Multidimensional Cheap Talk

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Abstract

In this paper we extend the cheap talk model of Crawford and Sobel (1982) to a multidimensional state space and policy space. We provide a characterization of equilibria. We focus on the question of feasibility of information transmission, for large degrees of conflict of interests between the sender and the receiver. We show that it is possible to construct equilibria with information transmission even for unboundedly large conflicts, but that any such equilibrium is based on knife-edge assumptions. We prove that influential equilibria are non-generic when the conflict between the sender and the receiver is large enough. Thus, adding more dimensions cannot improve upon information revelation when interests are too divergent.

1 Introduction

The seminal work of Crawford and Sobel (1982), henceforth CS, has paved the way for a vast literature applying cheap talk models to various contexts. Cheap talk is, for example, the model for explaining lobbying behavior; lobbies and interest groups are considered to exert political influence merely by convincing politicians, through casual communication, to take the 'right' policies.[1] The CS framework has led to an 'Informational theory of legislative committees' which explains the structure of committees in the Congress and the different rules that dictate how new bills can be amended.[2] In financial economics, cheap talk models explain the behavior of experts in financial markets and in particular the phenomenon of

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[1] On this line of research, see the survey in Grossman and Helpman (2001).

[2] See Gilligan and Krehbiel (1987), (1989), and Austen-Smith (1990).

herding.[3] Others explain social behavior such as political correctness.[4]

There are several reasons why the work of Crawford and Sobel (1982) is so influential. First, the model is easy to apply. The CS paper provides an algorithm to calculate informative equilibria and in particular to calculate the most informative equilibrium. More importantly, the CS framework allows to analyze the relation between the degree of conflict and the level of information that is transmitted between two individuals.[5] In particular, the model formalizes the intuition that as the preferences of two individuals diverge, the less they are able to communicate in a meaningful way. This feature of the CS model has been used to explain the observed patterns of lobbying activity. One prediction of this literature, which has also gained an empirical support, is that interest groups tend to lobby politicians whose preferences are relatively close to that of the interest group.[6]

The work of CS, however, as most of the applications of cheap talk models, focuses on a policy space and a state space of one dimension. In contrast, most political and economic decisions are concerned with more complex policy and state spaces, consisting of many relevant dimensions. One motivation behind analyzing a unidimensional space in the above mentioned applications, is that it may be an approximation for a multidimensional model. On the other hand, recent literature indicates that the predictions of models that assume multidimensional spaces may sharply differ from those that assume a unidimensional space. Although none of these papers extends the CS framework directly, this literature suggests that communication may be feasible, irrespective of the levels of conflict between the interested parties.[7]

In this paper we extend the basic cheap talk model, with one sender and one receiver, to a multidimensional environment. By doing so, we aim to study and isolate the effect of the dimensionality of the policy and state spaces in the cheap talk framework of CS. We find that, contrary to what one would expect from the above discussion, the multidimensional model does not yield results that are dramatically different from those of the unidimensional

[3] In this literature, investment decisions are cheap talk messages about experts' abilities, which allow them to acquire good reputation. See for example Trueman (1994) and Levy (2003).

[4] Morris (2001).

[5] It is important to note that the CS framework formalizes conflict in a particular way. This formulation hinges on the assumption that individuals have single peaked preferences over the policy space. The degree of conflict between two individuals is related to the distance between their respective ideal points in the different states of the world. In this paper we use the same notion of conflict as in the CS model.

[6] For a survey of this literature, and additional results on lobbying and information, see Austen-Smith (1995).

[7] Battaglini (2002) proves the existence of a fully revealing equilibrium when there are two or more senders who know the state of the world. Chakraborty and Harbaugh (2003) provide conditions on utilities such that informative equilibria exist for any level of conflict, in a simplified model (uniform distribution over the states and an equal level of conflict on each dimension).

model. In particular, we show that *generically*, when the level of conflict is high between two individuals, no information can be exchanged.

To be more specific we analyze the following model. As in CS, there are two players, a sender and a receiver. The receiver has to choose a policy in some policy space. The appropriate choice of policy depends on the realization of a state of the world. The receiver initially has a prior distribution on the state of the world. The sender on the other hand is informed about the state. the game has a simple form; first, the sender transmits a message about the state to the receiver. After observing the message, the receiver takes an action. The utility of both the sender and the receiver depend on the action and on the state of the world.[8] The sender and receiver differ in their optimal choice of action given the state of the world, i.e., there is a conflict of interests. In contrast to CS, we assume that the policy and state spaces are multidimensional.[9]

We focus on two main questions. First, we investigate the relation between the level of conflict and the ability of the players to communicate. In particular, we analyze whether the conflict between the sender and the receiver still is an obstacle for meaningful information transmission when the policy space is multidimensional. Second, we provide a general characterization of Perfect Bayesian equilibria and explore whether these equilibria retain the same attractive features as in the unidimensional model. These equilibria, as in the analysis of CS, are in partition form. That is, the state space is divided to convex sets of sender types, where in each set, all types of senders send the same message and thereby induce the same action. The actions reflect the (correct) expectations of the receiver over the set of senders who send the same message.

We then analyze whether information transmission is feasible, for large degrees of conflict of interests between the sender and the receiver. When the policy and state spaces consist of only one dimension, as CS have shown, there is no information transmission when the interests of the sender and the receiver are sufficiently divergent. When the conflict is too large, the sender would always prefer the receiver to take the most extreme action on this one dimension. Thus, no matter what is the real state of the world, he would always send the same message - the one which induces the receiver to take the most extreme action with the highest probability. This means that this message, or any other, cannot be informative about the type of the sender.

In the multidimensional world, however, there is some intuition which points otherwise.

[8] We focus on the case in which the utilities are functions of the Euclidean distances of the state from the actual action on each dimension. This is the common assumption in the literature; our results hold more generally, see section 6.

[9] We assume, as is common in the literature, that the state and policy space are the same. Our results can be generalized to the case in which the action space is a subset of the state space (and possibly of lower dimensionality).

Some 'bundling' of the dimensions may occur in equilibrium, even when the conflict is very large. For example, suppose that the sender prefers a higher action than the receiver on each dimension. An equilibrium may involve a message such as 'x is low but y is high'. This message can gain credibility (i.e., it would indeed be sent by low x and high y types) because it is unfavorable for these senders on the x dimension. It is nonetheless worthwhile for them to transmit it, since they get their way on the y dimension and, given their type, it is better for them than sending the other equilibrium message that translates as 'y is low and x is high'.

To be phrased more precisely, this intuition relies on the idea that when there are many dimensions, we can always span the space by a dimension (vector) of the conflict, and a dimension on which the sender and the receiver agree on, i.e., the interests of the sender and the receiver are aligned on this latter dimension. The sender should be willing to transmit any information on this dimension, disregarding the magnitude of the conflict. It may even be possible for him to transmit all the information on this dimension, implying an equilibrium with infinitely many credible messages.[10]

The following graph in the two-dimensional policy space is helpful for understanding the intuition for the possible existence of an equilibrium with infinitely many messages. Let us focus on Euclidean preferences, i.e., when a player puts the same weight on each dimension. Thus, the utility of a player decreases in the Euclidean distance of the action from his ideal policy. The receiver's ideal policy is the state of the world whereas the sender's ideal policy is removed from it by the vector $\mathbf{b}$, for any state of the world. The graph depicts the vector of conflict, $\mathbf{b}$. The orthogonal line to the vector $\mathbf{b}$ is the dimension on which the sender and the receiver 'agree on', the line BB. That is, if a sender could choose an action on the line BB, he would choose the action which is closest to the true state of the world. In this sense, the interests of the sender and the receiver are aligned on BB since the receiver also prefers the action which is closest to the state of the world (his ideal policy).

To see why the sender chooses the 'most truthful' action, take for example all senders who know that the state of the world is on some line AA, which is parallel to the vector $\mathbf{b}$, and orthogonal to BB. These senders would prefer action $\mathbf{a}$ among the points on BB. Their ideal policy, that is the state of the world removed by the vector $\mathbf{b}$, is also on the line AA or on its continuation, and the point $\mathbf{a}$ is closest to them on the line BB. The action

[10] The intuition relies on works by Austen-Smith (1993a) and more recently, Battaglini (2002) and Chakraborty and Harbaugh (2003). These papers are the exception in the literature, since they analyze a multidimensional policy space. We discuss the related literature at the end of this section. The intuition about multidimensional analysis is also indicated in Spector (2000). His model is very different from the rest of the literature as he analyzes a repeated game with continuum of agents, divergent priors and discrete policy space.

a is also the closest to the true state of the world among all actions in BB. This holds for any magnitude of the vector **b**, and in particular for very large conflicts in which the vector **b** could be imagined as stretching to infinity. Similarly, all senders on the line $A'A'$ would rather choose $\mathbf{a}'$ (ignore $\mathbf{a}''$ for the time being):

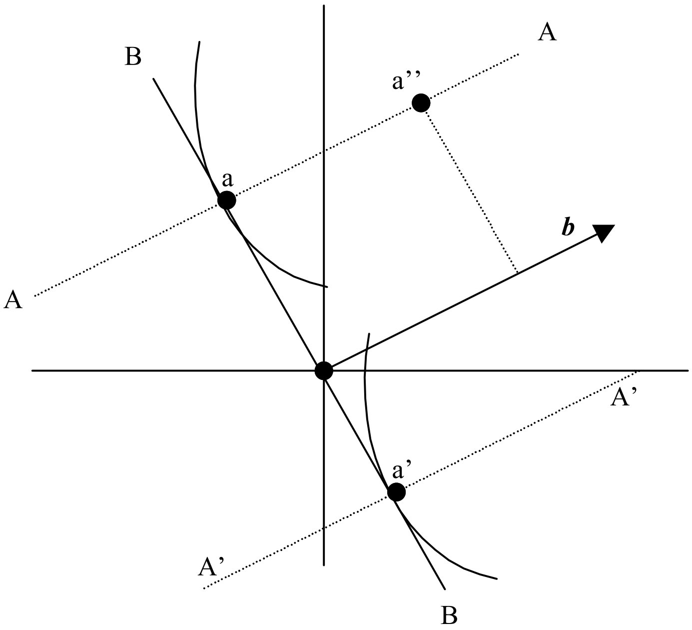

Figure 1: All senders on AA prefer a on the line BB whereas all senders on $A'A'$ choose a' on BB.

Obviously, one can construct infinitely many lines parallel to AA, or $A'A'$, or more generally, lines which have the slope of the vector of conflict **b**. All the senders who know that the state of the world is on one such particular line, would prefer one particular action on the line BB and these 'favorable' actions would span the line BB. Thus, it seems that in equilibrium, the sender may be willing to transmit all information on the line BB, i.e., that infinitely many credible messages can be transmitted in equilibrium.

A closer look at this suggested reporting strategy reveals a problem however in sustaining it in equilibrium. If the receiver observes a message advocating **a**, she understands that the real state of the world is somewhere on the AA line. She then updates her beliefs about the state of the world. These beliefs are surly on the line AA since these are expectations over this set of states. Her beliefs, however, do not necessarily coincide with the point **a**. Suppose that the receiver's expectations over the set of states in AA coincide with some point $\mathbf{a}''$ on the line AA and $\mathbf{a}'$ on the line $A'A'$.

But then such an equilibrium cannot exist; when the conflict is very large, as we show in the paper, the vector of the conflict becomes the *most* important dimension for the sender.

This dimension looms large and any two actions whose coordinates differ in the dimension of the conflict, $\mathbf{b}$, cannot be equilibrium actions. For large conflicts, as in the unidimensional world, the sender would always choose to induce the most extreme action *on the dimension of the conflict*. In the above case, senders both on AA and on $A'A'$ would prefer to induce $\mathbf{a}''$.[11]

In other words, there is indeed a dimension on which the sender and the receiver 'agree on'; but the sender would transmit information on this dimension only under the condition that this information would not affect the action of the receiver on the dimension of the conflict. These two dimensions, however, depend on the direction of the conflict (the vector $\mathbf{b}$) and the weights that the sender assigns to each of the dimensions in his utility (which are equal in the above example). Generically then, these two dimensions will not be independently distributed. Information about the dimension on which the sender and the receiver agree on, would imply how the state of the world is likely to be distributed on the dimension of the conflict. Thus, the receiver would change her choice on the dimension of the conflict upon observing information on other dimensions and as a result, the sender cannot transmit infinitely many messages in equilibrium.[12]

An equilibrium with infinitely many messages may not exist, but this may be too ambitious a requirement for very large conflicts. It may be that an equilibrium with few messages may hold, such as the 'bundling' equilibrium described above, with the sender just stating whether 'x is low and y is high' or the other way around. In the paper we show however that the existence of such equilibria, even with small or finite number of massages, is non generic.

The main result in this paper is that when the conflict between the sender and the receiver is large enough, informative equilibria generically do not exist. Although one can construct such equilibria for unboundedly large conflicts (we provide such examples in section 5), the point of the paper is to prove that any such equilibrium is based on knife-edge assumptions. In particular, we show that following a perturbation of either the underling distribution of the states, or the weights agents place on different dimensions, or the direction of the underling conflict between the sender and the receiver, these equilibria would cease to exist.

Our method of proof is the following. First, we characterize the conditions for the existence of informative equilibria in the limit case in which the conflict between the sender and the receiver grows to infinity. The conditions are that (i) the actions that are chosen by the receiver in equilibrium must be on a particular line which is parallel to the dimension of 'agreement'; (ii) on the line of actions, any two actions must be equidistant from an

[11] The coordinate of $\mathbf{a}'$ on the vector $\mathbf{b}$ is at the origin and that of $\mathbf{a}''$ is to the north-east of the origin.

[12] This indicates that commitment on behalf of the receiver would have facilitated information transmission. We discuss the issue of commitment in section 6.

indifferent type which is in between them; (iii) the line of indifferent types between any two action has the slope of the vector of conflict; (iv) all actions are expectations over the set of types which support these actions.

We then reformulate the model by spanning the state space using the two dimensions, that of the 'conflict' and that of an 'agreement'. Using the reformulation, we decompose the equilibrium conditions into two problems, such that the set of common solutions to these two problems is the set of all informative equilibria in the limit. We show that the two problems are independent, so that a common solution to the two problems is non generic. This proves that the existence of informative equilibrium in the limit is non generic.

Finally, we prove that the inexistence of an informative equilibrium in the limit implies that there is large enough levels of conflict for which informative equilibria cannot exist. This part of the proof is complicated by a particular feature of the equilibria in the multi-dimensional policy space. Whereas in the unidimensional policy space the existence of an equilibrium with n informative messages implies the existence of an equilibrium with $n-k$ informative messages for some integer $k \in (0, n)$, this is not true in the multidimensional policy space. Thus, to prove that no informative equilibrium exists it is not enough to show that an equilibrium with two different actions fails to exist (whereas this would be sufficient in the unidimensional model). We therefore have to show that informative equilibria do not exist for all sequences of equilibria when the level of conflict grows to infinity, those in which the number of induced actions is bounded by some finite number and those in which the number of induced actions converges to infinity. This is the final step of the proof.

Our paper also illustrates that the model is not easily applied, even for low levels of conflict. When the conflict is small, informative equilibria exist and can be characterized by partitions of the space, as in the unidimensional policy space. However, as indicated above, there is not necessarily a monotonous relation between the number of actions and the level of the conflict, as opposed to the unidimensional model. Moreover, although equilibria are characterized by partitions of the space, as in the unidimensional model, the elements of the partition are of different shapes, and the partition depends on the shape of the original state space. Thus, as opposed to the equilibria in the unidimensional model, it is very difficult - if not impossible - to find a general algorithm that can characterize informative equilibria when these exist, and moreover it is difficult to characterize the most informative equilibrium.

The papers most related to ours are the works of Battaglini (2002) and of Chakraborty and Harbaugh (2003). Battaglini (2002) also analyzes a model with a multidimensional state and policy spaces. In his model, there are two senders who both know the state of the world. He shows that in this case all information can be revealed in equilibrium, disregarding the magnitude of the conflict of interests. In the fully revealing equilibrium, the state space is spanned by two vectors; each of these vectors is the dimension on which the receiver has no conflict with one of the senders. Each sender then truthfully tells the receiver the coordinate of the state of the world on this dimension. This information revelation is feasible in a model with two senders, because one sender's message on a particular dimension indeed does not change the action of the receiver on other dimensions, and in particular on the dimension of the conflict. This is because the receiver 'already' knows the exact information on the dimension of the conflict, extracted from the report of the other sender. This result hinges on the assumption that both senders know exactly the same information.

Our work stresses that such a result cannot in general hold when there is only one sender. In particular, it is not only that full information transmission is not feasible, but that equilibria with any information transmission are not robust. We therefore disentangle the effect of increasing the dimensionality of the state space, and increasing the number of senders, in a CS framework.

The other relevant paper, by Chakraborty and Harbaugh (2003), studies the conditions on utility functions which allow the existence of the two actions 'bundling' equilibrium described above, for all degrees of conflict. Our analysis shows that for a subset of the utility functions they identify, that is, the ones that we analyze in this paper, this equilibrium is non generic.

Other related papers are concerned with extending additional features of the work of CS. Krishna and Morgan (2001) analyze the case of two senders and one dimension of conflict. They show that more information can be revealed when a receiver communicates with two senders instead of one, as long as the senders' interests are biased in opposite directions. Farrell and Gibbons (1989) show that the existence of two receivers may facilitate information transmission.[13] Aumann and Hart (2003) and Krishna and Morgan (2002) analyze models in which the receiver is also allowed to communicate, even though she is not informed. Her messages allow the players to conduct joint lotteries. These increase the degree of information transmitted, but still cannot induce informative equilibria when the interests are too divergent.

The rest of the paper is organized as follows. In the next section we present the model. In section 3 we characterize the equilibria of the model. Section 4 analyzes equilibria when there is a high degree of conflict between the sender and the receiver. In this section we present Theorem 1, our main result, about informative equilibria being non-generic. Section 5 puts forward examples that illustrate the implications of Theorem 1. In addition, we show in this section why the model is difficult to apply, with examples illustrating the characteristics of informative equilibria when the level of conflict is low. Section 6 concludes by discussing some extensions, notably to the case of an informed receiver and many senders.

[13] For an application of this idea to international relations see Levy and Razin (2004).

2 The model

An individual (the receiver) has to choose a policy in a multi-dimensional policy space, $\Re^d$. Denote the policy choice or action taken by the receiver by $\mathbf{a}$. The appropriate choice of policy depends on the realization of a state of the world $\boldsymbol{\theta}$, in a compact and convex subset of $\Re^d$ denoted by Θ.[14] The receiver initially holds an atomless and continuous prior distribution on the states in Θ denoted by F with a strictly positive density function f on Θ. We assume, without loss of generality, that Θ includes the origin and that the origin is the expectation of F on Θ.

A sender is fully informed about $\boldsymbol{\theta}$. Before the receiver takes his action $\mathbf{a}$, he observes a message about $\boldsymbol{\theta}$, transmitted by the sender. The sender chooses a message m in a set of messages $M = \Theta$. Upon observing the message, the receiver chooses her action $\mathbf{a}$ at the expectation of $\boldsymbol{\theta}$, $E(\boldsymbol{\theta})$, according to her posterior.[15]

The sender's preferences over the actions of the receiver are represented by the vector $\mathbf{b} = (b_1, b_2, ..., b_d)$ and by a strictly decreasing utility function, $U(\mathbf{a}|\boldsymbol{\theta}) = v(\Delta_\alpha(\mathbf{a}, \mathbf{b}|\boldsymbol{\theta}))$, defined over

$$\Delta_\alpha(\mathbf{a}, \mathbf{b}|\boldsymbol{\theta}) = \sum_{i=1}^{d} \alpha_i(a_i - (b_i + \theta_i))^2.$$

Note that the vector α (without loss of generality we assume that all its elements are strictly positive and sum up to one) denotes the relative importance of the different dimensions in the preferences of the sender.[16]

To summarize, the game has two stages. In stage 1 the sender observes $\boldsymbol{\theta}$ and then sends a message $m \in \Theta$. In stage 2, the receiver updates her beliefs about $\boldsymbol{\theta}$ and takes an action $\mathbf{a}$ at the expectations of $\boldsymbol{\theta}$. We analyze (weak) Perfect Bayesian equilibria of this game.

In what follows we focus our analysis on the case of $d = 2$. All our results carry over to the case in which $d > 2$.

3 Characterization of equilibria

A strategy of player type $\boldsymbol{\theta}$ is a probability distribution, m_θ, over the set of messages M. For any message $m \in M$, let $\mathbf{a}(m)$ denote the action chosen by the receiver.

An equilibrium is a pair of a strategy function $m : \Theta \to \Delta(M)$ (denoted by m_θ) for the sender and a belief function for the receiver, $f(\boldsymbol{\theta}|m)$, satisfying:

(1) $\forall\boldsymbol{\theta}$ and any $m', m'' \in M$ such that $m_\theta(m'), m_\theta(m'') > 0$,

[14] We discuss how to modify the assumption that Θ is compact in section 6.

[15] The assumption that the receiver chooses policy at the expectation is taken for simplicity. Moreover, it is consistent with the receiver maximizing a utility function that is quadratic in the distance of policy from the origin.

[16] We discuss the generalization of the preferences of the sender in section 6.

$$U(\mathbf{a}(m')|\boldsymbol{\theta}) = U(\mathbf{a}(m'')|\boldsymbol{\theta}) = \max_{m \in M} U(\mathbf{a}(m)|\boldsymbol{\theta})$$

(2)

$$\mathbf{a}(m) = E[\boldsymbol{\theta}|m] = \int_{\Theta} \boldsymbol{\theta} \cdot f(\boldsymbol{\theta}|m) d\boldsymbol{\theta}.$$

(3) $f(\boldsymbol{\theta}|m)$ is updated using Bayes rule whenever possible.

It is clear that an equilibrium exists; for example, one possible equilibrium is a 'babbling' equilibrium, when the sender sends the same message irrespective of $\boldsymbol{\theta}$. Proposition 1 below characterizes the equilibria of the model.

PROPOSITION 1 *Any equilibrium is almost surely equivalent in outcomes to an equilibrium in which the type space is partitioned into convex sets. All types of senders in the same set induce the same action, and each type induces it with probability one.*

Proposition 1 shows an analogy with the unidimensional model of Crawford and Sobel (1982). As in CS, any equilibrium in the multidimensional model is almost surely equivalent in terms of outcomes to an equilibrium in partition form. In any such equilibrium, the set of types is partitioned into convex sets and all agents in each element of the partition induce, with probability one, the same action.

Unlike the CS unidimensional model, the partition form of equilibria is not very useful for deriving other applicable results. Once the multidimensional state space is considered, there is no consistency in the shapes of the elements of typical equilibrium partitions, and the state space and its shape play an important role in determining the set of equilibria. Moreover, in contrast to the unidimensional model, if an equilibrium with k induced actions exists, it does not imply that an equilibrium with less than k induced actions also exists. These observations indicate that generally it is difficult to use algorithms to find equilibria in the multidimensional model, and that it is very difficult to characterize the most informative equilibrium. We illustrate these observations later on in section 5. We now turn to the characterization of equilibria with high levels of conflicts.

4 Equilibria with high levels of conflicts

In this section we analyze the set of equilibria when the conflict between the sender and the receiver is large. We focus on influential equilibria:

DEFINITION 1 An equilibrium is *influential* if there exist at least two messages m, m' such that $\mathbf{a}(m) \neq \mathbf{a}(m')$.

We derive our main result in three steps. First, we characterize the conditions for the existence of equilibria when the level of conflict between the sender and the receiver is large. In section 4.1 we take these limit conditions and reformulate them into two problems, denoted by problem A and problem B. We characterize the solutions to these two problems. Finally, section 4.2 presents the main result which shows that the solution to both problems is non-generic. We prove that the implication of this is that influential equilibria do not exist for large enough levels of conflict.

We now characterize the conditions for the existence of influential equilibria when the conflict is large. Recall that b_1 represents the conflict on the $x-$dimension, and b_2 represents the conflict on the $y-$dimension. Also, α_1 represents the relative importance of the $x-$dimension in the sender's preferences, and $\alpha_2 = 1-\alpha_1$ represents the relative importance of the $y-$dimension.

In what follows we increase the distance between the ideal point of the sender and the origin in the direction of the vector $\mathbf{b} =(b_1, b_2)$, a ray with slope $\frac{b_2}{b_1}$. Denote by b the norm of $\mathbf{b}$, i.e., $b = ||\mathbf{b}||$.

The two parameters that play a crucial role in what follows are $\beta^* = -\frac{\alpha_1}{\alpha_2}\frac{b_1}{b_2}$ and $\delta^* = -\frac{\alpha_1}{\alpha_2}\frac{1}{\beta^*} = \frac{b_2}{b_1}$. The first, β^*, is the slope of what we intuitively term the dimension of 'agreement' and the latter, δ^*, can be thought of as the slope of the dimension of 'conflict'. Assume without loss of generality that $|\beta^*| \in (0, \infty)$.

Our first step is an important building block of what will follow:

PROPOSITION 2 *For any $\varepsilon > 0$, there exists $\bar{b}$ such that for all $b > \bar{b}$ and any two distinct actions (a_1', a_2') and (a_1'', a_2'') induced in equilibrium, (i) $|\frac{a_2''-a_2'}{a_1''-a_1'} - \beta^*| \leq \varepsilon$; (ii) The set of sender types who are indifferent between these two actions is a line with slope δ such that $|\delta - \delta^*| \leq \varepsilon$.*

To see the intuition for part (i), note that when the conflict becomes very large, it implies that the distance between typical actions that the receiver may take and the sender's ideal point, increases. The indifference curves of the sender that go through these typical actions can be approximated by a line, with a slope approaching β^*. If several actions are induced in equilibrium, some sender types must be indifferent among them, hence the actions must lie on a line with this particular slope, β^*. Any action which is not on this line, must be strictly inferior or strictly superior relative to all the other induced actions for all types of senders. In other words, one dimension of the policy space becomes more and more important and induced actions cannot differ on this important dimension.

As for part (ii), suppose that in equilibrium there are two induced actions, $\mathbf{a}'$ and $\mathbf{a}''$. Some sender types must advocate $\mathbf{a}'$ and some must send messages to induce $\mathbf{a}''$, where the types who are indifferent compose the line which is the boundary between these two sets of types. However, the senders' ideal policies are actually removed from the state or action space by the vector $\mathbf{b}$ whose slope is δ^*. When the norm of this vector converges to infinity, and the distance between the typical actions and the sender's ideal point increases, the state space becomes 'small' relative to the magnitude of the vector of the conflict. Any other line, with a slope different from δ^*, would not constitute a separating hyperplane for the far removed senders' types; all senders' types would be on one side of the line or the other, so that all would support the same action, be it $\mathbf{a}'$ or $\mathbf{a}''$. Thus, if some types prefer $\mathbf{a}'$ and some prefer $\mathbf{a}''$, the slope of the line of the indifferent types must identify with the slope of the conflict.

Proof of Proposition 2: (i) For any b, if there are two distinct actions, $\mathbf{a}' = (a_1', a_2')$ *and* $\mathbf{a}'' = (a_1'', a_2'')$, induced in equilibrium, there must be some sender type $\boldsymbol{\theta}'$ that is indifferent between inducing either of these actions. For this type we have,

$$v(\Delta_\alpha(\mathbf{a}', \mathbf{b}|\boldsymbol{\theta}')) = v(\Delta_\alpha(\mathbf{a}'', \mathbf{b}|\boldsymbol{\theta}')) \Leftrightarrow \tag{1}$$
$$\sum_{i=1}^{2} \alpha_i(a_i' - (b_i + \theta_i'))^2 = \sum_{i=1}^{2} \alpha_i(a_i'' - (b_i + \theta_i'))^2 \Leftrightarrow$$
$$\alpha_1(a_1'^2 - a_1''^2) - 2\alpha_1(b_1 + \theta_1')(a_1' - a_1'') = \alpha_2(a_2''^2 - a_2'^2) - 2\alpha_2(b_2 + \theta_2')(a_2'' - a_2') \tag{2}$$

First suppose that $a_1' = a_1''$. This implies that the left-hand-side of (2) is 0 and therefore to satisfy the equation, it has to be that:

$$\alpha_2(a_2'' - a_2')((a_2'' + a_2') - 2(b_2 + \theta_2')) = 0$$

But since the actions are distant from one another, $a_2'' \neq a_2'$. Thus, for all

$$b_2 > \frac{(a_2'' + a_2')}{2} - \theta_2',$$

where $\frac{(a_2''+a_2')}{2} - \theta_2'$ is bounded, equation (2) is violated. Consider then the case of $a_1' \neq a_1''$ and let us re-arrange (2):

$$\frac{(a_2'' - a_2')}{(a_1' - a_1'')}\left(1 - \frac{\alpha_2(a_2'' + a_2')}{2\alpha_2(b_2 + \theta_2')}\right) - \frac{\alpha_1(b_1 + \theta_1')}{\alpha_2(b_2 + \theta_2')} = -\frac{\alpha_1(a_1' + a_1'')}{2\alpha_2(b_2 + \theta_2')} \Leftrightarrow \tag{3}$$

$$\frac{(a_2'' - a_2')}{(a_1' - a_1'')} = \frac{2\alpha_1(b_1 + \theta_1') - \alpha_1(a_1' + a_1'')}{2\alpha_2(b_2 + \theta_2') - \alpha_2(a_2'' + a_2')} \tag{4}$$

Note that any induced action must be in (the compact set) Θ. Moreover, it must be that both b_1 and b_2 are growing to infinity as b converges to infinity. This implies that the right hand side of (4) is converging to $\frac{\alpha_1 b_1}{\alpha_2 b_2}$ when b converges to infinity. Moreover this convergence is uniform (with respect to induced actions) by the compactness of Θ.[17] Thus, $-\frac{(a_2''-a_2')}{(a_1'-a_1'')} = \frac{(a_2''-a_2')}{(a_1''-a_1')} \to -\frac{\alpha_1 b_1}{\alpha_2 b_2} = \beta^*$.

[17] To see this one can define for any b, a triple $(\mathbf{a}'(b), \mathbf{a}'(b), \boldsymbol{\theta}(b)) \in \Theta^3$ which is a maximizer of $|\frac{2\alpha_1(b_1+\theta_1')-\alpha_1(a_1'+a_1'')}{2\alpha_2(b_2+\theta_2')-\alpha_2(a_2''+a_2')} - \beta^*|$ on Θ^3. By compactness such a maximizer exists. Furthermore, by compactness of Θ, $\lim_{b\to\infty} |\frac{2\alpha_1(b_1+\theta_1'(b))-\alpha_1(a_1'(b)+a_1''(b))}{2\alpha_2(b_2+\theta_2'(b))-\alpha_2(a_2''(b)+a_2'(b))} - \beta^*| = 0$. This last equality implies that the convergence is uniform.

(ii) For any two actions, (a_1', a_2') and (a_1'', a_2'') induced in equilibrium, look at a typical type, $\boldsymbol{\theta}'$, who is indifferent between the two actions. This type should satisfy (1) and therefore by rearranging we get,

$$\theta_2' = \theta_1' \frac{\alpha_1(a_1' - a_1'')}{\alpha_2(a_2'' - a_2')} - \frac{\alpha_1}{2\alpha_2} \frac{(a_1'^2 - a_1''^2)}{(a_2'' - a_2')} + \frac{2\alpha_1}{2\alpha_2} b_1 \frac{(a_1' - a_1'')}{(a_2'' - a_2')} + \frac{(a_2'' + a_2')}{2} - b_2, \qquad (5)$$

which shows that the set of indifferent sender types is a line with slope $\frac{\alpha_1(a_1'-a_1'')}{\alpha_2(a_2''-a_2')}$. By (i), $\frac{(a_1'-a_1'')}{(a_2''-a_2')} \to -\frac{1}{\beta^*}$ and so we have that the slope of the set of indifferent sender types converges to $-\frac{\alpha_1}{\alpha_2}\frac{1}{\beta^*} = \delta^*$.□

Proposition 2 puts very stringent conditions on the existence of equilibria; first, all induced actions will tend to be distributed on a particular line of slope β^*. But note that if all actions converge to be on the same line with a slope β^*, the expectations over all these actions must be on this line as well. These expectations must also coincide with the prior; as a result, this line must go through the prior expectations, which are normalized to be at the origin, $(0, 0)$. Thus, an implication of Proposition 2 is that all actions converge to the line $\theta_2 = \beta^*\theta_1$.

The second part states that the types who support each action will be in a subset of Θ with boundaries which tend to be lines with slope δ^*. In addition, it is easy to derive from part (i) that the sender type which is the mid-point between any two actions, $(\frac{a_1''+a_1'}{2}, \frac{a_2''+a_2'}{2})$, is also indifferent between the two actions, i.e., it is on the line of indifferent types. Trivially, $(\frac{a_1''+a_1'}{2}, \frac{a_2''+a_2'}{2})$ is equidistant from each action and also converges to the same line of the actions, that with a slope β^*. Thus, an implication of Proposition 2 is that on the line of actions, the actions are equidistant from the type who is indifferent between them. In other words, the actions are equidistant - on the β^* direction - from their respective line of indifferent types.

Remark 1 It is helpful to think of the preferences of the sender, when the conflict is very large, as *lexicographic preferences*. The relevant dimensions become the lines of slopes β^* and δ^*. First, the sender prefers the action which is on a line with a slope β^* that is closest to him, as if his indifference curves are linear with a slope β^*. If both actions are on the same line with a slope β^*, the sender prefers the action that is on the closest line with a slope δ^*.[18]

To pursue our analysis further, Corollary 1 summarizes Proposition 2 and its implications discussed above. In the next section we show that this tight structure summarized below implies that generically influential equilibria will not exist when the conflicts' levels are high.

[18] More precisely, the distance is measured on the β^* dimension, so that the preferred action is the one on the δ^* line which is closest *on the β^* direction.*

COROLLARY 1 *For any $\varepsilon > 0$, there exists $\bar{b}$ such that for all $b > \bar{b}$ and any two distinct actions $\mathbf{a}' = (a_1', a_2')$ and $\mathbf{a}'' = (a_1'', a_2'')$ induced in equilibrium, (i) $|a_2'' - \beta^* a_1''| \leq \varepsilon$ and $|a_2' - \beta^* a_1'| \leq \varepsilon$; (ii) The set of sender types who are indifferent between these two actions is a line with slope δ such that $|\delta - \delta^*| \leq \varepsilon$; (iii) The type $(\frac{a_1''+a_1'}{2}, \frac{a_2''+a_2'}{2})$ who is equidistant from the two actions is indifferent between the two actions.*

4.1 Equilibria with high levels of conflicts: a reformulation

Corollary 1 established the conditions for the existence of equilibria for very large conflicts. We now use these results to reformulate the problem, which would enable us to prove that generically, influential equilibria do not exist when the conflict is very large.

To make things simpler, we build a new coordinate system with respect to the slopes β^* and δ^*. We take the smallest parallelgam with slopes β^* and δ^* that contains Θ. We denote one of the points of intersection between the δ^* and the β^* lines of the parallelgam, say the south-west point, as (0,0) and let the two lines crossing at (0,0) span the space. Similarly one can denote the three other intersections by $(\bar{x}, 0)$ and $(0, \bar{y})$ and $(\bar{x}, \bar{y})$, with the convention that the new $x - axis$ is the dimension of slope β^* and the $y - axis$ that of slope δ^*. For any $\boldsymbol{\theta} \in \Theta$, let (x, y) be the same point expressed in the new coordinate system and let the set Θ be mapped into the set Θ^*.

Denote the marginal distribution on the $y - axis$ as $f^{\delta^*}(y)$ defined on $(0, \bar{y})$ and the marginal distribution on the $x - axis$ as $f^{\beta^*}(x)$ defined on $(0, \bar{x})$. Finally, we term the reaction curve, $\gamma(x)$, as the expectations over y for values of y for which $(x, y) \in \Theta^*$, i.e.,

$$\gamma(x) = E[y|(x, y) \in \Theta^*].$$

Figure 2 illustrates the new coordinate system, and depicts an example of $\gamma(x)$. The dashed lines in Θ^* are lines with slope δ^* and $\gamma(x)$ is the expectations over each of these lines, i.e., the expectations over y for a particular value of x :

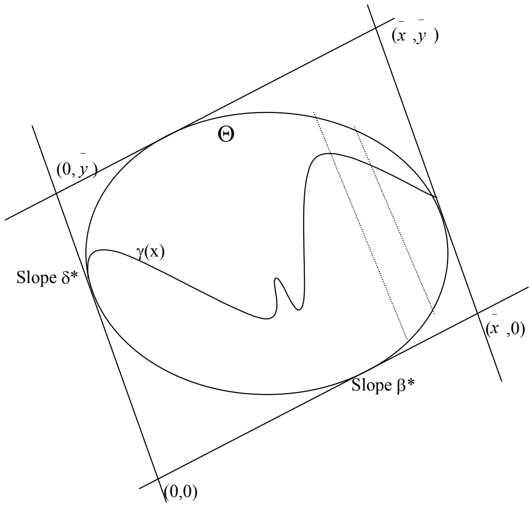

Figure 2: The new coordinate system.

In this new coordinate system, we propose to consider the following two problems. The first, problem A, is the following. Think of partitioning Θ^* into 'strips' with boundaries that are lines with a slope δ^*, such that the expectations taken on any two neighboring strips are equidistant (with respect to direction β^*) from the line that separates them. Figure 3a illustrates a solution to such a problem. It is a vector $x = (0, x_2, x_3, \bar{x})$, that induces the expectations (a_1, a_2, a_3), which are equidistant from x_2 and from x_3 respectively:

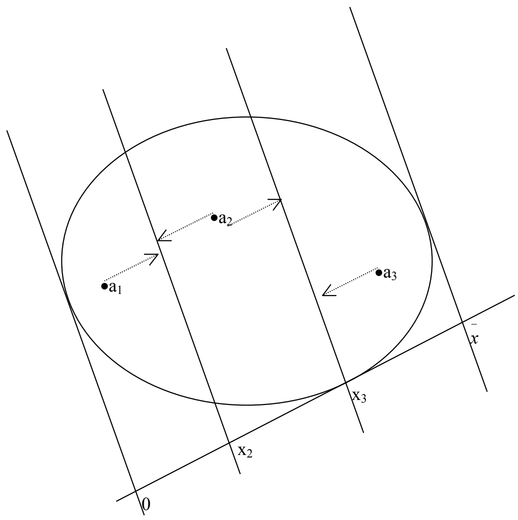

Figure 3a

This problem can be formalized as follows:

Problem A

A vector $(x_1, ..x_k) \in (0, \bar{x})^k$*, such that* $x_i \leq x_{i+1}$ *for all* $i \in (1, ..., k-1)$*, and* $k \geq 3$*, is a solution to problem A if* $x_1 = 0$, $x_k = \bar{x}$ and *for all* $i \in (2, ..., k-1)$,

$$|\int_{x_{i-1}}^{x_i} x f^{\beta^*}(x)dx - x_i| = |\int_{x_i}^{x_{i+1}} x f^{\beta^*}(x)dx - x_i|.$$

We proceed to consider the second problem, problem B. For any two points, $x, x' \in [0, \bar{x}]$, let

$$\mu^y_{x,x'} = E(y|x'' \in [x, x'] \text{ and } (x'', y) \in \Theta^*)$$

and let

$$\mu^y = \mu^y_{0,\bar{x}}$$

that is, μ^y is the prior expectations over the newly defined $y-$dimension. Problem B demands partitioning Θ^* into strips, with boundaries that are lines with a slope δ^*, but in a way so that the expectations taken within each strip are all on the same line of slope β^*, that is, they all have the same value of y, μ^y. Figure 3b illustrates a solution to such a problem: the vector $x' = (0, x'_2, x'_3, \bar{x})$ induces the expectations (a'_1, a'_2, a'_3) which are on a line with slope β^* that goes through the prior, μ^y. In the figure we also illustrate the reaction curve $\gamma(x)$, which is helpful in this problem; since $\gamma(x)$ is the expected value of y given a particular value of x, the expectations of the $y-$value over some strip (x_i, x_{i+1}) for $i \in \{1, 2, 3\}$ are essentially the expectations over $\gamma(x)$ in this strip, where in these expectations each value of $\gamma(x)$ is weighted by the marginal distribution over x.

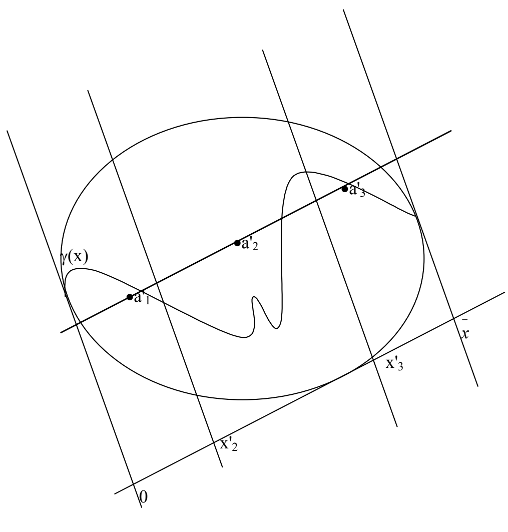

Figure 3b.

Formally,

Problem B

A vector $(x_1, ..x_k) \in (0, \bar{x})^k$*, such that* $x_i \leq x_{i+1}$ *for all* $i \in (1, ..., k-1)$*, and* $k \geq 3$*, is a solution to problem B if* $x_1 = 0$, $x_k = \bar{x}$ and *for all* $i \in (2, ..., k-1)$,

$$\int_{x_{i-1}}^{x_i} \frac{\gamma(x) f^{\beta^*}(x)}{\int_{x_{i-1}}^{x_i} f^{\beta^*}(x)dx} dx = \mu^y.$$

Problems A and B decompose therefore the conditions for the existence of equilibria. Problem A relates to the requirement that any two actions converge to be equidistant - in the β^* dimension - from the line of the indifferent types. Problem B relates to the requirement that all actions have actually to converge to the line with a slope β^* that goes through the prior. Both problems take into account that the lines of indifferent types converges to be with a slope δ^*. And, both problems take into account that each action is the expectations of the receiver over the set of sender types who prefer this action to the other possible actions. Therefore, there is a close relation between the solutions of problems A and B and the existence of equilibria.

In our main result, below, we show that the existence of a solution to both problems, A and B, is non-generic. This, as we will show, also implies that influential equilibria are non-generic for large conflicts. Even before we state and prove this formally, one can observe the following: a solution to problem A depends only on the marginal distribution over x, $f^{\beta^*}(x)$, whereas the solution to problem B depends on $f^{\beta^*}(x)$ as well as on $\gamma(x)$.[19] This implies that the set of solutions is independent. For example, if we alter the reaction curve $\gamma(x)$, without changing $f^{\beta^*}(x)$, the marginal distribution over x, then the set of solutions for problem B may change whereas the set of solutions for problem A remains fixed.

Our next step is to characterize the solutions of problems A and B.

Definition 2 *The reaction curve* $\gamma(x)$ *satisfies the* $l-$*crossing property, if there are* l *(finite) solutions to* $\gamma(x) = \mu^y$.

For example, in Figure 3b, the reaction curve satisfies the three-crossing property. Note that the $l-$crossing property, is a property of the relation between the distribution function $F(.)$ and the slopes δ^* and β^*. That is, it is not a property of a distribution function alone, but a joint property of the prior distribution and the other primitives of the model, the conflict on each dimension and the weight that the sender places on each of the dimensions.

[19]The solution to problem B depends therefore on the marginal on the $y - axis$ only through the expectations on that dimension.

Proposition 3

(i) Problem A: the set of solutions to problem A is isomorphic to the set of equilibrium outcomes of the model with $\Theta = [0, \bar{x}]$, *a density function* $f^{\beta^*}(x)$ *over* Θ *and* $\boldsymbol{b}$*=0 (see Crawford and Sobel, 1982). In particular, there exists a countable number of solutions.*

(ii) Problem B: (a) if $\gamma(x) = const$ *then any vector* $(x_1, ..x_k)$ *is a solution to problem B. (b) If the reaction curve* $\gamma(x)$ *satisfies the* $l-$*crossing property, then there are at most a finite number of solutions to problem B. (c) If the reaction curve* $\gamma(x)$ *satisfies the one-crossing property, then there does not exist a solution to problem B.*

Proof of Proposition 3:

(i) This follows from the analysis of Crawford and Sobel (1982).

(ii) (a)When $\gamma(x) = c$ for all x, then for all i, and x_i, x_{i-1}, $\mu^y_{x_i, x_{i-1}} = c$. Therefore any vector is a solution for B. In other words, if x and y are independently distributed then every vector x is a solution for B.[20] Recall however that x and y are not the original dimensions of the problem but correspond to the β^* and δ^* dimensions.

(ii) (b) We now construct an algorithm to compute the set of solutions for problem B. We will show both that the algorithm provides a finite number of solutions, and that it characterizes all the solutions to problem B.

Step 1: The algorithm

1. Start from $x_1 = 0$ and find the first $x \in (0, \bar{x}]$ such that $\mu^y_{0,x} = \mu^y$. If $x = \bar{x}$ the algorithm stops with no solution. If $x < \bar{x}$, denote $x_2 = x$. Suppose we have defined in this way x_m, $m \geq 2$. Continue this process starting from x_m. If there exists an $x \in (x_m, \bar{x})$ such that $\mu^y_{x_m,x} = \mu^y$ we define $x_{m+1} = x$ and proceed to the next step in the algorithm starting with x_{m+1}. If $x = \bar{x}$, we define $x_{m+1} = \bar{x}$ and the algorithm stops. Let k be the subscript of the last point defined in the algorithm.

By the $l-$ crossing property, the algorithm stops at a finite time. To see this, note that there cannot be two points x_i and x_{i+1} chosen by the algorithm that are between two neighboring solutions of the equation $\gamma(x) = \mu^y$ (i.e., two 'crossings'). If there were such two points than either $\gamma(x) > \mu^y$ or $\gamma(x) < \mu^y$ in the strip defined by (x_i, x_{i+1}), in contradiction to the construction of the algorithm. Thus, since there is a finite number of crossings, there must be finite number of solutions in the algorithm.

2. The set of solutions to problem B proposed by the algorithm is any (ordered) vector $(x'_1, ..., x'_m)$, $3 \leq m \leq k$, such that $x'_1 = 0, x'_m = \bar{x}$ and $x'_i \in \{x_2, ..., x_{k-1}\}$ for all $i = 2, ...m-1$.

Step 2: The solutions proposed by the algorithm are the only solutions for problem B.

[20] In particular, this would imply that when x and y are independently distributed, then there exist equilibria with any number of messages.

First note that since in any solution x of problem B $\mu^y_{x_i,x_{i+1}} = \mu^y$, then also $\mu^y_{0,x_i} = \mu^y$ for all x_i which are part of a solution x. Second, suppose that we find a solution $(x'_1, ..., x'_k)$ but it is not characterized by the algorithm above. In particular, suppose that x'_i is not part of any solution above. It is therefore in between some x_m and x_{m+1} which were characterized by the algorithm. In this case, $\mu^y_{0,x_m} = \mu^y$ and $\mu^y_{0,x'_i} = \mu^y$, which implies that $\mu^y_{x_m,x'_i} = \mu^y$. This is however in contradiction to the definition of x_{m+1}, since x_{m+1} is the smallest value of $x > x_m$ so that $\mu^y_{x_m,x} = \mu^y$. So $(x'_1, ..., x'_k)$ must be part of a solution found by the algorithm.

(ii) (c) If there is a single crossing of $\gamma(x)$ with μ^y this implies that the above algorithm will stop with no solution at the first step. The reason is that for all $x < \bar{x}$, either $\mu^y_{0,,x} < \mu^y$ or $\mu^y_{0,x} > \mu^y$. To see this, let x' be the unique solution for $\gamma(x) = \mu^y$. For any $x \leq x'$, $\mu^y_{0,x} \neq \mu^y$, and for any $x \geq x'$, $\mu^y_{x,\bar{x}} \neq \mu^y$, but in any solution x for the algorithm, $\mu^y_{x,\bar{x}} = \mu^y$ and $\mu^y_{0,\bar{x}} = \mu^y$ have to be satisfied. Thus, there is no solution. This concludes the proof.■

The implication of Proposition 3 is that the existence of a solution which solves both problems A and B simultaneously, seems highly unlikely. Part (c) of the Proposition already provides sufficient conditions for the non-existence of a solution to problem B, namely, the one-crossing property (in section 5 we show examples in which the one-crossing property is satisfied). For all other parameters, as we show below, a common solution to problem A and B is non-generic. We are now ready to state and prove our main result.

4.2 Equilibria with high levels of conflicts: the main result

Let $Solution(A)$ and $Solution(B)$ be the sets of solutions of problems A and B respectively.

Theorem 1 *(i) If* $Solution(A) \cap Solution(B) = \phi$ *then there exists a* $\bar{b}$ *such that for all* $b > \bar{b}$ *there exists no influential equilibrium; (ii) Suppose that Solution(A)∩Solution(B)≠ ∅. Then there exists a set of perturbations of the distribution function* $F(\boldsymbol{\theta}^*)$ *on* Θ^* *such that Solution(A)∩Solution(B)= ∅.*

The first part of the Theorem relates problems A and B to the existence of an equilibrium for high levels of conflict. That is, it states that if there is no common solution to problem A and B, then for high levels of conflict, indeed there is no influential equilibrium. The second part of Theorem 1 establishes that a solution to both problem A and problem B is non-generic. In the remaining of this section, we explain the intuition for the proof in two steps, starting with the second part.

A common solution to problems A and B is non-generic

To show that a common solution to problems A and B is non-generic, we construct perturbations to the distribution function which change the set of solutions of problem B while maintaining fixed the set of solutions for problem A. None of the new solutions of problem B would coincide with any of the solutions of problem A, so that a common solution would not exist following a perturbation.

First recall that a solution to problem B satisfies that for all i, and x_i, x_{i-1}, $\mu^y_{x_i,x_{i+1}} = \mu^y$, and moreover, by the algorithm that we have constructed, it also satisfies that for all i, $\mu^y_{0,x_{i+1}} = \mu^y$. We now focus on perturbations to $\gamma(x)$, while fixing the marginal density over x, $f^{\beta^*}(x)$. This type of perturbation will not alter the set of solutions of problem A.

Let us take for example any (small) local perturbation which alters $\gamma(x)$ in some strip x_i, x_{i+1}, which therefore changes the prior in that strip, $\mu^y_{x_i,x_{i+1}}$, and as a result changes also μ^y. Such a perturbation is described in the following figure, Figure 4:

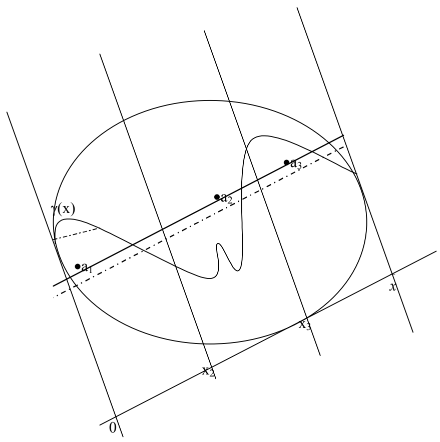

Figure 4: An example of a perturbation.

In the original solution, all the actions a_1, a_2 and a_3 are on a line with slope β^* that goes through μ^y. There is a small and local perturbation of the reaction curve $\gamma(x)$ represented by the dashed line, in the strip between 0 and x_2 which changes μ^y_{0,x_1} 'downward' to $\hat{\mu}^y_{0,x_1}$. This perturbation moves μ^y 'downward', and the dashed line with slope β^* goes through the new prior, denoted by $\hat{\mu}^y$. Note that μ^y must change less than the local change in μ^y_{0,x_2}.

It is now easy to see that the solution $(0, x_2, x_3, \bar{x})$ is not a solution to problem B any more: $\hat{\mu}^y_{0,x_2} \neq \hat{\mu}^y$, $\mu^y_{x_2,x_3} \neq \hat{\mu}^y$ and $\mu^y_{x_3,\bar{x}} \neq \hat{\mu}^y$ since the expectations on these two latter strips have not changed. Moreover, neither of the components of this solution can be part of a different solution, since also $\mu^y_{0,x_2} \neq \hat{\mu}^y$ and $\mu^y_{0,x_3} \neq \hat{\mu}^y$. This implies that any former solution to problem B is not a solution following the perturbation.

To complete the intuition for part (ii), note the following. Generically, whenever there is a slight change in the prior μ^y, any new solution to problem B is only slightly different from one of the original solutions. But then, following a perturbation, $Solution(A) \cap Solution(B)$ is empty; any previous solution to B corresponded to a solution of A, but since the set of solutions of A is not a continuum, none of the new solutions to B is a solution to A.

The inexistence of influential equilibria for large conflict

The final step of our analysis, is to establish the continuity of the problem. This is complicated because in the multidimensional policy space, if an equilibrium with k messages does not exist, it does not imply that an equilibrium with more than k messages does not exist (we illustrate this in section 5). Therefore, one cannot prove our result by focusing on equilibria with only two different actions (as opposed to the unidimensional model).

We must therefore show that the statement is true for all cases of sequences of equilibria when b converges to infinity, those in which the number of induced actions is bounded by some finite number and those in which the number of induced actions converges to infinity. We prove all these cases in the appendix; here we provide the intuition as to why an equilibrium with two different actions cannot exist. That is, we demonstrate now that when $Solution(A) \cap Solution(B) = \phi$, there is no sequence of equilibria in which the number of induced actions converges to two.

Assume, by way of contradiction, that such a sequence of equilibria exists, where along the sequence the two actions converge to some $\mathbf{a}$ and $\mathbf{a}'$. By Proposition 2, $\mathbf{a}$ and $\mathbf{a}'$ must lie on a line of slope β^* and the line of types who are indifferent between the two induced actions converges to a line of slope δ^*. Therefore, the sets of senders who support the two induced actions are converging to the part of Θ which is above and below the line of slope δ^*. Moreover, this line of indifferent types converges to go through the midpoint between the two limit actions $\mathbf{a}$ and $\mathbf{a}'$. Finally, any induced action is the expectation of the state of the world conditional on the state being in the relevant set of senders who induce this action.

But the above steps imply that along the sequence of equilibria, the two actions must be converging to one another, so that in the limit $\mathbf{a} = \mathbf{a}'$. The reason is that in the limit, the two induced actions cannot simultaneously be bounded away from each other, lie on some line of slope β^*, and on this line, be equidistant from the indifferent midpoint. If this would be the case, then $Solution(A) \cap Solution(B) \neq \phi$, a contradiction. Therefore, if such a sequence of equilibria exists, the two induced actions must converge to one another.

Finally, we arrive at a contradiction. It cannot be that in equilibrium, the two induced actions are arbitrarily close to each other. This would imply that each action is arbitrarily close to the line of types who are indifferent between the two actions. This line has slope δ^*, divides Θ to two convex subsets, is the border of each of these support sets, and must run between the two actions. But the measure of at least one of two support sets that cover Θ must be strictly positive. Since the density $f(.)$ is strictly positive on Θ, the expectations over this particular support set cannot converge to its borders, a contradiction.

5 Examples and Implications

In this section we discuss examples pertaining to high and to low level of conflicts. We first explore some implications of Theorem 1, which illustrate why influential equilibria do not exist in the multidimensional state space for high levels of conflict. We then shift our focus to low levels on conflict.

5.1 High levels of conflict

Let us consider first the state space of a square, $\Theta = [0,1]^2$, and a prior uniform distribution. It is easy to see the following. First, when $b_1 = b_2$ and $\alpha_1 = \alpha_2$, then $\delta^* = 1$ and $\beta^* = -1$. This implies that $\gamma(x) = const$, i.e., it is the diagonal with a slope -1 (recall that $\gamma(x)$ is the curve of expectations over lines with slope δ^*, such as the dotted lines shown in figure 5a).

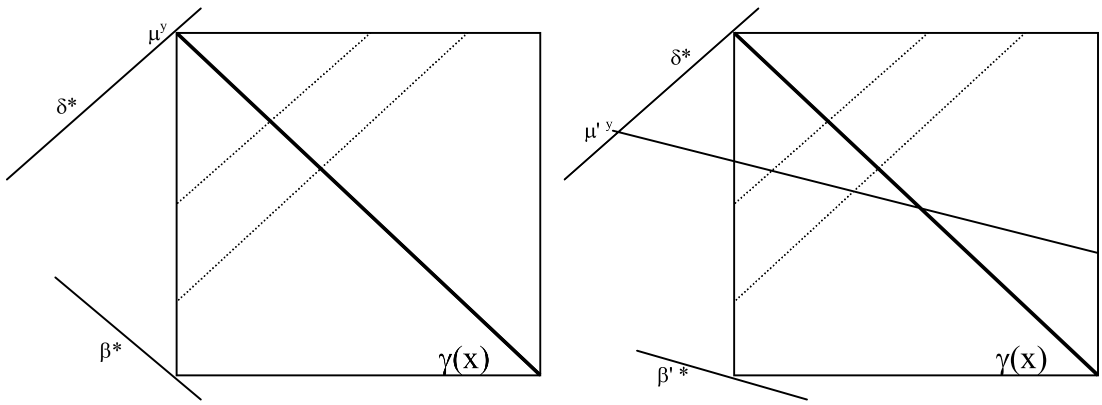

Figure 5a (left) in which every vector is a solution to problem B and figure 5b (right) which shows that generically in the square, problem B has no solution.

Second, when $b_1 = b_2$ and $\alpha_1 \neq \alpha_2$, the one-crossing property is satisfied. This is exemplified in figure 5b, because in this case $\beta^* \neq -1$. This implies that there does not exist a solution to problem B. Finally, for all other parameter values, the $l-$ crossing property is satisfied (in particular, it is easy to show that the three-crossing property is satisfied for all other parameters).

The implications of the above are as follows. Consider the case of the parameters $b_1 = b_2 \geq 0$ and $\alpha_1 = \alpha_2 = \frac{1}{2}$. Indeed, for any $b_1 = b_2$, there exists an influential equilibrium with infinitely many messages; all senders with the type $\theta_2 = \theta_1 + \lambda$ for $\lambda \in [-1, 1]$ send the same message λ and the receiver takes an action $(\frac{1-\lambda}{2}, \frac{1+\lambda}{2})$. This equilibrium can be approximated by the limit of a sequence of a common solutions for problems A and B, when we increase the size of the vector x, i.e., when we let k converge to infinity.

However, when the conflict is very large this equilibrium is non-generic, that is, it is not robust to small changes in the parameters of the model. For example, any local change in the uniform distribution that would slightly change $\gamma(x)$, will knock down this equilibrium, as Theorem 1 implies.

But moreover, if for example we replace the assumption of $\alpha_1 = \alpha_2$ by any other values for these parameters, there is no influential equilibrium at all. In this case, as in figure 5b, $\beta^* \neq -1$ and therefore $\gamma(x)$ satisfies the one-crossing property. From Proposition 3 we know that there is no solution to problem B in this case, which by Theorem 1 implies that no informative messages can be sustained in equilibrium.

We find this example important, first, because it demonstrates that other changes to the parameters of the model, not only to the distribution function as presented in Theorem 1, can upset influential equilibria. This is a consequence of the fact that the $l-$ crossing property that we have defined, is a property satisfied jointly by all parameters of the model and not by the distribution function alone.

Second, it may seem attractive in applications to use the symmetric case of $b_1 = b_2$ and $\alpha_1 = \alpha_2$. We emphasize here that this case may be misleading since results regarding information revelation are completely different for all other parameters.

The one-crossing property, which implies inexistence of influential equilibria for large conflicts, is also satisfied in other cases; consider for example the state space bounded by a circle, i.e., $\Theta = \{\theta_1, \theta_2 | \theta_1^2 + \theta_2^2 \leq r^2\}$, where the prior distribution is uniform. In this case, the reaction curve $\gamma(x)$, which is the expectations over lines with slope δ^* (as the dotted lines in figure 6), is always orthogonal to δ^* (this follows from the symmetry of the state space and the uniform distribution).

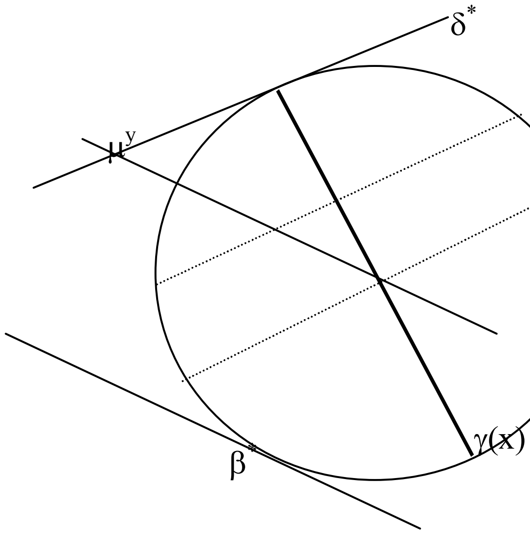

Figure 6: In the circle there are generically no solutions to probelm B.

Thus, $\gamma(x)$ is a line that goes through the prior with a slope $-\frac{1}{\delta^*}$. The prior μ^y is the β^*-projection on the δ^*-dimension, as described in the figure. Thus, it is clear that whenever $\beta^* \neq -\frac{1}{\delta^*}$, $\gamma(x)$ crosses μ^y, or the line with slope β^* that goes through the prior, only once. Since the one-crossing property is generically satisfied (whenever $\beta^* \neq -\frac{1}{\delta^*}$ or in other words whenever $\alpha_1 \neq \alpha_2$), there is no solution to problem B. Thus, in this state space, generically there is no influential equilibria for high degrees of conflict.

5.2 Low levels of conflict

When the levels of conflicts are low, influential equilibria can be supported. The examples presented in this section illustrate first that a multidimensional set up may actually improve upon information transmission relative to a unidimensional set up. However, we then illustrate the difficulties in characterizing influential equilibria in the multidimensional model.

Bundling and the possibility of improving information transmission Let us consider an equilibrium with two induced actions, for the state space $\Theta = [0, 1] \times [0, 1]$ and a uniform prior distribution over the state space. In the equilibrium, the square is divided to two groups of senders, each of them sending a different message. The actions are the expectations over the set of states represented by each group of senders, whereas given the actions, each sender indeed prefers to send the message that he is supposed to send.

Such an equilibrium exists when the level of the conflict is low. We have computed a particular equilibrium for $(b_1, b_2) = (.4, .3), (\alpha_1, \alpha_2) = (.5, .5)$. In this equilibrium, all types (θ_1, θ_2) below the line

$$\theta_2 = \lambda\theta_1 + \gamma$$

for $\{\gamma = -.2705, \lambda = 1.0325\}$, send the same message and induce the action $\mathbf{a} = (.754, .254)$, whereas all types (θ_1, θ_2) above this line send a different message and induce the action $\mathbf{a}' = (.4, .596)$. Moreover, this equilibrium, depicted in Figure 7, is robust to small changes in the parameters b and α, or in the prior distribution:

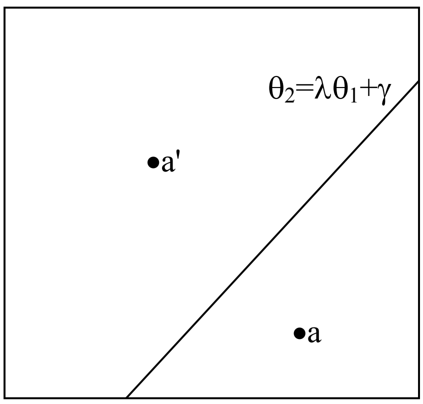

Figure 7: An equilibrium in the square, with two actions.

This example illustrates what we term the 'bundling' effect. The conflict has $b_1 \geq \frac{1}{4}$ and $b_2 \geq \frac{1}{4}$, which, as shown in Crawford and Sobel (1982), implies that a cheap talk game on each dimension separately results in babbling only and no credible information can be transmitted. However, information transmission is feasible once the two dimensions are bundled. In particular, the sender who always wants a higher action than the receiver would take, trades-off unfavorable information on one dimension (admitting it has a low value) with favorable information on the other dimension (that it has a relatively high value). Thus, bundling the two dimensions together in one communication game can improve upon information transmission.

The difficulty in characterizing equilibria for low levels of conflict Once the analysis becomes multidimensional, many useful features of the unidimensional analysis are lost. In the above 'bundling' example, the elements of the partitions differ; the two subsets do not have the same shape. This inconsistency of the shapes makes it harder to characterize equilibria. In addition, the shape of the state space Θ determines the characteristics of equilibria. To illustrate that, let us consider a state space bounded by a circle, and assume that the prior distribution over the states is uniform. We will show that generically, for all (b_1, b_2), there is no equilibrium with two induced actions.

Assume, to the contrary, that an equilibrium with two actions exists, for the actions $\mathbf{a} = (a_1, a_2)$ and $\mathbf{a}' = (a'_1, a'_2)$. The types of senders $\boldsymbol{\theta} = (\theta_1, \theta_2)$ who are indifferent between the two actions form a straight line, defined by the following equation (this is equation (5) in the previous section):

$$\theta_2 = \theta_1 \frac{\alpha_1(a_1 - a'_1)}{\alpha_2(a'_2 - a_2)} - \frac{\alpha_1}{2\alpha_2} \frac{(a_1^2 - a_1'^2)}{(a'_2 - a_2)} + \frac{\alpha_1}{\alpha_2} b_1 \frac{(a_1 - a'_1)}{(a'_2 - a_2)} + \frac{(a'_2 + a_2)}{2} - b_2$$

This line separates the two subsets of senders, each sending a different message and eliciting a different action, $\mathbf{a}$ or $\mathbf{a}'$. However, given the straight separating line, the symmetry of the square and the uniform distribution, the expectations over each such subset of senders must be on a line which is orthogonal to the line of the indifferent types (as in Figure 6). This implies that the following has to hold:

$$\frac{\alpha_1(a_1 - a'_1)}{\alpha_2(a'_2 - a_2)} = -\frac{1}{\frac{a'_2 - a_2}{a'_1 - a_1}}$$

where the left-hand-side is the slope of the line of indifferent types as defined in the previous equation, and the right-hand-side is the slope orthogonal to that between the two induced actions. As seen from this condition, this equation can hold iff $\alpha_1 = \alpha_2$.

Thus, equilibria with two induced actions generically do not exist in the circle when the prior distribution is uniform. However, we showed above that if the state space is a square, equilibria with two induced actions hold and are generic. Equilibria are therefore sensitive to the shape of the state space, which implies a difficult transition from the unidimensional model to the multidimensional model.

Finally, it is easy to see that if an equilibrium with k induced actions does not exist, it does not imply that an equilibrium with more than k induced actions doesn't exist. This marks an important difference with the analysis of the unidimensional model. To see why this is the case, note that we have shown that it is impossible to support an equilibrium with two different induced actions when the state space's boundary is a circle and the prior distribution is uniform. This is true for any vector of conflict. In particular, it is also true for $b_1 = b_2 = 0$. However, note that in this case, i.e., when there is no conflict, there exists an equilibrium with full information transmission, in which the sender reveals the true state and the receiver takes this as her action.

This feature is in particularly problematic for applications. First, it complicates the ability to prove the existence or inexistence of influential equilibria. In the unidimensional policy space, one can prove that influential equilibria do not exist simply by proving that an equilibrium with two different induced actions does not exist. This is not a feasible method of proof in the multidimensional policy space (see also the proof of Theorem 1 in the appendix). Second, it complicates the ability to characterize the most informative equilibrium (again, in the unidimensional policy space the most informative equilibrium is the one with most induced actions. This is not necessarily true in the multidimensional policy space).

6 Discussion

Our main result proves that influential equilibria are non-generic for large conflicts. We have shown that in the limit, for any equilibrium there is a set of perturbations that upsets its existence. We have focused on perturbations to the distribution function over the state space, but one can easily construct many other perturbations, such as to the state space itself, or to the other primitives of the model - the parameters describing the direction of the conflict or the relative weights of the different dimensions in the sender's utility function. In addition, we have established the continuity of the problem, so that if influential equilibria do not exist in the limit, there exist large enough degrees of the conflict so that influential equilibria cannot be sustained for these parameters as well. In what follows, we discuss some extensions of our results.

Informed receiver In our model the sender has a perfect private signal about the state of the world, whereas the receiver only knows its prior distribution function. Let us change this now and assume that the receiver also has a private signal about the state of the world, although, in order to keep things interesting, his signal is not perfect.

The main difference from our basic model now is that the sender, when transmitting his message, perceives the receiver's action *as a lottery.* This is because he does not know the exact signal of the receiver. In an influential equilibrium, different messages will induce lotteries with different support, whereas any two types of senders who send the same message, would face a lottery with the same support but with a different probability distribution over these lotteries (since they view differently the probability distribution over the signals of the receiver).

We conjecture that our results would hold for this environment as well. In particular, it is easy to extend the result in Proposition 2; if there are two different messages in equilibrium, then all types of senders who are indifferent between the two messages, must perceive the *expectations* over the lotteries induced by these two messages as converging to be on a line with a slope β^*.[21]

Many senders Another natural way to extend our paper is to increase the number of senders. In particular, it would be interesting to consider the case of the senders being imperfectly informed, i.e., each of them has a conditionally independent signal about the state of the world.[22]

Note that the case of many senders bears strategic resemblance to the case of an informed receiver described above. In particular, when one sender considers which messages to transmit, he perceives the receiver as informed and the receiver's action as a lottery, since other senders may have provided the receiver with some private information. We believe that we can therefore extend our results to this case as well.[23]

The inability of the receiver to commit In cheap talk models, the receiver cannot commit to implement a particular subset of policies.[24] This is reasonable in political economy models. A politician (the receiver) may not be able to commit to implement particular policies which accord with the interests of some lobby who provides him with information.

[21] We have extended Proposition 2 in this way for the case of a linear v.

[22] If all the senders are equally informed, then as Battaglini (2002) has shown, for any level of conflict there exists an equilibrium with full information revelation. The equilibrium has the particular feature that from the point of view of each sender, any of his messages will induce an action on the same line with a slope β^*.

[23] Several papers have analyzed models with many senders who have imperfect signals. Austen-Smith, (1990) and (1993b), analyzes the case of imperfectly informed senders and compares the information properties of the equilibria with either joint or sequential messages. Wolinsky (2002) shows how allowing communication among the experts may increase information revelation. These papers focus on a unidimensional state space. In Austen-Smith (1993a) the multidimensional state space is analyzed, to study experts' incentives to acquire information. Recently, Battaglini (2003) shows that it is possible to extract information from many senders who have imperfect signals, in a multidimensional environment, for some particular distribution functions. Whether this holds for other assumptions awaits future research.

[24] For a paper that takes the opposite assumption and analyzes communication as a mechanism design problem see Glazer and Rubinstein (2004).

The repeated game mechanisms may be relatively weak in such an environment of limited future interaction due to term limits and high turnover of politicians holding office. Moreover, the political decision making process leaves narrow possibilities for monitoring and enforcing commitment. If the politician is just one member of a many-members committee, it may be hard for the lobby to infer from the committee's decision whether the politician has acted in the right or wrong way.

In situations in which some commitment is feasible, our model points to how commitment ability should be designed. That is, when the conflict between the interested parties becomes too large, then the receiver can enhance information transmission by committing to a particular subset of policies, all on the same line with a slope β^*. This result contrasts with the unidimensional model, where for large conflicts, commitment is not beneficial for the receiver.[25]

Compactness of the state space In the model an important assumption is the compactness of the state space, Θ. This assumption is crucial in the proof of Proposition 2 and therefore for the results stated in Theorem 1. In particular, when the state space is R^2, influential equilibria may exist for any degree of conflict.

Theorem 1 may be generalized to cover this case also. Assume that the state space is R^2 but that the distribution F has finite expectations. We conjecture that as the degree of conflict increases, the probability that more than one action is induced in equilibrium converges to zero. In contrast to the case of a compact state space, in this environment our result about influential equilibria would be therefore in terms of probabilistic limits.

Generalizing the utility function The focus of this paper has been to show why the CS approach to model strategic information transmission, may be limited in a multidimensional state space. In this respect, the assumptions on the utility functions were chosen to correspond to those in the literature. This gives the model its best chance to yield applicable results. Although the results could be extended to more general utility specifications, our analysis proves that this approach may be futile.

The analysis in the paper highlights an interesting point about the CS modelling approach and modeling information transmission in general. It is easy to find specifications of utilities under which even in the one dimensional model it would be difficult to find influential equilibria. The assumptions underling the analysis in CS guarantee that this is not the case. One possible direction of future research, in the multidimensional state space, is to find other specifications of utilities, under which, influential equilibria may arise.

[25] For a paper modeling commitment in the unidimensional model, see Dessein (2002).

Appendix

Definitions and notation For any $\mathbf{a}', \mathbf{a}'' \in \Re^d$, let $d(\mathbf{a}', \mathbf{a}'') = ||\mathbf{a}' - \mathbf{a}''||$. For any $\mathbf{a} \in R^d$ and any set $A \subset R^d$, define the distance between $\mathbf{a}$ and A,

$$d(\mathbf{a}, A) = \inf_{\mathbf{a}' \in A} d(\mathbf{a}, \mathbf{a}').$$

Let $\mathbf{a}$,$\mathbf{a}' \in \Re^d$ be two actions. We define the set of agents that weakly and strictly prefer $\mathbf{a}$ to $\mathbf{a}'$, by, $R(\mathbf{a}, \mathbf{a}') = \{\boldsymbol{\theta} \in \Theta | U(\mathbf{a}|\boldsymbol{\theta}) \geqslant U(\mathbf{a}'|\boldsymbol{\theta})\}$ and $P(\mathbf{a}, \mathbf{a}') = \{\boldsymbol{\theta} \in \Theta | U(\mathbf{a}|\boldsymbol{\theta}) > U(\mathbf{a}'|\boldsymbol{\theta})\}$.

For any induced action $\mathbf{a}$, we define the *support set* of $\mathbf{a}$ as the set of types that induce $\mathbf{a}$ with a strictly positive density, i.e., for any $\mathbf{a} \in A$,

$$S(\mathbf{a}) = \{\boldsymbol{\theta} \in \Theta | \exists m \in M \text{ such that } \mathbf{a}(m) = \mathbf{a} \text{ and } m_\theta(m) > 0\}$$

For any equilibrium with a set of induced actions A, and any induced action $\mathbf{a}$, define the *potential support* set of $\mathbf{a}$, $\bar{S}(\mathbf{a})$, by $\bar{S}(\mathbf{a}) = \bigcap_{\mathbf{a}' \neq \mathbf{a}, \mathbf{a}' \in A} R(\mathbf{a}, \mathbf{a}')$. Types that are not in $\bar{S}(\mathbf{a})$ do not induce $\mathbf{a}$. Define the *definite support* set for $\mathbf{a}$ by $\underline{S}(\mathbf{a}) = \bigcap_{\mathbf{a}' \neq \mathbf{a}, \mathbf{a}' \in A} P(\mathbf{a}, \mathbf{a}')$. Types that are in $\underline{S}(\mathbf{a})$ will choose to induce $\mathbf{a}$ with probability one.

By Proposition 2, the set of indifferent types between $\mathbf{a}$ and $\mathbf{a}'$ is a line. Let $l^{\mathbf{a},\mathbf{a}'}$ denote the line of indifferent types between two induced actions, $\mathbf{a}$ and $\mathbf{a}'$. We say that two induced actions are *neighbors* if there exists a type, $\boldsymbol{\theta}$, satisfying $\boldsymbol{\theta} \in l^{\mathbf{a},\mathbf{a}'} \cap Bdry(S(\mathbf{a})) \cap Bdry(S(\mathbf{a}'))$.

Finally, let A be a measurable set, the measure of A is $M(A) = \int_A dF$.

Proofs of results PROOF OF PROPOSITION 1:

LEMMA 1 *In any equilibrium and any induced action* $\mathbf{a} \in A$,

$$\underline{S}(\mathbf{a}) \subseteq S(\mathbf{a}) \subseteq \bar{S}(\mathbf{a}).$$

Proof: Obviously if $\boldsymbol{\theta} \in \underline{S}(\mathbf{a})$, $\boldsymbol{\theta} \in S(\mathbf{a})$ as $\mathbf{a}$ is an induced action and it gives type $\boldsymbol{\theta}$ maximal utility. If $\boldsymbol{\theta} \notin \bar{S}(\mathbf{a})$ there must exist $\mathbf{a}' \in A$ such that $U(\mathbf{a}'|\boldsymbol{\theta}) > U(\mathbf{a}|\boldsymbol{\theta})$ and therefore by the equilibrium conditions, $\boldsymbol{\theta} \notin S(\mathbf{a})$.□

LEMMA 2 *In any equilibrium and any induced action* $\mathbf{a} \in A$, $M(\bar{S}(\mathbf{a})/\underline{S}(\mathbf{a})) = 0$.

Proof: For any $\mathbf{a}' \in A$, we have that $M(R(\mathbf{a}, \mathbf{a}')/P(\mathbf{a}, \mathbf{a}')) = 0$ as it is the set of types who are indifferent between the two actions. As preferences are Euclidean, this set is a line whose measure is zero in Θ. Therefore, $M(\bar{S}(\mathbf{a})/\underline{S}(\mathbf{a})) = M(\bigcap_{\tilde{a} \neq \mathbf{a}, \tilde{a} \in A} R(\mathbf{a}, \tilde{a}) / \bigcap_{\tilde{a} \neq \mathbf{a}, \tilde{a} \in A} P(\mathbf{a}, \tilde{a})) =$
$M(\bigcap_{\tilde{a} \neq \mathbf{a}, \tilde{a} \in A}(R(\mathbf{a}, \tilde{a})/P(\mathbf{a}, \tilde{a})) \leq M(R(\mathbf{a}, \mathbf{a}')/P(\mathbf{a}, \mathbf{a}')) = 0$.□

Finally, Lemma 3 characterizes the sets $\bar{S}(\mathbf{a})$ and $\underline{S}(\mathbf{a})$,

LEMMA 3 *(i)* $\bar{S}(\mathbf{a})$ *is a convex, closed set. (ii)* $\underline{S}(\mathbf{a})$ *is a convex set.*

Proof: (i) Since Θ is convex, for any $\tilde{a} \in A$, $R(\mathbf{a}, \tilde{a})$ is convex as it is the intersection of Θ and a convex hyperplane. Therefore $\bar{S}(\mathbf{a}) = \bigcap_{\tilde{a} \neq \mathbf{a}, \tilde{a} \in A} R(\mathbf{a}, \tilde{a})$ is convex. Moreover as $R(\mathbf{a}, \tilde{a})$ is a closed set for any $\tilde{a} \in Y$, $\bar{S}(\mathbf{a})$ is closed.

(iii) Since Θ is convex, for any $\tilde{a} \in A$, $P(\mathbf{a}, \tilde{a})$ is convex as it is the intersection of Θ and a convex hyperplane. Therefore $\bar{S}(\mathbf{a}) = \bigcap_{\tilde{a} \neq \mathbf{a}, \tilde{a} \in A} R(\mathbf{a}, \tilde{a})$ is convex.□

Lemmata 1,2 and 3 imply Proposition 1 as equilibria may differ from one another in outcomes only by having zero measure sets of types inducing different actions in equilibrium.■

PROOF OF THEOREM 1:

PROOF OF PART (i): we are considering a sequence of influential equilibria pertaining to $\{b_n\}_{n=0}^{\infty}$ with a corresponding set of induced actions A_n. We prove the following preliminary results:

LEMMA 4 *For any convex set C with strictly positive measure, $\eta > 0$, the distance of the expectation over C to the boundary of C is bounded from below by a strictly positive number.*

Proof: Specifically, we show that $d(E(C), Bdry(C)) > \frac{\lambda(\frac{\eta}{2})}{2} > 0$, where $\lambda(.)$ is an increasing, strictly positive function. Fix $\eta > 0$. Look at all possible convex subsets of Θ with measure η. For any such set C, define the width of C to be the smallest distance between any two parallel lines that are tangent to C. Note that the width of C is bounded below by some $\lambda(\eta) > 0$, where $\lambda(\eta)$ is increasing. This is evident by the compactness of Θ and the fact that $f(.)$ is atomless.

Fix a set C as above and find the closest point, p, on the border of C to $E(C)$. Let δ^C be the slope of the tangent to C at that point. Now divide C into two equal measured subsets by a line of slope δ^C. Denote the subset that includes p by U and the other by D. $E[C] = \frac{1}{2}E[U] + \frac{1}{2}E[D]$. Note that U and D are convex sets. The distance of p from D must be at least $\lambda(\frac{\eta}{2})$. This implies that $E[C]$ must be distanced from p by at least $\frac{\lambda(\frac{\eta}{2})}{2}$.□

LEMMA 5 *For any sequence of two neighboring actions, $\{\mathbf{a}_n, \mathbf{a}'_n\}$, either $\lim_{n\to\infty} M(S(\mathbf{a}_n)) = \lim_{n\to\infty} M(S(\mathbf{a}'_n)) = 0$ or $\lim_{n\to\infty} M(S(\mathbf{a}_n)) > 0$ and $\lim_{n\to\infty} M(S(\mathbf{a}'_n)) > 0$.*

Proof: First let us focus on a convergent sequence of $\mathbf{a}_n$ and $\mathbf{a}'_n$ (although we abuse notation by keeping the index as n). Suppose that along the sequence, $\lim_{n\to\infty} M(S(\mathbf{a}_n)) = 0$ but $\lim_{n\to\infty} M(S(\mathbf{a}'_n)) > 0$. By Lemma 4, this implies that $\lim_{n\to\infty} ||\mathbf{a}_n - \mathbf{a}'_n|| > 0$. Moreover, $l^{\mathbf{a}_n,\mathbf{a}'_n}$ is part of the boundary of $S(\mathbf{a}_n)$ and $S(\mathbf{a}'_n)$. By proposition 2 the boundaries all converge in slope to δ^* and therefore $S(\mathbf{a}_n)$ is converging to be a subset of $l^{\mathbf{a}_n,\mathbf{a}'_n}$. But this implies that $\lim_{n\to\infty} \mathbf{a}_n \in l^{\mathbf{a}_n,\mathbf{a}'_n}$ and $\lim_{n\to\infty} d(\mathbf{a}'_n, l^{\mathbf{a}_n,\mathbf{a}'_n}) > 0$ which is a contradiction as $l^{\mathbf{a}_n,\mathbf{a}'_n}$ converges to pass through the midpoint between $\mathbf{a}_n$ and $\mathbf{a}'_n$.□

The finite case:

Suppose that for all n, the number of actions induced in equilibrium is bounded by some finite $\bar{k}$, i.e., $2 \leq |A_n| \leq \bar{k}$ above some $\bar{n}$. We can then find a convergent subsequence of actions A_{n_k} in which the actions converge to some set A such that $|A| \geq 2$.

We show that all the actions along the sequence are bounded away from one another.

LEMMA 6 There exists K such that for all $k > K$, *the lines of indifferent types between any two neighboring actions do not intersect in* Θ.

Proof: We first show that no two induced actions, along the sequence, converge to one another. Suppose by way of contradiction that there exist two such actions, i.e., there exists a subsequence of equilibria and two induced actions $\mathbf{a}'_{n_k}, \mathbf{a}''_{n_k} \in A_{n_k}$ such that $||\mathbf{a}'_{n_k} - \mathbf{a}''_{n_k}|| \to 0$.

Note that for any two neighboring induced actions, $\mathbf{a}$ and $\tilde{a}$, $\mathbf{a} \notin IntS(\tilde{a})$. This can be seen as follows. $\mathbf{a} \in S(\mathbf{a})$ and therefore $\mathbf{a} \in R(\mathbf{a}, \tilde{a})$. Therefore, $\mathbf{a} \notin P(\tilde{a}, \mathbf{a})$ and thus $\mathbf{a} \notin Int(S(\tilde{a}))$. By Lemma 4 this implies that if $M(S(\mathbf{a}))$ is bounded below by some $\eta > 0$, then the two neighboring actions, $\mathbf{a}$ and $\tilde{a}$, are bounded away from each other. Then $M(S(\mathbf{a}'_{n_k})), M(S(\mathbf{a}''_{n_k})) \to 0$. By Lemma 5 this implies that $M(S(\mathbf{a}_{n_k})) \to 0$ for any $\mathbf{a}_{n_k} \in A_{n_k}$. To see this note that there are a bounded number of induced actions in equilibrium, and that all induced actions are connected under the "neighboring" relation. This contradicts the supposition that there exist two actions which converge.

We know that any line $l^{\mathbf{a},\mathbf{a}'}$ converges to pass through the midpoint on the line between $\mathbf{a}$ and $\mathbf{a}'$. Let us focus on any three induced actions $\mathbf{a}$, $\mathbf{a}'$ and $\mathbf{a}''$ such that $\mathbf{a}$ and $\mathbf{a}'$ are neighbors and so are $\mathbf{a}'$ and $\mathbf{a}''$. By Proposition 2 and the above, the distance between the midpoint between $\mathbf{a}$ and $\mathbf{a}'$and that between $\mathbf{a}'$ and $\mathbf{a}''$ must be bounded below by some strictly positive number. By the compactness of Θ and by Proposition 2, the lines must converge to be parallel and thus, for high enough k, never cross within Θ.[26]

Note also that any $\boldsymbol{\theta} \in \Theta$ that is strictly between two lines $l^{\mathbf{a},\mathbf{a}'}$ and $l^{\mathbf{a}',\mathbf{a}''}$ has the same ordering of preferences over A_{n_k} and in particular has the same optimal actions. Moreover, as any such point is not on any indifference line, it must be that there is a unique induced action that maximizes the type's preference.□

Using the reformulated coordinate system, let $x = (0, x_2, ..., x_{|A|-1}, \bar{x})$ represent the collection of corresponding limit-indifference lines between the elements of A, i.e., each x_i represents the line $\{y|(x_i, y) \in \Theta^*\}$. Let us order the elements of A respectively with the ordering of the $x's$.

LEMMA 7 $x \in Solution(A) \cap Solution(B)$.

[26] In particular, one can find a k' above which the lines do not intersect. Moreover this k' depends only on the bound $\bar{k}$.

Proof:

We will show that: (i) the distance of the β^*-axis projections of a^i and a^{i+1} to x^{i+1} are equal for $i = 1, ..., |A| - 1$, (ii) $a^i = E[\boldsymbol{\theta}_{|x,y}|x_i, x_{i+1}]$, and that (iii) all the a^i lie on a line of slope β^*.

Note that (iii) follows from proposition 2. By Proposition 2 the line of indifference between two actions converges to a slope of δ^* and thus converges to the relevant x_i. By (iii) we know that any two actions are on a line of slope β^* and thus the midpoint between the two actions is on the indifferent curve between the two actions. Therefore, both actions' projections on the β^* dimension are equidistant from x^i. This proves (i). We now prove part (ii). Suppose that $k > K$ (as in Lemma 6). On the sequence of equilibria represented by $\{A_{n_k}\}_{k=1}^{\infty}$ and for an induced action $\mathbf{a}_{n_k} \to a^i$, define the set $S^*(\mathbf{a}_{n_k})$ to be the largest convex set that is bordered by two lines of slope δ^* and the contours of Θ^* that is a subset of $S(\mathbf{a}_{n_k})$. Note that by Lemma 6, this is well defined. We now define a new sequence of actions. For any $k > K$,

$$\mathbf{a}^*_{n_k} = E(\boldsymbol{\theta}|\boldsymbol{\theta} \in S^*(\mathbf{a}_{n_k})).$$

We first show that $\lim_{k\to\infty} ||\mathbf{a}^*_{n_k} - \mathbf{a}_{n_k}|| = 0$. First, note that $M(S(\mathbf{a}_{n_k})/S^*(\mathbf{a}_{n_k})) \to 0$ as $S^*(\mathbf{a}_{n_k}) \subset S(\mathbf{a}_{n_k})$, $M(S(\mathbf{a}_{n_k}) > 0$ and by Proposition 2 the slopes of the boundaries of $S(\mathbf{a}_{n_k})$ converge to δ^* and so $S^*(\mathbf{a}_{n_k})$ is converging to $S(\mathbf{a}_{n_k})$. Finally note that $S^*(\mathbf{a}_{n_k})$ converges to the strip of Θ^* bounded by lines of slope δ^* passing through the points x_i and x_{i+1} on the reformulated $x - axis$.□

This completes the proof of the finite case since we reach a contradiction.

Proof for the infinite case:

We now prove the theorem for the case of infinite actions. That is, suppose that along the sequence of equilibria $\{A_n\}_{n=1}^{\infty}$ the number of induced actions is converging to infinity.

LEMMA 8 *When the number of induced actions converges to infinity along the sequence of equilibria, then for any induced action,* $\mathbf{a}_n$, $M(S(\mathbf{a}_n)) \to 0$.

Proof: Suppose that along the sequence there exists an induced action $\mathbf{a}_n$ such that $M(S(\mathbf{a}_n)) \nrightarrow 0$. This implies, by Lemma 5, that for all its neighboring actions, $\tilde{a}_n$, also $M(S(\tilde{a}_n)) \nrightarrow 0$ and so on. As a result for all actions induced in equilibrium the measure of the support set is bounded away from 0, a contradiction because then only a finite number of actions can be induced.□

Now note that by Proposition 2, for high conflicts, all actions induced in equilibria must be close to some line with slope β^*. Let l^{β^*} be the line of slope β^* passing through μ^y on the reformulated $y - axis$.

Let $D = \{\boldsymbol{\theta} \in \Theta^*|\delta^*\theta_1 + \gamma' \geq \theta_2 \geq \delta^*\theta_1 + \gamma''\}$ for some γ', γ'' such that $|\gamma' - \gamma''| > 0$ and $d(\boldsymbol{\theta}', Co(Graph(\gamma(x)) \cap D)) > \lambda$ for some $\lambda > 0$ for all $\boldsymbol{\theta}' \in D \cap l^{\beta^*}$. To find such a D

we need to know that there is an interval in which there is no infinite crossing between the line and the reaction curve $\gamma(x)$ which must follow from the fact that the intersection of the solution for A and B is empty. This follows from the continuity of $\gamma(x)$.

Let $A_n^D = \{\mathbf{a}_n \in A_n \cap D\}$. Note that A_n^D is not empty since that would be a contradiction to Lemma 8. Recall that these actions must be close to l^{β^*}. Since $M(S(\mathbf{a}_n)) \to 0$ for any induced $\mathbf{a}_n$, also $S(\mathbf{a}_n) \subset D$ almost surely for all $\mathbf{a}_n \in A_n^D$ and there are no $\mathbf{a}'_n \notin D$ such that $S(\mathbf{a}'_n) \subset D$. Denote the expectations over D by $E(D)$. We now take the expectation over actions in A_n^D. Each action is close to l^{β^*} and is weighted by the measure of its support group. But the difference between this expectations to $E(D)$ must converge to 0, i.e., $d(E(D), l^{\beta^*}) \to 0$.

On the other hand, $E(D)$ must converge to lie within the convex hull of $Graph(\gamma(x)) \cap D$. This follows from the fact that $Graph(\gamma(x)) \cap D$ is the set of expectations over all lines with slope δ^* that go through D. But then $d(E(D), Co(Graph(\gamma(x)) \cap D)) \to 0$. Taken together with $d(E(D), l^{\beta^*}) \to 0$, this is in contradiction to the choice of D. Therefore, an equilibrium with infinitely many induced actions cannot be sustained.

This complete the proof of part (i) of Theorem 1.□

PROOF OF PART (ii):

We define a local perturbation to $\gamma(x)$ as a perturbation in a strip $[x', x'']$ such that for any $x \in$*Solution(B)* there exists an i such that $[x', x''] \subset [x_{i-1}, x_i]$. Due to the fact that any solution to B is finite, this definition is not restrictive.

LEMMA 9 *Suppose that $x \in$Solution(A)$\cap$Solution(B). Following any perturbation of $F(.)$ which maintains the same $f^{\beta^*}(x)$ but is a local perturbation to $\gamma(x)$, then $x \notin$Solution(A)$\cap$Solution(B).*

Proof: Denote by $\hat{\mu}^y_{x_{i-1},x_i}$ the values after the perturbation and by $\mu^y_{x_{i-1},x_i}$ the values before the perturbation. With a local change in $\gamma(x)$, then it must be that for any $x \in$*Solution(A)$\cap$Solution(B),* then $\hat{\mu}^y_{x_{i-1},x_i} \neq \mu^y$ for some unique i (since the local perturbation is inside this strip). Thus, for any x' which was part of a solution x to the original problem, it is now the case that $\hat{\mu}^y_{0,x'} \neq \hat{\mu}^y_{x',\bar{x}}$ because by the algorithm of constructing solutions to problem B it is either the case that $x_i \leq x'$, in which case $\hat{\mu}^y_{x',\bar{x}} = \mu^y$ but $\hat{\mu}^y_{0,x'} \neq \mu^y$ or that $x_i \geq x'$ in which case $\hat{\mu}^y_{0,x'} = \mu^y$ but $\hat{\mu}^y_{x',\bar{x}} \neq \mu^y$. Thus, by the proof of Proposition 3, x' cannot be a part of a new solution.□

Denote by $\gamma'(x)$ the perturbed reaction curve. Let $\varepsilon = \max d(\gamma'(x), \gamma(x))$ and term the set of perturbations described above as $\varepsilon-$perturbations. Let B denote the original problem and B' denote the problem after an $\varepsilon-$perturbation. Similarly, let x denote an original solution and let x' denote a solution after an $\varepsilon-$perturbation.

LEMMA 10 *Following any $\varepsilon-$perturbation of $F(.)$ then generically for any $x' \in$Solution(B'),* *there exists an* $x \in$*Solution(B) such that* $\lim_{\varepsilon \to 0} |x_i - x_i'| = 0$ *for all* i,$x_i \in x$ *and* $x_i' \in x'$*.*

Proof: Consider the original solution $x \in$*Solution(B).* Generically, for any x_j which is part of this vector, then $\frac{d\gamma(x)}{dx}|_{x=x_j} > 0$ or $\frac{d\gamma(x)}{dx}|_{x=x_j} < 0$. Given a local perturbation in some strip $[x_{i-1}, x_i]$, the new prior is $\hat{\mu}^y$. Without loss of generality, assume that $\hat{\mu}^y > \mu^y$. Consider now the algorithm of finding a solution to problem B, as outlined in the proof of Proposition 3. In the new problem B', the first value of x', defined as x_2', has to satisfy $\hat{\mu}^y_{0,x_2'} = \hat{\mu}^y$. But when $\varepsilon \to 0$, then $\hat{\mu}^y \to \mu^y$ and hence $\hat{\mu}^y_{0,x_2'} \to \mu^y_{0,x_2}$ for x_2 which the smallest value of x which satisfies $\mu^y_{0,x_2} = \mu^y$ at the original problem. If then $\frac{d\gamma(x)}{dx}|_{x=x_2} > (<)0$ there is a value $x_2' > (<) x_2$, such that $\lim_{\varepsilon \to 0} |x_2 - x_2'| = 0$, which satisfies that $\hat{\mu}^y_{0,x_2'} = \hat{\mu}^y$. The same follows for all solutions of the algorithm.□

We can now complete the proof of this part. Note that the set of solutions for A does not change with an $\varepsilon-$perturbation. Following any such $\varepsilon-$perturbation, when ε is small enough, then each original solution for B changes infinitesimally. However, the set of solutions for A is not a continuum. Then, no new solution for B can coincide with an original solution for A. This complete part (ii) and the proof of Theorem 1.■

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