The Dynamics of Government.*

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Abstract

How does the size of the transfer system evolve in the short and in the long run? We model income redistribution as determined by voting among individuals of different types and income realizations. Taxation is distortionary because it discourages effort to accumulate human capital. Voters are fully rational, realizing that transfers have implications also for future economic decisions and taxation outcomes. In our economy, our politically driven redistribution provides insurance, and we investigate to what extent the democratic process provides it appropriately.

A general finding is that redistribution tends to be too persistent relative to what would have been chosen by a utilitarian planner under commitment. The difference is larger, the lower is the political influence of young agents, the lower is the altruistic concern for future generations, and the lower is risk-aversion. Furthermore, there tends to be too much redistribution in the political equilibrium. Finally, we find that the political mechanism is important: settings with smooth preference aggregation—we analyze probabilistic voting here—produce less persistence and do not admit multiple rational expectation equilibria, which occur under majority-voting aggregation.

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1 Introduction

In most currently developed countries, the size of government—say, as measured by the transfer system—has increased significantly over the last 100 years or so. Are we on a monotonic trajectory toward a (perhaps even higher) steady-state level of government, or is the size of government likely to experience significant swings ahead? Some empirical studies focusing on more recent periods and higher frequencies find what could be seeds of oscillatory behavior. In particular, in a number of instances, the provision of welfare programs is inversely related to the number of their beneficiaries.[1] Thus, low dependency ratios might induce generous redistributive policies, which in turn would lead to low incentives to become successful and thus contribute to subsequent increasing dependency ratios. This would then lead to a lower demand for redistribution, and so on: a "cyclical" pattern may emerge.

Based on our time series, it is difficult to immediately separate the inherent dynamics of government—which may or may not be oscillatory—from the "shocks" that we have been subject to, such as wars, structural change due to technology, demographics, and so on. One could imagine pure statistical characterizations of the data, but we do think that empirical analysis of the time series could profit significantly from the use of theory. A relevant parallel is the use of the neoclassical growth model as an organizing tool in much of the empirical work on economic growth (see, e.g., Barro and Sala-i-Martin, 1995). In the case of the dynamics of government, there is no "consensus theory" to build on; in fact, there are very few frameworks that even deliver endogenous dynamics of government.The purpose of this paper is to build in the direction of such a theory. We stop far short of a model that one could use in actual data analysis here, but we do incorporate what we think are important ingredients in the kind of model we envision building further toward in future work. In particular, it contains both forces for persistence and for oscillations—along the lines of the above data discussion—together allowing nontrivial and rich dynamics of government. What we mainly see as missing is a neoclassical connection; there is no physical capital accumulation, and the only state variable in this paper is the size of the welfare state itself.

What are the two forces we model? First, a standard argument suggests that welfare state policies should be persistent; once a constituency of net recipients of transfers is built up, this constituency is self-generating and perpetuates the transfer system. The second element formalizes the story above suggesting oscillatory dynamics. In particular, with a large group of net recipient of transfers, the group from which the transfers originate must be correspondingly small, which means that per-unit transfers are more costly—distortionary—to effectuate: the cost of the welfare state is large when the group in need is large, and vice versa. If, further, the size of the group in need depends on the incentives to succeed—which depend on the social insurance provided by the welfare state—then this means that a cyclical pattern could emerge. Our model delivers dynamics that are a resolution of the forces toward monotonic dynamics and toward oscillations. Using this model, we also examine the role efficiency plays in the political process determining the size of transfers, and we investigate the direction politics distort the dynamics of government programs.

In our model, redistribution is a pure wealth transfer motivated by selfish concerns. While agents are ex-ante identical, luck and effort make them, ex post, rich or poor, and to the extent that the poor have political influence, they achieve net redistribution in their favor. Because insurance markets for individual risk are missing, these transfers provide ex-ante valuable insurance. However, the return to effort depends on the net wage premium of success. Thus, redistribution distorts effort,

[1]For example, Di Tella and MacCullough (2004) find that unemployment benefits (replacement rates) fall as the unemployment rate increases in a panel of OECD countries. Similarly, Razin et al. (2002) document that pension benefits fall as the dependency ratios increase.

and is therefore costly. The political system considered presumes that current voters set current policies but cannot bind the hands of future voters. Finally, we focus on cases where reputational mechanisms are absent (Markov equilibria).

Our main finding is that the political mechanism tends to stabilize the dynamics of government. Our political equilibrium always settles down in a steady state, though possibly in an oscillatory manner. In contrast, the constrained optimum may entail oscillations that do not die out. However, such political stabilization is welfare-reducing. Relative to a "constrained optimal" allocation determined by a benevolent planner who can commit to future transfer policies but is subject to redistribution being distortionary, the political system dampens, and for some parameter values even completely eliminates, the cycles that would be present in the optimal allocation. This stands in sharp contrast to the predictions of the recent literature on politically driven cycles (see Alesina, Roubini and Cohen 1997 for a survey), which predict that politics create fluctuations.

Two questions need to be answered to understand these findings. First, how can oscillations in taxation and GDP be efficient, in contrast with the standard tax-smoothing argument (see, e.g., Barro, 1979)? The key assumption is that human capital investments increase earnings all periods of the remaining work-life, but fully depreciate as agents pass away.[2] Thus, it is the present value of taxes over individuals' life-cycle that determines the distortion on their investments – the time-path of taxes is irrelevant. The planner tries to smooth the distortions, and not necessarily taxes, and it follows that optimal allocations tend to cycle. If it were optimal to tax at a high rate at a point in time, e.g., in order to redistribute to the initial old or to exploit the low tax-elasticity of the initial old, then one can reduce the distortionary impact of this tax hike by lowering taxes the next period. Because taxes are lowered the following period, however, in order to smooth the distortions by keeping the present value of taxes similar, two periods later taxes have to be increased again, and so on: a one-time splash produces ripples.

Second, why does the political system reduce these (constrained-optimal) oscillations? We assume individual preferences to be aggregated by a probabilistic-voting mechanism à la Lindbeck-Weibull (1987), which allows, as a particular case, that the utilities of all agents bear an equal weight in the political decision. We choose this political mechanism since it resembles most closely the choice of a benevolent utilitarian planner, avoiding that inefficient outcomes be driven by the political preponderance of some agents or groups (e.g., the median voter in Downsian models with majority voting). Moreover, our agents are fully rational, and choose taxation by taking into full account the efficiency considerations discussed above. Therefore, the political determination of redistribution has no built-in irrationality. However, the political mechanism (i) lacks commitment and (ii) may under-represent the interests of future generation. As far as (i) is concerned, elected policy makers cannot automatically adjust future – or past – taxes to reduce the distortionary impact of redistribution and social insurance. Thus, oscillations are partly or fully offset with politically determined taxation. As far as (ii) is concerned, when voters are less than perfectly altruistic they do not put sufficient weight on the degree to which current redistribution distorts current investments and, subsequently, the burden on future generations.

While the comparison between the equilibrium with the constrained optimum yields clear-cut

[2] Tax cycles do not hinge on the overlapping generation structure of the model **per se** – the human capital accumulation studied **here** is a particular case of a more general point on which we elaborate in a paper in progress (Hassler et al. 2003). There, we show that tax cycles also arise in dynastic models under realistic assumptions on the depreciation structure of (physical or human) capital. The key assumption is that investments decay at a faster rate in the long than in the short run. This happens naturally in our overlapping generations model, since agents cannot transmit their human capital to their offspring.

Also, in this paper we abstract from the ability of governments to run deficits. **However**, introducing deficit financing (as we do in Hassler et al. 2003) does not eliminate tax-driven cycles.

results, the exact nature of the equilibrium dynamics of taxation (monotonic or oscillatory) is in general ambiguous. This ambiguity is per se interesting as it arises from the interplay of two forces. First, a large such group makes the tax cost per unit of benefits high. Thus, redistribution is more costly the larger the group of poor agents, which speaks for lower taxes and redistribution. We call this the *tax-base effect*; the optimum under commitment discussed above trades off these tax-base effects over time. Second, with probabilistic voting, an increase in the number of poor voters leads to larger political power of the group favoring redistribution. This *constituency effect*counteracts the tax-base effect: more poor agents speaks for higher taxes. Depending on parameter values, the equilibrium tax rate can be either increasing or decreasing in the number of poor voters. The dynamics are oscillatory when all living agents have equal influence on the political process and when the insurance value of redistribution is large (i.e. large risk aversion), relative to the distortionary effect of taxation. If, however, the insurance value is low and old agents are politically over-represented in the determination of transfer policies, then the constituency effect may dominate and the size of government may be characterized by monotonic rather than oscillating dynamics. Our analysis also unravels the effects of a number of characteristics of the political system on the level of long-run redistribution. The long-run size of government is higher with higher risk aversion and lower when the distortionary impact of taxes is large. Moreover, larger political weight of the young agents weakens the pure ex-post redistribution motive and reduces the long-run level of redistribution.

We emphasize lack of commitment in the political mechanism by focusing entirely on Markov-perfect equilibria. Absence of reputation mechanisms is operationalized by focusing on equilibria which are limits of the corresponding finite-horizon equilibria. Of course, if the horizon is literally infinite and there is sufficiently low discounting, one could construct a large variety of equilibria (for this approach, see, e.g., Bernheim and Nataraj, 2002). We think, however, that it is useful to carefully examine the implications of a full lack of commitment. Moreover, in models with state variables there are channels that allow current voters to influence the future, thus not replicating commitment but imperfectly replacing it, as in the strategic-debt literature (see e.g. Persson and Svensson, 1989). Here, the state variable is the initial group of unlucky agents: a large such group tends to lead to high redistribution in the current period (assuming equilibrium redistribution is driven by the constituency effect). As a consequence, next period's redistribution can be influenced today by using current taxes to influence current effort and, hence, affecting the set of unlucky agents in the beginning of next period.

Once attention has been limited to Markov equilibria, we must still face the important question of whether our political Markov equilibria are unique: can we expect "stability" in the size of government in democracies? A similar setup, considered in Hassler et al. (2003), henceforth HRSZ, assumes Downsian majority voting and finds that Markov equilibria are not unique: in one equilibrium the welfare state survives, while in another it collapses. Multiplicity arises, there, from a stark feature of majority voting models; the equilibrium tax rate increases discontinuously as the number of poor exceeds 50%. This opens the possibility of voting strategically over redistribution in order to induce future changes of majority. The probabilistic-voting mechanism, in contrast, features a smooth mapping from group sizes to tax outcomes. In fact, we show here that multiple equilibria cannot occur with probabilistic voting in the finite- and infinite-horizon equilibria of our baseline setup, whereas they do with majority voting.

On a purely methodological level, this paper contributes to the tools for analyzing dynamic politico-economic issues. All results are analytical, due to a convenient linear-quadratic formulation, similar to the approach addressing strategic concerns in the literature on time-consistent policies and differential games (see e.g. Cohen, 1988, and Miller and Salmon, 1985). The theoretical literature on the dynamics of government is, arguably, scant and we believe the main reason is a lack of convenient analytical tools. The study of economic dynamics is perhaps hard, but there is a large body of work on the subject: for a given policy environment, it is textbook material how to analyze an economy's behavior over time when the economic actors are fully rational. Similarly, pure political theory has worked on dynamic policy determination. The combination of politics and economics is what poses a difficulty; one needs to model strategic voting interactions, where political agents consider the consequences of their choice on future political outcomes, as well as appeal to dynamic equilibrium theory to ensure that all economic agents – consumers, firms and government – maximize their respective objective functions under rational expectations, and resource constraints. Prior to HRSZ, the only nontrivial dynamic models (that is, that are not repeated static frameworks or purely "backward-looking" setups) relied essentially on numerical solution (see, e.g., Krusell and Ríos-Rull (1999)). HRSZ provided a tractable linear-quadratic framework where voters are influenced both by the state of the economy – the current income distribution – and foresee effects of the current policy outcomes on both future income distributions and future voting outcomes, which they care about. The present paper uses some tools from HRSZ but extends it in a technically non-trivial and economically important way by introducing risk aversion and a social insurance motive.[3]

The paper is organized as follows. Section 2 presents the economic structure of the model. Section 3 analyzes the constrained optimum: the allocation chosen by a planner who cares about future generations and has commitment. Section 4 describes the political decision making and analyzes politically determined redistribution. Section 4.4 discusses uniqueness of equilibrium under a finite horizon and the connection between our Markov-perfect equilibrium and the limit of finite-horizon equilibria. Because the political equilibrium differs from the constrained optimum both in lacking commitment and in lacking a concern for future generations, Section 4.5 finally studies a case where voters are altruistic toward future generations (but cannot commit to future policy). Section 5 concludes. All proofs are provided in the appendix.

2 The model

2.1 Population, preferences, technology, and policy

The model economy has a continuum of two-period lived agents, who work in both periods. Upon birth, agents are subject to an ability shock. With probability μ, an agent is high-skilled, and with probability $1-\mu$ she is low-skilled. We label high-skilled agents as "entrepreneurs" and low-skilled agents as "workers". Entrepreneurs undertake a risky investment in human capital yielding a stochastic return. With probability e the investment is successful and the entrepreneur earns a labor income $w+\underline{w}$ each period, where $w \leq 1$. With probability $1-e$, the investment is unsuccessful, and the labor income is $\underline{w}$, again each period. The cost of investment is e^2, and we interpret it as the disutility of educational effort. Workers earn an income normalized to zero, which cannot be affected by human capital investments.

In order to make the problem interesting, we assume the component $\underline{w}$ of the entrepreneurial income to be not verifiable. Therefore, insurance agencies, whether private or public, cannot

[3] In HRSZ, redistribution is by construction socially wasteful, as it distorts incentives, while agents are risk neutral, so insurance has no value. Therefore, the constrained-optimal allocation always entails zero redistribution, and the paper does not yield interesting normative implications. Apart from the different assumption about the political mechanism (majority vs. probabilistic voting), the model presented here encompasses HRSZ as the particular case in which agents are risk neutral, as the subsequent discussion will show.

condition payments on agents' skills, but can only discriminate between successful entrepreneurs (who have a verifiable income equal to w) and the rest of the population (who has a verifiable income normalized to zero).

Agents' preferences are given by

$$V_t^y = E_t[u(c_t) + \beta u(c_{t+1}) - e_t^2],$$

where $\beta \in [0,1]$ is the discount factor and

$$u(c) = \begin{cases} ac - (a-1)x & \text{if } c < x \\ c & \text{if } c \geq x \end{cases},$$

with $a \geq 1$. Thus, felicity is concave and piecewise linear in consumption; marginal utility drops discretely at a threshold consumption level x and is constant everywhere else, as shown in Figure 1. The kink in preferences allows us to maintain analytical tractability while allowing ex-ante risk aversion. a parameterizes the concavity of the utility function; if $a = 1$ agents are risk neutral, while if $a > 1$ they are risk averse. For convenience, we introduce the variable $R \equiv (1-\mu)(a-1) \geq 0$, which we use as a measure of aggregate risk aversion.

We assume that $\underline{w} > x$, implying that, after the realization of the ability shock, high-skill agents are effectively risk-neutral. In particular, the marginal utility of income for high-skill agents is equal to unity independently of their income realization. Since agents cannot sign contracts before their skill level is realized, this implies that no private insurance market can exist. The government can, however, increase the ex-ante utility of agents through redistributive programs providing insurance "behind the veil of ignorance". Like private insurers, governments can only condition transfers on observable income. Unlike private insurers, however, they can force agents to be part of the insurance scheme by setting compulsory taxes. In particular, in each period, the government can levy a lump-sum tax τ on all agents and transfer the proceeds to individuals with low observable income (either workers or unsuccessful entrepreneurs).[4] We denote by $b \in [0,1]$ the transfer rate, implying that all agents but the successful entrepreneurs receive an amount bw. The government budget is assumed to balance in every period, and the government cannot issue age-dependent taxes and transfers (see Hassler et al. (2003) for an extension where age-dependent programs are allowed).

[4] The assumption of lump-sum taxes is immaterial. It can be shown that the model is isomorphic to one where transfers are financed by taxation levied on the observable component of labor income. The proof is available upon request.

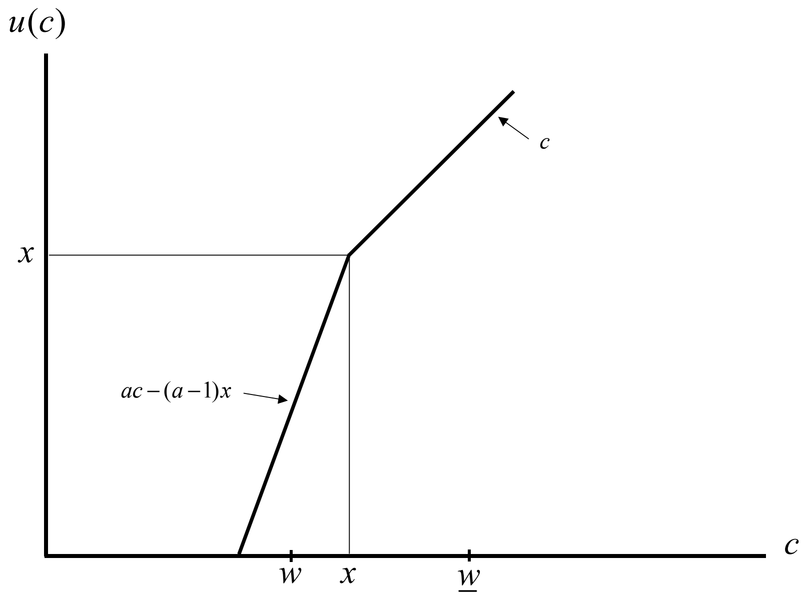


Figure 1. Worker felicity

We assume, additionally, that $w < x$. This assumption simplifies the analysis, since it implies that the marginal utility of low-skill agents is $a > 1$ irrespectively of the redistribution policy (recall that $b \leq 1$).[5] Thus, in summary, we assume that

$$w < x < \underline{w}. \tag{1}$$

Finally, we assume that the subjective discount rate, $(1-\beta)/\beta$, equals the market interest rate. Under this assumption, the savings decisions can be abstracted from since income is the same in both periods of life for all individuals.

2.2 Discussion of assumptions

We assume that only high skilled individuals have hidden income and make an effort choice and that individual ability is revealed already in the beginning of life. These assumptions are stark, but we believe that they provide a reasonable shortcut description of important real-world features: (i) in terms of its effect on productivity, the effort and human capital investments of some workers are more important than that of others; (ii) it is likely that those agents with high entrepreneurial ability are also well-endowed in other dimensions, therefore having higher income than workers also if they are less successful; and (iii) already before entering college, individuals have a good idea of their prospects in life.[6]

Our model abstracts from physical capital. The effects of redistribution on the accumulation of physical capital may of course be important, but the distortion to human capital accumulation we consider here captures the same kind of dynamic trade-offs that are present in a standard consumption-savings decision.

[5] In a previous version of this paper, we assumed that even workers had a stochastic income, but could not affect its distribution through investments. This generalization yields qualitatively identical results.

[6] For example, Kean and Wolphin (1998) argue that up to 90% of the variance of agents' lifetime utility can be explained by information known at age 16 of an individual's life.

Why are insurance markets missing in our model, and why is there a role for government provided redistribution? Clearly, parents are in reality able to shelter their offspring against some types of verifiable ability shocks. In the model, we abstract from this possibility by assuming that parents are not altruistic. However, even though parental altruism should deliver some intergenerational insurance (for example that successful parents make transfers to unskilled children), we believe that such insurance will never be perfect. Thus, democratic constitutions, allowing a possibility to vote over transfers, provide redistribution with an *ex-ante* insurance value that neither altruism nor private insurance markets can deliver.

The policy instruments available to the government are quite limited by design; in most political-economy setups, and this one is no exception, the policy instruments are restricted so as to yield a nontrivial and interesting choice situation for voters/the government. We do not allow budget deficits and surpluses and, more importantly, we introduce some impediment to the ability of governments to target transfers to specific groups. In particular, we assume unlucky entrepreneurs and workers to be pooled in the same program, while it would be beneficial to separate them. While this is an extreme characterization, it captures the realistic feature that welfare state programs are plagued by informational problems reducing their effectiveness and increasing their cost. The absence of government debt is instead due to tractability consideration, and we plan to extend our analysis in this direction in future research.

2.3 The determination of effort as a function of government policy

The utilities of the agents alive at time t can be expressed as a function of government policy variables (benefits and taxes) and human capital investments:

$$\begin{aligned}
\tilde{V}^{oes}(b_t, \tau_t) &= w + \underline{w} - \tau_t \\
\tilde{V}^{oeu}(b_t, \tau_t) &= b_t w + \underline{w} - \tau_t \\
\tilde{V}^{ow}(b_t, \tau_t) &= a(b_t w - \tau_t) - (a-1)x \\
\tilde{V}^{ye}(e_t, b_t, b_{t+1}, \tau_t, \tau_{t+1}) &= e_t(1+\beta)w + (1-e_t)(b_t + \beta b_{t+1})w \\
&\quad - e_t^2 - (\tau_t + \beta\tau_{t+1}) + (1+\beta)\underline{w} \\
\tilde{V}^{yw}(b_t, b_{t+1}, \tau_t, \tau_{t+1}) &= a(b_t w - \tau_t + \beta(b_{t+1}w - \tau_{t+1})) - (a-1)(1+\beta)x
\end{aligned} \tag{2}$$

where the superscripts oes, oeu, ow, ye and yw denote old successful entrepreneurs, old unsuccessful entrepreneurs, old workers, young entrepreneurs and young workers, respectively.

The optimal investment choice of the young entrepreneurs, given b_t and b_{t+1}, is

$$e_t^* = e(b_t, b_{t+1}) \equiv \frac{1+\beta-(b_t+\beta b_{t+1})}{2}w. \tag{3}$$

Since the realization of the investment is *i.i.d.* across entrepreneurs, and all choose the same level of effort, then $e_t(b_t, b_{t+1})$ is also the proportion of entrepreneurs who become successful. Moreover, since success is persistent, this is also the proportion of successful old entrepreneurs in period $t+1$. It is useful to denote by $u_{t+1} = 1 - e_t(b_t, b_{t+1})$ the proportion of unsuccessful entrepreneurs.[7]

The government budget constraint is $2\tau_t = (2(1-\mu) + \mu u_t + \mu(1-e_t^*))wb_t$. Using, (3), we have

$$\begin{aligned}
\tau_t &= \tau(b_t, b_{t+1}, u_t) \\
&\equiv \left(1 + \frac{\mu}{2}\left(u_t - 1 - (1+\beta)\frac{w}{2} + (b_t + \beta b_{t+1})\frac{w}{2}\right)\right)b_t w.
\end{aligned} \tag{4}$$

[7]The restrictions $0 \le b \le 1$ and $w \le 1$ imply that $u_{t+1} \in \left[\frac{1-\beta}{2}, 1\right]$.

The marginal tax cost of redistribution in period t, $\frac{\partial \tau_t}{\partial b_t}$, increases in u_t (because more old entrepreneurs are benefit recipients) and in b_t and b_{t+1} (because more young entrepreneurs become unsuccessful). Since the old in period t cannot enjoy any benefits in period $t+1$, their equilibrium utility will therefore be decreasing in b_{t+1}.

Preliminary remarks about preferences for redistribution are as follows. The old successful entrepreneurs prefer zero benefits, since redistribution implies positive taxes without providing them with any benefits. Benefit recipients (workers and unsuccessful entrepreneurs), in contrast, are better off with some redistribution, even though their preferences for redistribution may be non-monotonic, as net benefits may fall with b at high levels of taxation, due to a Laffer curve effect. Note also that the Laffer curve is dynamic, depending both on historical investment levels and expectations about future taxation.

After the ability shock is realized, young workers like redistribution more than young entrepreneur. However, one should note that the government transfer programs entail some intergenerational redistribution, since the proportion of old and young successful entrepreneurs are in general different. The preferences of the different group of young agents depend therefore on the balance between inter- and intra-generational effects.

3 The Ramsey allocation with commitment

In this section, we show that optimal policy in the present model involves oscillations in taxation and redistribution. We will accomplish this goal by characterizing the full commitment solution in the case where the planner's weights on generation t is β^t, i.e., the planner discounts across consumption of different generations as the private agents discount their own consumption over time. In this particular case, the solution is relatively simple. We also extend briefly the analysis to the case in which the planner discount future at a general rate $\lambda \in (0,1)$. In the general case, however, of the optimal tax sequence is more involved. Since the main focus of this paper is on political economy, we limit attention in this case to long-run properties.

3.1 Statement of the commitment problem

The choice set of the planner is the set of sequences of benefits, $\{b_t\}_{t=0}^{\infty}$, that are feasible for some sequence of taxes and associated private effort choices. We assume that the planner can commit to future benefits; we sometimes refer to this problem as the Ramsey problem and to its solution as the Ramsey allocation. The planner is assumed to be perfectly utilitarian when evaluating the utility of ex-ante identical agents. To simplify, we assume that she discounts future generations at a constant rate by attaching a weight λ^t to agents born at time t. In the analysis of the political allocation, we will assume balanced budgets in each period and to be able to compare the two allocations, we impose this assumption also on the Ramsey allocation.[8] The planner chooses the sequence of b_t $\forall t \geq 0$ in order to maximize

$$\begin{aligned} W(u_0) \equiv\ & \beta(1-\mu)\tilde{V}^{ow}(b_0,\tau_0) + \beta\mu(1-u_0)\tilde{V}^{oes}(b_0,\tau_0) + \beta\mu u_0 \tilde{V}^{oeu}(b_0,\tau_0) \\ & + \sum_{t=0}^{\infty} \lambda^{t+1}\left(\mu\tilde{V}^{ye}\left(e(b_t,b_{t+1}),b_t,b_{t+1},\tau_t,\tau_{t+1}\right) + (1-\mu)\tilde{V}^{yw}(b_t,b_{t+1},\tau_t,\tau_{t+1})\right), \end{aligned} \tag{5}$$

[8]Interestingly, it can be shown that when $\beta = \lambda$, the balanced budget restriction does not bind for the Ramsey planner. That is, the planner would choose the same allocation even if she were allowed to accumulate debt or savings.

subject to

$$\begin{aligned} b_t &\in [0,1], \\ \tau_t &= \begin{cases} \tau(b_0, b_1, u_0) & \text{for } t = 0 \\ \tau(b_t, b_{t+1}, 1 - e(b_{t-1}, b_t)) & \text{for } t \geq 1. \end{cases} \end{aligned} \tag{6}$$

3.2 Characterizing the solution: a recursive formulation

Let us now analyze the Ramsey allocation. The planner's problem, (5), does not admit a standard recursive formulation since its solution is time-inconsistent (see Section 4.5 below for more details). Intuitively, the choice of b_{t+1} takes into account how the effort choice at t is influenced, but this effort choice is bygone when the time comes to implement b_{t+1}. It is well known that Ramsey problems admit a two-stage formulation whereby future decisions, in stage two, can be described as coming from a recursive problem with an additional state variable whereas the time-zero decisions, in stage one, can be derived from a "static" problem whose payoffs are given by the value function associated with the solution to the recursive problem.[9] In this framework we will show that the second-stage recursive problem is particularly simple in that the choice of b_{t+1} involves only one state variable: *current period's*level of transfers b_t – knowledge of b_t is sufficient to determine the optimal b_{t+1}. This result follows from the fact that since individuals live for two periods only, a benevolent planner who can commit to benefits one period ahead would choose the same level of redistribution as a planner who could commit all future periods. Specifically, if the planner at period t chooses b_{t+1}, she would have chosen the same b_{t+1} if she had had the ability to commit at any period $s < t$ (assuming b_t is held constant). Furthermore, although the flow of felicity in period t is affected by both the predetermined variables u_t and b_t, the optimal choice of b_{t+1} is only affected by b_t. Therefore, the recursive program has only b_t as a state variable, with b_{t+1} being the choice variable. As to the choice in the initial period, the planner is not subject to earlier pre-commitments and thus chooses b_0 and b_1 simultaneously. We thus prove that

Lemma 1 *The utilitarian planner program (5) is equivalent to the following recursive program:*

$$W(u_0) = \max_{b_0 \in [0,1]} \{Y_0(u_0, b_0) + V(b_0)\} \tag{7}$$

$$V(b_t) = \max_{b_{t+1} \in [0,1]} \{Y(b_t, b_{t+1}) + \lambda V(b_{t+1})\} \quad \textit{for } t \geq 0, \tag{8}$$

where $Y_0(u_0, b_0)$ is a linear-quadratic function, defined in appendix, and

$$\begin{aligned} Y(b_t, b_{t+1}) = \left(\frac{\mu w^2}{4}\right) \cdot \{ & 2\left((1+\beta)(\beta+\lambda) R - \beta^2\right) b_t \\ & - \left((1+\beta)(\beta+\lambda)(R+1) - \lambda - 2\beta^2\right) b_t^2 \\ & - \left((\beta+\lambda)^2 (R+1) - 2\lambda\beta\right) b_t b_{t+1} \\ & + 2\lambda\beta^2 b_{t+1} - \beta^2 \lambda b_{t+1}^2] + Q, \end{aligned}$$

and where Q is a constant defined in the appendix. Moreover, the mapping $\Gamma(v) = \max_{b' \in [0,1]} \{Y(b, b') + \lambda v(b')\}$ is a contraction mapping with V as the unique fixed point.

[9] See, e.g., Marcet and Marimon (1999), where the additional state variable is marginal utility or the Lagrange multiplier associated with the incentive constraint.

The recursive formulation of Lemma 1 shows that the optimal policy can be represented in terms of two policy rules. The first rule, which sets the initial choice of redistribution, maps the initial (predetermined) proportion of unsuccessful entrepreneurs into initial choices of redistribution. The second rule applies from period one onwards and maps previous period's benefits into current benefits, $b_t = f(b_{t-1})$.

3.2.1 Convergence to any steady state must be oscillatory

For the purposes of the discussion here, the main implication of the above result is the fact that we can use the function $Y(b_t, b_{t+1})$ to discuss dynamics. If the constraint $b_{t+1} \in (0,1)$ is not binding, the optimal allocation must satisfy the following first-order condition: $Y_2(b_t, b_{t+1}) + \lambda Y_1(b_{t+1}, b_{t+2}) = 0$. This follows from a standard envelope argument and the first-order condition on the Bellman equation (8). Calculating the derivatives and simplifying terms yields the following dynamic system:

$$\eta_0 + \eta_1 b_t + \eta_2 b_{t+1} + \lambda \eta_1 b_{t+2} \leq 0, \tag{9}$$

where

$$\begin{aligned}
\eta_0 &\equiv 2\lambda (1+\beta)(\beta+\lambda) R \geq 0 \\
\eta_1 &\equiv -\left((\beta+\lambda)^2 (R+1) - 2\lambda\beta\right) < 0 \\
\eta_2 &\equiv -2\lambda \left((1+\beta)(\beta+\lambda)(R+1) - \lambda - \beta^2\right) < 0
\end{aligned}$$

This dynamic system is exactly linear, and it has the important property that the coefficients on b_t, b_{t+1}, and b_{t+2} are all negative. This follows from Y being strictly concave in each of its arguments separately and from b_t and b_{t+1} displaying "substitutability". Intuitively, concavity follows from the convex cost function for effort and from the fact that taxation is more costly on the margin, the higher its level. The substitutability reflects the fact that effort depends on both b_t and b_{t+1}, so if one of these variables is high, the cost of increasing the other one marginally is high. Finally, note that if the solution is converging to a steady-state, an interior solution has to obtain along the transition.

Now consider the functional-equation version of the first-order condition for benefits: letting f denote the policy rule mapping b_t into an optimal b_{t+1}, we have $Y_2(b_t, f(b_t)) + \lambda Y_1(f(b_t), f(f(b_t))) = 0$ for all b_t in the neighborhood of a hypothetical steady state. Thus, locally, we are able to rule out an increasing f: if b_t is increased, Y_2 as well as λY_1 will decrease, since the coefficients on b_t, $f(b_t)$, and $f(f(b_t))$ are all negative and $f' > 0$.[10] Therefore, we have established the following:

Lemma 2 *The utilitarian planner program (5) leads to a solution which either does not converge to a steady state or, if it does, converges in an oscillatory fashion.*

It might seem surprising that the planner would not opt for benefit smoothing but it turns out that oscillating benefits reduce the distortion associated with redistribution. As noted in the introduction, the reason is that the investments young agents make in period t has an effect on the tax cost of redistribution both in period t and in period $t+1$. More precisely, if benefits in period $t-1$ were large (small), the young entrepreneurs will make a small (large) investment effort in that period. Thus, in period t, the old entrepreneurs will be relatively unsuccessful (successful), so there will be many (few) benefit recipients that period, and the cost of redistribution per dollar

[10] The case where f is a constant is ruled out from concavity.

of transfer in period t will therefore be relative large (small). Thus, the planner will set relatively small (large) benefits in period t. Applying a similar logic for period $t+1$, it is clear why the optimal sequence of benefits might be oscillatory. Intuitively, the planner reduces the distortion of benefits in period t by choosing lower benefits next period, as the investment decision of the young in period t depends on redistribution both in period t and in period $t+1$. When the planner sets at particular value b_t, she takes into account the effects this has both on felicity in $t-1$, t, and $t+1$ and the decision is, therefore, both backward- and forward-looking. Without commitment, the backward-looking aspect disappears, having, as we will see below, qualitative consequences for redistribution dynamics.

3.2.2 Full characterization of the Ramsey problem when $\lambda = \beta$.

The solution to the Ramsey problem delivers a policy rule which is not globally linear, except in the case $\lambda = \beta$. This case is a natural benchmark, as it implies that the planner discounts future felicities at the same rate at which agents discount future within their life-horizon. In this case, it is possible to attain a simple closed-form solution, summarized in the following Proposition.

Proposition 1 *The optimal solution to the planner program (5) in the case $\lambda = \beta$ is*

$$b_t = b^p - (b_{t-1} - b^p), \forall t \geq 1,$$

and

$$b_0 = \left(1 + \frac{(1-u_0)}{\frac{w}{2}(1-\beta)}\right) b^p,$$

where

$$b^p \equiv \frac{R}{1+2R}.$$

The result in Lemma 2 was that the equilibrium law of motion for benefits, f, cannot be increasing around a steady state. In the case $\lambda = \beta$ here, f indeed is decreasing and the slope of the policy function is -1. That is, initial conditions are persistent. To show that there really is a limit cycle in this case, one also needs to study the first-stage problem. As is clear from the proposition, the period-0 choice of the planner depends negatively on the initial condition on u_0, since this variable influences the marginal costs of providing benefits (see Hassler et al. (2003) for a more detailed discussion). Thus, the planner will choose initial benefits larger than or equal to b^p, and then oscillate forever between this level and another level on the other side of b^p.

When $\lambda \neq \beta$, dynamics continue to be oscillatory. Benefits may converge to a steady state or to a 2-period cycle. Convergence to a steady-state occur whenever one of the root of the characteristic equation associated with the difference equation (9) lies within the unit interval. This occurs for a non-empty range of λ's strictly larger than β.[11] When $\lambda < \beta$, instead, the optimal plan never converges to a steady state, and redistribution moves in the long-run between positive and zero redistribution. Intuitively, one can see that a reason for divergence in benefits is that in order to keep a constant distortion in present-value terms, $b_t + \beta b_{t+1}$ needs to be constant, and to keep this number constant and positive while there are oscillations, b_{t+1} must explode since we assume that $\beta < 1$. Before that occurs, however, b_t hits its lower bound. Thus, the summary description of optimal redistribution is that oscillations always occur and that divergence to a 2-period cycle occurs over the range of the parameter space in which we are interested. Since we will show that

[11]However, the dynamics are once again diverging to a 2-period limit cycle for a range λ sufficiently large.

when the benefit sequence is determined through political choice oscillations are possible, but they always converge to a steady state, an important result of this paper is that the political mechanism dampens efficient fluctuations and, in some cases, even generate policy persistence (i.e., monotonic convergence).

4 Politically determined redistribution

4.1 The political game

In the political equilibrium, the benefit policy is chosen in each period through a voting mechanism. In the benchmark case, we assume that agents vote over next period redistribution at the end of each period, after the uncertainty about individual entrepreneurial earnings has been realized. Since the old have no interest at stake, they are assumed to abstain. This is equivalent to assuming that agents vote over the current benefit policy before the effort choice of the entrepreneurs is made, and that only the old agents are entitled to vote. We later extend the analysis to the case in which both the young and the old vote on current benefits.[12]

4.1.1 Probabilistic voting

We assume a two-candidate political model of probabilistic voting a la Lindbeck-Weibull (1987) and restrict attention to Markov-perfect equilibria. In this model, whose features are discussed extensively in Persson and Tabellini (2000) and are therefore not detailed here, agents cast their votes on one of two candidates, who maximize their probability of becoming elected. Voters have heterogeneous preferences not only over redistribution, but also over some non-economic-policy dimension that is orthogonal to redistribution and over which the candidates cannot make binding commitments. We refer to this additional dimension as "ideology" Persson and Tabellini (2000). Voters differ in their evaluation of the candidates' ideology and their preferences over this dimension are subject to an aggregate shock whose realization is unknown to the candidates when platforms over redistribution are set.[13] In the equilibrium of this model, both candidates choose the same platform over redistribution and each has a fifty percent probability of winning. More importantly, the impact of each group on the equilibrium policy outcome increases with the relative weight in utility of the policy variable. Intuitively, if agents in a group have a lower concern for ideology, a candidate making a small change in redistribution in favor of this group will trigger a larger increase in her political support. In other terms, groups with many "swing-voters" are more attractive to power-seeking candidates and exert a stronger influence on the equilibrium political outcome. We will assume that the relative concern for ideology versus redistribution is the same within cohorts but may vary between cohorts. Under this assumption, it is straightforward to show that in equilibrium, the candidates' platforms simply maximize a weighted sum of individual utilities, where the weights are the same for all agents within a cohort but may differ between cohorts. Thus, the equilibrium policy maximizes a "political objective function" which is a weighted average utility of all voters. We will consider the cases when the political weight on the old is normalized to unity and the weight on the young is $\omega \in [0, 1]$.

[12]In this case, it turns out to be irrelevant whether young agents vote before or after their ability has been revealed.

[13]Since candidates have no intrinsic preferences over redistribution, they are assumed to implement their promised platform.

4.1.2 Definition of equilibrium

It is convenient to define the indirect utilities

$$
\begin{aligned}
& V^y\left(b_t, b_{t+1}, b_{t+2}, u_t, u_{t+1}\right) \\
\equiv\ & \mu \tilde{V}^{ye}\left(e\left(b_t, b_{t+1}\right), b_t, b_{t+1}, \tau\left(b_t, b_{t+1}, u_t\right), \tau\left(b_{t+1}, b_{t+2}, u_{t+1}\right)\right) \\
& +(1-\mu) \tilde{V}^{yw}\left(b_t, b_{t+1}, \tau\left(b_t, b_{t+1}, u_t\right), \tau\left(b_{t+1}, b_{t+2}, u_{t+1}\right)\right),
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
& V^j\left(b_t, b_{t+1}, u_t\right) \\
\equiv\ & \tilde{V}^j\left(b_t, \tau\left(b_t, b_{t+1}, u_t\right)\right), \quad \text{for } \ j \in \{oes, oeu, ow\}.
\end{aligned}
$$

We construct equilibria whose policy functions are linear (except for kinks implied by upper and lower bounds) in the aggregate state variable: the proportion of current unsuccessful old entrepreneurs (u_t). The political equilibrium is defined as follows.

Definition 1 *A political equilibrium is defined as a pair of functions $\langle B, U\rangle$, where $B:[0,1]\rightarrow[0,1]$ is a public policy rule, $b_t = B\left(u_t\right)$, and $U:[0,1]\rightarrow[0,1]$ is a private decision rule, $u_{t+1} = U\left(b_t\right)$, such that, given the political weight $\omega\in[0,1]$ on each young, the following functional equations hold:*

1. $B\left(u_t\right) = \arg\max_{b_t\in[0,\bar{b}]} V\left(b_t, b_{t+1}, b_{t+2}, u_t, u_{t+1}\right)$ subject to $u_{t+1} = U\left(b_t\right)$, $b_{t+1} = B\left(U\left(b_t\right)\right)$, and $b_{t+2} = B(U(B\left(U\left(b_t\right)\right)))$, and
2. $U\left(b_t\right) = 1 - e\left(b_t, b_{t+1}\right)$ with $b_{t+1} = B\left(U\left(b_t\right)\right)$,
   where

$$
\begin{aligned}
& V\left(b_t, b_{t+1}, b_{t+2}, u_t, u_{t+1}\right) \\
\equiv\ & \mu\left((1-u_t)V^{oeu}\left(.\right) + u_t V^{oes}\left(.\right)\right) \\
& +(1-\mu)V^{ow}\left(.\right) \\
& +\omega\left(V^y\left(.\right)\right).
\end{aligned} \tag{11}
$$

The first equilibrium condition requires the political mechanism to choose b_t to maximize V, taking into account that future redistribution depends on the current policy choice via the equilibrium private decision rule and future equilibrium public policy rules. Furthermore, it requires $B(u_t)$ to be a fixed point in the functional equation (1). In other words, suppose that agents believe that future benefits are set according to the function $b_{t+j} = B\left(u_{t+j}\right)$. Then we require that the same function $B(u_t)$ define optimal benefits today. We should note that in the case $\omega = 0$, the political objective is given by the first two rows of (11) only, depending only on b_t, b_{t+1}, and u_t.

The second equilibrium condition states that all young individuals choose their investment optimally, given b_t and b_{t+1}, and that agents have rational expectations about future benefits and distributions of types. In general, U could be a function of both u_t and b_t, but in our particular model u_t has no direct effect on the investment choice of the young. Thus, in our equilibria the equilibrium investment choice of the young is fully determined by the current benefit level.

The function V entails the assumption that all agents within a given generation exert the same political influence, irrespective of their type. In the general case where two generations participate into each election, however, we allow for age-specific differences in the concern for the ideological dimension. This is parameterized by $\omega\in[0,1]$. In particular, $\omega<1$ means that the old care less, on average, about ideology and have more "swing-voters" than the young. Hence, their preferences carry more weight in the political objective function, V. The opposite would be true if $\omega > 1$, a case that we do not consider. When $\omega = 1$ all voters are equally represented.

4.2 An economy where all agents are risk-neutral

In this section, we consider the particular case when $a = 1$, i.e., when agents are risk-neutral and have therefore, ex-post, the same preference intensity for economic policy. Here, the welfare state entails no insurance value.

It is instructive to see how the equilibrium is constructed. Let us therefore sketch the method we use to find the political equilibrium in the simplest case when $\omega = 0$, leaving some details for the appendix. Substituting the discounted value functions (10) into the political objective function (11), we obtain (with a slight abuse of notation)

$$\begin{aligned} V\left(b_t, b_{t+1}, u_t\right) &= \frac{\mu}{2}\left(u_t b_t w - \left(1 - e\left(b_t, b_{t+1}\right)\right) b_t w\right) \\ &\quad + \mu(1 - u_t) w \end{aligned} \tag{12}$$

Notice that the last term is exogenous from the perspective of the voter: it is predetermined. Omitting this and the proportionality factor $\mu/2$, and noting that $1 - e\left(b_t, b_{t+1}\right) = u_{t+1}$, the political objective can therefore be written as $(u_t - u_{t+1}) b_t w$: positive benefits help the current old only if the number of unsuccessful old entrepreneurs exceeds the number of unsuccessful young entrepreneurs. The latter, of course, is determined by policy. Thus, the workers do not enter this expression: since they are of equal number in each cohort, any transfers between them will net to zero.

Disregarding constants and using the expression for $e\left(b_t, b_{t+1}\right)$, the political objective in (12) can be written as

$$u_t b_t w - \left(1 - (1+\beta)\frac{w}{2} + \frac{w}{2}\left(b_t + \beta b_{t+1}\right)\right) b_t w. \tag{13}$$

We need to find two functions $B(u_t)$ and $U(b_t)$ satisfying the two equilibrium conditions in Definition 3. Guided by the linear-quadratic form of the objective function, we guess on the functional form for B: $B(u_t) = \alpha_0 + \alpha_1 u_t$, for some yet undetermined coefficients α_0 and α_1. Using this guess, the second equilibrium condition can be written as

$$U\left(b_t\right) = 1 - \frac{1 + \beta - \left(b_t + \beta\left(\alpha_0 + \alpha_1 U\left(b_t\right)\right)\right)}{2} w. \tag{14}$$

Solving for $U(b_t)$ we obtain

$$U\left(b_t\right) = \frac{2 - w\left(1 + \beta\left(1 - \alpha_0\right)\right) + b_t w}{2 - \beta\alpha_1 w}.$$

Substituting the expression for $U(b_t)$ and the guess of $B(u_t)$ into the first-order condition and solving for b_t gives

$$b_t = \frac{1}{2w}\left(-2 + w\left(1 + \beta\left(1 - \alpha_0\right)\right)\right) + \frac{2 - \beta\alpha_1 w}{2w} u_t,$$

which verifies the tentative guess as a fixed-point of equilibrium condition 1 if $\alpha_1 = \frac{2}{w(2+\beta)}$ and $\alpha_0 = -\alpha_1\left(1 - \frac{1}{2}(1+\beta) w\right)$,heuristically establishing the following proposition.[14]

[14]Given the quadratic objective, it is straightforward to check that the first-order condition will be sufficient for a maximum.

Proposition 2 *Assume $a = 1$ and $\omega = 0$ (risk neutrality, "n", and only the old vote, "o"). The political equilibrium is characterized as follows:*

$$B^{no}(u_t) = \begin{cases} \frac{2}{w(2+\beta)}(u_t - \underline{u}) & \text{if } u_t \geq \underline{u} \\ 0 & \text{else} \end{cases}$$

$$U^{no}(b_t) = \underline{u} + \frac{w}{2}\left(1 + \frac{\beta}{2}\right) b_t,$$

where where $\underline{u} \equiv 1 - e(0,0) = 1 - \frac{1+\beta}{2}w$. Given any $u_0 > \underline{u}$, the equilibrium law of motion is

$$u_{t+1} = \underline{u} + \frac{1}{2}(u_t - \underline{u}),$$

$$b_{t+1} = \frac{1}{2} b_t,$$

and the economy converges monotonically to a unique steady state with $b = 0$ and $u = \underline{u}$. For $u_0 \leq \underline{u}$, $u_t = \underline{u}$ $\forall t > 0$.

In the equilibrium of Proposition 2, redistribution occurs along the transition path, i.e., as long as $u_0 > \underline{u}$. In the long run, however, there is no redistribution. Unlike in the Ramsey allocation, here we see monotone convergence: the dynamics are characterized by a positive root equal to 1/2. The speed of convergence is thus independent of w and β.

Figure 2 represents the equilibrium policy function and law of motion. The left-hand panel shows that when $u_t > \underline{u}$ redistribution is positive in equilibrium. Moreover, the equilibrium level of b_t increases linearly with u_t. The right-hand panel illustrates how the equilibrium law of motion implies monotonic asymptotic convergence to the steady state as long as $u_0 > \underline{u}$.

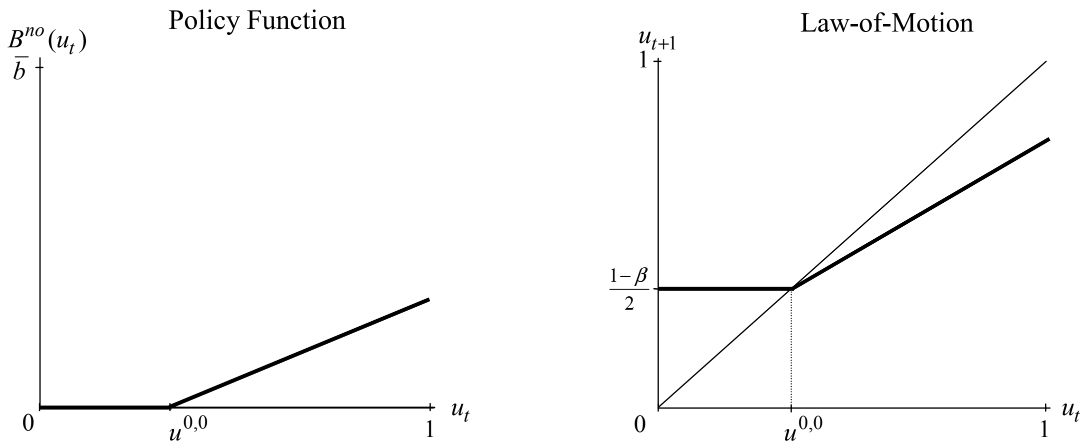


Figure 2. Risk neutrality and only the old vote.

Our results can be interpreted as follows: when only the old influence the political outcome, the equilibrium redistribution, $B^{no}(u_t)$, maximizes the average income of the old. This implies maximizing the intergenerational transfer from young to old individuals without concern for intra-generational redistribution. Intergenerational transfers benefitting the current voters can, however, be achieved by setting $b_t > 0$ only if the proportion of old unsuccessful agents is higher than the proportion of young unsuccessful, i.e., if $u_t > u_{t+1}$. In particular, no redistribution can occur in steady state. The results of Proposition 2 generalize to the case of $\omega \in [0,1]$.

Turning to voter participation of the young, we have

Proposition 3 *For $a = 1$ and any $\omega \in [0,1]$, the political equilibrium is characterized as follows:*

$$\begin{aligned} B^n(u_t) &= \begin{cases} \frac{\rho}{w}(u_t - \underline{u}) & \text{if } u_t \geq \underline{u} \\ 0 & \text{else} \end{cases} \\ U^n(b_t) &= \underline{u} + \frac{w}{2-\beta\rho} b_t, \end{aligned}$$

where

$$\rho = \frac{2Z}{1+\beta Z}$$

and $Z \in [0, 1/2]$ is the real solution to

$$Z\left(1 + \omega\beta Z^2\right) = \frac{1-\omega}{2},$$

which is decreasing in ω and β. Given any $u_0 > \underline{u}$, the equilibrium law of motion is

$$\begin{aligned} u_{t+1} &= \underline{u} + Z(u_t - \underline{u}), \\ b_{t+1} &= Z b_t, \end{aligned}$$

and the economy converges monotonically to a unique steady state with $b = b^n = 0$ and $u = \underline{u}$. For $u_0 \leq \underline{u}$, $u_t = \underline{u}$ $\forall t > 0$. Finally, ρ is decreasing in ω and β; if $\omega = 1$, we obtain $\rho = 0$, implying immediate convergence to the steady state.

In the case when both young and old agents vote on current benefits, the equilibrium has the same qualitative features as in the benchmark case ($\omega = 0$), provided that $\omega < 1$. In particular, redistribution occurs along the transition path, but there is no welfare state in the long run. Since $\omega < 1$, the old are politically preponderant and the political equilibrium therefore favors redistribution from young to old. Such redistribution can be achieved via positive benefits if and only if $u_t > u_{t+1}$. Therefore, redistribution is positive only along the transition to the steady state. For any $\omega < 1$, dynamics are characterized by a positive root $Z \leq 1/2$. The higher is ω, the lower are transfers and the flatter are the equilibrium policy function and the law of motion in Figure 1. This is due to the fact that the young exert political pressure against redistribution. With $\omega = 1$, benefits are zero regardless of u_t and system jumps to the steady state immediately.

4.3 The case of risk-averse agents

HRSZ find that, in a model of majority voting, the welfare state can survive in the long run even though agents are risk-neutral. The previous section shows, however, that under probabilistic voting redistribution must die off in the long run. That is, for the same economic environment, the long-run state of the transfer system can depend critically on the form of the democratic process. Moreover, the transitional dynamics here are characterized by monotonic rather than oscillatory convergence. In this section, we show that the political equilibrium features the long-run survival of the welfare state under probabilistic voting, provided that a positive proportion of agents in society are risk-averse. The convergence to the steady state may be oscillatory or monotonic depending on the extent of risk aversion and the political influence of the young.

As in the previous subsection we will initially assume that the young agents have no influence in the voting process, i.e., that $\omega = 0$. When $a \geq 1$, the political objective function, $V(b_t, b_{t+1}, u_t)$, can be expressed (up to scaling and excluding constants) as follows:

$$\begin{aligned} V(b_t, b_{t+1}, u_t) &= \frac{2R}{1+R}(1-u_t)\, b_t \\ &\quad + u_t b_t - \left(1 - (1+\beta)\frac{w}{2} + \frac{w}{2}(b_t + \beta b_{t+1})\right) b_t. \end{aligned} \tag{15}$$

The political objective is derived as in the case of risk neutrality, differing by the first term (the second line, as above, equals $(u_t - u_{t+1}) b_t$), which reflects a positive effect from redistribution whenever aggregate risk aversion is positive: the higher marginal utility of workers makes any redistributed dollar pay off more the higher is R. The part $(1 - u_t)$ is the fraction of old successful entrepreneurs, representing the size of the inelastic tax base. This term inversely reflects the distortionary cost of redistribution. We can now characterize the equilibrium as follows.

Proposition 4 *Assume $\omega = 0$ and risk aversion ("a"). Then if $R \in [0, R_{\max}]$, where $R_{\max} > 1$ is defined below, the political equilibrium is characterized as follows:*

$$\begin{aligned} B^{ao}(u_t) &= \begin{cases} b^{ao} + \frac{\rho}{w}(u_t - \underline{u}^{ao}) & \text{if } u_t \geq \underline{u}^{ao} - \frac{w}{\rho} b^{ao} \\ 0 & \text{otherwise} \end{cases} \\ U^{ao}(b_t) &= \underline{u}^{ao} + \frac{w}{2 - \beta\rho}(b_t - b^{ao}), \end{aligned}$$

where

$$\begin{aligned} \rho &= \frac{2Z}{1+\beta Z} \\ Z &= \frac{1}{2}\frac{1-R}{1+R} \in \left[-\frac{1}{2}, \frac{1}{2}\right] \\ b^{ao} &= \frac{4R}{1+3R}\frac{1+\beta}{2+\beta}, \\ \underline{u}^{ao} &= 1 - \frac{w}{2}(1+\beta)(1 - b^{ao}), \end{aligned}$$

and $R_{\max}$ is defined as R such that $B^{ao}(0) = 1$. If $R > 0$, then $b^{ao} > 0$ and $b_t > 0 \ \forall\ t > 0$ (redistribution is positive after at most one period). Furthermore, b^{ao} and $\underline{u}^{ao}$ increase in R. For $t > 0$, the equilibrium law of motion is

$$\begin{aligned} u_{t+1} &= \underline{u}^{ao} + Z(u_t - \underline{u}^{ao}), \\ b_{t+1} &= b^{ao} + Z(b_t - b^{ao}). \end{aligned}$$

Given u_0, the economy converges to a unique steady state with $b = b^{ao}$ and $u = \underline{u}^{ao}$ following an oscillating (monotone) path if $R > (<)\, 1$. If $R = 1$, convergence is immediate.

Imposing the upper bound on aggregate risk aversion, $R < R_{\max}$, ensures that the constraint $b \leq 1$ is not binding in equilibrium. This is necessary in order for the equilibrium policy function and private decision rule to be linear and, hence, for the analytical characterization of the political equilibrium to be viable.

Proposition 4 establishes that the dynamics of redistribution involve convergence to a unique steady state characterized by a positive benefit rate, provided some agents are risk averse ($R > 0$).

Steady-state benefits, b^{ao}, increase in risk aversion and in the share of workers, while they decrease in the wage rate because the distortionary effect of benefits increases with the return to effort.

The equilibrium policy function and the dynamics of u_t are depicted in Figure 3. As long as $R < 1$, dynamics are characterized by a positive root, implying monotone convergence. If $R > 1$ instead, the benefit rate is a decreasing function of u_t and the root Z is negative, implying convergence following an oscillatory pattern. In the particular case where $R = 1$, convergence to the steady state occurs in one period.

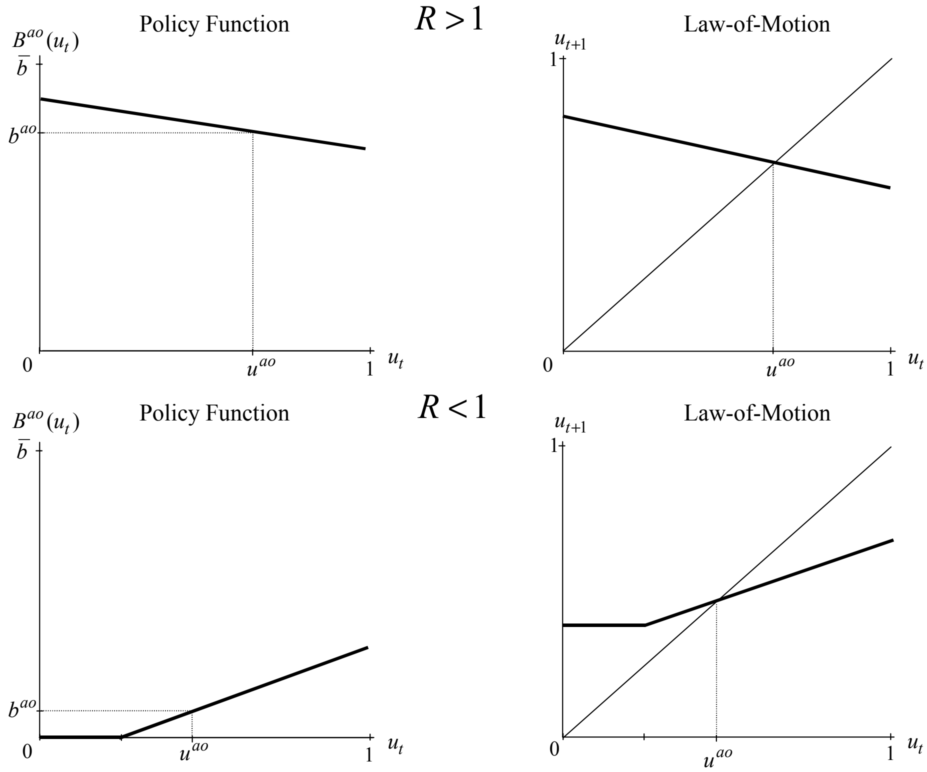


Figure 3. The equilibrium policy function $B(u_t)$ and u_t dynamics under risk aversion ($R \geq 0$).

The dynamics are characterized by two opposing forces. On the one hand, the larger the current share of unsuccessful entrepreneurs, u_t, the higher the tax cost per unit of benefits. This is captured by the fact that the first term of (15) falls in u_t, reflecting the higher dependency ratio associated with a higher share of unsuccessful old entrepreneurs. We label this the *tax-base effect.* Through this effects, a higher u_t reduces the marginal (political) value of benefits, tending to generate a negative relationship between b and u. On the other hand, the larger is u_t, the larger is the second term of (15), reflecting a stronger political pressure for redistribution since more individual entrepreneurs benefit from redistribution. We label this the *constituency effect* and note that a higher u_t increases the marginal political value of redistribution, tending to generate a positive relationship between b and u. When aggregate risk aversion is low, the latter effect dominates, while the opposite is true when aggregate risk aversion is high. The reason for this is that when R is high, the political influence of the entrepreneurs diminish, as workers become, on average, more sensitive to the issue of redistribution, due to their higher individual risk aversion. Since with a probabilistic voting model preference intensity matters, this implies that the policy implemented in equilibrium reflects the will of the average worker more closely, i.e., there is more redistribution. Consequently, the policy outcome becomes less sensitive to the share of *unsuccessful entrepreneurs,* who want positive redistribution. Thus, the dynamics are dominated by the cost effect. In sum, higher aggregate risk aversion therefore *increases* steady state benefits and makes the *tax-base effect* stronger.

Proposition 4 can be generalized to the case in which the young participate in the political decision: $\omega \in [0,1]$. The equilibrium has the same form as Proposition 4. However, the expression of Z is involved and we only state here its main properties. The detailed proof of this proposition is omitted and is available upon request.

Proposition 5 *Assume $0 \leq R \leq R_{\max}$ and $\omega \in [0,1]$. The political equilibrium is then characterized as follows:*

$$\begin{aligned} B^a(u_t) &= \begin{cases} b^a + \frac{\rho}{w}(u_t - \underline{u}^a) & \text{if } u_t \geq \underline{u}^a - \frac{w}{\rho} b^a \\ 0 & \text{otherwise} \end{cases} \\ U^a(b_t) &= \underline{u}^a + \frac{w}{2-\beta\rho}(b_t - b^a), \end{aligned}$$

where

$$\rho = \frac{2Z}{1+\beta Z}$$

and Z is a constant with the following properties: (i) $Z \in (-4/7, 1/2]$, (ii) $dZ/d\omega < 0$, (iii) $Z > (<) \, 0$ iff $R < (>) \, \frac{1-\omega}{1+\omega}$; and (iv) if $Z > 0$, $dZ/dR < 0$.
For $t \geq 0$, the equilibrium law of motion is

$$\begin{aligned} u_{t+1} &= \underline{u}^a + Z(u_t - \underline{u}^a), \\ b_{t+1} &= b^a + Z(b_t - b^a), \end{aligned}$$

where $\underline{u}^a$ and b^a are functions of Z defined in the appendix. Given any u_0, the economy converges to a unique steady state following an oscillating (monotone) path if $R > (<) \, \frac{1-\omega}{1+\omega}$. If $R = \frac{1-\omega}{1+\omega}$, convergence is immediate to a steady state where $b = R(1+\omega)^2 / [(2+\beta)/(1+\beta) + \omega(1-\omega)] \geq 0$.

We note that an increase in the political participation of the young decreases the slope of the policy function. In particular, the sign of the slope coefficient and whether dynamics are oscillating or not depend on whether $R \lesseqgtr \frac{1-\omega}{1+\omega}$. When dynamics are monotone, the persistence of redistributive policies falls in R, since, then, $dZ/dR < 0$. This condition nests the result of Proposition 4 that the policy function is upward- (downward-)sloping if and only if $R < 1$ ($R > 1$) when $\omega = 0$. If instead the young are as politically influential as the old ($\omega = 1$), then the policy function becomes downward-sloping and dynamics are oscillatory for any positive level of risk aversion.

As far as the young are concerned, both the cost effect and the intergenerational redistribution motive imply that benefits should fall with u_t (recall that the larger u_t the larger the transfer from the young to the old). Therefore, as the influence of the young increases, the intergenerational redistribution motive is mitigated. If $\omega < 1$, the old retain some political preponderance, and intergenerational transfers towards the old carry some weight in the political decision. If $\omega = 0$, however, this motive disappears and the dynamics of redistribution is determined by the cost effect alone. Since the tax-base effect implies a negative relation between benefits and the number of old unsuccessful entrepreneurs, stronger influence of young voters reduces the slope of the policy function and tends to make dynamics oscillatory.

Unfortunately, due to the complicated expression of Z, we have not been able to sign the effect of an increase of the participation of the young on steady-state redistribution, although numerical analysis suggests that an increase in ω reduces redistribution in the long run.

4.4 Finite-horizon results

In this section we seek to answer two related questions. First, we aim to show that the Markov-perfect equilibria derived above are indeed limits of finite-horizon equilibria. Second, and more substantially, we wish to find out whether there can be more than one finite-horizon equilibrium, i.e., whether there can be a role for "coordination", and perhaps "reputation", even in finite-horizon versions of this model. The uniqueness question is a substantial one not only formally, but in a very applied sense: it touches on the "stability" of government redistribution schemes, which was challenged recently in HRSZ. In that paper, a simple version of the present model with majority voting was shown to robustly produce multiple equilibria independently of the time horizon. I.e., a "belief in the welfare system" seemed necessary to support the system. For brevity, we will not cover all cases in this section; concentrating on our baseline setup where only the old vote and where $w = 1$. We will first study the limit of the finite-horizon case and then discuss uniqueness.

We assume the economic environment to be identical to that of previous sections except in a final period T where the newborn young make an effort investment but live only for one period. In the finite horizon economy, the equilibrium policy function will in general be time-dependent. For $t < T$, equilibrium condition 1 is thus modified to

Definition 2 $B^t(u_t) = \arg\max_{b_t \in [0,\bar{b}]} V(b_t, b_{t+1}, u_t)$ *subject to* $b_{t+1} = B^{t+1}(u_{t+1})$ *with* $u_{t+1} = 1 - e(b_t, b_{t+1})$.

Guessing preliminarily that $B^{t+1}(u)$ is linear, i.e., $B^{t+1}(u) = A_{t+1} + B_{t+1}u$, it is straightforward to show that an interior solution to the maximization problem at period t yields a linear policy function $B^t(u) = A_t + B_t u$. Let $j \equiv T - t$ denote the number of periods until the horizon, then the coefficients A_j and B_j satisfy[15]

$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = \begin{bmatrix} -\frac{\beta}{2} & -\frac{R}{1+R}\beta \\ 0 & -\frac{1-R}{1+R}\frac{\beta}{2} \end{bmatrix} \begin{bmatrix} A_{j-1} \\ B_{j-1} \end{bmatrix} + \begin{bmatrix} \frac{2R}{1+R} - \frac{1-\beta}{2} \\ \frac{1-R}{1+R} \end{bmatrix}. \tag{16}$$

The solution to this linear system of difference equations is

$$\begin{aligned} A_j &= \left(-\frac{\beta}{2}\right)^j (A_0 - A) + \left(-\frac{\beta}{2}\right)^j \left(1 - \left(\frac{1-R}{1+R}\right)^j\right)(B_0 - B) + A, \\ B_j &= \left(-\frac{\beta}{2}\frac{1-R}{1+R}\right)^j (B_0 - B) + B, \end{aligned} \tag{17}$$

where A_0 and B_0 now define the policy rule in the last period ($t = T$). As this system is stable (the roots are $-\frac{\beta}{2}$and $-\frac{\beta}{2}\frac{1-R}{1+R}$), then, as $j \to \infty$, the coefficients of the policy rule converge to

$$\begin{aligned} A &\equiv \frac{8R}{(2+\beta)(2+\beta+R(2-\beta))} - \frac{1-\beta}{2+\beta}, \\ B &\equiv 2\frac{1-R}{2(1+R)+\beta(1-R)}, \end{aligned}$$

and the policy rule is identical in the limit to infinite horizon case detailed in Proposition 4 for the case $\omega = 0$ and $w = 1$.

[15] This equation follows from using $b_{t+1} = A_{j-1} + B_{j-1}u_{t+1}$ in the political objective function (15).

Next, we derive the final period policy function, $B^T(u)$ and show that it is unique and linear, with $b_T = A_0 + B_0 u_T$, for all u_T in the reachable range $\left[\frac{1-\beta}{2}, 1\right]$. This provides the initial condition for the difference equation (17). Given the result above, it follows that the finite horizon equilibrium is unique.

We propose a parameterization of the final period which we regard as reasonable, although the argument does not hinge on the specific choice. In particular, we assume that the young born in the final period live for one period only and that they make an effort choice. To make the final period comparable to the previous periods, we compensate for the fact that the young in the last period get return on effort only in one period by scaling down their disutility of effort by $(1+\beta)^{-1}$. Thus, the effort cost is equal to $e^2/(1+\beta)$, implying that the optimal effort is equivalent to that which agents living for two periods would have chosen if they had faced the benefit level b_T in both periods. This optimal effort level is $e_T^* = (1-b_T)\frac{1+\beta}{2}$, implying that taxes in the final period are given by

$$\tau_T = \frac{1}{2}\left(2(1-\mu) + \mu u_T + \mu\left(\frac{1-\beta}{2} + b_T\frac{1+\beta}{2}\right)\right)b_T.$$

Substituting in τ_T into the utility function yields the following political objective function:

$$V_T(b_T, u_T) = \mu((1-u_T) + u_T b_T) + ((1-\mu)b_T + Rb_T) - (1+R)\tau_T.$$

Maximizing $V_T(b_T, u_T)$ with respect to b_T yields $b_T^* = A_0 + B_0 u_T$, with

$$\begin{aligned} A_0 &= -\frac{1}{2}\frac{1-\beta}{1+\beta} + \frac{2R}{(1+R)(1+\beta)}, \\ B_0 &= \frac{1}{1+\beta}\frac{1-R}{1+R}, \end{aligned}$$

The analysis has so far ignored, for the sake of simplicity, the constraint that $b \in [0,1]$. Characterizing the sequence of policy functions when these constraints are present is more complicated and the details of the results depend critically on the exact form of the effort function in the last period, about which we do not have strong prior information.[16]

4.5 Voting with benevolence toward future generations

There are two qualitative differences between the Ramsey allocation in Section 3 and the political allocations in Section 4. First, the Ramsey planner has commitment. Second, the planner cares about future, yet unborn, generations. In order to isolate the importance of commitment for the different characteristics of the two allocations, in this section we shall study the intermediate case where policy is chosen by a benevolent social planner who has no access to a commitment technology. One can think of this case as forever electing to office an agent with altruism toward future generations but without the ability to commit future policy. Alternatively, it can be thought

[16] That economies where the constraint that $b \in [0,1]$ never binds exist can be shown easily in some special cases, such as when $R=1$. In this case, $B_j = B = 0$ for all $j \geq 0$, and, for all t,

$$B^t(u) = A\left(1 - \left(-\frac{\beta}{2}\right)^j \frac{\beta}{2(1+\beta)}\right) > 0$$

since $A = (1+\beta+2s(1-\mu)/\mu)/(2+\beta)$. So the constraint is not binding when $R=1$. By continuity, the same argument carries over for values of R sufficiently close to one. For more general values of the parameters, we also encountered no multiplicity, though some of our analysis here relies on numerical methods.

of as a case when old voters are altruistic: the vote in order to maximize a welfare function which is a weighted average of their own felicity and the welfare of the next generation. Either way, this case allows us to differentiate between the effects of altruism and commitment.

In order to derive a recursive formulation of the problem, we define the "weighted average felicity" across both young and old agents at time t as

$$\begin{aligned} F_0(u_t, b_t, b_{t+1}) \equiv\ & \beta(1-\mu)\left(\tilde{V}^{ow}(b_t, \tau_t)\right) \\ & + \beta\mu\left((1-u_t)\tilde{V}^{oes}(b_t, \tau_t) + u_t\tilde{V}^{oeu}(b_t, \tau_t)\right) \\ & + \lambda(1-\mu)(a(wb_t - \tau_t) - (a-1)x) \\ & + \lambda\mu\left(e_t^* w + (1-e_t^*) wb_t - (e_t^*)^2 + \underline{w} - \tau_t\right), \end{aligned} \tag{18}$$

where $e_t^* = e(b_t, b_{t+1})$ and $\tau_t = \tau(b_t, b_{t+1}, u_t)$.

As above, we let $\beta \in [0,1]$ and $\lambda \in [0,\beta]$, respectively, be the weights on the old and young currently alive. When $\beta = \lambda$, the old are "perfectly altruistic" and weight equally their old-age felicity and that of the young. If, on the other hand, $\lambda = 0$, we obtain the case analyzed in Proposition 2. We restrict attention to economies where $\lambda \leq \beta$, so that altruism cannot "exceed 100%".

Since the problem is autonomous when policies are in the Markov class, a recursive formulation of the political objective can be written as

$$\begin{aligned} W(u_t) &\equiv \max_{b_t \in [0,\bar{b}]} \{F_0(u_t, b_t, b_{t+1}) + \lambda W(u_{t+1})\}, \\ \text{s.t. } b_{t+1} &= B(U(b_t)), \\ u_{t+1} &= U(b_t). \end{aligned} \tag{19}$$

In direct analogy with our equilibrium definition above, we provide

Definition 3 *A political equilibrium with altruistic voting is defined as a pair of functions* $\langle B, U\rangle$, *where* $B : [0,1] \to [0,1]$ *is a public policy rule,* $b_t = B(u_t)$, *and* $U : [0,1] \to [0,1]$ *is a private decision rule,* $u_{t+1} = U(b_t)$.

1. $B(u_t) = \arg\max_{b_t} \{F_0(u_t, b_t, b_{t+1}) + \lambda W(u_{t+1})\}$ subject to $u_{t+1} = U(b_t)$, $b_{t+1} = B(U(b_t))$ and $b_t \in [0, \bar{b}]$,
2. $U(b_t) = 1 - e(b_t, b_{t+1})$ with $b_{t+1} = B(U(b_t))$,
   where $W(u_{t+1})$ is defined by the Bellman equation (19).

The following can then be established.

Proposition 6 *Assume* $\lambda \leq \beta$ *(altruism, "al") and* $0 \leq R \leq R_{\max}$. *The political equilibrium with altruistic voting is characterized as follows:*

$$\begin{aligned} B^{alo}(u_t) &= \begin{cases} b^{alo} + \frac{\rho}{w}\left(u_t - \underline{u}^{alo}\right) & \text{if } u_t \geq \underline{u}^{alo} - \frac{w}{\rho}b^{alo} \\ 0 & \text{otherwise} \end{cases} \\ U^{alo}(b_t) &= \underline{u}^{alo} + \frac{w}{2-\beta\rho}\left(b_t - b^{alo}\right), \end{aligned}$$

where

$$\rho = \frac{2Z}{1+\beta Z}$$

and Z is a constant (details in the proof) with the following properties: (i) $Z \in [-1/(2+\lambda), 1/2]$, (ii) $dZ/dR < 0$, (iii) $dZ/d\lambda < 0$, and (iv) $Z > 0$ if and only if $R > (\beta - \lambda)/(\beta + \lambda)$.
For $t > 0$, the equilibrium law of motion is

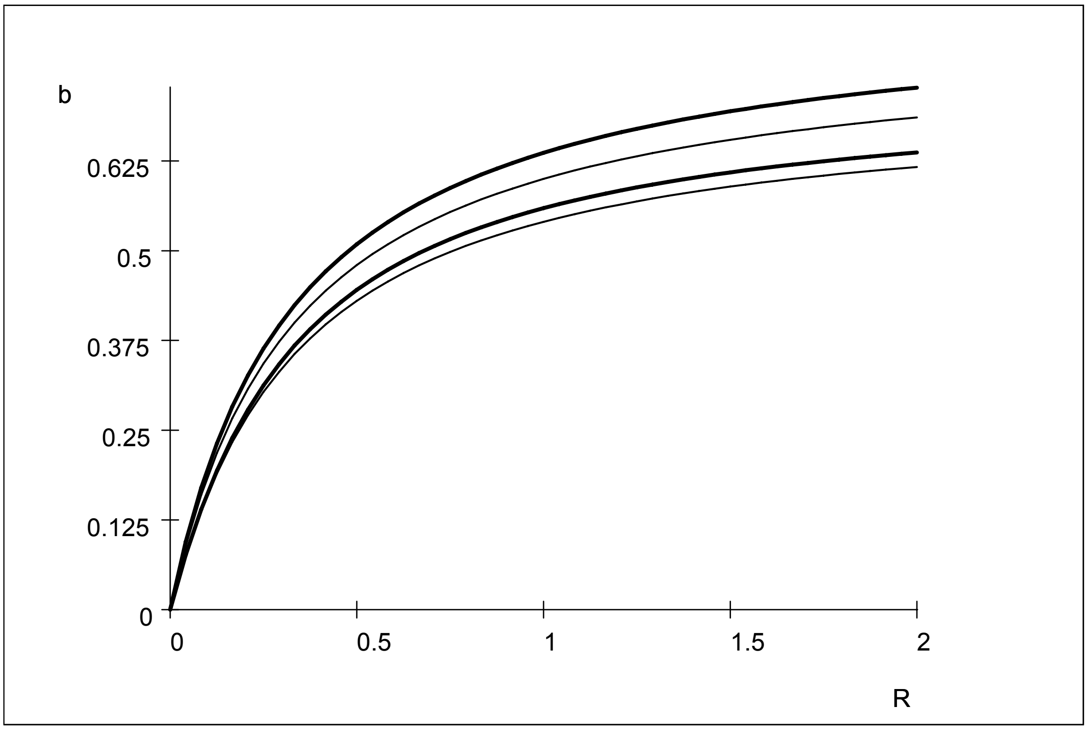
$$\begin{aligned} u_{t+1} &= \underline{u}^{alo} + Z\left(u_t - \underline{u}^{alo}\right), \\ b_{t+1} &= b^{alo} + Z\left(b_t - b^{alo}\right). \end{aligned}$$

Given any u_0, the economy converges to a unique steady state such that $b = b^{alo} \leq b^{ao}$ and $u = \underline{u}^{alo} \leq \underline{u}^{ao}$ following an oscillating (monotone) path if $R > (<) \frac{\beta-\lambda}{\beta+\lambda}$. If $R = 0$, $Z = \frac{1}{2}\left(1 - \frac{\lambda}{\beta}\right) \geq 0$ and $b^{alo} = 0$. If $R = \frac{\beta-\lambda}{\beta+\lambda}$, convergence is immediate to a steady state with $\frac{1+\beta}{\beta(2+\beta)-\lambda}(\beta - \lambda) \geq 0$.

The proposition establishes that the slope coefficient of the policy function is decreasing in λ. Namely, the altruistic motive in the political equilibrium reduces the interest in intergenerational redistribution and tends to make efficiency considerations more important (similarly to when we considered political participation of the young). This strengthens the tax-base effect and tends to make dynamics oscillatory. In particular, if $\lambda = \beta$ and $R > 0$, the dynamics are oscillatory, indicating that the tax-base effect dominates the constituency effect. However, oscillations need not occur: for $\lambda < \beta$, when R is sufficiently small, dynamics are monotone—contrasting the Ramsey case—and we therefore conclude that the lack of commitment does play an important role in dampening tax cycles. This section, however, also shows that voters' concern about future generations tends to contribute to oscillatory dynamics.

One would suspect that steady-state redistribution is decreasing in λ, since the distortionary costs of taxation are larger for young than for old individuals. We have not been able to establish this relation formally, however, although we have not found any numerical counterexamples. Figure 4 considers the parametric case where $\beta = 1/2$, showing the steady-state benefits in the case when $\lambda = 0$ (no altruism, upper curve) and the steady-state benefits in the case when $\lambda = \beta$ (lower curve) for different values of the risk-aversion parameter R. As we see, the difference increases with R. It also turns out that this increases with β.

One would suspect that steady-state redistribution is decreasing in λ, since the distortionary costs of taxation are larger for young than for old individuals. We have not been able to establish this relation formally, however, although we have not found any numerical counterexamples. To compare the case of full altruism to the case when the old voters have no concern at all for future generations, we plot b^{ao} (solid curves) and b^{alo} (dashed curves) against R when $\lambda = \beta$ and Figure 4.The curves are drawn for two values of β, namely $\beta = 1/2$ (thin curves) and $\beta = 3/4$ (thick curves). As is evident from the figure, the benefits are larger when there is less altruism. Moreover, this effect is larger as β and R increases.



Steady state benefits with (dashed) and without (solid) altruism for $\beta = 1/2$ (thin) and $\beta = 3/4$ (thick).

5 Conclusion

At least some, and we believe many, questions in political economy call for an analysis that uses a dynamic setting. The literature, however, lacks analytical frameworks where voting and economic decision making both are rational and forward-looking and where the dynamic mechanisms are fully operative; in most of the existing papers that respect rationality of all agents, the dynamics are either mutated by preferences that mimic myopia (under some conditions, logarithmic utility has this feature) or by a lack of dynamic decision variables that call for forward-looking (such as investment), or they are only possible to analyze using numerical methods. In this paper, we have constructed a positive model where redistribution and social insurance take place in a dynamic setting: the taxation underlying these expenditures distorts human capital accumulation. In our economy, current taxation thus sets off nontrivial political and economic dynamics and the agents take these dynamics into account when making decisions. We are thus able to conduct both exercises of "comparative statics"—analyzing the effects of primitives on long-run outcomes—and of "comparative dynamics"—analyzing the effects of primitives on short-run outcomes. We focus on the case where the political system—a setting where policy decisions are made through probabilistic voting—cannot, either formally or through reputation effects, commit to future policy decisions.

The model is analytically tractable, making the mechanisms that determine the dynamics transparent. Relative to "constrained-optimal" allocations, i.e., allocations which would result if a planner could set all taxes and transfers at time 0 to maximize some weighted utility of all agents, we find that the political system has a stabilizing role. In particular, the lack of commitment makes (optimal) oscillatory responses to disturbances become weaker or disappear. This effect can be qualitatively important: for a large range of parameter values, the constrained optimum prescribes limit cycles, whereas the political equilibrium never does.

We have identified two opposing mechanisms underlying the determination of redistribution and of how it evolves over time. One of these mechanisms is the *constituency effect*. According to this effect, an increase in redistribution induces a change in individual actions that increases future demand for redistribution. Thus, the constituency effect tends to induce positive feedback, and therefore persistence, in the size of government.

We believe that such an effect may be at work also in other areas of government activity. For example, an expansion of government employment may induce educational choices suited for government jobs and therefore make future reductions in investments politically costly, an effect stressed by, among others Lindbeck (1995). In this paper, we have used probabilistic voting as the political aggregator of preferences. This voting model provides a smooth mapping from the distribution of preferences to political outcomes. This means that constituency effect is smooth, operating over a large range of the domain of the state-variable. In particular, in this paper, the constituency effect generates persistence in the level of redistribution, but eventually redistribution always returns to a unique steady state.

In contrast to the smooth operation of probabilistic voting, Downsian majority voting may lead to abrupt changes in policy when the preferences of the median voter changes. Therefore, the constituency effect can be stronger under majority voting than under probabilistic voting, leading not only to persistence but to complete hysteresis as in Hassler et al. (2003), where a temporary shock to the demand for redistribution may lead to indefinitely high levels of redistribution. Our model therefore predicts that, *ceteris paribus*, countries with a political system closer in line with the smooth (discontinuous) preference aggregation of probabilistic voting (majority voting) should have weaker (stronger) policy persistence. Another difference between these institutions that follows from this logic and that we analyze in this paper is that majority voting can lead to expectational equilibria—beliefs that the government will (continue to) be large in the future can be self-fulfilling—whereas probabilistic voting cannot.

The second mechanism behind the dynamics of government that we identify is the *tax-base effect*. According to this effect, positive redistribution today leads to higher future costs of redistribution since higher levels of redistribution reduces investments and thereby shrinks the future size of the tax base. Since higher costs of redistribution reduce the attractiveness and therefore political viability of redistribution, the tax-base effect produces a negative feedback inducing oscillating dynamics. The tax-base effect is the only active channel underlying the constrained-optimal allocation.

We have showed that there are several factors that can strengthen the relative importance of the constituency and tax-base effects in our political equilibrium, thereby determining the extent to which equilibrium dynamics are persistent or oscillatory.

First, increased political influence of individuals behind the veil of ignorance, or by young agents in our terminology, tends to increase the relative importance of the costs of redistribution and thus of the tax-base effect. This is a natural consequence of the fact that conflicts of redistribution strengthen with age when individuals are exposed to idiosyncratic shocks. Individuals who ex-ante have coinciding interests on social insurance may later in life be divided into "losers" and "winners" from redistribution. As the political influence of the latter is diminished, the relative size of winners and losers become less important as, instead, the common ex-ante interest gets the political upper hand. Furthermore, the distortionary costs of redistribution are partly borne in the future, since current redistribution reduces the future size of the tax base. This is a greater concern for young individuals with a longer remaining lifetime. However, whenever the ex-post interest is politically preponderant, a case which we deem the most likely, dynamics are monotone and redistribution persistent.

Second, more concern about the welfare of future generations also strengthens the tax-base effect. When the old voters are altruistic vis-à-vis the young, they appreciate the ex-ante interest and not only their own ex-post interest. Therefore, altruism vis-à-vis future generations tends to generate less persistence; in fact, when the old place the same weight on their own utility as on that of their offspring, dynamics are always oscillatory.

Third, an ability to commit future levels of redistribution strengthens the tax-base effect. Since future benefits distorts current investment choices, increasing the current cost of redistribution, agents have an interest in curtailing future redistribution. This interest is particularly strong if current redistribution is chosen to be high. Therefore, commitment tends to induce a negative feedback and oscillating dynamics. In particular, when the discount factor on future cohorts equal the private intertemporal discount factor, dynamics are characterized by a unitary negative root, producing everlasting oscillations of constant amplitude.

Finally, we have found that higher levels of risk-aversion also strengthens the tax-base effect. In our model, we have separated the insurance value of redistribution from its distortive costs by assuming that individuals in need of redistribution do not make choices that are distorted by redistribution. In a more general setting, an increase in risk-aversion might also affect the marginal utility of unsuccessful agents whose investment decisions are sensitive to the amount of redistribution. In such a case, the constituency effect should also be strengthened by higher risk aversion, making the total effect on dynamics ambiguous.

In conclusion, we have identified several parameters that affect the dynamics of redistribution. Identifying the inherent dynamic patterns of government size in the data and tying them to observables—possibly using a model like the one we develop here—is a demanding task but one that we think may be a very profitable one in the not too distant future.

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6 Appendix

6.1 Proof of Lemma 1

Consider the Ramsey-problem as formulated in (5). Define the planner's period t felicity, i.e., the "weighted average felicity" across young and old agents at time t for $t \geq 1$, as

$$\begin{aligned} F(b_{t-1}, b_t, b_{t+1}) \equiv\ & \beta(1-\mu)\tilde{V}^{ow}(b_t, \tau_t) \\ & +\beta\mu\left(e^*_{t-1}\tilde{V}^{oes}(b_t, \tau_t) + \left(1-e^*_{t-1}\right)\tilde{V}^{oeu}(b_t, \tau_t)\right) \\ & +\lambda\mu\left(we^*_t + (1-e^*_t)b_t w - (e^*_t)^2 - \tau_t + \underline{w}\right) \\ & +\lambda(1-\mu)(a(b_t w - \tau_t) - (a-1)x), \end{aligned} \tag{20}$$

where $e^*_j = e(b_j, b_{j+1})$ is defined in (3), $\tau_t = \tau(b_t, b_{t+1}, 1 - e(b_{t-1}, b_t))$ is defined in (4) and the functions $\tilde{V}^i$ are defined in (2). Note that the function $F(b_{t-1}, b_t, b_{t+1})$ is additively separable in (b_{t-1}, b_t) and (b_t, b_{t+1}). More formally, there exist (linear-quadratic) functions G and H such that $F(b_{t-1}, b_t, b_{t+1}) = G(b_{t-1}, b_t) + H(b_t, b_{t+1})$, where

$$G(b_{t-1}, b_t) = -\frac{\mu w^2}{4}(((\beta+\lambda)(R+1) - 2\beta)b_t b_{t-1} + 2\beta b_{t-1})$$

and

$$\begin{aligned} H(b_t, b_{t+1}) =\ & \left(\frac{w^2\mu}{4}\right)\cdot[2((1+\beta)(\beta+\lambda)(R+1) - (1+2\beta)\beta - \lambda)b_t \\ & -\left((1+\beta)(\beta+\lambda)(R+1) - \lambda - 2\beta^2\right)b_t^2 \\ & +2\lambda\beta^2 b_{t+1} - \beta^2\lambda b_{t+1}^2 - \beta(\beta+\lambda)(R+1)b_t b_{t+1}\big] + Q, \end{aligned}$$

where the constant Q is given by

$$Q \equiv \frac{w^2}{4}\mu(1+\beta)(2\beta + \lambda(1-\beta)) + (\beta+\lambda)(\mu\underline{w} - Rx).$$

Define now $Y(b_t, b_{t+1}) \equiv \lambda G(b_t, b_{t+1}) + H(b_t, b_{t+1})$, where the expression for Y appears in the text (in Lemma 1). Using this and the definition of the function $F_0(u_0, b_0, b_1)$ from equation (18), the sequential problem (5) can be expressed as

$$\begin{aligned} W(u_0) &= \max_{\{b_t\}_{t=0}^{\infty}}\left\{F_0(b_0, b_1, u_0) + \sum_{t=1}^{\infty}\lambda^t F(b_{t-1}, b_t, b_{t+1})\right\} \\ &= \max_{\{b_t\}_{t=0}^{\infty}}\left\{F_0(b_0, b_1, u_0) - H(b_0, b_1) + \sum_{t=0}^{\infty}\lambda^t(\lambda G(b_t, b_{t+1}) + H(b_t, b_{t+1}))\right\} \\ &= \max_{\{b_t\}_{t=0}^{\infty}}\left\{Y_0(b_0, u_0) + \sum_{t=0}^{\infty}\lambda^t Y(b_t, b_{t+1})\right\}, \end{aligned} \tag{21}$$

where $Y_0 \equiv F_0(b_0, b_1, u_0) - H(b_0, b_1)$ is given by

$$\begin{aligned} Y_0(b_0, u_0) &= \frac{\mu w}{2}\left(((\beta+\lambda)(R+1)-2\beta)\left(1-u_0-\frac{w}{2}(1+\beta)\right)+w\beta^2\right)b_0 \\ &+\frac{\mu w^2}{4}\beta((\beta+\lambda)(R+1)-2\beta)b_0^2+\mu\beta w\left(1-u_0-\frac{w}{2}(1+\beta)\right). \end{aligned}$$

Define now the value function V for the continuation problem, i.e., for $t \geq 1$ in (21), via the functional equation

$$V(b_t) = \max_{b_{t+1}\in[0,1]}\{Y(b_t, b_{t+1}) + \lambda V(b_{t+1})\}.$$

Since Y is bounded by the fact that $b \in [0,1]$ and since $0 \leq \lambda < 1$, this is a contraction mapping with a unique solution, which must also be the solution to the sequential continuation problem (Theorem 4.3 in Stokey and Lucas, 1989). Given V, it follows from the sequential formulation (21) that benefits in the initial period can be determined from the static problem

$$W(u_0) = \max_{b_0\in[0,1]}\{Y_0(u_0, b_0) + V(b_0)\}.$$

6.2 Proof of Proposition 1

Start[17] by guessing that the value function, $V(b_t)$, is linear-quadratic in b, i.e., that

$$V(b_t) = \frac{\mu w^2}{4}\left(A_0 + A_1 b_t + A_2 b_t^2\right), \tag{22}$$

where

$$\begin{aligned} A_2 &\equiv -\beta(1-\beta+2R) \\ A_1 &\equiv 2\beta(2R-\beta) \\ A_0 &\equiv \frac{\left(64R^2-9\beta^2\right)\beta^2}{16(1-\beta)(2R+1)}+\frac{4}{(1-\beta)\mu w^2}\tilde{Q} \\ \tilde{Q} &\equiv \frac{w^2}{4}\mu(1+\beta)\beta(3-\beta)+2\beta(\mu\underline{w}-Rx). \end{aligned}$$

When $\lambda = \beta$, the current felicity Y in the recursive formulation (8) simplifies to

$$\begin{aligned} Y(b_t, b_{t+1}) &= \left(\frac{\mu w^2}{4}\right)\cdot\{2\beta(2R(1+\beta)-\beta)b_t \\ &\quad -\beta(1+2(1+\beta)R)b_t^2 - 2\beta^2(1+2R)b_t b_{t+1} \\ &\quad + 2\beta^3 b_{t+1} - \beta^3 b_{t+1}^2] + \tilde{Q}. \end{aligned}$$

Note that the right-hand side in the bellman equation is concave in b_{t+1} since the coefficient on b_{t+1}^2 is negative $(0 > -\beta^2(1+2R))$. The first-order condition $0 \geq \frac{d}{db_{t+1}}\{Y(b_t, b_{t+1}) + \beta V(b_{t+1})\}$ is then sufficient for optimality and is given by

$$b_{t+1} = f(b_t) = b^* - (b_t - b^*),$$

[17] An alternative strategy for this proof would have been to guess a linear-quadratic functional form for the value function, with *unknown* parameters A_0, A_1, and A_2. We would then assume that the first-order condition be necessary and sufficient for optimality, compute the optimal policy $f(b)$ as a function of the A_i's. Finally, we would compute the A_i's by equating coefficients through the Bellman equation $V(b) = Y(b, f(b)) + \beta V(f(b))$.

where $b^* = R/(1+2R)$. It is now straightforward to verify that $V(b) = Y(b, f(b)) + \beta V(f(b))$. This proves that V is a fixed-point of the functional mapping $\Gamma(v) = \max_{b' \in [0,1]} \{Y(b, b') + \lambda v(b')\}$. Since Γ is a contraction mapping, f must be the unique optimal policy.

Consider now the first-period problem (7). Inserting the expression for Y_0 and V and simplifying yields

$$\begin{aligned} & \max_{b_0 \in [0,1]} \{Y_0(u_0, b_0) + V(b_0)\} \\ = \; & \mu w \beta R \left(1 - u_0 + \frac{w}{2}(1-\beta)\right) b_0 - \frac{\mu w^2}{4} \beta (1-\beta)(2R+1) b_0^2 + \hat{Q}, \end{aligned}$$

where $\hat{Q}$ is a constant. This problem is convex, since the coefficient on b_0^2 is negative. The first-order condition $\frac{d}{db_0}\{Y_0(u_0, b_0) + V(b_0)\} \leq 0$ yields

$$b_0 = \min\left\{\frac{R}{(2R+1)}\left(1 + \frac{(1-u_0)}{\frac{w}{2}(1-\beta)}\right), 1\right\},$$

which is monotone falling in u_0.

6.3 Proof of Proposition 2

In addition to what is stated in the text, the constraint $b_t \in [0,1]$ remains to be verified. Without risk aversion, $B(u_t) = \frac{2}{w(2+\beta)}(u_t - \underline{u})$ yields an interior solution for any $u_t \geq \underline{u}$. However, if $u_t < \underline{u}$, the restriction $b_t \geq 0$ will bind. Thus, we modify the guess to $B(u_t) = \frac{2}{w(2+\beta)}(u_t - \underline{u})$ if $u_t \geq \underline{u}$ and 0 otherwise. The new guess will still maximize the political objective. To see this, note that (13) remains unchanged under the new guess of $B(u_t)$, since the guess is modified only for values of u_{t+1} that are infeasible (this follows directly from the fact that, for any feasible pair (b_t, b_{t+1}), $u_{t+1} = 1 - \frac{1+\beta-(b_t+\beta b_{t+1})}{2} w \geq 1 - \frac{1}{2}(1+\beta) w$). For the same reason, $U(b_t)$ in (14) is unaffected.

6.4 Proof of Proposition 3

When $R = 0$, and for $\omega \in [0,1]$, the political objective function can be written as

$$\begin{aligned} V \; = \; & \mu(1-u_t) w + \mu u_t b_t w + (1-\mu) b_t w \\ & + \omega(1-\mu)(b_t + \beta b_{t+1}) w \\ & + \omega \mu e_t (1+\beta) w + \omega \mu (1-e_t)(b_t + \beta b_{t+1}) w \\ & - \omega \mu (e_t)^2 \\ & - (1+\omega) \tau_t - \omega \beta \tau_{t+1}. \end{aligned}$$

Here, it is convenient to guess that $b_t = \alpha_1 (u_t - \underline{u})$ for a coefficient α_1 yet to be determined. We first solve for $U(b_t)$ to obtain

$$U(b_t) = \underline{u} + \frac{w}{2 - \beta \alpha_1 w} b_t,$$

delivering $B(U(b_t)) = \frac{w\alpha_1}{2-\beta\alpha_1 w} b_t$. Defining $Z = \frac{\alpha_1 w}{2-\beta\alpha_1 w}$, this implies $b_{t+1} = Z b_t$ and $b_{t+2} = Z^2 b_t$ and

$$\begin{aligned} e_t &= (1+\beta-(1+\beta Z) b_t) \frac{w}{2} \\ \tau_t &= \left(1 - \frac{\mu}{2}\left(1 - u_t + (1+\beta-(1+\beta Z) b_t) \frac{w}{2}\right)\right) b_t w \\ \tau_{t+1} &= \left(1 - \frac{\mu w}{4}(2(1+\beta) - (1 + Z(1+\beta+\beta Z)) b_t)\right) Z b_t w. \end{aligned}$$

We now note that

$$\begin{aligned}
\frac{db_{t+1}}{db_t} &= Z, \frac{db_{t+2}}{db_t} = Z^2, \frac{de_t}{db_t} = -\left(1+\beta Z\right)\frac{w}{2} \\
\frac{d\tau_t}{db_t} &\equiv T_0 = \left(2 - \mu\left(1 - u_t - w\left(1+\beta Z\right)b_t + \frac{w\left(1+\beta\right)}{2}\right)\right)\frac{w}{2} \\
\frac{d\tau_{t+1}}{db_t} &\equiv T_1 = \frac{1}{2}\mu w^2 Z\left(1+Z\right)\left(1+\beta Z\right)b_t + wZ - \mu w\frac{1}{2}w\left(1+\beta\right)Z.
\end{aligned}$$

The first-order condition for maximizing the political objective is then

$$0 = \mu u_t w + \left(1 + \omega\left(1+\beta Z\right)\right)\left(1-\mu\right)w + \omega\mu\left(1-e_t\right)\left(1+\beta Z\right)w - \left(1+\omega\right)T_0 - \omega\beta T_1$$

yielding

$$b_t = \frac{1}{w}\frac{1-\omega}{1+Z\left(\beta+\omega\beta Z\left(1+\beta Z\right)\right)}\left(u_t - \underline{u}\right)$$

verifying the guess, provided

$$\alpha_1 = \frac{1}{w}\frac{1-\omega}{1+Z\left(\beta+\omega\beta Z\left(1+\beta Z\right)\right)}.$$

From the definition of Z we obtain $\alpha_1 = \frac{2Z}{(1+\beta Z)w}$. Thus,

$$\frac{2Z}{\left(1+\beta Z\right)w} = \frac{1}{w}\frac{1-\omega}{1+Z\left(\beta+\omega\beta Z\left(1+\beta Z\right)\right)}, \tag{23}$$

which can be rewritten

$$\left(\omega\beta Z^2 + 1\right)Z = \frac{1-\omega}{2}.$$

The left-hand side is monotonically increasing from zero as Z increases from zero. Furthermore, an increase in ω increases the left-hand side while it reduces the right-hand side. The solution in terms of Z therefore falls. Similarly, an increase in β increases the left-hand side, provided $\omega > 0$, implying that that solution in terms of Z falls, unless it is zero. Since α_1 is monotonically increasing in Z, we have established that α_1 is decreasing in ω and β. From this it also follows that the constraint $B\left(u\right) \in [0,1]$ is satisfied for any $u \geq \underline{u}$ and we set $B\left(u\right) = 0$ for $u < \underline{u}$, which, as in the previous proof, neither affects the objective function nor $U\left(b_t\right)$.

6.5 Proof of Proposition 4

As above, guess $B\left(u_t\right) = \alpha_0 + \alpha_1 u_t$. Then

$$B\left(U\left(b_t\right)\right) = \alpha_0 + \alpha_1\frac{2-\left(1+\beta\left(1-\alpha_0\right)\right)w}{2-\beta\alpha_1 w} + b_t\frac{\alpha_1 w}{2-\beta\alpha_1 w}. \tag{24}$$

Using this in the first-order condition for maximizing (11) over b_t and solving for b_t yields

$$b_t = \frac{\left(2-\alpha_1 w\beta\right)R}{w\left(1+R\right)} - \frac{1}{w} + \frac{1+\beta\left(1-\alpha_0\right)}{2} + \frac{1}{2}\frac{\left(1-R\right)\left(2-\beta\alpha_1 w\right)}{\left(1+R\right)w}u_t,$$

which is linear, as conjectured. We thus need to set

$$\begin{aligned}\alpha_0 &= -\frac{2-w(1+\beta)}{w(2(1+R)+\beta(1-R))} \\ &\quad +R\frac{(\beta(1-\beta)+2)w+8-2(2-\beta)}{(2(1+R)+\beta(1-R))w(2+\beta)} \\ \alpha_1 &= \frac{2(1-R)}{w(2+\beta+R(2-\beta))}=\frac{\rho}{w}.\end{aligned}$$

Noting that $B(U(b_t))$ and $U(B(u))$ have a fixed point, respectively, at

$$\begin{aligned}b^{ao} &= R\frac{4(1+\beta)}{(2+\beta)(1+3R)}, \\ \underline{u}^{ao} &= 1-\frac{w}{2}(1+\beta)(1-b^{ao})\end{aligned}$$

we can write

$$\begin{aligned}B(u_t) &= \alpha_0+\alpha_1 u_t \\ &= b^{ao}+\frac{\rho}{w}(u_t-\underline{u}^{ao}).\end{aligned}$$

Next we need to prove that if $R<R_{\max}$ then the constraint $b\leq\bar{b}$ is never binding in equilibrium. Clearly, since $B(u)$ is linear, its maximum is attained at either $u=0$ or $u=1$. Hence, if $\bar{b}\geq\phi(R)\equiv\max\{B(0;R),B(1;R)\}$, the constraint does not bind. Note that $\bar{b}$ is independent of R and that $\phi'(R)>0$ since algebraic manipulations show yield $\frac{\partial B(0;R)}{\partial R},\frac{\partial B(1;R)}{\partial R}>0$. Hence, $\phi(.)$ is a one-to-one mapping and admits use of the inverse function $\phi^{-1}(.)$ so there must exist an R such that $\phi(.)\leq\bar{b}$. Now, we must show that $\bar{b}\geq\phi(R)$ for all $R\geq 0$. For this purpose we consider $\phi(0)=\alpha_0+\alpha_1$ since B is upward-sloping when $R=0$. We focus on the case when $w=1$. In that case, $\phi(0)=\frac{1+\beta}{2+\beta}$ while $\bar{b}=\frac{1}{\mu\beta}\left(\sqrt{\left((\mu+2)^2+4\mu\beta\right)}-(\mu+2)\right)$, which is decreasing in μ.[18] We therefore require μ to be sufficiently small, i.e., specifically that

$$\mu\leq 4\frac{2+\beta}{4+7\beta+4\beta^2+\beta^3}\in[3/4,2].$$

Note also that when $\beta\leq 0.65$, this condition is satisfied for any $\mu\in[0,1]$. We also need to make sure that oscillations can occur, i.e., that $R>1$. In that case, we first note that $\bar{b}$ is independent of R and

$$\phi(1)=\frac{1+\beta}{2+\beta},$$

implying that for sufficiently large μ, $\bar{b}\geq\phi(1)$.

To prove that benefits are strictly positive after at most one period, observe that, in equilibrium, the law of motion $b_{t+1}=B^{ao}(U^{ao}(b_t))$, conditioned on $u_{t+1}\geq\underline{u}^{ao}-\frac{w}{\rho}b^{ao}$, can be written

$$\begin{aligned}b_{t+1} &= b^{ao}+\frac{1}{2}\frac{1-R}{1+R}(b_t-b^{ao}) \\ &= b^{ao}\frac{1}{2}\frac{1+3R}{1+R}+\frac{1}{2}\frac{1-R}{1+R}b_t.\end{aligned}$$

[18] The derivative is

$$\frac{\partial\bar{b}}{\partial\mu}=-2\frac{(2+\mu+\mu\beta)-\sqrt{(2+\mu+\mu\beta)^2-\mu^2\beta(2+\beta)}}{\mu^2\beta\sqrt{(2+\mu)^2+4\mu\beta}}<0.$$

Clearly, whenever $\frac{1}{2}\frac{1-R}{1+R} > 0$, $b_t \geq 0$ implies $b_{t+1} \geq 0$, since $b^{ao} \geq 0$, implying that $u_{t+1} \geq \underline{u}^{ao} - \frac{w}{\rho} b^{ao}$. Consider the other case, i.e., consider $\frac{1}{2}\frac{1-R}{1+R} \leq 0$. Then, since $b_t \leq 1$, we have

$$\begin{aligned} b_{t+1} &\geq b^{ao}\frac{1}{2}\frac{1+3R}{1+R} + \frac{1}{2}\frac{1-R}{1+R} \\ &= \frac{1}{2}\frac{R(2+3\beta)}{(2+\beta)(1+R)} + \frac{1}{2(1+R)} > 0. \end{aligned}$$

Whenever $b_t \geq 0$, then $b_{t+1} > 0$ for any $t \geq 0$. Since we assume that u_0 is arbitrary, we cannot rule out that the constraint that $b_0 \geq 0$ binds.

6.6 Proof of Proposition 6

The weighted average felicity $F(u_t, b_t, b_{t+1})$ is

$$\begin{aligned} F(u_t, b_t, b_{t+1}) &= \beta(\mu(1-u_t)w + \mu u_t b_t w) \\ &\quad + (\beta+\lambda)((1-\mu)b_t w + R b_t w) \\ &\quad + \lambda\left(\mu e(b_t, b_{t+1})w + \mu(1-e(b_t, b_{t+1}))b_t w - \mu e(b_t, b_{t+1})^2\right) \\ &\quad - (\lambda+\beta)(1+R)\tau(u_t, b_t, b_{t+1}). \end{aligned}$$

The usual guess, $b_t = \alpha_0 + \alpha_1 u_t$, together with the equation for optimal effort choice yields $b_{t+1} = X + Zb_t$, implying $b^{alo} = \frac{X}{1-Z}$ and $\underline{u}^{alo} = 1 - e\left(\frac{X}{1-Z}, \frac{X}{1-Z}\right)$ where, as above, $X \equiv \alpha_0 + \alpha_1 \frac{2-(1+\beta(1-\alpha_0))w}{2-\beta\alpha_1 w}$ and $Z \equiv \frac{\alpha_1 w}{2-\beta\alpha_1 w}$, which implies $u_{t+1} = 1 - (1+\beta(1-X) \; - (1+\beta Z)b_t)\frac{w}{2}$. The problem admits a recursive formulation of the following type:

$$\begin{aligned} W(u_t) &= \max_{b_t\in[0,\bar{b}]} \{F(u_t, b_t, b_{t+1}) + \lambda W(u_{t+1})\}, \qquad (25) \\ \text{s.t. } b_{t+1} &= X + Zb_t, \\ u_{t+1} &= 1 - (1+\beta(1-X) - (1+\beta Z)b_t)\frac{w}{2}. \end{aligned}$$

Given the guess, the first-order condition for maximizing the RHS of the Bellman equation is

$$\frac{\partial F}{\partial b_t} + \frac{\partial F}{\partial b_{t+t}}Z + \lambda W\prime(u_{t+1})(1+\beta Z)\frac{w}{2} = 0,$$

where

$$\begin{aligned} \frac{\partial F}{\partial b_t} &= \beta\mu u_t w + (\beta+\lambda)((1-\mu)w + Rw) + \lambda(\mu(1-e_t)w) \\ &\quad - (\lambda+\beta)(1+R)\frac{\partial \tau}{\partial b_t} + \lambda\mu(w - b_t w - 2e_t)\frac{\partial e_t}{\partial b_t}, \\ \frac{\partial F}{\partial b_{t+t}} &= -(\lambda+\beta)(1+R)\frac{\partial \tau}{\partial b_{t+t}} + \lambda\mu(w - wb_t - 2e_t)\frac{\partial e_t}{\partial b_{t+1}}. \end{aligned}$$

We also know that

$$\begin{aligned} \frac{\partial e_t}{\partial b_t} &= -\frac{w}{2}, \frac{\partial e_t}{\partial b_{t+1}} = -\beta\frac{w}{2} \\ \frac{\partial \tau}{\partial b_t} &= \left(1 - \frac{\mu}{2}\left(1 - u_t + (1+\beta)\frac{w}{2} - (b_t + \beta(X + Zb_t))\frac{w}{2}\right)\right)w + \mu b_t \frac{w^2}{4} \\ \frac{\partial \tau}{\partial b_{t+1}} &= \frac{1}{4}\beta\mu w^2 b_t. \end{aligned}$$

Using the envelope condition and the fact that $\frac{\partial \tau(u_t, b_t, b_{t+1})}{\partial u_t} = \frac{1}{2}\mu b_t w$, we obtain

$$\begin{aligned} W'(u_t) &= \beta(-\mu w + \mu b_t w) - (\lambda+\beta)(1+R)\frac{\partial \tau(u_t, b_t, b_{t+1})}{\partial u_t} \\ &= \left(\frac{\beta-\lambda}{2} - \frac{\lambda+\beta}{2}R\right)\mu w b_t - \beta\mu w, \end{aligned}$$

implying that we can write the first-order condition as

$$\begin{aligned} &(\beta\mu u_t + (\beta+\lambda)(1-\mu+R) + \lambda\mu(1-e_t))\, w \\ &- (\lambda+\beta)(1+R)\frac{\partial\tau}{\partial b_t} - \lambda\mu(w - b_t w - 2e_t)\frac{w}{2} \\ &-Z(\lambda+\beta)(1+R)\frac{\partial\tau}{\partial b_{t+1}} - Z\lambda\mu(w - wb_t - 2e_t)\beta\frac{w}{2} \\ &+\lambda\left(\left(\frac{\beta-\lambda}{2} - \frac{\lambda+\beta}{2}R\right)\mu w b_t - \beta\mu w\right)(1+\beta Z)\frac{w}{2} \\ = \;& 0. \end{aligned}$$

Collecting terms and using $e_t = (1+\beta(1-X) \;\; - (1+\beta Z)\, b_t)\,\frac{w}{2}$ yields

$$\begin{aligned} 0 &= \frac{1}{2}\mu w^2(1+Z\beta)\left(\frac{1}{2}\lambda(\beta-\lambda) - \beta(1+\lambda Z) - R(\beta+\lambda)\left(1+\frac{1}{2}\lambda\right)\right) b_t \\ &\quad + \frac{\mu w}{2}(\lambda+\beta)\left(\frac{\beta-\lambda}{\lambda+\beta} - R\right) u_t + C, \end{aligned} \tag{26}$$

where

$$\begin{aligned} C &= \left(1+\frac{1}{2}(1+\beta(1-X))\, w\right)\mu\frac{w}{2}(\beta+\lambda) R - (\beta - \lambda(1-w))\,\mu\frac{w}{2} \\ &\quad -\frac{1}{2}\lambda\beta\mu w^2(1+Z(1+\beta)) + \left(Z\lambda\beta + \frac{\beta+\lambda}{2}\right)\frac{\mu}{2}(1+\beta(1-X))\, w^2. \end{aligned}$$

The guess $b_t = \alpha_0 + \alpha_1 u_t$ and the FOC (26) can be used to solve, by equating coefficients, for α_0 and α_1. Next, using the definition $Z \equiv \frac{\alpha_1 w}{2-\beta\alpha_1 w}$ implies that Z must satisfy

$$Q^{alo}(Z;R) \equiv Z\left(\beta(1+\lambda Z) - \lambda\frac{\beta-\lambda}{2} + R\frac{\beta+\lambda}{2}(2+\lambda)\right) = \frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta} - R\right). \tag{27}$$

and Z belongs to the unit interval. We then obtain equilibrium Z as the unique stable solution to the quadratic equation $Q^{alo}(Z;R) - \frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta} - R\right) = 0$. This is given by

$$\begin{aligned} Z &= \frac{-\left(\beta - \lambda\frac{\beta-\lambda}{2} + R\frac{\beta+\lambda}{2}(2+\lambda)\right)}{2\lambda\beta} \\ &\quad + \frac{\sqrt{\left(\beta - \lambda\frac{\beta-\lambda}{2} + R\frac{\beta+\lambda}{2}(2+\lambda)\right)^2 + 4\lambda\beta\frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta} - R\right)}}{2\lambda\beta}. \end{aligned}$$

The solution is always real, since the discriminant increases in R and equals $\frac{1}{4}\left(\lambda\left(\beta-\lambda\right)+2\beta\right)^{2}>0$ when $R=0$.[19] To see that the other root is smaller than -1, we note that when $R=0$ it is given by $-\frac{1}{\lambda}<-1$ and that it decreases in R. Straightforward algebra then shows that for any $R\geq 0$, there is a solution to (27) in $\left[-\frac{1}{2+\lambda},\frac{1}{2}\right]$, which decreases in R and λ, and that $Z<(\geq)0$ if $R>(\leq)\frac{\beta-\lambda}{\lambda+\beta}$.

Finally, from the definition of α_0 and α_1 we have $\alpha_0=\frac{Xw-Z(2-w(1+\beta))}{w(1+\beta Z)}$. Using this in the solution for the constant in (26), we obtain

$$\left(Xw-Z\left(2-w\left(1+\beta\right)\right)\right)\frac{1}{2}\mu w\left(\frac{1}{2}\lambda\left(\beta-\lambda\right)-\beta\left(1+\lambda Z\right)-R\left(\beta+\lambda\right)\left(1+\frac{1}{2}\lambda\right)\right)=-C,$$

from which we find

$$\begin{aligned} X &= -\frac{-\left(\left(2+\left(1+\beta\right)w\right)\left(\beta+\lambda\right)\right)R}{w\left(\left(2+\lambda+\beta\right)\left(\beta+\lambda\right)R+\lambda^{2}+\beta\left(2+\beta\right)+2Z\lambda\beta\left(1+\beta\right)\right)} \\ &\quad +\frac{\left(Z+\frac{1}{2}\left(\frac{2-\lambda}{\lambda}+\frac{\lambda}{\beta}\right)+\frac{1}{2}\frac{2+\lambda}{\lambda\beta}\left(\beta+\lambda\right)R\right)2\left(2-w\left(1+\beta\right)\right)\lambda\beta Z}{w\left(\left(2+\lambda+\beta\right)\left(\beta+\lambda\right)R+\lambda^{2}+\beta\left(2+\beta\right)+2Z\lambda\beta\left(1+\beta\right)\right)} \\ &\quad -\frac{\left(\beta-\lambda\right)\left(2-w\left(1+\beta\right)\right)}{w\left(\left(2+\lambda+\beta\right)\left(\beta+\lambda\right)R+\lambda^{2}+\beta\left(2+\beta\right)+2Z\lambda\beta\left(1+\beta\right)\right)}. \end{aligned}$$

To see the derivations of the claims on Z in more detail, we note

$$\begin{aligned} Q^{alo}\left(0;R\right) &= 0, \\ \frac{dQ^{alo}\left(Z;R\right)}{dZ} &= R\frac{\beta+\lambda}{2}\left(2+\lambda\right)-\frac{1}{2}\lambda\left(\beta-\lambda\right)+\beta\left(1+2\lambda Z\right), \\ Q^{alo}\left(\frac{1}{2};R\right)-\frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta}-R\right) &= \left(\beta+\lambda\right)\left(1+\frac{1}{4}\lambda\right)R+\frac{1}{2}\lambda\left(1+\frac{1}{2}\lambda\right) \\ &\geq 0, \end{aligned}$$

with equality only if $\lambda=R=0$. Therefore, when $R=\frac{\beta-\lambda}{\lambda+\beta}$, it must be that $Z=0$ and that $b^{alo}=X=\frac{1+\beta}{\beta(2+\beta)-\lambda}\left(\beta-\lambda\right)$.

In addition, $\frac{dQ^{alo}(Z;R)}{dZ}>0$ if $Z>0$ and $\frac{\beta-\lambda}{\lambda+\beta}>R$. Thus, $\frac{\beta-\lambda}{\lambda+\beta}>R$ implies $Z\in\left(0,\frac{1}{2}\right]$.

Furthermore, since both

$$\frac{d\left(Q^{alo}\left(Z;R\right)-\frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta}-R\right)\right)}{d\lambda} = \lambda+2\beta Z+\frac{1}{2}\left(1-\beta\right)+R\left(\frac{3}{2}+\lambda+\frac{1}{2}\beta\right)$$

and

$$\frac{d\left(Q^{alo}\left(Z;R\right)-\frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta}-R\right)\right)}{dR} = \left(\beta+\lambda\right)\frac{1}{2}\left(1+Z\left(2+\lambda\right)\right)$$

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[19] The discriminant is

$$\left(\beta-\lambda\frac{\beta-\lambda}{2}+R\frac{\beta+\lambda}{2}\left(2+\lambda\right)\right)^{2}+4\lambda\beta\frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta}-R\right)$$

with a derivative

$$\frac{1}{2}\left(\beta+\lambda\right)\left(\left(\lambda+\beta\right)\left(4\left(1+\lambda\right)+\lambda^{2}\right)R+4\beta\left(1-\lambda\right)+\lambda^{2}\left(2-\left(\beta-\lambda\right)\right)\right).$$

are larger than zero when $Z > 0$, equilibrium Z is decreasing in λ and R.

Next, consider the case when $\frac{\beta-\lambda}{\lambda+\beta} < R$. Clearly, now equilibrium Z is negative. Furthermore, for any $Z > -\frac{1}{2+\lambda}$,

$$\begin{aligned}
\frac{d\left(Q^{alo}(Z;R) - \frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta} - R\right)\right)}{dZ} &= R\frac{\beta+\lambda}{2}(2+\lambda) - \frac{1}{2}\lambda(\beta-\lambda) + \beta(1+2\lambda Z) \\
&> \frac{\beta-\lambda}{\lambda+\beta}\frac{\beta+\lambda}{2}(2+\lambda) - \frac{1}{2}\lambda(\beta-\lambda) + \beta(1+2\lambda Z) \\
&= 2\beta(1+Z\lambda) - \lambda > 0,
\end{aligned}$$

and since

$$\begin{aligned}
&\left(Q^{alo}\left(-\frac{1}{2+\lambda};R\right) - \frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta} - R\right)\right) \\
&= -\left(\frac{1}{2+\lambda}\beta + \frac{1}{2}(\beta-\lambda)\right)\left(1 - \frac{\lambda}{2+\lambda}\right) \le 0,
\end{aligned}$$

equilibrium $Z \ge -\frac{1}{2+\lambda}$. Finally,

$$\begin{aligned}
\frac{d\left(Q^{alo}(Z;R) - \frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta} - R\right)\right)}{d\lambda} &= \lambda + 2\beta Z + \frac{1}{2}(1-\beta) + R\left(\frac{3}{2} + \lambda + \frac{1}{2}\beta\right) \\
&\ge \left(\lambda - 2\beta\frac{1}{2+\lambda} + \frac{1}{2}(1-\beta) + \frac{\beta-\lambda}{\lambda+\beta}\left(\frac{3}{2} + \lambda + \frac{1}{2}\beta\right)\right) > 0
\end{aligned}$$

and

$$\frac{d\left(Q^{alo}(Z;R) - \frac{\lambda+\beta}{2}\left(\frac{\beta-\lambda}{\lambda+\beta} - R\right)\right)}{dR} = (\beta+\lambda)\frac{1}{2}(1+Z(2+\lambda)),$$

implying that equilibrium Z is decreasing in R and in λ.