Electoral Competition With Private Information*

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Abstract

This paper analyses a two-candidate-one-dimension election model in which candidates possess private information over the preferences of the electorate. I show that in equilibrium candidates' positions are jointly determined by their beliefs over the electorate's preferences and their beliefs over their opponents' signals. Depending on the information structure, they may choose positions that are more moderate or more extreme than their expectations of the ideal point of the median voter. Information that are less widely known tend to have smaller impact on candidates' positions. More informative private signals may lower electorate welfare.

JEL Classification D72

1 Introduction

To politicians seeking public office, there are few things more important than figuring out what the electorate wants. But accessing the opinion of a large electorate is not easy. Consulting every voter is certainly impossible for any constituency larger than a few thousands. While opinion polls allow candidates to survey a large sample of voters, it is infeasible to ask questions in depth, and results may be influenced by the way questions are framed. Besides, even the most accurate polls can capture only the public opinion at the moment and could not predict how it would change over the course of a campaign. As candidates come from different backgrounds, gather information through different channels, and consult with different advisors, their knowledge about their constituency is diverse, as well as fragmentary. We often assume that candidates choose different platforms out of

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ideological reasons. In fact, they may do so simply because they have divergent beliefs about the preferences of the electorate.

Existing models of electoral competition assume that candidates have either complete or common information. In this paper I show that incorporating private information into a formal model can yield new insights about electoral competition. In standard Downsian models of two-candidate electoral competition, where candidates have complete information about the electorate, competition for votes compels the candidates to adopt the median voter's favorite policy as their platforms. The same obviously is not true when candidates have only incomplete information (as they do not know which policy the median voter prefers). More surprisingly, candidates generally do not choose as platforms the position they *expect* the median voter to prefer. Thus, there is no natural generalization of the median-voter theorem in electoral competition with incomplete information. Furthermore, electorate welfare is not monotone in the quality of the candidates' signals. Even though each candidate has an incentive to select a popular position, increasing their ability do so may actually lower welfare.

The model follows the standard two-candidate Downsian model except that the candidates are uncertain about the preferences of the electorate. Each candidate receives a private signal and updates his belief over the electorate's preferences, and on the basis of which selects a policy platform from a one-dimensional issue space. Then, the election takes place. The candidate who receives a plurality of votes wins and implements his position. Candidates have no policy preferences, and their sole objective is to win the election. Voters care about policy, as well as the identity of the winner. Their non-policy, candidate-specific preferences are represented by a random variable. Candidates know the distribution but not the realization of this random variable, and hence, from their perspective, voters vote probabilistically. In equilibrium, each candidate selects a position that maximizes his chance of winning, based on his beliefs about the voters, as well as his opponent's strategy. Because signals are private, *ex post* the candidates may not choose the same position.

To understand why candidates do not choose their respective expected median-voter positions in equilibrium, note that in two-candidate elections, the optimal position of a candidate lies *strictly* between the expected median-voter position and the position of his opponent.[1] The expected median-voter position is usually not a best response, because a candidate can capture some of the votes between him and his opponent by leaning toward his opponent. The standard policy convergence result relies on the additional condition that candidates have the *same* expectation over the median-voter position, which is true only when candidates have complete or common information. In that case, each candidate tries to choose a position closer to the common expectation than the other does; hence, in equilibrium their positions converge to that position. The same argument does not work when candidates have different expectations. As a result, a candidate's position and his expectation are usually correlated but not identical.

It is interesting to know the way in which candidates' equilibrium positions are different from the expected median-voter positions. Using the posterior expected median-voter position as a benchmark, I call a candidate's position "moderate," if it lies between the prior and posterior expected median-voter positions and "extreme," if it is farther away from the prior expectation than the posterior. Given the tendency for candidates to choose positions close to each other, one would expect them to choose moderate positions (in which case candidates having opposite beliefs will be closer to each other). In fact, I find that depending on the information structure candidates's equilibrium position can go either way. I consider two specific information structures. In the first one, there is a common informative signal, but each candidate observes the signal only with probability less than one. In the second, the candidates' signals are equally informative and conditional independent. In equilibrium, candidates under the first information structure choose moderate positions, while those under the second may choose either moderate or extreme positions.

There are two opposing effects at work. The law of iterated expectation implies that candidates believe that their opponent's *average* expectation of the median-voter position lies between his own prior and posterior expectations. Since candidates' equilibrium positions are correlated with their

[1] I use the term "expected median-voter position" to mean a candidate's expectation of the ideal point of the median voter, conditional on his own signal. The two candidates may have different expected median-voter positions.

expectations, they would also believe that their opponent's position is on average more moderate than the expected median-voter position. This "average effect" pushes candidates toward moderate positions. On the other hand, candidates care also about the distribution of their opponents' beliefs. Specifically, candidates put more weight on the event that their opponents receive signals similar to their own. It turns out that the best response against such an opponent is to choose an extreme position. If this "distribution effect" is sufficient strong, the candidates may choose extreme positions in equilibrium.

Apart from candidate positioning, the model also raises an interesting point about the desirability of divergent platforms. The welfare of the voters is a function of the distance between the winner's position and their ideal points, and it is maximized when the median-voter position is adopted.[2] In models with complete information, there is no loss in efficiency for both candidates to choose the same position, as long as it is the ideal point of the median voter. But when candidates have only incomplete information, choosing the median-voter position is no longer feasible. In this case, voters have no choice when candidates choose the same position, whereas they can choose the position they prefer when candidates choose distinct ones.

Individual candidates do not internalize the gains from divergent platforms. Each of them has an incentive to choose a platform so as to maximize his own chance of election. As a result, their positions tend to be too close together. Because of this conflict between individual popularity and platform diversity, electorate welfare is not monotone in the quality of the signals. As both signals predict the preferences of the same electorate, making them more accurate necessarily increases their correlation. As candidates with similar beliefs choose similar positions, having more accurate signals may reduce platform diversity. In section 5, I show that adding noise to candidates' signal actually can raise electorate welfare. Many commentators have noted that the increasing reliance on opinion polls has resulted in candidates taking similar positions in most major issues. My result suggests that even though better polls allow individual candidates to select more popular platforms,

[2]This is true when all voters have the same utility function up to the ideal point, and the utility function is concave in the distance between the ideal point and the policy. See Coughlin and Nitzan (1981).

they need not enhance social welfare.

Downs (1957) first establishes the platform-convergence result. Subsequent works, extending the model in many directions, largely confirm Downs' original insight, but also identify conditions under which platforms may diverge. An early model of probabilistic voting is Hinich (1977), which shows that candidates' platforms need not converge to the median-voter position. The model I use is derived from Anderson et al (1992). Several papers study election models in which candidates have private information on factors, such as their own ability (Rogoff, 1990) and policy preferences (Alesina and Cukierman, 1990; Banks, 1990; Harrington, 1992, 1993). These papers examine how the candidates' incentive to reveal their private information to voters affects their platforms and electoral outcomes. Heidhues and Lagerlöf (2000) study in a two-alternative election model in which candidates have private information over the efficacy of the two alternatives. They show that in equilibrium candidates under-use their own information and conform to the prior belief of the voters. My results are different in that candidates in my model may under- or over-use their information. More importantly, their results, like the rest of the literature, arise because candidates cannot credibly signal their private information to the voters. Since voters in my model know their own preferences, signalling is not the issue. Instead, my objective is to understand how electoral competition determines a candidate's incentive to use his private information.

The rest of the paper is organized as follows. In sections 2 and 3, I introduce the formal model and derive sufficient conditions for the existence of pure-strategy equilibrium. Section 4 examines candidate positioning. Section 5 shows that more informative signals can lower welfare. Section 6 concludes.

2 A Model

Two candidates compete for an elective office. Each receives a private signal about preferences of the electorate and announces a policy platform. Voters then cast their votes. The candidate who receives a majority of the votes wins the election and implements his platform.

There is a continuum of voters with unit mass; each has a policy preference over an one-dimensional issue space, denoted by a finite interval. If candidate i with platform z wins, a voter with ideal point x receives

$$u(i,z,x) = -\tau|x-z| + \varepsilon_i \tag{1}$$

where τ is a strictly positive constant and ε_i a candidate-specific random term. The difference between the two random terms, $\varepsilon_1 - \varepsilon_2$, is logistically distributed with zero mean and a variance of $\frac{\pi^2}{3}$. The random term captures voters' preferences over the candidates' non-policy characteristics, such as communication skills and trustworthiness. In general, one should expect voters' non-policy preferences to be correlated. Here, in order to simplify the analysis, I assume that they are identical, so that ε_i is common among all voters. The voters know ε_i when they cast their ballots, while the candidates know only the distribution of ε_i and not its realization when they pick their platforms. Voters vote for the candidate whose election yields a higher utility. The importance of the random term is determined by τ. When τ is small, the vote largely depends on the random term. The converse holds true when τ is large.

The preferences of the voters are distributed over the issue space according to a density function g_α indexed by α, the ideal point of the median voter. The candidates know the functional form of g but are uncertain about α. Their prior beliefs over α is represented by a density function f. Let x_m denote the expected ideal point of the median voter. I assume that $\alpha \in [0,1]$ and f is continuously differentiable, symmetric, and single-peaked.

Before choosing their platforms, each candidate receives a private signal, $s_i \in S \equiv \{l,c,r\}$. Write $h(s_1,s_2|x)$ for the conditional probability that candidates 1 and 2 receive s_1 and s_2, respectively, given ideal point x. Similarly, write $h(s_i|x)$ for the conditional probability that candidates i receives s_i. The information structure can be fully characterized by a function $A : [0,1] \to [0,1]^9$ where

$$A(x) = \begin{bmatrix} h(l,l|x) & h(l,c|x) & h(l,r|x) \\ h(c,l|x) & h(c,c|x) & h(c,r|x) \\ h(r,l|x) & h(r,c|x) & h(r,r|x) \end{bmatrix}.$$

Given A, candidates update their beliefs over the ideal point of the median voter according to the

Bayes' rule. I use $f(.|s_i)$ and $F(.|s_i)$ to denote the posterior density and distribution distributions of α conditional on s_i. Formally

$$f(x|s_i) = \frac{h(s_i|x)f(x)}{\int_0^1 h(s_i|x)f(x)dx}.$$

Henceforth, if y is the expectation of some random variable over f, then $y(.|s)$ and $y(s)$ denotes y conditional on $f(.|s)$. Note that A also defines a candidate's posterior belief regarding his opponent's signal. Candidate i receiving s_i would believe that there is a probability $\mu\left(s_j|s_i\right)$ that candidate j receives s_j, where

$$\mu\left(s_j|s_i\right) = \frac{\int_0^1 h(s_i,\, s_j|x)f(x)dx}{\sum_{s_j \in S_j} \int_0^1 h(s_i,\, s_j|x)f(x)dx}.$$

Throughout, I assume that A satisfies the following assumptions.

Assumption 1 *For all* $x \in [0,1]$, $A(x) = A'(x)$.

Assumption 2 *For all* $x \in [0,1]$, $h(l,l|x) = h(r,r|1-x)$, $h(l,c|x) = h(r,c|1-x)$, $h(l,r|x) = h(l,r|1-x)$, *and* $h(c,c|x) = h(c,c|1-x)$.

Assumption 3 *For all* $x < y$, $\frac{h(l|x)}{h(l|y)} \geq \frac{h(c|x)}{h(c|y)} \geq \frac{h(r|x)}{h(r|y)}$.

Assumption 4 *For all* $s \in S$ *and for all* $x < y$, $\frac{h(l|x,s)}{h(l|y,s)} \geq \frac{h(c|x,s)}{h(c|y,s)} \geq \frac{h(r|x,s)}{h(r|y,s)}$.

Assumption 1 means that the candidates are treated identically. For any pair of signal (a, b), the probability that candidate 1 receives a and candidate 2 receives b is the same as the probability that candidate 1 receives b and candidate 2 receives a. Assumption 2 implies that the probability that signal l is drawn when $\alpha = x$ is the same as the probability that signal r is drawn when $\alpha = 1 - x$, and signal c is equally likely to be generated by ideal points x and $1 - x$. It follows that $x_m(c)$, the expected median-voter position conditional on c, is 0.5. Assumptions 3 and 4 are monotone-likelihood-ratio conditions. They imply that, first, for all $x \in [0,1]$, $F(x|l) \geq F(x|c) \geq F(x|r)$, and, second, for all $x \in [0,1]$ and for all $s \in S$, $F(x|s,l) \geq F(x|s,c) \geq F(x|s,r)$ (Milgrom 1982). They also mean that $x_m(l,l) \square x_m(l) \square x_m(c) \square x_m(r) \square x_m(r,r)$. That is, a candidate receiving l shifts his posterior belief to the left, while one receiving r shifts his to the right. Furthermore, learning that the other candidate's signal is l does not move a candidate's belief to the right.

3 Equilibrium

After receiving their private signals, the candidates each choose a position $z \in [0,1]$ as their election platform. A pure strategy of candidate i, denoted by σ_i, is a function from S to $[0,1]$. Let $V_i(z_i, z_j|s_i, s_j)$ denote the probability that candidate i with platform z_i wins the election when candidate j chooses z_j and the signals are s_i and s_j. Assume that the winner's payoff is one and the loser's payoff is zero. A candidate i choosing z_i receives an expected payoff of

$$U_i(z_i, \sigma_j|s_i) = \sum_{s_j \in S_j} \mu(s_j|s_i) V_i(z_i, \sigma_j(s_j)|s_i, s_j).$$

A pair of pure strategies (σ_1, σ_2) constitutes a Bayesian Nash equilibrium if, for $i \in \{1,2\}$ and for all $s_i \in S_i$, $\sigma_i(s_i) \in \arg\max_{z_i \in [0,1]} U_i(z_i, \sigma_j|s_i)$.

The vote for a candidate generally depends on g, the distribution of voters conditional on α, as well as on f, the distribution of α. However, for any z_i and z_j, the identity of the winner depends only on f.

Lemma 1 *A candidate wins the election if the median voter strictly prefers him to the other candidate.*

Proof of Lemma 1: Assume without loss of generality that $z_i < z_j$. Suppose $-\tau|\alpha - z_i| + \varepsilon_i > -\tau|\alpha - z_j| + \varepsilon_j$; that is, the median voter strictly prefers candidate i to j. Then, for all $x < \alpha$,

$$\begin{aligned} -\tau\left(|x - z_i| - |x - z_j|\right) &= -\tau\left(|x - \alpha + \alpha - z_i| - |x - \alpha + \alpha - z_j|\right) \\ &\geq -\tau\left(|\alpha - z_i| - |\alpha - z_j|\right), \end{aligned}$$

with equality holds either when $\alpha < z_i$ or when $x - z_j > 0$. Thus, any voter with ideal point x less than α strictly prefers candidate i to j. By continuity, voters with ideal point slightly larger than α strictly prefers candidate i to j as well. A majority of voters therefore prefer candidate i to j, as α is the ideal point of the median voter. The proof for the case where the median voter strictly prefers candidate j to i is similar and hence omitted.

Lemma 1 follows from the assumption that all voters have the same non-policy preferences. It should be noted that the lemma applies only to situations where the median voter *strictly* prefers one candidate to the other, and the election need not be tied when the median voter is indifferent between the two candidate.[3] However, since this event occurs with zero probability, as the distribution of ε_i is atomless, the outcome of the election is almost surely determined by the preference of the median voter, and whoever wins his vote wins the election. For any given α, the probability of candidate i winning is

$$\pi(q) = \frac{e^{\tau q}}{1 + e^{\tau q}}, \tag{2}$$

where $q = |\alpha - z_j| - |\alpha - z_i|$ measures the extent to which candidate i's position is closer to the median voter's ideal point than candidate j's. $V_i(z_i, z_j|s_i)$, the average probability of winning, obtained by integrating π over f, can be written as

$$\begin{aligned} V_i(z_i, z_j|s_i) &= \int_0^1 \pi(|x - z_j| - |x - z_i|) f\,(x|s_i)\,dx \\ &= F(z_i|s_i)\pi(z_j - z_i) + \int_{z_i}^{z_j} \pi(z_i + z_j - 2x) f(x|s_i) dx + (1 - F(z_j|s_i))\pi(z_i - z_j). \end{aligned}$$

The assumption of probabilistic voting guarantees that V_i is continuous in z_i and z_j. For any τ, π is increasing in q, convex in the negative domain, and concave in the positive one, and its first derivative is symmetric over zero (i.e. $\frac{d\pi(t)}{dq} = \frac{d\pi(-t)}{dq}$).[4] The shape of π means that a change in a candidate's position has a larger impact on indifferent voters (with small q) than partisan ones (with larger q). The size of τ and the precise functional forms of (1) and (2) are not crucial to the results.

[Figure 1 here.]

[3]For example, suppose $\alpha > z_j > z_i$ and the median voter is indifferent between the two candidates. Voters with ideal point $x \geq z_j$ are indifferent, while those with $x < z_j$ strictly prefer candidate i. Hence, candidate i wins certainly.

[4]More precisely, $\frac{d\pi}{dq} = \frac{\tau e^{\tau q}}{(1+e^{\tau q})^2}$, $\frac{d^2\pi}{dq^2} = \frac{\tau^2 e^{\tau q}(1-e^{\tau q})}{(1+e^{\tau q})^3}$, and $\frac{d^3\pi}{dq^3} = \frac{\tau^3 e^{\tau q}(1-4e^{\tau q}+e^{2\tau q})}{(1+e^{\tau q})^4}$.

The following two lemmas play important roles in the results that follow in the next section. (As both lemmas hold for any signal s, I suppress s in the equations.)

Lemma 2 *For all z_j, there exists a unique z^* such that (i) $\frac{\partial V_i(z^*,z_j)}{\partial z_i} = 0$ and (ii) for all $z_i \neq z^*$, $\frac{\partial V_i(z_i,z_j)}{\partial z_i}(z^* - z_i) > 0$. For all z_j, $\frac{dz^*}{dz_j} > 0$. When $z_j = x_m$, $z^* = x_m$. When $z_j \neq x_m$, $z^* \in (\min(x_m, z_j), \max(x_m, z_j))$.*

Lemma 3 *Let z' be the position such that $1 - F(z') = F(z)$. Then, for all $z < x_m$ and for all $y \in [0, z']$, $\frac{\partial V_i(z,z)}{\partial z_i} > \frac{\partial V_i(z,y)}{\partial z_i}$; and for all $z > x_m$ and for all $y \in [z', 1]$, $\frac{\partial V_i(z,z)}{\partial z_i} < \frac{\partial V_i(z,y)}{\partial z_i}$.*

Lemma 2 reflects a well-known feature of two-candidate-one-dimensional electoral competitions, namely the tendency for candidates to choose similar positions. It says that for any opponent's position a candidate always has a unique best response, which moves in the same direction as his opponent does. If the opponent's position is the same as the candidates' expectation of the median-voter position, then the candidate's best response is to choose that position; otherwise, his best response is to choose a position strictly between the two. In other words, candidates want to be near their opponents but on the popular side of the public opinion. By moving toward his opponent from the median-voter position, a candidate captures some of the votes between he and his opponent without losing much of his base, as voters indifferent between the two candidates are more sensitive to a minor change in position than those who strongly prefer one candidate to the other.[5] It follows immediately from Lemma 1 that candidates choose the expected median-voter position in equilibrium when they have common information. Figure 1 shows $\frac{\partial V_i(z_i,z_j)}{\partial z_i}$ when voters are distributed according to a truncated normal distribution with a mean of 0.5 and a variance of 0.09. The best response z^* is 0.52 when $z_j = 0.6$ and 0.56 when $z_j = 0.7$.

Lemma 3 describes how a candidate's marginal incentive to change his position is affected by his opponent's position. It says that for a candidate not choosing x_m, the marginal gains for moving toward x_m is greater when his opponent's position is z than any other in $[0, z']$ if $z < x_m$ (or $[z', 1]$ if

[5] That is, $\frac{d\pi}{dq}$ reaches the maximum at $q = 0$.

$z > x_m$). Since we have already learned from Lemma 2 that there is a tendency for a candidate to move toward his opponent, it is no surprise that a candidate has a lower marginal incentive to move toward x_m as his opponent moves away from it. What is not obvious is that the candidate may still have a lower marginal incentive to move toward x_m even as his opponent moves toward x_m. For example, in figure 1 $\frac{\partial V_i(0.7,0.7)}{\partial z_i} < \frac{\partial V_i(0.7,0.6)}{\partial z_i}$. This is because competition is more intense when candidates' positions are close to each other. While a small shift in position may make two similar candidates look significantly different, it would not have the same effect when the candidates are already far apart.[6] By moving toward x_m, a candidate widens the difference between he and his opponent, making voters less sensitive to any further change in position.

It is clear from figure 1 that V_i is generally not concave. As a result U_i need not be quasi-concave, and the candidates' best-response sets may be non-convex. For example, if a candidate believes, first, that the distribution of the ideal point is either skewed to the left or the right, and, second, that his opponent (owing to superior information) will choose left in the first occasion and right in the second, then the candidate's best response is to choose either left or right but not in between, where he will lose surely. Because of the non-convexity, the game does not satisfy standard sufficient conditions for the existence of pure-strategy Bayesian Nash equilibrium. (Since U_i is continuous and the candidates' strategy set is compact, mixed-strategy Nash equilibrium always exists.) This does not mean that pure-strategy equilibrium does not exist, only that there may not be one. The following proposition shows that pure-strategy equilibrium exists when the private signals are weak.

Proposition 1 *Consider a sequence of conditional probability functions $\{h_n(s_i, s_j|x)\}_{n=1}^{\infty}$ converging uniformly to an non-informative probability function $h^*(s_i, s_j|x)$ that is constant in x. Let Γ_n be the election game associated with $h_n(s_i, s_j|x)$. There exists n^* such that for all $n \geq n^*$, Γ_n has a pure-strategy equilibrium.*

The proof relies on the fact the payoff function U_i is "locally" concave, which means that for

[6] Imagine how Patrick Buchanan will look compared to Ralph Nader if the former becomes slightly more liberal.

any point $x \in [0,1]$, there is some interval I containing x such that for all $s_i \in S$, $U_i(.|s_i)$ is concave for all z_i contained in I, provided that the other candidate's action is also contained in I. Here I provide an outline of the proof. First, I show that as the signals become weaker, both $x_m(l,l)$ and $x_m(r,r)$ converge to 0.5, so that for sufficiently weak signals, U_i is concave in the interval $[x_m(l,l), x_m(r,r)]$. It then follows from Glicksberg (1952) that pure-strategy equilibrium exists in a modified game in which candidates are restricted to positions in the interval $[x_m(l,l), x_m(r,r)]$. Finally, I show that any position outside of $[x_m(l,l), x_m(r,r)]$ cannot be a best response against a position inside of $[x_m(l,l), x_m(r,r)]$. Hence, the pure-strategy equilibrium of the restricted game is also an equilibrium in the original game.

4 Candidate Positioning

Downs (1957) shows that in two-candidate-one-dimensional election models both candidates select the median-voter position in equilibrium. An analogue to the celebrated Median Voter Theorem holds true in my model when candidates have common information. But, as we shall see, when candidates have private information and, hence, different posterior expectations over the ideal point of the median voter, they generally do not choose their respective expected median-voter positions in equilibrium.

For the rest of the paper, I assume that U_i is quasi-concave and pure-strategy equilibrium exists.[7] A candidate's strategy is symmetric if $\sigma_i(l) = 1 - \sigma_i(r)$ and $\sigma_i(c) = 0.5$. A pure-strategy equilibrium is symmetric if the candidates adopt identical symmetric strategies. (Hence, I shall drop the subscript i and use σ to denote the equilibrium strategy.) The following proposition shows that symmetric equilibrium exists and is unique.

Proposition 2 *There exists a unique symmetric equilibrium σ^*, and*

$$\sigma^*(l) \begin{cases} < x_m(l) \\ = x_m(l) \\ > x_m(l) \end{cases} \quad \text{if} \quad L(x_m(l)) \begin{cases} < 0 \\ = 0 \\ > 0 \end{cases} .$$

[7]Proposition 1 shows that these conditions hold when the signals are weak.

where

$$L(z) = \mu(l|l)\frac{\partial V_i(z,z|l,l)}{\partial z_i} + \mu(c|l)\frac{\partial V_i(z,0.5|l,c)}{\partial z_i} + \mu(r|l)\frac{\partial V_i(z,1-z|l,r)}{\partial z_i}. \quad (3)$$

Suppose the candidates choose the symmetric strategy profile $(z, 0.5, 1-z)$. Then $L(z)$ is the marginal gains a candidate with signal l will receive for moving slightly to the right from z. If $L(z)$ is equal to zero, z is a best response for the candidate, provided that the other candidate's strategy is $(z, 0.5, 1-z)$.[8] On the other hand, if $L(z)$ is not zero, the candidate can raise his payoff by slightly altering his position. Proposition 2 says that in the unique symmetric equilibrium, a candidate receiving l chooses a position less than $x_m(l)$ when $L(x_m(l)) < 0$, and greater than $x_m(l)$ when $L(x_m(l)) < 0$.

To understand the idea of the proof, first consider the strategy profile $(0.5, 0.5, 0.5)$. As signal l shifts the posterior belief over the ideal point to the left, if the candidates choose this strategy, then a candidate with signal l, expecting $x_m(l) < 0.5$, will want to defect to the left. Hence $L(0.5) < 0$. Now, consider the strategy profile $(x_m(l,l), 0.5, x_m(r,r))$. If the candidates choose this strategy, then a candidate with signal l will want to defect to the right. Note that $x_m(l,l)$ is a best response only against l. Against the other two signals, the best response is larger than $x_m(l,l)$. Hence $L(x_m(l,l)) > 0$. In the proof given in the appendix, I show that L is decreasing in z. As a result, L intersects the x-axis once, and the intersection is larger than $x_m(l)$ if and only if $L(x_m(l)) > 0$. See figure 2.

[Figure 2 here.]

Henceforth, I focus exclusively on the symmetric equilibrium. I call a candidate's strategy or position "moderate" if $\sigma(l) > x_m(l)$ and $\sigma(r) < x_m(r)$, and "extreme" if $\sigma(l) < x_m(l)$ and $\sigma(r) > x_m(r)$. When candidates choose moderate positions, their positions are closer to each other than are their underlying beliefs. The converse is true when candidates choose extreme positions.

[8] The first order conditions are also sufficient because V is concave in the first argument. Since 0.5 is a best response for a candidate with signal c, $L(z) = 0$ is necessary and sufficient for $(z, 0.5, 1-z)$ to be a Nash equilibrium.

By Proposition 2, we can determine whether the candidates' equilibrium positions are moderate or extreme without computing the equilibrium explicitly.

The crucial difference between common and private signals is that candidates do not observe each other's signal in the latter. In equilibrium a candidate's position is determined by his beliefs over his opponent's signal, as well as that over the ideal point of the median voter. Depending on the information structure, the candidates may choose moderate or extreme positions in equilibrium. Below, I illustrate the connection between information structure and candidates' positioning by means of two examples. There are two reasons why candidates may receive different information. It is possible that one candidate has superior information, knowing something the other does not, or their information are obtained through independent channels. The two examples roughly correspond to these two situations. In the first one, there is a common signal, which each candidate observes with probability less than one. Because the signal is common, a candidate's belief over the ideal point is not be affected by his knowledge of his opponent's signal. In the second example, the candidates receive independent signals so that they can infer additional information from their opponents' signals. I show that while candidates in the first example always choose moderate positions, those in the second may choose either way.

4.1 Example One (Common Signals)

For concreteness, imagine that there is a study on the demographic characteristics of the voters that has important implications on their preference. The candidates know that the study exists and are free to purchase a copy of the report, but there is only a probability $p < 1$ that each will actually do so.[9] The study has two possible conclusions. Let l denote the one which implies the voters are liberal and r the one which implies the voters are conservative. Let c denote the event that a candidate does not have the report. Whether a candidate purchases the report is private information. Let $\rho(s_i|x)$ denote the probability that the ideal point is x when the conclusion is s_i.

[9]Some candidates do not read.

I assume that for all $x < y$,

$$\frac{\rho(l|x)}{\rho(l|y)} \geq \frac{\rho(r|x)}{\rho(r|y)}.$$

The complete information structure is given by

$$A(x) = \begin{bmatrix} p^2\rho(l|x) & p(1-p)\rho(l|x) & 0 \\ p(1-p)\rho(l|x) & (1-p)^2 & p(1-p)\rho(r|x) \\ 0 & p(1-p)\rho(r|x) & p^2\rho(r|x) \end{bmatrix}.$$

In this example, a candidate who knows the conclusion of the report cannot infer any additional information from knowing what the other candidate knows. As there is only one source of information, the candidate knows that his opponent either knows the conclusion or not, and in neither case would the candidate update his belief. It means that for $s \in \{l, r\}$, $F(.|s, s) = F(.|s, c) = F(.|s)$. Proposition 3 says that under such circumstances the candidates choose positions closer to the center than their beliefs, and, furthermore, the equilibrium position is farther away from the expected median-voter position when the chance that candidates purchase the report is low.

Proposition 3 *Let $\sigma^*(p) = (\sigma^*(l, p), 0.5, \sigma^*(r, p))$ denote the symmetric equilibrium in Example 1. Then, for all $p \in [0, 1)$, $\sigma^*(l, p) > x_m(l)$ and $\frac{d\sigma^*(l,p)}{dp} < 0$.*

Proof of Proposition 3:

For a candidate receiving l, there is a probability p that his opponent receives l and a probability $1 - p$ that his opponent receives c. As the signal is common, $V(.|l, l) = V(.|l, c) = V(.|l)$ and $\mu(r|l) = 0$. We can write

$$L(x_m(l)) = p\frac{\partial V_i(x_m(l), x_m(l)|l)}{\partial z_i} + (1-p)\frac{\partial V_i(x_m(l), 0.5|l)}{\partial z_i} > 0.$$

The inequality holds as, by Lemma 2, the first term in the middle expression is zero, and the second term is positive. To a candidate with signal l, a small deviation from $x_m(l)$ to the right causes a second-order loss against an opponent choosing $x_m(l)$ and a first-order gain against one choosing 0.5. By definition, $L(\sigma^*(l, p)) = 0$. Differentiating both sides with respect to p gives

$$\frac{d\sigma^*(l,p)}{dp} = \frac{\frac{\partial V_i(\sigma^*(l,p),0.5|l,c)}{\partial z_i} - \frac{\partial V_i(\sigma^*(l,p),\sigma^*(l,p)|l,l)}{\partial z_i}}{p\frac{\partial V_i^2(\sigma^*(l,p),\sigma^*(l,p)|l,l)}{\partial z_i} + (1-p)\frac{\partial V_i^2(\sigma^*(l,p),0.5|l,c)}{\partial z_i}} < 0.$$

In the above expression, the numerator is positive as $\sigma^*(l,p) \in (x_m(l), 0.5)$ and the denominator is negative as V_i is concave.

Proposition 3 provides an alternative interpretation to the well-known policy-convergence result. According to the standard interpretation, candidates forego their own idiosyncratic preferences and converge to a common position popular among voters. See Calvert (1985). Under this interpretation, policy convergence is desirable as it ensures that the will of the people is carried out. In my model, the candidates, however, have no policy preferences of their own. Instead, they "under-use" their private information and partially converge to the conventional wisdom. Instead of choosing a position that they think the electorate will like, they choose one that is closer to what they think the other candidate thinks the electorate will like. The equilibrium position $\sigma^*(l,p)$ is increasing in p. In other words, the less widely known a signal, the smaller its impact on a candidate's position.

In this example a candidate who knows the conclusion of the study would expect his opponent to have a belief that is "on average" between his own posterior belief and the common prior. For example, suppose candidate 1 is the one who knows the conclusion. If candidate 2 also knows it, his posterior belief will be the same as candidate 1's. On the other hand, if candidate 2 does not know, his posterior belief will be the same as the prior. Since candidate 1 does not know whether candidate 2 knows, he will believe that candidate 2's belief is on average between the prior and his own posterior belief. More generally, the law of iterated expectation implies that for any information structure A that satisfies Assumptions 1 to 4 and for all $x \in [0,1]$,

$$F(x|l) = \sum_{s_j \in S} \mu(s_j|l) F(x|l, s_j) \geq \sum_{s_j \in S} \mu(s_j|l) F(x|s).$$

Intuitively, candidates do not expect his opponent to know what they know, as signals are private. If a candidate receives a l signal, he would expect his opponent to have a moderate belief between $F(.|l)$ and $F(.)$. Now, since candidates' positions are correlated with beliefs, this candidate would also expect his opponent to take a position more moderate than his expectation of the median-voter position. Hence, there is a tendency for candidates to choose moderate positions.

4.2 Example Two (Independent Signals)

Suppose the candidates independently conduct their own polls, which are identically designed, and in which a respondent is asked to identify herself as liberal (l), centrist (c), or conservative (r). Suppose $\xi(s_i|x)$, the probability that a respondent with ideal point x says s_i in candidate i's poll, satisfies the monotone-likelihood-ration condition, such that for all $x < y$,

$$\frac{\xi(l|x)}{\xi(l|y)} > \frac{\xi(c|x)}{\xi(c|y)} > \frac{\xi(r|x)}{\xi(r|y)}.$$

Furthermore, assume that for all $x \in [0,1]$ $\xi(s_i|x)$ can be written as,

$$\xi(l|x) = (1-d)\gamma\left(l|x\right), \xi(c|x) = d, \text{and } \xi(r|x) = (1-d)\gamma\left(r|x\right),$$

for some function $\gamma\left(.|.\right)$ and constant d. The assumption means that the response c is non-informative, and candidates' posterior beliefs over the ideal point are not affected by d. The information structure of the game is

$$A(x) = \begin{bmatrix} \xi^2(l|x) & \xi(l|x)d & \xi(l|x)\xi(r|x) \\ d\xi(l|x) & d^2 & d\xi(r|x) \\ \xi(r|x)\xi(l|x) & \xi(r|x)d & \xi^2(r|x) \end{bmatrix}.$$

As the polls are conducted independently, a candidate can infer additional information from the polling outcome of his opponent. Specifically, a candidate with signal l updates his belief to the left, from $F\left(.|l\right)$ to $F\left(.|l,l\right)$, if he knows the other candidate's signal is l, and he updates his belief to the right, from $F\left(.|l\right)$ to $F(.|l,r)$, if he knows the other candidate's signal is r. His belief is unchanged if the other candidate's signal is c. In summary,

$$x_m(l,l) < x_m(l,c) = x_m(l) < x_m(l,r).$$

Proposition 4 says that in this case candidates choose moderate positions when d is large and extreme positions when d is small.

Proposition 4 *In Example 2, there exists* $\underline{d}$ *and* $\overline{d} \in (0,1)$ *such that* $\sigma^*(l) \Box x_m(l)$ *when* $d \Box \underline{d}$ *and* $\sigma^*(l) \geq x_m(l)$ *when* $d \geq \overline{d}$.

Proof of Proposition 4:

By definition,

$$L(x_m(l)) = \mu(l|l)\frac{\partial V_i(x_m(l), x_m(l)|l,l)}{\partial z_i} + \mu(c|l)\frac{\partial V_i(x_m(l), 0.5|l,c)}{\partial z_i} + \mu(r|l)\frac{\partial V_i(x_m(l), 1-x_m(l)|l,r)}{\partial z_i} \tag{4}$$

Recall from Lemma 2 that $x_m(l)$ is a best response for a candidate with signal l if his opponent chooses the same position, meaning that

$$\mu(l|l)\frac{\partial V_i(x_m(l), x_m(l)|l,l)}{\partial z_i} + \mu(c|l)\frac{\partial V_i(x_m(l), x_m(l)|l,c)}{\partial z_i} + \mu(r|l)\frac{\partial V_i(x_m(l), x_m(l)|l,r)}{\partial z_i} = 0. \tag{5}$$

Subtracting (5) from (4) gives

$$L(x_m(l)) = \mu(c|l)C + \mu(r|l)D$$

where

$$\begin{aligned} C &= \frac{\partial V_i(x_m(l), 0.5|l,c)}{\partial z_i} - \frac{\partial V_i(x_m(l), x_m(l)|l,c)}{\partial z_i}, \\ D &= \frac{\partial V_i(x_m(l), 1-x_m(l)|l,r)}{\partial z_i} - \frac{\partial V_i(x_m(l), x_m(l)|l,r)}{\partial z_i}. \end{aligned}$$

C and D measure the change in $\frac{\partial V_i(x_m(l), z_j|l)}{\partial z_i}$, the marginal gains to move right, when z_j is 0.5 and $1-x_m(l)$, respectively, rather than $x_m(l)$. As $f(.|l,c) = f(.|l)$, by Lemma 2, C is positive. By Assumption 2, the posterior distribution of α conditional on l and r is symmetric, meaning that $F(x_m(l)) = 1 - F(1-x_m(l))$. Hence, by Lemma 3, D is negative.

By definition,

$$\mu(c|l) = \frac{d\int_0^1 \gamma(l|x)f(x)dx}{d\int_0^1 \gamma(l|x)f(x)dx + (1-d)\int_0^1 \gamma^2(l|x)f(x)dx + (1-d)\int_0^1 \gamma(l|x)\gamma(r|x)f(x)dx}.$$

It is straightforward to show that $\lim_{d\to 0}\mu(c|l) = 0$ and $\lim_{d\to 1}\mu(c|l) = 1$. Thus, when d is close to one,

$$L(x_m(l)) \sim \frac{\partial V_i(x_m(l), 0.5|l)}{\partial z_i} - \frac{\partial V_i(x_m(l), x_m(l)|l)}{\partial z_i} > 0,$$

and when d is close to zero,

$$L(x_m(l)) \sim \mu(r|l) \left(\frac{\partial V_i(x_m(l), 1 - x_m(l)|l, r)}{\partial z_i} - \frac{\partial V_i(x_m(l), x_m(l)|l, r)}{\partial z_i} \right) < 0.$$

It follows from Proposition 2 that candidates choose moderate positions in the first case and extreme ones in the second.

When d is close to 1, the prior probability for a candidate to receive l or r is extremely low, and a candidate, even when his own signal is l or r, expects his opponent to receive c. The situation is similar to Example 1. In equilibrium, the candidate receiving l chooses a position between $x_m(l)$ and 0.5, the equilibrium position of an opponent with signal c, so as to capture the votes between $x_m(l)$ and 0.5.

When d is small, a candidate expects his opponent to receive either l or r. In either case, his posterior belief over α changes substantially when he finds out which signal his opponent receives. Intuitively, this is a situation where there is a lot of uncertainty and the candidates are easily swayed by their opponents' beliefs. If a candidate's own signal is l, he will think it is more likely that his opponent receives l than r and, therefore, he will still believe that his opponent's average belief is more moderate than his own. But, unlike his counterpart in Example 1, he will not choose a moderate position. The candidate knows that the opponent receiving r will choose a position far to the right, significantly different from his own. As their positions are already far apart, the candidate is not going to lose much against an opponent with signal r by moving marginally to the left. But doing so will help him gain significantly against the opponent with signal l, whose position is close to his. As a result, in equilibrium a candidate will try to outflank the opponent with the same signal by choosing an extreme position.[10]

[10] Candidate 1 is like a goalkeeper defending a penalty kick in soccer. As the shooter aims either at the left or the right corner, the goalkeeper should dive all the way to one side even when he thinks the chance is almost 5–50. Diving ninety-five instead of a hundred percent to the left would not increase his chance of blocking a shot to the right.

4.3 Discussion

Examples 1 and 2 illustrate two effects that determine equilibrium outcomes. On the one hand, candidates expect the mean of their opponents' beliefs over the median-voter position to be more moderate than their own. As candidates prefer to choose a position close to their opponents', this "average effect" pushes candidates toward moderate positions. On the other hand, candidates care more about opponents who are close to them. Since such opponents must have received a similar signal, candidates put more weight on the event that the distribution of median-voter position is more extreme than what their own signal indicates. This "distribution effect" pushes candidates toward extreme positions. Depending on the relative strength of these two effects, the candidates may choose moderate or extreme positions. While both results are robust, it should be noted that the "distribution effect" is operative only when candidates (conditional on their own signal) find their opponent's signal informative. When they do not, the "average effect" dominates and the candidates choose moderate positions in equilibrium.

In the model, candidates can infer their opponents' private information through their action, and, hence, they may want to revise their positions as soon as their original positions are announced. However, if candidates could change their positions, they would want to conceal their information by not committing to a particular position in the beginning. Furthermore, changing position is costly as it harms a candidate's credibility. Thus, allowing candidates to revise their platforms may not lead to full information revelation.

Thus far, I have assumed that differences in beliefs arise out of asymmetric information. In reality, they may also reflect ideological and not informational differences. When candidates have heterogeneous beliefs, they do not update their beliefs on the basis of each other's signal. For example, a left-wing candidate who believes voters are willing to pay higher taxes in return for better social service are unlikely to change his mind upon learning that his right-wing counterpart believes the opposite. It is far more likely that he will simply conclude that the right-wing candidate is wrong. Such a cases is essentially the same as Example 1, where candidates choose moderate positions relative to their beliefs.

When candidates receive multiple signals, the results suggest that candidates may react differently to equally informative signals. In particular, information are widely known tend to have a larger impact on a candidate's platform. Polling data, for example, may have a strong influence on a candidate's positions, as they are likely to be known by both candidates. In comparison, a candidate may have strong personal beliefs over what the voters want (through his own research on the issues and experience with voters), but since these beliefs are less likely to be shared by his opponent, they tend to have smaller impact.

Aragones and Palfrey (2000) show that in a standard one-dimensional Downsian model of two-candidate elections, the candidate with an advantage (say, better image) adopts a more moderate position than the disadvantaged candidate, for the favored candidate wins when the policy positions of the two candidates are close. The logic behind Proposition 3 suggests that in a similar model with private information, the favored candidate may use his private information less than the disadvantaged candidate. It also suggests that in contest with two leading candidates and one or more also-rans, the leading candidates are likely to be less responsive to private signals than minor candidates who have little to lose.[11]

5 Signal Quality and Electorate Welfare

In a standard complete-information Downsian model (with linear or concave utility function), the average utility of the voters is maximized when the median-voter position is implemented. In equilibrium, both candidates choose the position, and the outcome is efficient. In such a model, there is no efficiency loss for having identical platforms.

However, when candidates are uncertain about the preferences of the electorate, selecting the median-voter position is no longer feasible (as they do not know its location). In that case, the outcome is usually inefficient when candidates choose identical positions. Consider the case where

[11]Lewis 1996 covering the 1996 Republican primaries made a similar observation.

voters have no non-policy preferences. If both candidates choose the same positions, then that position would be implemented, regardless of the preferences of the voters. But if one candidate changes his position, then the voters can now choose between two positions. When they prefer the original position, they can still elect the candidate holding that position and, hence, will not be any worse off. When they prefer the new position, they will be strictly better off, as they can now implement that position by electing the other candidate.

Individual candidates however do not internalize the benefits of divergent platforms. IAs a result, as we shall see, electorate welfare is not monotone in the quality of the signals. Before I proceed, I need to introduce a measure of electorate welfare. In my model, while the identity of the winner depends only on α, the average utility of the voters depends on both g, the distribution of voters conditional on α, as well as f, the distribution of α. In the following, I measure welfare by the average utility of the median voter, which depends only on f. This is equivalent to assuming that all voters have the same ideal point α. The assumption, while restrictive, is sufficient for my present purpose, which is to show that having more accurate signals *can* lower welfare. The welfare of the electorate is measured by

$$\begin{aligned} W(z_1, z_2) &= \int_0^1 E\left(\max(u(1, z_1), u(2, z_2))\right) f(x)dx \\ &= \int_0^1 \ln\left(\sum_{i=1}^{2} e^{-\tau|x-z_i|}\right) f(x)dx. \end{aligned}$$

W decreases linearly with the distance between the winning position and the ideal point of the median-voter.[12] When τ is large, the vote is mainly determined by the candidates' positions, and the electorate welfare is approximately equal to $\int_0^1 \max_{i\in\{1,2\}}\left(-\tau\left|x - z_i\right|\right) f(x)dx$. When τ is small, the vote is mainly determined by non-issue characteristics, and the electorate welfare is approximately equal to $\ln 2$, the expectation of $\max(\varepsilon_1, \varepsilon_2)$.[13]

Lemma 4 *For any z, $z' \neq z$, there exists τ^* such that for all $\tau \geq \tau^*$, $W(z, z') > W(z, z)$.*

[12] In general, the average utility of the voters also decreases as the distance the winning position and the median-voter position increases, but the relation is not linear.

[13] See Anderson, de Palma, and Thisse (1992) pp.60-61.

Lemma 4 formalizes the argument I made in the beginning of the section. It says that if the voters are care mainly about policy, then for any pair of distinct platforms z, z', the voters are better off when one candidate choose z and the other z' than when both choose z or when both choose z'.

5.1 Example Three (Partially Correlated Signals)

I illustrate the connection between signal quality and electorate welfare through a model of partially correlated signals. For concreteness, imagine that each candidate, like those in Example 2, conducts a private poll. But the polls, instead of being fully independent, are both derived from some common underlying poll denoted by $s_0 \in S = \{l, c, r\}$. Let

$$P(\beta) = \begin{bmatrix} p(l|l,\beta) & p(c|l,\beta) & p(r|l,\beta) \\ p(l|c,\beta) & p(c|c,\beta) & p(r|c,\beta) \\ p(l|r,\beta) & p(c|r,\beta) & p(r|r,\beta) \end{bmatrix}$$

denote a family of garbling matrixes indexed by a positive real parameter β. $P(0)$ is the identity matrix. For all β and for all $s_0 \in S$, $\sum_{s_i \in S_i} p(s_i|s_0,\beta) = 1$. Given s_0 and β, candidate i receives s_i with probability $p(s_i|s_0,\beta)$. For all s_i, $s_0 \in S$, $p(s_i|s_0,\beta)$ is continuously differentiable in the parameter β. When β is zero, the candidates receive identical signals. When β is positive, the candidates' signals will be correlated conditional on α. This would be the case if, for example, all opinion polls using the same technique tend to elicit similar responses. Finally, I assume that $\psi(.|.)$, the probability distribution function of the common poll s_0, satisfies the monotone-likelihood-ratio condition. The information structure can be represented by:

$$A(x,\beta) = P'(\beta) \times \begin{bmatrix} \psi(l|x) & 0 & 0 \\ 0 & \psi(c|x) & 0 \\ 0 & 0 & \psi(r|x) \end{bmatrix} \times P(\beta).$$

$A(x,\beta)$ obviously satisfies Assumptions 1 and 2. For the remainder of this section, I assume that β is sufficiently small so that Assumptions 3 and 4 are satisfied as well.

Let $\sigma^*(\beta)$ denote the symmetric equilibrium. The equilibrium electorate welfare is given by

$$W^*(\beta) = \sum_{s_0 \in S} \sum_{s_1 \in S_1} \sum_{s_2 \in S_2} prob(s_0) p(s_1|s_0,\beta) p(s_2|s_0,\beta) W(\sigma^*(s_1,\beta), \sigma^*(s_2,\beta)|s_0)$$

where $W(\sigma^*(s_1,\beta), \sigma^*(s_2,\beta)|s_0)$ denote the social welfare given s_0, s_1, and s_2.

Following Blackwell (1950), one signal is more informative than another if the former is a sufficient statistic of the latter. In the example, the common signal s_0 is more informative than an individual signal s_i when β is greater than zero. A more informative signal enables a candidate to access the preference of the electorate more accurately and, hence, reduce the average distance between his position and the ideal point of the median voter.

The objective of the analysis is to compare $W^*(\beta)$ with $W^*(0)$. First, I consider the case where candidates have common information. To that end, let us assume for the time being that the candidates receive the same noisy signal $\widetilde{s}$ derived from s_0, rather than independent ones. It is important to distinguish between the expected median-voter position conditional on the underlying signal s_0 and that conditional on the candidates' noisy signal $\widetilde{s}$. I shall write $x_m(s_0)$ for the former and $x_m(\widetilde{s}|\beta)$ for the latter. In equilibrium both candidates receive the same signal $\widetilde{s}$ and choose the same position $x_m(\widetilde{s}|\beta)$. The equilibrium welfare is thus given by

$$W^*(\beta) = \sum_{s_0 \in S} \sum_{s \in S} prob(s_0) p(\widetilde{s}|s_0,\beta) W(x_m(\widetilde{s}|\beta), x_m(\widetilde{s}|\beta)|s_0).$$

Proposition 5 *In Example 3, if candidates have common information, then $W^*(0) > W^*(\beta)$ for all $\beta > 0$.*

Proposition 5 says that under common information the electorate welfare is increasing in the informativeness of the signal. The key of the proposition is to show that

$$\forall z \in [0,1]/x_m \text{ and} \forall s_0,\ W(x_m(s_0), x_m(s_0)|s_0) > W(z,z|s_0), \tag{6}$$

which means that given that the candidates's positions are identical, the electorate welfare is maximized when $z_1 = z_2 = x_m(s_0)$. When $\beta = 0$, the candidates' signal $\widetilde{s}$ is always the same as the common signal s_0 and their expected median-voter position $x_m(\widetilde{s}|\beta)$ is equal to $x_m(s_0)$. When $\beta > 0$, there is a positive probability that $\widetilde{s} \neq s_0$, and $x_m(l|\beta)$ is more moderate than $x_m(l)$, as the candidates' signal is less informative than the underlying signal. Hence, the equilibrium position is different from $x_m(s_0)$. It then follows from (4) that $W^*(0) > W^*(\beta)$.

I now return to the case where candidates have independent signals. When $\beta = 0$, the equilibrium strategy is still $x_m(s_i|\beta)$. But the situation is different when $\beta > 0$. Not only is it true that the candidates's signals may not be the same as s_0, they may also be different from each other. For example, when $s_0 = l$, it is possible that one candidate receives l while the other receives r. The garbling therefore creates two welfare effects. On the one hand, it makes the candidates' signals less accurate, reducing their individual ability to select a popular position. On the other hand, it leads to more variety in positions. Whether electorate welfare is higher when $\beta > 0$ or when $\beta = 0$ depends on the size of the two effects. The following proposition shows that when divergent platforms are desirable, adding a small amount of the noise increases electorate welfare.

Proposition 6 *Suppose for all $s \in S$ and for all $s' \neq s$, $W(x_m(s), x_m(s') > W(x_m(s), x_m(s'))$. Then $\frac{dW^*}{d\beta}|_{\beta=0} > 0$.*

To illustrate the intuition behind Proposition 6, I compare the welfare of the electorate between $\beta = 0$ and $\beta > 0$ when the underlying signal s_0 is l. The same logic applies to the other two signals. To simplify notation, I write $W(s_1, s_2|\beta, s_0)$ for $W(\sigma^*(s_1|\beta), \sigma^*(s_2|\beta)|s_0)$. When $\beta = 0$, both candidates observe l and the welfare is $W(l, l|0, l)$. When $\beta > 0$, there are three possible outcomes, namely both candidates receive l, only one receives l, and neither receives l. The electorate welfare $W(l, l|\beta, l)$ can be written as

$$p(l, l|l, \beta)W(l, l|\beta, l) + \sum_{s_2 \in \{c,r\}} 2p(l, s_2|l, \beta)W(l, s_1|\beta, l) + \sum_{(s_1,s_2) \in \{c,r\}^2} p(s_1, s_2|l, \beta)W(s_1, s_2|\beta, l). \quad (7)$$

When β approaches 0, for each candidate the probability of receiving l approaches 1. As the candidates' signal are conditionally independent, the probability that neither candidate receives l

is of second order, insignificant compared to the probability that at least one candidate receives l. As a result, we can ignore the last term and concentrate on the first two in (5).

First, consider the event where both candidates receive l. We know from (4) that $x_m(l)$ maximizes welfare given the candidates choose the same position. As their signals become less informative when $\beta > 0$, the candidates' equilibrium position $\sigma^*(l|\beta)$ would be more moderate than $x_m(l)$, and electorate welfare, as a result, would be lower than it would have been if $\beta = 0$. However, as β approaches 0, $\sigma^*(l|\beta)$ converges to $x_m(l)$, and since $\frac{dW^*(z,z)}{dz}|_{z=x_m} = 0$, choosing $\sigma^*(l, \beta)$ instead of $x_m(l)$ causes only a second-order loss in welfare.

Finally, consider the event where one candidate receives l and the other c or r. As β converges to 0, $\sigma^*(s|\beta)$ converges to $x_m(s)$. Compared to the case of $\beta = 0$, where both candidates choose $x_m(l)$, voters have a first-order gain as they can now choose between $x_m(l)$ and $x_m(r)$ (or between $x_m(l)$ and $x_m(c)$).

In summary, a small increase of β will result in a first-order gain and a second-order loss; therefore, when β is sufficiently small, $W^*(\beta) > W^*(0)$.

5.2 Welfare Effects of Opinion Polls

Despite its popularity among pundits, the claim that the democratic process is harmed by candidates' increasing reliance on polls is puzzling form the perspective of rational choice.[14] Candidates gain votes by adopting popular platforms. By providing a more accurate assessment of the public opinion, opinion polls ensure that the winning platform is closely aligned with the preferences of the electorate. As a result, they should increase welfare.

Conventional arguments against opinion polls are often problematic. One typical argument is simply that politicians should adopt the "right" policy rather than the one the public prefers. Another argument is that opinion polls are biased and do not reflect the preference of the electorate. However, if that were the case, then it would be irrational for candidates to devote so much resources to them.

[14] For example, see Geer (1996), Lewis (1998), and Sabato (1981).

Proposition 6 provides an alternative explanation as to why better opinion polls may lead to inferior outcomes. When candidates are uncertain about the preferences of the electorate, it is socially efficient for them to adopt divergent platforms. In this world, having more accurate opinion polls have two effects. Each candidate can choose a position that is on average more popular. But, the candidates are also more likely to choose similar positions. While the first effect raises social welfare, the second does the opposite. As a result, welfare may go down when the second effect dominates the first. Note that this can happen only in a model with private information. In a standard complete-information model, divergent platforms do not increase welfare.

According this view, polls are harmful, not because they are biased, but because they lead to premature platform convergence. Polls capture current public opinion but do not predict how it would change when voters acquire new information. Even when the public currently favor a particular policy, it may still be socially beneficial for candidates to explore different alternatives. However, the more accurate the polls, the higher the potential price a candidate has to pay for taking a currently unpopular position. Instead of doing so, they may settle on a consensus prematurely. For example, in the 2000 US Presidential election, the Democrats have largely endorsed the Republican's position on the missile defense program, which is popular among voters, despite open questions regarding the feasibility of the program. Even if we believe missile defense should be built so long as a majority supports it, the fact remains that the society will benefit from the public debate that would have occurred had the Democrats chosen a different position.

6 Concluding Remarks

Models of electoral competition usually begin with the assumption that candidates have complete information over the preferences of the electorate. In reality candidates always have to settle for incomplete information collected through public and private channels. In this paper, I analyze the problem facing such candidates in a one-dimension Downsian model. My results identify two new properties of electoral competition that do not exist in models with complete information. First, candidates generally do not select the expected median-voter position in equilibrium, and, second, electorate welfare is not monotone with signal quality. My result explains why better opinion polls may actually lower welfare.

I have assumed that signals are exogenously given. A natural extension is to allow candidates to collect information from multiple channels and decide the how much information to collect. Throughout, I have focused on the properties of the symmetric pure-strategy equilibrium. But as I mention in section 3 such an equilibrium do not always exist, and it is important to analyze the properties of mixed-strategy equilibria. More work is also needed to determine the prevalence of each type of outcomes. While both moderate and extreme outcomes are robust, causal observations seem to suggest that the first type is far more common. Note that the two types of outcomes are not mutually exclusive. In models with more than three signals, candidates may choose moderate positions for some signals and extreme positions for others.

7 Appendix

Proof of Lemma 2:

By definition:

$$\frac{\partial V_i}{\partial z_i} = (1 - F(z_i) - F(z_j))\frac{d\pi(z_i - z_j)}{dq} + \int_{z_i}^{z_j} \frac{d\pi(2x - z_i - z_j)}{dq} f(x)dx.$$

When $z_j = x_m$, $\frac{\partial V_i(x_m, z_j)}{\partial z_i} = 0$. When $z_j > x_m$, $\frac{\partial V_i(x_m, z_j)}{\partial z_i} > 0$ and $\frac{\partial V_i(z_j, z_j)}{\partial z_i} < 0$. When $z_j < x_m$, $\frac{\partial V_i(x_m, z_j)}{\partial z_i} < 0$ and $\frac{\partial V_i(z_j, z_j)}{\partial z_i} > 0$. In either case, there exists $z^* \in (\min(z_j, x_m), \max(z_j, x_m))$ such that $\frac{\partial V_i(z^*, z_j)}{\partial z_i} = 0$. By definition,

$$\frac{\partial^2 V_i}{\partial z_i^2} = -2f(z_i)\frac{d\pi(z_i - z_j)}{dq} + (1 - F(z_i) - F(z_j))\frac{d^2\pi(z_i - z_j)}{dq^2} - \int_{z_i}^{z_j} \frac{d^2\pi(2x - z_i - z_j)}{dq^2} f(x)dx.$$

Using the fact that $\frac{d^2\pi}{dq^2} = \frac{d\pi}{dq}\left(\frac{1-e^{\tau q}}{1+e^{\tau q}}\right)$, we can write $\frac{\partial^2 V_i}{\partial z_i^2}$ as

$$\frac{\partial^2 V_i}{\partial z_i^2} = -2f(z_i)\frac{d\pi(z_i - z_j)}{dq} + \tau\frac{\partial V_i}{\partial z_i}\left(\frac{1 - e^{\tau(z_i - z_j)}}{1 + e^{\tau(z_i - z_j)}}\right) \tag{8}$$

$$- \tau \int_{z_i}^{z_j} \frac{\partial\pi(2x - z_i - z_j)}{\partial q}\left(\frac{1 - e^{\tau(z_i - z_j)}}{1 + e^{\tau(z_i - z_j)}} + \frac{1 - e^{\tau(2x - z_i - z_j)}}{1 + e^{\tau(2x - z_i - z_j)}}\right) f(x)dx.$$

The first term in (6) is negative and the second term is zero when $\frac{\partial V_i}{\partial z_i} = 0$. As $\frac{d\left(\frac{1-e^{\tau q}}{1+e^{\tau q}}\right)}{dq} = \frac{-\tau e^{2\tau x}}{(1+e^{\tau x})^2} < 0$, and $\frac{1-e^{\tau(z_i-z_j)}}{1+e^{\tau(z_i-z_j)}} + \frac{1-e^{\tau(z_j-z_i)}}{1+e^{\tau(z_j-z_i)}} = 0$,

$$\frac{1-e^{\tau(z_i-z_j)}}{1+e^{\tau(z_i-z_j)}} + \frac{1-e^{\tau(2x-z_i-z_j)}}{1+e^{\tau(2x-z_i-z_j)}} > 0 \text{ for all } x \in [\min(z_i, z_j), \max(z_i, z_j)],$$

and hence the third term in (6) is negative. We can therefore conclude that $\frac{\partial^2 V_i}{\partial z_i^2}$ is negative when $\frac{\partial V_i}{\partial z_i} = 0$. It implies that z^* is unique. Furthermore, $\frac{\partial V_i}{\partial z_i} > 0$ when $z_i < z^*$, and $\frac{\partial V_i}{\partial z_i} < 0$ when $z_i > z^*$.

Proof of Lemma 3:

Recall that z' is the position such that $1 - F(z') = F(z)$. Suppose $z < x_m$. For all $y \in [z, z']$,

$$\begin{aligned}
\frac{\partial V_i}{\partial z_i}(z,z) &= (1-2F(z))\frac{\tau}{4} \\
&= (1-F(z)-F(y))\frac{\tau}{4} + (F(y)-F(z))\frac{\tau}{4} \\
&> (1-F(z)-F(y))\frac{d\pi(z-y)}{dq} + \int_z^y \frac{d\pi(2x-y-z)}{dq} f(x)dx \\
&= \frac{\partial V_i}{\partial z_i}(z,y).
\end{aligned}$$

For all $y < z$,

$$\begin{aligned}
\frac{\partial V_i}{\partial z_i}(z,z) &> (1-2F(z))\frac{d\pi(y-z)}{dq} - \int_y^z \left(\frac{d\pi(2x-y-z)}{dq} - \frac{d\pi(y-z)}{dq}\right) f(x)dx \\
&= \frac{\partial V_i}{\partial z_i}(z,y).
\end{aligned}$$

The proof when $z > x_m$ is analogous.

Proof of Proposition 1:

For any conditional probability function $h_n(.|x)$, let $x_m^n(s_1, s_2)$ denote the expected median-voter position conditional on s_1 and s_2. If $h_n(.|x)$ converges uniformly to some $h^*(.|x)$ constant in x, then $\lim_{n\to\infty} x_m^n(l,l) = \lim_{n\to\infty} x_m^n(r,r) = 0.5$. Thus, for any k, there is some n^* such that for all $n \geq n^*$, $x_m^n(r,r) - x_m^n(l,l) < 2k$. It follows from Lemma 5 that for sufficiently large n, $\frac{\partial^2 V_i(z_i,z_j)}{\partial z_i^2} < 0$ for all z_i, $z_j \in [x_m(l,l), x_m(r,r)]$. Let Γ_n denote such a game. Consider a restricted version of Γ_n in which the candidates' choice sets are restricted to $[x_m(l,l), x_m(r,r)]$. As V_i is concave in this restricted game, pure-strategy equilibrium exists. Let σ^* denote such an equilibrium. From Lemma

2, any $z \notin [x_m(l,l), x_m(r,r)]$ is never a best response against σ^*; hence, σ^* is also an equilibrium of the original game, Γ_n.

Lemma 5 *For any τ and for any $y \in [0,1]$, there exists k such that $\frac{\partial^2 V_i(z_i,z_j)}{\partial z_i^2} < 0$ for all z_i, $z_j \in [y-k, y+k]$.*

Proof of lemma 5:

Pick some $k > 0$. Let $\underline{f} = \min\{f(x) : x \in [y-k, y+k]\}$, and let $\xi(k) = \max\left\{\left|\frac{d^2\pi(r)}{dq^2}\right| : r \in [0, 2k]\right\}$. It is straightforward to verify that for all $\overline{z}_i$, $\overline{z}_j \in [y-k, y+k]$,

$$\frac{\partial^2 V_i(\overline{z}_i, \overline{z}_j)}{\partial z_i^2} < -2\underline{f}\frac{d\pi(2k)}{dq} + (|1 - F(\overline{z}_i) + F(\overline{z}_j)| + |F(\overline{z}_j) - F(\overline{z}_i)|)\,\xi(l).$$

Since $\lim_{k\to 0} \frac{d\pi_i(2k)}{dq} = \frac{\tau}{4}$ and $\lim_{k\to 0} \xi(k) = 0$, $\frac{\partial^2 V_i(\overline{z}_i,\overline{z}_j)}{\partial z_i^2}$ is strictly negative when k is sufficiently small.

Proof of Proposition 2:

By Lemma 2, $L(0.5) = \frac{\partial V_i(0.5, 0.5|l)}{\partial z_i} < 0$. Next, consider $L(x_m(l,l))$. The position $x_m(l,l)$ is the best response against $x_m(l,l)$ if the other candidate's signal is also l. But against the other two signals, the best response is larger than $x_m(l,l)$. As $x_m(l,l)$ is less than both $\min(0.5, x_m(l,c))$ and $\min(x_m(l,r), 1-x_m(l,l))$, it follows from Lemma 2 that both $\frac{\partial V_i(x_m(l,l),0.5|l,c)}{\partial z_i}$ and $\frac{\partial V_i(x_m(l,l),1-x_m(l,l)|l,r)}{\partial z_i}$ are strictly positive. It follows that

$$\begin{aligned} L(x_m(l,l)) &= \mu(c|l)\frac{\partial V_i(x_m(l,l), 0.5|l,c)}{\partial z_i} + \mu(r|l)\frac{\partial V_i(x_m(l,l), 1 - x_m(l,l)|l,r)}{\partial z_i} \\ &> 0. \end{aligned}$$

By continuity, there exists z^* such that $L(z^*) = 0$.

To show that z^* is unique, note that L is monotonic: for all $z \in [0,1]$

$$\begin{aligned} \frac{dL(z)}{dz} &= \mu(l|l)\left(\frac{\partial V_i^2(z, z|l,l)}{\partial z_i^2} + \frac{\partial V_i^2(z, z|l,l)}{\partial z_i \partial z_j}\right) + \mu(c|l)\frac{\partial V_i^2(z, 0.5|l,c)}{\partial z_i^2} \\ &+ \mu(r|l)\left(\frac{\partial V_i^2(z, 1-z|l,r)}{\partial z_i^2} - \frac{\partial V_i^2(z, 1-z|l,r)}{\partial z_i \partial z_j}\right). \end{aligned}$$

By definition,

$$\frac{\partial^2 V_i}{\partial z_i z_j} = -(1 - F(z_i) - F(z_j))\frac{d^2\pi(z_i - z_j)}{dq^2} - \int_{z_i}^{z_j} \frac{d^2\pi(2x - z_i - z_j)}{dq^2} f(x)dx.$$

It is straightforward to verify that both $\frac{\partial V_i^2(z,z|l,l)}{\partial z_i \partial z_j}$ and $\frac{\partial V_i^2(z,1-z|l,r)}{\partial z_i \partial z_j}$ equal 0. The latter is true because $f(x|l,r)$ is symmetric over 0.5. We can therefore write

$$\begin{aligned}\frac{dL(z)}{dz} &= \mu(l|l)\frac{\partial V_i^2(z,z|l,l)}{\partial z_i^2} + \mu(c|l)\frac{\partial V_i^2(z,0.5|l,c)}{\partial z_i^2} + \mu(r|l)\frac{\partial V_i^2(z,1-z|l,r)}{\partial z_i^2}\\ &< 0.\end{aligned}$$

As a result L can intersect 0 only once. Since V_i is concave, so is U_i is concave in the relevant range, and hence $L(z^*) = 0$ is necessary and sufficient for $(z^*, 0.5, 1 - z^*)$ to be a Nash equilibrium.

Proof of Lemma4:

Note that $W(z, z') > \int_0^1 \max_{i\in\{1,2\}} (-\tau |x - z_i|) f(x)dx - \ln 2$. It follows that

$$W(z, z') - W(z, z) = \int_{\frac{z+z'}{2}}^1 -\tau \left(|z' - x| - |z - x|\right) f(x)dx - 2\ln 2.$$

The expression is strictly positive when τ is sufficiently large.

Proof of Proposition 5

First, I show that for all $s_0 \in S$, $x_m(s_0) = \arg\max_{z\in[0,1]} W(z, z|s_0)$. By definition,

$$W(z, z|s_0) = \ln 2 + \int_0^z -\tau(z - x)f(x|s_0)dx + \int_z^1 -\tau(x - z)f(x|s_0)dx.$$

It is straightforward to show that $\frac{dW(z,z|s_0)}{dz} = -\int_0^z \tau f(x|s_0)dx + \int_z^1 \tau f(x|s_0)dx$ and $\frac{d^2W(z,z|s_0)}{dz^2} = -2\tau f(z) < 0$. Thus, $\frac{dW(z,z|s_0)}{dz} = 0$ if and only if $z = x_m(s_0)$. To see that the Proposition is true, note that

$$\begin{aligned}W^*(\beta) &= \sum_{s_0\in S} prob(s_0) \sum_{s\in S} p(s|s_0, \beta) \int_0^1 -\tau |x - x_m(s)| f(x|s_0)dx,\\ &< \sum_{s_0\in S} prob(s_0) \int_0^1 -\tau |x - x_m(s_0)| f(x|s_0)dx,\\ &= W^*(0).\end{aligned}$$

Proof of Proposition 6:

By definition:

$$\frac{dW}{d\beta} = \sum_{s_0 \in S} prob(s_0) \sum_{s_1 \in S_1} \sum_{s_2 \in S_2} \{ \frac{d(p(s_1|s_0,\beta)p(s_2|s_0,\beta))}{d\beta} W(z_1^*(s_1,0), z_2^*(s_2,0)|s_0)$$
$$+ p(s_1|s_0,0)p(s_2|s_0,0) \frac{dW(z_1^*(s_1,\beta), z_2^*(s_2,\beta)|s_0)}{dz} \left(\frac{dz_1^*(s_1,\beta)}{d\beta} + \frac{dz_2^*(s_1,\beta)}{d\beta} \right) \}.$$

For all $s \in S$, $\lim_{\beta \to 0} z_1^*(s,\beta) = x_m(s)$. Proposition 5 thus implies that $\lim_{\beta \to 0} \frac{dW(z_1^*(s_1,\beta), z_2^*(s_2,\beta)|s_0)}{dz} = 0$. Second, for all $s_0 \in S$ and for all $s \in S/s_0$, $\lim_{\beta \to 0} p(s|s_0,\beta) = 0$. That is, the probability for a candidate to receive a signal other than the underlying signal s_0 goes to zero as β goes to zero. By definition

$$\frac{d(p(s_1|s_0,\beta)p(s_2|s_0,\beta))}{d\beta} = p(s_1|s_0,\beta)\frac{dp(s_2|s_0,\beta)}{d\beta} + p(s_2|s_0,\beta)\frac{dp(s_1|s_0,\beta)}{d\beta}.$$

Hence, for all $s_1, s_2 \neq s_0$, $\lim_{\beta \to 0} \frac{d(p(s_1|s_0,\beta)p(s_2|s_0,\beta))}{d\beta}$. We can therefore write $\frac{dW}{d\beta}|_{\beta=0}$ as

$$\frac{dW}{d\beta}|_{\beta=0} = \sum_{s_0 \in S} prob(s_0) 2 \{ \frac{dp(s_0|s_0,\beta)}{d\beta}|_{\beta=0} W(x_m(s_0), x_m(s_0)|s_0)$$
$$+ \sum_{s_1 \in S/s_0} \frac{dp(s_1|s_0,\beta)}{d\beta}|_{\beta=0} W(x_m(s_0), x_m(s_1)|s_0) \}$$
$$= \sum_{s_0 \in S} prob(s_0) \sum_{s_1 \in S/s_0} \frac{dp(s_1|s_0,\beta)}{d\beta} \{ W(x_m(s_0), x_m(s_0)|s_0) - W(x_m(s_0), x_m(s_1)|s_0) \}$$
$$> 0.$$

The second equality holds as $\sum_{s_1 \in S} \frac{dp(s_1|s_0,\beta)}{d\beta} = 0$, and the last holds as $W(x_m(s_0), x_m(s_0)|s_0)$ is strictly less than $W(x_m(s_0), x_m(s_1)|s_0)$.

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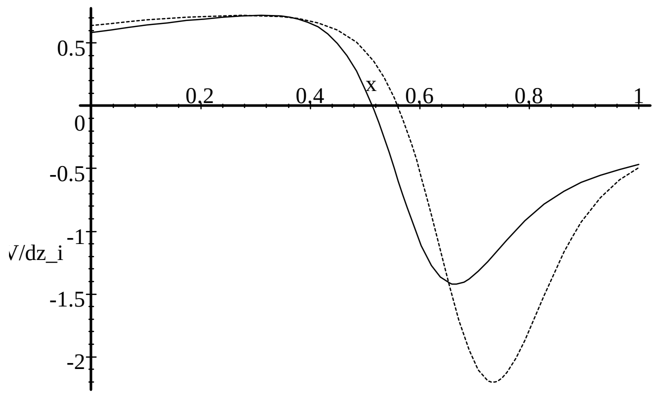

Figure 1: Solid line: $z_j = 0.6$, Dotted line: $z_j = 0.7$.

Figure 2: