Partisan Politics and Aggregation Failure with Ignorant Voters[†]

Faruk Gul

and

Wolfgang Pesendorfer

Princeton University

August 2004

Abstract

We explore the implications of voter ignorance on policy selection and policy outcomes in a model of party competition. We show that voter ignorance of the parties' policy choice has no effect on the outcome of a large election in a model where voters know the distribution of preferences. We then explore a model where voters are ignorant of policy choices and of the distribution of preferences in the electorate. We characterize the limit equilibria (as the number of voters gets large) and show that parties may fail to choose the median favored policy (*partisan politics*) and that voters may reject the median preferred alternative among the available options (*aggregation failure*). We show that these non-Downsian conclusions are most pronounced if parties have weak policy preferences and mostly care about winning the election.

[†] Financial support from the National Science Foundation is gratefully acknowledged.

1. Introduction

Surveys routinely find that the American electorate is poorly informed about the policy positions of candidates. (For an early reference, see Berelson, Lazarsfeld and McPhee (1954)). In a 1990/1991 survey only 57% of voters could correctly identify relative ideological positions of the Republican and Democratic parties on a left/right spectrum and only 45% of voters could correctly identify the parties' relative position on federal spending (Delli-Carpini and Keeter (1993), Table 2). The same surveys show that voters are equally ill informed about the electorate itself. For example, the 1990/1991 survey shows that only 47% of voters correctly identify the party that holds the majority of seats in the Senate (Delli-Carpini and Keeter (1993), Table 2).

In this paper, we explore the implications of voter ignorance on policy selection and policy outcomes. We consider a very simple and stylized candidate competition model. There are two candidates; one candidate (the *committed candidate*) has a fixed and known policy position (r) while the other candidate (the *opportunistic candidate*) must choose between a partisan policy (denoted l) and a moderate policy (denoted m). The median voter prefers m to r and r to l. Hence, the median voter ranks the opportunistic candidate's moderate policy at the top and the partisan policy at the bottom. The opportunistic candidate is motivated both by winning the election and by the resulting policy outcome. In particular, we assume that the opportunistic candidate prefers the partisan to the moderate policy.

The standard model of party competition (Downs 1957) considers two candidates who seek to maximize the probability of getting elected. Both candidates choose policies prior to the election. Voters observe the policy choice and vote for the candidate who offers the more attractive policy. The model predicts that the median preferred policy will be implemented. Candidate competition is similar to Bertrand competition in oligopoly models. If a candidate chooses a policy that is not median preferred to his opponent's policy then he will lose the election. As a result, the Downsian prediction of median preferred outcomes holds even when the candidates have policy preferences.

Our model differs from the standard Downsian model in three ways: first, we assume that some voters are ignorant of the opportunistic candidate's policy choice. We incorporate this ignorance into a strategic model of candidate competition by assuming that each voter is independently informed of the realized policy choice with a probability between zero and one.

Second, we assume that voters are ignorant of other voters' preferences. We model this ignorance by introducing a state variable that determines the distribution of preferences and whose realization is unobserved by voters.

Third, the (opportunistic) candidate learns the distribution of preferences prior to selecting a policy. Hence, the candidate makes his policy choice knowing the distribution of preferences in the electorate while voters are ignorant of the distribution of preferences when they cast their vote. Our assumptions are motivated by the evidence on voter ignorance and by the fact that candidates often take (secret) opinion polls prior to selecting a policy. These opinion polls may provide good information about the distribution of preferences.

Our focus is on large elections. Therefore, we study limit equilibria when the number of voters goes to infinity. The parameters are chosen so that for a large electorate the moderate policy m is median preferred with probability close to one for every realization of the distribution of voter preferences. We normalize the (von Neumann-Morgenstern) utility function of the opportunistic candidate so that his utility of winning the election with the partisan policy is 1, his utility of losing the election is 0, and his utility of winning with the moderate policy is $\mu \in (0,1)$. Hence, μ close to zero describes a candidate who derives utility from winning only if he can implement his favored policy while μ close to 1 describes a candidate who is motivated primarily by winning the election. When μ is close to one we refer to the candidate as an "office seeker". Many of our results focus on the case where the opportunistic candidate is an office seeker since we find this to be the more descriptive case.

Our main results establish the following departures from the standard Downsian model:

(1) *Non-median Policy Choice:* The policy choice of the opportunistic candidate need not reflect the preferences of the median voter. Rather, in some states of the world the opportunistic candidate chooses his favored policy. We will refer to this departure from the predictions of the Downsian model as *partisan politics.*

(2) *Non-median Election Outcomes:* The candidate who offers the median preferred policy may lose the election. We will refer to this phenomenon as *aggregation failure.*

(3) *Partisan politics and aggregation failure when the opportunistic candidate is an office seeker:* We show that the probability of partisan politics is maximized and the probability of aggregation failure is positive when the opportunistic candidate is an office seeker. Moreover, increasing μ leads to an increase in the likelihood of partisan politics under appropriate curvature assumptions. Under additional distributional assumptions, we show that the probability of aggregation failure is independent of μ.

Item (3) above distinguishes our analysis from other models that yield partisan politics.[1] In our model, the less the candidate cares about policy, the more likely he is to engage in partisan politics. This suggests that we should expect partisan politics and aggregation failure even in a situation where parties select candidates with the objective of maximizing the probability of winning the election.

The key for the opportunistic candidate's success in our model is his willingness to choose a moderate policy when he expects the election to be close. An office seeker is reluctant to choose the partisan policy and risk losing the election if he expects the election to be close. Therefore, conditional on a vote being pivotal an office seeker is expected to choose the partisan policy with a small probability. In contrast, a candidate with a strong preference for the partisan policy is expected to choose the partisan policy even if by doing so he risks losing the election with significant probability. As a result, the office seeker receives a much larger share of the uninformed vote than a candidate with a strong partisan preference.

The willingness of a candidates to choose a median preferred policy when the election is close does not imply that this candidate will have a high ex ante probability of choosing the median preferred policy. In fact, the probability that the opportunistic candidate implements the partisan policy is maximal if the candidate is an office seeker. Because the office seeker receives a large share of the uninformed vote he can afford to choose the partisan policy and win the election in situations where the candidate with a strong partisan preference would lose the elections. Conditioning on being pivotal creates a wedge

[1] Such a model is presented by Calvert (1985).

between voting behavior and (unconditional) policy choices. Candidates with a weak policy preference benefit from this effect.

To isolate the effects of our three main assumptions, we consider two alternative versions of our main model. First, we study a benchmark model in which voters are ignorant of the candidate's policy choice but both the voters and the candidate know the distribution of preferences. We show that in that model Downsian prediction are attained: in large elections, the median preferred policy is implemented with probability one and therefore neither partisan politics nor aggregation failures can occur. Despite the fact that only a fraction of voters are informed of the opportunistic candidate's policy choice the outcome is *as if* all voters are perfectly informed. This result echoes the information aggregation results in Feddersen and Pesendorfer (1996, 1997).

Our analysis also relies on the assumption that the opportunistic candidate is well-informed of the distribution of voter preferences while voters are not. We show this by introducing a model where both the voters and the opportunistic candidate are ignorant of the distribution of preferences. We show that there are two possibilities if the candidate is an office seeker. If the fraction of informed voters is sufficiently small, then the opportunistic candidate chooses the partisan policy but loses the election. If the fraction of informed voters is sufficiently high than the median preferred outcome is implemented. Hence, aggregation failure is never observed. If very few voters are informed, equilibrium reflects the opportunistic candidate's credibility problem. He cannot commit to the median preferred policy and therefore he cannot win the election. Otherwise, equilibrium resembles the equilibrium of a Downsian model.

The evidence of voter ignorance may be considered puzzling since one might expect political competition to force candidates the devote resources to informing the voters of their position. In section 5, we investigate this hypothesis. We find that giving the candidate the opportunity to increase the proportion of informed voters has no effect on election outcomes when the candidate is an office seeker. Hence, adding voluntary disclosure to the model does not mitigate partisan politics or aggregation failure. This is true even though informing voters is assumed to be costless. Our analysis suggests, however, that informing voters about the opponent's position, provided such information can be revealed in a credible way, may be an effective remedy for partisan politics and aggregation failure. Hence, we find a role for "negative campaigning".

1.1 Related Literature

Several authors have examined the robustness of the Downsian prediction by introducing uncertainty about the electorate and policy preferences of candidates. For example, Calvert (1985) analyzes the case where two candidates are symmetrically informed of the uncertain distribution of voter preferences. Bernhard, Duggan and Squintani (2003) and Chan (2001) analyze the case of asymmetric information. In their models, there is no voter ignorance but policy outcomes differ from the predicted median's preferred policy because candidates have policy preferences and are uncertain about the distribution of voter preferences. Hence, their model is similar to the comparison model we provide in section 4, and yields similar results: if candidates have weak policy preferences and mostly care about winning the election then the Downsian is attained. Otherwise, the candidates trade-off probability of losing the election against winning with a less desired policy. In all cases, there is no aggregation failure since voters know the policy choices of the candidates.

There is a long tradition of models examining the information aggregation properties of elections. A classic result in this area is the Condorcet Jury theorem. (See, for example, Young (1988)). Traditional jury models assume that voters do not behave strategically. Austen Smith and Banks (1995) and Feddersen and Pesendorfer (1996) analyze models closely related to the jury model under the assumption that voters act strategically. The information aggregation literature assumes that voters are uncertain about a state variable that affects their ranking of candidates. In our context, this corresponds to a situation where the opportunistic candidate's policy is chosen by some exogenous random draw. The difference here is that the candidate's policy choice is a strategic choice. Our analysis of the benchmark model (i.e., when both the voters and the candidate are informed of the electorate), show that the information aggregation result can be extended to the case where candidates choose policies strategically.

In a series of papers, McKelvey and Ordeshook (1985, 1986) argue that even if voters are ignorant of policy choices they may still be able to infer correctly which candidate offers the preferred policy from polling data, endorsements, and other public information.

In other words, McKelvey and Ordeshook argue that ignorance about policy choices alone may not lead to non-median outcomes. We provide a similar result. If all voters have access to accurate opinion polls that identify the distribution of preference types then we are in the benchmark case where information is aggregated and voter ignorance about policy choice is irrelevant. However, if some fraction of voters remains uninformed about the distribution of preferences then the election cannot aggregate information and non-median outcomes will result.

2. The Benchmark Model

In this section we present a model of electoral competition where some voters are ignorant of one candidate's policy choice. We assume that the candidates and voters know the distribution of voter preferences. In the next section we present our main model where voters (but not the opportunistic candidate) do not know the distribution of voter preferences and are uncertain of the policy of a candidate. The analysis of this section indicates that voter ignorance regarding policies alone can neither explain partisan politics nor aggregation failure.

In the benchmark model V_n^0, two candidates stand for election. Candidate a is committed to a fixed policy (denoted r) while candidate b must choose between a moderate policy (denoted m) and a partisan policy (denoted l). Let $O = \{l, m, r\}$ denote the set of possible policy outcomes.

The payoff of candidate b is 1 if he is elected and chooses the partisan policy l, $\mu \in (0, 1)$ if he is elected and chooses the moderate policy m, and 0 if he is not elected.

In the game V_n^0 there are $2n + 1$ voters. Voters are expected utility maximizers whose preference depends on the (policy) outcome of the election. We assume that all voters prefer m to r and r to l. Hence, all voter's prefer the moderate policy of b to the policy of a and the policy of a to the partisan policy of b. We normalize the voters' von Neumann-Morgenstern utility function so that the utility of policy l is zero, the utility of policy m is 1, and the utility of policy r is $\lambda \in T = [0, 1]$ which is the type of the voter. Therefore, if candidate b chooses m with probability λ (and l with probability $1 - \lambda$), then the voter type λ is indifferent between a and the lottery over policies offered by b. Types of voters are drawn according to the probability distribution F with support $[0,1]$. We assume that F admits a continuous density f on $[0,1]$ such that $f(\lambda) > 0$ for all $\lambda \in [0,1]$.

Candidate b does not observe the preference type of individual voters. Voters observe their own type but not the type of other voters. Candidate b chooses a policy $p \in \{l, m\}$. Each voter is independently informed of the policy choice with probability $\delta \in (0,1]$. If the candidate chooses a mixed action then informed voters observe the realization of the mixed action. Every voter must vote for one of the candidates. The candidate who receives $n+1$ or more votes wins the election.

A strategy for an uninformed voter specifies for every preference type the probability that the voter votes for candidate b. Hence, a strategy for an uninformed voter is a measurable function $\sigma^v : T \to [0,1]$ where $\sigma^v(\lambda)$ denotes the probability with which an uninformed voter of type λ votes for b. A strategy for b specifies a probability of choosing the moderate policy, denoted $\sigma^b \in [0,1]$.

We analyze symmetric Nash equilibria in weakly undominated strategies of the game V_n^0. Hence, we assume that in equilibrium all uninformed voters use the same strategy σ^v. The assumption that voters choose a weakly undominated strategy implies that informed voters always choose their preferred candidate. Note that all informed voters of type $\lambda \in (0,1)$ strictly prefer b if b chooses m and strictly prefer a if b chooses l. Since F is continuous the types $\lambda = 0$ and $\lambda = 1$ occur with probability 0. Therefore, with probability one there is a unique weakly undominated strategy for an informed voter: vote for a if b chooses l and for b if b chooses m.

Below, "equilibrium" refers to a symmetric Nash equilibrium in weakly undominated strategies. Suppressing the strategy of informed voters, an equilibrium can be characterized by a pair $\sigma = (\sigma^v, \sigma^b)$. Every strategy pair induces a probability distribution over outcomes, denoted ϕ where ϕ^o denotes the probability that policy $o \in \{l, m, r\}$ is implemented.

Let θ denote the probability that b chooses m conditional on a vote being pivotal given the strategy profile σ. For σ^v to be optimal all uninformed voters with $\lambda < \theta$ must vote for b while all voters with $\lambda > \theta$ must vote for a. (Note that $\lambda = \theta$ occurs with probability 0). Hence, in order to describe the equilibria of the election game, if is sufficient to consider strategies (σ^v, σ^b) consisting of two numbers; σ^b, the probability that candidate b chooses the moderate policy and σ^v, the cutoff level for uninformed voters.

Consider any voter strategy σ^v. Let $\pi^o(\sigma)$ denote the probability that a randomly selected voter votes for b conditional on the policy $o \in \{l, m\}$ of candidate b. That is;

$$\begin{aligned} \pi^l(\sigma^v) &= F(\sigma^v)(1-\delta) \\ \pi^m(\sigma^v) &= \pi^l(\sigma^v) + \delta \end{aligned} \tag{1}$$

For $x \in [0,1]$, let $B_n(x)$ be the binomial probability of at least $n+1$ successes out of $2n+1$ trials given that the probability of success in each trial is x. Hence,

$$B_n(x) = \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} x^k (1-x)^{2n+1-k} \tag{2}$$

Then, $B_n(\pi^o(\sigma^v))$ is the probability that candidate b wins the election V_n^0 given that b chooses policy $o \in \{l, m\}$ and voters use strategy σ^v. Hence, if σ is an equilibrium of V_n^0 then the corresponding outcome function is given by

$$\begin{aligned} \phi^l &= (1-\sigma^b) B_n(\pi^m(\sigma^v)) \\ \phi^m &= \sigma^b B_n(\pi^m(\sigma^v)) \\ \phi^r &= 1 - \phi^m - \phi^l \end{aligned} \tag{3}$$

For a fixed F, μ, δ satisfying the assumptions above, let $\mathcal{E}_n^0$ denote the set of all equilibria of V_n^0 (for the parameters F, μ, δ). Let $\Phi_n(\sigma)$ denote the outcome ϕ of σ as defined by equation (3) above. Let $\mathcal{E}^0$ denote the set of limit equilibria; that is, $\sigma \in \mathcal{E}^0$ if there exists a sequence $\{\sigma_n\}$ converging to σ such that $\sigma_n \in \mathcal{E}_n^0$ for all n. For $\sigma \in \mathcal{E}^0$, we use $\Phi(\sigma)$ to denote the set of possible outcomes associated with the limit equilibrium σ. That is, $\phi \in \Phi(\sigma)$ if there exists $\sigma_n \in \mathcal{E}_n^0$ for all n such that σ_n converges to σ and $\Phi_n(\sigma_n)$ converges to ϕ.

Proposition 1 characterizes equilibrium outcomes for large electorates. It states that in a limit equilibrium the moderate policy m is implemented with probability 1. Hence, the fact that a (possibly large) fraction of voters are ignorant of b's policy choice has virtually no impact on the election outcome when n is large.

Proposition 1: *If $\sigma \in \mathcal{E}^0$ and $\phi \in \Phi(\sigma)$ then $\phi^m = 1$.*

Proof: It is straightforward to show that a symmetric equilibrium in weakly undominated strategies exists. Here we show that as n becomes large, the probability of the moderate policy being implemented converges to one along any sequence of equilibrium outcomes.

If $\sigma_n \in \mathcal{E}_n^0$ and $\sigma_n^b = 0$ then b receives no votes and therefore a deviation to m strictly increases b's expected payoff. Hence, for all n, $\sigma_n^b > 0$. Next, note that if $\sigma_n^b = 1$ then a receives no votes and m is implemented with probability 1. Hence, it remains to consider the case where (along some subsequence) $\sigma_n^b \in (0,1)$ for all n.

Since b chooses both policies with strictly positive probability he must be indifferent between them. The indifference of b implies

$$\mu B_n(\pi^l(\sigma^v)) = B_n(\pi^m(\sigma^v)) \tag{4}$$

Next we show that (4) implies $\lim \pi^l(\sigma_n^v) = 1/2$ along any sequence $\sigma_n \in \mathcal{E}_n^0$. If $\pi^l(\sigma_n^v) \geq 1/2 + \epsilon$ along any (sub)sequence then $\lim B_n(\pi^l(\sigma_n^v))/B_n(\pi^m(\sigma_n^v) = 1$ violating (4). Similarly, $\pi^l(\sigma_n^v) \leq 1/2 - \epsilon$ for all n implies $\lim B_n(\pi^l(\sigma_n^v))/B_n(\pi^m(\sigma_n^v)) = 0$, again violating (4). Since $\pi^l(\sigma_n^v)$ converges to 1/2, equation (1) implies $\lim \sigma_>^v 0$. As we noted when proving that voters must use a cutoff strategy, voter optimality requires that σ_v equal the conditional probability that candidate b has chosen m given that the voter is pivotal. Hence,

$$\sigma_n^v = \frac{\sigma_n^b \binom{2n+1}{n+1} \pi^m(\sigma_n^v)^n (1 - \pi^m(\sigma_n^v))^n}{\sigma_n^b \binom{2n+1}{n+1} \pi^m(\sigma_n^v)^n (1 - \pi^m(\sigma_n^v))^n + \sigma_n^b \binom{2n+1}{n+1} \pi^l(\sigma_n^v)^n (1 - \pi^l(\sigma_n^v))^n} \tag{5}$$

Some simplification of (5) yields

$$\frac{\sigma_n^b}{1 - \sigma_n^b} \cdot \frac{\alpha_n^m}{\alpha_n^l} = \frac{\sigma_n^v}{1 - \sigma_n^v}$$

where $\alpha^o = \pi^o(\sigma^v)^n (1 - \pi^o(\sigma^v))^n$ for $o \in \{m, l\}$. Note that α_n^m / α_n^l converges to 0 since $\pi(\sigma_n^v)$ converges to 1/2. Therefore, $\lim \sigma_n^v > 0$ implies $\lim \sigma_n^b = 1$. Also, equation (1) yields $\lim \pi^m(\sigma_n^v) = 1/2 + \delta$. Hence the probability of b winning conditional on choosing m converges to 1, which completes the proof. $\square$

Proposition 1 establishes that neither partisan politics nor aggregation failure can occur in our benchmark model; the median preferred policy is chosen by candidate b and b wins the election. Proposition 1 is related to earlier information aggregation results in Feddersen and Pesendorfer (1997). There it is shown that for a fixed mixed strategy of b, as n goes to infinity, the probability that b is elected if he chooses l goes to zero and the probability that b is elected if he chooses m goes to one. This result does not imply Proposition 1 because the strategy here is endogenous, not fixed. Proposition 1 pins down both the behavior of the voters and the candidate.

To understand the intuition behind Proposition 1, consider the case where $\delta < 1/2$. The first step in the argument is to note that for large electorates, if b's expected vote share when choosing l is less than 1/2, then he strictly prefers choosing m to l. This is clear if b's vote share is greater than 1/2 conditional on m, which would mean that he wins for sure with m and loses for sure with l. If his vote share is less than 1/2 in both cases, then his probability of winning goes to zero with either policy, but it goes to zero much faster with l than with m. Hence, in both cases b strictly prefers m to l.

The second step is to note that for large n, candidate b must mix in equilibrium. If in equilibrium b were to choose l for sure, then his vote share would be less than 1/2 with both strategies. In that case, the argument above establishes that he strictly prefers m. If in equilibrium b were to choose m for sure then the support of the uninformed voters would guarantee victory for b irrespective of the policy choice. In that case, b strictly prefers l.

The third step is to observe that in order to maintain b's indifference between l and m, it is necessary for his expected equilibrium vote share to converge to 1/2 when b chooses l. If b's vote share when choosing l were greater than 1/2 then he wins for sure with l and hence would never choose m. If his vote share when choosing l is less than 1/2, then the argument above establishes that b would strictly prefer m.

Finally, since the probability of winning with l converges to 1/2 (and therefore the probability of winning with m converges to 1), conditional on a vote being pivotal it is much more likely that b has chosen l than m. To maintain the incentives for uninformed voters it must therefore be the case that b chooses l with vanishing probability as n goes to infinity. Hence, in large electorates b will choose m almost all the time and almost always win when he does so.

3. Uncertainty about the Electorate

The model in the previous section assumes that the distribution of preferences is known to all players. In this section, we analyze a game V_n with uncertainty about the distribution of preferences. Our purpose is to investigate a situation where voters are ignorant both of the policy choice and of the distribution of preferences in the electorate.

In the game V_n, each voter is independently assigned a preference type. The probability distribution of voter preferences depends on the state $s \in S = [0, 1]$ in the following way: with probability $(1-s)/2$ the voter is a partisan who always prefers b; with probability $s/2$ the voter is a partisan who always prefers a; and with probability $1/2$ the voter is a swing voter. The preference types of swing voters are drawn independently according to the distribution F with support $T = [0, 1]$. We assume that F admits a continuous density f with $f > 0$ on T. As in the previous section, all swing voters receive a utility of 1 if m is implemented and a utility of 0 if l is implemented. The utility of type λ when policy r is implemented is λ. Note that the probability that a voter is a swing voter is independent of the state s.

The timing of events is as follows. Nature draws a state $s \in S$ according to the distribution G. We assume that G admits a continuous density g such that $g > 0$ on S. Then, nature independently assigns each voter a preference type according to the probability distribution defined above. Candidates observe the state s but not the preference types of individual voters. Upon observing the state, b chooses a policy $o \in \{l, m\}$. Voters observe their own type but not the type of other voters or the state s. Each voter is independently informed of the policy choice of b with probability $\delta \in (0, 1)$. We assume that informed voters observe the *realization* of the policy choice if b chooses a mixed action. Voters must vote for one of the candidates. The candidate who receives $n + 1$ or more votes wins the election.

As in the benchmark game, a strategy for the uninformed swing voter specifies the probability that a voter of type λ votes for b, for every λ. Hence, a strategy for an uninformed swing voter is a function $\sigma^v : T \to [0, 1]$ where $\sigma^v(\lambda)$ denotes the probability with which an uninformed swing voter of type λ chooses the b. A strategy for b specifies a probability distribution over policies for each realization of the state s. Hence, a strategy for b is a measurable function $\sigma^b : S \to [0,1]$ where $\sigma^b(s)$ denotes the probability of choosing the moderate policy (policy m.)

We analyze symmetric Nash equilibria in weakly undominated strategies. Therefore, equilibrium strategies of uninformed voters are described by a single function σ^v. As before, informed swing voters (with probability 1) vote for b if b chooses m and for a otherwise. Similarly, partisan voters always choose their preferred candidate. Below, "equilibrium" refers to a symmetric Nash equilibrium in weakly undominated strategies. We suppress the behavior of partisan and informed voters and describe equilibria by a pair $\sigma = (\sigma^v, \sigma^b)$.

Given a strategy profile σ, any swing voter, regardless of whether he is informed or not, assigns strictly positive probability to being pivotal irrespective of the strategy of the other swing voters. Since this is the only event in which a voter can affect the election outcome, the optimal behavior of uninformed voters depends on the probability that b chooses policy m conditional on a vote being pivotal. Let $\theta \in [0,1]$ denote this probability. Recall that for $\lambda < \theta$ the voter strictly prefers b and for $\lambda > \theta$ the voter strictly prefers the a. Hence, as in our benchmark model, without loss of generality, we can assume that voters' strategy σ^v is cutoff strategy (i.e., $\sigma^v \in T = [0,1]$).

For any voter strategy σ^v, state $s \in S$, and policy $p \in \{l, m\}$, let $\pi^o(\sigma^v, s)$ denote the probability that a randomly selected voter votes for b in state s conditional on candidate b choosing policy o. That is;

$$\begin{aligned} \pi^l(\sigma^v, s) &= (1-s)/2 + (1-\delta)F(\sigma^v)/2 \\ \pi^m(\sigma^v, s) &= \pi^l(\sigma^v, s) + \delta/2 \end{aligned} \tag{6}$$

Hence, the probability that candidate b wins the election with $2n+1$ voter given that choose policy o in state s is $B_n(\pi^o(\sigma^v, s))$. (Recall that $B_n(x)$ is the binomial probability of at least $n = 1$ successes in $2n+1$ trials.) Hence, if σ is an equilibrium of the voting game with $2n+1$ voters then the corresponding outcome function is given by

$$\begin{aligned} \phi^l(s) &= (1-\sigma^b(s))B_n(\pi^m(\sigma^v, s)) \\ \phi^m(s) &= \sigma^b(s)B_n(\pi^m(\sigma^v, s)) \\ \phi^r(s) &= 1 - \phi^m(s) - \phi^l(s) \end{aligned} \tag{7}$$

For a fixed F, G, u, μ, δ satisfying the assumptions above, let $\mathcal{E}_n$ denote the set of equilibria of the game V_n. For $\sigma \in \mathcal{E}_n$, let $\Phi_n(\sigma)$ denote the corresponding outcome. We abuse notation and write $\mathcal{E}_n(\mu)$ or $\mathcal{E}_n(\mu, \delta)$ when wish to be explicit about a particular set of parameters.

We have already noted that every optimal voter strategy is characterized by a single cutoff type $\lambda = \sigma^v$. Our first result establishes that every equilibrium strategy of candidate b is also described by a cutoff. That is, in any equilibrium there is a state $s \in S$ such that b chooses l at states $s' < s$ and m at $s' > s$ (with probability 1). We call an equilibrium in which voters and candidate b use a cutoff strategy a cutoff equilibrium. Hence, a cutoff equilibrium consists of a pair numbers $(\sigma^v, \sigma^s) \in T \times S$. Since the probability of any single $\lambda = \sigma^v$ or any single $s = \sigma^b$ is zero, these two numbers provide a sufficient description of a cutoff equilibrium. Proposition 2 establishes the existence of an equilibrium. It also shows that every equilibrium is a cutoff equilibrium.

Proposition 2: *For all n, $\mathcal{E}_n$ is non-empty and every $\sigma \in \mathcal{E}_n$ is a cutoff equilibrium.*

Proof: see Appendix.

Since we know that every optimal strategy of the voters is a cutoff strategy, to establish that all equilibria are cutoff equilibria, it is enough to show that b's best response to any cutoff strategy is also a cutoff strategy. In particular, we will show that given a cutoff strategy for the voters, if it is optimal for b to choose m at state s, then the only optimal action for him at higher states is m. Note that candidate b chooses l if

$$B_n(\pi^l(\sigma^v, s)) > \mu B_n(\pi^m(\sigma^v, s))$$

and m if this inequality is reversed. Hence, to prove that m is optimal at s implies it is the only optimal action at $s' > s$, we need to show that $B_n(\pi^l(\sigma^v, s))/B_n(\pi^m(\sigma^v, s))$ is decreasing in s. In Lemma 1, we show that B_n is log-concave. Since $\pi^m(\sigma^v, s) = \pi^l(\sigma^v, s) + (1 - \lambda)\delta$ and $\pi^m(\sigma^v, s)$ is a linear and decreasing function of s, log-concavity of B_n implies that $B_n(\pi^l(\sigma^v, s))/B_n(\pi^m(\sigma^v, s))$ is decreasing in s. We use a fixed-point argument to establish the existence of a cutoff strategy equilibrium.

Proposition 3 below characterizes the set of limit equilibria $\mathcal{E}$. We say that $s^o(\lambda)$ is the critical state for policy o at cutoff λ if $\pi^o(\lambda, s) = 1/2$. Hence, the critical state of policy $o \in \{l, m\}$ is the state at which a randomly drawn voter chooses b with probability 1/2 if b chooses policy o. Equation (5) establishes that critical states satisfy

$$\begin{aligned} s^l(\lambda) &= (1-\delta)F(\lambda) \\ s^m(\lambda) &= (1-\delta)F(\lambda) + \delta \end{aligned} \tag{8}$$

Clearly both s^l and s^m are increasing functions of λ.

Proposition 2 above enables us to define the limit equilibria and their outcomes for the current model as we have done for the benchmark model in the previous section. Let $\mathcal{E}$ denote the set of limit equilibria; that is $\mathcal{E}$ is the set of σ such that $\sigma = \lim_{n\to\infty} \sigma_n$ for $\sigma_n \in \mathcal{E}_n$ for all n. Then, define $\Phi(\sigma)$ as the set of all ϕ such that for some sequence σ_n with $\sigma_n \in \mathcal{E}_n$ for all n, converging to σ, $\Phi_n(\sigma_n)$ converges to ϕ.

Proposition 3 provides a simple formula for the equilibrium cutoff of voters and establishes that the equilibrium cutoff of candidate b is equal to the critical state for policy l at the voters equilibrium cutoff. Hence, there is a unique outcome $\phi \in \Phi(\sigma)$ associated with any limit equilibrium σ.

Proposition 3: *(i) If $\sigma \in \mathcal{E}$ and $\phi \in \Phi(\sigma)$ then*

$$\begin{aligned} \sigma^b &= s^l(\sigma^v) \\ \sigma^v &= \frac{g(s^m(\sigma^v))}{g(s^m(\sigma^v)) + g(s^l(\sigma^v))(1-\mu)} \end{aligned}$$

$$\begin{aligned} \phi^l(s) &= 1 \qquad \textit{if } s < s^l(\sigma^v) \\ \phi^m(s) &= 1 \qquad \textit{if } s^l(\sigma^v) < s < s^m(\sigma^v) \\ \phi^r(s) &= 1 \qquad \textit{if } s^m(\sigma^v) < s \end{aligned}$$

(ii) If g is log-concave then there is a unique $\sigma \in \mathcal{E}$.

Proof: see Appendix

The intuition behind the description of the outcome associated with a limit equilibrium σ is straightforward. The probability that a randomly selected voter will prefer candidate

b even if b chooses l is greater than $1/2$ at any state $s < s^l(\sigma^v)$. Hence, with a large electorate, b wins the election with probability one when b chooses l. Since b strictly prefers l to m it follows that l must be chosen. Hence, $\sigma^b(s) = 0$ and $\phi^l(s) = 1$ for states $s < s^l(\sigma^v)$. Conversely, at states $s > s^m(\sigma^v)$ the probability that a randomly selected voter will prefer b is less than $1/2$ even when b chooses m. However, as n grows, the probability winning with l goes to zero faster than the probability of winning with m. Hence, at such states $\sigma^b(s) = \phi^r(s) = 1$. While at states $s \in (s^l(\sigma^v), s^m(\sigma^v)$ candidate b wins the election if and only if he chooses m. Hence, $\sigma^b(s) = \phi^m(s) = 1$.

–Insert figure 1 here–

Next, we provide intuition for the characterization of the limit voter cutpoint in Proposition 3. Note that as the number of voters becomes very large, the probability of being pivotal is concentrated around states in which the election is expected to be tied. There are two such states, $s^l(\sigma^v)$ and $s^m(\sigma^v)$; Hence, conditional on being pivotal, a voter knows that the state is in one of two small "critical intervals" around the critical states. The inference problem for the uninformed voter therefore reduces to determining the relative likelihoods of $s^l(\sigma^v)$ and $s^m(\sigma^v)$ conditional on a vote being pivotal.

Consider the incentives for candidate b with these intervals. If the probability of winning with l is less than μ-times the probability of winning with m then b strictly prefers the moderate policy m. Therefore, the critical interval around $s^l(\sigma^v)$ is truncated at the point where the probability of winning drops below μ. (The probability of winning the election with m in a neighborhood of $s^l(\sigma^v)$ is close to one). Hence, the closer the parameter μ is to 1 the smaller the critical interval. The key step in the proof is to show that the relative likelihood of the $s^l(\sigma^v)$ and $s^m(\sigma^v)$ is related to μ by the simple formula given in Proposition 3.

To understand part (ii) of the proposition, note that by equation (6), $s^m(\sigma^v) - s^l(\sigma^v) = \delta > 0$ does not depend on σ^v. Since $s^l(\sigma^v)$ is increasing in λ, $\frac{g(s^m(\sigma^v))}{g(s^l(\sigma^v))}$ is non-increasing whenever $\frac{g(x+k)}{g(x)}$ is a non-increasing function of x for $k > 0$; that is, whenever g is log-concave. Hence, the equation that defines σ^v in Proposition 3 can be satisfied by at most one σ^v. There is a unique $s^l(\sigma^v)$ corresponding to this σ^v. Since any sequence of σ_n lies in a compact set, the existence of a limit equilibrium is ensured.

Our objective is to investigate the likelihoods of partisan politics and aggregation failure in limit equilibria. Let $L(\sigma), M(\sigma)$ and $R(\sigma)$ denote the ex ante probabilities of the policies l, m and r being implemented given the limit equilibrium σ. By Proposition 3 (see figure 1), these probabilities can be computed as follows:

$$\begin{aligned} L(\sigma) &= G(\sigma^v) \\ M(\sigma) &= G(s^m(\sigma^v)) - G(\sigma^v) \\ R(\sigma) &= 1 - G(s^m(\sigma^v)) \end{aligned} \tag{9}$$

Candidate b wins the election when policy l or policy m are implemented. Let $W(\sigma)$ denote the probability that b wins the election in the limit equilibrium σ. We have

$$W(\sigma) := L(\sigma) + M(\sigma) \tag{10}$$

Partisan politics occurs when b chooses a policy that does not reflect the preferences of the median voter. In our model, partisan politics occurs when b chooses l. Let $P(\sigma)$ denote the probability of partisan politics in a limit equilibrium σ. Note that by Proposition 3 (see also Figure 1) b wins the election whenever he chooses l and therefore

$$P(\sigma) = L(\sigma) \tag{11}$$

An aggregation failure occurs when the candidate who offers the median preferred policy loses the election. Let $A(\sigma)$ denote the probability of aggregation failure in a limit equilibrium σ. Aggregation failure occurs if b choose policy l and wins or if b chooses policy m and loses. Hence,

$$A(\sigma) = L(\sigma) + R(\sigma) \tag{12}$$

In Proposition 4 below, we analyze how changes in the parameter μ affect limit equilibria. Note that μ measures the strength of the partisan preference of candidate b. An increase in μ means that candidate b's preference for a partisan policy becomes weaker.

For example, as μ approaches 1, the candidate cares only about winning the election and does not care whether he wins with the partisan or the moderate policy. We describe this situation as the candidate being an office seeker.

Proposition 4(i) shows that an office seeker wins the election for sure despite the fact that he maximizes the probability of partisan politics. In Proposition 4(ii) we consider the case where the uniqueness of the limit equilibrium is guaranteed and assume that g is differentiable. Under these assumption an increase in μ (and hence a decrease in the partisan preference of b) implies an increase in the probability of b winning and an increase in the probability of partisan politics. Finally, Proposition 4(iii) shows that when G is uniform the likelihood of aggregation failure does not depend on μ.

Proposition 4: *(i) Let $\lim \mu^k = 1$, $\sigma^k \in \mathcal{E}(\mu^k)$ for all k and $\sigma \in \mathcal{E}(\mu)$ for $\mu \in (0,1)$. Then*

$$1 = \lim W(\sigma^k) > W(\sigma) \quad \text{and} \quad \lim P(\sigma^k) > P(\sigma)$$

(ii) If g is log-concave and differentiable, and $\mu' > \mu$ then

$$W(\sigma') > W(\sigma) \quad \text{and} \quad P(\sigma') > P(\sigma)$$

whenever $\sigma' \in \mathcal{E}(\mu')$ and $\sigma \in \mathcal{E}(\mu)$. (iii) If G is uniform then

$$A(\sigma) = 1 - \delta$$

for all $\mu \in (0,1)$ and $\sigma \in \mathcal{E}(\mu)$.

Proof: It follows from Proposition 3 that $\sigma^k \in \mathcal{E}(\mu^k)$ and $\lim \mu^k = 1$ implies $\lim \sigma^{vk} = 1$. Then equations (9) – (11) prove part (i). To prove part (ii) note that by Proposition 3(i) and equation (8), any equilibrium voter cutoff σ^v must solve

$$\sigma^v(1 + \rho(\sigma^v)(1 - \mu)) = 1$$

where

$$\rho(\sigma^v) = \frac{g((1-\delta)F(\sigma^v))}{g((1-\delta)F(\sigma^v) + \delta)}$$

Since F is strictly increasing, g is differentiable and log-concave, it follows that $\rho'(\sigma^v) := \partial\rho(\sigma^v)/\partial\sigma^v \geq 0$. Taking a total derivative therefore yields

$$\frac{d\sigma^v}{d\mu} = \frac{\sigma^v \rho(\lambda)}{1 + \rho(\sigma^v(1-\mu)) + \sigma^v \rho'(\sigma^v)(1-\mu)} > 0$$

as desired. When G is uniform, equations (8), (9) and (12) establish that $A(\sigma) = 1 - \delta$ for all μ and all $\sigma \in \mathcal{E}(\mu)$. □

To understand Proposition 4, note that the second line of Proposition 3(i) ensure that σ^v approaches 1 as μ approaches 1. Hence, when candidate b is an office seeker all uninformed voters voter for him. Then, candidate b wins the election for sure if he chooses m. Hence, candidate b never loses. Note that candidate b can never win the election with policy l if informed voters together with the partisan voters constitute a majority. On the other hand, if all uninformed voter vote b, the informed voters and the partisans of a are the only ones voting for a. Hence, an office seeker maximizes the probability of winning with l which is equal to $P(\sigma)$. Part (ii) of the proposition is related to the uniqueness result in Proposition 3. Given differentiability, the log-concavity ensures that σ^v is increasing in μ, which yields the desired comparative statics.

Note that while an increase in σ^v leads to an unambiguous increase in $P(\sigma)$, its effect on $A(\sigma)$ is not clear. To see this recall that aggregation failure does not occur if and only if the outcome is m.

$$\begin{aligned} A(\sigma) =& 1 - M(\sigma) \\ =& 1 - G(\sigma^v) + G(s^m(\sigma^v)) \end{aligned}$$

An increase in σ^v increases $s^m(\sigma^v)$ and hence the net affect on $A(\sigma)$ is unclear. However, when G is uniform, we have

$$\begin{aligned} A(\sigma) =& 1 - M(\sigma) \\ =& 1 - G(\sigma^v) + G(s^m(\sigma^v)) \\ =& 1 - \delta \end{aligned}$$

for all $\mu \in (0, 1)$ and $\sigma \in \mathcal{E}(\mu)$.

Proposition 4(i) implies that an office seeker is elected with probability 1, as in the benchmark model V^o. However, in contrast to the benchmark model the probability of partisan politics and the probability of aggregation failure do not go to 0. In the case where G is uniform, the probability of aggregation failure is $1-\delta$, the fraction of swing voters that are uninformed, regardless of μ.

Next, we analyze the effect of a change in the distribution of swing voter preferences on the limit equilibrium outcomes. Let $F_\alpha, \alpha \in [0,1]$ denote a family of probability distributions over preference types $T := [0,1]$. Assume that $F_\alpha(\lambda)$ is increasing and differentiable in α. Hence, a decrease in α implies a first order stochastically dominant shift in the distribution of preferences (in favor of candidate b).

Proposition 5 asserts that shifting swing voter preferences towards candidate b has the same effect as increasing μ: candidate b will win more frequently and choose the partisan policy more frequently.

Proposition 5: *If g is log-concave and differentiable, $\alpha > \alpha'$, then*

$$P(\sigma') > P(\sigma) \quad \text{and} \quad W(\sigma') > W(\sigma)$$

whenever $\sigma' \in \mathcal{E}(F_{\alpha'})$ and $\sigma \in \mathcal{E}(F_\alpha)$.

Proof: By Proposition 3 and equation (8), an equilibrium σ^v must solve

$$\sigma^v(1 + \rho(\sigma^v, \alpha)(1-\mu)) = 1$$

where

$$\rho(\sigma^v, \alpha) := \frac{g((1-\delta)F_\alpha(\sigma^v))}{g((1-\delta)F_\alpha(\sigma^v) + \delta)}$$

Since g is differentiable and log-concave and F_α is strictly increasing, it follows that

$$\partial\rho/\partial\sigma^v \geq 0, \partial\rho/\partial\alpha \leq 0$$

Therefore, taking a total derivative yields

$$\frac{d\sigma^v}{d\alpha} = \frac{-\sigma^v(1-\mu)\partial\rho/\partial\alpha}{1 + (1-\mu)(\rho + \sigma^v\partial\rho/\partial\sigma^v)} \geq 0$$

□

As in the case of an increase in μ, a shift in the distribution of swing voters' preferences towards candidate b increases the probability that an uninformed voter votes for b. Therefore the probability of a partisan outcome and similarly, the probability that b is elected is strictly increasing in α.

Next, we analyze to role of δ, the probability that a voter is informed, on limit equilibrium outcomes. Of particular interest is the case when μ approaches 1 and δ approaches 0; that is, when an office seeker faces a (almost) completely ignorant electorate. Proposition 6 establishes that in this case, both partisan politics and aggregation failure are observed with probability 1. The case where F and G are both uniform leads to unambiguous comparative statics in δ for any μ. In this case, a more ignorant electorate implies more partisan politics and greater aggregation failure.

Proposition 6: *(i) If* $\lim(\mu^k, \delta^k) = (1, 0)$ *and* $\sigma^k \in \mathcal{E}(\mu^k, \delta^k)$ *for all* k*, then*

$$\lim P(\sigma^k) = \lim A(\sigma^k) = 1$$

(ii) If both F, G *are uniform and* $\delta > \delta'$

$$P(\sigma') > P(\sigma) \quad \text{and} \quad A(\sigma') > A(\sigma)$$

whenever $\sigma' \in \mathcal{E}(\delta')$ *and* $\sigma \in \mathcal{E}(\delta)$.

Proof: Proposition 3 ensures that $\lim \sigma^{vk} = 1$. Then, (i) follows from equations $(9)-(12)$. Part (ii) follows from Proposition 3 and equations $(9) - (12)$ as well. □

In our analysis of Proposition 4, we noted that all uninformed voters vote for b (i.e. $s^l(\sigma^v) = 1$) when b is an office seeker. This observation suffices to explain part (i) of Proposition 6. It is straightforward to verify using the characterization of limit equilibria in Proposition 3 and equations $(8)-(12)$ then when F, G are both uniform $P(\sigma) = (1-\delta)\sigma^v$. By Proposition 4, $A(\sigma) = 1 - \delta$ when G is uniform. The latter observation establishes that A is decreasing in δ for all μ. Proposition 3 also yields $\sigma^v = 1/(2 - \mu)$ and hence $P(\sigma)$ is also decreasing in δ.

4. Uninformed Candidates

In the analysis of the previous section, we assumed that the voters are ignorant of both the policy choice of candidate b and the electorate (i.e., the distribution of preferences). However, we also assumed that the candidates are informed. To illustrate the importance of this last assumption we briefly examine a version of the model where candidate b is also uninformed of the distribution of preferences. Consider an election game with an ignorant candidate b, V^b which is identical to the game in section 3 except in V^b candidate b cannot observe the parameter s. For simplicity assume that F and G are uniform on $[0, 1]$.

In this modified model, the strategy of the candidate cannot depend on s. As before, we consider symmetric equilibria in weakly undominated strategies. It is straightforward to show that the equilibrium strategy of voters is a cutoff strategy σ^v. Hence, we can define the set of limit equilibria $\mathcal{E}^c$ for V^b the same way that we defined $\mathcal{E}^0$ for V^0. Proposition 7 below characterizes the unique limit equilibrium of the game V^b. In this equilibrium, candidate b mixes between l and m with probability σ^b. The voter strategy σ^v is equal to σ^b. Hence, voters with type $\lambda < \sigma^b$ vote for b while voters with type $\lambda > \sigma^b$ vote for a.

Proposition 7: *For F, G uniform there is a unique limit equilibrium $\sigma \in \mathcal{E}^c(\mu, \delta)$ where,*

$$\sigma^v = \sigma^b = \begin{cases} 1 & \textit{if } \mu \geq \frac{1-2\delta}{1-\delta} \\ \frac{\delta}{(1-\mu)(1-\delta)} & \textit{if } \mu < \frac{1-2\delta}{1-\delta} \end{cases}$$

Proof: We note that in any limit equilibrium, candidate b wins at any state $s < s^l(\sigma^v) = (1-\delta)F(\sigma^v)$ if he adopts l while he wins with policy m at states $s < s^m(\sigma^v) = (1-\delta)F(\sigma^v)$ and he loses at any state $s > s^m(\sigma^v)$ no matter what policy he chooses. This follows from equation (7) and the fact that at any state where a random voter votes for $x \in \{a, b\}$ with probability greater than 1/2, the probability of x winning goes to 1 as the number of voters goes to infinity. Hence, (since F, G are uniform), the probability that b wins if he chooses m is $(1-\delta)\sigma^v + \delta$, while the probability that b wins if he chooses l is $(1-\delta)\sigma^v$. Hence, for a mixed limit equilibrium strategy to be optimal, we must have

$$\mu(1-\delta)\sigma^v + \mu\delta = (1-\delta)\sigma^v \tag{13}$$

Solving (13) for σ^v yields

$$\sigma^v = \frac{\delta}{(1-\mu)(1-\delta)} \tag{14}$$

It is easy to verify that (13 defines the unique equilibrium strategy for b if $\mu < (1-2\delta)/(1-\delta)$ and $\sigma^v = 1$ is the unique equilibrium strategy if $\mu \geq (1-2\delta)/(1-\delta)$.

To complete the proof we need to show that $\sigma^b = \sigma^v$. As we have argued in section 3, conditional on being pivotal, the voter assigns probability 1 that the state is either $s^l(\sigma^v)$ or $s^m(\sigma^v)$ and since G is uniform, he assigns equal probability to both events.[2] Optimality of an uninformed voter's strategy requires that σ^v equal the probability of b choosing m conditional on the voter being pivotal. Hence,

$$\sigma^v = \frac{\sigma^b/2}{\sigma^b/2 + (1-\sigma^b)/2} = \sigma^b$$

□

Let L^b, M^b, R^b be the ex ante probability of the outcomes l, m, r respectively and let P^b be the probability of partisan politics and A^b denote the probability of aggregation failure in V^b. Recall that for candidate b to win with the partisan policy l, the state has to be less than $s^l(\sigma^v)$ while with policy m, he wins at any state less than $s^m(\sigma^v)$. Hence, the ex ante probabilities of each outcome are easily computed to be

$$\begin{aligned} L^b(\sigma) &= (1-\sigma^b)G(s^l(\sigma^v)) \\ M^b(\sigma) &= \sigma^b(G(s^m(\sigma^v)) - G(s^l(\sigma^v))) \\ R^b(\sigma) &= 1 - G(s^m(\sigma^v)) \end{aligned}$$

Recall that partisan politics arise if candidate b chooses l while aggregation failure occurs if candidate b chooses l and wins or choose m and loses. Hence,

$$\begin{aligned} P^b(\sigma) &= 1 - \sigma^v \\ A^b(\sigma) &= (1-\sigma^b)G(s^l(\sigma^v)) + \sigma^b(1 - G(s^m(\sigma^v))) \end{aligned}$$

Proposition 8 establishes that in the game V^b, aggregation failure never occurs when an office seeker confronts a completely ignorant electorate. If δ goes to zero slower than

[2] For a formal statement and proof of a similar assertion, see the proof of Proposition 2.

$1-\mu$, the standard Downsian outcome prevails. Otherwise, the candidate (almost) always chooses the partisan policy and (almost) always loses.

Proposition 8: *If* (μ^k, δ^k) *converges to* $(1, 0)$, $\alpha := \lim(1-\mu^k)/\delta^k \neq 1$, *and* $\sigma^k \in \mathcal{E}(\mu^k, \delta^k)$ *for all* k, *then*

$$\begin{aligned} &\lim A^b(\sigma^k) = 0 \qquad \textit{and} \\ &\lim P^b(\sigma^k) = \lim R^b(\sigma^k) = 1 \qquad \textit{if } \alpha > 1 \\ &\lim M^b(\sigma^k) = 1 \qquad \textit{if } \alpha < 1 \end{aligned}$$

The intuition behind Proposition 8 is as follows. When an office seeker confronts almost completely ignorant electorate, there are two possibilities: either the fraction of informed voters is high enough to discipline candidate b, in which case he behaves as if there is perfect information and standard Downsian outcome attains, or there are two few informed voters in which case candidate b cannot commit to policy m and hence loses the election. The more partisan the candidate, the greater the fraction of informed voters needed to get the Downsian outcome. The simple relationship between δ and μ needed to ensure the Downsian outcome is due to our assumption of uniform F, G.

5. Control of Information

This section considers a situation where b can control the information about policy choices. As in the previous section, we assume that a has a fixed policy r and b chooses a policy $p \in \{l, m\}$. In addition, b chooses the probability $\delta^* \in \{\delta, \Delta\}$ (where $0 < \delta < \Delta < 1$) with which voters are informed of the policy choice. We assume that the choice of δ^* is not observed by voters.

One interpretation of this model is the following. Suppose b runs two campaign commercials. One commercial is uninformative about the policy choice while the other commercial is informative. The candidate runs a certain fixed number of commercials but must choose what proportion of the commercials are informative. Voters sample one (or more) of the commercials at random. If a voters has the informative commercial in his sample he observes the policy choice. The voter remains uninformed if the informative commercial is not in the sample.

Since all swing voters strictly prefer m to r and voters never use weakly dominated strategies, b will choose $\delta = \Delta$ whenever he chooses the moderate policy m. Note also that all swing voters prefer r to l. Therefore choosing Δ when the policy l is chosen cannot be optimal unless all uninformed swing voters vote for a. If all uninformed swing voters vote for a then the choice of δ^* does not affect voting behavior when a chooses l. Therefore, the analysis below suppresses the choice of δ and assumes that the probability that a voter is informed of the policy choice is δ if b chooses l and Δ if b chooses m.

It is straightforward to adapt the analysis of the previous section to this new game V^c and to adopt the corresponding definitions for $\mathcal{E}^c_n$, $\mathcal{E}^c$. Since the candidate either chooses (δ, l) or (Δ, m), the definitions of the critical states are modified as follows:

$$s^l(\sigma^v) = F(\sigma^v)(1-\delta)$$
$$s^m(\sigma^v) = F(\sigma^v)(1-\Delta) + \Delta$$

With this modified definition of critical states, Proposition 3 is easily adopted to V^c.

Proposition 9: *If $\sigma \in \mathcal{E}^c$ and $\phi \in \Phi(\sigma)$ then*

$$\sigma^b = s^l(\sigma^v)$$
$$\sigma^v = \frac{(1-\Delta)g(s^m(\sigma^v))}{(1-\Delta)g(s^m(\sigma^v)) + (1-\delta)g(s^l(\sigma^v))(1-\mu)}$$

$$\phi^l(s) = 1 \qquad \text{if } s < s^l(\sigma^v)$$
$$\phi^m(s) = 1 \qquad \text{if } s^l(\sigma^v) < s < s^m(\sigma^v)$$
$$\phi^r(s) = 1 \qquad \text{if } s^m(\sigma^v) < s$$

Note that uninformed voters must take into account the fact that b can choose the informativeness of the campaign. The terms $1-\delta$ and $1-\Delta$ in Proposition 9 reflect this effect. However, for μ close to one this term has little effect on σ^v and hence has little effect on the probability that b is elected. Proposition 10 below makes this precise.

In order to the address the question, "Does providing the opportunistic candidate with means to inform voters of his action curtail partisan politics and aggregation failure?" we compare the limit equilibrium of the game in section 3 with the corresponding limit equilibrium of the current game with the same parameters (including δ) and any $\Delta > \delta$. We focus on the case of where b is an office seeker. Proposition 10 below establishes that limit equilibria of V and the corresponding V^c are identical when b is an office seeker.

Proposition 10: *Let* $\lim \mu^k = 1$, $\sigma^k \in \mathcal{E}(\mu^k, \delta)$, $\hat{\sigma}^k \in \mathcal{E}^c(\mu^k, \delta, \Delta)$ *for all* k, *then*

$$\lim L(\sigma^k) = G(1-\delta) = \lim L(\hat{\sigma}^k)$$
$$\lim M(\sigma^k) = 1 - G(1-\delta) = \lim M^c(\hat{\sigma}^k)$$

Proposition 10 shows that when candidate b is an office seeker the fact that he can choose to inform voters has no impact on election outcomes.

To understand Proposition 10 note in the model with δ fixed all uninformed voter vote for b when b is an office seeker (μ^k converges to 1). This result is independent of δ and extends to the game where b can choose to run a more informative campaign. As a result, s^m converges to 1 in both games. But if all uninformed voters vote for b, b choose l unless the informed voters and the a partisans form a majority (i.e., $s > 1 - \delta$). Otherwise, he chooses m and wins. Therefore, the behavior of candidate b converges to the same limit strategy in both games.

When b has control over the informativeness of his campaign, he chooses the partisan policy and runs an uninformative campaign (i.e., chooses δ^* as small as possible) in states $s < 1-\delta$. The probability of partisan politics is therefore equal to $G(1-\delta)$ and unaffected by b's ability to provide information. For states $s > 1 - \delta$ candidate b runs an informative campaign and chooses $\delta^* = \Delta$ together with the moderate policy m. As in the benchmark case with a fixed δ, candidate b wins the election and implements the moderate policy in states $s > 1 - \delta$. The effect of the informative campaign in this case is to increase the margin of victory of candidate b.

This section demonstrates that the findings of partisan politics and aggregation failure continue to hold in a setting where the candidate can choose to inform voters of their own policy choices.

6. Conclusion

We analyzed how candidate competition is altered when some voters are informed of the candidate's policy choice. We show that when a candidate is an office seeker with a weak partisan preference then the ignorance of voters will permit him to win with more partisan policies. At the same time the candidate will be treated by uninformed voters as if he has chosen the moderate (median preferred) policy.

One consequence of this effect is that candidates have little incentive to spend resources to inform voters of their policy choices. When the opportunistic candidate is an office seeker providing the candidate with the opportunity to freely inform voters has no effect on the equilibrium outcome. As long as voters are convinced that a candidate will "do what it takes" to get elected, his chance of getting elected is not harmed by the ignorance among voters. At the same time, a less well informed electorate allows the candidate to choose policies that closer match his policy preference.

7. Appendix

Lemma 1: *(i)* $B_n(x) = \frac{\int_0^x \theta^n(1-\theta)^n d\theta}{\int_0^1 \theta^n(1-\theta)^n d\theta}$; *(ii)* B_n *is strictly log-concave.*

Proof: (i) The binomial theorem implies that

$$\begin{aligned}\int_0^x \theta^n(1-\theta)^n d\theta = \int_0^x (\theta-\theta^2)^n d\theta = \int_0^x \sum_{k=0}^{n} \binom{n}{k} (-1)^k \theta^{n+k} \\ = \sum_{k=0}^{n} \binom{n}{k} \frac{x^{n+k+1}}{n+k+1}(-1)^k\end{aligned} \tag{A1}$$

Next, we show that

$$B_n(x) = \frac{(2n+1)!}{n!n!} \sum_{k=0}^{n} \binom{n}{k} \frac{x^{n+k+1}}{n+k+1}(-1)^k \tag{A2}$$

The binomial theorem yields

$$\begin{aligned}B_n(x) &= \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} x^k(1-x)^{2n+1-k} \\ &= \sum_{k=n+1}^{2n+1} \sum_{m=0}^{2n+1-k} x^{k+m}(-1)^m \binom{2n+1-k}{m}\binom{2n+1}{k}\end{aligned}$$

Hence, letting $t = m + k$ and rearranging terms yields

$$\begin{aligned}
B_n(x) &= \sum_{k=n+1}^{2n+1} \sum_{t=k}^{2n+1} x^t (-1)^{t-k} \binom{2n+1-k}{t-k} \binom{2n+1}{k} \\
&= \sum_{t=n+1}^{2n+1} x^t \sum_{k=n+1}^{t} \binom{2n+1-k}{t-k} (-1)^{t-k} \binom{2n+1}{k} \\
&= \sum_{t=n+1}^{2n+1} \frac{(2n+1)!\, x^t}{(2n+1-t)!} \sum_{m=0}^{t-(n+1)} \frac{(-1)^m}{m!(t-m)!}
\end{aligned}$$

Feller (1967) pg 65 provides the followign identity:

$$\binom{a}{k} - \binom{a}{k-1} + - \dots \binom{a}{0} = \binom{a-1}{k}$$

Hence, the last equation implies

$$\begin{aligned}
B_n(x) &= \sum_{t=n+1}^{2n+1} \frac{(2n+1)!\, x^t}{(2n+1-t)!} \frac{(-1)^{t-(n+1)}}{t!} \frac{(t-1)!}{(t-(n+1))! n!} \\
&= \frac{(2n+1)!}{n! n!} \sum_{t=n+1}^{2n+1} \frac{(-1)^{t-(n+1)} n!\, x^t}{(2n+1-t)!(t-(n+1))!\, t} \\
&= \frac{(2n+1)}{n! n!} \sum_{k=0}^{n} \binom{n}{k} \frac{x^{n+k+1}}{n+k+1} (-1)^k
\end{aligned}$$

We conclude from $(A1)$ and $(A2)$ that

$$A \cdot \frac{\int_0^x \theta^n (1-\theta)^n d\theta}{\int_0^1 \theta^n (1-\theta)^n d\theta} = B_n(x)$$

for some constant $A > 0$. Clearly, $A = 1$ since

$$1 = B_n(1) = A \cdot \frac{\int_0^1 \theta^n (1-\theta)^n d\theta}{\int_0^1 \theta^n (1-\theta)^n d\theta} = A$$

which proves part (i)

(ii) We must show that

$$\frac{d}{dx} \left(\frac{B_n'(x)}{B_n(x)} \right) < 0 \qquad (A3)$$

Substituting the left hand side expression from part (i) and computing the derivative, straightforward computation shows that inequality (A3) is equivalent to

$$n(1-2x)\left(\int_0^x \theta^n(1-\theta)^n d\theta\right) - x^{n+1}(1-x)^{n+1} < 0 \qquad (A4)$$

For $x \geq 1/2$ the inequality is obviously correct. To see that it holds for $x < 1/2$ note that for $x < 1/2$ we have

$$(1-2x)\left(\int_0^x \theta^n(1-\theta)^n\right) \leq \left(\int_0^x \theta^n(1-\theta)^n(1-2\theta)d\theta\right) = \frac{x^{n+1}(1-x)^{n+1}}{n+1} \qquad (A5)$$

Substituting $(A5)$ into $(A4)$ proves part (ii). □

Lemma 2: *Assume (i)* $\lim a_n = 1/2$, $\lim \alpha_n = \alpha > 1/2$, $\lim b_n = b < 1/2$, *and* $\lim \beta_n = \beta > 1/2$. *(ii)* $\{f_1, h_1, f_2, h_2, \ldots\}$ *are equicontinuous functions on* $[0,1]$ *such that for some* $c, C \in I\!R_+$ $c \leq f_n \leq C$, $c \leq h_n \leq C$ *for all* n, *and (iii)* $\lim f_n(1/2)$, $\lim h_n(1/2)$, $\gamma := \lim \frac{\int_{a_n}^1 x^n(1-x_n)^n dx}{\int_0^1 x^n(1-x_n)^n dx}$ *exist. Then,*

$$\lim \frac{\int_{a_n}^{\alpha_n} x^n(1-x_n)^n f_n(x)dx}{\int_{b_n}^{\beta_n} x^n(1-x_n)^n h_n(x)dx} = \gamma \lim \frac{f_n(1/2)}{h_n(1/2)}$$

Proof: Define

$$q_n(x) = x^n(1-x)^n$$
$$z_n = \frac{\left(\frac{1}{2}-\epsilon\right)^n\left(\frac{1}{2}+\epsilon\right)^n}{\left(\frac{1-\epsilon}{2}\right)^n\left(\frac{1+\epsilon}{2}\right)^n}$$
$$X_n(r,t) = \int_r^t q_n(x)dx$$

Step 1: $\lim r_n = r < t = \lim t_n$ and $1/2 \notin [r,t]$ implies

$$\lim \frac{X_n(r_n,t_n)}{X_n(0,1)} = 0$$

Assume that $1/2 < r$ and choose $\epsilon \in (0, a - 1/2)$. (The proof for the $1/2 > t$ is symmetric and omitted.) Let Note that q is a strictly quasiconcave function on $[0,1]$ which attains its unique maximum at $1/2$. Let $y = r - \epsilon$ and $z = \min\{t+\epsilon, 1\}$. Hence,

$q_n(x) \leq q_n(y)$ for all $x \in [y, z]$ and $q_n(x) \geq q_n((1-\epsilon)/2)$ for all $x \in [(1-\epsilon)/2, (1+\epsilon)/2]$. Therefore, for n sufficiently large

$$\begin{aligned}\frac{X_n(r_n, t_n)}{X_n(0,1)} &\leq \frac{X_n(y,z)}{X_n(1-\epsilon)/2, (1+\epsilon)/2} \\ &\leq \frac{(A-a)z_n}{\epsilon}\end{aligned}$$

Since $\lim z_n = 0$ step 1 follows.

Step 2: $\lim r_n = r < t = \lim t_n$ and $1/2 \in (r, t)$ implies

$$\lim \frac{X_n(r_n, t_n)}{X_n(0,1)} = 1$$

Choose $\epsilon \in (0, \min\{1/2 - r, t - 1/2\}$. Then, for n large enough

$$1 \geq \lim \frac{X_n(r_n, t_n)}{X_n(0,1)} \geq \frac{X_n(1/2-\epsilon, 1/2+\epsilon)}{X_n(0,1)} = \frac{1}{1 + \frac{X_n(0,1/2-\epsilon)}{X_n(1/2-\epsilon,1/2+\epsilon)} + \frac{X_n(1/2+\epsilon,1)}{X_n(1/2-\epsilon,1/2+\epsilon)}}$$

By step 1, the second and third terms in the denominator go to 0 as n goes to 0, proving step 2.

Let

$$\begin{aligned}N_n &= \int_{a_n}^{\alpha} q_n(x) f_n(x) dx \\ D_n &= \int_{b_n}^{\beta_n} q_n(x) h_n(x) dx \\ T_n &= \frac{N_n}{D_n}\end{aligned}$$

Step 3: $\lim T_n = \frac{\gamma \lim f_n(1/2)}{\lim h_n(1/2)}$.

The equicontinuity of f_n, h_n ensures that for any $\epsilon > 0$ there exists $\epsilon' > 0$ such that for n large enough

$$[f_n(1/2) - \epsilon] X_n(a_n, 1/2 + \epsilon') \leq N_n \leq [f_n(1/2) + \epsilon] X_n(a_n, 1/2 + \epsilon') + C X_n(1/2 + \epsilon', 1)$$

$$[f_n(1/2) - \epsilon] X_n(1/2 - \epsilon', 1/2 + \epsilon') \leq$$

$$D_n \leq [f_n(1/2) + \epsilon] X_n(1/2 - \epsilon', 1/2 + \epsilon') + C X_n(0, 1/2 - \epsilon') + C X_n(1/2 + \epsilon', 1)$$

Using the expressions above to bound N_n/D_n, then dividing terms by $X_n(0,1)$, letting n go to infinity and applying steps 1 and 2 yields

$$\frac{\lim f_n(1/2)-\epsilon}{\lim h_n(1/2)+\epsilon}\cdot\lim\frac{X_n(a_n,1/2+\delta)}{X_n(1/2-\epsilon',1/2+\epsilon')}\leq$$
$$\lim T_n \leq \frac{\lim f_n(1/2)+\epsilon}{\lim h_n(1/2)-\epsilon}\cdot\lim\frac{X_n(a_n,1/2+\epsilon')}{X_n(1/2-\epsilon',1/2+\epsilon')}$$

Applying step 1 and step 2 again yields

$$\frac{\lim f_n(1/2)-\epsilon}{\lim h_n(1/2)+\epsilon}\cdot\frac{X_n(a_n,1)}{X_n(0,1)}\leq \lim T_n \leq \frac{\lim f_n(1/2)+\epsilon}{\lim h_n(1/2)-\epsilon}\cdot\frac{X_n(a_n,1)}{X_n(0,1)}$$

Since the equation above holds for any ϵ, we conclude that

$$\lim T_n = \frac{\lim f_n(1/2)}{\lim h_n(1/2)}\cdot\lim\frac{X_n(a_n,1)}{X_n(0,1)} = \frac{\gamma \lim f_n(1/2)}{\lim h_n(1/2)}$$

as desired.

7.1 Proof of Proposition 2

In the text, we have shown that in any equilibrium the voters must use a cutoff strategy. Hence to prove that every equilibrium is a cutoff equilibrium, we show that b's best response to any cutoff strategy is also a cutoff strategy.

If σ^b is a best response to voters' cutoff strategy λ then $\sigma^b(s) = 1$ whenever

$$\frac{B_n(\pi^m(\lambda,s))}{B_n(\pi^l(\lambda,s))} > \mu$$

and $\sigma^b(s) = 0$ if this inequality is reversed. To show that this yields a cutoff strategy, it suffices to show that $\frac{B_n(\pi^m(\lambda,s))}{B_n(\pi^l(\lambda,s))}$ is strictly increasing in s or equivalently

$$\ln B_n(\pi^m(\lambda,s)) - \ln B_n(\pi^l(\lambda,s))$$

is strictly increasing in s. Recall that $\pi^l(\lambda,s)$ is a strictly decreasing linear function of s and $\pi^m(\lambda,s) = \pi^l(\lambda,s) + \frac{\delta}{2}$. Lemma 1 shows that $\ln B_n$ is strictly concave and therefore $\ln B_n(x+\delta) - \ln B_n(x)$ is strictly decreasing in x. Hence, $\ln B_n(\pi^m(\lambda,s)) - \ln B_n(\pi^l(\lambda,s))$ is strictly increasing in s.

To prove that equilibrium exists, let $h : T \times S \to T$ be defined as

$$h_n(\lambda, s) := \left[1 + \frac{\int_0^s \pi^l(\lambda, s)^n (1 - \pi^l(\beta, s))^n g(s) ds}{\int_s^1 \pi^m(\lambda, s)^n (1 - \pi^m(\lambda, s))^n g(s) ds}\right]^{-1} \qquad (A7)$$

Note that h_n is continuous and $h(\lambda, s)$ is the probability that candidate b chooses m given that voters cutoff strategy is λ, candidate b's cutoff strategy is s and a voter is pivotal. Hence $h(\lambda, s)$ describes the optimal cutoff of a voter if other voters are using the symmetric cutoff strategy λ and b is using the cutoff strategy s.

Let $k : T \to S$ be defined as follows:

$$k(\lambda) := \begin{cases} 1 & \text{if } \frac{B_n(\pi^l(\lambda,1)))}{B_n(\pi^m(\lambda,1))} > \mu \\ 0 & \text{if } \frac{B_n(\pi^l(\lambda,0)))}{B_n(\pi^m(\lambda,0))} < \mu \\ \{s \in S | \frac{B_n(\pi^l(\lambda,s)))}{B_n(\pi^m(\lambda,s)))} = \mu\} & \text{otherwise.} \end{cases}$$

Note that k is in fact a function since $\frac{B_n(\pi^l(\lambda,s)))}{B_n(\pi^m(\lambda,s)))}$ is decreasing and continuous in s. Since $\frac{B_n(\pi^l(\lambda,s)))}{B_n(\pi^m(\lambda,s)))}$ is jointly continuous in (λ, s), k is also continuous. The cutoff strategy with cutoff $k(\lambda)$ is the best response of b to the cutoff strategy λ by voters. We conclude that a fixed-point of $(h, k) : S \times T \to S \times T$ is an equilibrium in cutoff strategies. Since both h, k are continuous and S, T are compact this mapping has a fixed point. $\square$

7.2 Proof of Proposition 3

Lemma 3: *Let (σ_n^v, σ_n^b) be a convergent sequence of equilibria with limit (σ^v, σ^b). Then, $\sigma^b = s^l(\sigma^v)$.*

Proof: Let $s < s^l(\sigma^v)$. Then there is $\epsilon > 0$ such that $\pi^l(\sigma^v, s) \geq 1/2 + \epsilon$ for n sufficiently large. This implies that b wins the election with probability close to one if he chooses policy l. Since $\mu < 1$ this implies that l must be the unique optimal choice at s and hence $\sigma^v > s$. We conclude that $\sigma^b \geq s^l(\sigma^v)$.

If $s^l(\sigma^v) > s > s^m(\sigma^v)$ then there is $\epsilon > 0$ such that $\pi^m(\sigma_n^v, s') > 1/2 + \epsilon$ and $\pi^l(\sigma_n^v, s') < 1/2 - \epsilon$. This implies that b wins with probability close to one if he chooses m but loses with probability close to one if he chooses l. Since $0 < \mu$ it follows that the unique optimal choice is m. If follows that $\sigma^b \leq s^l(\sigma^v)$ and hence $\sigma^b = s^l(\sigma^v)$. $\square$

Lemma 4: *Let (σ_n^v, σ_n^b) be a convergent sequence of equilibria with limit (σ_n^v, σ_n^b). Then, $0 < \sigma_n^b < 1$ for large n and $\lim B_n(\pi^l(\sigma_n^v, \sigma_n^b)) = \mu$.*

Proof: (i) Suppose there exists a (sub)sequence of equilibria (σ_n^v, σ_n^b) such that $\sigma_n^b = 1$ for all n (and hence $\sigma^b = 1$). Then since conditional on a vote being pivotal b chooses l with probability 1, $\sigma_n^v = 0$ for all n. Hence, $\sigma^v = 0$ and by Lemma 3, $\sigma^b = s^l(\sigma^b) = 0 \neq 1 = \sigma^b$ a contradiction. If there exists a (sub)sequence of equilibria (σ_n^v, σ_n^b) such that $\sigma_n^b = 0$ for all n (and hence $\sigma^b = 0$), then since conditional on a vote being pivotal, b chooses m with probability 1, $\sigma_n^v = 1$ for all n and therefore $\sigma^v = 1$. By Lemma 3, $\sigma^b = s^l(\sigma^v) = 1 - \delta > 0 \neq \sigma^b$ a contradiction.

(ii) For $0 < \sigma_n^b < 1$ we must have

$$\mu B_n(\pi^m(\sigma_n^v, \sigma_n^b)) = B_n(\pi^l(\sigma_n^v, \sigma_n^b))$$

This follows since for $o \in \{l, m\}$, $\pi^o(\sigma_n^v, \cdot)$ is continuous for all n. Further observe that $\pi^m = \pi^l + \delta/2$ with $\pi^l(\sigma_n^v, \sigma_n^b) = 1/2$. Therefore, it follows that for large n, $\pi^m(\sigma_n^v, \sigma_n^b) > 1/2 + \epsilon$ for some $\epsilon > 0$ and hence $\lim B_n(\pi^m(\sigma_n^v, \sigma_n^b)) = 1$. This yields the Lemma. □

Lemma 5: *Let (σ_n^v, σ_n^b) be a sequence of equilibria converging to (σ^v, σ^b). Then, $0 < \sigma^b < 1$.*

Proof: Optimality of voters' strategies ensures that $\sigma_n^v = h_n(\sigma_n^v, \sigma_n^b)$. First, we show that $h_n(\sigma_n^v, \sigma_n^v) \leq 1 - \epsilon$ for some $\epsilon > 0$ and n sufficiently large. If $\sigma_n^v \leq 1/2$ we are done. Hence, assume that $\sigma^v > 1/2$ and hence $\sigma_n^v > 1/2$ for large n. Since g is continuous and $g > 0$ on S, there exists constants $C \geq c > 0$ such that

$$\begin{aligned}\frac{1 - h_n(\sigma_n^v, \sigma_n^b)}{h_n(\sigma_n^v, \sigma_n^b)} &= \frac{\int_0^{\sigma_n^b} \pi^l(\sigma_n^v, s)^n (1 - \pi^l(\sigma_n^v, s))^n g(s) ds}{\int_{\sigma_n^b}^1 \pi^m(\sigma_n^v, s)^n (1 - \pi_m(\sigma_n^v, s))^n g(s) ds} \\ &\geq \frac{c}{C} \cdot \frac{\int_0^{\sigma_n^b} \pi^l(\sigma_n^v, s)^n (1 - \pi^l(\sigma_n^v, s))^n ds}{\int_{\sigma_n^b}^1 \pi^m(\sigma_n^v, s)^n (1 - \pi_m(\sigma_n^v, s))^n ds}\end{aligned}$$

A change of variables using the fact that $\pi^m = \pi^l + \delta/2$ and π^l is a linear function of s establishes

$$\frac{1 - h_n(\sigma_n^v, \sigma_n^b)}{h_n(\sigma_n^v, \sigma_n^b)} \geq \alpha \cdot \frac{\int_{\pi^l(\sigma_n^v, 0)}^{\pi^l(\sigma_n^v, \sigma_n^b)} x^n (1 - x)^n ds}{\int_0^1 x^n (1 - x)^n ds}$$

for some $\alpha > 0$.

Optimality of candidate b's behavior requires that $\pi^l(\sigma_n^v, \sigma_n^b) \leq b_n$ where $B_n(b_n) = \mu$. By the law of large numbers, $\lim b_n = 1/2$. Let $a_n := \pi^l(\sigma_n^v, 0) = 1/2 + F(\sigma_n^v)(1-\delta)/2$ and note that $a_n > 1/2$ for n large since $\lambda_n \geq 1/2$. By Lemma 2, we conclude that for large n,

$$\frac{\int_{\pi^l(\sigma_n^v,\sigma_n^b)}^{\pi^l(\sigma_n^v,1)} x^n(1-x)^n ds}{\int_0^1 x^n(1-x)^n ds} = \frac{\int_{b_n}^{a_n} x^n(1-x)^n dx}{\int_0^1 x^n(1-x)^n dx} \geq \frac{1}{2}\frac{\int_{b_n}^{1} x^n(1-x)^n dx}{\int_0^1 x^n(1-x)^n dx}$$

By Lemma 1,

$$\frac{\int_{b_n}^{1} x^n(1-x)^n dx}{\int_0^1 x^n(1-x)^n dx} = 1 - \frac{\int_0^{b_n} x^n(1-x)^n dx}{\int_0^1 x^n(1-x)^n dx} = 1-\mu$$

Since $1-\mu > 0$ we conclude that $1-h(\lambda_n, s_n)$ is bounded away from 0 and hence $h(\sigma_n^v, \sigma_n^b)$ is bounded away from 1 for all n. This shows that $\sigma^v < 1-\epsilon$ for some $\epsilon > 0$.

Next we show that $\sigma^v > \epsilon$ for some $\epsilon > 0$. Let $a_n := \pi^m(\sigma_n^v, 1)$ and note that $a_n = 1/2 - (1-\delta)F(1-\sigma^v)/2$. Since σ^v stays bounded away from 1 we conclude that there is $\epsilon > 0$ such that $a_n < 1/2 - \epsilon$ for large n. Let $b_n := \pi^m(\sigma_n^v, \sigma_n^b)$ and note that $b_n = \pi^l(\sigma_n^v, \sigma_n^b) + \delta/2$. Since $\lim \pi^l(\sigma_n^v, \sigma_n^b) = 1/2$ we conclude that there is $\epsilon > 0$ such that $b_n \geq 1/2 + \epsilon$ for some $\epsilon > 0$.

By an analogous argument to the one above, we conclude that there is a constant $\alpha > 0$ such that for large n

$$\frac{1-h_n(\sigma_n^v, \sigma_n^b)}{h_n(\sigma_n^v, \sigma_n^b)} \geq \alpha \cdot \frac{\int_0^1 x^n(1-x)^n ds}{\int_{a_n}^{b_n} x^n(1-x)^n ds} \geq \alpha$$

This shows that $h(\sigma_n^v, \sigma_n^b)$ stays bounded away from zero for all n and therefore $\sigma^v > 0$. □

Lemma 6: *Let (σ_n^v, σ_n^b) be a sequence of equilibria and converging to (σ^v, σ^b). Then,*

$$\sigma^v = \frac{g(s^m(\sigma^v))}{g(s^m(\sigma^v)) + g(s^l(\sigma^v))(1-\mu)}$$

Proof: Note that

$$T_n := \frac{1-\sigma_n^v}{\sigma_n^v} = \frac{\int_0^{\sigma_n^b} \pi^l(\sigma_n^v, s)^n (1-\pi^l(\sigma_n^v, s))^n g(s)}{\int_{\sigma_n^b}^1 \pi^m(\sigma_n^v, s)^n (1-\pi^m(\sigma_n^v, s))^n g(s)}$$

To prove the Lemma, we will show that

$$\lim T_n = \frac{g(s^l(\sigma^v))}{g(s^m(\sigma^v))}(1-\mu) \tag{A8}$$

Let $a_n := \pi^l(\sigma_n^v, \sigma_n^b), \alpha_n := \pi_l(\sigma_n^v, 0), b_n := \pi_m(\sigma_n^v, 1), \beta_n := \pi^m(\sigma_n^v, \sigma_n^v)$. Since $[0,1]$ is compact, we can assume without loss of generality, that $(a_n, \alpha_n, b_n, \beta_n)$ converges to some (a, α, b, β). By Lemma 5 and the fact that $\delta > 0$, we have $\alpha, \beta > 1/2$ and $b < 1/2$. Moreover, $a = 1/2$ by Lemma 3.

Recall that $q_n(x) := x^n(1-x)^n$. A change of variables yields

$$T_n = \frac{\int_{a_n}^{\alpha_n} q_n(x) h_n^l(x) dx}{\int_{b_n}^{\beta_n} q_n(x) h_n^m(x) dx}$$

where $h_n^o(x) = g(z_n^o(x))$ for $o \in \{l, m\}$ and $z_n^o(x)$ is the unique solution to $\pi^o(\sigma_n^v, z_n^o)) = x$.

First, seach the collection of linear functions $\{h_n^l, h_n^m\}$ for $n = 1, \ldots$ all have the same slope, they are equicontinuous. Also, by Lemma 3, $\sigma^b = a = s^l(\sigma^v)$ and therefore $\lim h^l(1/2) = g(s^l(\sigma^v))$. Similarly, $\lim h^m(1/2) = g(s^m(\sigma^v))$. Then, by Lemma 2,

$$\lim T_n = \frac{\lim h^l(1/2)}{\lim h^m(1/2)} \cdot \lim \frac{\int_{a_n}^1 q_n(x) dx}{\int_0^{\beta_n} q_n(x) dx} = \frac{g(s^l(\sigma^v))}{g(s^m(\sigma^v))} \cdot \lim \frac{\int_{a_n}^1 q_n(x) dx}{\int_0^{\beta_n} q_n(x) dx} \tag{A9}$$

But by Lemma 1,

$$\lim \frac{\int_{a_n}^1 q_n(x) dx}{\int_0^1 q_n(x) dx} = 1 - \lim \frac{\int_0^{a_n} q_n(x) dx}{\int_0^1 q_n(x) dx} = 1 - \lim B_n(a_n)$$

and since $a_n = \pi^l(\sigma^v, \sigma^b)$, Lemma 4 yields $B_n(a_n) = \mu$. Then, $(A9)$ establishes $\lim T_n = \frac{g(s^l(\sigma^v))}{g(s^m(\sigma^v))}(1-\mu)$ as desired. □

Lemma 7: *If $\sigma \in \mathcal{E}$ and $\phi \in \Phi(\sigma)$ then*

$$\begin{aligned} \phi^l(s) &= 1 \qquad \text{if } s < s^l(\sigma^v) \\ \phi^m(s) &= 1 \qquad \text{if } s^l(\sigma^v) < s < s^m(\sigma^v) \\ \phi^r(s) &= 1 \qquad \text{if } s^m(\sigma^v) < s. \end{aligned}$$

Proof: Suppose $\sigma_n \in \mathcal{E}_n$ for all n. If $s < s^l(\sigma^v)$ then by Lemma 3, there exists N, $s^* \in (s, s^l(\sigma^v))$ such that for all $n \geq N$, $\sigma_n^b \geq s^*$. Hence, in equilibrium (for all $n \geq N$) candidate b chooses l at s. Also, for $\epsilon > 0$, we can choose N sufficently large so that $B_n(\pi^l(\sigma_n^v, s)) \geq B_n(\pi^l(\sigma_n^v, s^*)) - \epsilon$. Since, $\lim \pi^l(\sigma_n^v, s^*) > 1/2$ we conclude that $\lim B_n(\pi^l(\sigma_n^v, s)) \geq 1 - \epsilon$. Since this statement holds for any ϵ, we have $\lim B_n(\pi^l(\sigma_n^v, s)) = 1$, as desired. The proofs of the other two cases are similar and omitted. $\square$

Lemmas 3, 6 and 7 prove part (i) of the proposition. To prove part (ii) note that by Lemma 5, if $\sigma \in \mathcal{E}$ then σ_v is a fixed-point of the mapping h defined by

$$h(\lambda) = \frac{g(s^m(\lambda))}{g(s^m(\lambda)) + g(s^l(\lambda))(1 - \mu)}$$

But, as we noted in the text, h is non-increasing when g is log-concave. Hence, h has a unique-fixed point when g is log-concave. Then, Lemma 3 ensures that $\mathcal{E}$ is a singleton when g is log-concave. $\square$

References

1. Austen-Smith D. and J. Banks, (1995) “Information Aggregation, Rationality and the Condorcet Jury Theorem”, American Political Science Review, 90, 34-45.

2. Berelson, B. R., P. Lazarsfeld, and W. N. McPhee. 1954. *Voting: A Study of Opinion Formation in a Presidential Campaign.* Chicago: University of Chicago Press.

3. Bernhardt, Duggan and Squintani (2003), “Electoral Competition with Privately Informed Candidates”, mimeo, University of Rochester.

4. Calvert, R. (1985), “Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence.” American Journal of Political Science 29 (February 1985), pp. 69-95.

5. Chan, J. (2001), “Electoral Competition with Private Information”, mimeo, Johns Hopkins University.

6. Delli Carpini, M. X., and S. Keeter (1993) “Measuring Political Knowledge: Putting First Things First,” *American Journal of Political Science* 37:1179-1206.

7. Downs, A., *An Economic Theory of Democracy*, New York, Harper, 1957.

8. Feddersen, T and W. Pesendorfer, (1996) “The Swing Voter’s Curse”, American Economic Review 86, 381-398.

9. McKelvey, R. D., and P. C. Ordeshook (1985), “Elections with Limited Information: A Fulfilled Expectations Model using Contemporaneous Poss and Endorsement Data as Information Sources. *Journal of Economic Theory*, 36, 55-85.

10. McKelvey, R. D., and P. C. Ordeshook (1986), “Information, Electoral Equilibria, and the Democratic Ideal”, *The Journal of Politics*, 48, no 4, 909-937.

11. Young, P., (1988) Condorcet’s Theory of Voting, American Political Science Review, 82, 1231-1244.

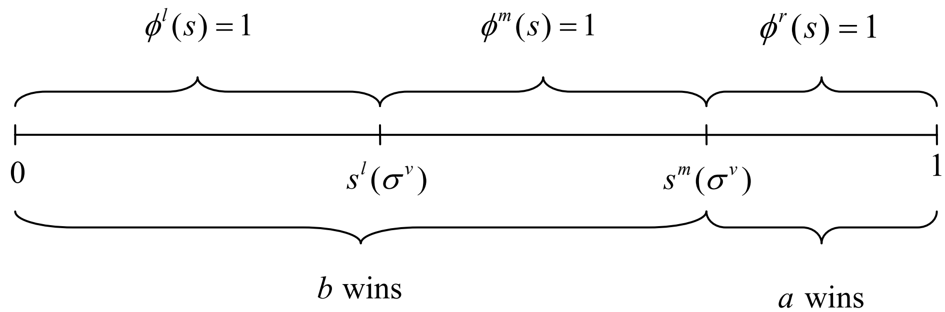

Figure 1