random variables, where $\eta_i^n(\theta)\,x_i(\theta_i) \in [0,\overline{\theta}]$ for every i and n. Hence there exists some $\sigma^2 < \infty$ such that the variance of $\eta_i^n(\theta)\,x_i(\theta_i)$ is less than σ^2 for all i and Chebyshevs inequality implies that

$$\Pr\left[\frac{\sum_i \eta_i^n(\theta)\,x_i(\theta_i)}{n} \geq k_n\right] \leq \Pr\left[\left|\sum_i \eta_i^n(\theta)\,x_i(\theta_i) \Leftrightarrow \sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]\right| \geq \delta n\right] \leq \frac{\sigma^2}{\delta^2 n} \tag{A2}$$

CASE 2: Next, suppose $k_n < \sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n + \delta$ and define $H(n) = \{\theta \mid \sum_i \eta_i^n(\theta)\,x_i(\theta_i)/n > \sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n + \delta\}$ and $L(n) = \{\theta \in \Theta \mid k_n \leq \sum_i \eta_i^n(\theta)\,x_i(\theta_i)/n \leq \sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n + \delta\}$. Since $\rho^n(\theta) = 1$ if and only if $\theta \in H(n) \cup L(n)$ the integral constraint evaluated at the optimal rule is

$$\begin{aligned} 0 \quad &\leq \quad \int_{\theta\in H(n)}\left(\frac{\sum_i \eta_i^n(\theta)\,x_i(\theta)}{n} \Leftrightarrow \frac{C(n)}{n}\right)dF^n(\theta) + \int_{\theta\in L(n)}\left(\frac{\sum_i \eta_i^n(\theta)\,x_i(\theta)}{n} \Leftrightarrow \frac{C(n)}{n}\right)dF^n(\theta) \\ &\leq \quad \left(\overline{\theta} \Leftrightarrow C(n)/n\right)\Pr(H(n)) + \left(\sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n + \delta \Leftrightarrow C(n)/n\right)\Pr(L(n)), \end{aligned} \tag{A3}$$

after observing that $\sum_i \eta_i^n(\theta)\,x_i(\theta_i)/n \leq \sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n + \delta$ for all $\theta \in L(n)$ and that $\eta_i^n(\theta)\,x_i(\theta_i) \leq \overline{\theta}$ for all i and θ_i. By the same application of Chebyshevs inequality as in (A2),

$\Pr\left(H\left(n\right)\right) \leq \frac{\sigma^2}{\delta^2 n}$. Moreover, $\lim_{n\to\infty} \sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n \Leftrightarrow C\left(n\right)/n = \Leftrightarrow 2\delta$ and $\lim_{n\to\infty} C\left(n\right)/n = c^*$, so there exists N such that $\sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n + \delta \Leftrightarrow C\left(n\right)/n \leq \Leftrightarrow \frac{\delta}{2}$ and $C\left(n\right)/n \leq c^* \Leftrightarrow \delta$ for $n \geq N$. Combining with (A3) and rearranging shows that $\Pr\left(L\left(n\right)\right) \leq \frac{2\left(\overline{\theta}-c^*+\delta\right)}{\delta}\frac{\sigma^2}{\delta^2 n}$ for $n \geq N$. Hence,

$$E\rho^n\left(\theta\right) = \Pr\left(H\left(n\right)\right) + \Pr\left(L\left(n\right)\right) \leq \frac{\sigma^2}{\delta^2 n}\left(1 + 2\left(\overline{\theta} \Leftrightarrow c^* + \delta\right)/\delta\right). \tag{A4}$$

CASE 1 and **CASE 2** are exhaustive, so (A2) and (A4) implies that there is $N < \infty$ such that

$$E\rho^n\left(\theta\right) \leq \max\left[\frac{\sigma^2}{\delta^2 n}, \frac{\sigma^2}{\delta^2 n}\left(1 + 2\left(\overline{\theta} \Leftrightarrow c^* + \delta\right)/\delta\right)\right] \qquad \forall n \geq N \tag{A5}$$

Since the right hand side of (A5) goes to zero as $n \to \infty$ it follows that $\lim_{n\to\infty} E\rho^n\left(\theta\right) = 0$. ■

A.2 Proposition 3

Lemma A1 *Let (ρ^n, η^n) be a sequence of feasible mechanisms, where for all n and i there exists θ_i^n such that $\eta_i^n\left(\theta_i\right) = 1$ if $\theta_i \geq \theta_i^n$ and $\eta_i^n\left(\theta_i\right) = 0$ for $\theta_i < \theta_i^n$. Moreover, let ρ^n be the best provision rule associated with η^n for every n. Then, 1) $E\rho^n\left(\theta\right) \to 1$ as $n \to \infty$ if $\lim_{n\to\infty} \sum_i \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n > c^*$, 2) $E\rho^n\left(\theta\right) \to 0$ as $n \to \infty$ if $\lim_{n\to\infty} \sum_i \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n < c^*$.*

Proof. (**CASE 1**) For any fixed η^n it is ex post optimal to provide if and only if $\sum_i \eta_i^n\left(\theta_i\right)\theta_i \geq C\left(n\right)$. Since $\theta_i \geq x_i\left(\theta_i\right)$ it follows that $\rho^n\left(\theta\right) = 1$ for all θ such that $\sum_i \eta_i\left(\theta_i\right) x_i\left(\theta_i\right) \geq C\left(n\right)$ and $\rho^n\left(\theta\right) \leq 1$ for all θ such that $\sum_i \eta_i\left(\theta_i\right) x_i\left(\theta_i\right) < C\left(n\right)$, so

$$\begin{aligned}
\int_{\theta\in\Theta}\left(\frac{\sum_i \eta_i^n\left(\theta\right) x_i\left(\theta_i\right) \Leftrightarrow C\left(n\right)}{n}\right)\rho^n\left(\theta\right) dF^n\left(\theta\right) &\geq \int_{\theta\in\Theta}\left(\frac{\sum_i \eta_i^n\left(\theta\right) x_i\left(\theta_i\right) \Leftrightarrow C\left(n\right)}{n}\right) dF^n\left(\theta\right) \\
= \sum_i \frac{\theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]}{n} \Leftrightarrow \frac{C\left(n\right)}{n} &\to \lim_{n\to\infty}\sum_i \frac{\theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]}{n} \Leftrightarrow c^* > 0
\end{aligned} \tag{A6}$$

The integral constraint of (17) thus holds strictly for n large enough, so (ρ^n, η^n) is feasible. Since $\theta_i \geq x_i\left(\theta_i\right)$ it follows that $1 \Leftrightarrow E\rho^n\left(\theta\right) = \Pr[\sum_i \eta_i^n\left(\theta_i\right)\theta_i \leq C\left(n\right)] \leq \Pr[\sum_i \eta_i^n\left(\theta_i\right) x_i\left(\theta_i\right) \leq C\left(n\right)]$. By hypothesis, for each $\epsilon > 0$ there exists N such that $C\left(n\right)/n \leq \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n \Leftrightarrow \epsilon$ for every $n \geq N$. An application of Chebyshevs inequality completes the proof.

(**CASE 2**) The probability of provision for the best rule conditional on η^n is bounded above by the maximal probability of provision conditional on η^n. Although the inclusion rules are different from the ones in Proposition 2, ρ^n still has the same form as in (A1). Replacing δ with $\delta' = 1/2\left(c^* \Leftrightarrow \lim_{n\to\infty}\sum_i \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n\right)$ one can proceed as in the proof of Proposition 2. ■

Proof of Proposition 3. Suppose for contradiction that $\{\rho^n, \eta^n\}$ is a sequence of solutions to (17) such that $E\rho^n(\theta)$ does not converge to 1. Every element of $\{\sum_{i=1}^n \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n\}_{n=1}^{\infty}$ belongs to a compact set, since $\theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)] < \overline{\theta}$, and optimality of the inclusion rules requires that $\theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)] \geq 0$ (see (16)). The provision rule in the solution to (17) must optimize the objective function conditional on the inclusion rule in the optimal solution. Taking a subsequence if necessary, Lemma A1 implies that $\lim_{n\to\infty} \sum_{i=1}^n \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n = c^*$ and that there is some $\delta > 0$ and $N < \infty$ such that $E\rho^n(\theta) < 1 \Leftrightarrow \delta$ if $E\rho^n(\theta)$ fails to converge to unity: $\lim_{n\to\infty} E\rho^n(\theta) = 1$ if $\lim_{n\to\infty} \sum_{i=1}^n \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n > c^*$ and $\lim_{n\to\infty} E\rho^n(\theta) = 0$ if $\lim_{n\to\infty} \sum_{i=1}^n \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n < c^*$. The latter cannot be optimal since the surplus converges to zero and the best mechanism with $\theta_i^n = \theta_i^*$ for every i and n generates a strictly positive surplus. Pick some $\epsilon_1 > 0$ and partition the set of θ for which provisions occur into $H(n) = \{\theta | \rho(\theta) = 1 \text{ and } \sum_i \eta_i^n(\theta)\theta_i/n \geq \sum_i E\eta_i^n(\theta)\theta_i/n + \epsilon_1\}$ and $L(n) = \{\theta | \rho(\theta) = 1 \text{ and } \sum_i \eta_i^n(\theta)\theta_i/n < \sum_i E\eta_i^n(\theta)\theta_i/n + \epsilon_1\}$. Let $S(\rho^n, \eta^n)$ denote the associated per capita surplus in the nth economy, which satisfies

$$\begin{aligned} S(\rho^n, \eta^n) &= \int_{\theta\in H(n)} \frac{[\sum_i \eta_i^n(\theta)\theta_i \Leftrightarrow C(n)]}{n} dF^n(\theta) + \int_{\theta\in L(n)} \frac{[\sum_i \eta_i^n(\theta)\theta_i \Leftrightarrow C(n)]}{n} dF^n(\theta) \\ &\leq \Pr(H(n))\left(\overline{\theta} \Leftrightarrow \frac{C(n)}{n}\right) + \Pr(L(n))\left(\frac{\sum_i E\eta_i^n(\theta)\theta_i}{n} \Leftrightarrow \frac{C(n)}{n} + \epsilon_1\right) \end{aligned} \quad \text{(A7)}$$

An application of Chebyshevs inequality shows that $\lim_{n\to\infty} \Pr(H(n)) = 0$ for any $\epsilon_1 > 0$ and $\Pr(L(n)) \leq \Pr(L(n)) + \Pr(H(n)) \leq 1 \Leftrightarrow \delta$. Hence, for any $\epsilon_1, \epsilon_2 > 0$ there is a finite N such that

$$S(\rho^n, \eta^n) \leq (1 \Leftrightarrow \delta)\left(\frac{\sum_i E\eta_i^n(\theta)\theta_i}{n} \Leftrightarrow \frac{C(n)}{n} + \epsilon_1\right) + \epsilon_2. \quad \text{(A8)}$$

Consider an alternative (sub-) sequence of mechanisms $\{\widehat{\rho}^n, \widehat{\eta}^n\}$ where

$$\widehat{\rho}^n(\theta) = 1 \quad \forall \theta \in \Theta \qquad \widehat{\eta}_i^n(\theta_i) = \begin{cases} 1 & \text{if } \theta_i \geq \theta_i^* \\ 1 \Leftrightarrow \frac{\delta}{2} & \text{if } \theta_i^n \leq \theta_i \leq \theta_i^* \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in I, n, \quad \text{(A9)}$$

and θ_i^* is the threshold that extracts the maximal transfer from the agent defined in (19. The expected per capita surplus from this mechanism is

$$S(\widehat{\rho}^n, \widehat{\eta}^n) = \frac{\frac{(1-\delta)}{2}\sum_i E\eta_i^n(\theta)\theta_i + \frac{\delta}{2}\sum_i \int_{\theta_i^*}^{\overline{\theta}_i} \theta_i f_i(\theta_i)\, d\theta_i \Leftrightarrow C(n)}{n}. \quad \text{(A10)}$$

Together, (A8) and (A10) imply that there exists N' such that $S(\rho^n, \eta^n) < S(\widehat{\rho}^n, \widehat{\eta}^n)$ for every

$n \geq N'$. Moreover, since $\lim_{n\to\infty} \sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n > \lim_{n\to\infty} \sum_i \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n = c^*$,

$$\int_{\theta\in\Theta} \left(\sum_i \frac{\widehat{\eta}_i^n(\theta)\, x_i(\theta_i) \Leftrightarrow C(n)}{n} \right) \widehat{\rho}^n(\theta)\, dF^n(\theta) = \sum_i \frac{E\widehat{\eta}_i^n(\theta)\, x_i(\theta_i)}{n} \Leftrightarrow \frac{C(n)}{n} \tag{A11}$$

$$= \frac{\sum_i \left[\frac{(1-\delta)}{2} \theta_i^n [1 \Leftrightarrow F_i(\theta_i^n)] + \frac{\delta}{2} \theta_i^* [1 \Leftrightarrow F_i(\theta_i^*)] \right]}{n} \Leftrightarrow \frac{C(n)}{n} \to \frac{\delta}{2} \left(\lim_{n\to\infty} \frac{\sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]}{n} \Leftrightarrow c^* \right)$$

as $n \to \infty$. Hence there is N'' such that $(\widehat{\rho}^n, \widehat{\eta}^n)$ is feasible for $n > N''$. We conclude that mechanism (A9) is feasible and better than the hypothetical optimal mechanism for $n \geq \max\{N, N', N''\}$. ■

A.3 Proposition 4

Proof. If $\lim_{n\to\infty} \sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n < c^*$, Proposition 4 is trivial since the per capita surplus converges to zero in the efficient mechanism. Hence, I now assume that $\lim_{n\to\infty} \sum_i \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n > c^*$. Let $\gamma \in (0,1)$ and define $\widetilde{\theta}_i^n \equiv \gamma\theta_i^n + (1 \Leftrightarrow \gamma)\theta_i^*$, where θ_i^n is the threshold from the surplus maximizing mechanism for every i and n and θ_i^* is defined in (19) for every i. Consider a sequence $\left\{ (\widehat{\rho}^n, \widehat{\eta}^n, \widehat{\xi}^n) \right\}$ of fixed fee mechanisms where for each n

$$\widehat{\eta}_i^n(\theta) = \begin{cases} 1 & \text{if } \theta_i \geq \widetilde{\theta}_i^n \\ 0 & \text{otherwise} \end{cases} \qquad \widehat{\rho}^n(\theta) = \begin{cases} 1 & \text{if } \sum_i \widehat{\eta}_i(\theta)\, \widetilde{\theta}_i^n \geq C(n) \\ 0 & \text{otherwise} \end{cases} . \tag{A12}$$

$$\widehat{\xi}_i^n(\theta) = \begin{cases} \widetilde{\theta}_i^n & \text{if } \theta_i \geq \widetilde{\theta}_i^n \text{ and } \sum_j \widehat{\eta}_j(\theta)\, \widetilde{\theta}_j^n \geq C(n) \\ 0 & \text{otherwise} \end{cases}$$

The feasibility constraint (3) holds (also ex post) by construction and is payoff from announcement $(\widehat{\theta}_i, \widehat{\theta}_{-i})$ is

$$\widehat{\rho}^n(\widehat{\theta}_i, \widehat{\theta}_{-i})\widehat{\eta}_i^n(\widehat{\theta}_i, \widehat{\theta}_{-i}) \left(\theta_i \Leftrightarrow \widehat{\xi}_i^n(\widehat{\theta}_i, \widehat{\theta}_{-i}) \right) = \begin{cases} \widehat{\rho}^n(\widehat{\theta}_i, \widehat{\theta}_{-i}) \left(\theta_i \Leftrightarrow \widehat{\theta}_i \right) & \text{if } \widehat{\theta}_i \geq \widetilde{\theta}_i^n \\ 0 & \text{otherwise} \end{cases} . \tag{A13}$$

The participation constraint (4) is thus satisfied and truth-telling is a dominant strategy and therefore also incentive compatible in the Bayesian sense. Mechanism $(\widehat{\rho}^n, \widehat{\eta}^n, \widehat{\xi}^n)$ is thus incentive feasible for every n, with probability of provision given by $\Pr\left[\sum_i \widehat{\eta}_i^n(\theta)\, \widetilde{\theta}_i^n/n \geq C(n)/n \right]$. For each n let λ^n be the Lagrange multiplier associated with the surplus maximizing mechanism.

CASE 1: Suppose that (taking a subsequence if necessary) $\lim_{n\to\infty} \lambda^n/(1+\lambda^n) = 1$. Then we see from (16) that $\lim_{n\to\infty} \theta_i^n = \theta_i^*$ for every i, implying that $\lim_{n\to\infty} \widetilde{\theta}_i^n = \theta_i^*$ for any $\gamma \in [0,1]$. Hence, $\lim_{n\to\infty} \sum_i \widetilde{\theta}_i^n[1 \Leftrightarrow F_i(\widetilde{\theta}_i^n)]/n = \lim_{n\to\infty} \theta_i^*[1 \Leftrightarrow F_i(\theta_i^*)]/n > c^*$ and there exists $\epsilon > 0$ and finite N such that $\sum_i \widetilde{\theta}_i^n[1 \Leftrightarrow F_i(\widetilde{\theta}_i^n)]/n \Leftrightarrow C(n)/n \geq \epsilon$ for all $n \geq N$. The probability of provision for mechanism (A12) is $\Pr\left[\sum_i \widehat{\eta}_i^n(\theta)\widetilde{\theta}_i^n/n \geq C(n)/n\right]$ and $E\widehat{\eta}_i^n(\theta)\widetilde{\theta}_i^n = \widetilde{\theta}_i^n[1 \Leftrightarrow F_i(\widetilde{\theta}_i^n)]$, so another application of Chebyshevs inequality implies that

$$\begin{aligned} 1 \Leftrightarrow E\widehat{\rho}^n(\theta) &= \Pr\left[\sum_i \widehat{\eta}_i(\theta)\widetilde{\theta}_i^n \leq C(n)\right] \leq \Pr\left[\frac{\sum_i \widehat{\eta}_i(\theta)\widetilde{\theta}_i^n}{n} \Leftrightarrow \frac{\sum_i \widetilde{\theta}_i^n(1 \Leftrightarrow F_i(\widetilde{\theta}_i^n))}{n} \leq \Leftrightarrow \epsilon\right] \\ &\leq \Pr\left[\left|\sum_i \widehat{\eta}_i(\theta)\widetilde{\theta}_i^n \Leftrightarrow \sum_i \widetilde{\theta}_i^n\left(1 \Leftrightarrow F_i(\widetilde{\theta}_i^n)\right)\right| \leq \epsilon n\right] \leq \frac{\sigma^2}{\epsilon^2 n}. \end{aligned} \tag{A14}$$

Thus, $\lim_{n\to\infty} E\widehat{\rho}^n(\theta) = 1$. Since $\lim_{n\to\infty}\widetilde{\theta}_i^n = \lim_{n\to\infty}\theta_i^n = \theta_i^*$ it is easy to check that the per capita surplus in the fixed fee mechanism approaches that of the optimal mechanism for any γ.

CASE 2: Suppose instead (taking a subsequence if necessary) that $\lim_{n\to\infty}\lambda^n/(1+\lambda^n) = \beta < 1$. With some abuse of notation, let $\theta_i(\lambda)$ be the inclusion threshold from (16) associated with multiplier λ. We notice that $\lim_{n\to\infty}\theta_i^n = \theta_i(\lambda) < \theta_i^*$. Therefore, $\lim_{n\to\infty}\sum_i \int_{\theta_i^n}^{\overline{\theta}_i}\theta_i f_i(\theta_i)\,d\theta_i/n = \lim_{n\to\infty}\sum_i \int_{\theta_i^n(\lambda)}^{\overline{\theta}_i}\theta_i f_i(\theta_i)\,d\theta_i/n$ and $\lim_{n\to\infty}\sum_i \int_{\gamma\theta_i^n+(1-\gamma)\theta_i^*}^{\overline{\theta}_i}\theta_i f_i(\theta_i)\,d\theta_i/n = \lim_{n\to\infty}\sum_i \int_{\gamma\theta_i^n(\lambda)+(1-\gamma)\theta_i^*}^{\overline{\theta}_i}\theta_i f_i(\theta_i)\,d\theta_i/n$ for any γ. Moreover, the second limit converges to the first as $\gamma \to 1$. Together, this implies that for any $\epsilon > 0$ there exists $\gamma < 1$ and $N < \infty$ such that

$$\sum_i \int_{\theta_i^n}^{\overline{\theta}_i}\theta_i f_i(\theta_i)\,d\theta_i/n \Leftrightarrow \sum_i \int_{\gamma\theta_i^n+(1-\gamma)\theta_i^*}^{\overline{\theta}_i}\theta_i f_i(\theta_i)\,d\theta_i/n < \epsilon/2. \tag{A15}$$

for all $n \geq N$. Next, $\theta_i^n = \theta_j^n$ and $\theta_i^* = \theta_j^*$ for any (i,j) such that $F_i = F_j$ and $F_i \in \mathcal{F}$ for any i, where $\mathcal{F}$ is finite. Hence there exists $\delta > 0$ and $N' < \infty$ such that $(\widetilde{\theta}_i^n \Leftrightarrow \theta_i^n) = (1 \Leftrightarrow \gamma)(\theta_i^* \Leftrightarrow \theta_i^n) > \delta$ for $n \geq N'$. Since $\theta_i(1 \Leftrightarrow F_i(\theta_i))$ is strictly increasing on $[\underline{\theta}_i, \theta_i^*]$ this in turn implies that there is $\epsilon_i > 0$ such that $\widetilde{\theta}_i^n[1 \Leftrightarrow F_i(\widetilde{\theta}_i^n)] \geq \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)] + 2\epsilon_i$. Because $\{\epsilon_1, ..., \epsilon_n\}$ takes on at most as many values as the cardinality of $\mathcal{F}$ this in turn establishes existence of some $\widetilde{\epsilon} > 0$ such that $\widetilde{\epsilon} \leq \epsilon_i$ for all i. Lemma A1 implies that $\lim_{n\to\infty}\sum_i \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n \geq c^*$ since otherwise there is no provision in the limit. Hence there exists N'' such that $\sum_i \widetilde{\theta}_i^n[1 \Leftrightarrow F_i(\widetilde{\theta}_i^n)]/n \geq C(n)/n + \widetilde{\epsilon}$ for every $n \geq N''$. Applying (A14) again we conclude that $\lim_{n\to\infty} E\widehat{\rho}^n(\theta) = 1$ also along such subsequence. Let $S(\rho^n, \eta^n)$ and $S(\widehat{\rho}^n, \widehat{\eta}^n)$ be the per capita surplus generated by the optimal and fixed fee mechanisms respectively. Since the probability of provision converges to one in both mechanisms it is easy to verify that there exists N''' such that

$$S(\rho^n, \eta^n) \Leftrightarrow S(\widehat{\rho}^n, \widehat{\eta}^n) \leq \sum_i \int_{\theta_i^n}^{\overline{\theta}_i}\theta_i f_i(\theta_i)\,d\theta_i/n \Leftrightarrow \sum_i \int_{\gamma\theta_i^n+(1-\gamma)\theta_i^*}^{\overline{\theta}_i}\theta_i f_i(\theta_i)\,d\theta_i/n + \epsilon/2, \tag{A16}$$

and using (A15) this implies that $S(\rho^n, \eta^n) \Leftrightarrow S(\widehat{\rho}^n, \widehat{\eta}^n) < \epsilon$. Since ϵ was arbitrary the result follows. ■

A.4 Proposition 5

For the proofs of Proposition 5,6 and 7 it is convenient to define the functions

$$G^n(y^n,\eta^n) \equiv \int_{\theta_1}\cdots\int_{\theta_n}\left(v(y^n(\theta))\frac{\sum_i \eta_i^n(\theta)\,x_i(\theta_i)}{n} \Leftrightarrow \frac{y^n(\theta)\,C(n)}{n}\right)\Pi_k f_k(\theta_k)\,d\theta_k \qquad \text{(A17)}$$

$$S^n(y^n,\eta^n) \equiv \int_{\theta_1}\cdots\int_{\theta_n}\left(v(y^n(\theta))\frac{\sum_i \eta_i(\theta)\,\theta_i v(y^n(\theta))}{n} \Leftrightarrow \frac{y^n(\theta)\,C(n)}{n}\right)\Pi_k f_k(\theta_k)\,d\theta_k. \qquad \text{(A18)}$$

Multiplying everything with $1/n$ doesn't change the programming problem, so $S^n(y^n,\eta^n)$ may be taken as the objective function to (17) and the constraint is satisfied if and only if $G^n(y^n,\eta^n) \geq 0$.

Lemma A2 *Let $\{y^n,\eta_i^n\}_{n=1}^{\infty}$ be a sequence of incentive feasible mechanisms, where for each n and $i \leq n$, η_i^n is a threshold rule with inclusion threshold θ_i^n (independent of θ_{-i}). Then, for each $\epsilon > 0$ there exists some finite N such that*

$$v(Ey^n(\theta))\frac{\sum_i \theta_i^n(1 \Leftrightarrow F_i(\theta_i^n))}{n} \geq Ey^n(\theta)\,\frac{C(n)}{n} \Leftrightarrow \epsilon \qquad \forall n \geq N. \qquad \text{(A19)}$$

Proof. Fix $\epsilon > 0$ and let $H(n) = \{\theta | \sum_i \eta_i^n(\theta)\,x_i(\theta_i)/n \Leftrightarrow \sum_i \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n > \delta\}$, where $\delta = \epsilon/2v(\overline{y})(\overline{\theta}+1) > 0$. Decompose $G^n(y^n,\eta^n)$ in (A17) as

$$\begin{aligned} G^n(y^n,\eta^n) &= \int_{\theta\in H(n)} v(y^n(\theta))\frac{\sum_i \eta_i(\theta)\,x_i(\theta_i)}{n}dF^n(\theta) \qquad \text{(A20)} \\ &\quad + \int_{\theta\in\Theta\backslash H(n)} v(y^n(\theta))\frac{\sum_i \eta_i(\theta)\,x_i(\theta_i)}{n}dF^n(\theta) \Leftrightarrow \frac{C(n)}{n}Ey^n(\theta) \end{aligned}$$

Observing that $\theta \in \Theta\backslash H(n)$ implies that $\sum_i \eta_i(\theta)\,x_i(\theta_i)/n \leq \sum_i \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]/n + \delta$ we have that the second term in the right hand side of (A20) satisfies

$$\begin{aligned} &\int_{\theta\in\Theta\backslash H(n)} v(y^n(\theta))\frac{\sum_i \eta_i(\theta)\,x_i(\theta_i)}{n}dF^n(\theta) \qquad \text{(A21)} \\ \leq\ & \left(\frac{\sum_i \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]}{n} + \delta\right)\int_{\theta\in\Theta\backslash H(n)} v(y^n(\theta))dF^n(\theta) \\ \leq\ & \left(\frac{\sum_i \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]}{n} + \delta\right)\int_{\theta\in\Theta} v(y^n(\theta))dF^n(\theta) \leq v(Ey^n(\theta))\left(\frac{\sum_i \theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]}{n} + \delta\right) \end{aligned}$$

where the last inequality comes from concavity of v. Since $\delta > \epsilon/2v(\overline{y})$ and v is increasing, so $\delta v(Ey^n(\theta)) \leq \delta v(\overline{y}) \leq \frac{\epsilon}{2}$. Combined with (A21) this implies that

$$\int_{\theta\in\Theta\backslash H(n)} v(y^n(\theta))\frac{\sum_i \eta_i(\theta)\,x_i(\theta_i)}{n}dF^n(\theta) \leq v(Ey^n(\theta))\frac{\theta_i^n[1 \Leftrightarrow F_i(\theta_i^n)]}{n} + \frac{\epsilon}{2} \qquad \text{(A22)}$$

Finally, we note that $v(y^n(\theta)) \leq v(\overline{y})$ for every $\theta \in \Theta$ and $\eta_i(\theta)\, x_i(\theta_i) \leq \overline{\theta}$ for every i, so

$$\int_{\theta \in H(n)} v(y^n(\theta)) \frac{\sum_i \eta_i(\theta)\, x_i(\theta_i)}{n} dF^n(\theta) \ \leq v(\overline{y})\overline{\theta} \int_{\theta \in H(n)} dF^n(\theta) = v(\overline{y})\overline{\theta}\, \Pr(H(n)), \tag{A23}$$

where $\Pr(H(n)) = \Pr\left[\sum_i \eta_i^n(\theta)\, x_i(\theta_i)/n \Leftrightarrow \sum_i \theta_i^n [1 \Leftrightarrow F_i(\theta_i^n)]/n > \delta\right]$. By Chebyshevs inequality there exists N such that $\Pr(H(n)) < \delta$ for $n \geq N$ and since $\delta < \epsilon / 2v(\overline{y})\overline{\theta}$ we have that

$$\int_{\theta \in H(n)} v(y^n(\theta)) \frac{\sum_i \eta_i(\theta)\, x_i(\theta_i)}{n} dF^n(\theta) < \frac{\epsilon}{2} \tag{A24}$$

for every $n \geq N$. The conclusion follows by substituting (A22) and (A24) back into (A20) and noting that $G^n(y^n, \eta^n) \leq 0$ for feasibility. ■

Lemma A3 *Fix any $\epsilon > 0$ and let $\{\eta^n\}_{n=1}^{\infty}$ be a sequence of threshold inclusion rules. Then there exists some finite N such that $S^n(y^n, \eta^n) > S^n(y'^n, \eta^n)$ for every $n \geq N$ and any provision rules y^n, y'^n satisfying*

$$\frac{\sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i)\, d\theta_i}{n} \left(Ev(y^n(\theta)) \Leftrightarrow Ev(y'^n(\theta))\right) \Leftrightarrow \left(Ey^n(\theta) \Leftrightarrow Ey'^n(\theta)\right) \frac{C(n)}{n} > \epsilon. \tag{A25}$$

Proof. Fix $\epsilon > 0$ and define $H(n) = \left\{\theta \left| \sum_i \eta_i^n(\theta)\, \theta_i / n \geq \sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i)\, d\theta_i / n \Leftrightarrow \delta \right.\right\}$, where $\delta = \epsilon / 2v(\overline{y})(\overline{\theta} + 1)$. For mechanism (y^n, η^n) the per capita surplus can be decomposed as

$$\begin{aligned} S^n(y^n, \eta^n) &= \int_{\theta \in H(n)} \frac{\sum_i \eta_i^n(\theta)\, \theta_i v(y^n(\theta))}{n} dF^n(\theta) \\ &\quad + \int_{\theta \in \Theta \setminus H(n)} \frac{\sum_i \eta_i^n(\theta)\, \theta_i v(y^n(\theta))}{n} dF^n(\theta) \Leftrightarrow Ey^n(\theta) \frac{C(n)}{n} \\ &\geq \left(\sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \frac{\theta_i f_i(\theta_i)\, d\theta_i}{n} \Leftrightarrow \delta\right) \int_{\theta \in H(n)} v(y^n(\theta)) dF^n(\theta) \Leftrightarrow Ey^n(\theta) \frac{C(n)}{n} \end{aligned} \tag{A26}$$

where

$$\begin{aligned} \int_{\theta \in H(n)} v(y^n(\theta)) dF^n(\theta) &= Ev(y^n(\theta)) \Leftrightarrow \int_{\theta \in \Theta \setminus H(n)} v(y^n(\theta)) dF^n(\theta) \\ &\geq Ev(y^n(\theta)) \Leftrightarrow v(\overline{y})\left(1 \Leftrightarrow \Pr(H(n))\right). \end{aligned} \tag{A27}$$

Together, (A27) and (A26) imply

$$S^n(y^n, \eta^n) \geq \left(\sum_i \frac{\int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i)\, d\theta_i}{n} \Leftrightarrow \delta\right) \left[Ev(y^n(\theta)) \Leftrightarrow v(\overline{y})\left(1 \Leftrightarrow \Pr(H(n))\right)\right] \Leftrightarrow Ey^n(\theta) \frac{C(n)}{n} \tag{A28}$$

Let $L(n) = \left\{ \theta \left| \sum_i \eta_i^n(\theta)\,\theta_i/n \le \sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i)\, d\theta_i/n + \delta \right. \right\}$. A symmetric argument shows

$$
\begin{aligned}
S^n(y'^n, \eta^n) \;\le\; & \left(\sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i)\, d\theta_i/n + \delta \right) \int_{\theta \in L(n)} v(y'^n(\theta))\, dF^n(\theta) \\
& + \frac{\sum_i \overline{\theta}_i v(\overline{y})}{n} \left[1 \Leftrightarrow \Pr(L(n)) \right] \Leftrightarrow E y'^n(\theta) \frac{C(n)}{n} \\
\le\; & \left(\sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i)\, d\theta_i/n + \delta \right) E v(y'^n(\theta)) + \frac{\sum_i \overline{\theta}_i v(\overline{y})}{n} \left[1 \Leftrightarrow \Pr(L(n)) \right] \Leftrightarrow E y^n(\theta) \frac{C(n)}{n}.
\end{aligned}
\tag{A29}
$$

Now, $\delta > 0$ and $E\{\sum_i \eta_i^n(\theta)\,\theta_i\} = \sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i)\, d\theta_i/n$, so, by Chebyshevs inequality, there exists N such that $\Pr(H(n)) \ge 1 \Leftrightarrow \delta$ for all $n \ge N$ and $\Pr(L(n)) \ge 1 \Leftrightarrow \delta$ for all $n \ge N$. Together with (A28),(A29),(A25) and our choice of $\delta = \epsilon/2v(\overline{y})(\overline{\theta} + 1)$ this implies that

$$
\begin{aligned}
& S^n(y^n, \eta^n) \Leftrightarrow S^n(y'^n, \eta^n) \\
\ge\; & \underbrace{\frac{\sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i)\, d\theta_i}{n} \left(E v(y^n(\theta)) \Leftrightarrow E v(y'^n(\theta)) \right) \Leftrightarrow \left(E y^n(\theta) \Leftrightarrow E y'^n(\theta) \right) \frac{C(n)}{n}}_{>\epsilon \text{ by (A25)}} \\
& \Leftrightarrow \delta \underbrace{\left[E v(y^n(\theta)) \Leftrightarrow v(\overline{y}) \left(1 \Leftrightarrow \Pr(H(n)) \right) \right]}_{<\delta[v(\overline{y}) \Pr(H(n))] < \delta v(\overline{y})} \Leftrightarrow v(\overline{y}) \underbrace{\left(1 \Leftrightarrow \Pr(H(n)) \right)}_{<\delta} \underbrace{\sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i)\, d\theta_i/n}_{<\overline{\theta}} \\
& \Leftrightarrow \delta \underbrace{E v(y'^n(\theta))}_{<v(\overline{y})} \Leftrightarrow \underbrace{\frac{\sum_i \overline{\theta}_i v(\overline{y})}{n}}_{<\overline{\theta} v(\overline{y})} \underbrace{\left[1 \Leftrightarrow \Pr(L(n)) \right]}_{<\delta} \;>\; \epsilon \Leftrightarrow \delta v(\overline{y}) \Leftrightarrow \delta \overline{\theta} v(\overline{y}) \Leftrightarrow \delta v(\overline{y}) \Leftrightarrow \delta \overline{\theta} v(\overline{y}) \;=\; 0.
\end{aligned}
\tag{A30}
$$

Since ϵ was arbitrary, the result follows. ■

Lemma A4 *Suppose v is strictly concave. Then, for each $\epsilon_1, \epsilon_2 > 0$ there exists some $\delta > 0$ such that $v(Ey^n(\theta)) \ge Ev(y^n(\theta)) + \delta$ for every $y^n(\cdot)$ such that $\Pr(|y^n(\theta) \Leftrightarrow E(y^n(\theta))| \ge \epsilon_1) \ge \epsilon_2$.*

Proof. Omitted. ■

Lemma A5 *Consider a sequence of incentive feasible mechanisms $\{y^n, \eta^n\}$. Suppose there are $\epsilon_1, \epsilon_2 > 0$ and N such that $\Pr(|y^n(\theta) \Leftrightarrow Ey^n(\theta)| \ge \epsilon_1) \ge \epsilon_2$ for every $n \ge N$. Consider the alternative sequence $\{\overline{y}^n, \eta^n\}$ where $\overline{y}^n(\theta) = Ey^n(\theta)$ for all $\theta \in \Theta$ and every n and the inclusion rules are unchanged. Then, there exists N' such that $\{\overline{y}^n, \eta^n\}$ is incentive feasible for every $n \ge N'$.*

Proof. By Lemma A4 there exists $\delta > 0$ such that $v(Ey^n(\theta)) \ge Ev(y^n(\theta)) + \delta$. Moreover under the hypothesis of the Lemma there exists δ' such that, $Ey^n(\theta) > \delta'$ for all $n \ge N$. Applying Lemma

A2 this implies that there exists $K > 0$ such that $\sum_i \theta_i^n [1 \Leftrightarrow F_i(\theta_i^n)]/n \geq K$ for all $n \geq N$ since otherwise $Ey^n(\theta) \to 0$ (since (A19) would be violated otherwise). Define $\widetilde{\delta} = \delta K / \overline{\theta} v(\overline{y}) > 0$ and let $\widetilde{H}(n) = \left\{ \theta | \sum_i \eta_i^n(\theta) x_i(\theta_i)/n \geq \sum_i \theta_i^n [1 \Leftrightarrow F_i(\theta_i^n)]/n + \widetilde{\delta} \right\}$. A decomposition of G^n in (A17) evaluated at (y^n, η^n) along the same lines as the decomposition of S^n in Lemma A3 yields

$$G^n(y^n, \eta^n) \leq Ev(y^n(\theta)) \left(\frac{\sum_i \theta_i^n [1 \Leftrightarrow F_i(\theta_i^n)]}{n} + \widetilde{\delta} \right) + \overline{\theta} v(\overline{y}) \Pr(\widetilde{H}(n)) \Leftrightarrow Ey^n(\theta) \frac{C(n)}{n}. \qquad \text{(A31)}$$

For the mechanism $\{\overline{y}^n, \eta^n\}$, a direct calculation shows that

$$G^n(\overline{y}^n, \eta^n) = v(Ey^n(\theta)) \frac{\sum_i \theta_i^n [1 \Leftrightarrow F_i(\theta_i^n)]}{n} \Leftrightarrow Ey^n(\theta) \frac{C(n)}{n}. \qquad \text{(A32)}$$

$E \sum_i \eta_i^n(\theta) x_i(\theta_i) = \sum_i \theta_i^n [1 \Leftrightarrow F_i(\theta_i^n)]$, so an application of Chebyshevs inequality shows that there exists $N \leq N' < \infty$ such that $\Pr(\widetilde{H}(n)) < \widetilde{\delta}$. Using this and that $v(Ey^n(\theta)) \geq Ev(y^n(\theta)) + \delta$ together with (A32) shows that for $n \geq N'$

$$\begin{aligned} G^n(\overline{y}^n, \eta^n) &\geq G^n(y^n, \eta^n) + \delta \underbrace{\frac{\sum_i \theta_i^n [1 \Leftrightarrow F_i(\theta_i^n)]}{n}}_{>K} \Leftrightarrow \widetilde{\delta} \underbrace{Ev(y^n(\theta))}_{<v(\overline{y})} \Leftrightarrow \overline{\theta} v(\overline{y}) \underbrace{\Pr(\widetilde{H}(n))}_{<\widetilde{\delta}} \\ &> G^n(y^n, \eta^n) + \delta K \Leftrightarrow \widetilde{\delta} v(\overline{y})(1 + \overline{\theta}) = G^n(y^n, \eta^n) \geq 0, \end{aligned} \qquad \text{(A33)}$$

where the final inequality follows because (y^n, η^n) is incentive feasible. Hence $(\overline{y}^n, \eta^n)$ is also incentive feasible for $n \geq N'$.

Proof of Proposition 5. If the proposition would fail, then (taking a subsequence if necessary) there exists some $\epsilon_1 > 0$ and $\epsilon_2 > 0$ such that $\Pr(|y^n(\theta) \Leftrightarrow Ey^n(\theta)| \geq \epsilon_1) \geq \epsilon_2$ for all n. Consider a sequence of alternative mechanisms $\{\overline{y}^n, \eta^n\}$, where for each n the only difference with the initial mechanism is that $\overline{y}^n(\theta) = Ey^n(\theta)$. By Lemma A4 we have that there exists δ such that $v(Ey^n(\theta)) \geq Ev(y^n(\theta)) + \delta$ holds for every n in the sequence. Moreover, for the same reasons as in Lemma A5 there exists $K > 0$ such that $\sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i) d\theta_i / n \geq K$. Hence,

$$\frac{\sum_i \int_{\theta_i^n}^{\overline{\theta}_i} \theta_i f_i(\theta_i) d\theta_i}{n} (Ev(\overline{y}^n(\theta)) \Leftrightarrow Ev(y^n(\theta))) \Leftrightarrow (E\overline{y}^n(\theta)) \Leftrightarrow Ey^n(\theta))) \frac{C(n)}{n} > \delta K > 0, \qquad \text{(A34)}$$

which by application of Lemma A3 implies that there exists N such that $(\overline{y}^n, \eta^n)$ generates a higher surplus than $(\overline{y}, \eta^n)$. By Lemma A5 there exists N' such that $(\overline{y}^n, \eta^n)$ is incentive feasible for $n \geq N'$. Hence we have contradicted that $\{\overline{y}, \eta^n\}$ is a sequence of optimal mechanisms. ■

A.5 Proposition 6

Lemma A6 *$\Phi(\cdot), \Psi(\cdot)$ and $Q(\cdot)$ defined in (26), (26) and (28) satisfy the following properties:*

1. *$\Phi(\cdot), \Psi(\cdot)$ and $Q(\cdot)$ are continuous functions of β*
2. *$\Phi(\beta) > \Psi(\beta)$ for every $\beta \in [0,1]$*
3. *$\Phi(\cdot), \Psi(\cdot)$ and $Q(\cdot)$ are weakly decreasing in β*
4. *$\Phi(\beta)v(Q(\beta)) \Leftrightarrow Q(\beta)c^*$ is weakly decreasing in β. Moreover, if $\beta' < \beta''$ is such that $Q(\beta') > Q(\beta'')$, then $\Phi(\beta')v(Q(\beta')) \Leftrightarrow Q(\beta')c^* > \Phi(\beta'')v(Q(\beta'')) \Leftrightarrow Q(\beta'')c^*$.*

Proof. *(PART 1)* $(1 \Leftrightarrow \beta)\theta_i + \beta x_i(\theta_i)$ is strictly increasing in θ_i, implying that there is a unique threshold $\widetilde{\theta}_i(\beta) \in [\underline{\theta}_i, \overline{\theta}_i]$ such that $(1 \Leftrightarrow \beta)\theta_i + \beta x_i(\theta_i) \geq 0$ if and only if $\theta_i \geq \widetilde{\theta}_i(\beta)$. Moreover, $(1 \Leftrightarrow \beta)\theta_i + \beta x_i(\theta_i) < 0$ for every $\theta_i < \widetilde{\theta}_i(\beta)$ and $(1 \Leftrightarrow \beta)\theta_i + \beta x_i(\theta_i) > 0$ for every $\theta_i > \widetilde{\theta}_i(\beta)$, so if $\{\beta^n\}$ is a sequence with $\lim_{n\to\infty} \beta^n \to \beta$, then $\lim_{n\to\infty} \widetilde{\eta}_i(\theta_i, \beta^n) = \widetilde{\eta}_i(\theta_i, \beta)$ for all $\theta_i \neq \widetilde{\theta}_i(\beta)$. Continuity of $\Phi(\cdot)$ and $\Psi(\cdot)$ follows by Lebesgues' monotone convergence theorem and the theorem of the maximum guarantees that $Q(\cdot)$ is an upper-hemi-continuous correspondence. Strict concavity of v implies that $Q(\beta)$ is unique for every $\beta \in [0,1]$, implying that $Q(\cdot)$ is a continuous function.
(PART 2) Each inclusion rule $\widetilde{\eta}_i(\cdot, \beta)$ can be characterized by an inclusion threshold $\widetilde{\theta}_i(\beta)$ (possibly equal to $\underline{\theta}_i$) for each β. Since $x_i(\theta_i)$ is continuous and $x_i(\overline{\theta}_i) = \overline{\theta}_i \Leftrightarrow (1 \Leftrightarrow F_i(\overline{\theta}_i)/f_i(\theta_i) = \overline{\theta}_i$ there exists $\theta_i' < \overline{\theta}_i$ such that $x_i(\theta_i) > 0$ for all $\theta_i \geq \theta_i'$ and $\widetilde{\theta}_i(\beta) \leq \widetilde{\theta}_i(1) \leq \theta_i'$ for every $\beta \in [0,1]$. Hence,

$$\begin{aligned}\Phi(\beta) \Leftrightarrow \Psi(\beta) &= \int_{\theta_1} \cdots \int_{\theta_r} \frac{\sum_{i=1}^r \widetilde{\eta}_i(\theta_i, \beta)\left(\theta_i \Leftrightarrow x_i(\theta_i)\right)}{r} dF^n(\theta) \qquad \text{(A35)}\\ &= \frac{1}{r}\sum_{i=1}^r \int_{\widetilde{\theta}_i(\beta)}^{\overline{\theta}_i} \left(\theta_i \Leftrightarrow x_i(\theta_i)\right) f_i(\theta_i)\, d\theta_i = \frac{1}{r}\sum_{i=1}^r \int_{\widetilde{\theta}_i(\beta)}^{\overline{\theta}_i} (1 \Leftrightarrow F_i(\theta_i)\, d\theta_i > 0\end{aligned}$$

(PART 3) The proof which is omitted (see Norman [20]) uses that $Q(\beta'))$ is better than $Q(\beta'')$ for $\beta = \beta'$ and vice versa for $\beta = \beta''$ together with PART 2 of the claim.
(PART 4) To show that $\Phi(\beta)v(Q(\beta)) \Leftrightarrow Q(\beta)c^*$ is weakly decreasing, suppose for contradiction that there exists $\beta' < \beta''$ such that $\Phi(\beta')v(Q(\beta')) \Leftrightarrow Q(\beta')c^* < \Phi(\beta'')v(Q(\beta'')) \Leftrightarrow Q(\beta'')c^*$. Then,

$$\begin{aligned}&v(Q(\beta'))\left[(1 \Leftrightarrow \beta')\Phi(\beta') + \beta'\Psi(\beta')\right] \Leftrightarrow Q(\beta')c^* \qquad \text{(A36)}\\ < \;& \Phi(\beta'')v(Q(\beta'')) \Leftrightarrow Q(\beta'')c^* + v(Q(\beta'))\beta' \underbrace{\left(\Psi(\beta') \Leftrightarrow \Phi(\beta')\right)}_{<0}\end{aligned}$$

$$
\begin{aligned}
&\leq \quad \Phi(\beta'')v(Q(\beta'')) \Leftrightarrow Q(\beta'')c^* + v(Q(\beta''))\beta' \left(\Psi(\beta') \Leftrightarrow \Phi(\beta')\right) \\
&= \quad v(Q(\beta'')) \left[(1 \Leftrightarrow \beta')\Phi(\beta') + \beta'\Psi(\beta')\right] \Leftrightarrow Q(\beta'')c^*,
\end{aligned}
$$

contradicting the assumption that $Q(\beta')$ solves (28) for $\beta = \beta'$. Finally, if $Q(\beta') > Q(\beta'')$ then $v(Q(\beta'))\beta' \left(\Psi(\beta') \Leftrightarrow \Phi(\beta')\right) < v(Q(\beta''))\beta' \left(\Psi(\beta') \Leftrightarrow \Phi(\beta')\right)$, implying that (A36) generates a contradiction also if from a weak inequality, thus validating the final claim. ■

Lemma A7 *Consider a sequence of replications of a given finite economy* $\{v, [0, \overline{y}], C(r), F^r\}$. *Let* $\{y^n, \eta^n\}_{n=1}^{\infty}$ *be a sequence of optimal mechanisms and* λ^n *be the Lagrange multiplier associated with the mechanism* (y^n, η^n) *and* $\beta^n = \frac{\lambda^n}{1+\lambda^n}$ *for each* n. *Then, for any subsequence* $\{n_k\}$ *such that* $\lim_{n_k \to \infty} \beta^{n_k} = \beta$ *we have that*

$$
\lim_{n_k \to \infty} \int_{\theta_1} \dots \int_{\theta_{n_k}} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} \Pi_k f_k(\theta_k)\, d\theta_k = \Phi(\beta) \tag{A37}
$$

$$
\lim_{n_k \to \infty} \int_{\theta_1} \dots \int_{\theta_{n_k}} \frac{\sum_i \eta_i^{n_k}(\theta)\, x_i(\theta_i)}{n_k} \Pi_k f_k(\theta_k)\, d\theta_k = \Psi(\beta) \tag{A38}
$$

Proof. Omitted (see Norman [20]). ■

Lemma A8 *Suppose the hypotheses of Lemma A7 are fulfilled. Then* $\lim_{n_k \to \infty} E y^{n_k}(\theta) = Q(\beta)$ *for any subsequence such that* $\lim_{n_k \to \infty} \beta^{n_k} = \beta$

Proof. Define $\chi_i(\beta, \theta_i) = (1 \Leftrightarrow \beta)\theta_i + \beta x_i(\theta_i)$ and let

$$
\begin{aligned}
Y^{n_k} &\equiv \arg\max_{y \in [0, \overline{y}]} \frac{v(y) E\left(\sum_{i=1}^{n_k} \eta_i^{n_k}(\theta)\, \chi_i(\beta^{n_k}, \theta_i)\right) \Leftrightarrow y C(n_k)}{n_k} \qquad \text{(A39)} \\
y^{n_k}(\theta) &= \arg\max_{y \in [0, \overline{y}]} \frac{\sum_{i=1}^{n_k} v(y) \eta_i^{n_k}(\theta)\, \chi_i(\beta^{n_k}, \theta_i) \Leftrightarrow y C(n_k)}{n_k},
\end{aligned}
$$

where Y^{n_k} is well-defined for every n_k by strict concavity of v and $y^{n_k}(\theta)$ is an alternative way to express the provision rule in (15) for economy n_k. Pick an arbitrary $\epsilon > 0$ and let $m = 2(\overline{y}+1) < \infty$. By continuity of $y^{n_k}(\theta)$ in the parameters of the problem (the theorem of the maximum) there exists $\delta > 0$ such that $|y^{n_k}(\theta) \Leftrightarrow Y^{n_k}| < \epsilon/m$ for all θ such that $|C(n)/n \Leftrightarrow c^*| \leq \delta$ and

$$
\left| \frac{\sum_{i=1}^{n_k} \eta_i^{n_k}(\theta)\, \chi_i(\beta^{n_k}, \theta_i) \Leftrightarrow E \sum_{i=1}^{n_k} \eta_i^{n_k}(\theta)\, \chi_i(\beta^{n_k}, \theta_i)}{n_k} \right| \leq \delta \tag{A40}
$$

Moreover, $\{\chi_i(\beta^{n_k}, \theta_i)\}_{i=1}^{n_k}$ is a sequence of independent random variables with bounded variance, so an application of Chebyshevs inequality to implies that there exists N such that

$$
\Pr\left[\left| \sum_{i=1}^{n_k} \eta_i^{n_k}(\theta)\, \chi_i(\beta^{n_k}, \theta_i) \Leftrightarrow E \sum_{i=1}^{n_k} \eta_i^{n_k}(\theta)\, \chi_i(\beta^{n_k}, \theta_i) \right| \geq \delta n_k \right] < \frac{\epsilon}{m}. \tag{A41}
$$

for every $n_k \geq N$. It follows that $y^{n_k}(\theta) \geq Y^{n_k} \Leftrightarrow \frac{\epsilon}{m}$ with probability of at least $\left(1 \Leftrightarrow \frac{\epsilon}{m}\right)$, so $Ey^{n_k} \geq \left(1 \Leftrightarrow \frac{\epsilon}{m}\right)\left(Y^{n_k} \Leftrightarrow \frac{\epsilon}{m}\right)$ for $n_k \geq N$. Symmetrically, $y^{n_k}(\theta) \leq Y^{n_k} + \frac{\epsilon}{m}$ with probability of at least $\left(1 \Leftrightarrow \frac{\epsilon}{m}\right)$ and $y^{n_k}(\theta) \leq \overline{y}$ for all θ, so $Ey^{n_k} \leq \left(1 \Leftrightarrow \frac{\epsilon}{m}\right)\left(Y^{n_k} + \frac{\epsilon}{m}\right) + \frac{\epsilon}{m}\overline{y}$ for all $n_k \geq N$. Since $m = 2(\overline{y}+1)$ we have that

$$\begin{aligned} Ey^{n_k} &\geq \left(1 \Leftrightarrow \frac{\epsilon}{m}\right)\left(Y^{n_k} \Leftrightarrow \frac{\epsilon}{m}\right) \Leftrightarrow Ey^{n_k} \Leftrightarrow Y^{n_k} \geq \Leftrightarrow \frac{\epsilon}{m}\left(Y^{n_k} \Leftrightarrow 1 \Leftrightarrow \frac{\epsilon}{m}\right) \\ &\geq \Leftrightarrow \frac{\epsilon}{m} Y^{n_k} \geq \Leftrightarrow \frac{\epsilon}{2(\overline{y}+1)} Y^{n_k} \geq \Leftrightarrow \frac{\epsilon}{2} \\ Ey^{n_k} &\leq \left(1 \Leftrightarrow \frac{\epsilon}{m}\right)\left(Y^{n_k} + \frac{\epsilon}{m}\right) + \frac{\epsilon}{m}\overline{y} \Leftrightarrow Ey^{n_k} \Leftrightarrow Y^{n_k} \leq \frac{\epsilon}{m}\left(\overline{y} \Leftrightarrow Y^{n_k} \Leftrightarrow \frac{\epsilon}{m} + 1\right) \\ &\leq \frac{\epsilon}{m}(\overline{y}+1) \leq \frac{\epsilon}{2(\overline{y}+1)}(\overline{y}+1) \leq \frac{\epsilon}{2}, \end{aligned} \tag{A42}$$

so $|Ey^{n_k} \Leftrightarrow Y^{n_k}| \leq \frac{\epsilon}{2}$ for all $n_k \geq N$. To complete the argument we observe that given a subsequence $\{\beta^{n_k}\}$ such that $\beta^{n_k} \to \beta$ as $n_k \to \infty$ we may apply Lemma A7 to conclude that

$$\lim_{n_k \to \infty} \frac{E\sum_{i=1}^{n_k} \eta_i^{n_k}(\theta)\,\chi_i(\beta^{n_k}, \theta_i)}{n_k} = (1 \Leftrightarrow \beta)\Phi(\beta) + \beta\Psi(\beta) \tag{A43}$$

and $\lim_{n_k \to \infty} C(n_k)/n_k = c^*$ by assumption. The theorem of the maximum assures that there exists some finite N' such that $|Y^{n_k} \Leftrightarrow Q(\beta)| \leq \epsilon/2$ for $n_k \geq N'$. Picking $N'' = \max\{N, N'\}$ the triangle inequality implies that $|Ey^{n_k} \Leftrightarrow Q(\beta)| \leq \epsilon$. Since $\epsilon > 0$ was arbitrary the result follows. ■

Lemma A9 *Suppose the hypotheses of Lemma A7 are fulfilled. Then* $\lim_{n_k \to \infty} S^{n_k}(y^{n_k}, \eta^{n_k}) = \Phi(\beta)v(Q(\beta)) \Leftrightarrow Q(\beta)c^*$ *for any subsequence* $\{n_k\}$ *such that* $\lim_{n_k \to \infty} \beta^{n_k} = \beta$

Proof. Pick an arbitrary $\epsilon > 0$ and let

$$\widetilde{S}^{n_k} = v\left(Ey^{n_k}(\theta)\right) \int_{\theta \in \Theta} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^n(\theta) \Leftrightarrow Ey^{n_k}(\theta)\frac{C(n_k)}{n_k} \tag{A44}$$

Combining (A37) in Lemma A7 with Lemma A8 we conclude there exists some finite N such that $\left|\widetilde{S}^{n_k} \Leftrightarrow \Phi(\beta)v(Q(\beta)) \Leftrightarrow Q(\beta)c^*\right| < \epsilon/2$ for all $n_k \geq N$. Let $m = 2\overline{\theta}(1 + v(\overline{y})) < \infty$. Continuity of v implies that there exists $\delta > 0$ such that $|v(Ey^{n_k}(\theta)) \Leftrightarrow v(y)| < \frac{\epsilon}{m}$ for all y such that $|y \Leftrightarrow Ey^{n_k}(\theta)| \leq \delta$. Moreover, Proposition 5 guarantees that for every $\delta > 0$ there exists N' such that $\Pr(|y^{n_k}(\theta) \Leftrightarrow Ey^{n_k}| \geq \delta) \leq \frac{\epsilon}{m}$ for all $n_k \geq N'$. For each n_k let $H(n_k) = \{\theta|\, y^{n_k}(\theta) \geq Ey^{n_k}(\theta) \Leftrightarrow \delta\}$ and observe that $\Pr(H(n_k)) \geq 1 \Leftrightarrow \frac{\epsilon}{m}$ and $\frac{\sum_i \eta_i^{n_k}(\theta)\theta_i}{n_k} \leq \overline{\theta}$ for every $\theta \in \Theta$, so

$$\begin{aligned} \int_{\theta \in H(n_k)} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^n(\theta) &= \int_{\theta \in \Theta} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^n(\theta) \Leftrightarrow \int_{\theta \in \Theta \setminus H(n_k)} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^n(\theta) \\ &\geq \int_{\theta \in \Theta} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^n(\theta) \Leftrightarrow \frac{\epsilon\overline{\theta}}{m} \end{aligned} \tag{A45}$$

Since $y^{n_k}(\theta) \geq Ey^{n_k}(\theta) \Leftrightarrow \delta$ for every $\theta \in H(n_k)$ it follows that for all $n_k \geq N'' = \max\{N, N'\}$,

$$\begin{aligned}
S^{n_k}(y^{n_k}, \eta^{n_k}) &= \int_{\theta\in\Theta} \left(\frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i v(y^{n_k}(\theta)) \Leftrightarrow y^{n_k}(\theta)\, C(n_k)}{n_k} \right) dF^{n_k}(\theta) &\text{(A46)}\\
&\geq v(Ey^{n_k}(\theta) \Leftrightarrow \delta) \int_{\theta\in H(n_k)} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^{n_k}(\theta) \Leftrightarrow Ey^{n_k}(\theta)\, \frac{C(n_k)}{n_k}\\
&\geq \left[v(Ey^{n_k}(\theta)) \Leftrightarrow \frac{\epsilon}{m} \right] \int_{\theta\in H(n_k)} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^{n_k}(\theta) \Leftrightarrow Ey^{n_k}(\theta)\, \frac{C(n_k)}{n_k}\\
&\geq \left[v(Ey^{n_k}(\theta)) \Leftrightarrow \frac{\epsilon}{m} \right] \left[\int_{\theta\in\Theta} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^{n_k}(\theta) \Leftrightarrow \frac{\epsilon\overline{\theta}}{m} \right] \Leftrightarrow Ey^{n_k}(\theta)\, \frac{C(n_k)}{n_k}\\
&= \widetilde{S}^{n_k} \Leftrightarrow \frac{\epsilon}{m} \left(\int_{\theta\in\Theta} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^{n_k}(\theta) + \overline{\theta} v(Ey^{n_k}(\theta)) \Leftrightarrow \frac{\epsilon\overline{\theta}}{m} \right)\\
&\geq \widetilde{S}^{n_k} \Leftrightarrow \frac{\epsilon\overline{\theta}[1 + v(\overline{y})]}{m} = \widetilde{S}^{n_k} \Leftrightarrow \frac{\epsilon}{2},
\end{aligned}$$

where the last inequality comes from $m = 2\overline{\theta}(1 + v(\overline{y}))$. Next, let $L(n_k) = \{\theta |\, y^{n_k}(\theta) \leq Ey^{n_k} + \delta\}$. Observing that $\Pr(L(n_k)) \geq 1 \Leftrightarrow \frac{\epsilon}{m}$ for $n_k \geq N''$, $y^{n_k}(\theta) \leq \overline{y}$ for all θ,and that $\sum_i \eta_i^{n_k}(\theta)\,\theta_i / n_k \leq \overline{\theta}$ for all θ we have that

$$\begin{aligned}
S^{n_k}(y^{n_k}, \eta^{n_k}) &= \int_{\theta\in\Theta} \left(\frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i v(y^{n_k}(\theta)) \Leftrightarrow y^{n_k}(\theta)\, C(n_k)}{n_k} \right) dF^{n_k}(\theta) &\text{(A47)}\\
&\leq v(Ey^{n_k}(\theta) + \delta) \int_{\theta\in L(n_k)} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^{n_k}(\theta)\\
&\qquad + v(\overline{y}) \int_{\theta\in\Theta\setminus L(n_k)} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^{n_k}(\theta) \Leftrightarrow Ey^{n_k}(\theta)\, \frac{C(n_k)}{n_k}\\
&\leq \left[v(Ey^{n_k}(\theta)) + \frac{\epsilon}{m} \right] \int_{\theta\in\Theta} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^{n_k}(\theta) + \frac{\overline{\theta} v(\overline{y})\epsilon}{m} \Leftrightarrow Ey^{n_k}(\theta)\, \frac{C(n_k)}{n_k}\\
&= \widetilde{S}^{n_k} + \frac{\epsilon}{m} \left(\int_{\theta\in\Theta} \frac{\sum_i \eta_i^{n_k}(\theta)\,\theta_i}{n_k} dF^{n_k}(\theta) + \overline{\theta} v(\overline{y}) \right) \leq \widetilde{S}^{n_k} + \frac{\epsilon\overline{\theta}\,(1 + v(\overline{y}))}{m}\\
&\leq \widetilde{S}^{n_k} + \frac{\epsilon}{2}.
\end{aligned}$$

Hence, $\left| S^{n_k}(y^{n_k}, \eta^{n_k}) \Leftrightarrow \widetilde{S}^{n_k} \right| \leq \frac{\epsilon}{2}$ for all $n_k \geq N''$ and since $\left| \widetilde{S}^{n_k} \Leftrightarrow [\Phi(\beta) v(Q(\beta)) \Leftrightarrow Q(\beta) c^*] \right| < \frac{\epsilon}{2}$ for all $n_k \geq N''$ it follows from the triangle inequality that $|S^{n_k}(y^{n_k}, \eta^{n_k}) \Leftrightarrow [\Phi(\beta) v(Q(\beta)) \Leftrightarrow Q(\beta) c^*]| < \epsilon$. Since ϵ was arbitrary the result follows. ■

Proof of Proposition 6. If there exists no y^* such that $\lim_{n\to\infty} Ey^n(\theta) = y^*$ Lemma A8 implies that $\{\beta^n\}_{n=1}^{\infty}$ cannot be a converging sequence. Since each $\beta^n \in [0, 1]$, a compact set, it must then exist at least two accumulation points β^L, β^H and corresponding subsequences $\{\beta^{n_{k(L)}}\}, \{\beta^{n_{k(H)}}\}$ with $\lim_{n_{k(J)}\to\infty} \beta^{n_{k(J)}} = \beta^J$ for $J = L, H$ where $Q(\beta^L) \neq Q(\beta^H)$. Label β^L, β^H such that $\beta^L < \beta^H$

and observe that parts 3 and 4 of Lemma A6 imply that $Q(\beta^L) > Q(\beta^H)$ and

$$\Phi(\beta^L)v(Q(\beta^L)) \Leftrightarrow Q(\beta^L)c^* > \Phi(\beta^H)v(Q(\beta^H)) \Leftrightarrow Q(\beta^H)c^*. \tag{A48}$$

Pick some $\lambda \in (0,1)$ and let $Q^\lambda = \lambda Q(\beta^H) + (1 \Leftrightarrow \lambda)\, Q(\beta^L)$. Consider a sequence of mechanisms $\{\overline{y}^n, \overline{\eta}^n\}_{n=1}^\infty$, where for each n

$$\overline{y}^n\,(\theta) = Q^\lambda \text{ for all } \theta \in \Theta \qquad \overline{\eta}_i^n\,(\theta) = \eta_i^n(\theta_i, \beta^L) \text{ for all } \theta \in \Theta \tag{A49}$$

This mechanism generates a per capita surplus

$$S^n(\overline{y}^n, \overline{\eta}^n) = v(Q^\lambda) \int_{\theta_1} \dots \int_{\theta_n} \frac{\sum_i \eta_i^n(\theta, \beta^L)\theta_i}{n} dF^n\,(\theta) \Leftrightarrow Q^\lambda \frac{C\,(n)}{n}, \tag{A50}$$

implying (by Lemma A7) that $\lim_{n\to\infty} S^n(\overline{y}^n, \overline{\eta}^n) = v(Q^\lambda)\Phi(\beta^L) \Leftrightarrow Q^\lambda c^*$,while

$$\lim_{n_{k(H)}\to\infty} S^{n_{k(H)}}(y^{n_{k(H)}}, \eta^{n_{k(H)}}) = \Phi(\beta^H)v(Q(\beta^H)) \Leftrightarrow Q(\beta^H)c^*. \tag{A51}$$

Strict concavity implies that

$$\begin{aligned} v(Q^\lambda)\Phi(\beta^L) \Leftrightarrow Q^\lambda c^* &> \lambda\left[v(Q(\beta^H)) + (1 \Leftrightarrow \lambda)v(Q(\beta^L))\right]\Phi(\beta^L) \Leftrightarrow Q^\lambda c^* \qquad \text{(A52)}\\ &= \lambda\left[\Phi(\beta^L)v(Q(\beta^H)) \Leftrightarrow Q(\beta^H)c^*\right] + (1 \Leftrightarrow \lambda)\left[\Phi(\beta^L)v(Q(\beta^L)) \Leftrightarrow Q(\beta^L)c^*\right]\\ \Big/\Phi(\beta^L) \geq \Phi(\beta^H)\Big/ \quad &\geq \lambda\left[\Phi(\beta^H)v(Q(\beta^H)) \Leftrightarrow Q(\beta^H)c^*\right] + (1 \Leftrightarrow \lambda)\left[\Phi(\beta^L)v(Q(\beta^L)) \Leftrightarrow Q(\beta^L)c^*\right]\\ &\geq \Phi(\beta^H)v(Q(\beta^H)) \Leftrightarrow Q(\beta^H)c^*, \end{aligned}$$

where the final inequality comes from (A48). Hence there exists N such that $S^{n_{k(H)}}(\overline{y}^{n_{k(H)}}, \overline{\eta}^{n_{k(H)}}) > S^{n_{k(H)}}(y^{n_{k(H)}}, \eta^{n_{k(H)}})$ for all $n_{k(H)} \geq N$. Finally, we will establish feasibility of $(\overline{y}^n, \overline{\eta}^n)$ for large enough n. Since $\{y^{n_{k(L)}}, \eta^{n_{k(L)}}\}$ and $\{y^{n_{k(H)}}, \eta^{n_{k(H)}}\}$ are sequences of optimal mechanisms, they must also be sequences of feasible mechanisms, which by Lemma A2 requires that for every $\epsilon > 0$ there exists a finite N such that (A19) holds for $j = L, H$ and every $n_{k(J)} \geq N$. From (A38) in Lemma A7 we have that $\lim_{n_{k(J)}\to\infty} \frac{\theta_i^{n_{k(J)}}(1-F_i(\theta_i^{n_{k(J)}}))}{n_{k(J)}} = \Psi(\beta^J)$ and $\lim_{n_{k(J)}\to\infty} Ey^{n_{k(J)}} = Q(\beta^J)$ by virtue of Lemma A8. Combining this with (A19) it follows that a necessary condition for $\{y^{n_{k(L)}}, \eta^{n_{k(L)}}\}$ and $\{y^{n_{k(H)}}, \eta^{n_{k(H)}}\}$ to satisfy feasibility everywhere in each sequence is that

$$v(Q(\beta^J))\Psi(\beta^J) \geq Q(\beta^J)c^* \tag{A53}$$

holds for $J = L, H$. The mechanism $S^n(\overline{y}^n, \overline{\eta}^n)$ is feasible if

$$v(Q^\lambda) \int_\theta \frac{\sum_i \eta_i^n(\theta, \beta^L)x_i\,(\theta_i)}{n} dF^n\,(\theta) \Leftrightarrow Q^\lambda \frac{C\,(n)}{n} \geq 0, \tag{A54}$$

where (A38) in Lemma A7 implies that $\lim_{n\to\infty}\int_{\theta_1}...\int_{\theta n}\sum_i \eta_i^n(\theta,\beta^L)x_i(\theta_i)/n\Pi_k f_k(\theta_k)\,d\theta_k =$ $\Psi(\beta^L)$ and $\Psi(\beta^L)\geq\Psi(\beta^H)$, so from (A53) it follows that

$$\begin{aligned} v\left(Q(\beta^H)\right)\Psi(\beta^L) &\geq v\left(Q(\beta^H)\right)\Psi(\beta^H)\geq Q(\beta^H)c^* \text{ and } v\left(Q(\beta^L)\right)\Psi(\beta^L)\geq Q(\beta^L)c^* \quad \text{(A55)}\\ \Rightarrow v\left(Q^\lambda\right)\Psi(\beta^L) &> \lambda v\left(Q(\beta^H)\right)\Psi(\beta^L)+(1\Leftrightarrow\lambda)\,v\left(Q(\beta^L)\right)\Psi(\beta^L)\\ &\geq \lambda Q(\beta^H)c^*+(1\Leftrightarrow\lambda)\,Q(\beta^L)c^*=Q^\lambda c^*. \end{aligned}$$

Using (A55) we see that the limit of the left hand side in (A54) is strictly positive, so there exists N' such that $S^n(\overline{y}^n,\overline{\eta}^n)$ is feasible for $n\geq N'$. Hence $S^{n_{k(H)}}(\overline{y}^{n_{k(H)}},\overline{\eta}^{n_{k(H)}})$ is both feasible and better than the hypothetical optimal solution for $n_{k(H)}\geq N''=\max\{N,N'\}$. The result follows. ■

A.6 Proposition 7

Proof. *(PART 1)* Any feasible mechanism (y^n,η^n) satisfies $G(y^n,\eta^n)\geq 0$ and $v(y^n(\theta))\leq v(0)+v'(0)y^n(\theta)=v'(0)y^n(\theta)$ by concavity of v. Hence,

$$G(y^n,\eta^n)\leq\int_{\theta\in\Theta}\left(v'(0)\,y^n(\theta)\,\frac{\sum_i\eta_i^n(\theta)\,x_i(\theta_i)}{n}\Leftrightarrow\frac{y^n(\theta)\,C(n)}{n}\right)dF^n(\theta)$$

Defining $p^n(\theta)=y^n(\theta)/\overline{y}$, $\widetilde{C}(n)=\frac{1}{v'(0)}C(n)$, and $c^{**}=\frac{1}{v'(0)}c^*$ we conclude that a necessary condition for the constraint to be satisfied is that $\int_{\theta\in\Theta}\left(\sum_i\eta_i^n(\theta)\,x_i(\theta_i)\Leftrightarrow\widetilde{C}(n)\right)p^n(\theta)\,dF^n(\theta)\geq 0$ and $\lim_{n\to\infty}\sum_i\theta_i^*(1\Leftrightarrow F_i(\theta_i^*)/n<c^{**}$. Proposition 2 can be thus be applied to conclude that $Ep^n(\theta)\to 0$ as $n\to\infty$. The conclusion follows since $Ey^n(\theta)\leq\overline{y}Ep^n(\theta)$ and $\overline{y}$ is finite.

(PART 2-SKETCH) No matter whether $v'(\epsilon)\to\infty$ as $\epsilon\to 0$ or if $\sum_{i=1}^r v'(0)\,\theta_i^*(1\Leftrightarrow F_i(\theta_i^*))/n>c^*$ there exists $\epsilon>0$ such that $\sum_{i=1}^r v'(\epsilon)\,\theta_i^*(1\Leftrightarrow F_i(\theta_i^*))/n>\epsilon c^*$. If the hypothesis is false $y^n(\theta)$ converges in probability to $y^*=0$, in which case the per capita surplus goes to zero as $n\to\infty$. A feasible mechanism is to provide $y^n(\theta)=\epsilon$ for all θ and include agent i if and only if $\theta_i\geq\theta_i^*$. Plugging into the constraint and taking limits we see that the constraint is satisfied for n large enough and since this generates a positive per capita surplus it is better than the hypothetical optimal mechanism. ■

A.7 Proposition 9

Proof. Set $\eta_i^n(\theta)=1$ for all i and all θ and proceed as in Part 1 of the proof of Proposition 7. ■

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