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Abstract

Why would a political elite voluntarily dilute its political power by extending the franchise? This paper develops a dynamic recursive framework for studying voter enfranchisement. We study properties of *dynamic enfranchisement games*, dynamic games in which political rights evolve over time. Each period, private decisions of citizens co-mingle with government policies to act upon a state variable such as capital stock, a public good, or the likelihood of an insurrection. Policies are determined by the median voter of a potentially restricted franchise. The enfranchised group can choose, through its median voter, to expand the set of citizens with voting rights. In this way, each period's median voter can effectively delegate decision authority to a new median in the next period. We characterize the equilibria of a dynamic enfranchisement game by its Euler equations. In certain games, the equilibria generate paths that display a gradual, sometimes uneven history of enfranchisement that is roughly consistent with observed patterns of extensions. Our main result shows that extensions of the franchise occur in a given period *if and only if* the private decisions of the citizenry have a net positive spillover to the dynamic payoff of the current median voter. The size of the extension depends on the size of the spillover. Since the class of games we study can accommodate a number of proposed explanations for franchise extension (e.g., the threat of insurrection, or ideological or class conflict within the elite, etc), the result suggests a common causal mechanism for these seemingly different explanations. We describe a number of parametric environments that correspond to the various explanations, and show how the mechanism works in each.

Keywords: Dynamic games, voter enfranchisement, franchise extension equilibria

JEL codes: C73, D72, D78.

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"There is no more invariable rule in the history of society. The further electoral rights are extended, the greater the need for extending them; for after each concession, the strength of the democracy increases, and its demands increase with its strength."

Alexis de Tocqueville[1]

1 Introduction

Voluntary expansion of political rights by a ruling elite is at first glance paradoxical. The elite, after all, dilutes its power when it extends these rights to others. Yet, significant extensions of the voting franchise took place in Europe throughout the 19th and early 20th centuries. Instances of franchise extensions date back, in fact, much further. The constitutional reforms of Cleisthenes in 508 BC in Athens was arguably an early form of franchise extension.[2] Another early instance occurred in 494 BC, when the patricians in the early Roman Republic conceded the right of the plebs (the "commoners") to participate in the election of magistrates.

This paper examines the determinants of franchise extension. We have two goals in mind. First, rather than describing a specific, stylized model to "explain" the history of voting rights, we propose a general framework in which competing explanations of franchise extention can be usefully evaluated. The few existing models tend to differentiate themselves by whether franchise extensions are modeled as externally (i.e., the threat of revolution) or internally (i.e., political competition between members of the elite) driven conflicts. We seek a "canonical" framework that can accommodate, as special cases, the essential elements of most existing models and existing explanations of the franchise. We review some of these later in Section 2. Second, we want a model which produces outcomes that are consistent, broadly speaking, with the observations on franchise extensions. Though the "data" of enfranchisements are often hard to interpret, we identify certain tendencies associated with many or most observed extensions of rights. These observations are also discussed in Section 2.

To address these goals, we specify a class of dynamic games in which the set of eligible voters is endogenously determined each period. We refer to these as *dynamic enfranchisement games*. Dynamic enfranchisement games are dynamic games in which political rights evolve as a fully dynamic recursive phenomenon.

Specifically, we posit a society of n infinitely lived citizens. Each period, private decisions of citizens co-mingle with a government policy to determine the value of an "economic" state variable in the subsequent period. This economic state, which may be a capital stock, a public good, or the likelihood of an insurrection, evolves according to a simple non-stochastic transition function of the previous period's state, private actions, and the policy decision. The government policy such as a tax, a public expenditure, or public investment, is determined

[1] Alexis de Tocqueville, *Democracy in America*, Vol. 1, ch.4

[2] Among other things, these reforms delineated citizenship and allowed for participation in the citizen Assembly. See Fine, (1983).

each period by the median voter from a possibly restricted set of eligible voters. Each citizen's private decision may affect others. For example, the decision may be labor effort, or savings, or voluntary contributions to a public good, or participation in a popular revolt. These decisions spill over to others' payoffs either directly by entering their preferences or indirectly through changes in the state. Payoffs in a given period depend on these decisions and on the current state.

A key characteristic of the model is that political rights are explicitly chosen to solve a strategic delegation problem. Initially, the franchise is restricted — a subset of the citizens has voting rights. The current median voter of the restricted group, however, may choose to expand the set of citizens with voting rights. In this way, each period's median voter can effectively delegate decision authority to a new median in the next period by changing the set of eligible voters. Because the franchise option is a carefully calibrated instrument in the hands of the currently enfranchised, universal suffrage need not result. A current median may choose to extend the vote to only a subset of the remaining citizens.

Moreover, this strategic delegation is *recursive*. Since no commitment is attributed to a current franchise extension, an extension of rights is not a once-and-for-all decision. A franchise extension now does not preclude the future enfranchised group from extending even further later on. Consequently, extension may be a slow, gradual process by which elites extend to lessor elites, who proceed eventually to elites-in-waiting, and so forth.

With the effective delegation of decision authority from one median voter to another, a complete description of the state in each period is given by the economic state variable and the identity of the current median voter (the "political state" variable). A *Franchise Extension equilibrium* is a state-contingent profile of private actions, and public policies and franchise decisions that constitutes a Markov Perfect equilibrium of the game in which, in each state, the policy and franchise choice is determined by a median voter.

Franchise Extension equilibria are shown to exhibit partial, gradual, and possibly uneven franchise extensions. The unevenness may be due to the particular evolution of the economic state variable, or it may be due to peculiarities in the distribution of heterogeneous citizens. We provide a characterization of franchise extension equilibrium in terms of its Euler equations, each corresponding to a participant's decision problem. These Euler equations are analogous to those in dynamic politico-economic models of policy such as found in the "Generalized Euler equation" approach of Klein, Krusell, and Ríos Rull (2002). However, Euler equations in dynamic enfranchisement games contain strategic interaction terms not present in politico-economic models.

In fact, these extra terms are the key to understanding franchise extension. Our main result shows that an extension of the franchise occur in a given period *if and only if* the private decisions of the citizenry have a net positive (marginal) spillover to the dynamic payoff of the current median voter. The size of the extension depends on the size of the spillover.

Among other things, the result implies that in the absence of private decisions of the citizens, policies alone cannot cause a pivotal decision maker to relinquish power. Private decisions of the citizenry represent an implicit "policy-relevant" externality that the pivotal decision maker does not control. Because of the dynamic nature of the problem, current "policy-bribes" cannot induce the appropriate effort from the public since they do not guarantee favorable policies in the future.

The franchise extension, however, does offer a guarantee. A change in voting rights places decision authority in the hands of a different pivotal voter in the future. Hence, franchise extensions represent a credible commitment to future policies that are closer to those preferred by the disenfranchised citizens. If this elicits a positive spillover in their effort choices, the pivotal voter today is willing to sacrifice his power.

The idea that franchise is a commitment device was also explored in a seminal paper on the franchise by Acemoglu and Robinson (2000). They posit a model in which a ruling elite can choose whether in any period to make a once-and-for-all, universal extension of voting rights to the rest of the population. The motive is to pre-empt a threat of uprising or revolution. We refer to this pre-emption motive as the "external conflict" explanation. The external conflict explanation contrasts with an "internal conflict" explanation, an example of which is a recent paper by Lizzeri and Persico (2003). According to "internal conflict" explanation, rights are extended to build support in an ideological or class conflict within the elite.

Section 2 discusses similarities and differences between our approach and these and other models of franchise extension. Section 3 describes the basic framework. We show by means of several examples that dynamic enfranchisement games are broad enough to accommodate both "internal" and "external conflict" explanations. The results therefore suggest *a common causal mechanism that underlies both types of rationale*. In Section 4 we characterize Franchise Extension equilibria that admit a *first order characterization*, i.e, that satisfy and are fully characterized by interior Euler equations. The main results are described there. Section 5 finds explicit analytical solutions in a number of distinct environments. These cases illustrate how the model can produce franchise extension paths that are roughly consistent with observed political reforms. Section 6 contains concluding remarks. Section 7 is an Appendix with proofs of the main results.

2 Three Observations and Two Types of Models

Many of the franchise extensions observed throughout history have common characteristics. There are three qualitative characteristics of observed franchise extensions that the present framework should confront.

(I) Most extensions are *partial* extensions. Historically, ruling elites have not had to choose exclusively between dictatorship and universal suffrage. More often, voting rights are offered to the "adjacent" group in the social hierarchy. Often the restricted franchise was defined by wealth.[3] Finer (1997, p. 336) writes of nascent democracy in the Greek city states:

[3] The term *timocracy* was introduced by Aristotle to characterize systems restricted in this way (see

"In the earliest forms of restricted participation, that is, in the oligarchies, a property qualification constituted the basis for full citizenship. Later, in some cities, all sources of wealth were put on equality with land, and citizens' rights and duties were gradated according to one's riches."

In the 19th century, England partially expanded along lines of wealth or property ownership as well. However, in Italy, the franchise was granted to citizens who passed certain educational as well as financial criteria in 1849. 19th century Prussia presents an interesting case: in 1849, voting rights were extended to most citizens, but these rights were accorded proportionately to the percentage of taxes paid.[4] Finally, even today in most countries the franchise is usually restricted in some way.[5]

(II) Extensions are typically *gradual* processes, not one shot decisions. England's history bears this out. A brief chronology of 19th and early 20th century franchise extensions in the U.K. indicates a gradual broadening of political rights.[6]

1830	Voting franchise restricted to some 2% of population
1832	Reform Act extends franchise to 3.5% of population
1867	Second Reform Act extends to some 7.7% of population
1884	Extension to 15% of population
1918	Universal male (over 21) suffrage and female (over 30) suffrage
1928	Universal suffrage (over 21)

Franchise extension in England had, in fact, a longer history whose beginnings predated these extensions. In a number of other European countries, gradual extensions corresponded to technological innovations such as those of the industrial revolution. In ancient Rome, extensions occurred as the state's boundaries gradually expanded.

(III) Extensions are often *uneven*. In many countries, large delays, lasting decades or longer have occurred between successive extensions. Again, England's chronology is an example. Rights were extended in fits and starts. In the Netherlands, voting rights were extended in 1857 from 2% to 14% of the population. The next major expansion occurred in 1894 when rights were extended to all males. In Italy, universal male suffrage in 1912 was preceded by an extension in 1882 (14%) which, in turn, was preceded by the partial extension

http://classics.mit.edu/Aristotle/nicomachaen.html).

[4] The electorate was divided into three groups, each group given equal weight in the voting. The wealthiest individuals who accounted for the first third of taxes paid accounted for 3.5% of the population. The next wealthiest group — the "middle class" — accounted for 10-12% of the population. The rest of the population (about 85%) accounted for the remaining third of the voting power.

[5] In the U.S., convicted felons cannot typically vote, and, until recently, "on-site" registration in some states effectively limits voting rights of the immobile and the mentally ill.

[6] Finer (1997), p. 1638.

in 1849. In the ancient Roman Republic, various extensions not associated with territorial expansion occurred in 494 BC, 336 BC, and 287 BC.[7]

Very little is known about whether and what types of models can accommodate these criteria. There is a sizable informal literature in political science. For this we refer the reader to the useful surveys in Acemoglu and Robinson (AR) (2000) and Lizzeri and Persico (2003). We concentrate instead on the much sparser formal modeling that has been done, starting with Acemoglu and Robinson's work (2000, 2001), itself.[8] The essential claim in Acemoglu and Robinson's work is that the primary force behind, at least, the 19th century extensions was the desire by the elite to head off social unrest. AR postulate a dynamic game in which the timing of an all-or-nothing franchise extension is determined by the median voter of a ruling elite. A state variable evolves stochastically which determines the rate of success of any popular revolt. In the absence of a franchise decision, the disenfranchised mob, acting as a unitary actor, revolts in certain states of the world, and refrains in others. Redistribution to the disenfranchised is not a credible deterrent since it will only be used in threatening states of the world. By contrast, an extension of voting rights to the entire population puts the decision in the hands of the population median who chooses redistribution in all states. Extensions are then a credible way to buy-off the populace. Hence, franchise extensions pre-empt revolutions.

A somewhat similar motive for extending the franchise appears in a model by Conley and Temimi (2001). They examine a game in which extension of voting rights occurs because of the potential for the disenfranchised group to impose costs on the elite through rioting and protest if the franchise is not extended. These costs induce a trade-off not unlike that in the dynamic counterpart by AR. Expansion entails a loss of decision making power. However, it also pre-empts the costly social unrest. Unlike AR, the CT model cannot address dynamic issues such as timing of the franchise decision. However, unlike AR, they do address the explicit free rider problems in the decision to revolt.

These "external conflict" models may be contrasted with an alternative "internal conflict" story in which political competition within the elite leads one or another faction to reach out to disenfranchised citizens. Lizzeri and Persico (2003) formulate a game with elements of this story. They examine a static, random voter model of spatial competition between two candidates who vie for votes among a restricted franchise. The competition creates an inefficiency when there are relatively few eligible voters. A franchise extension is shown to lead to a more efficient electoral process in terms of the allocation of expenditure between public goods and private transfers.

In a similar vein, Barbera, Maschler, and Shalev (2001) examine a finite horizon dynamic

[7] In 336 BC, one of the consulships became available for election by plebians. In 287 BC the Hortensian Law was introduced which gave resolutions in the plebian council the force of law. Again, see Finer (1997).

[8] We limit our attention to models in which franchise decisions are explicitly endogenous. In particular, we acknowledge but do not discuss a large literature that examines the *consequences* of the expansion of rights. To name one example, Husted and Kenny (1997) examine the effect of extensions on the size of government expenditures.

game in which any member of a society can unilaterally invite a desirable outsider to join the society from abroad. Though theirs is essentially a model of immigration rather than of franchise extension (since outsiders are not members of society before they enter), it shares the feature that internal frictions are the driving force influencing "who gets in."

In the subsequent section, we describe a class of dynamic enfranchisement games that can accommodate many of the key elements of these diverse models. In contrast to existing models that focus on one source of conflict (external threat) or another (internal political competition), our model allows conflicting objectives across *all* members of society. We also require a rich enough class of environments that can produce dynamic paths of extensions consistent with the aforementioned facts regarding the evolution of the franchise.

3 Dynamic Enfranchisement Games

In this section we first describe a class of abstract, dynamic games that admit the possibility of franchise extension. We then provide several examples subsumed by the general framework.

3.1 Abstract formulation

There are n citizens in a society, each labeled $i = 1, ..., n$. Citizens are assumed to differ according to a taste, productivity, or income parameter. These differences induce a natural ordering of citizens which, in turn, coincides with the ordering defined by the index i.[9] The population of all citizens is denoted N.

Time is discrete: $t = 0, 1, 2, \ldots$. At time t, each individual chooses some action e_{it} that describes a private decision taken by citizen i at date t. We let E denote the set of feasible private decisions for each citizen, and denote the vector of efforts by

$$e_t = (e_{1t}, \ldots, e_{nt}).$$

These decisions may capture any number of activities, including labor effort, savings, or investment activities. They may also include "non-economic" activities such as religious worship. To simplify language, we refer to the decision as simply the *effort choice*.

Also at time t, a policy variable p_t, chosen from some feasible set P. For example, p may be a flat tax rate on income which generates revenue to produce a public good. We assume that policies, whatever they happen to be, are outcomes of a political process. This political process aggregates the preferences of a subset of the population $M_t \subseteq N$. That is, political rights are restricted to the citizens in M_t. If, for example, M_t is a single individual, then this

[9] For example, if citizens differ in exogenous wealth y, then we shall assume $y_i < y_j$ whenever $i > j$.

individual is a dictator with respect to policy choices.[10]

The natural interpretation here is that political rights are *voting rights*, and so, to fix ideas, a median voter aggregation is assumed here. The model could subsume a number of other political aggregation processes without fundamental changes in the analysis. Let $i = m_t$ denote the median voter of the restricted franchise in period t. The population is ordered so that the enfranchised citizenry is described by the set, $M_t = \{1, 2, \ldots, 2m_t - 1\}$.

By their vote, certain citizens have the right to choose current policies. However, they can also choose to *extend* these rights to others in the future. This may be done for a number of reasons, some of which were outlined in the Introduction. Each period, therefore, the currently enfranchised group chooses, along with the policy p_t, a group next period that will have the same rights and choices in period $t + 1$. Specifically, citizens in M_t choose, via the median voter, to enfranchise a group M_{t+1} next period.

Because policies and franchise choices are determined by the median voter (the so-called "Condorcet Winner"), we can equivalently think of the median voter in period t, denoted by $i = m_t$, as choosing the median voter in the following period, that is m_{t+1}.[11] At the beginning of the game, the median voter is denoted by m_0.

At each date t, effort and policy choices interact to influence a physical state variable denoted by $\omega_t \in \Omega$. In most of the analysis the state is one-dimensional, i.e., $\Omega \subset \mathbb{R}$. This state may represent a level of capital stock or a stock of natural resource. Alternatively, it could represent aggregate wealth or another moment of the distribution of income, or the strength of an overthrow threat. This physical state ω_t is assumed to evolve according to a transition function Q where

$$\omega_{t+1} = Q(\omega_t,\ e_t,\ p_t)$$

and ω_0 is given exogenously in order to begin the process. A complete description of the state of the game at date t is given, then, by (ω_t, m_t).

The payoff to each individual, $i \in N$, is a time separable function,

$$\sum_{t=0}^{\infty} \delta^t\ u_i(\omega_t,\ e_t,\ p_t\ )$$

where δ is a common discount factor, and the stage payoff is u_i. Note that since a citizen's private decision can affect others, his decision may be subject to a "free rider" problem in the sense that under (or over) provision of e_i, relative to some socially optimum benchmark, is likely.

[10] We not model how these rights are enforced or preserved. Of course, the state variable could capture some the technology for of preserving these restricted rights.

[11] In principle, if franchise *contraction* is permitted, any individual in the population can be made the median voter by suitable choice of M_{t+1}. However, we restrict the analysis to (weak) franchise *extensions*. For example, if the initial enfranchised group is of the form $M_0 = \{1, 2, ..., n_0\}$, then this restriction means that the median voter can never have an index $m < n_0/2$.

A Dynamic Enfranchisement Game, G, is summarized by the collection

$$G = \langle (u_i)_{i=1}^n, Q, \Omega, E, P, \omega_0, m_0; N \rangle$$

This specification has made two key simplifying modeling assumptions. First, we assume that all decisions are one dimensional. This is clearly made for reasons of tractability. Second, we have assumed that the dynamic game is deterministic. This could easily be modified to allows for shocks and other stochastic features. In general, one would ordinarily use the language of stochastic games (where Q evolves according to a Markov kernel). The deterministic assumption is made, not so much for tractability, but for ease of illustration. The basic ideas are expressed most directly in the deterministic case.

We examine dynamic enfranchisement games that satisfy the following assumptions.

(A1) $\Omega \subset \mathbb{R}$ and both E and P are compact, convex intervals in $\mathbb{R}_+$.

(A2) For each i, u_i and Q are twice continuously differentiable and strictly, jointly concave in all variables.

3.2 Examples

In this Section, we show that the class of Dynamic Enfranchisement Games is broad enough to cover a large number of interesting political/policy examples including environments in which internal or external conflicts exist.

3.2.1 Conflict over Wealth Accumulation

A classic dynamic policy game problem involves wealth accumulation with public capital. Citizens in the population exert costly labor effort e_i in a given period which generates income in the following period. We denote the income earned by individual i in the current period t by ω_{it}, and the current economic state is aggregate income $\omega_t = \sum_i \omega_{it}$. The policy p_t is a proportional tax on income, revenue from which, $p_t\omega_t$, is used to maintain productive infrastructure for next period. That is, the level of infrastructure available for use is therefore the tax revenue generated in the previous period. There is no private saving. An individual's income generation depends on his labor and the level of public investment Hence, $\omega_{i\,t+1} = f_i(p_t\omega_t, e_{it})$ where f is a concave, increasing production function. The physical state therefore evolves according to the transition function

$$\omega_{t+1} = Q(\omega_t, e_t, p_t) = \sum_i f_i(p_t\omega_t, e_{it})$$

In a given period, individual i cares about after-tax income (i.e., consumption) and about leisure according to a utility function

$$u_i(\omega_t, e_t, p_t) \;=\; u((1-p_t)\omega_{it}, e_{it})$$

for a utility u which is increasing in after tax income and decreasing in labor effort.

Here, citizens differ according to initial income, $\omega_{i\,0}$. Clearly, to the extent that initial income differences create or preserve continued income differences, different preferred tax rates are induced even if all citizens have the same utility function, u, since their realized utility each period, $u((1-p_t)\omega_{it}, e_{it})$, differs by income, ω_{it} in period t. The voter with the median income in the enfranchised group may wish to induce additional effort from certain citizens in the population by committing to a different tax rate than he, the median voter would choose by himself. One way to do this would be to grant authority for future tax policy to a different median voter.

3.2.2 Internal Conflicts over Public Goods

This example captures elements of the "internal conflict" explanation of franchise extension.[12] Tax revenue is again used to invest in an asset, but now it yields a public consumption good. Each citizen holds wealth in the form of land. The land endowment, y_i, of citizen i is exogenous, and it does not vary over time. Aggregate income is $Y = \sum_i y_i$. The policy p_t in period t is a flat tax on land, yielding revenue p_tY. Aggregate individual effort, $\sum_i e_{it}$, instead of augmenting personal incomes, increase the value of the public good next period. That is, at each date t,

$$\omega_{t+1} \;=\; f(p_tY, \sum_i e_{it}, \omega_t)$$

where ω_{t+1} is the public good produced next period. Finally, citizen i cares about after-tax wealth, about leisure, and about the public good. His payoff in period t is

$$u_i(\omega_t, e_t, p_t) \;=\; u(y_i(1-p_t), e_{it}, \alpha_i\omega_t)$$

Here, citizens in the population could differ in at least two ways. First, they could differ according to a taste parameter $\alpha_i \in [0,1]$. Citizens with higher values of α may place higher value of on public good. Examples of this type of conflict include views on of state-supported religion, or the support of certain social policies, such as opposition to scientific theories of evolution, the promotion of liberal attitudes towards race and sexual preference issues, and the enactment and enforcement of anti-abortion laws. One would expect in this case that

[12] However, the specifics here are very different from the "internal conflict" explanation in Lizzeri and Persico (LP) (2003). We discuss the differences between LP and the present work at some length in Section 5.3.

preferred tax rates will differ across the population. We refer to cases of taste heterogeneity such as this as cases of *ideological conflict*.

Second, citizens may differ in the amount of land wealth, y_i, they have. We refer to cases of income or wealth heterogeneity as cases of *class conflict*. Class conflict of this type is common in public economics, and can be shown to induce differences in voting behavior regarding redistribution, public goods, and tax policies generally.

3.2.3 The Threat of Insurrection

According to the "external conflict" explanation, franchise expansion occurs to head off the threat of revolution, uprising, or insurrection. Implicitly, such threats arise from the non-satisfaction of the preferences of the disenfranchised by the policies chosen by the elite,[13] and franchise extension may be an effective means of reducing the incentives of agents to engage in uprising.

In this example, a class conflict coupled with the threat of insurrection is the driving force behind a franchise extension. In some sense, this example is close to Acemoglu's and Robinson's model of "threat of revolt" as an explanation for the 19th century extensions.

To simplify things there are two distinct groups, referred to concretely as the nobility (Group A) and the peasantry (Group B), respectively. There are J such peasants, and $n - J$ noblemen. The franchise belongs to a subset of the nobility. A nobleman with index i has a quantity of land y_i where, as before, y_i is constant across time. Each period, a unit of land generates a unit of a consumption good, so that y_i acres generate y_i units of potential consumption. By contrast, peasants are completely disenfranchised and possess no land.

Each period t, there is a possibility that the peasants may successfully revolt and confiscate the nobility's aggregate return, $Y \equiv \sum_{i \in A} y_i$ from land. Each peasant $j = 1, \ldots, J$, contributes e_{jt} toward this effort, while each nobleman $i = J + 1, \ldots, n$ contributes effort e_{it} toward suppressing the revolt. As before, effort is costly to all citizens.

Let $E_{At} = \sum_{i \in A} e_{it}$ and $E_{Bt} = \sum_{j \in B} e_{jt}$ denote the aggregate effort by nobility and peasantry, respectively, in period t. The state variable, ω_t, is the probability in period t that the confiscation by the peasants is unsuccessful. Formally,

$$\omega_{t+1} = f(E_{At}, E_{Bt}, \omega_t)$$

so that the success likelihood depends on the aggregate effort of each group, presumably increasing in E_{At} (less likely confiscation) and decreasing in E_{Bt} (more likely confiscation).

[13] One motivation for rebellion that we do not consider is the simple desire to be part of the decision-making process, independent of whether existing decisions are in accordance with a disenfranchised individual's preferences. That is, there is no explicit utility gained from "having the vote".

If a confiscation is successful, then the entire return Y is expropriated by the peasantry who split it evenly. On the other hand, if the revolt is unsuccessful, then peasants receive a redistributive subsidy chosen by the median voter in the restricted franchise before the state revolt's success is known. Roughly, the idea is that redistribution is used to "buy off" the peasants by inducing them to reduce their effort toward the uprising.[14]

Each period t, the median nobleman chooses a redistributive tax rate p_t which produces revenue p_tY. However, the technology for redistribution is concave — implying that some of the revenue is potentially lost in the redistributive process. Formally, revenue p_tY produces $g(p_tY)$ available to be equally distributed to all members of society if there is no confiscation, where g is a concave function.

All citizens have von Neumann Morganstern utility u defined on consumption and effort. Members of the nobility have expected utility in period t of

$$u_{it} \;=\; \omega_t \; u\left((1-p_t)y_i + g(p_tY)/n,\; e_{it}\;\right) \;+ (1-\omega_t)u(0, e_{it})$$

while members of the peasantry have utility

$$u_{jt} \;=\; \omega_t \; u\left(g(p_tY)/n,\; e_{jt}\right) \;+\; (1-\omega_t)\; u\left(Y/J,\; e_{jt}\right)$$

To summarize, individuals in the nobility differ by income, and the policy instrument is a redistributive tax. Individuals can either be supportive of the current policy or they can undermine it. Their current efforts determine the likelihood that the currently enfranchised group remains in power.

3.3 Franchise Extension Equilibria

Fix a dynamic enfranchisement game G. We assume that all citizens condition their behavior only on payoff relevant information. The payoff relevant state is a pair (ω, m). Here, ω is interpreted as the "economic" state while m represents the "political" state. Strategies that condition only on the state are commonly referred to as *Markov strategies.* A Markov strategy profile is a triple $\Pi \equiv (\sigma, \psi, \mu)$ where

$$\sigma = (\sigma_1, \ldots, \sigma_n)$$

and $\sigma_i : \Omega \times N \to E$ for each i. Here, $\sigma_i(\omega_t, m_t) = e_{i\,t}$ is the action taken by citizen i when the physical state is ω_t and the current median voter is m_t in period t.

Analogously, $\psi : \Omega \times N \to P$ where $\psi(\omega_t, m_t) = p_t$ is the policy chosen by the current median voter m_t when the physical state is ω_t in period t.

[14] Hence, the example has features (deliberately) similar to the model of Acemoglu and Robinson (2000).

Finally, $\mu : \Omega \times N \to N$ where $\mu(\omega_t, m_t) = m_{t+1}$ is next period's pivotal voter chosen by the current median voter m_t when the physical state is ω_t. Recall our earlier remark on the relation between median m_t and the enfranchised group, M_t. While the franchise decision should logically correspond to a set M_{t+1} of citizens who are given the right to vote next period, there is no loss of generality in presuming that the current median voter m_t selects next period's median voter m_{t+1} directly as long as a Median Voter Theorem holds.

To summarize, σ is a profile of individual behavioral rules of the citizenry; ψ is the policy rule; μ is the franchise rule. The last two are determined by the median voter in each period.

The payoffs to each individual i of a Markov strategy profile, $\Pi = (\sigma, \psi, \mu)$ in state (ω_t, m_t) can be expressed recursively as

$$V_i(\omega_t, m_t;\ \Pi\ ) \ \equiv\ u_i(\omega_t, \sigma(\omega_t, m_t), \psi(\omega_t, m_t)\ )\ +\ \delta\ V_i(\omega_{t+1}, m_{t+1};\ \Pi\ ) \tag{1}$$

where

$$\omega_{t+1}\ =\ Q(\omega_t, \sigma(\omega_t, m_t), \psi(\omega_t, m_t)\ ) \tag{2}$$

and

$$m_{t+1}\ =\ \mu(\omega_t, m_t) \tag{3}$$

The following Lemma identifies a sufficient condition, familiar in static models, to imply the existence of a median voter in the dynamic model.

Lemma (Median Voter Theorem) *Fix some profile $\Pi = (\sigma, \psi, \mu)$. For any state (ω_t, m_t) in period t, and for any pair of policy and voting franchise choices (p_t, m_{t+1}) and $(\hat{p}_t, \hat{m}_{t+1})$ in which $\omega_{t+1} = Q(\omega_t, \sigma(\omega_t, m_t), p_t)$ and $\hat{\omega}_{t+1} = Q(\omega_t, \sigma(\omega_t, m_t), \hat{p}_t)$, resp., suppose for some i*

$$[u_i(\omega_t, \sigma(\omega_t, m_t), p_t) + \delta\ V_i(\omega_{t+1}, m_{t+1};\ \Pi\ )] - [u_i(\omega_t, \sigma(\omega_t, m_t), \hat{p}_t) + \delta\ V_i(\hat{\omega}_{t+1}, \hat{m}_{t+1};\ \Pi\ )]\ >\ 0. \tag{4}$$

Suppose that (4) holds for citizen i implies that (4) holds for every citizen, $j > i$. Then, in each such state, (ω_t, m_t), there exists a policy choice and franchise decision, (p^*_t, m^*_{t+1}), such that no other pairing of policy choice and franchise decision is strictly preferred to (p^*_t, m^*_{t+1}) by a strict majority of the voters in M_t.

The Lemma is an immediate consequence of a well known result by Gans and Smart (1996), in which a single crossing property on voter preferences, namely (4) holds for i implies (4) holds for all $j > i$, implies a Condorcet Winner exists and coincides with the individual with the median index, m. Because this ordering of citizens does not vary across states, the hypothesis of the Lemma is actually stronger than necessary if the goal is merely to produce a median voter in each state. In the absence of a change in the franchise, the same median voter will prevail each period. Though strong, this assumption will prove useful for isolating the effect of a change in the franchise due to a deliberate decision rather than due to an environmental change.

Definition A *Franchise Extension equilibrium (FEE)* is a Markov profile, $\Pi = (\sigma, \psi, \mu)$, consisting of state contingent efforts, policies, and enfranchisement choices such that at each date $t = 0, 1, 2, \ldots$, the following hold.

(i) *Optimal effort decisions* For any state (ω_t, m_t), each i, and each $\hat{\sigma}_i$,

$$V_i(\omega_t, m_t;\ \Pi\ ) \ \geq\ V_i(\omega_t, m_t;\ \hat{\sigma}_i, \sigma_{-i}, \psi, \mu\ )$$

(ii) *Single Crossing Property ensuring a Median Voter* The hypothesis of the Median Voter Lemma applies.

(iii) *Optimal policy and franchise decisions* For any state (ω_t, m_t) and for any $\hat{e}_{m_t}$, $\hat{p}_t$, and $\hat{m}_{t+1}$,

$$V_{m_t}(\omega_t, m_t;\ \Pi\ ) \ \geq\ u_{m_t}(\omega_t, \sigma_{-m_t}(\omega_t, m_t), \hat{e}_{m_t}, \hat{p}_t) \ +\ \delta\ V_{m_t}(\hat{\omega}_{t+1}, \hat{m}_{t+1};\ \Pi\ )$$

where $\hat{\omega}_{t+1} \ =\ Q(\omega_t, \sigma_{-m_t}(\omega_t, m_t), \hat{e}_{m_t}, \hat{p})$.

(iv) (Consistency of Voting Franchise Rule) For any state (ω_t, m_t), there exists a finite set $M = \{1, 2, \ldots, n'\} \subset N$ for which $\mu(\omega_t, m_t)$ is the median voter in M.

A Franchise Extension equilibrium (FEE) is a Markov Perfect equilibrium with a well defined Condorcet Winner that makes policy and franchise decisions. Each citizen chooses his own effort optimally given the state and his (correct) forecast of others' effort rules and the policy/franchise rules. Median or pivotal voters exist in each state, and the pivotal voter chooses policy, effort, and the future franchise optimally given the state and his (correct) forecast of the effort rules of the rest of the citizenry. Property (iv) requires that the delegation decision is equivalent to some alteration of the current franchise under a median voter. The question of general existence of FEE is taken up in a companion paper (Lagunoff (2003)). In the present paper, we construct FEE equilibria in a number of parametric examples in Section 5.

It is worth noting that a delegated choice of $m_{t+1} > m_t$ made by the current median, m_t, need not correspond to a larger set of voters. However, to keep things simple, the sets M_t will always be assumed to have the form $\{1, \ldots, n'\}$, and so a choice of $m_{t+1} > m_t$ will aways correspond to an expansion of voting rights, i.e., $M_{t+1} \supset M_t$.

3.4 Finite Agents versus the Continuum

We make two further modeling assumptions which deserve comment. First, it will prove more tractable to relax the last Property (iv) of the equilibrium and treat the voter type as chosen from a continuum rather than from a discrete set M. Specifically, let $N \subset [0,1]$. If the finite set of voters is sufficiently dense in the continuum, then the resulting franchise choices constitute an approximation of the actual equilibrium.

An alternative modeling strategy might have posited a continuum of voters from the beginning. However, the continuum presents a problem. In much of the history of voter enfranchisement, the effort choices of citizens correspond to voluntary decisions in a collective action problem such as volunteering to take part in a protest or public insurrection. But with the continuum, free rider problems in these decisions are extreme. An individual in a continuum would never choose to riot or threaten the status quo, or alternatively, to defend the status quo. The finite agent assumption is therefore critical to prevent the unreasonable boundary solution $e_i = 0$ in effort choices of citizens. Indeed, we later show that for franchise extension to exist, these boundary solutions *must not* occur. To sum up, franchise choices are characterized in the next sections as if the current median could choose the subsequent median from a continuum of types, but in the citizens' private decisions, the finite agent assumption is taken literally.

Second, though we treat the indices m as choice variables for voters, the Markov strategies are actually functions of the *types* of players, rather than their identities. For example, in the class conflict examples, individuals are ordered by wealth, $y_1 \geq y_2 \geq \cdots \geq y_n$. In that case, the strategy $\sigma_i(\omega, m)$ is just notational shorthand for $\sigma_i(\omega, y_m)$. [15]

4 First Order Characterization

In this Section we characterize necessary conditions for an FEE assuming differentiability of the value function. Later we establish conditions under which differentiability holds. In all that follows, we drop the time notation, t, and adopt the usual convention in which primes, e.g., ω', are used to denote variables in the subsequent period $t+1$, and double primes, e.g., ω'', used to denote the variable two periods ahead $t+2$.

Let $\Pi = (\sigma, \psi, \mu)$ be a Franchise Extension equilibrium (FEE). Consider, first, a citizen's effort decision. One can write the recursive payoff evaluated at a FEE as the functional

[15] Of course, it must be assumed that citizens are sufficiently dense in the type space to justify the assumption that wealth is a continuous variable. Also, if types are not uniformly distributed, then there will be differences between the type contingent strategy, $\sigma_i(\omega, y_m)$, and the strategy $\sigma_i(\omega, m)$ that merely keeps track of player index. In order to examine these explicit distributional considerations, we will use the "type" notation explicitly when parametric examples are examined.

equation:

$$V_i(\omega, m;\ \Pi\ ) \ = \ \max_{e_i} \quad [u_i(\omega, e_i, \sigma_{-i}(\omega, m), \psi(\omega, m)) \ + \ \delta\ V_i(\omega', \mu(\omega, m);\ \Pi\ )\ ] \tag{5}$$

subject to $\omega' = Q(\omega, e_i, \sigma_{-i}(\omega, m), \psi(\omega, m)\ )$. If this value function is differentiable, then the (interior) Euler equation is

$$\frac{\partial u_i}{\partial e_i} + \delta \frac{\partial V_i}{\partial \omega'} \frac{\partial Q}{\partial e_i} \ = \ 0 \tag{6}$$

As for the pivotal voter's problem, recall that the pivotal voter makes two choices. He chooses a policy in the current period given the state ω. He also chooses next period's median voter by making a franchise decision in the current period. That is, a median voter with index m chooses next period's median, m'. The functional equation resulting from the dual choice of policy and franchise is

$$V_m(\omega, m;\ \Pi\ ) \ = \ \max_{e_m, p, m'} \quad [u_m(\omega, e_m, \sigma_{-m}(\omega, m), p) \ + \ \delta\ V_m(\omega', m';\ \Pi\ )\ ]$$

subject to $\omega' = Q(\omega, e_m, \sigma_{-m}(\omega, m), p\ )$. Derived from this value function, the interior Euler equation for the policy decision, p is

$$\frac{\partial u_m}{\partial p} + \delta \frac{\partial V_i}{\partial \omega'} \frac{\partial Q}{\partial p} \ = \ 0 \tag{7}$$

whereas, the interior Euler equation for franchise decision, m', made by pivotal voter m is

$$\delta \frac{\partial V_m}{\partial m'} \ = \ 0 \tag{8}$$

Definition We will say that a Franchise Extension equilibrium, $\Pi = (\sigma, \psi, \mu)$, admits a *first order characterization* if for each citizen i and each voter m, in every state (ω, m), (i) the profile $\Pi = (\sigma, \psi, \mu)$ satisfies the Equations (6), (7), and (8) , (ii) the expression in (6) is strictly decreasing in e_i, and (iii) if the matrix of second derivatives of the system formed by (the left-hand sides of) (6), (7), and (8) is negative semi-definite.

Any FEE that admits a first order characterization is fully characterized by its Euler equations. Among them, Equation (8), is the most relevant for understanding franchise expansion.

Expressed in terms of a useful decomposition of marginal effects, Equation (8) is given by,

$$
\begin{aligned}
\frac{\partial V_m}{\partial m'} &= \overbrace{\left[\frac{\partial u_m}{\partial p'} + \delta \frac{\partial V_m}{\partial \omega''}\frac{\partial Q}{\partial p'}\right]\frac{\partial \psi}{\partial m'}}^{\text{effect of } m' \text{ on future policy}} + \overbrace{\delta \frac{\partial V_m}{\partial m''}\frac{\partial \mu}{\partial m'}}^{\text{effect of } m' \text{ on future franchise decision}} \\
&+ \overbrace{\sum_{j \neq m}\left[\frac{\partial u_m}{\partial e'_j} + \delta \frac{\partial V_m}{\partial \omega''}\frac{\partial Q}{\partial e'_j}\right]\frac{\partial \sigma_j}{\partial m'}}^{\text{effect of } m' \text{ on future citizen behavior}} \\
&= 0
\end{aligned} \tag{9}
$$

Clearly, a voter m chooses to expand the current franchise only if (9) is satisfied at values $\mu(\omega, m) > m$. The decomposition illustrates the various marginal effects that a change in the future pivotal voter has on the payoff of the current pivotal voter. This means that the current pivotal voter, m, rationally anticipates his choice of m' on future effort choices of the citizenry, and future policies and franchise decisions of subsequent median voters (including himself, should he choose to retain political power). Among other things, the current median realizes that his choice of franchise expansion may not be the end of the process. Since next period's pivotal voter, m', also satisfies his Euler equations, (7) and (8), if the current pivotal over, m, extends the franchise to $m' > m$, then the Single Crossing Property implies

$$
\overbrace{\left[\frac{\partial u_m}{\partial p'} + \delta \frac{\partial V_m}{\partial \omega''}\frac{\partial Q}{\partial p'}\right]\frac{\partial \psi}{\partial m'}}^{\text{effect of } m' \text{ on future policy}} + \overbrace{\delta \frac{\partial V_m}{\partial m''}\frac{\partial \mu}{\partial m'}}^{\text{effect on future franchise decision}} \leq 0 \tag{10}
$$

A franchise extension, therefore, implies that the marginal payoff from other citizens' effort responses to the extension be nonnegative, i.e.,

$$
\overbrace{\sum_{j \neq m}\left[\frac{\partial u_m}{\partial e'_j} + \delta \frac{\partial V_m}{\partial \omega''}\frac{\partial Q}{\partial e'_j}\right]\frac{\partial \sigma_j}{\partial m'}}^{\text{effect on future citizen behavior}} \geq 0 \tag{11}
$$

Hence, an optimal enfranchisement for voter m balances the positive marginal effect from future effort choices (11) from the citizenry with the negative marginal effect of putting future policy and franchise decisions in the hand of other agents (10). This is illustrated by the two solid lines in Figure 1. If the current median voter is m, retaining the franchise results in no loss of control - that is, a zero marginal cost. On the other hand, extending the franchise to a median $\widehat{m}'$ generates maximal benefits associated with effort inducement, but imposes

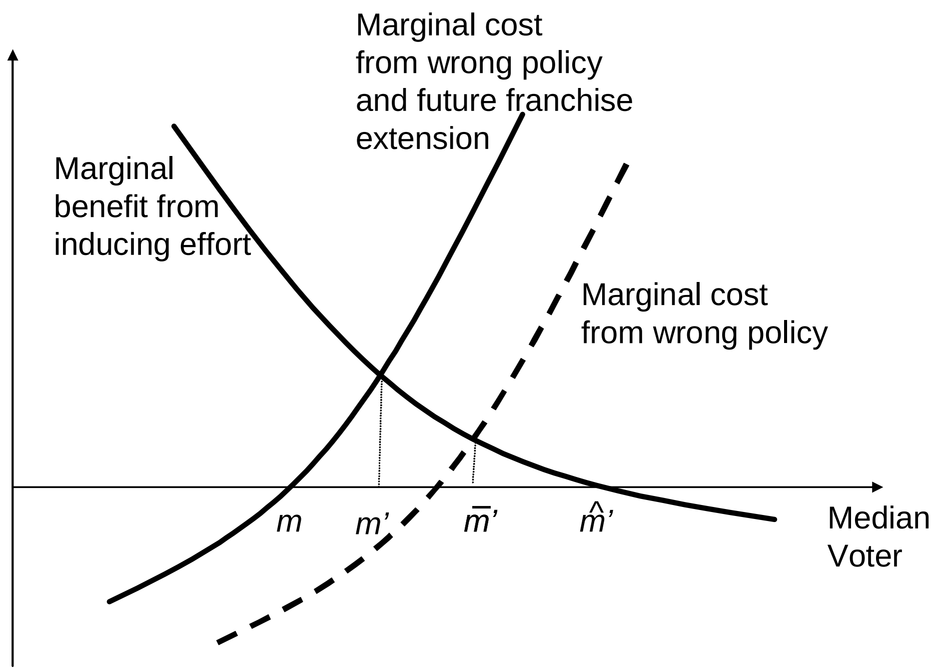

Figure 1: Optimal enfranchisement equates marginal benefits from preferable effort decisions with marginal costs of future policy and franchise distortions

large costs in terms of future policy and franchise decisions. The index m' balances these two effects.

In fact, the logic can be extended to obtain the following necessary and sufficient condition for franchise extension.

Proposition 1 *In any Franchise Extension equilibrium that admits a first order characterization, the franchise is extended in state* (ω, m), *i.e.,* $\mu(\omega, m) = m' > m$ *if and only if*

$$\sum_{j \neq m} \left[\frac{\partial u_m}{\partial e'_j} - \left(\frac{\partial u_m / \partial e'_m}{\partial Q / \partial e'_m} \right) \frac{\partial Q}{\partial e'_j} \right] \frac{\partial \sigma_j}{\partial m'} > 0 \tag{12}$$

holds at $m = m'$.

Though the result is a straightforward application of the Envelope Theorem, we include the complete proof in the Appendix. Roughly, the idea is that franchise extension requires the spillover of effort choices of ordinary citizens, without which a current policy maker would preserve his own power to make future policy decisions into perpetuity. This is true regardless of whether the effort choices are directed toward investment in public goods or the investment in political upheaval.

This last point is worth emphasizing. Specifically, *the same causal mechanism underlies both the so-called "internal conflict" and "external conflict" explanations for franchise extension.* In the internal conflict story, disagreements within the elite over public goods create a motive by some to extend voting rights to "sympathetic outsiders." The effort choice is, for instance, a private input needed to produce the controversial good. In the external conflict story, the threat of uprising or insurrection creates a "buy off" motive for expansion of rights. The effort choice, in that case, is one's contribution either to the cause of overturning or to the cause of defending the current regime. In either case, the franchise is extended if and only if the aggregate effect of these spillovers are positive. Presumably, the larger the spillover effect, the larger is the extension.

In the presence of spillovers, a franchise extension can accomplish what a policy change cannot. Namely, the franchise extension is a credible commitment to future policy changes. The pivotal voter cannot credibly use current policy instruments to change future behavior except through (blunt) changes in the physical state. Since current policy changes do not imply future policy changes, citizens with preferences that differ widely from those of the pivotal voter expect the same median voter to continue to produce poor policy choices ("poor" from their point of view) in the future.

By contrast, an extension delegates authority to a different pivotal voter tomorrow. This guarantees that future policies in subsequent periods are closer to those that the current median voter would like to be able to commit to. Since this elicits a positive spillover in their effort choices, the pivotal voter today is willing to sacrifice his power. In this sense, the role of franchise extension is a familiar one in time-consistent models of policy. Extensions are credible since they delegate policy-making authority to a median whose tastes are closer to the large group of citizens.

Notice, however, that while the enfranchisement option may improve things, it is not generally a perfect substitute for the optimal, time inconsistent policy sequence. With recursive enfranchisement, the initial voter cannot limit future franchise extensions. A future median may delegate beyond the point at which the first median would choose if the first median could make a once-and-for-all franchise decision. In turn, this possibility distorts the current decision. To see this, consider an optimal once-and-for-all extension. A once-and-for-all extension trades off the marginal benefits of extra effort against the marginal costs of future policy changes (the dashed curve) as illustrated in Figure 1. Since these costs do not include the costs of future extensions, the new median is $\overline{m}' > m'$. Since $m' < \bar{m}'$ in Figure 1, the current median limits the extension of the franchise below that of a once-and-for-all decision.

An immediate corollary of the Proposition is: *absent the spillovers in private decisions, the level of voter enfranchisement remains fixed.* This statement has predictive content. Consider an example of a policy that subsidizes a particular "state religion." Current subsidies determine, say, the subsequent available stock of churches. Citizen i's church attendance does not affect others' payoffs, and it does not affect the technology for building churches. In this case, the current median voter will not delegate authority to another. Though conflicts over state-funded religion may, in fact, create serious social conflict, it would not then lead to broader political rights.

Proposition 1 provides a relatively simple way to check if an expansion of the franchise occurs in equilibrium. To make full use of it, however, requires practical use of all the Euler equations, since the Inequality (12) depends on knowing both values of the equilibrium strategies, and their curvature. Consequently, the Euler equations (6)-(8) require a reformulation that depends, to the extent possible, only on the "primitives" of the problem.

Proposition 2 *Let $\Pi = (\sigma, \psi, \mu)$ denote a profile of continuously differentiable Markov strategies such that in every state (ω, m), the values $\sigma(\omega, m)$, $\psi(\omega, m)$, and $\mu(\omega, m)$ lie in the interior of their respective strategy sets. Then Π is a Franchise Extension equilibrium that admits a first order characterization if and only if it satisfies the following.*

I. In every state (ω, m), the profile $\Pi = (\sigma, \psi, \mu)$ satisfies:

For median voter $i = m$,

$$G_m^1 \equiv \frac{\partial u_m}{\partial p} - \left(\frac{\partial u_m / \partial e_m}{\partial Q / \partial e_m}\right)\frac{\partial Q}{\partial p} = 0 \tag{E-1}$$

and

$$\begin{aligned} G_m^2 \equiv & \left[G_m^{1\,\prime}\right]\frac{\partial \psi}{\partial m'} - \delta\frac{\partial \mu / \partial m'}{\partial \mu / \partial \omega'}\left(\frac{\partial u_m / \partial e_m}{\partial Q / \partial e_m} + \Lambda_m(\omega'; \Pi)\right) \\ & + \sum_{j \neq m}\left[\frac{\partial u_m}{\partial e_j'} - \left(\frac{\partial u_m / \partial e_m'}{\partial Q / \partial e_m'}\right)\frac{\partial Q}{\partial e_j'}\right]\frac{\partial \sigma_j}{\partial m'} = 0 \end{aligned} \tag{E-2}$$

where $G_m^{1\,\prime}$ is the left side of Equation (E-1) iterated forward one period, and where

$$\Lambda_i(\omega; \Pi) \equiv \frac{\partial u_i}{\partial \omega} + \frac{\partial u_i}{\partial p}\frac{\partial \psi}{\partial \omega} + \sum_{j \neq i}\frac{\partial u_i}{\partial e_j}\frac{\partial \sigma_j}{\partial \omega} - \frac{\partial u_i / \partial e_i}{\partial Q / \partial e_i}\left[\frac{\partial Q}{\partial \omega} + \frac{\partial Q}{\partial p}\frac{\partial \psi}{\partial \omega} + \sum_{j \neq i}\frac{\partial Q}{\partial e_j}\frac{\partial \sigma_j}{\partial \omega}\right],$$

and for each citizen i,

$$G_i^3 \equiv \frac{\partial u_i}{\partial e_i} + \delta\, \Lambda_i(\omega'; \Pi)\frac{\partial Q}{\partial e_i} + \delta^2 \left[G_i^{2\prime}\right] \frac{\partial \mu}{\partial \omega'}\frac{\partial Q}{\partial e_i} = 0, \quad \forall i \in N \tag{E-3}$$

II. The expression G_i^3 for each i is strictly decreasing in e_i.

III. The matrix of second derivatives of the system formed by (G_m^1, G_m^2, G_m^3) is negative semi-definite.

IV. For all i ,if $G_i^1 dp \ + G_i^2 dm' > 0$, then $G_j^1 dp \ + G_j^2 dm' > 0$ for all $j > i$

The Euler equations (E-1), (E-2), and (E-3) are reformulated from the original Euler equations (7), (8), and (6), respectively, in order to eliminate their functional dependence on the value functions. What remains is a collection of n partial differential equations in the strategy profile Π. Klein, Krusell, and Ríos Rull (KKR) (2002) examine properties of similar "Envelope-adjusted" Euler equations in recursively competitive equilibrium models of policy. They refer to these equations as Generalized Euler Equations. As in their reformulation, the Euler equations above differ substantially from those of single agent, dynamic programming problems. Unlike in dynamic programming problems, these Euler equations depend on one's equilibrium decision rules in the future, and on others' equilibrium decision rules in the present and in the future. Hence, they cannot be reduced to pure expressions of primitives as is typical of Euler equations in DP problems.[16] Despite their apparent complexity, the primary virtue of Properties I-IV is that they provide a computationally tractable characterization of Franchise Extension equilibria. We make further use of these properties in examples below.

5 Parametric Environments

Here we examine a series of parametric cases. These cases illustrate how the first order characterization may be used to understand enfranchisement. They also illustrate how enfranchisement may exhibit many of the qualitative features outlined in Section 2. The equilibria in each of these environments are stationary in the economic state, ω. This means $\partial\psi/\partial\omega = 0$ and $\partial\sigma/\partial\omega = 0$, and $\partial\mu/\partial\omega = 0$. Consequently, the Euler Equation, (E-3), for franchise decision collapses to:

$$\frac{\partial u_i}{\partial e_i} \ + \ \delta \left[\frac{\partial u_i}{\partial \omega'} \ - \ \left(\frac{\partial u_i/\partial e_i'}{\partial Q/\partial e_i'} \right) \frac{\partial Q}{\partial \omega'} \right] \frac{\partial Q}{\partial e_i} \ = \ 0 \tag{13}$$

[16] Moreover, the politico-economic and policy models used by KKR and others in the literature are not, strictly speaking, dynamic games since individual behavior is filtered out in those models by the competitive price mechanism. Even without a franchise decision, the Euler equations in the present paper contain a number of extra terms not found in the competitive, "hybrid" models. See Lagunoff (2003) for an extended discussion of dynamic political games. A related, though simpler (no policy or franchise decisions), version of the Euler equations in (I) also shows up in Basar and Olsder (1995, Theorem 6.5).

5.1 Conflict without Franchise Extension

As the results show, extensions are not useful unless they change future behavior. To illustrate this idea, we revisit the example with ideological conflict in wealth accumulation — Section 3.2.1. Recall that an individual's income, ω_i', next period is generated by one's own labor effort, e_i in the current period and by current tax revenue. Consider the Cobb Douglas accumulation technology

$$\omega_i' \;=\; (p\omega)^{\theta} {e_i}^{1-\theta}.$$

Individuals differ by the relative weights they attach to labor and the consumption:

$$u(\omega, e, p; \alpha_i) \;=\; (1-\alpha_i)\log(\omega_i(1-p)) + \alpha_i \log(1-e_i).$$

These weights are ordered from lowest to highest so that $\alpha_1 \geq \cdots \geq \alpha_n$. By associating the indices of citizens to the ordering of marginal utility types, we will verify that the single crossing property holds. If α_{m_0} is smaller than the population median weight, then the initial franchise is restricted. An extension of the franchise therefore represents a delegation of decision authority to citizens with higher marginal utility of leisure than that of the initial median voter. Notationally, we can express all decision rules as functions of type α_m.

The basic intuition of the model is that by commiting to a different future policy, current (and future) effort choices can be influenced in a way that is beneficial to the current median voter. In the current example, next period's tax rate determines how much of next period's income is available for consumption, which in turn determines the net productivity of today's effort, so there is a potential link between p' and e_i for individual i. However, if we write out the relevant parts of i's dynamic payoff, we find

$$\begin{aligned} u_i &= \alpha_i \log(1-e_i) + \delta(1-\alpha_i)\log(\omega_i'(1-p')) + \delta^2(1-\alpha_i)\log(\omega_i''(1-p'')) \\ &= \alpha_i \log(1-e_i) + \delta(1-\alpha_i)\log([(p\omega)^{\theta} e_i^{1-\theta}](1-p')) + ... \end{aligned}$$

which is additively separable in e_i and p'.[17] We therefore expect that individual i's effort choice will be independent of his (correct) prediction of future tax rates, so there is no benefit to the current median in trying to induce a change in behavior by extending the franchise. Indeed, even if the current median could commit to a different tax policy, it would not do so.

We confirm this intuition, by first verifying that this game has a Franchise Extension equilibrium that admits a first order characterization. The equilibrium is invariant in the economic state, ω. In this environment, Equation (13), which characterizes the ω-stationary Euler equation for private effort, becomes

[17] The $''$ terms and beyond include e_i and p' due to the evolution of the physical state, but since the transition equation is mulitipicatively separable in its arguments, and utility is log, the additive separability is propagated too.

$$-\frac{\alpha_i}{1-e_i} \ + \ \delta\left[\left(\frac{1-\alpha_i}{\omega_i'} \ + \ \alpha_i\frac{\theta}{1-\theta}\frac{e_i'^{\,\theta}}{1-e_i'}\frac{\sum_j e_j'^{\,1-\theta}}{\omega'}\right)\frac{(p\omega)^\theta}{e_i^\theta}\right] \ = \ 0$$

Using the fact that $\omega_i' = (p\omega)^\theta e_i^{1-\theta}$ and $\omega' = (p\omega)^\theta \sum_j e_j^{1-\theta}$ and using the consistency condition $e_i' = e_i$ in a stationary FEE, we rewrite this equation as

$$-\frac{\alpha_i}{1-e_i} \ + \ \delta\left[\frac{1-\alpha_i}{e_i} \ + \ \alpha_i\frac{\theta}{1-\theta}\frac{1}{1-e_i}\right] \ = \ 0$$

Solving for e_i gives the stationary FEE behavioral rule,

$$\sigma_i(\omega, \alpha_m) \ = \ e_i \ = \ \frac{\delta(1-\alpha_i)}{\alpha_i(1-\delta\frac{\theta}{1-\theta}) + \delta(1-\alpha_i)}. \tag{14}$$

Notice that the behavior in equilibrium is invariant to current policy as well as being invariant to the state, (ω, α_m). The equilibrium policy rule is

$$\psi(\omega, \alpha_m) \ = \ p \ = \ \delta\frac{\theta}{1-\theta} \tag{15}$$

so that tax rates are also state-invariant. In particular, they do not vary with the index of the median voter. Evidently, the separability implied by logarithmic preferences means that even with the inter-temporal link between tax revenue and the productivity of effort, current effort levels are independent of current tax rates.

Now observe that by Proposition 1, a necessary condition for franchise extension is that $\frac{\partial\sigma_j}{\partial m} > 0$, or, as a direct function of type, $\frac{\partial\sigma_j}{\partial\alpha_m} \ < \ 0$ for some $j \neq m$. However, as is indicated by (14), $\frac{\partial\sigma_j}{\partial\alpha_m} = 0$ unless $j = m$. Hence, this example does not admit a franchise extension. The current type, α_{m_0} retains decision authority forever. This is not surprising given the nature of the policy and effort rules.

5.2 Internal Class Conflict Generates Franchise Extension

Recall the examples in Section 3.2.2 where the environment exhibits a class conflict over a public good. y_i is each individual's exogenous land endowment. The aggregate endowment is $Y = \sum_i y$. A flat tax rate p on land is chosen by the median enfranchised voter, and it finances a public good, ω. Citizens differ in their endowments. Specifically, the citizens are ordered so that $y_1 \geq y_2 \geq \cdots y_n > 0$. Citizen 1 is the wealthiest while Citizen n is the poorest.[18]

[18] We have "reversed" the ordering so that higher indices correspond to lower wealth classes. This maintains consistency with the earlier notation in which extension proceeds to citizens with higher indices.

Payoffs each period are given by

$$u_i(\omega, e, p) \;=\; y_i(1-p) \;+\; \omega \;-\; ce_i^2$$

All citizens value the public good the same way, however each differs in income, y_i. These income differences induce differences in the way that rich or poor citizens view a tax increase. For simplicity, assume that the population median is $\bar{y}$, and assume that $y_{m_0} > \bar{y}$. This means the initial franchise is restricted. Notice that if $y_{m_0} < y_1$, then internal class conflict exists within the enfranchised elite. Citizens with wealth levels on the outer fringe of the elite may have more in common with their neighbors just below them in the income strata than with other members of the elite.

The transition law for the public good assumes that the public good fully depreciates each period. Parametrically, it is given by

$$\omega' = (pY)^\theta E$$

where $E = \sum_i e_i$. Since the public good does not accumulate, decision rules do not vary with the current level of the good. The first order condition gives an individual's effort choice as a function of tax p: $e_i = \frac{\delta(pY)^\theta}{2c}$. As before, the effort and policy rules can be expressed as direct functions of an individual's type, in this case, y_i. The policy rule is given by

$$\psi(y_m) \;=\; C\left(\frac{1}{y_m}\right)^{\frac{1}{1-2\theta}}$$

where $C = \left(\frac{n\delta^2 Y^{2\theta}}{2c}\right)^{\frac{1}{1-2\theta}}$ is a positive constant. Effort levels do not vary with one's own land wealth, policy preferences do. If $\theta < 1/2$, then preferred tax rates decrease in one's own wealth. Wealthier individuals prefer lower taxes. Substituting the policy rule into the effort choices, we derive the effort rule for each individual as

$$\sigma(y_m) \;=\; K\left(\frac{1}{y_m}\right)^{\frac{\theta}{1-2\theta}}. \qquad (16)$$

where $K = \frac{\delta Y^\theta}{2c}C^\theta$, another positive constant. Notice that optimal efforts are the same for all individuals, and depend only on the identity of the median voter. According to σ, effort choices decrease in the wealth of the median voter — wealthier voters indirectly induce lower effort. Since one's effort contributes public capital to the creation of the public good. franchise extension is a mechanism by which a current decision maker can, by delegating his authority, change the level of public capital.

Now observe that $\frac{\partial \sigma_j}{\partial y_m} < 0$ whenever $\frac{\partial \sigma_j}{\partial m} > 0$ since y_m is ordered from highest wealth type to lowest. A franchise extension therefore requires a movement of the median evaluation toward lower, rather than higher, land endowment, y_m. This means that the inequality in Proposition 1 is reversed, using y_i as an individual's type. Hence, from Proposition 1, the equilibrium admits a franchise extension iff

$$\sum_{j\neq m} \frac{\partial \sigma_j}{\partial y_m} \; < \; 0 \tag{17}$$

Therefore, differentiating the behavioral rule σ with respect to the state y_m, we see that (17) holds iff $\theta < 1/2$. Hence, if $\theta < 1/2$, then $\mu(y) < y$, for all y, meaning that the franchise is extended to successively lower classes in the income strata. Each extension elicits a higher effort from the citizens. Extension also produces a higher tax rate since taxes and effort are complementary inputs in the production of public goods.

We now turn to the issue of derivation of an equilibrium franchise rule. Formally, the current median delegates to a voter with the land endowment $y_{m'}$, which is chosen to satisfy the Euler equation $\frac{\partial V_m}{\partial y_{m'}} = 0$. Expanding this equation gives

$$\begin{aligned}
\frac{\partial V_m}{\partial y_{m'}} &= \left[\frac{\partial u_m}{\partial p'} - \left(\frac{\partial u_m/\partial e'_m}{\partial Q/\partial e'_m}\right)\frac{\partial Q}{\partial p'}\right]\frac{\partial \psi}{\partial m'} + \sum_{j\neq m}\left[\frac{\partial u_m}{\partial e'_j} - \left(\frac{\partial u_m/\partial e'_m}{\partial Q/\partial e'_m}\right)\frac{\partial Q}{\partial e'_j}\right]\frac{\partial \sigma_j}{\partial m'} + \delta\frac{\partial V_m}{\partial y_{m''}}\frac{\partial \mu}{\partial y_{m'}} \\
& \left[-y_m + \frac{2cK\left(\frac{1}{y'_m}\right)^{\frac{\theta}{1-2\theta}}}{C\left(\frac{1}{y'_m}\right)^{\frac{1}{1-2\theta}}}\left(nK\left(\frac{1}{y'_m}\right)^{\frac{\theta}{1-2\theta}}\right)\right] C\frac{-1}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{1}{1-2\theta}+1} \\
&\quad + \sum_{i\neq m} 2c\left[K\left(\frac{1}{y_{m'}}\right)^{\frac{\theta}{1-2\theta}}\right]\left[K\frac{-\theta}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{\theta}{1-2\theta}+1}\right] + \delta\frac{\partial V_m}{\partial y_{m''}}\frac{\partial \mu}{\partial y_{m'}} \\
&= \left[-y_m + y_{m'}\right] C\frac{-1}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{1}{1-2\theta}+1} \\
&\quad + \sum_{i\neq m} 2c\left[K\left(\frac{1}{y_{m'}}\right)^{\frac{\theta}{1-2\theta}}\right]\left[K\frac{-\theta}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{\theta}{1-2\theta}+1}\right] + \delta\frac{\partial V_m}{\partial y_{m''}}\frac{\partial \mu}{\partial y_{m'}} \\
&= C\left[-y_m + y_{m'}\right]\frac{-1}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{1}{1-2\theta}+1} + 2c(n-1)K^2\frac{-\theta}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{1}{1-2\theta}} + \delta\frac{\partial V_m}{\partial y_{m''}}\frac{\partial \mu}{\partial y_{m'}} \\
&= 0
\end{aligned} \tag{18}$$

The first equality in (18) follows by definition of the Euler equation (9). The second and third equalities are just algebra. The final equality is the first order condition. By iterating forward (18), we obtain the Euler equation for an initial median voter $m = m_0$:

$$
\begin{aligned}
\frac{\partial V_{m_0}}{\partial y_{m_1}} &= \sum_{t=1}^{\infty} \delta^{t-1} \left\{ [-y_{m_0} + y_{m_t}] \frac{-C}{1-2\theta} \left(\frac{1}{y_{m_t}} \right)^{\frac{1}{1-2\theta}+1} + \frac{-2c(n-1)K^2\theta}{1-2\theta} \left(\frac{1}{y_{m_t}} \right)^{\frac{1}{1-2\theta}} \right\} \prod_{\tau=0}^{t-1} \frac{\partial \mu}{\partial m_\tau} \\
&= 0
\end{aligned} \tag{19}
$$

We now verify a "guess" that there exists an equilibrium franchise rule which is linear. Specifically, verify $y_{m'} = \mu y_m$ with $0 < \mu < 1$. Given this form, the Euler equation, (19) can be expressed as

$$
\begin{aligned}
\frac{\partial V_m}{\partial y_{m'}} &= \sum_{t=1}^{\infty} (\delta\mu)^{t-1} \left\{ C \left[\mu^t - 1 \right] \left(\frac{1}{\mu^t} \right)^{\frac{1}{1-2\theta}+1} + 2c(n-1)K^2\theta \left(\frac{1}{\mu^t} \right)^{\frac{1}{1-2\theta}} \right\} \\
&= 0
\end{aligned} \tag{20}
$$

Note that Equation (20) no longer depends on the wealth, y_m, of the decision maker. Hence, Equation (20) is an equation in one unknown, namely μ, the proposed coefficient of a linear franchise rule. Using the definitions of C and K, Equation (20) reduces to

$$
\sum_{t=1}^{\infty} \left(\frac{\delta}{\mu^{\frac{2\theta}{1-2\theta}}} \right)^t \left\{ (\mu^t - 1) + \theta(n-1)/n \right\} = 0
$$

which, provided that $\delta < \mu^{\frac{2\theta}{1-2\theta}}$, reduces to

$$
\mu^{\frac{2\theta}{1-2\theta}} (1 - \mu) = \delta\theta(n-1)/n \tag{21}
$$

Hence, any $\mu \in (0, 1)$ which solves (21) is the coefficient in a linear, equilibrium franchise rule. That is, if μ solves (21), then a FEE enfranchisement rule is given by $y_{m'} = \mu y_m$. Since the rule is linear, extensions occur until universal suffrage is attained. The linear rule is illustrated in Figure 2.

To get a better sense of how this rule on land endowment types translates back into enfranchisement of individuals, let F denote the land distribution. Specifically, let $F(y)$ denote the fraction of citizens with wealth levels lower than y, that is, $F(y) = \frac{1}{n}|\{i : y_i < y\}|$. Then, the fraction of enfranchised citizens corresponding to pivotal voter type y_m is given by

$$
z(y_m) = 2(1 - F(y_m))
$$

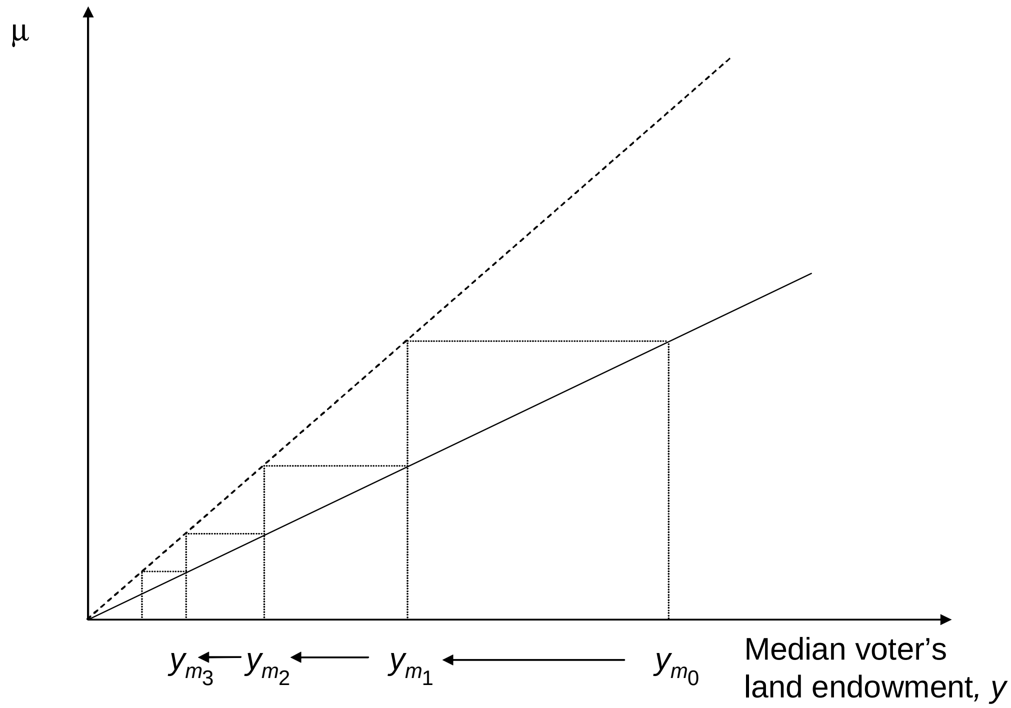

Figure 2: Linear franchise extension rule

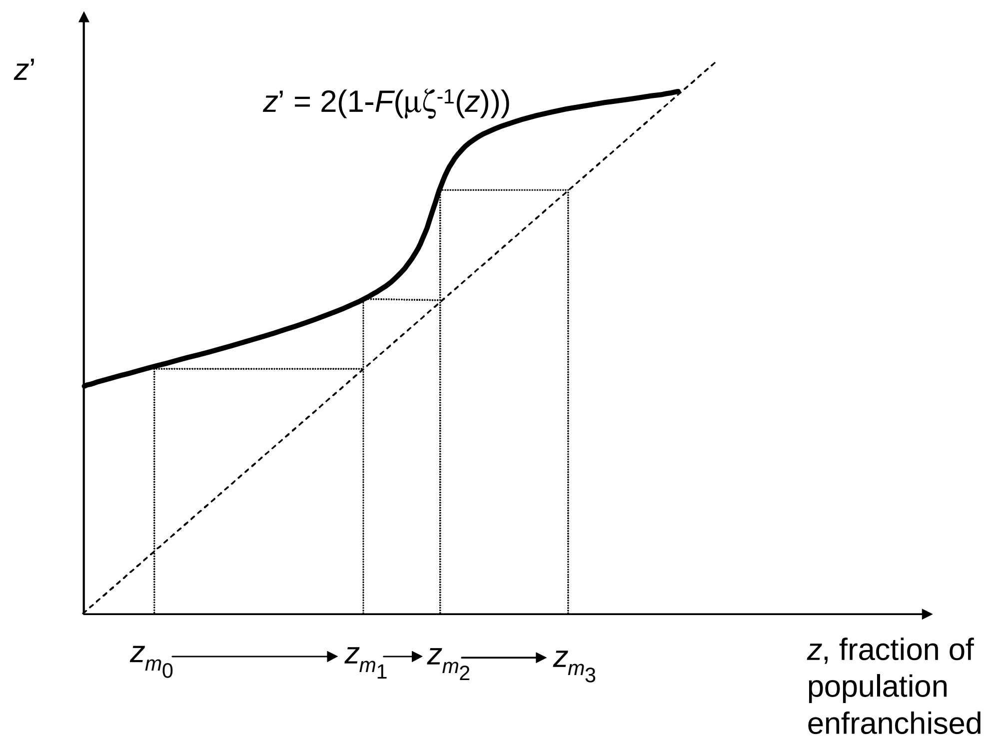

Figure 3: An uneven dynamic path of enfranchised fraction of population given wealth distribution F.

The actual path of enfranchised individuals as a fraction of the population[19] is

$$z(y_m), z(\mu y_m), z(\mu^2 y_m), \ldots$$

Figure 3 demonstrates that this law of motion may, in fact, induce an expansion of rights which is quite uneven.

5.3 Internal Ideological Conflict Generates Partial Franchise Extension

This environment is the same as the one before, except that, instead of wealth differences, citizens differ in ideological views toward a public good. While land holdings are now identical for all citizens, valuations of the public good may differ. Indeed, some citizens may value it negatively. Thus, there are two groups who have fundamentally conflicting views toward the good, with differing preference intensities within each group. We use this case to compare the "internal" mechanism for extension with that of Lizzeri and Persico (2003), who model this type of mechanism as well.

Here, tax revenue is used to provide the public good, but the production is augmented (reduced) by individual contributions of positive (negative) effort. Worldly examples of this type of conflict are given in Section 3.

Formally, preferences are given by

$$u_i(\omega, e, p) = y(1-p) - ce_i^2 + \alpha_i \omega$$

where each land, y, is the same across all citizens, however, α_i is a citizen's utility weight on the public good. We order the weights so that $\alpha_1 \geq \alpha_2 \geq \cdots \alpha_J > 0 > \alpha_{J+1} \geq \cdots \geq \alpha_n$. Citizens for whom $\alpha_i > 0$ support the provision of the public good, while others suffer a utility cost from it. Assume that $\sum_i \alpha_i > 0$ so that, on balance, positive feeling toward the good is more intense than negative feeling toward it. Assume also that $\alpha_{m_0} > 0$ so that political power initially rests with the "positives." For simplicity, assume that the population median is 0 so that $\alpha_{m_0} > 0$ represents a restricted franchise. Since these individuals may disagree about intensity of preference even if they agree that the public good is a "good," internal conflicts that lead to franchise extension are possible.

Each individual can choose costly effort to either increase or decrease the public good in the subsequent period. Let $e_i \in [-b, b]$ where b is a large enough bound so that interior solutions always exist. Hence if $e_i > 0$ is chosen by i then this individual invests in the public good, whereas if $e_i < 0$, then he exerts effort to resist such investments.

[19] In general one would expect the optimal franchise extension rule to depend on the distribution function F. But in this example, since efforts depend only on the identity of the median voter (see equation 16), the spill-over, and hence the optimal extension, is independent of F.

Aggregate effort provision is $E = \sum_i e_i$, and the size of the public good is multiplicative in this effort and tax revenue. Benefits from the public good accrue in the following period, so the law of motion is given by

$$\omega' = (pY)^\theta E$$

where $Y = \sum_i y = ny$ is the aggregate land in this society. The public good is assumed to fully depreciate each period. For reasons that will be clear later on, we assume $1/2 > \theta > 1/4$. If $E > 0$, then tax revenue is used to produce positive amounts of the public good. If, however, $E < 0$, then revenue is used to produce negative amounts of the good. In such a case, a voter with a positive marginal evaluation, $\alpha_m > 0$, would prefer a tax rate of 0.

Once again, franchise extension may represent a mechanism by which a decision maker can commit to a certain tax rate, and the relative strength of preferences now determines whether this is desirable. For example, if aggregate effort is decreasing in the tax rate (opponents of the public good dissent more strongly than supporters), then it may be possible to commit to a lower tax rate and thus increase net effort, by delegating the choice of tax policy to an agent with a smaller value of α.

Suppose, initially that $E > 0$ is forecast. Then an individual's effort choice, as a function of tax p, is given by $e_i = \frac{\delta \alpha_i (pY)^\theta}{2c}$. As expected, this effort is positive iff $\alpha_i > 0$.

As before, all decision rules can be expressed as direct functions of type, α_i. It is not hard to show that the optimal policy rule for positive intensity types is given by

$$\psi(\alpha_m) \; = \; \bar{C} \alpha_m^{\frac{1}{1-2\theta}}$$

where $\bar{C} = \left(\frac{\delta^2 Y^{2\theta} \sum_i \alpha_i}{2cy} \right)^{\frac{1}{1-2\theta}}$ is a positive constant. Substituting the policy rule into the effort choices, we derive the behavioral rule for each individual as

$$\sigma_i(\alpha_m) \; = \; \alpha_i \bar{K} \alpha_m^{\frac{\theta}{1-2\theta}} \tag{22}$$

where $\bar{K} = \frac{\delta Y^\theta}{2c} \bar{C}^\theta$, another positive constant.

Just as in the previous example, types are ordered so that $\frac{\partial \sigma_j}{\partial \alpha_m} < 0$ whenever $\frac{\partial \sigma_j}{\partial m} > 0$. A franchise extension therefore requires a movement of the median evaluation toward lower, rather than higher, weights, α_m. This means that inequality in Proposition 1 is reversed, using α_i as an individual's type. Hence, from Proposition 1, the equilibrium admits a franchise extension iff

$$\sum_{j \neq m} \frac{\partial \sigma_j}{\partial \alpha_m} \; < \; 0 \tag{23}$$

Next, observe that $\frac{\partial Q}{\partial e_i} > (<) 0$ iff $\alpha_i > 0$ ($\alpha_i < 0$). Therefore, differentiating the behavioral rule (22) with respect to the state α_m, we see that (23) holds iff $\sum_{j \neq m} \alpha_j \; < 0$, or, in other words, $\sum_j \alpha_j < \alpha_m$. This implies that extension occurs only if the median lies above the aggregate welfare weight for the project. But since $\sum_j \alpha_j > 0$ then this means, among other things, that universal suffrage is not achieved — extension stops when $\alpha_m = \sum_j \alpha_j$.

A franchise expansion occurs then if the relative transfer from the "positive" group (less that of the pivotal voter) is outweighed by the relative gain to the "negative" group when taxes are lowered due to a smaller median weight on the public good. Roughly, the idea is that if the dominant group extend the franchise to at least some of the outsiders, then the tax burden is lower. Consequently, the outsiders do not fight as hard to resist taxation. If $\sum_j \alpha_j < \alpha_m$ holds, then, evidently, the drop in outsider effort outweighs the drop in insider effort.

This "internally driven" explanation appears to be different from the "internally driven" explanation underlying the model by Lizzeri and Persico (LP) (2003). LP consider a random voting model with two groups of citizens. Preferences over policy are uniform within each group, but individuals differ along an ideological dimension (which correlates with inherent policy-independent support for one of two political parties). In their model, the equilibrium policy choice reflects the relative electoral strengths of ideologically neutral voters in both groups, and is a kind of weighted average of the most preferred policy of each group. Extending the franchise to members of one of the groups (but not the other) can have the effect of shifting the equilibrium in the direction of that group's most preferred policy.[20]

There are clear differences between LP's set up and ours. Initial members of the elite are, on average, no different to the disenfranchised; policy is determined by party-political competition rather than median voter preferences; and policy choices and franchise extension are determined by different political processes (while in our model they are both determined by the median voter). However, there are some more fundamental similarities: in particular, in both models, the economic outcome of the political process is in general not that which is most preferred by the individual who has the option of extending the franchise (in both models this is the median voter), and by doing so this individual can move the outcome in a desirable direction.

In LP, extension effectively strengthens the current median's voice in the policy decision, but it comes at a cost of diluting his share of redistributive transfers within the elite. In our model, extension has beneficial static efficiency effects, but these are traded off against the costs of loss of control over future decisions.

Unfortunately, an analytical solution to the equilibrium franchise rule in the present model is not tractable. However, if the discount factor δ is low, then an approximate solution is given by the one period Euler equation. Replacing ∞ with $T = 1$ in the infinite horizon Euler

[20] Note that the optimality of such an extension is not automatic, since by expanding the size of the elite, per capita resources are reduced. However, if initially members of the expanding group were receiving no private transfers, this dilution effect is absent, and they can be made better off by the expansion.

equation (see Appendix B), the one period or "terminal" solution is given by μ^* where

$$\alpha_{m'} = \mu^*(\alpha_m) = \alpha_m \left(1 + \theta(1 - \frac{\alpha_m}{\sum_j \alpha_j})\right)$$

This solution is consistent with the requirement that $\sum_j \alpha_j \leq \alpha_m$.

5.4 External Conflict Generates Franchise Extension

Here we recall the "external conflict" example in which franchise expansion occurs to head off the threat insurrection. Recall that there $n - J$ noblemen and J peasants. The franchise belongs within the former group. A nobleman i has y_i land which generates y_i units of potential consumption each period. Peasants are disenfranchised and possess no land.

Each peasant chooses e_j which contributes toward an uprising which, if successful, confiscates the nobility's aggregate return, $Y \equiv \sum_{i \in A} y_i$ from land. A nobleman i chooses effort e_i to suppress the revolt. As before, effort is costly to all citizens, and $E_A = \sum_{i \in A} e_i$ and $E_B = \sum_{j \in B} e_j$. Finally, recall that the state variable, ω, is the probability in the current period that the confiscation by the peasants is unsuccessful. The initial state is $\omega_0 \in (0, 1)$. Next period's confiscation likelihood depends on current effort and the current state according to

$$\omega' = \begin{cases} 1 - (\frac{E_B - E_A}{n}) - \omega & if \quad 0 < (\frac{E_B - E_A}{n}) + \omega < 1 \\ 0 & if \quad (\frac{E_B - E_A}{n}) + \omega > 1 \\ 1 & if \quad otherwise \end{cases}$$

The negatively serial dependence means that a very likely confiscation in the current period reduces the likelihood in the subsequent period.[21]

A successful confiscation splits the return Y evenly among the peasants. The peasants receive a redistributive subsidy chosen by the median voter in the restricted franchise before the state revolt's success is known to "buy off" the peasants.

Each period, the median nobleman chooses a redistributive tax rate p which produces revenue pY. However, the technology for redistribution is concave — implying that some of the revenue is potentially lost in the redistributive process. Formally, revenue pY produces $(pY)^\theta$ available to be equally distributed to all members of society if there is no confiscation. , We assume $0 < \theta < 1$ so that if θ is close to one then very little is lost by the redistributive technology.

Members of nobility have stage utility function

$$u_i \;=\; \omega[(1-p)y_i + \frac{(pY)^\theta}{n}] - ce_i^2, \; i \in A$$

[21] The idea is that "close calls" today lead to better deterrence tomorrow. Conversely, confiscation is more likely when one's guard is down.

while members of the peasantry have utility

$$u_j \; = \; \omega \frac{(pY)^\theta}{n} \; + \; (1-\omega)\frac{Y}{J} \; - \; ce_j^2, \; j \in B$$

Using the Euler equations in a first order characterization, it can be shown, once again, that behavior and policy rules are invariant to the physical state, ω. The rules are given by

$$\psi(y_m) = \left(\frac{\theta Y^\theta}{n y_m}\right)^{\frac{1}{1-\theta}}$$

for policy, and

$$\sigma_i(y_m) = \frac{1}{2cn}(\frac{\delta}{1+\delta}) \left[y_i + (\frac{1}{\theta} y_m - y_i) \left(\frac{\theta Y^\theta}{n y_m}\right)^{\frac{1}{1-\theta}} \right], \; i \in A$$

and

$$\sigma_j(y_m) = \frac{1}{2cn}(\frac{\delta}{1+\delta}) \left[\frac{Y}{J} - \frac{1}{\theta} y_m \left(\frac{\theta Y^\theta}{n y_m}\right)^{\frac{1}{1-\theta}} \right], \; j \in B$$

for noblemen and peasants, respectively.

Using the same techniques as before, it is not hard to show that the franchise is extended by median income voter y_m iff

$$\sum_{i \in A, \, i \neq m} \frac{\partial \sigma_i}{\partial y_m} \; < \; \sum_{j \in B} \frac{\partial \sigma_j}{\partial y_m} \tag{24}$$

Using the expressions for effort rules above, it is not hard to show that (24) is equivalent to $y_m > Y/n$. Hence, a landowner with endowment y_m extends the franchise iff his land value is larger than average.

Despite the stationarity of the equilibrium, its analytical solution is again not tractable. However, it is instructive to see how the franchise decision affects the trajectory of the state — the likelihood of insurrection — by changing the private effort of citizens. Figure 4 illustrates the effect of repeated extensions. The extension effectively lowers the success rate of an insurrection. When authority is given to a lower income nobleman, the low income nobleman chooses a larger redistributive tax. This induces a relatively greater effort toward the defense of the status quo compared to the effort directed toward its demise. The "buy-off" is therefore successful.

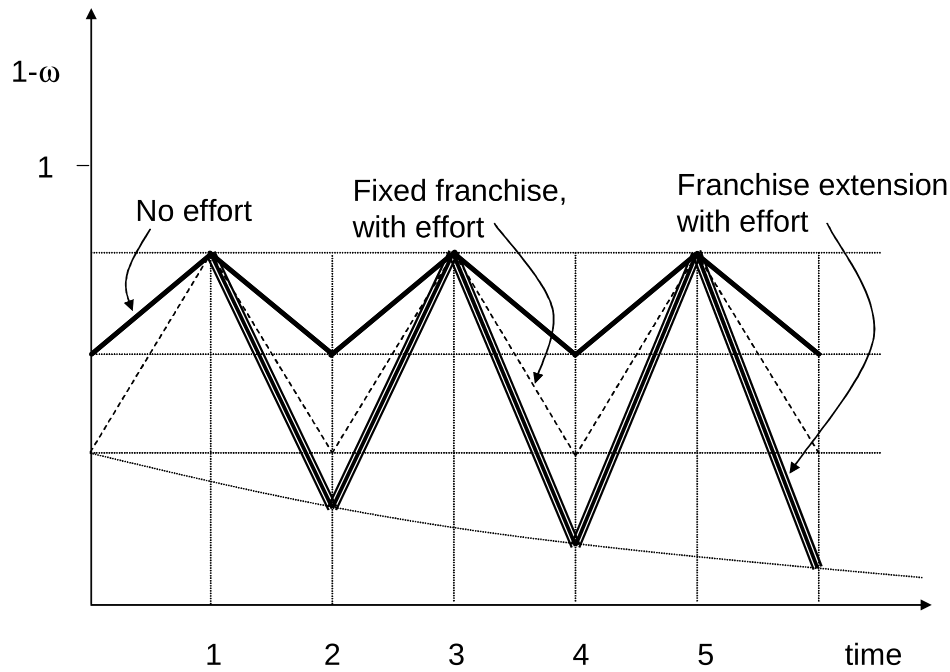

Figure 4: Path of insurrection likelihoods with and without a one-time franchise extension

6 Summary

This paper introduces a class of dynamic enfranchisement games. In these games, private decisions of all citizens are explicitly modeled, and voluntary extension of the voter franchise is a dynamically recursive phenomenon. We know of no other model with these features.

We characterize franchise extension equilibria of these games. These are Markov Perfect equilibria in which franchise extensions effectively delegate authority to a new median voter in the subsequent period. The current median is motivated by a desire to permanently change the policy-relevant private decisions of ordinary citizens. The franchise extension is therefore used as a commitment device to change private behavior through permanent changes in policy. This underlying causal mechanism is at the heart of both "internal" and "external" explanations of observed franchise extensions.

The framework is shown to cover a variety of policy environments. However, the usual caveats regarding limitations apply. The present environment is deterministic and assumes simple, single dimensional policies and private decisions. Naturally, the framework can be extended to include environments with multi-dimensional policies and multi-dimensional private decisions. The framework can also be extended to stochastic games — games in which the transitions are stochastic. Moreover, issues of equilibrium existence are also omitted, although some of these issue are taken up in a companion paper, Lagunoff (2003). Future research might be directed toward computational methods for generating franchise extension equilibria. It is hoped that a broader understanding of the dynamic enfranchisement game leads to a broader understanding of the mechanisms that sustain and extend democracy.

7 Appendix A: Proofs of the Propositions

Proof of Proposition 1 Let Π admit a first order characterization. If the current median voter, m, chooses to keep the current franchise, i.e., if $\mu(\omega, m) = m' = m$, then the Envelope Theorem implies:

$$\delta \frac{\partial V_m}{\partial m''} = 0 \text{ and } \frac{\partial u_m}{\partial p'} + \delta \frac{\partial V_m}{\partial \omega''_m} \frac{\partial Q}{\partial p'} = 0$$

Next, since the citizen's Euler equation, (6), must hold each period and in each state, we obtain

$$\frac{\partial V_i}{\partial \omega''} = -\frac{\partial u_i / \partial e'_i}{\delta \ \partial Q / \partial e'_i} \tag{25}$$

Substituting these three equations in the franchise Euler equation, (9), if Π admits a franchise extension then (12) must hold at $m' = m$. To obtain the converse, observe that since Π admits a first order characterization then equation (9) is decreasing, and so if (12) holds at $m' = m$, then by the Envelope Theorem, the solution to (9) entails a choice $m' > m$. ∎

Proof of Proposition 2 First we show (E-1)-(E-3) are equivalent to the original Euler equations, (6)-(8). The techniques for showing this are fairly standard. We differentiate the value function in Equation (5) with respect to the state using the Envelope Theorem wherever possible. This gives

$$\frac{\partial V_i}{\partial \omega} = \frac{\partial u_i}{\partial \omega} + \frac{\partial u_i}{\partial p}\frac{\partial \psi}{\partial \omega} + \sum_{j \neq i} \frac{\partial u_i}{\partial e_j}\frac{\partial \sigma_j}{\partial \omega} + \delta \frac{\partial V_i}{\partial \omega'} \left[\frac{\partial Q}{\partial \omega} + \frac{\partial Q}{\partial p}\frac{\partial \psi}{\partial \omega} + \sum_{j \neq i} \frac{\partial Q}{\partial e_j}\frac{\partial \sigma_j}{\partial \omega} \right] + \delta \frac{\partial V_i}{\partial m'}\frac{\partial \mu}{\partial \omega} \tag{26}$$

From the citizen's Euler equation, (6), we obtain

$$\frac{\partial V_i}{\partial \omega'} = -\frac{\partial u_i / \partial e_i}{\delta \ \partial Q / \partial e_i} \tag{27}$$

Then, substitute (27) for $\frac{\partial V_i}{\partial \omega'}$ in the expression (26) and iterate $\frac{\partial V_i}{\partial \omega}$ one period forward to obtain

$$
\begin{aligned}
\frac{\partial V_i}{\partial \omega'} &= \frac{\partial u_i}{\partial \omega'} + \frac{\partial u_i}{\partial p'}\frac{\partial \psi}{\partial \omega'} + \sum_{j \neq i} \frac{\partial u_i}{\partial e'_j}\frac{\partial \sigma_j}{\partial \omega'} - \frac{\partial u_i / \partial e'_i}{\partial Q / \partial e'_i}\left[\frac{\partial Q}{\partial \omega'} + \frac{\partial Q}{\partial p'}\frac{\partial \psi}{\partial \omega'} + \sum_{j \neq i} \frac{\partial Q}{\partial e'_j}\frac{\partial \sigma_j}{\partial \omega'}\right] + \delta \frac{\partial V_i}{\partial m''}\frac{\partial \mu}{\partial \omega'} \\
&\equiv \Lambda_i(\omega'; \Pi) + \delta \frac{\partial V_i}{\partial m''}\frac{\partial \mu}{\partial \omega'}
\end{aligned} \tag{28}
$$

where Λ_i is defined in the statement of the Proposition.

Now substitute (28) into the original first order condition (6) to obtain

$$
\frac{\partial u_i}{\partial e_i} + \delta \left[\Lambda_i(\omega'; \Pi) + \delta \frac{\partial V_i}{\partial m''}\frac{\partial \mu}{\partial \omega'}\right] \frac{\partial Q}{\partial e_i} = 0 \tag{29}
$$

Using (29), we solve for $\delta \frac{\partial V_m}{\partial m''}$ to obtain

$$
\delta \frac{\partial V_m}{\partial m''} = -\frac{1}{\partial \mu / \partial \omega'}\left(\frac{\partial u_m / \partial e_m}{\partial Q / \partial e_m} + \Lambda_m(\omega'; \Pi)\right) \tag{30}
$$

Equations (28)-(30) can now be used to obtain the adjusted Euler equations for the pivotal voter's policy and franchise decision, and all citizens' effort decisions, resp. To this end, write the pivotal voter m's Euler equation for the optimal policy choice p as

$$
\frac{\partial u_m}{\partial p} + \delta \left[\Lambda_m(\omega'; \Pi) + \delta \frac{\partial V_m}{\partial m''}\frac{\partial \mu}{\partial \omega'}\right] \frac{\partial Q}{\partial p} = 0 \tag{31}
$$

Then substitute (30) in place of $\frac{\partial V_m}{\partial m''}$ in (31) to obtain (E-1), the Euler equation for the pivotal voter's policy. Next, recall the Euler equation for the franchise decision expressed in terms of its explicit decomposition of effects, Equation (9). Using Equation (27) to substitute for $\frac{\partial V_m}{\partial \omega'}$ and Equation (30) to substitute for $\frac{\partial V_m}{\partial m''}$ we rewrite Equation (9) to obtain (E-2), the Euler equation for the franchise decision. Finally, for any ordinary citizen i, we iterate the left side of (E-2) one period forward. This yields (E-3), the Euler equation for the behavior decision rule of citizens.

Consequently, a continuously differentiable, interior profile $\Pi = (\sigma, \mu, \psi)$ satisfies properties I-III if and only iff these same properties apply to the original Euler equations, (6)-(8). But these are the conditions for which there exists an equilibrium that admits a first order characterization, save for the single crossing property, (ii). As stated earlier, Property IV is equivalent to the single crossing property when profiles are differentiable. ∎

8 Appendix B: Equilibrium with Internal Ideological Conflict

As in the case of class conflict, we derive the an Euler equation for franchise extension, which is

$$\begin{aligned}
\frac{\partial V_m}{\partial \alpha_{m'}} &= \left[\frac{\partial u_m}{\partial p'} - \left(\frac{\partial u_m/\partial e'_m}{\partial Q/\partial e'_m}\right)\frac{\partial Q}{\partial p'}\right]\frac{\partial \psi}{\partial m'} + \sum_{j\neq m}\left[\frac{\partial u_m}{\partial e'_j} - \left(\frac{\partial u_m/\partial e'_m}{\partial Q/\partial e'_m}\right)\frac{\partial Q}{\partial e'_j}\right]\frac{\partial \sigma_j}{\partial m'} + \delta\frac{\partial V_m}{\partial \alpha_{m''}}\frac{\partial \mu}{\partial \alpha_{m'}} \\
&= \left[-y + y\frac{\alpha_m}{\alpha_{m'}}\right]\frac{\bar{C}}{1-2\theta}\alpha_{m'}^{\frac{2\theta}{1-2\theta}} + \alpha_{m'}^{\frac{\theta}{1-2\theta}}\left[\sum_{i\neq m}\alpha_i \bar{K}\frac{\theta 2c\alpha_m \bar{K}}{1-2\theta}\alpha_{m'}^{\frac{\theta}{1-2\theta}-1}\right] + \delta\frac{\partial V_m}{\partial \alpha_{m''}}\frac{\partial \mu}{\partial \alpha_{m'}} \\
&= \left[\frac{\alpha_m}{\alpha_{m'}} - 1\right]\frac{y\bar{C}}{1-2\theta}\alpha_{m'}^{\frac{2\theta}{1-2\theta}} + \alpha_m\left[\frac{\theta 2c\bar{K}^2}{1-2\theta}\sum_{i\neq m}\alpha_i\right]\alpha_{m'}^{\frac{2\theta}{1-2\theta}-1} + \delta\frac{\partial V_m}{\partial \alpha_{m''}}\frac{\partial \mu}{\partial \alpha_{m'}} \quad = \quad 0
\end{aligned} \tag{32}$$

Recall that since $\theta > 1/4$ and $\sum_{i\neq m}\alpha_i < 0$, then $\frac{\partial V_m}{\partial \alpha_{m'}}$ is decreasing in $\alpha_{m'}$, and so the solution to (32) is a maximizer. The optimal franchise extension is, therefore, the median voter, m', that solves (32). By iterating forward (32), the infinite horizon Euler equation for an initial median voter $m = m_0$ is

$$\begin{aligned}
\frac{\partial V_{m_0}}{\partial \alpha_{m_1}} &= \sum_{t=1}^{\infty}\delta^{t-1}\left\{\left[\frac{\alpha_{m_0}}{\alpha_{m_t}} - 1\right]\frac{y\bar{C}}{1-2\theta}\alpha_{m_t}^{\frac{2\theta}{1-2\theta}} + \alpha_{m_0}\left[\frac{2c\bar{K}^2\theta}{1-2\theta}\sum_{i\neq m_0}\alpha_i\right]\alpha_{m_t}^{\frac{2\theta}{1-2\theta}-1}\right\}\prod_{\tau=0}^{t-1}\frac{\partial \mu}{\partial \alpha_{m_\tau}} \\
&= \sum_{t=1}^{\infty}\delta^{t-1}\left\{\left[\frac{\alpha_{m_0}}{\alpha_{m_t}} - 1\right]\alpha_{m_t}^{\frac{2\theta}{1-2\theta}} + \frac{\alpha_{m_0}}{\alpha_{m_t}}\left[\theta(\sum_{i\neq m_0}\alpha_i/\sum_j \alpha_j\right]\alpha_{m_t}^{\frac{2\theta}{1-2\theta}}\right\}\prod_{\tau=0}^{t-1}\frac{\partial \mu}{\partial \alpha_{m_\tau}} \\
&= \sum_{t=1}^{\infty}\delta^{t-1}\left\{\frac{\alpha_{m_0}}{\alpha_{m_t}}\alpha_{m_t}^{\frac{2\theta}{1-2\theta}}\left(1 + \theta\frac{\Sigma - \alpha_{m_0}}{\Sigma}\right) - \alpha_{m_t}^{\frac{2\theta}{1-2\theta}}\right\}\prod_{\tau=0}^{t-1}\frac{\partial \mu}{\partial \alpha_{m_\tau}} \\
&= 0
\end{aligned} \tag{33}$$

where the third equality comes from the fact that $\sum_{i\neq m_0}\alpha_i = \Sigma - \alpha_{m_0}$ where we define $\Sigma \equiv \sum_j \alpha_j$. While this expression does not give tractable analytical solution, we do find an approximate solution when δ is small by finding the solution to the truncated game when $T = 1$.[22]

[22] Extending this one period problem, we tried to find the infinite horizon rule by taking limits of the time-dependent rules derived in a T period, truncated game. While this approach probably feasible computationally,

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it is not feasible analytically. For instance, even in a *two* period game, the initial period's decision rule is the solution to

$$\alpha_{m'}^{\frac{2\theta}{1-2\theta}}\left[\frac{\alpha_m(1+\theta)-\frac{\theta}{\Sigma}\alpha_m^2}{\alpha_{m'}}-1\right] \;=\; \delta\left(\alpha_{m'}(1+\theta)-\frac{\theta}{\Sigma}\alpha_{m'}^2\right)^{\frac{2\theta}{1-2\theta}}\left[1-\frac{\alpha_m(1+\theta)-\frac{\theta}{\Sigma}\alpha_m^2}{\alpha_{m'}(1+\theta)-\frac{\theta}{\Sigma}\alpha_{m'}^2}\right]\left[(1+\theta)-2\frac{\theta}{\Sigma}\alpha_{m'}\right]$$