ON THE FREQUENCY OF RETENTION DECISIONS: A USEFUL TOOL FOR UNDERSTANDING POLITICAL BUSINESS CYCLES?*

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VERY PRELIMINARY - COMMENTS WELCOME

Abstract

The Political Business Cycles literature studies the effects of elections on the real economy and on policy choices. Most studies in this literature assume that the incumbent policymaker can only affect election outcomes with actions at election periods (this explains the cycles). This paper explores alternative explanations in a model without this assumption and shows that such assumption is not necessary for explaining the cycles. If this assumption is removed, results from comparative–statics exercises are different. Moreover, in contrast with previous literature, this paper proposes empirical implications that are conditional on the beliefs about the incumbent policymaker's ability.

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1 Introduction

Empirical evidence (see, for example, Persson and Tabellini [2003]) indicates that (to some extent) both the real economy and policy choices are affected by the proximity of elections (the so–called Political Business and Budget Cycles). What explains these cycles? There are two key components in the answers provided in the recent literature (for a review of this literature see Drazen [2000] and Shi and Svensson [2003]). Firstly, policymakers want to increase their reelection probability. Secondly, only their actions at election periods can affect their reelection probabilities. This second assumption seems extreme and is adopted mainly for tractability reasons. Is it possible to explain Political Business or Budget Cycles without this assumption? Would the predictions in the Political Business and Budget Cycles literature be affected if an incumbent policymaker can affect election outcomes with his actions at any period? The purpose of this paper is to provide an answer to these questions. Alternative explanations for these cycles are provided here showing that such assumption is not necessary for explaining Political Business (or Budget) Cycles. Moreover, eliminating this extreme assumption produces important changes in the predictions of the model. Thus, the paper has a second important contribution to previous studies of Political Business Cycles that use models of career concerns: the results presented in this paper are conditional on the beliefs about the incumbent's ability producing a new set of empirically testable implications (for empirical analysis, past performance could be used as an indication of the beliefs about the incumbent's ability).[1] Moreover, the roles the beliefs about the incumbent's ability play in this paper are different from the ones they play in the adverse–selection literature.

As discussed by Martinez [2003], studying the incentives a policymaker has because he wants to win elections is not (fundamentally) different from studying the incentives any worker has if he

[1] Ability or talent refers to a persistent characteristic of a policymaker. Competence refers to a realization of this ability in a certain period. A policymaker's ability determines the distribution for his competences.

wants to keep his job. The model presented here could be reinterpreted to study the way in which the importance of moral hazard problems depends on the proximity of the retention decision in any principal–agent relationship (if the agent is concerned about being fired). Let us consider, for example, the end of a contract that commits the principal to work with a certain agent. If the contract ends, the principal can choose to replace this agent with a new one.

The model presented here is an extension of the model of career concerns presented by Martinez [2003]. The main difference is that here voters decide whether they want to reelect the incumbent every two periods (i.e., after two output observations). The incumbent's ability is unknown and may be learned through the quantity of output delivered every period. Thus, voters try to learn the incumbent's ability and vote according to this. If the incumbent is not reelected, a challenger of unknown ability is appointed. The incumbent decides his action by in part trying to manipulate the voters' beliefs. The incumbent's action can be interpreted as an unobservable variable (like effort as in most principal–agent models or stealing as in the models of career concerns presented by Persson and Tabellini [2000]) or as an observable variable in a model with uninformed voters (as in the model presented by Shi and Svensson [2002] where the incumbent manipulates fiscal policy producing Political Budget Cycles).

If the beliefs about the incumbent's ability depend on all the observations of his performances, it is necessary to analyze the links between these beliefs and the incumbent's optimal decisions. This is a truly dynamic problem since the incumbent's optimal decisions depend on the beliefs about his ability and these decisions influence the future equilibrium beliefs. In the existing literature on Political Business (or Budget) Cycles, dynamics are typically sidestepped. This paper studies these links providing a new set of empirically testable implications. The analysis of closed–form solutions is presented for a finite–horizon framework. An infinite–horizon framework is characterized as well using in part numerical methods (there is an important non–stationarity in this problem that makes an infinite–horizon analysis interesting).

Why are there Political Business (and Budget) Cycles if a policymaker can affect election results with his actions at every period? Firstly, the effectiveness of a policymaker's action in affecting election results is different at different periods. This effectiveness is endogenous in this paper (it depends on the way in which the incumbent's equilibrium actions change if the beliefs about his ability change) while in most of the previous literature it is assumed that only actions at election periods are effective. Secondly, in a period without elections, the incumbent knows that he can affect the voters' next–period posterior beliefs (and the election results) with his actions in the current period or in the next period. In deciding his action in the current period, the incumbent compares the marginal utility of his current action with the expected marginal utility of his action next period (the next–period equilibrium strategy is a function of the next–period beliefs about the incumbent's ability and these beliefs are unknown in the current period). In evaluating this expected marginal utility, it is important to consider the concavity of the incumbent's equilibrium strategy as a function of the beliefs.

As explained by Martinez [2003], in election periods, equilibrium strategies depend on the beliefs about the incumbent's ability. The election–period beliefs depend on previous beliefs. Consequently, in a period without elections, the expected relative effectiveness of the incumbent's actions and the expected concavity of his future equilibrium strategy depend on the beliefs about the incumbent's ability. Therefore, in a period without elections, the incumbent's action depends on the beliefs about his ability and the differences between actions in periods with and without elections (and the Political Business and Budget Cycles) also depend on these beliefs.

Given that a policymaker's strategy depends not only on the proximity of the elections but also on the beliefs about his ability, this should be taken into account for empirical analysis. Differences in policymakers' behavior for given beliefs about their ability are called here Political Business (or Budget) Cycles (let us note that the beliefs about a policymaker's ability may change while he is in office). The importance and the sign of these cycles depends on these beliefs.

In this framework, in order to understand comparative–statics exercises, it is crucial to understand the way in which changes in a parameter value change the "relative effectiveness" in altering election outcomes of the policymaker's actions in periods with and without elections. As explained before, this relative effectiveness is endogenous here while in the previous literature it is assumed that the incumbent's actions are not effective before the election period. Therefore, the predictions presented here differ markedly from the ones in previous studies. For example, in contrast with the result by Shi and Svensson [2002], a change in the per–period value a policymaker assigns to being in office has almost no effect in the (relative) importance of the cycles (if this change is not large enough to produce a large change in the relative effectiveness). Moreover, in the model presented here, the predictions of comparative–statics exercises are conditional on the beliefs about the incumbent's ability inviting to revisit the existing empirical literature.

The rest of this paper is structured as follows. Section II describes the model and defines equilibrium. Section III characterizes the results. Section IV presents comparative–statics exercises. Section V concludes and suggests possible extensions.

2 The Model

For expositional simplicity, an extension of the model presented by Martinez [2003] in Section IV (which, in turn, follows closely the one presented in Holmström [1999]) is discussed here. The implications of the assumptions in this framework are discussed by Martinez [2003]. The existence of conflicts among voters is not considered.[2, 3]

Voters lack the capacity to commit to an output–contingent reelection rule. The lack–of–

[2] This paper may be interpreted as considering situations where the decisive voter cares about future performance and not about ideology.

[3] The model presented here could be extended to include probabilistic voting as done by Shi and Svensson [2002].

commitment case is an interesting benchmark and voters' behavior is not linked to output–contingent contracts.[4]

The incumbent policymaker wants to produce more public goods for the voters because his ability is learned through the quantity of public goods he delivers and voters decide if they want to reelect him depending on their beliefs about his ability.

The ability level of policymaker i is denoted by $\bar{\eta}_{it}$. The amount of public good available is a stochastic function of the incumbent's ability and his effort level, a_t. In particular,

$$g_t = a_t + \bar{\eta}_{it} + \varepsilon_t,$$

where ε_t is a normally distributed random variable with expected value 0 and precision h_ε (the variance equals $\frac{1}{h_\varepsilon}$).

The voters' utility in each period depends on the production of public goods and it is denoted by $u(g_t)$. In particular, it is considered here that $u(g_t) = g_t$. A policymaker's utility is normalized to zero when he is not in office. The value of holding office for the incumbent is R and, in office, he may choose any non–negative effort level. There is a cost of effort, given by c, with $c' > 0$, $c'' > 0$, and $c'(0) = 0$.

The players (the voters and the incumbent) are ignorant of the incumbent's ability.[5,6] An

[4]Martinez [2003] discusses alternative voting models by Ashworth [2001], Banks and Sundaram [1990, 1993, and 1998], Ferejohn [1986], and Persson and Tabellini [2000].

[5]Other papers in the literature assume that the policymakers know their ability. Having the incumbent not know his ability allows us to consider situations where a policymaker in a new position may be ignorant of his ability when met with new tasks. This assumption also helps to understand situations where a policymaker's success does not only depend on his individual ability but also on the ability of others working with him.

[6]In adverse–selection models, a more talented incumbent may increase the reelection probability by deteriorating future possibilities (because doing this is a signal of high ability). This may not be the most appealing way of explaining Political Business (or Budget) Cycles.

incumbent is said to be more competent if he obtained more public goods with a given effort level (in a certain period). Thus, an incumbent's competence, η_t, is defined as the sum of his ability, $\bar{\eta}_t$, and ε_t, i.e.,

$$\eta_t \equiv \bar{\eta}_t + \varepsilon_t.$$

The prior distribution for the beliefs about a new incumbent's ability is normally distributed with mean x_0 and precision h_0. A policymaker's ability evolves as a random walk, $\bar{\eta}_{it+1} = \bar{\eta}_{it} + \beta_{it}$. β_{it} is assumed to be normally distributed with mean 0 and precision h_β. The precision of β_{it}, is chosen to avoid having tenure as a determinant of the players' decisions (the precision in the posterior distributions is always equal to the priors' one),

$$h_\beta = \frac{h_0^2 + h_0 h_\varepsilon}{h_\varepsilon}.$$

Thus, the mean is sufficient to characterize the beliefs and it is denoted by x.[7] Let x_{at} and x_{pt} denote the expected competence in the incumbent and the voters' beliefs, respectively. The precision in the belief about a policymaker's competence is equal to

$$H = \frac{h_0 h_\varepsilon}{h_\varepsilon + h_0}.$$

The mean in the posterior distribution is a weighted sum of the prior mean and the inferred competence. Let

$$\mu = \frac{h_0}{h_0 + h_\varepsilon} \tag{1}$$

[7] The tenure effect presented by Holmström [1999] is clear. With more observations of a policymaker's competences, the prior belief about his ability becomes more precise and the observation at the current period has less weight in the posterior. Given that the policymaker can only use his action to affect the observation at the current period, the incentives to exert effort in production are lower when a policymaker has been in office longer. Thus, the equilibrium effort level declines with tenure. The links between the beliefs about the incumbent's ability and the optimal decisions of the players are explored here.

denote the weight of the prior mean.

At the beginning of every period, the incumbent decides his effort level. This paper concentrates on symmetric equilibria. An incumbent's optimal effort level is a function of both x_{at} and x_{pt}, and depends also on the presence of elections. The equilibrium effort level is denoted by $\hat{a}_t(x_{at}, x_{pt})$ for a period without elections and $a_t(x_{at}, x_{pt})$ if there is an election at the end of the period. $a_t(x_t) \equiv a_t(x_t, x_t)$ and $\hat{a}_t(x_t) \equiv \hat{a}_t(x_t, x_t)$ denote the incumbent's optimal effort levels if the voters and the incumbent's beliefs are coincidental (for example, on the equilibrium path).

After the incumbent chooses his action, ε_t is realized. The players do not observe η_t directly, but they do observe g_t. Observing g_t allows them to infer η_t by using the production function and their knowledge about the incumbent's effort, a_t. The incumbent knows a_t and is always able to infer η_t correctly. Voters infer η_t using the incumbent's equilibrium effort implied by their beliefs ($a_t(x_{pt})$ or $\hat{a}_t(x_{pt})$). The voters' inferred competence is given by

$$\eta_{pt}(\eta_t, a_t, x_{pt}) = \eta_t + a_t - a_t(x_{pt}) \tag{2}$$

in a period with elections and

$$\hat{\eta}_{pt}(\eta_t, a_t, x_{pt}) = \eta_t + a_t - \hat{a}_t(x_{pt})$$

in a period without elections. They make the right inference on the equilibrium path. Their inference may be wrong, however, if deviations from the equilibrium behavior are analyzed.

Players use their inferred competence to update their beliefs. x'_{at} and x'_{pt} denote the expected competence in the incumbent and the voters' posterior beliefs.

Both finite–horizon and infinite–horizon frameworks are considered here. Martinez [2003] establishes that in considering retention decisions, the number of periods considered in a model is specially important. In order to understand what framework is the most adequate for a particular situation, it is important to consider who are the players in the game. For example, in electoral competitions, one may think that policymakers are politicians with finite life, or political parties

with infinite life (even in the case of politicians, one would have to consider their ability to choose his successor, and to extract rents from him).

At the end of an election period, voters decide if they want to reelect the incumbent. Voters always believe to be on the equilibrium path where the incumbent and the voters' beliefs are coincidental. Let their reelection strategy be denoted by $z_t(x'_{pt})$, where $z_t(x'_{pt})$ equals one if the incumbent is reelected, and zero if otherwise. For expositional simplicity, voters are restricted to replacing the incumbent only with policymakers that were not in office before. This is an interesting starting point and it simplifies the analysis. The main results would not change much if this assumption is removed.[8]

Let δ denote the discount factor,

$$V_t(x_{pt}) = a_t(x_{pt}) + x_{pt} + \delta E\left[\max_{z\in\{0,1\}}\left\{z\hat{V}_{t+1}(x'_{pt}) + (1-z)\hat{V}_{t+1}(x_0)\right\}; x_{pt}\right] \tag{3}$$

denotes a voter's expected lifetime utility at the beginning of a period with elections, where E denotes the expectation operator.

$$\hat{V}_{t+1}(x_{pt+1}) = \hat{a}_t(x_{pt+1}) + x_{pt+1} + \delta E[V_{t+2}(x'_{pt+1}); x_{pt+1}] \tag{4}$$

denotes a voter's expected lifetime utility at the beginning of a period without elections.

For expositional simplicity, it is useful to define the random variable $y \equiv \sqrt{H}\,(\eta - x_a)$. y is distributed as a standard normal random variable. Let f denote the standard normal density function.

[8] Previous studies analyze models of optimal retention of agents where the optimal action for the principal does not involve hiring a previously tried agent other than the incumbent (see, for example, Banks and Sundaram [1990]). Martinez [2003] shows that, in models of career concerns, this is not always true. On the other hand, this is true in most of the cases analyzed. Furthermore, this would be true if there are many periods ahead and, because of that, an agent's ability is more important.

Let $W_t(x_{at}, x_{pt})$ and $\hat{W}_t(x_{at}, x_{pt})$ denote the incumbent's expected lifetime utility at the beginning of a period with and without elections, respectively (in a T–period model, $W_{T+1}(x_{at}, x_{pt}) = 0$ for any x_{at}, x_{pt}). The incumbent's problems can be written as follows:

$$\begin{aligned} W_t(x_{at}, x_{pt}) &= \max_{a_t} \left\{ R - c(a_t) + \delta \int \hat{W}_{t+1}(x'_{at}, x'_{pt}) z_t(x'_{pt}) f(y_t)\, dy_t \right\} && (5) \\ s.t\ x'_{pt} &= \mu x_{pt} + (1-\mu)\, \eta_{pt}(\eta_t, a_t, x_{pt}) && (6) \\ x'_{at} &= \mu x_{at} + (1-\mu)\, \eta_t. && (7) \end{aligned}$$

$$\begin{aligned} \hat{W}_t(x_{at}, x_{pt}) &= \max_{a_t} \left\{ R - c(a_t) + \delta \int W_{t+1}(x'_{at}, x'_{pt}) f(y_t)\, dy_t \right\} && (8) \\ s.t\ x'_{pt} &= \mu x_{pt} + (1-\mu)\, \hat{\eta}_{pt}(\eta_t, a_t, x_{pt}) && (9) \\ x'_{at} &= \mu x_{at} + (1-\mu)\, \eta_t. \end{aligned}$$

$W_t(x_t)$ and $\hat{W}_t(x_t)$ denote the incumbent's expected lifetime utilities if the beliefs about his ability are coincidental and are represented by x_t.

Definition 1 *A symmetric equilibrium for this model consists of the functions $V_t(x_{pt})$, $\hat{V}_t(x_{pt})$, $W_t(x_{at}, x_{pt})$, and $\hat{W}_t(x_{at}, x_{pt})$; and strategies $z_t(x'_{pt})$, $a_t(x_{at}, x_{pt})$and $\hat{a}_t(x_{at}, x_{pt})$; such that, for every period in the game:*

1. *$V_t(x_{pt})$ and $W_t(x_{at}, x_{pt})$ satisfy voters and incumbents' problems in periods with elections.*
2. *$\hat{V}_t(x_{pt})$ is as defined in equation 4.*
3. *$\hat{W}_t(x_{at}, x_{pt})$ satisfies the incumbents' problems in periods without elections.*
4. *$z_t(x'_{pt})$ solves the incumbents' problems in equation 3 (in periods with elections).*
5. *$a_t(x_{at}, x_{pt})$ solves the incumbents' problems in equation 5 (in periods with elections).*
6. *$\hat{a}_t(x_{at}, x_{pt})$ solves the incumbents' problems in equation 8 (in periods without elections).*

7. *The posterior beliefs are obtained from the priors using the equilibrium strategies, and Bayes' Rule as indicated in equations 6, 7, and 9.*

3 Results

Differences between incumbents' actions in periods with and without elections are studied here. These differences imply differences in the quantity of public goods (g_t) and might be used for explaining Political Business Cycles. Alternatively, the model presented here could easily be modified to study Political Budget Cycles if the incumbent's action represents a fiscal policy choice (as in the model presented by Shi and Svensson [2002]).

It will be shown that policymakers make intertemporal–effort–allocation decisions. In order to understand why policymakers may decide to exert a lower effort at periods without elections (for given beliefs about their ability), it is important to consider the relative effectiveness in increasing reelection probabilities of the efforts exerted each period. Moreover, the concavity (or convexity) of the incumbent's equilibrium strategies as functions of the incumbent's expected competence has to be considered.

For expositional simplicity, equilibrium in the last five periods of a finite–horizon framework is described first. In order to have a better understanding of the way in which the results would change if more periods were considered or policymakers have infinite lives, an infinite–horizon version of the model is discussed later.

3.1 A T–period Framework

Let us consider first a T–period framework in which there are retention decisions every two periods with the last one at period $T-1$. The solution for the last two periods is exactly as discussed by Martinez [2003]. At period T a policymaker has no incentives to exert effort and $a_T(x_T) = 0$

for all x_T. Therefore, a voter's expected utility at the beginning of period T is giving by x_T and a voter wants to reelect the incumbent at the end of period $T-1$ if and only if $x'_{pT-1} > x_0$, i.e., $z_{T-1}(x'_{pT-1}) = 1$ if and only if $x'_{pT-1} > x_0$.

In characterizing equilibrium decisions at $T-1$, only situations where $x_{pT-1} = x_{aT-1}$ need to be considered. If x_{T-1} represents this common belief, the incumbent's problem at period $T-1$ reads:

$$\begin{aligned} W_{T-1}(x_{T-1}) &= \max_{a_{T-1}} \left\{ R - c\left(a_{T-1}\right) + \delta P[x'_{pT-1} > x_0] R \right\} \\ s.t.\ \ x'_{pT-1} &= \mu x_{T-1} + (1-\mu)\left[\eta_{T-1} + a_{T-1} - a_{T-1}(x_{T-1})\right]. \end{aligned} \tag{10}$$

This problem can be written making explicit that what the incumbent can modify is the minimum competence realization required for reelection (let us note that equation 10 is a particular case of equation 5). The next equation shows this:

$$W_{T-1}(x_{T-1}) = \max_{a_{T-1}} \left\{ R - c\left(a_{T-1}\right) + \delta P\left[\eta_{T-1} > \frac{x_0 - \mu x_{T-1}}{1-\mu} + a_{T-1}(x_{T-1}) - a_{T-1}\right] R \right\}. \tag{11}$$

The first–order condition for this problem, evaluated in equilibrium (where $a_{T-1} = a_{T-1}(x_{T-1})$) is given by

$$c'\left[a_{T-1}(x_{T-1})\right] = \delta\sqrt{H} R f\left[\sqrt{H}\frac{x_0 - x_{T-1}}{1-\mu}\right]. \tag{12}$$

Given that the marginal cost is increasing with respect to effort; it can be seen from this equation that $a_{T-1}(x_{T-1})$ has the shape of a Gauss kernel with mean x_0.[9,10] In order to illustrate this, let us

[9] Functional–form assumptions on c' can make this problem concave and the first–order condition in equation 12 necessary and sufficient to characterize the optimal action. In particular, the problem is concave in the examples discussed in the paper.

[10] The generality of this result is discussed in Martinez [2003]. The additive production function used here implies that the derivative of the minimum competence required for reelection with respect to the incumbent's effort is independent of the beliefs about the incumbent ability (as shown in equation 11). The density function in equation 12 is increasing with respect to x_p if and only if $x_p > x_0$ because of the normality assumption. On the other hand, that this density function is increasing with respect x_a if and only if $x_p > x_0$ holds for a larger set of assumptions.

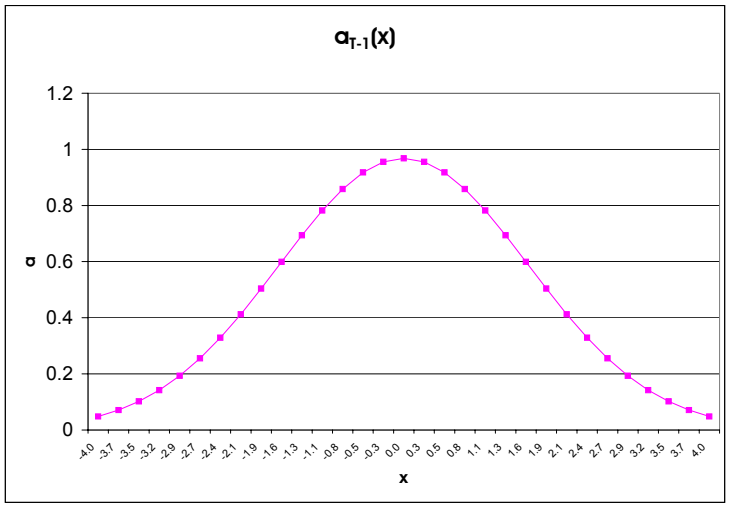

Figure 1: Optimal effort level at $T-1$.

consider the same example discussed by Martinez [2003]: $c(a) = a^n$, $n = 5$, $\delta = .9$, $R = 20$, $x_0 = 0$, $h_0 = .75$, and $h_\varepsilon = .75$. Figure 1 shows the optimal effort level for this example.

3.1.1 $T-2$: The last Cycle

In a period without elections, incentives come from the tradeoff between exerting effort in the current or the next period. Exerting a higher effort at $T-2$ may allow the incumbent to exert a lower effort next period and still have the same reelection probability at the end of $T-1$.

The problem the incumbent faces is as described in equation 8. The first–order condition for this problem is given by

$$\begin{aligned} c'\left(a_{T-2}\right) &= \delta \int \frac{\partial W_{T-1}(x'_{aT-2}, x'_{pT-2})}{\partial x'_{pT-2}} \frac{\partial x'_{pT-2}}{\partial a_{T-2}} f\left(y_{T-2}\right) dy_{T-2} \\ s.t\ x'_{pT-2} &= \mu x_{pT-2} + (1-\mu)\left[\frac{y_{T-2}}{\sqrt{H}} + x_{aT-2} + a_{T-2} - a(x_{pT-2})\right] \\ x'_{aT-2} &= x_{aT-2} + (1-\mu)\frac{y_{T-2}}{\sqrt{H}}. \end{aligned} \tag{13}$$

The next equation presents the Euler Equation for this problem evaluated in equilibrium.

$$
\begin{aligned}
c'\left[a_{T-2}(x_{T-2})\right] &= \delta \int r_{T-2}\left(x'_{T-2}\right) c'\left[a_{T-1}(x'_{T-2})\right] f\left(y_{T-2}\right) dy_{T-2} \qquad (14)\\
r_{T-2}\left(x'_{T-2}\right) &\equiv \left[\mu - (1-\mu)\, a'_{T-1}(x'_{T-2})\right].
\end{aligned}
$$

This equation represents the typical intertemporal tradeoff in dynamic models: having less today allows a policymaker having more next period. In this case, the marginal cost from a higher effort level today is compensated with an expected lower effort level next period.

In this intertemporal–effort–allocation decision, an incumbent considers the relative effectiveness of the efforts exerted at $T-2$ and at $T-1$ in changing the voters' posterior belief at $T-1$ (and, therefore, the reelection decisions). This relative effectiveness is represented in equation 14 by $r_{T-2}\left(x'_{T-2}\right)$. $r_{T-2}\left(x'_{T-2}\right)$ indicates the way in which an increase in the expected competence in the voters' posterior beliefs at $T-2$ (that is equal to the expected competence in their prior beliefs at $T-1$, x_{pT-1}) affects the expected competence in the voters' posterior beliefs at $T-1$ (x'_{pT-1}). If $r_{T-2}\left(x'_{T-2}\right)$ is less than one, it implies that the effort exerted at $T-1$ is more effective (than the effort exerted at $T-2$). The value of the relative effectiveness is endogenous here (it depends on $a'_{T-1}(x'_{T-2})$) and it is assumed to be zero in most of the previous models of Political Business (or Budget) Cycles. In those models, given that the relative effectiveness is assumed to be zero, the incumbent only tries to influence the election outcome with his action in the election period and this is why these models produce cycles. This is not the case in the model presented here.

If a higher a_{T-2} implies a lower x'_{pT-1}, $r_{T-2}\left(x'_{T-2}\right)$ is negative. As shown in equation 13, a higher a_{T-2} implies a higher x_{pT-1} (that is equal to x'_{pT-2}). A higher x_{pT-1} affects x'_{pT-1} (and the reelection decisions) in two ways. Firstly, for any competence inferred by the voters at $T-1$ (η_{pT-1}), a higher expected competence in the voters' prior (x_{pT-1}) implies a higher expected competence in their posterior (x'_{pT-1}). This is represented in $r_{T-2}\left(x'_{T-2}\right)$ by μ, the weight of x_{pT-1} in x'_{pT-1}. Secondly, a higher x_{pT-1} affects the competence inferred by the voters at $T-1$ (η_{pT-1}) through the implied change in the equilibrium incumbent's effort expected by the voters ($a_{T-1}\left(x_{pT-1}\right)$), as

shown in equation 2. Because a higher x_{pT-1} may imply a higher $a_{T-1}(x_{pT-1})$, a higher a_{T-2} may imply a lower η_{pT-1} and, therefore, it may imply a lower x'_{pT-1}. This effect is weighted in $r_{T-2}\left(x'_{T-2}\right)$ by $(1-\mu)$, the weight of competence in the posterior expected competence.

How large (or small) can the expected relative effectiveness be? It cannot be too large (or too small). In order to find the equilibrium effort level for $T-2$ (or at least to find an equilibrium without corner solutions), the right–hand side in equation 14 needs to be positive (for all x_{T-2}), i.e., the incumbent should not expect to be worse off if he exerts a higher effort at $T-2$. The relative effectiveness can be negative if the optimal effort level at $T-1$ increases a lot if the incumbent is believed to be more talented, i.e., if $a'_{T-1}(x'_{T-2})$ is high. Therefore, in order to assure the existence of equilibrium in this problem, it is necessary to consider parameter values such that $a'_{T-1}(x'_{T-2})$ is not too high (Martinez [2003] discusses the way in which parameter values affect the optimal effort level). In order to have an idea of how high this derivative can be, let us consider that for the relative effectiveness to be positive (for a given x'_{T-2}) the following inequality needs to hold:

$$a'_{T-1}(x'_{T-2}) < \frac{\mu}{1-\mu}.$$

Let us note that, because of the symmetry in $a'_{T-1}(x'_{T-2})$ (shown in equation 12), this implies that

$$a'_{T-1}(x'_{T-2}) > -\frac{\mu}{1-\mu}$$

and, therefore, the relative effectiveness cannot be higher than 2μ for all x'_{T-2}.

Considering the expected relative effectiveness is not enough for understanding the differences between equilibrium effort levels with and without elections. In fact, the effort at $T-2$ may be higher than the effort at $T-1$ (for the same equilibrium beliefs) even if the expected relative effectiveness is lower than one.[11] It is also important to consider the concavity (or convexity) of the

[11] This is illustrated by the example considered in Figure 2. In this example, the expected relative effectiveness is lower than one for all x and the effort level at $T-2$ is higher than the effort level at $T-1$ if x is far enough from x_0.

marginal cost function and $a_{T-1}(x'_{T-2})$. Equation 14 shows that $c'[a_{T-2}(x)]$ is equal to the expected marginal cost of a_{T-1} (weighted by the relative effectiveness and discounted by δ). In order to understand the way in which this expected marginal cost (and, therefore, $c'[a_{T-2}(x)]$) compares with $c'[a_{T-1}(x)]$ (the marginal cost evaluated at the expected competence) Jensen's Inequality has to be considered.

In order to compare the effort levels at $T-1$ and $T-2$, let us note first that $a_{T-1}(x_0) > a_{T-2}(x_0)$, i.e., for equilibrium beliefs that imply the maximum effort level at $T-1$, the model predicts a "positive" Political Business Cycle (a higher effort level at the election period). This is stated in the following proposition:

Proposition 1 *Let us assume that $a_{T-1}(x_0)$ and $a_{T-2}(x_0)$ are well defined. Then,*

$$c'[a_{T-2}(x_0)] = \delta\mu \int c'[a_{T-1}(x'_0)] f(y_{T-2}) dy_{T-2}$$

and, therefore, $a_{T-1}(x_0) > a_{T-2}(x_0)$.

Proof. *Let us recall that $a_{T-1}(x'_{T-2})$ is a symmetric function with maximum at $x'_{T-2} = x_0$ ($y_{T-2} = 0$), and, therefore, $c'\left[a_{T-1}\left(x'_{T-2}\right)\right]$ is a symmetric function with maximum at $x'_{T-2} = x_0$. f is a symmetric function with maximum at $y_{T-2} = 0$. Let us note that $r_{T-2}(x_0) = \mu$ and $r_{T-2}(x_0 + A) - r_{T-2}(x_0) = r_{T-2}(x_0) - r_{T-2}(x_0 - A)$ for all $A \in \Re$. Consequently,*

$$\delta \int r_{T-2}(x'_0) c'[a_{T-1}(x'_0)] f(y_{T-2}) dy_{T-2} = \delta\mu \int c'[a_{T-1}(x'_0)] f(y_{T-2}) dy_{T-2}.$$

Given that $\delta\mu < 1$,

$$c'[a_{T-2}(x_0)] < \int c'[a_{T-1}(x'_0)] f(y_{T-2}) dy_{T-2}.$$

$c'[a_{T-1}(x_0)] \leq c'[a_{T-1}(x)]$ for all x. Therefore,

$$c'[a_{T-1}(x_0)] > \int c'[a_{T-1}(x'_0)] f(y_{T-2}) dy_{T-2}.$$

Consequently, $c'[a_{T-1}(x_0)] > c'[a_{T-2}(x_0)]$, and $a_{T-1}(x_0) > a_{T-2}(x_0)$ (by $c'' > 0$). ■

On the other hand, it is known that for values of x further from x_0, $a_{T-1}(x)$ is a convex function and, therefore, Jensen's Inequality would imply that $a_{T-1}(x) < a_{T-2}(x)$. Thus, in order to understand the way in which the importance of the cycles depends on the beliefs about the incumbent's ability, both the relative effectiveness and the concavity (convexity) of $a_{T-1}(x)$ need to be considered. $a_{T-1}(x)$ is concave for x close to x_0, implying that the Political Business Cycles would be more important for these beliefs ($a_{T-1}(x)$ is convex if x is further from x_0). In order to understand the role of the relative effectiveness, $a'_{T-1}(x')$ has to be considered. The relative effectiveness is increasing for x' close to x_0, and decreasing for extreme values of x' (as illustrated in Figure 6).

The discussion above implies a set of empirically testable implications that are not considered in the previous literature: the size (and the sign) of the Political Business (or Budget) Cycles depends on the beliefs about the incumbent's ability. Moreover, in analyzing the way in which the presence of election affects the behavior of a particular policymaker, it is important to consider that the beliefs about this policymaker's ability may be different at different periods and this may be what explains differences in behavior. Figure 2 allows comparing the optimal effort level at $T-2$ with the optimal effort level at $T-1$ presented before.

3.1.2 $T-3$ and $T-4$: The Previous Cycle

Voters' decisions at the end of $T-3$ are not as simple as the ones at the end of period $T-1$. At $T-1$ voters know that the future effort level is independent of the beliefs about the incumbent's ability and, therefore, they always prefer a policymaker with higher expected competence. This does not need to be the case at $T-3$ because a policymaker who is believed to be more talented may exert a lower effort level at $T-2$ and/or $T-1$. Consequently, at the end of $T-3$, voters could decide not to reelect an incumbent who is believed to be more talented than an untried policymaker. Martinez [2003] presents examples where voters prefer not to reelect an incumbent who is more talented than an untried policymaker (in a model with elections in every period) but points out that in general this

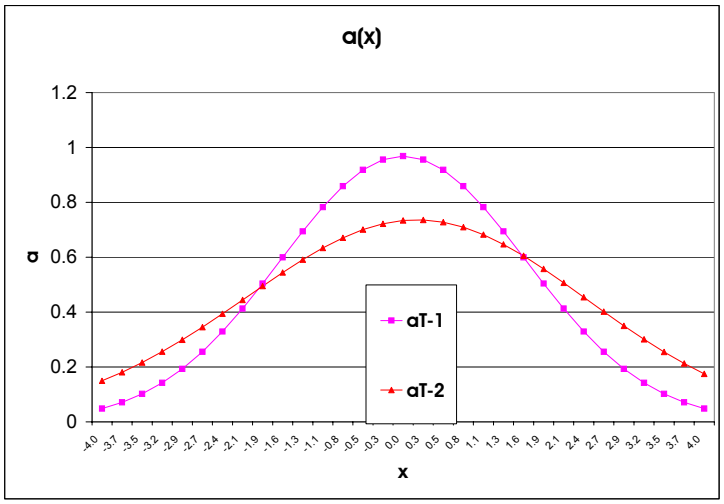

Figure 2: Effort Levels With and Without Elections.

would not be the case. It is more difficult to find such an example in the framework presented here. Firstly, in the examples presented by Martinez [2003] there are two periods remaining in the game when voters dismiss a talented incumbent. Here, there are three periods remaining and, therefore, ability is more important. Secondly, in the examples presented by Martinez [2003], the marginal productivity of the incumbent's competence is decreased if the incumbent decides to behave against the voters' interest (and this is not the case here). Consequently, the paper proceeds by assuming that at $T-3$ voters reelect the incumbent if and only if his expected competence is higher than the one for an untried policymaker. When specific examples are considered, it is checked that this is true.

The incentives an incumbent faces at $T-1$ are a special case of the incentives an incumbent faces at $T-3$ (and at any election period before $T-1$). At $T-3$ the incumbent does not only care about affecting the reelection probability in the current period but also the future reelection probabilities. These incentives are similar to the one an incumbent faces at $T-2$: in order to increase future reelection probabilities, the incumbent may decide to exert effort now or in the

future. The difference is that at $T-3$ the incumbent may not enjoy the future benefit because he may lose the election. The following Euler Equation describes these incentives.

$$\begin{aligned} c'\left[a_{T-3}(x_{T-3})\right] &= RPE_{T-3}(x_{T-3}) + JVE_{T-3}(x_{T-3}) \\ RPE_{T-3}(x_{T-3}) &\equiv \delta\sqrt{H}W_{T-2}\left(x_0\right) f\left[\sqrt{H}\frac{x_0 - x_{T-3}}{1-\mu}\right] \\ JVE_{T-3}(x_{T-3}) &\equiv \delta\int_{\sqrt{H}\frac{x_0-x_{T-3}}{1-\mu}} r_{T-3}(x'_{T-3})c'\left[a_{T-2}(x'_{T-3})\right] f\left(y_{T-3}\right) dy_{T-3}. \end{aligned} \tag{15}$$

Following Martinez [2003], the two terms in the right–hand side of equation 15 are called "retention probability effect" and "job value effect." Let us note that the incentives described in equation 12 represent a special case of the ones described in equation 15: in equation 12 the job value effect is zero (there is no reelection probability to affect at period T) and the office value in the retention probability effect is equal to $W_T(x_0) = R$. The job value effect represents the intertemporal–effort–allocation incentives. It looks like the right–hand side in equation 14 but it has a lower bound in the integral because the incumbent may lose the election.

At $T-4$ the incumbent makes an intertemporal–effort–allocation decision as the one described in equation 14. The main difference between the Political Business Cycles in periods $T-4$ and $T-3$ and the one described before is given by the job value effect in equation 15. Let us note that, for a high enough x_{T-3}, the lower bound in the integral in $JVE_{T-3}(x_{T-3})$ is very low (the incumbent is very likely to win the next election), and the incumbent has at $T-3$ an almost "full" job value effect (as at $T-4$). Therefore, for a high enough x_{T-3}, the difference between the effort level with and without elections is lower than in the last cycle of the game. Figure 3 illustrates this using the parameter values discussed before for presenting the differences between effort levels with and without elections as a percentage of the effort level in the election period for the two cycles described. Let us note that, even if an x high enough is considered, and there is an almost full job value effect at $T-3$, the expected next–period effort is higher at $T-4$ than at $T-3$ and, therefore, current effort is higher at $T-4$ than at $T-3$ (the retention probability effect is low if x is high).

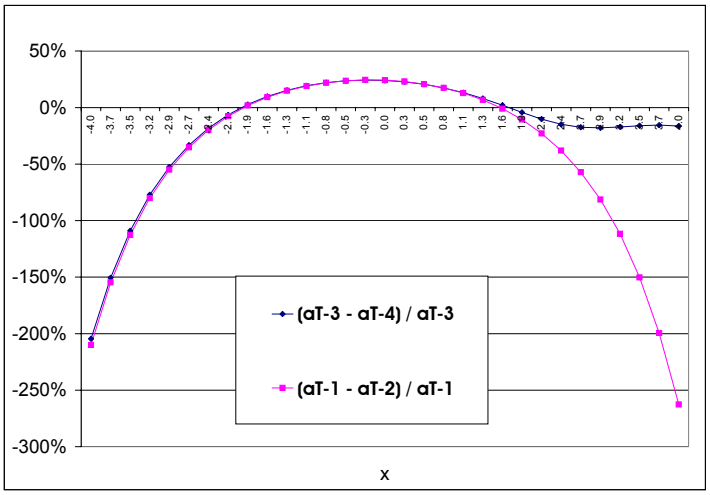

Figure 3: Differences between Effort Levels With and Without Elections as a Percentage of the Effort Level in the Election Period.

Thus, the importance of the cycles depends on the proximity of the end of the game. This could produce empirically testable implications if policymakers at different stages of their careers are considered. Term limits could also be studied.[12] Political Business Cycles would be different if the incumbent is closer to the term limit or the end of his career. Moreover, these differences would depend on the beliefs about the incumbent's ability.

How would the differences between effort levels with and without elections be further from the termination of the game? This question is answered next.

3.2 An Infinite–Horizon Framework

There are two important reasons for analyzing an infinite–horizon framework. First, it was established above that Political Business Cycles are different depending on the proximity of the end

[12] This implies introducing another difference (besides the expected competence) between the incumbent and the challenger.

of the game. The results presented here can be viewed as the limits of the results from a finite–horizon framework if players are far enough from the termination of the game. Moreover, an infinite–horizon framework may be more adequate if the incumbent represents the interest of political parties with infinite lives and, even if the incumbent represents his own interest, it would be important to consider his ability to choose his successor and to extract rents from him.

Situations where voters reelect the incumbent if and only if his expected competence is higher than the one for an untried policymaker are considered here. This is more likely to be true in an infinite–horizon framework where, if voters find a talented policymaker, they have more periods to enjoy his ability. When specific examples are considered, it is checked that this is true in equilibrium.

Let us consider now the incentives an incumbent has. In a period with elections, incentives are as described for $T-3$: the incumbent wants to increase the current and the future reelection probabilities. These incentives are represented in the following Euler Equation:

$$\begin{aligned} c'\left[a(x_t)\right] &= RPE(x_t) + JVE(x_t) \\ RPE(x_t) &\equiv \delta\sqrt{H}\hat{W}\left(x_0\right) f\left[\sqrt{H}\frac{x_0 - x_t}{1-\mu}\right] \\ JVE(x_t) &\equiv \delta\int_{\sqrt{H}\frac{x_0-x_t}{1-\mu}} \left[\mu - (1-\mu)\,\hat{a}'(x_t')\right] c'\left[\hat{a}(x_t')\right] f\left(y_t\right) dy_t. \end{aligned} \tag{16}$$

In a period without elections, incentives are as described for $T-2$ and $T-4$: an incumbent makes an intertemporal–effort–allocation decision. These incentives are represented in the following Euler Equation:

$$c'\left[\hat{a}(x)\right] = \delta\int \left[\mu - (1-\mu)\,a'(x')\right] c'\left[a(x')\right] f\left(y\right) dy. \tag{17}$$

Given the complexity of the problem studied here, a numerical approach is necessary to have a better understanding of the problem. It is easy to see that the results from the numerical approach presented here are very close to the results from the finite–horizon framework if periods that are far from the termination of the game are considered. The Euler Equations described in equations

16 and 17, and the value function for an incumbent in a period without elections evaluated in equilibrium (when the incumbent and the voters' beliefs are coincidental) constitute a system of three functional equations with three unknowns (the functions $\hat{W}(x)$, $\hat{a}(x)$, and $a(x)$). Numerical methods allow finding these functions. After that, $\hat{V}(x)$ has to be computed and it is necessary to check that $\hat{V}(x) > \hat{V}(x_0)$ if and only if $x > x_0$, as is assumed to find the results.

In order to perform this calculation, the parameter values discussed in the example presented before are used. Comparative–statics exercises on these values are conducted later on. Let us note that the solutions found here are very close to the closed–form solutions found in the finite–horizon framework for a period that is far from the termination of the game and the solutions of the infinite–horizon framework can be viewed as the limit of the solutions of the finite–horizon framework (if a period far from the termination of the game is considered). For all the parameter values tried, $\hat{V}(x)$ is an increasing function. Moreover, $\hat{a}(x) + x$ is often increasing with respect to x.

It was explained before that, if the incumbent's expected competence is high enough, and the effort at the election period is a convex function of the beliefs, a finite–horizon framework predicts a "negative" Political Business (or Budget) Cycle, i.e., it predicts that the incumbent's effort level is higher in a period without elections. Moreover, this is true even if periods $T-3$ and $T-4$ are compared and there is an almost full job value effect at $T-3$. It was explained that this is because of the non–stationarity present in the finite–horizon framework and this should not be observed if an infinite–horizon version of the model is considered or if two periods far enough from the termination of the game are compared in a finite–horizon framework. Figure 4 illustrates this: in the infinite–horizon framework, if x is high enough, effort is higher in election periods. Let us note that there are some equilibrium values for the expected competence higher than x_0 such that the effort level is slightly higher without elections. Figure 4 shows that the numerical approach used for computing the solutions in the infinite–horizon framework produces results that are coherent with the ones obtained from closed–form solutions in the finite–horizon version of the model.

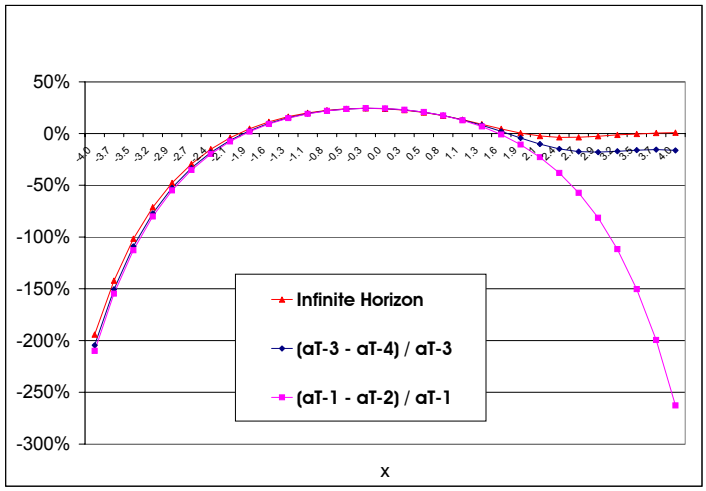

Figure 4: Differences between Effort Levels With and Without Elections as a Percentage of the Effort Level in the Election Period.

4 Comparative Statics

Under what circumstances are the Political Business (or Budget) Cycles more important? This section shows the answers this framework provides to this question through the analysis of comparative–statics exercises.[13] Differences in the per–period office value (R), the marginal cost of effort, the level of uncertainty about the policymakers' ability, and the amount of noise in the production process are analyzed here.[14] It is shown here that if the incumbent's actions in every period affect election outcomes, results from comparative–statics exercises may be different from what is found in the previous literature (if only actions at election periods can affect election outcomes). Moreover,

[13] In conducting comparative–statics exercises, is necessary to consider possible changes in the voters' reelection rule when parameter values are changed.

[14] In a model with uniformed voters, where a policymaker's action is observable (and can only affect the election outcome in election periods), Shi and Svensson [2002] analyze the effect of differences in the proportion of uninformed voters. The same could be done here if the model is reformulated.

in contrast with the previous literature, this model produces empirically testable implications that are conditional on the beliefs about the incumbent's ability.

4.1 Office Value

In a model where only policymakers' actions at election periods affect election outcomes, Shi and Svensson [2002] show that if the per–period office value (denoted here by R) is higher, the size of the Political Budget Cycle is larger. Moreover, they find empirical evidence that supports this prediction. The intuition behind this result is simple. A higher R implies that there are more incentives to increase reelection probabilities. Given that reelection probabilities can only be increased with actions at election periods, an increase in R increases the importance of the cycles.

What can be learned about this relationship from the model presented here? If the incumbent can affect election results with his actions in every period, a higher R implies a higher effort level in every period. For example, let us compare the effort levels at $T-1$ and $T-2$. Equation 12 shows that a higher R implies a higher effort level at $T-1$ for any x_{T-1}. Equation 12 shows that if a higher effort is expected at $T-1$, the incumbent decides to exert a higher effort at $T-2$. In particular, if c' is a homogeneous function, the next proposition shows that at x_0 (if changes in the relative effectiveness do not have to be considered) the difference between the effort levels with and without elections as a percentage of the effort level in election periods is independent of R.

Proposition 2 *Let us assume that $a_{T-1}(x_0)$ and $a_{T-2}(x_0)$ are well defined and c' is a homogeneous function of order j. Then, $\frac{a_{T-1}(x_0)-a_{T-2}(x_0)}{a_{T-1}(x_0)}$ is independent of R.*

Proof. Let us consider any office value $R=R_0$. Let us suppose that there is a change in the office value from R_0 to $R_1=\lambda R_0$. $a_{T-1}\left(x_0;R_0\right)$ satisfies

$$c'\left[a_{T-1}\left(x_0;R_0\right)\right]=\delta\sqrt{H}R_0 f\left(0\right).$$

$a_{T-1}(x_0; R_1)$ satisfies

$$c'\left[a_{T-1}(x_0; R_1)\right] = \delta\sqrt{H}R_1 f(0).$$

Therefore, $c'\left[a_{T-1}(x_0; R_1)\right] = \lambda c'\left[a_{T-1}(x_0; R_0)\right]$. Given that c' is homogenous of order j,

$$\lambda c'\left[a_{T-1}(x_0; R_0)\right] = c'\left[\lambda^{\frac{1}{j}} a_{T-1}(x_0, R_0)\right]$$

and $a_{T-1}(x_0; R_1) = \lambda^{\frac{1}{j}} a_{T-1}(x_0; R_0)$.

$$c'\left[a_{T-2}(x_0; R_0)\right] = \delta\mu \int c'\left[a_{T-1}(x_0; R_0)\right] f(y_{T-2})\, dy_{T-2}$$

and

$$c'\left[a_{T-2}(x_0; R_1)\right] = \delta\mu \int c'\left[a_{T-1}(x_0; R_1)\right] f(y_{T-2})\, dy_{T-2}.$$

Therefore, $c'\left[a_{T-1}(x_0; R_1)\right] = \lambda c'\left[a_{T-1}(x_0; R_0)\right]$ and $a_{T-2}(x_0; R_1) = \lambda^{\frac{1}{j}} a_{T-2}(x_0; R_0)$. Thus,

$$\frac{a_{T-1}(x_0; R_0) - a_{T-2}(x_0; R_0)}{a_{T-1}(x_0; R_0)} = \frac{a_{T-1}(x_0; R_1) - a_{T-2}(x_0; R_1)}{a_{T-1}(x_0; R_1)}$$

and $\frac{a_{T-1}(x_0) - a_{T-2}(x_0)}{a_{T-1}(x_0)}$ is independent of R. ■

This is illustrated in Figure 5 that represents the same example discussed above for different values of R. Figure 5 shows that when considering cycles for equilibrium beliefs different from x_0, the way in which changes in R change the relative effectiveness has to be considered. It can be seen that the differences in effort levels with and without elections as a percentage of the effort level at $T-1$ do not change much with small changes in R. On the other hand, if the change in R is important enough, the implied change in the relative size of the cycles may be important. This change is explained by differences in the relative effectiveness. If R is higher, $a_{T-1}(x_{T-1})$ is more responsive to changes in x_{T-1} and, therefore, the relative effectiveness of a_{T-2} is higher (lower) if x'_{T-2} is higher (lower) than x_0 (and $a_{T-1}(x'_{T-2})$ is a decreasing function). This is illustrated in Figure 6.

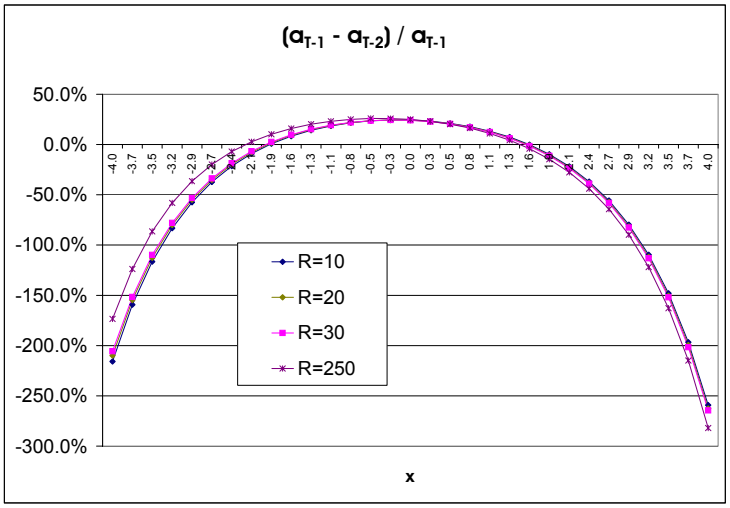

Figure 5: Differences between Effort Levels With and Without Elections as a Percentage of the Effort Level in the Election Period.

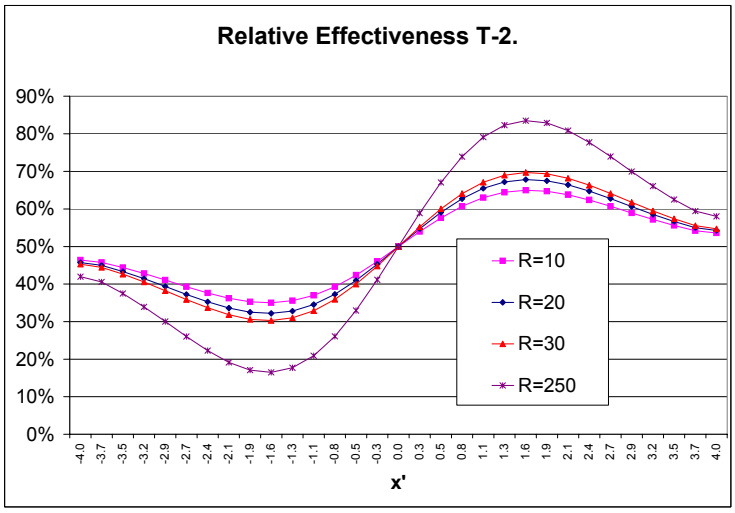

Figure 6: e_{T-2} Relative Effectiveness as a Function of Next Period Equilibrium Beliefs.

The discussion above implies empirically testable implications that allow distinguishing the model in this paper from the one presented by Shi and Svensson [2002] that predicts that differences in policymakers' actions with and without elections as a percentage of their actions in election periods are increasing with respect to R.

On the other hand, let us note that the results discussed above do not contradict the empirical findings in Shi and Svensson [2002] because the discussion presented above does not refer to differences in policymakers' actions with and without elections but to these differences as a percentage of the action levels in election periods. In particular, if c' is a homogeneous function, the next proposition shows that at x_0 (when changes in the relative effectiveness do not have to be considered and the action level is higher in election periods) the difference between the effort levels with and without elections is increasing with respect to R.

Proposition 3 *Let us assume that $a_{T-1}(x_0)$ and $a_{T-2}(x_0)$ are well defined and c' is a homogeneous function of order j. Then, $a_{T-1}(x_0) - a_{T-2}(x_0)$ is increasing with respect to R.*

Proof. Let us consider any office value $R = R_0$. Let us suppose that there is an increase in the office value from R_0 to $R_1 = \lambda R_0$ (where $\lambda > 1$). It was established before that $a_{T-1}(x_0, R_1) = \lambda^{\frac{1}{j}} a_{T-1}(x_0, R_0)$ and $a_{T-2}(x_0, R_1) = \lambda^{\frac{1}{j}} a_{T-2}(x_0, R_0)$. Let us recall that $a_{T-1}(x_0) - a_{T-2}(x_0) > 0$ (for any R). Thus,

$$a_{T-1}(x_0, R_0) - a_{T-2}(x_0, R_0) - [a_{T-1}(x_0, R_1) - a_{T-2}(x_0, R_1)] = [a_{T-1}(x_0, R_0) - a_{T-2}(x_0, R_0)](1 - \lambda) < 0$$

and $a_{T-1}(x_0) - a_{T-2}(x_0)$ decreases if R increases. ■

This is illustrated in Figure 7 that presents the same examples discussed above. Let us note that, given that small changes in R imply almost the same proportional changes in $a_{T-1}(x)$ and $a_{T-2}(x)$, in general, for certain equilibrium beliefs (x) the difference between the equilibrium effort level with and without elections $(a_{T-1}(x) - a_{T-2}(x))$ is increasing (decreasing) with respect to (a small change in) R if this difference is positive (negative). The previous conclusion does not always

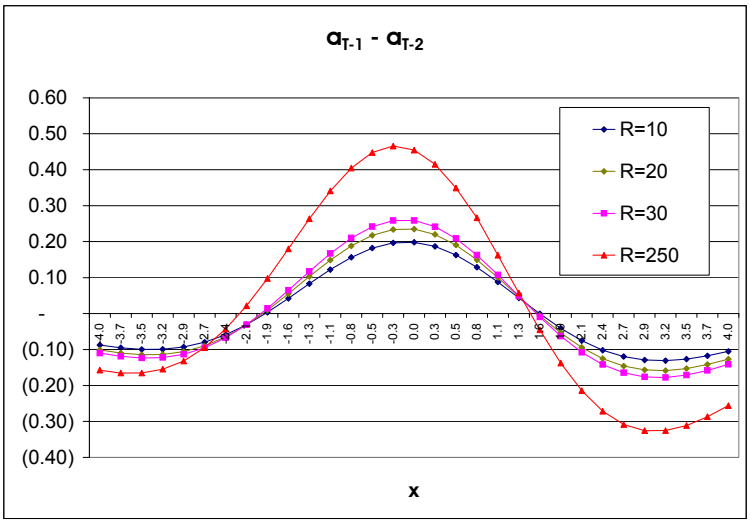

Figure 7: Differences between Effort Levels With and Without Elections.

hold because of the changes in the relative effectiveness that imply that there exist some values of x (far from x_0) such that the Political Business Cycle is positive (negative) and it is increasing (decreasing) with respect to R.

Again, the empirical implications of the model presented here are conditional on the beliefs about the incumbent's ability what allows distinguishing this model from the previous literature.

Results are different if cycles further from the termination of the game or in the infinite–horizon version of the model are considered. In these cases, there is a job value effect in periods with elections and, therefore, an increase (decrease) in the relative effectiveness increases (decreases) the optimal effort level in election periods. The effects of a change in the relative effectiveness on the incentives the incumbent has in election periods are closer to the one in periods without elections if x is high and the incumbent is likely to win the next election and to enjoy the future benefits from the current effort he exerts (the lower bound in the integral in the job value effect is low). Figure 8 illustrates this showing the differences between effort levels with and without elections as a percentage of the effort level in the election period. It shows that, for high values of x, changes

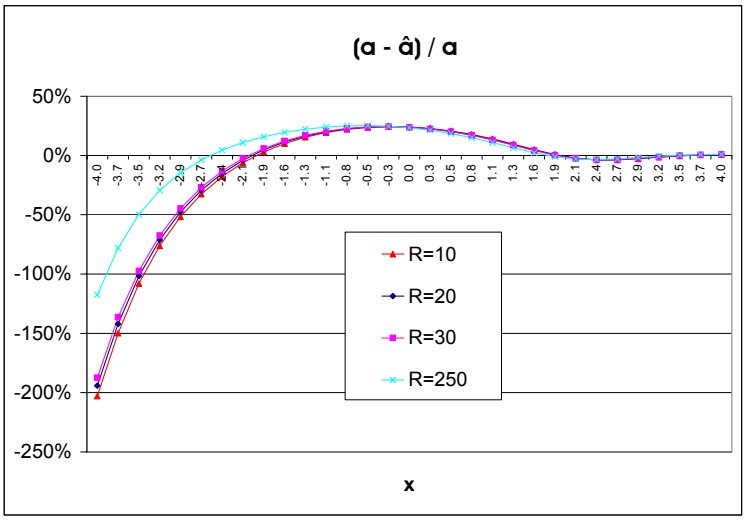

Figure 8: Differences between Effort Levels With and Without Elections as a Percentage of the Effort Level in the Election Period.

in R imply small changes in the "relative" cycles even if large changes in R are considered (let us recall that, in this framework, $\hat{a}(x)$ is only observable for x higher than or equal to x_0).

4.2 Marginal Cost of Effort

This section studies the way in which the framework developed here predicts that changes in the marginal cost of effort affect the Political Business Cycles. The analysis presented here could be reinterpreted as a study of the effects of variations in the marginal cost (or benefits) of stealing (or deviating resources) in political agency models like the ones presented by Persson and Tabellini [2000] or variations in the marginal cost of borrowing (or the interest rate) in models like the one presented by Shi and Svensson [2002].

For expositional simplicity, the last cycle in a finite–horizon framework is analyzed.[15] As shown

[15] In analyzing any previous cycle in a finite–horizon framework, or an infinite–horizon framework, it is necessary to consider that a change in the marginal cost of effort also affects the office value and, therefore, the marginal benefit

in equation 12, for any x_{T-1}, the equilibrium marginal cost of effort at $T-1$ ($c'[a_{T-1}(x_{T-1})]$) does not change with changes in the marginal cost function. Therefore, an increase in the marginal cost of effort implies a decrease in the optimal effort level.

There are two effects to consider at $T-2$. Firstly, a change in the equilibrium effort function at $T-1$ may imply a change in the relative effectiveness of the effort exerted at $T-2$. Secondly, as in $T-1$, an increase in the marginal cost of effort would imply a decrease in the optimal effort level.

In order to understand the way in which changes in the marginal cost of effort affect the cycles, it is necessary to consider the importance of the change in the marginal cost for different effort levels. Let us consider, for example, variations in n in the exponential cost function discussed in the examples above ($c(a) = a^n$). It is easy to show that an increase in n implies a decrease in the marginal cost of effort if and only if a is high enough ($1 + n \ln a$ is positive). Moreover, the change in a implied by an increase in n is less important for a higher a if a is higher than a threshold level ($1 + n \ln a + n$ is positive) and a is higher than this threshold level if there is an inverse relationship between a and n.

In particular, the next proposition shows that, without a change in the expected relative effectiveness, for equilibrium beliefs that imply a higher effort level in election periods, if effort levels are not too low, the difference between effort levels with and without elections is less important if n is higher.

Proposition 4 *Let us consider a number N such that $1 + n \ln N = 0$ and a number M such that $1 + n \ln M + n = 0$. Let us assume that $a_{T-1}(x_0)$ and $a_{T-2}(x_0)$ are well defined, $c(a) = a^n$, and*

of exerting effort (the office value affects the retention probability effect through $W_{t+1}(x_0)$ which depends on the policymaker's future equilibrium efforts which depend on the marginal cost of effort). The changes in the Political Business Cycles implied by changes in the marginal cost of effort are similar if changes in the office value have to be considered.

$n > 1$. *If* $a_{T-2}(x_0) \geq M$ *or* $a_{T-1}(x_0) \geq N$. *Then,* $a_{T-1}(x_0) - a_{T-2}(x_0)$ *is decreasing with respect to* n.

Proof. Firstly, let us suppose that $a_{T-2}(x_0) \geq M$, and, therefore, for all effort levels higher than $a_{T-2}(x_0)$, a higher equilibrium effort level implies that the change in the equilibrium effort level implied by a change in n is lower. Consequently, given that $a_{T-1}(x_0) > a_{T-2}(x_0)$,

$$\frac{da_{T-2}(x_0)}{dn} > \frac{da_{T-1}(x_0)}{dn}$$

and $a_{T-1}(x_0) - a_{T-2}(x_0)$ is decreasing with respect to n.

Secondly, let us suppose that $a_{T-1}(x_0) \geq N$. If $a_{T-2}(x_0) < N$, then

$$\frac{da_{T-2}(x_0)}{dn} > 0.$$

$a_{T-1}(x_0) \geq N$ implies

$$\frac{da_{T-1}(x_0)}{dn} \leq 0$$

and, therefore, $a_{T-1}(x_0) - a_{T-2}(x_0)$ is decreasing with respect to n. If $a_{T-2}(x_0) \geq N$, $N > M$ implies that $a_{T-1}(x_0) - a_{T-2}(x_0)$ is decreasing with respect to n. ■

This is illustrated in Figure 9. For other beliefs about the incumbent's ability, the same arguments used for x_0 (and the changes in the relative effectiveness) explain the way in which changes in n affect the differences between effort levels with and without elections. In particular, in Figure 9, for x close to x_0, either $a_{T-2}(x) \geq M$, $a_{T-1}(x) \geq N$, or both, and in general (if the change in the relative effectiveness does not reverse the result) an increase in n implies that a cycle is more important if the cycle is positive and a cycle is less important if the cycle is negative. For extreme values of x, $a_{T-2}(x) < M$ may imply that, even though a cycle is negative, the cycle is less important if n is higher.

In order to understand the way in which n affects the relative effectiveness, let us recall that for any x such that $a_{T-1}(x)$ is high enough, $a_{T-1}(x)$ is decreasing with respect to n, and for any

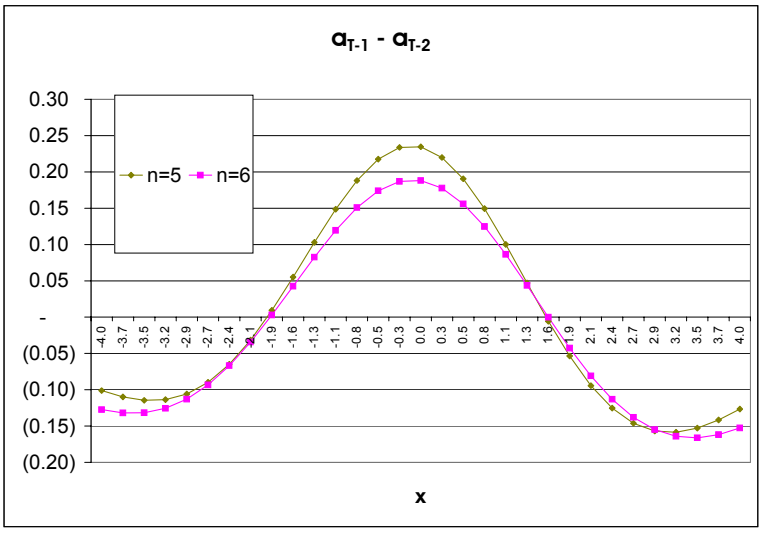

Figure 9: Differences between Effort Levels With and Without Elections.

x such that $a_{T-1}(x)$ is low enough, $a_{T-1}(x)$ is increasing with respect to n. Consequently, for x higher (lower) than x_0, an increase in n implies a decrease (increase) in the relative effectiveness for x close to x_0 and an increase (decrease) in the relative effectiveness for extreme values of x. This is illustrated in Figure 10.

4.3 Uncertainty about the Policymakers' ability (TO BE WRITTEN)

4.4 Uncertainty in the Production Process (TO BE WRITTEN)

5 Conclusions and Extensions

The paper presents a model of Political Business (or Budget) Cycles that contrast with most of the previous literature because, in the framework discussed here, a policymaker can alter election outcomes with his actions in every period (and not only in periods with elections). It is shown here that the extreme assumption adopted in most of the previous literature is not necessary for a

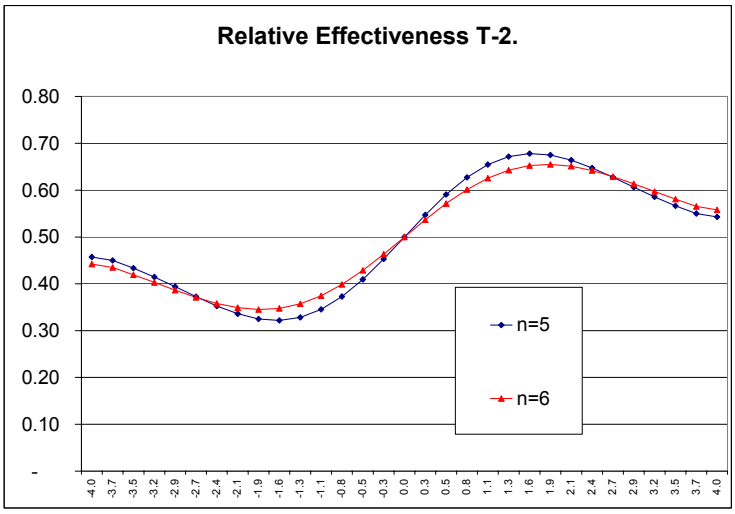

Figure 10: e_{T-2} Relative Effectiveness as a Function of Next Period Equilibrium Beliefs.

political agency model to produce Political Business (or Budget) Cycles. Moreover, this assumption has important consequences for the conclusions derived in this literature.

The model in this paper is a truly dynamic framework where policymakers' optimal decisions depend on the beliefs about their ability and these decisions influence future equilibrium beliefs. In the existing literature on Political Business (or Budget) Cycles, dynamics are sidestepped. The framework developed here can explain both positive Political Business Cycles and negative Political Business Cycles (the so–called post electoral cycles) something that is not possible with models in the previous literature.

Cycles are explained here by the endogenous relative effectiveness of the policymakers' actions in altering future reelections probabilities, and the concavity (or convexity) of the incumbent's strategies as functions of the equilibrium beliefs about his ability.

In contrast with most of the previous literature, this paper produces empirical implications on the sign and the size of cycles that are conditional on the beliefs about the incumbent policymaker's ability inviting to revisit the empirical findings in this area.

It is shown that if policymakers can affect reelection probabilities with his actions at any period, the results from comparative–statics exercises are different from what is presented in the previous literature allowing to distinguish the model presented in this paper. Moreover, in this paper, comparative–statics results are conditional on the beliefs about the incumbent's ability producing a new set of empirically testable implications.

The next step in this research project is to test the empirical implications of the model revising the findings of previous studies.

An interesting extension of the model presented here is to study a framework with term limits and/or retirement for the policymakers. This would produce empirically testable implications about Political Business (or Budget) Cycles.

Analyzing the way in which the framework developed here could help explaining differences in the frequency of elections (or the length of contracts) is also an interesting extension. In this framework, an incumbent policymaker prefers to postpone elections while voters prefer to increase the frequency of elections. This suggests that the frequency of elections may be decided in a bargaining process. In this framework, the intensity of the players' preferences about the frequency of elections depends on the equilibrium beliefs about the incumbent's ability and parameter values. Differences in these variables could help explaining differences in the frequency of elections. Moreover, the dynamic model presented here, may help understanding changes in the frequency of elections.

Additional natural extensions are analyzing cases with asymmetries in the learning processes, and a finite number of policymakers (political parties) participating in elections and studying situations in which players may decide the information structure.

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