**Inequality, Environmental Protection and Growth**

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# INEQUALITY, ENVIRONMENTAL PROTECTION AND GROWTH

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We analyze how, in representative democracies, income distribution influences the stringency of environmental policy and economic growth. Individuals (who differ in abilities) live for two periods, working when young and owning capital when old. Externalities are caused by a polluting factor. The revenue from pollution taxation, as well as capital-income taxation, is redistributed lump-sum to the old. The fiscal decision, at each point in time, is taken by a majority elected representative. In politico-economic equilibrium, more inequality (in terms of the skewness of the distribution) yields a lower pollution tax, a larger capital tax, and lower growth.

*Keywords:* Environmental policy, redistribution, inequality, political economy, growth.

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## 1. INTRODUCTION

Why do Scandinavian countries tend to protect the environment more than other developed countries? A new index of environmental performance, developed by the Yale Centre for Environmental Law and Policy and the Center for International Earth Science Information Network at Columbia University ranks a number of countries according to the effectiveness of environmental policies.[1] Scandinavian countries appear to be the most environmentally friendly, while Italy, and Spain are among the least so. France, the Netherlands, the UK and the US perform in between.

Since policies actually carried out in a democracy reflect the preferences of the public, evidence can also be looked for at individual level. Empirical evidence suggests that there is an array of individual social, political and demographic characteristics, such as age, education, gender, race, ideology, party affiliation and urbanisation, together with economic variables, including work status and individual income, which are relevant for public support towards environmental protection. In particular, there is compelling evidence that individual income plays an important role. Fishel (1979) finds that high income earners, professionals and college educated individuals were more likely to oppose the construction of a new wood-processing pulp mill in New Hampshire. More recently, a US study by Elliot et al. (1997) find that both socio-demographic and economic factors, including individual income, are influential for individual support on environmental spending; Kahn and Matsusaka (1997) find that individual income and the price of the environmental good can explain most of the variation in voting on environmental policies in California. This literature suggests that poorer individuals tend to prefer less stringent environmental policy (i.e. lower environmental taxes, regulation and environmental spending). These findings are consistent with the hypothesis that greater income inequality causes lower environmental protection. The reason is that a more right-skewed distribution of income implies that the median individual is poorer relative to the average. In the political-economy literature, the median would typically be the decisive individual, when individuals vote over policy. Expectedly, the poorer the decisive individual is, the lower would the level of environmental protection be, everything else equal. Indeed, from casual observations, we can observe that societies in which income is distributed more

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[1] See *http://www.ciesin.columbia.edu/indicators/ESI*. The Environmental Performance Index (EPI) is structured in underlying indicators and variables related to air quality, water quality, climate change and land protection. The EPI takes into account current conditions and the rate of change since 1990.

equally, such as the Scandinavian countries, are typically characterised by a higher environmental protection than more unequal societies, such as Italy, Spain, the UK and the US.[2] Nevertheless, little attention has been devoted to the analysis of how income inequality can influence the political decisions regarding the protection of the environment.[3] Related papers are Oates and Schwab (1988) and Marsiliani and Renström (2000a). The former develop a static model in which individuals are distinguished in wage and non-wage earners and the median voter takes decisions over a capital tax and a standard for local environmental quality, to focus on the issue of tax competition across jurisdictions.[4] The latter analyses the role of earmarking of environmental tax revenue to environmental abatement, in a two-period economy where a majority elected individual takes the tax- and spending decisions.

On the contrary, an extensive literature already exists on the links from income distribution to economic growth, through the political-economy channel. The main idea is that more unequal societies, in terms of skewness of the distribution, prefer more redistribution, which in turn discourages investment and growth (see Persson and Tabellini, 1994, and Benabou, 1996). Furthermore, there is empirical evidence of a negative correlation between inequality and growth in developed countries (see the survey by Benabou, 1996).[5]

The relationship between environmental policy reforms and growth has also been substantially explored. The common view among policymakers and industrialists is that environmental policy hampers growth, see e.g. van der Ploeg and Withagen (1991), and Ligthart and van der Ploeg (1994). The reason is quite intuitive. Environmental protection

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[2] Easily readable data on income inequality are available at *http://www.lisproject.org/keyfigures/ineqtable.htm*.

[3] Another body of literature focuses on the influence of lobby groups on environmental policy (see, among others, Fredriksson, 1997 and Aidt, 1998). In the current paper we do not model lobbies in order to focus more clearly on the role of income inequality.

[4] Oates and Schwab (1988) find that if the decisive individual is a wage earner, she will chose a negative capital tax and a higher environmental standard than the the first-best optimal level. If the decisive individual is a non-wage earner, she will prefer a positive capital tax (for redistributive reasons); however, no-clear cut answer is provided for whether the environmental standard is higher or lower than the first-best optimum. The reason for their, at first sight, counter-intuitive results regarding capital taxation has to do with the open-economy model. A capital subsidy attracts capital from abroad and increases the wage of the wage earners. The subsidy is paid for through a lump-sum tax which also falls on the capital owners.

[5] The empirical evidence about a negative correlation between inequality and growth has recently been questioned by Forbes (2000) and reconfirmed by Banerjee and Duflo (2000).

comes at the expense of production possibilities and lower the return on the accumable factor. However, there are also some papers which show that, under some conditions, environmental policy can boost economic growth. Gradus and Smulders (1993) show, in an endogenous growth model with human capital, that if clean environment affects the learning ability of the household, then a stricter environmental policy may give rise to greater growth. Bovenberg and Smulders (1995, 1996) and Gradus and Smulders (1996) find that a (in a first-best environment without distortionary taxes) an increase in the environmental tax can boost growth. The reason is that, in their models, clean environment is also a production factor. Nielsen, Pedersen and Sorensen (1995) and Bovenberg and de Mooij (1997) find that an increase in the environmental tax and a reduction of a distortionary tax can enhance growth. The reason for the result in the former is the presence of a market imperfection (union wage bargaining), while in the latter there are untaxed profits in the model (so the conditions for the standard Diamond-Mirrlees (1971) production-efficiency theorem do not hold) and the environmental tax acts as an implicit profit tax, and gives a "double dividend."

There has also been a theoretical investigation of so called Environmental Kuznets Curves (where one is mainly interested in the relationship between environmental quality and aggregate income over time) by John and Pecchenino (1994), Fisher and van Marrewijk (1998), Stokey (1998), and Jones and Manuelli (2001). These studies model optimised environmental policies in first-best situations (no distortionary taxes) and do not deal with income inequality. On the empirical side, there is evidence for an inverted U-shape relationship between per capita income and pollution (see, among others, Selden and Song, 1994, and Grossman and Krueger, 1995).

The purpose of our paper is twofold: first to analyse how, in democracies, individual income distribution influences political decisions about environmental protection, and, second, to determine how environmental protection and economic growth are interrelated in politico-economic equilibrium. The main hypothesis of this paper is that if we observe a negative correlation between inequality and growth, and between inequality and environmental protection, we can explain a positive correlation between environmental protection and growth.

In our paper, the level of environmental protection is determined endogenously, by a majority elected representative. Therefore our paper distinguishes itself from most of the related literature on growth and the environment in that it focuses on endogenous taxation rather than on environmental tax reforms. In order to address the growth issue, we need a dynamic framework. It is very difficult to solve politico-economic equilibria in dynastic models because individuals voting today would have to predict all future politico-economic equilibria, which will be a function of how individuals vote today. Such a model can only be solved if one resorts to computation. We therefore choose a two-period economy, where individuals (because of two-period lives) do not have to know all future politico-economic equilibria. We can then solve a sequence of political equilibria and still allow for dynamics of the underlying economy. The model we present can also be interpreted as a static economy and an overlapping-generation economy. We will discuss these alternative interpretations at the end of the paper.

A similar two-period model has been used in the analysis of inequality and growth by Persson and Tabellini (1994) and Benabou (1996), among others. We augment their framework by including a polluting factor of production, the use of which is taxed by the government. In our model, the young generation work and the old generation own the capital. Individuals in the young generation differ in ability to earn labour income. We will focus on one type of benefit only: lump-sum transfer to the old, which can be thought of as social security. Furthermore, we will explicitly model environmental policy which consists of taxation of a polluting factor (for example, energy). The fiscal decision is taken by a majority-elected representative, a period in advance, and is thus endogenised.

This framework permits us to answer the following questions: How do individuals' characteristics such as ability and, consequently, income inequality influence the decisions regarding pollution taxes? And how does the preferred environmental policy affect the economic growth of a country?

There are two driving forces, working in the same direction. First, environmental policy results in loss of production possibilities. Different individuals evaluate the production loss differently. Individuals with a higher marginal utility of consumption (the poorer ones) have a lower marginal rate of substitution between environment and private consumption if environment is a non-inferior good. Second, a poorer individual typically wishes to redistribute (using tax instruments on income) from richer individuals. The redistribution causes the consumption-possibilities frontier to move inwards (due to efficiency losses). In such an equilibrium, if the environment is a normal good, the marginal rate of substitution between environment and private consumption decreases (for all individuals).

We find that if inequality is high (in terms of skewness) so the median voter has a lower ability (i.e. is poorer), then in politico-economic equilibrium, redistribution is higher, environmental policy laxer and growth is lower.

The paper is organised as follows. Section 2 presents the model. In section 3 the economic equilibrium is solved for. Section 4 characterises preferences over policy. Section 5 solves for the politico-economic equilibrium as a function of individuals' abilities. Section 6 extends the analysis to two additional endogenous policy models: a static model where individuals differ in productive time, supply labour, and labour is taxed (as in Meltzer and Richard, 1981) and an overlapping-generation model, where individuals differ in period-one labour (as in Renström, 1996). Section 7 summarises and interprets the results.

## 2. THE ECONOMY

We specify a sequence of two-period economies. Individuals live for two periods, consuming in both periods, but only working when they are young. This is the same set-up used by Persson and Tabellini (1994), but augmented to allow for pollution. The period-one good is produced by labour (exogenous in supply), and the period-two good by capital (saved from the previous period) and pollution. Taxes are levied on capital income and on pollution, and a lump-sum transfer is given when the individuals are old. We allow for endogenous growth (period-one wage being a function of last generation's capital accumulation).

Denote the two consumption goods (consumed by individual *i*) as $c_1^i$ and $c_2^i$, respectively. The individual may transfer some of commodity 1 ( $k_1^i$ ) into commodity 2 at the after-tax rate *p*. The individual has an endowment of commodity 1, $w_0^i$, and receives a government transfer in terms of commodity 2, *S*. $c_1^i$ and $c_2^i$ are period 1 and 2 consumption respectively, $k_1^i$ is savings, *p* is the after-tax return on savings, and $w^i$ is period-1 labour income. We assume that $w_0^i=\gamma^i w_0$, and that the distribution of $\gamma^i$ (denoted $\Gamma(\gamma^i)$) is continuous and stationary over time. $\Gamma(\gamma^i)$ is also normalised so that the average $\gamma^i$ equals unity, and so that averages equals aggregates. We will denote averages/aggregates by omitting superscript *i*. Production takes place in the second period by using capital and pollution *x*.

Throughout we will make one separability assumption: the pollution externality enters the individuals' utility functions in a weakly separable way. This will make the individuals' marginal rates of substitutions between private commodities (and consequently the private consumption decisions) independent of the pollution externality. Without such a separation, the problem becomes intractable and one would have to resort to computation. The weak separability will *not*, however, make the individuals' evaluation of the environment independent of their private consumption, and, consequently, we may explore this interaction in the analysis. We next state the assumptions made.

### 2.1 Assumptions

**A1 Individuals' preferences**

First, we assume weak separability between private consumption and pollution

$$V^i = V\left(u(c_1^i, c_2^i), x\right) \tag{1}$$

where $V$ is concave, and $V_1>0$, $V_2<0$. Second we assume that utility of private consumption is additively separable and homothetic[6]

$$u = \ln(c_1^i) + \beta \ln(c_2^i) \tag{2}$$

where $\beta>0$.

**A2 Individuals' constraints**

The individuals' budget constraints are

$$c_1^i + k^i = w\gamma^i \qquad (3) \qquad\qquad c_2^i = pk^i + S, \quad p \equiv (1-\tau^k)R \tag{4}$$

where $R$ is the before-tax price of capital.

**A3 Production**

A large number of firms are operating under identical *constant-returns-to-scale* technologies. Therefore aggregate production can be calculated as if there was a representative firm employing the aggregate quantity of the capital supplied by the individuals, $k\equiv\int k^i d\Gamma(\gamma^i)$ and

[6] It is desirable to analyse a situation where the competitive equilibrium is invariant with respect to the underlying distribution and only the political channel is at work. This happens when the individual utility function is such that aggregation occurs, which logarithmic preferences guarantee. In a working paper version of this paper (Marsiliani and Renström, 2000b), we also conduct an analysis for general preferences and (constant returns to scale) production technologies in a neighborhood of no inequality. The results of this paper, regarding the effect of inequality on environmental taxation, hold locally in a neighborhood of no inequality.

the polluting factor.[7] For analytical tractability we assume a Cobb-Douglas production technology

$$F(k,x) = A\,k^{\alpha} x^{\mu} \tag{5}$$

where $0<\alpha<1$, $0<\mu<1$ and further $\alpha+\mu\leq 1$.[8] Firms take the factor prices of capital ($R$), and the pollution tax $\tau^x$, as given.

**A4 Individual heterogeneity**

Individuals differ in $\gamma^j$, which is distributed between $\gamma^{min}$ and $\gamma^{max}$ according to the time-invariant continuous distribution function $\Gamma(\gamma^j)$.[9] Furthermore $\gamma^{min} > 0$, and

$$\gamma^{\max} - 1 \;\leq\; \min\left\{\frac{1-\gamma^{\min}}{\beta}\,\frac{(1-\alpha)\beta + (1+\alpha\beta)\gamma^{\min}}{\alpha+\beta+(1-\alpha)\gamma^{\min}} \;,\; \left[2\alpha\beta\sqrt{\frac{1/\alpha+\beta}{1+\beta}} + \frac{\beta}{1+\beta}(1+\alpha+2\alpha\beta)\right]^{-1}\right\}$$

**A5 Government's constraint**

The tax receipts are fully used for the lump-sum transfer

$$S = \tau^k R k + \tau^x x \tag{6}$$

**A6 Representative democracy**

The tax rates, $\tau^k$ and $\tau^x$, are determined by a majority-elected representative one period in advance. We assume that one candidate of each type runs for office, and that candidacy is costless.

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[7] The polluting factor is provided at no cost. Thus, in absence of a government taxing or regulating it, this factor would be used up to the satiation point.

[8] If $\alpha+\mu<1$ there are rents to a hidden factor. In the section 6, dealing with extensions, this factor is labour supplied by the young generation.

[9] The conditions on $\gamma^{max}$ guarantee an interior solution with respect to economic policy. The first condition avoids the corner where the individuals of very high abilities would want to implement a capital subsidy so large that the lump-sum tax cannot be afforded by the poorest individuals. The second condition guarantee interior solution with respect to the capital subsidy for the individuals of high abilities. Details are in auxiliary appendix P.

## 3. ECONOMIC EQUILIBRIUM

In this section, individual and aggregate economic behaviour are solved for any given arbitrary sequences of tax rates.

### 3.1 Individual and aggregate economic behaviour

Maximisation of (2) subject to (3)-(4) gives the individuals' optimal decision over $k$. The equilibrium is

$$k^i = \frac{\beta}{1+\beta} w\gamma^i - \frac{1}{1+\beta}\frac{S}{p} \quad (7) \qquad\qquad k = \frac{\beta}{1+\beta} w - \frac{1}{1+\beta}\frac{S}{p} \quad (8)$$

for individual $i$ and the average/aggregate, respectively.

### 3.2 Firms' behaviour

Firms take prices as given. Profit maximisation implies that the before-tax price is given by $R=F_k$. The first-order condition for the use of factor $x$, $F_x(k,x)=\tau^x$, gives (aggregate/average) $x$ as a function of (aggregate/average) $k$ and $\tau^x$, with the following property

$$dx = d\tau^x/F_{xx} - (F_{xk}/F_{xx})\,dk \tag{9}$$

### 3.3 Government's budget

The budget may alternatively be written as

$$S = (\alpha + \mu)F - pk \tag{10}$$

From (10) and the above equilibrium conditions, we see that a pollution tax and selling pollution permits are *equivalent instruments*. We will define environmental strictness as the level of $\tau^x$, which implies that if the government sells emissions permits, the strictness measure is the (equilibrium) marginal product of pollution, $F_x$.

## 4. PREFERENCES OVER POLICY

Any individual elected into office will choose policy so as to maximise her own utility. The policy chosen is then a function of the type of the individual, say $\gamma^j$. We need first to find the properties of these policy functions. Later, in section 5, we will substitute the most preferred policy of a hypothetical policy maker into the other individuals' utility functions to obtain indirect utility functions, of $\gamma^i$ only. If individuals' indirect utilities over $\gamma^i$ are single peaked (see Lemma 1, in section 5), then, since individuals differ only in one dimension, the median individual cannot lose against any other candidate in a binary election, i.e. the median is the Condorcet winner.

The first step in solving for the equilibrium is to characterise the decision of an arbitrary candidate, *i*. It is instructive to first keep the general notation regarding *u* and *F*, and later on substitute for the functional specifications. The problem of the decisionmaker *i* is to

$$\max_{p,s,x} \; V\left(u(w^i - k^i, pk^i + S), x\right) + \lambda\left[(\alpha+\mu)F(k,x) - pk - S\right] \tag{11}$$

The problem is written as if the individual was to choose *x* directly (for example, imposing an emissions standard); however, it is just an equivalent representation of the situation where the pollution tax is chosen. This holds because firms all have the same production technology, and therefore no extra informational requirements are needed. The first-order conditions are

$$V_1 u_2 k^i + \lambda\left[\left((\alpha+\mu)F_k - p\right)\frac{\partial k}{\partial p} - k\right] = 0 \tag{12}$$

$$V_1 u_2 + \lambda\left[\left((\alpha+\mu)F_k - p\right)\frac{\partial k}{\partial S} - 1\right] = 0 \tag{13}$$

$$V_2 + \lambda(\alpha+\mu)F_x = 0 \tag{14}$$

We may observe the following. Since the pollution tax is pollution's marginal product, (14) may be written as $\tau^x = -V_2/((\alpha+\mu)\lambda)$. Everything being equal, an increase in $\lambda$ (the decisive individual's marginal utility of lump-sum income at the optimum) reduces the pollution tax. Environmental policy comes at the expense of production possibilities. This tends to make poorer individuals (with lower marginal rate of substitution between environment and private consumption) wanting a lower pollution tax. Furthermore, $\lambda$ is also evaluated at equilibrium production. If the individual is relatively poor and uses redistributive tax instruments, this tends to increase $\lambda$ further, because of the loss of efficiency.

The argument put forth above is just to illustrate what we believe are the driving forces. We need to prove that $\lambda$ is larger for a poorer individual if she was to choose policy than it would be for a richer individual if the latter were to choose policy. We also need to take into account how individuals evaluate the environment. If $V$ is not additively separable, then $V_2$ depends on the private consumption of the decisive individual (at the optimum) as well. For example, it could be the case that a poorer individual values the environment more (for example, $-V_2$ could be larger for poorer individuals). In order to formally prove the link between the income of the decisive individual and environmental protection, we need to take into account the whole system (12)-(14). We will do so by performing comparative statics, by changing $\gamma^i$ of the decisionmaker, and evaluating the consequences on $\tau^x$. We can then see the consequences of making the decisionmaker poorer or richer than average.

Combining (12) and (13) gives

$$k - k^{i} = \big((\alpha+\mu)F_k - p\big)\big(\partial k/\partial p - k^{i}\partial k/\partial S\big) \tag{15}$$

Then the capital tax is positive (zero/negative) if the decisive individual supplies less (equal/more) of $k$ than the average.

Next, we combine (13) and (14) to obtain the optimality condition for $\tau^x$

$$\frac{V_1 u_2}{-V_2} = \frac{1 - \big((\alpha+\mu)F_k - p\big)\dfrac{\partial k}{\partial S}}{(\alpha+\mu)\,\tau^x} \tag{16}$$

We need to know how the marginal rate of substitution between private consumption and the environment changes with the underlying variables. Let $V_j$ denote the derivative of $V$ with respect to argument $j$={1,2}, we then have

$$dV_j = V_{j1}\left[u_1 dw_0^{i} + u_2\big(k^{i}dp + dS\big)\right] + V_{j2}dx \tag{17}$$

Using the production technology, we may write the transfer (equation (10)) as

$$S = \left[\alpha + \mu - \alpha(1-\tau^k)\right]F(k, x) \tag{18}$$

Substituting for the transfer (18) into (8) gives $k$ as a function of $\tau^k$ and $w$

$$k = \alpha\beta(1-\tau^k)w_0 \big/ [\alpha + \mu + \alpha\beta(1-\tau^k)] \tag{19}$$

Taking the derivatives of (8) with respect to $p$ and $S$ and substituting into (15), and using (7), (18) and (19), in (15), gives the capital tax as a function of the endowment of the decisionmaker

$$(1-\gamma^{i})\left[\alpha+\mu+\beta(1-\tau^{k})\right][\alpha+\mu+\alpha\beta(1-\tau^{k})] \;=\; (1+\beta)(\alpha+\mu)\,[\alpha+\mu-(1-\tau^{k})] \tag{20}$$

which gives, as expected, $\partial\tau^k/\partial\gamma^i < 0$.[10] To find the relationship between the decisionmaker's $\gamma$ and the pollution tax, we need to evaluate (16) (taking into account (20)). The right-hand side of (16) is (by using (8) and the relation $p=(1-\tau^k)F_k$)

$$\left[1-\left((\alpha+\mu)F_k-p\right)\frac{\partial k}{\partial S}\right]\Big/\left[(\alpha+\mu)\,\tau^x\right] \;=\; \frac{\alpha+\mu+\beta(1-\tau^k)}{(1+\beta)(1-\tau^k)(\alpha+\mu)\,\tau^x} \tag{21}$$

Next, from the utility specification we have

$$1\big/u_2^{i} \;=\; \frac{pk^{i}+S}{\beta} \;=\; pw_0\left(\frac{\gamma^{i}-1}{1+\beta}+\frac{\alpha+\mu}{\alpha+\mu+\alpha\beta(1-\tau^k)}\right) \;=\; pw_0H \tag{22}$$

where

$$H \;\equiv\; (\alpha+\mu)(1+\beta)(1-\tau^k)[\alpha+\mu+\beta(1-\tau^k)]^{-1}[\alpha+\mu+\alpha\beta(1-\tau^k)]^{-1} \;>\; 0 \tag{23}$$

The second equality in (22) follows by using (7), (8), (18), the relation $\alpha F=F_k k$, and (19). The last equality in (22) follows from (20). Substituting (21) and (22) into (16), and also using (19) and (23), gives $V_1/(-V_2)=F/(\beta\tau^x)$, which differentiated becomes (by using (17) and $\tau^x x=\mu F$)

$$p\,dw_0^{i}+k^{i}dp+dS \;=\; -u_2^{-1}\Omega\left(dF/F-d\tau^x/\tau^x\right) \tag{24}$$

where

$$\Omega \;\equiv\; \frac{1+xV_{22}/V_2-xV_{12}/V_1}{V_{21}/V_2-V_{11}/V_1} \tag{25}$$

Note that $\Omega>0$ if private consumption and the environment are non-inferior goods. Next, we have $pdw_0^i+k^idp+dS = pw_0d\gamma^i+p\gamma^i dw_0+k^idp+dS = pw_0d\gamma^i+p\gamma^i dw_0+(k^i-k)dp+kdp+dS$. Use (7) and (8) to substitute for $k^i-k$, and differentiate (10), then we have

Differentiating $p=\alpha(1-\tau^k)F/k$, differentiating (19), using (5) and (9), substituting into (26) and

[10] This is because we have limited ourselves to interior solutions, by requiring the upper limit on $\gamma$. Otherwise, for $\gamma$ greater that the limit, the individual would want the maximum capital subsidy that could be funded, i.e. that drives consumption of the lowest productive individual to zero. Consequently above that limit $\partial\tau^k/\partial\gamma^i=0$.

$$p\,dw_0^i + k^i dp + dS \;=\; pw_0 d\gamma^i + p\gamma^i dw_0 + \beta[1+\beta]^{-1}w(\gamma^i - 1)dp + (\alpha+\mu)dF - p\,dk \tag{26}$$

combining with (24) gives (see the appendix)

$$\frac{1-\mu}{H}d\gamma^i + (\beta\mu + \Omega)\left[\frac{\alpha(\alpha+\mu)}{\alpha+\mu+\alpha\beta(1-\tau^k)}\,\frac{d(1-\tau^k)}{(1-\tau^k)} - \frac{d\tau^x}{\tau^x}\right] \tag{27}$$
$$+ \left[1-\mu+\alpha\beta+\alpha\Omega\right]dw_0/w_0 + (\beta+\Omega)dA/A = 0$$

Equation (20) gives the capital tax as a function of $\gamma^i$ (the identity of a decision maker). The capital tax is monotonically decreasing in $\gamma^i$. Equation (27) gives the environmental tax as a function of $\gamma^i$, the capital tax, and the underlying fundamentals $w_0$ and $A$. Notice that when private consumption and the environment are non-inferior (i.e. $\Omega>0$), then there is a positive relationship between $\gamma^i$ and the desired pollution tax. In the next section we will characterise the political equilibria.

## 5. POLITICO-ECONOMIC EQUILIBRIUM

We have characterised how a hypothetical decisionmaker will choose policy (equations (20) and (27)). We will now characterise the individuals' preferences over candidates. Denote the ability of an arbitrary candidate by $\gamma^*$. Substitute for policy as a function of $\gamma^*$ into the utility function of another individual, $i$, to get an indirect utility function in terms of $\gamma^*$, say $\tilde{V}^i(\gamma^*)$. We can establish that this function is single peaked with the maximum at $\tilde{V}^i(\gamma^i)$:

**Lemma 1** *Assume* A1, A2, A3, A4, A5, A6, *then any individual's preferences over representatives are single peaked.*
*Proof*: See Appendix.

Since we have a one-dimensional choice space (the identity of the decision maker), and preferences are single peaked, then an individual endowed with the median $\gamma$ cannot lose against any other candidate in a binary election. We have a median-voter equilibrium. Throughout this section we denote the median $\gamma$ by $\gamma^*$, and set $i$=* in equations (20) and (27). We will now examine the relationship between inequality (in terms of mean-median distance) and the environmental tax, and (where appropriate) economic growth, when policy is endogenous. We begin with the no-growth case first.

**Proposition 1** *Assume* A1, A2, A3, A4, A5, A6, *and* $w_0$ *fixed, and that private consumption and the environment are non-inferior; then the poorer the median is in relation to the mean in terms of first-period labour income, the lower the pollution tax is, the higher the capital tax, and the lower the aggregate supply of capital in politico-economic equilibrium. More productive economies (higher A) have a higher pollution tax in politico-economic equilibrium. The economy is always at the steady state.*

*Proof*: $\tau^k$ is decreasing in $\gamma^*$, then the result follows from (27). QED.

In the no-growth economy, income inequality reduces the environmental tax and a higher technology level increases the environmental tax. We will now turn to the endogenous-growth case, where past generation's capital accumulation improves (linearly) the productivity of present generation's labour.

**Proposition 2** *Assume* A1, A2, A3, A4, A5, A6, *and* $w_0=\omega k_{-1}$, *and that private consumption and the environment are non-inferior, then the poorer the median is in relation to the mean in terms of first-period labour income, the higher the capital tax, and the lower the growth rate in politico-economic equilibrium. For any given capital stock, the poorer the median is in relation to the mean, the lower the pollution tax is. The economy is always on the steady state growth path.*

*Proof*: Substitute for $w_0$ by using $w_0=\omega k_{-1}$ in (19). This gives

$$k/k_{-1} \;=\; \alpha\beta(1-\tau^k)\,\omega\,/[\alpha+\mu+\alpha\beta(1-\tau^k)] \tag{28}$$

Since $\tau^k$ is decreasing in $\gamma^*$, then the result follows from (28) and (27). QED

Thus, the model allowing for growth produces lower growth for higher inequality. At the same time the model shows a negative relationship between inequality and environmental protection. Therefore as a corollary of Proposition 2, economic growth and environmental protection are positively related in politico-economic equilibrium, when varying the underlying distribution of abilities.

## 6. MODEL VARIATIONS

The model presented above can be used to describe two additional economies. The first one (case I) is a static economy in which output is produced by labour and pollution. Labour and pollution are taxed at possibly different rates, and the tax receipts are redistributed lump-sum to the individuals. Individuals differ in time endowments. This implies that individuals with less productive time will supply less labour (than those with more productive time) if consumption is a normal good. There will then be a redistributive conflict, since the less endowed individuals gain from taxation of labour. This is similar to the Meltzer-Richard (1981) model, but augmented for pollution.

The second one (case II) is an overlapping-generations economy (similar to Renström, 1996, but augmented for pollution). Output in each period is produced by labour (inelastically supplied by the young), capital (supplied by the old), and pollution. The decision about taxes is taken one period in advance (the young decide on taxes to be implemented when they are old). Taxes are levied on capital income and on pollution, and the transfer is given to the old generation. Note that in case (II) ($l \equiv \int \gamma^i \, l d\Gamma(\gamma^i)$) and technology is:

**A3′ Production**

$$F(k, x, l) = A l^{1-\alpha-\mu} k^{\alpha} x^{\mu} \tag{29}$$

where $0<\alpha<1$, $0<\mu<1$, and $\alpha+\mu<1$. $A$ may depend on $k$ in the previous period.

Profit maximisation leads to $w=F_l$, where $w$ is the wage received by the next generation (the present generation receives $w_0$, which is labour's marginal product in the previous period). We can now derive the following results:

*Case I - static model*

**Proposition 3** *Assume* A1, A2, A3, A4, A5, A6, *and* $\alpha+\mu=1$, $w_0$ *fixed, and that private consumption and the environment are non-inferior; then the poorer the median is in relation to the mean (in terms of time endowment), the lower the pollution tax and the higher is the tax on factor k in politico-economic equilibrium. For a given distribution, the greater the productivity is (greater A), the larger the pollution tax is in politico-economic equilibrium.*

*Proof*: $\tau^k$ is decreasing in $\gamma^*$, then the result follows from (27). QED

*Case II - dynamic model; overlapping generations*

In the no endogenous-growth case (i.e. $A$ is constant), we have

**Proposition 4** *Assume* A1, A2, A3′, A4, A5, A6, *and* $\alpha+\mu<1$, *then the following holds*:

(a) *Sufficient for global stability of the economy under the endogenous tax programme that*

$$\beta\mu + \frac{1 + xV_{22}/V_2 - xV_{12}/V_1}{V_{21}/V_2 - V_{11}/V_1} > 0 \tag{30}$$

(b) *If* (30) *holds, then at the steady state, the poorer the median is in relation to the mean, the lower is the pollution tax, the greater is the capital tax, and the smaller is the capital stock.*

(c) *If private consumption and the environment are non-inferior, then out of the steady state (on the growing trajectory) at any level of $k_t$, the poorer the median is in relation to the mean, the lower is the pollution tax, the greater is the capital tax, and the lower is the growth rate.*

*Proof*: Differentiate (29) to obtain $dF/F = dA/A + \alpha dk/k + \mu dx/x$. Differentiating the relation $\tau^x = \mu F/x$ gives $d\tau^x/\tau^x = dF/F - dx/x$. Consequently

$$\frac{dF}{F} = \frac{\alpha}{1-\mu}\frac{dk}{k} + \frac{1}{1-\mu}\left(\frac{dA}{A} - \mu\frac{d\tau^x}{\tau^x}\right) \tag{31}$$

Next, $dw_0/w_0 = dF/F$. Index $k$ in $w_0$ with time-subscript $t$ and substitute (27) into (31) (to eliminate $d\tau^x/\tau^x$), then we have

$$\frac{dw_0}{w_0} = \frac{\alpha}{\alpha+m}\frac{dk}{k} + \frac{1-\mu}{(\alpha+\mu)(\beta\mu+\Omega)}\left(\frac{dA}{A} - \frac{\mu}{H}d\gamma^i\right) - \frac{\mu}{\alpha+m}\,\frac{\alpha(\alpha+\mu)}{\alpha+\mu+\alpha\beta\theta}\,\frac{d(1-\tau^k)}{1-\tau^k} \tag{32}$$

where

$$m \equiv (1-\mu)[(1+\beta)\mu + (1-\alpha)\Omega]/(\beta\mu+\Omega) \tag{33}$$

N.B. $m>0$ if $\beta\mu+\Omega\geq 0$. Use (19) with time index, so the left-hand side equals $k_{t+1}$, differentiate and use (32) (use also (20) to substitute for $d\gamma^*$) to obtain

$$\frac{dk_{t+1}}{k_{t+1}} = \frac{\alpha}{\alpha+m}\frac{dk_t}{k_t} + \frac{1-\mu}{\beta\mu+\Omega}\frac{\alpha+\mu}{\alpha+m}\frac{\Omega+(1-\tau^k)\left[\frac{\alpha\beta\mu}{\alpha+\mu}+\frac{(1-\alpha)(1+\beta)\beta\mu}{\alpha+\mu+\beta(1-\tau^k)}\right]}{\alpha+\mu+\alpha\beta(1-\tau^k)}\frac{d(1-\tau^k)}{1-\tau^k} \tag{34}$$

This equation is useful in proving the rest of the results.

(a) Along the dynamic path $\gamma^*$ and $\tau^k$ (as well as $A$) are constant, then (34) gives $d\ln(k_{t+1})/d\ln(k_t) = \alpha/(\alpha+m)$ which is smaller than unity if $\beta\mu+\Omega>0$ (see (33)). The definition of $\Omega$ (equation (25)) gives (30).

(b) We may drop the time subscripts in (34) and solve for $dk/k$. Then $\partial k/\partial(1-\tau^k) > 0$ if $\beta\mu+\Omega>0$. Consequently $\partial k/\partial\gamma^* > 0$. To prove $\partial\tau^x/\partial\gamma^* > 0$, we proceed as follows. Since $dw_0/w_0 = dF/F$, use (31) (with $dA=0$) to establish that $dw_0/w_0$ is positively related to $dk/k$ and negatively related to $d\tau^x/\tau^x$. Then, the only way (27) can hold (if $\beta\mu+\Omega>0$), as $\gamma^*$ increases [and consequently $k$ and $(1-\tau^k)$ increase], is that $\tau^x$ increases.

(c) The left-hand side of (34) is the increase in next period's capital stock, the larger it is the larger is the growth rate, given the level of $k_t$. Consequently, from (34), $\partial k_{t+1}/\partial(1-\tau^k) > 0$, given $k_t$. That $\partial\tau^x/\partial\gamma^* > 0$ is shown in the same way as in part (b). QED

Thus, the economy is always stable if private consumption and the environment are non-inferior. Furthermore, out of steady state, at any $k_t$, the level of the environmental tax and the growth rate are positively correlated when varying the mean-median distance.

Finally we will examine endogenous growth in the overlapping-generations economy. The source of growth is a capital externality generated by the capital accumulation by the past generation. Not all preferences are able to produce steady-state growth paths, therefore we will further restrict the utility function. We shall assume

**A1′ Individuals' preferences**

Additive separability between private consumption and pollution[11]

$$V^i = \ln(c_1^i) + \beta\ln(c_2^i) - D(x) \tag{35}$$

where $D'(x)>0$, $D''(x)\geq 0$.

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[11] In this case $\Omega\to+\infty$, which does not violate any of the previous equations. E.g. (27) still holds, but the terms multiplied by $\Omega$ sum to zero.

**Proposition 5** *Assume* A1′, A2, A3′, A4, A5, A6, $\alpha+\mu<1$, *and* $A_t=\bar{A}k_{t-1}^{1-\alpha}$. *Define the growth rate as* $g_t = k_t/k_{t-1} - 1$. *Then the following hold.*

(a) *There is a globally stable steady-state growth path, where capital and GDP grow at the same rates.*

(b) *The poorer the median individual is, in relation to the mean, the lower is the steady-state growth rate. For any given capital stock, the pollution tax is smaller.*

(c) *Out of steady state (as well as at the steady state), the growth rate* $g_t$ *and the environmental tax rate* $\tau_t^x$ *are positively related, for any* $k_{t-1}$.

*Proof*: For additively separable preferences, the optimal level of $x$ is constant over time and independent of the identity of the decisive individual. To see this substitute (21) and (22) into (16), and use (19), which gives $V_1/(-V_2)=F/(\beta\tau^x)$. Using the utility function in A1′, as well as the relation $\tau^x x=\mu F$, gives $D'(x)x = \beta\mu$.

(a) Use $w_0=F_l=(1-\alpha-\mu)A_t k_t^{\alpha} x^{\mu} l^{-\alpha-\mu}$, and substitute for $A_t=\bar{A}k_{t-1}^{1-\alpha}$, and insert into (19), to obtain

$$1+g_{t+1} = \tilde{A}\frac{\alpha\beta(1-\tau^k)}{\alpha+\mu+\alpha\beta(1-\tau^k)}(1+g_t)^{\alpha-1} \tag{36}$$

where $\tilde{A}=(1-\alpha-\mu)\bar{A}x^{\mu}l^{-\alpha-\mu}$. Let $z_t=\ln(1+g_t)$, then (36) is a first-order difference equation in $z$. Global stability follows from $1-\alpha<1$ (in fact the growth rate cycles toward its steady-state level). Steady state is when $g_{t+1}=g_t=g$. From (36) we have

$$1+g = \left[\frac{\tilde{A}\alpha\beta(1-\tau^k)}{\alpha+\mu+\alpha\beta(1-\tau^k)}\right]^{1/(2-\alpha)} \tag{37}$$

(b) Since $\partial g/\partial(1-\tau^k) > 0$ by (37), and $\partial(1-\tau^k)/\partial\gamma^*$ by (20), the first part holds. The second part is proven in (c) below.

(c) Since $\tau_t^x = F_x = \mu A_t k_t^{\alpha} x^{\mu-1} l^{1-\alpha-\mu}$. Substituting for $A_t=\bar{A}k_{t-1}^{1-\alpha}$ gives $\tau_t^x = \mu\bar{A}k_{t-1}g_t^{\alpha}x^{\mu-1}l^{1-\alpha-\mu}$, so given $k_{t-1}$, $\partial\tau_t^x/\partial g_t > 0$, which also proves the second part of (b) above. QED

Note that non-inferiority is not stated in Proposition 5, or appears explicitly in the proof. The reason is that the utility function in (35) is additively separable ($V_{12}=0$), which automatically gives normality. Thus, assumption A1′ already assumes normality.

To conclude section 6, the various cases produce the same predictions regarding inequality and environmental protection, and the models allowing for growth, produce lower growth for higher inequality, implying that growth and environmental protection are positively related in politico-economic equilibrium.

## 7. CONCLUSIONS

The paper has explored whether the income distribution within a country is a determinant in shaping political decisions regarding the protection of the environment. We have presented a two-period model where individuals differ in period-1 labour income. This model could also be interpreted as (I) a static model, where individuals differ in productive time, supply labour, and labour is taxed, or (II) an overlapping-generations model, where individuals differ in period-1 labour. In the various modifications, we found a relationship between inequality in terms of median-mean distance and pollution. The driving forces are two-fold. A poorer individual has a lower marginal rate of substitution between the environment and private consumption (if environment is a non-inferior good). This causes a poorer individual to protect the environment less (if she was to decide policy). The second force is that a poorer individual wishes to redistribute, thereby creating inefficiency. If the environment is a non-inferior good, this causes any individual to prefer more private consumption in relation to the environment. These forces work in the same direction.

We also explored the issue of growth. A poorer individual wishes to redistribute more and levy higher capital taxes. This, in our model, hampers capital accumulation and growth. Since we have proved a negative relation between inequality and growth on one hand, and between inequality and environmental protection on the other, we have verified a positive relationship between growth and environmental protection. Our model provides guidance for empirical analysis: it is important to include a measure of income inequality in the studies of growth and the environment. This is left to future research.

## APPENDIX

**Proof of Lemma 1**

Let $\theta$=1-$\tau^k$. We treat $\theta$ and $x$ as functions of the gamma of the decisionmaker, $\gamma^*$. First we prove that $\partial x/\partial\gamma^* < 0$. Use equation (9) to obtain

$$\frac{\partial x}{\partial\gamma^*} = \frac{1}{F_{xx}}\frac{\partial\tau^x}{\partial\gamma^*} - \frac{F_{xk}}{F_{xx}}\frac{\partial k}{\partial\gamma^*} = \frac{F_x}{F_{xx}}\left[\frac{1}{\tau^x}\frac{\partial\tau^x}{\partial\gamma^*} - \alpha\frac{\theta}{k}\frac{\partial k}{\partial\theta}\frac{1}{\theta}\frac{\partial\theta}{\partial\gamma^*}\right] \tag{38}$$

where the second equality follows by $F_{xk}$=$\alpha F_x/k$. Use the derivative of (19) to substitute for $\partial k/\partial\theta$ and we have

$$\frac{\partial x}{\partial\gamma^*} = \frac{F_x}{F_{xx}}\left[\frac{1}{\tau^x}\frac{\partial\tau^x}{\partial\gamma^*} - \alpha\frac{\alpha+\mu}{\alpha+\mu+\alpha\beta\theta}\frac{1}{\theta}\frac{\partial\theta}{\partial\gamma^*}\right] = \frac{F_x}{F_{xx}}\frac{1-\mu}{H(\beta\mu+\Omega)} \tag{39}$$

where the second equality follows by using (27) (for a change in the policy maker's $\gamma^*$, setting d$w_0$= d$A$ = 0). Thus $\partial x/\partial\gamma^* < 0$, at least as long as $\Omega\geq 0$ (i.e. at least as long as environment and private consumption are non-inferior).

Next, using (18) to substitute for $S$, and the relation $p$=$\alpha\theta F/k$, in (4) we have $c_2^i$ = [$\alpha\theta(k^i/k$-1)+$\alpha$+$\mu$]$F$, which substituted into (1) gives

$$V^i = V\left(u(w\gamma^i - k^i, [\alpha\theta(k^i/k-1)+\alpha+\mu]F), x\right) \tag{40}$$

Differentiating with respect to $\gamma^*$, we have

$$\begin{aligned}\frac{\partial V^i}{\partial\gamma^*} = {} & V_1^i u_2^i\left\{\alpha\left(\frac{k^i}{k}-1\right)F - \alpha\frac{k^i}{k}\frac{\theta}{k}\frac{\partial k}{\partial\theta}F + \left[\alpha\theta\left(\frac{k^i}{k}-1\right)+\alpha+\mu\right]F_k\frac{\partial k}{\partial\theta}\right\}\frac{\partial\theta}{\partial\gamma^*} \\ & + V_1^i\left\{u_2^i\left[\alpha\theta\left(\frac{k^i}{k}-1\right)+\alpha+\mu\right]F_x + \frac{V_2^i}{V_1^i}\right\}\frac{\partial x}{\partial\gamma^*}\end{aligned} \tag{41}$$

Equation (41) can be modified as follows: Use the relation $F_k$=$\alpha F/k$, use (7), (8), and (19) to substitute for $k^i/k$, use the derivative of (19) to substitute for $\partial k/\partial\theta$, and use (20) evaluated at $\gamma^*$. Then we have

$$\frac{\partial V^i}{\partial\gamma^*} = V_1^i u_2^i\alpha F\frac{\alpha+\mu+\beta\theta}{1+\beta}(\gamma^i-\gamma^*)\frac{1}{\theta}\frac{\partial\theta}{\partial\gamma^*} + \left\{V_1^i u_2^i\left[\alpha\theta\left(\frac{k^i}{k}-1\right)+\alpha+\mu\right]F_x + V_2^i\right\}\frac{\partial x}{\partial\gamma^*} \tag{42}$$

The first term is positive (zero, negative) when $\gamma^i$> (=,<) $\gamma^*$, since $\partial\theta/\partial\gamma^* > 0$. The term within curly brackets is the first-order variation of $V^i$ with respect to $x$, and is zero when the individual type coincides with that of the decision maker (i.e. when $\gamma^i=\gamma^*$). When it is positive (negative) the individual wishes larger (smaller) level of $x$. Since $\partial x/\partial\gamma^* < 0$, this is accomplished by reducing (increasing) $\gamma^*$. That is, the term in square brackets is positive (negative) when $\gamma^* - \gamma^i$ >(<) 0. Since $\partial x/\partial\gamma^* < 0$, the whole second term takes the same sign as sign($\gamma^i$-$\gamma^*$). Thus sign($\partial V^i/\partial\gamma^*$) = sign($\gamma^i$-$\gamma^*$). QED

**Derivation of (27)**

Let $\theta=(1-\tau^k)$. Differentiating the equality $p=\alpha\theta F/k$ gives

$$\frac{dp}{p} = \frac{d\theta}{\theta} + \frac{dF}{F} - \frac{dk}{k} \tag{43}$$

Differentiating (19) gives

$$\frac{dk}{k} = \frac{dw_0}{w_0} + \frac{\alpha+\mu}{\alpha+\mu+\alpha\beta\theta}\frac{d\theta}{\theta} \tag{44}$$

Using (44) to substitute for $d\theta/\theta$ in (43) gives

$$\frac{dp}{p} = \frac{\alpha\beta\theta}{\alpha+\mu}\frac{dk}{k} - \frac{\alpha+\mu+\alpha\beta\theta}{\alpha+\mu}\frac{dw_0}{w_0} + \frac{dF}{F} \tag{45}$$

Using (45) in (26) to substitute for $dp/p$, and equation (19) to substitute for $k/w_0$, and using (20) to substitute for terms involving $\gamma^i$-1, gives

$$p\,dw_0^i + k^i dp + dS = p w_0 H\left[\frac{d\gamma^i}{H} + (1+\alpha\beta)\frac{dw_0}{w_0} - \alpha\beta\frac{dk}{k} + \beta\frac{dF}{F}\right] \tag{46}$$

where $H$ is defined as in (23). Use (31) in (46)

$$p\,dw_0^i + k^i dp + dS = p w_0 H\left[\frac{d\gamma^i}{H} + (1+\alpha\beta)\frac{dw_0}{w_0} + \frac{\beta}{1-\mu}\left(\frac{dA}{A} - \mu\frac{d\tau^x}{\tau^x}\right) + \frac{\alpha\beta\mu}{1-\mu}\frac{dk}{k}\right] \tag{47}$$

Use (31) in the right-hand side of (24)

$$p\,dw_0^i + k^i dp + dS \;=\; -\frac{\Omega}{u_2}\left[\frac{1}{1-\mu}\left(\frac{dA}{A} - \frac{d\tau^x}{\tau^x}\right) + \frac{\alpha}{1-\mu}\frac{dk}{k}\right] \tag{48}$$

Using (44) in (47) and (48) to substitute for *dk*/*k*, and combining (47) and (48) gives

$$\begin{aligned} & p w_0 d\gamma^i + \frac{\beta\mu p w_0 H + \Omega/u_2}{1-\mu}\left[\frac{\alpha(\alpha+\mu)}{\alpha+\mu+\alpha\beta\theta}\frac{d\theta}{\theta} - \frac{d\tau^x}{\tau^x}\right] \\ & + \frac{(1-\mu+\alpha\beta)p w_0 H + \alpha\Omega/u_2}{1-\mu}\frac{dw_0}{w_0} + \frac{p w_0 H\beta + \Omega/u_2}{1-\mu}\frac{dA}{A} \;=\; 0 \end{aligned} \tag{49}$$

Next, use the last equality of (22) and divide through by $pw_0H$, and premultiply by (1-μ), gives (27).

# AUXILIARY APPENDICES

## APPENDIX P - Restrictions for interior solution

We shall find conditions under which we have interior solutions regarding policy choice. At the same time we will show that $\partial\tau^k/\partial\gamma^i < 0$ for $\tau^k$ given by equation (20) for interior solutions (if there is a corner then $\partial\tau^k/\partial\gamma^i = 0$). Define $\theta=1-\tau^k$. Take indirect utility as in (40) as starting point. Differentiate with respect to $\theta$, to obtain the first-order variation

$$\begin{aligned}\frac{\partial V^i}{\partial\theta} &= V_1^i u_2^i\left\{\alpha\left(\frac{k^i}{k}-1\right)F - \alpha\frac{k^i}{k}\frac{\theta}{k}\frac{\partial k}{\partial\theta}F + \left[\alpha\theta\left(\frac{k^i}{k}-1\right)+\alpha+\mu\right]F_k\frac{\partial k}{\partial\theta}\right\} \\ &= F V_1^i u_2^i\left\{\alpha\left(\frac{k^i}{k}-1\right)\left(1-\frac{\theta}{k}\frac{\partial k}{\partial\theta}\right) - \alpha\frac{\theta}{k}\frac{\partial k}{\partial\theta} + \left[\alpha\theta\left(\frac{k^i}{k}-1\right)+\alpha+\mu\right]\frac{F_k}{F}\frac{\partial k}{\partial\theta}\right\} \\ &= \alpha F V_1^i u_2^i\left\{\left(\frac{k^i}{k}-1\right)\left(1-\frac{\theta}{k}\frac{\partial k}{\partial\theta}\right) - \frac{\theta}{k}\frac{\partial k}{\partial\theta} + \left[\alpha\theta\left(\frac{k^i}{k}-1\right)+\alpha+\mu\right]\frac{1}{k}\frac{\partial k}{\partial\theta}\right\}\end{aligned} \tag{P1}$$

where the last equality follows by using the relation $F_k=\alpha F/k$.
Use (7) and (8) to substitute for $k^i/k$ -1,

$$\frac{\partial V^i}{\partial\theta} = \alpha F V_1^i u_2^i\left\{\frac{\beta}{1+\beta}(\gamma^i-1)\frac{w_0}{k}\left(1-\frac{\theta}{k}\frac{\partial k}{\partial\theta}\right) - \frac{\theta}{k}\frac{\partial k}{\partial\theta} + \left[\frac{\alpha\beta\theta}{1+\beta}(\gamma^i-1)\frac{w_0}{k}+\alpha+\mu\right]\frac{1}{k}\frac{\partial k}{\partial\theta}\right\} \tag{P2}$$

Use the derivative of (19) to substitute for $\partial k/\partial\theta$ in (P2), then we have

$$\begin{aligned}\frac{\partial V^i}{\partial\theta} &= \frac{\alpha F V_1^i u_2^i}{\alpha+\beta+\alpha\beta\theta}\left\{\frac{\beta(\gamma^i-1)}{1+\beta}\frac{w_0}{k}\alpha\beta\theta - (\alpha+\mu) + \left[\frac{\alpha\beta\theta}{1+\beta}(\gamma^i-1)\frac{w_0}{k}+\alpha+\mu\right]\frac{\alpha+\mu}{\theta}\right\} \\ &= \frac{\alpha F V_1^i u_2^i}{\alpha+\beta+\alpha\beta\theta}\left\{\frac{\alpha\beta(\gamma^i-1)}{1+\beta}\frac{w_0}{k}[\alpha+\mu+\beta\theta] + (\alpha+\mu)[\alpha+\mu-\theta]\right\}\end{aligned} \tag{P3}$$

Use (19) to substitute for $k$ in (P3)

$$\frac{\partial V^i}{\partial\theta} = \frac{\alpha}{\theta}F V_1^i u_2^i\left\{(\gamma^i-1)\frac{\alpha+\mu+\beta\theta}{1+\beta} + (\alpha+\mu)\frac{\alpha+\mu-\theta}{\alpha+\mu+\alpha\beta\theta}\right\} \tag{P4}$$

When the first-order variation is zero we have an interior solution, and equation (20) holds.

This is always the case for $\gamma^i \leq 1$. However, for arbitrarily large $\gamma^i$ the first-order variation will be positive, implying that the capital-subsidy ($\theta>1$) goes to a corner. The capital-subsidy is financed by a lump-sum tax, *-S*, and the corner is characterized by the maximum implementable lump-sum tax, which in turn is the one which drives the consumption of the lowest-ability person to zero. To focus on interior solutions we need to find the maximum $\gamma^i$ for which the first-order variation (FOV) is zero. We may write the FOV (P4) as

$$\frac{\partial V^i}{\partial \theta} = \frac{\alpha}{\theta} F(\alpha + \mu + \beta\theta) V_1^i u_2^i \left\{ \frac{\gamma^i - 1}{1+\beta} - M \right\} \tag{P5}$$

where

$$M \equiv \frac{\alpha + \mu}{\alpha + \mu + \beta\theta} \frac{\theta - \alpha - \mu}{\alpha + \mu + \alpha\beta\theta} \tag{P6}$$

Since $M$ reaches a maximum with respect to $\theta$ (see below) there is a largest value of $\gamma^i$ for which the FOV is zero. To find the maximum of $M$, differentiate (P6) with respect to $\theta$

$$\begin{aligned}
\frac{\partial M}{\partial \theta} &= \frac{(\alpha + \mu)\left[(\alpha + \mu)(1+\beta)(\alpha + \mu + \alpha\beta\theta) - \alpha\beta(\theta - \alpha - \mu)(\alpha + \mu + \beta\theta)\right]}{(\alpha + \mu + \beta\theta)^2 (\alpha + \mu + \alpha\beta\theta)^2} \\
&= \frac{(\alpha + \mu)^3 \alpha \left[ \frac{1 + \beta + \alpha\beta}{\alpha} + 2\beta \frac{\beta\theta}{\alpha + \mu} - \left( \frac{\beta\theta}{\alpha + \mu} \right)^2 \right]}{(\alpha + \mu + \beta\theta)^2 (\alpha + \mu + \alpha\beta\theta)^2} \\
&= \frac{(\alpha + \mu)^3 \alpha \left( \beta + \sqrt{(1+\beta)(1/\alpha + \beta)} - \frac{\beta\theta}{\alpha + \mu} \right) \left( \sqrt{(1+\beta)(1/\alpha + \beta)} - \beta + \frac{\beta\theta}{\alpha + \mu} \right)}{(\alpha + \mu + \beta\theta)^2 (\alpha + \mu + \alpha\beta\theta)^2}
\end{aligned} \tag{P7}$$

Clearly $M$ reaches a global maximum at

$$\frac{\beta\theta}{\alpha + \mu} = \beta + \sqrt{(1+\beta)(1/\alpha + \beta)} \tag{P8}$$

since for $\theta$ smaller (greater) than the level in (P8), $M$ is increasing (decreasing).

Inserting (P8) into (P6) gives the maximum $M$:

$$M^{\max} = \frac{1}{\beta}\frac{\sqrt{(1+\beta)(1/\alpha+\beta)}}{\left(1+\alpha\beta+\alpha\sqrt{(1+\beta)(1/\alpha+\beta)}\right)\left(1+\beta+\sqrt{(1+\beta)(1/\alpha+\beta)}\right)}$$

$$= \frac{1}{\beta}\frac{\sqrt{(1+\beta)(1/\alpha+\beta)}}{2(1+\beta)(1+\alpha\beta) + (1+\alpha+2\alpha\beta)\sqrt{(1+\beta)(1/\alpha+\beta)}} \tag{P9}$$

$$= \frac{1}{\beta}\frac{1}{2\alpha\sqrt{(1+\beta)(1/\alpha+\beta)} + 1+\alpha+2\alpha\beta}$$

where the second equality follows by multiplying the factors in the denominator.
This implies (combining (P5) and (P9))

$$\gamma^{\max} - 1 = \frac{1}{2\alpha\beta\sqrt{\dfrac{1/\alpha+\beta}{1+\beta}} + \dfrac{\beta}{1+\beta}(1+\alpha+2\alpha\beta)} \tag{P10}$$

For any $\gamma^i \leq \gamma^{\max}$ we have an interior solution (provided the consumption of the lowest ability does not go negative, which we investigate below).

Furthermore, for $\gamma^i < \gamma^{\max}$ we have, by (20), $\partial\theta/\partial\gamma^i > 0$ (i.e. $\partial\tau^k/\partial\gamma^i < 0$). To see this take the differental of (20)

$$\frac{d\gamma^i}{1-\gamma^i} = \left[\frac{\beta}{\alpha+\mu+\beta\theta} + \frac{\alpha\beta}{\alpha+\mu+\alpha\beta\theta} + \frac{1}{\alpha+\mu-\theta}\right]d\theta$$

$$= \frac{(\alpha+\mu)^2\alpha\left[\dfrac{1+\beta+\alpha\beta}{\alpha} + 2\beta\dfrac{\beta\theta}{\alpha+\mu} - \left(\dfrac{\beta\theta}{\alpha+\mu}\right)^2\right]}{(\alpha+\mu+\beta\theta)(\alpha+\mu+\alpha\beta\theta)(\alpha+\mu-\theta)}d\theta \tag{P11}$$

$$= \frac{(\alpha+\mu)^2\alpha\left(\beta+\sqrt{(1+\beta)(1/\alpha+\beta)} - \dfrac{\beta\theta}{\alpha+\mu}\right)\left(\sqrt{(1+\beta)(1/\alpha+\beta)} - \beta + \dfrac{\beta\theta}{\alpha+\mu}\right]}{(\alpha+\mu+\beta\theta)(\alpha+\mu+\alpha\beta\theta)(\alpha+\mu-\theta)}d\theta$$

Since sign($1-\gamma^i$)=sign($\alpha+\mu-\theta$), $\partial\theta/\partial\gamma^i > 0$ if $\theta < \alpha+\mu + \frac{\alpha+\mu}{\beta}\sqrt{(1+\beta)(1/\alpha+\beta)}$, which is the case if $\gamma^i < \gamma^{\max}$ (see (P8),(P9),(P10)).

We will next investigate the condition guaranteeing $c_1^i = w_0\gamma^i - k^i > 0$, for all $i$. The condition may bite at the lowest ability. Using (7) we write the inequality

$$c_1^{\min} = \frac{1}{1+\beta}\left(w_0\gamma^{\min} + \frac{S}{p}\right) > 0 \tag{P12}$$

As long as the term within parentheses is positive we have an interior solution.

$$w_0\gamma^{\min} > -\frac{S}{p} = \frac{\alpha\theta-\alpha-\mu}{\theta}\frac{F}{F_k} = \frac{\alpha\theta-\alpha-\mu}{\alpha\theta}k \tag{P13}$$

where the first equality follows by (18), and the second by using $\alpha F=F_k k$. Use (19) to substitute for $k$ in (P13), then we have

$$\gamma^{\min} > \beta\frac{\alpha\theta-\alpha-\mu}{\alpha+\mu+\alpha\beta\theta} \tag{P14}$$

which alternatively may be written as

$$\theta < \frac{\alpha+\mu}{\alpha\beta}\frac{\beta+\gamma^{\min}}{1-\gamma^{\min}} \tag{P15}$$

Substituting (P15), but with equality, into (20) gives

$$\gamma^{\max}-1 = \frac{1-\gamma^{\min}}{\beta}\frac{(1-\alpha)\beta+(1+\alpha\beta)\gamma^{\min}}{\alpha+\beta+(1-\alpha)\gamma^{\min}} \tag{P16}$$

Consequently, if $\gamma^{\min} \leq \gamma^i < \gamma^{\max}$, for all $i$, and $\gamma^{\max}$ equals the lowest value of either (P10) or (P16), then we have interior solutions.

**APPENDIX Q - Derivation of second equality of (22)**

First

$$\frac{pk^i + S}{\beta} = \frac{1}{\beta}\left(p(k^i - k) + pk + S\right)$$
$$= \frac{p}{\beta}\left(\frac{\beta}{1+\beta} w_0(\gamma^i - 1) + k + \frac{S}{p}\right) \tag{Q1}$$

where the last equality follows by using (7) and (8) to substitute for $k^i$-$k$.

Next, (18) gives

$$\frac{S}{p} = \frac{\alpha + \mu - \alpha(1-\tau^k)}{p} F$$
$$= \frac{\alpha + \mu - \alpha(1-\tau^k)}{1-\tau^k} \frac{F}{F_k} \tag{Q2}$$
$$= \frac{\alpha + \mu - \alpha(1-\tau^k)}{1-\tau^k} \frac{k}{\alpha}$$

where the last equality follows by using $F_k k = \alpha F$.

Substituting (Q2) into (Q1) gives

$$\frac{pk^i + S}{\beta} = \frac{p}{\beta}\left(\frac{\beta}{1+\beta} w_0(\gamma^i - 1) + \frac{\alpha + \mu}{\alpha(1-\tau^k)} k\right) \tag{Q3}$$

Using (19) in (Q3) to substitute for $k$ gives second equality of (22).

## APPENDIX R - Derivation of (46)

Using (45) in (26) to substitute for $dp/p$ gives

$$
\begin{aligned}
& p\,dw_0^i + k^i dp + dS \\
&= p w_0 d\gamma^i + p\gamma^i dw_0 + \frac{\beta}{1+\beta} w_0(\gamma^i - 1)p\left[\frac{\alpha\beta\theta}{\alpha+\mu}\frac{dk}{k} - \frac{\alpha+\mu+\alpha\beta\theta}{\alpha+\mu}\frac{dw_0}{w_0} + \frac{dF}{F}\right] \\
&\quad + (\alpha+\mu)dF - p\,dk \\
&= p w_0 d\gamma^i + p\left(\gamma^i - (\gamma^i - 1)\frac{\beta}{1+\beta}\frac{\alpha+\mu+\alpha\beta\theta}{\alpha+\mu}\right)dw_0 \\
&\quad + \left(\alpha+\mu + \frac{\beta}{1+\beta}\frac{w_0(\gamma^i - 1)p}{F}\right)dF + p\left(\frac{\beta}{1+\beta}(\gamma^i - 1)\frac{w_0}{k}\frac{\alpha\beta\theta}{\alpha+\mu} - 1\right)dk \\
&= p w_0\left\{d\gamma^i + A\frac{dw_0}{w_0} + B\frac{dF}{F} + C\frac{dk}{k}\right\}
\end{aligned}
\tag{R1}
$$

where

$$
\left.
\begin{aligned}
A &\equiv \gamma^i - (\gamma^i - 1)\frac{\beta}{1+\beta}\frac{\alpha+\mu+\alpha\beta\theta}{\alpha+\mu} \\
B &\equiv \frac{(\alpha+\mu)F}{w_0 p} + \frac{\beta}{1+\beta}(\gamma^i - 1) \\
C &\equiv \frac{\beta}{1+\beta}(\gamma^i - 1)\frac{\alpha\beta\theta}{\alpha+\mu} - \frac{k}{w_0}
\end{aligned}
\right\}
\tag{R2}
$$

Next

$$
\begin{aligned}
A &= 1 + \gamma^i - 1 - (\gamma^i - 1)\frac{\beta}{1+\beta}\frac{\alpha+\mu+\alpha\beta\theta}{\alpha+\mu} \\
&= 1 + (\gamma^i - 1)\left(1 - \frac{\beta}{1+\beta}\frac{\alpha+\mu+\alpha\beta\theta}{\alpha+\mu}\right) \\
&= 1 + (\gamma^i - 1)\frac{\alpha+\mu-\alpha\beta^2\theta}{(1+\beta)(\alpha+\mu)}
\end{aligned}
\tag{R3}
$$

Use equation (20) to substitute for $\gamma^i$-1 in (R3), then we have

$$
\begin{aligned}
A &= 1 - \frac{(\alpha + \mu - \alpha\beta^2\theta)[\alpha + \mu - \theta]}{[\alpha + \mu + \beta\theta][\alpha + \mu + \alpha\beta\theta]} \\
&= \frac{\theta(\alpha + \mu)[\beta + \alpha\beta + \alpha\beta^2 + 1]}{[\alpha + \mu + \beta\theta][\alpha + \mu + \alpha\beta\theta]} \\
&= \frac{\theta(1 + \beta)(\alpha + \mu)[1 + \alpha\beta]}{[\alpha + \mu + \beta\theta][\alpha + \mu + \alpha\beta\theta]} \\
&= (1 + \alpha\beta)H
\end{aligned}
\tag{R4}
$$

where the last equality follows from (23).

To modify the expression for $B$ in (R2), notice that $F/p = F/(\theta F_k) = Fk/(\theta F_k k) = Fk/(\theta\alpha F) = k/(\theta\alpha)$. Then

$$
B = \frac{\alpha + \mu}{\alpha\theta}\frac{k}{w_0} + \frac{\beta}{1 + \beta}(\gamma^i - 1)
\tag{R5}
$$

Use equations (19) and (20) to substitute for $k/w_0$ and $\gamma^i$-1, respectively, in (R5), then we have

$$
\begin{aligned}
B &= -\frac{\beta(\alpha + \mu)[\alpha + \mu - \theta]}{[\alpha + \mu + \beta\theta][\alpha + \mu + \alpha\beta\theta]} + \frac{(\alpha + \mu)\beta}{\alpha + \mu + \alpha\beta\theta} \\
&= \frac{\beta\theta(1 + \beta)(\alpha + \mu)}{[\alpha + \mu + \beta\theta][\alpha + \mu + \alpha\beta\theta]} \\
&= \beta H
\end{aligned}
\tag{R6}
$$

where the last equality follows from (23).

Use equations (19) and (20) to substitute for $k/w_0$ and $\gamma^i$-1, respectively, in the expression for $C$ in (R2), then we have

$$
\begin{aligned}
C &= -\frac{\alpha\beta^2\theta[\alpha + \mu - \theta]}{[\alpha + \mu + \beta\theta][\alpha + \mu + \alpha\beta\theta]} - \frac{\alpha\beta\theta}{\alpha + \mu + \alpha\beta\theta} \\
&= -\frac{\alpha\beta\theta(1 + \beta)(\alpha + \mu)}{[\alpha + \mu + \beta\theta][\alpha + \mu + \alpha\beta\theta]} \\
&= -\alpha\beta H
\end{aligned}
\tag{R7}
$$

where the last equality follows from (23).

Substituting the last lines of (R4), (R6), and (R7) into (R1) gives

$$p\,dw_0^i + k^i dp + dS \;=\; p\,w_0 \left\{ d\gamma^i \;+\; (1+\alpha\beta)\,H\frac{dw_0}{w_0} \;+\; \beta H\frac{dF}{F} \;-\; \alpha\beta H\frac{dk}{k} \right\} \qquad \text{(R8)}$$

which is equation (46).

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