B Technical appendix (omitted from paper)

Proof of Lemma 9

Rewriting, we must show that

$$\frac{\int^{\sigma} f\left(\tilde{\sigma}|L\right) \frac{f(\tilde{\sigma}|H)}{f(\tilde{\sigma}|L)} d\tilde{\sigma}}{\int^{\sigma} f\left(\tilde{\sigma}|L\right) d\tilde{\sigma}}$$

is increasing in σ. Differentiating, we must show

$$f\left(\sigma|L\right) \frac{f\left(\sigma|H\right)}{f\left(\sigma|L\right)} \int^{\sigma} f\left(\tilde{\sigma}|L\right) d\tilde{\sigma} > f\left(\sigma|L\right) \int^{\sigma} f\left(\tilde{\sigma}|L\right) \frac{f\left(\tilde{\sigma}|H\right)}{f\left(\tilde{\sigma}|L\right)} d\tilde{\sigma},$$

which is immediate from MLRP. The proof that $\left(1 - F\left(\sigma|L\right)\right) / \left(1 - F\left(\sigma|H\right)\right)$ is decreasing is exactly parallel; its lower bound follows from l'Hôpital's rule. ∎

Equilibrium selection

Lemma 10 ***(Rejection equilibrium)***

Fix beliefs b, a voting rule $\alpha > \frac{1}{2} + \frac{1}{2n}$ and preferences λ. Let $(\underline{x}_n, \bar{x}_n)$ be the interval defined in Lemma 1. Then if $x \leq \underline{x}_n$ the only trembling-hand perfect equilibrium is the non-responsive equilibrium in which each responder always rejects.

Proof: Let Z be as defined in the proof of Lemma **1**. Since $x \leq \underline{x}_n$, from the proof of Lemma **1**

$$\Delta^H\left(x,\sigma\right) \frac{b}{1-b} \frac{f\left(\sigma|H\right)}{f\left(\sigma|L\right)} \left(\frac{F\left(\sigma|H\right)}{F\left(\sigma|L\right)}\right)^{n-n\alpha} \left(\frac{1-F\left(\sigma|H\right)}{1-F\left(\sigma|L\right)}\right)^{n\alpha-1} + \Delta^L\left(x,\sigma\right) \leq 0$$

for all σ. It follows that

$$\Delta^H\left(x,\bar{\sigma}\right) \frac{b}{1-b} \frac{f\left(\bar{\sigma}|H\right)}{f\left(\bar{\sigma}|L\right)} \left(\frac{f\left(\bar{\sigma}|H\right)}{f\left(\bar{\sigma}|L\right)}\right)^{n\alpha-1} + \Delta^L\left(x,\bar{\sigma}\right) \leq 0.$$

It then follows that for any σ_i,

$$\Delta^H\left(x,\sigma_i\right) \frac{b}{1-b} \frac{f\left(\sigma_i|H\right)}{f\left(\sigma_i|L\right)} \left(\frac{f\left(\bar{\Sigma}|H\right)}{f\left(\bar{\sigma}|L\right)}\right)^{n\alpha-1} + \Delta^L\left(x,\sigma_i\right) \leq 0$$

(for this expression could only be strictly positive if $\Delta^H(x, \sigma_i)$ were strictly positive; but then by MLRP it would be strictly positive at $\sigma_i = \bar{\sigma}$). Finally, since $\frac{f(\bar{\sigma}|H)}{f(\bar{\sigma}|L)} > 1$, for any $m < n\alpha - 1$,

$$\Delta^H(x, \sigma_i) \frac{b}{1-b} \frac{f(\sigma_i|H)}{f(\sigma_i|L)} \left(\frac{f(\bar{\sigma}|H)}{f(\bar{\sigma}|L)} \right)^m + \Delta^L(x, \sigma_i) < 0.$$

In words, this last inequality says that responder i, having observed his own signal σ_i, will reject the offer x even if he conditions on the event that $m < n\alpha - 1$ other responders observe the most pro-acceptance signal $\bar{\sigma}$. This has two implications.

First, the equilibrium in which all responders reject always is a trembling-hand perfect equilibrium: for if all responders tremble and accept with probability ε independent of their own signal, it remains a best response to reject the offer. This follows since it is a best response to reject the offer given the information that $m < n\alpha - 1$ responders have observed $\bar{\sigma}$, it is certainly a best response to reject given no information.

Second, we claim that the equilibrium in which all responders accept is not trembling-hand perfect. Recall that responder i's vote only matters if exactly $n\alpha - 1$ other responders vote to accept. This event only arises if at least $n - n\alpha$ of the $n - 1$ other responders tremble. As the probability of trembles converges to zero, responder i's best response is determined entirely by the event in which exactly $n - n\alpha$ other responders tremble. But by assumption $n - n\alpha < n\alpha - 1$, and so even if responder i infers from $n - n\alpha$ trembles that $n - n\alpha$ responders have observed $\bar{\sigma}$, his best response is to reject. So the equilibrium in which all creditors accept always cannot be trembling-hand perfect.

∎

Proof of Proposition 5

For each n and λ and the majority voting rule α, fix an equilibrium π_n^* and β_n, and define $\hat{x}_n(\sigma_0; \lambda) = \pi_n^*(\sigma_0; \lambda, \alpha)$ and $\hat{b}_n(\sigma_0; \lambda) = \beta_n(\pi^*(\sigma_0; \lambda, \alpha), \lambda, \alpha)$ for the equilibrium offers and beliefs given voting rule α, and $\hat{A}_n(\sigma_0, \lambda) = A_n\left(\hat{x}_n(\sigma_0; \lambda), \hat{b}_n(\sigma_0; \lambda), \lambda, \alpha\right)$ for the associated acceptance event. Define $x_n(\sigma_0; \lambda)$, $b_n(\sigma_0; \lambda)$ and $A_n(\sigma_0, \lambda)$ similarly for unanimity rule. By Assumptions 3 and 4,

$$U^H(x=1,\sigma_i,\lambda=0)+V^H(x=1,\sigma_0)>\bar{U}^H(\sigma_i,\lambda=0)+\bar{V}^H(\sigma_0);$$

and Assumption 7 combined with the hypothesis that $x_L(\lambda,\alpha)=\infty$ imply

$$U^L(x=1,\sigma_i,\lambda=0)+V^L(x=1,\sigma_0)<\bar{U}^L(\sigma_i,\lambda=0)+\bar{V}^L(\sigma_0).$$

Under Assumption 7, there exists a linear transformation of the payoff functions such that (after the transformation) $\kappa^\omega\equiv U^\omega(x,\sigma_i,\lambda=0)+V^\omega(x,\sigma_0)-\left(\bar{U}^\omega(\sigma_i,\lambda=0)+\bar{V}^\omega(\sigma_0)\right)$ is independent of the offer x. From above, $\kappa^H>0>\kappa^L$. From Proposition 4, there exists $\bar{\lambda}>0$ and a γ_Π such that $\Pi_n^{*R}(\lambda,1)\geq\Pi_n^{*R}(\lambda,\alpha)+\gamma_\Pi$ whenever $\lambda\leq\bar{\lambda}$ and n is greater than some $N_\Pi(\lambda)$. By continuity, for any $\varepsilon>0$ there exists $\Lambda(\varepsilon)>0$ such that for any σ_0,σ_i and $\lambda\in[0,\Lambda(\varepsilon)]$,

$$\left|U^\omega(\hat{x}_n(\sigma_0;\lambda),\sigma_i,\lambda)+V^\omega(\hat{x}_n(\sigma_0;\lambda),\sigma_0)-\bar{U}^\omega(\sigma_i,\lambda)-\bar{V}^\omega(\sigma_0)\right|\geq|\kappa^\omega|-\varepsilon.$$

By Lemma 3, when λ is close enough to 0 the acceptance probabilities under the equilibrium offer given the the majority rule α converge to 0 and 1 in $\omega=L,H$ respectively, i.e., $\Pr\left(\hat{A}_n(\sigma_0,\lambda)|L\right)\to 0$ and $\Pr\left(\hat{A}_n(\sigma_0,\lambda)|H\right)\to 1$ as $n\to\infty$. As such, there exists $\Lambda>0$ and $N(\lambda)$ for each $\lambda\leq\Lambda$ such that if $n\geq N(\lambda)$, the expression

$$\begin{aligned}&\int_{\sigma_0}\Pr(A_n(\sigma_0,\lambda)|\omega)\left(E_{\sigma_i}\left[U^\omega(x_n(\sigma_0;\lambda),\sigma_i,\lambda)|\omega,A_n(\sigma_0,\lambda)\right]+V^\omega(x_n(\sigma_0;\lambda),\sigma_0)\right)dF(\sigma_0)\\&-\int_{\sigma_0}\Pr\left(\hat{A}_n(\sigma_0,\lambda)|\omega\right)\left(E_{\sigma_i}\left[U^\omega(\hat{x}_n(\sigma_0;\lambda),\sigma_i,\lambda)|\omega,\hat{A}_n(\sigma_0,\lambda)\right]+V^\omega(\hat{x}_n(\sigma_0;\lambda),\sigma_0)\right)dF(\sigma_0)\end{aligned}$$

is bounded above by $\gamma_\Pi/2$ for both $\omega=L,H$. That is, since (as noted in the main text) the outcome under the majority voting rule is approximately Pareto efficient, and payoffs are linear, then under the unanimity rule the sum of the payoffs is generally lower, and is certainly not significantly higher.

From this, it follows immediately that when $\lambda\leq\Lambda$ and $n\geq N(\lambda)$,

$$\Pi_n^{*R}(\lambda,1)+\Pi_n^{*P}(\lambda,1)\leq\Pi_n^{*R}(\lambda,\alpha)+\Pi_n^{*P}(\lambda,\alpha)+\frac{\gamma_\Pi}{2}.$$

Consequently, when $\lambda \leq \min\left\{\bar{\lambda}, \Lambda\right\}$ and $n \geq \max\left\{N_\Pi(\lambda), N(\lambda)\right\}$, then $\Pi_n^{*P}(\lambda, 1) \leq \Pi_n^{*P}(\lambda, \alpha) - \gamma_\Pi/2$. ∎

Private values material

Assumption 8 $\Delta^H(x = 1, \underline{\sigma}, \lambda = 1) < 0$.

Assumption 8 simply says that at the private values extreme, a responder who received the most pro status quo realization of σ_i prefers the status quo to even the most generous offer. When it holds, we obtain:

Lemma 11 ***(Private values responder payoffs under unanimity)***
Suppose unanimity rule is in effect ($\alpha = 1$), and that responders have preferences sufficiently close to private values such that $\Delta^H(x = 1, \underline{\sigma}, \lambda) < 0$. Then under any sequence of offers $x_n \leq 1$, the acceptance probability converges to 0 and the responder payoff converges to $E\left[\bar{U}^\omega(\sigma_i, \lambda)\right]$.

Proof: Let b_n be the corresponding sequence of beliefs. If the equilibrium is responsive, the equilibrium cutoff value is a solution to $Z(x_n, \sigma) = 0$. But by the assumption $\Delta^H(x = 1, \underline{\sigma}, \lambda)$, it is immediate that there exists $\varepsilon > 0$ independent of n such that $Z(x_n, \cdot)$ is strictly negative over $[\underline{\sigma}, \underline{\sigma} + \varepsilon]$ and so $\sigma_n^* > \underline{\sigma} + \varepsilon$. On the other hand, if the equilibrium is unresponsive then since $\bar{x}_n(\alpha, \lambda, b_n) = 1$ it is the rejection equilibrium and $\sigma_n^* = \bar{\sigma}$. In either case, the probability that each responder votes to accept is bounded away from 1, and so the probability that the offer x_n is accepted converges to 0. ∎

Lemma 11 says that when Assumption 8 holds, agreement is impossible to obtain when responders use unanimity rule. In contrast, given Assumption 3, there is *some* majority rule $\hat{\alpha}$ under which the responders will accept at least some offers. Clearly the proposer is better off if responders adopt such a rule. However, because of the private values component of responder preferences, it is quite possible that $E\left[\Delta^\omega(x, \sigma_i, \lambda) | A_n\right]$ is negative: that is, even though an offer is only accepted if it makes the marginal responder better off than the status quo, on average the coalition of responders may be worse off.

A pair of conditions that are sufficient to guarantee that there exists a majority rule that makes the responder as well as the proposer better off than unanimity rule are:

$$\Delta^H (1, E[\sigma_i|H], \lambda = 1) \;>\; 0 \tag{48}$$
$$\frac{\partial^2}{\partial \sigma_i^2}\Delta^H (x, \sigma_i, \lambda) \;\geq\; 0 \text{ for all } \lambda. \tag{49}$$

Of these, (49) is close to a normalization: for any given λ, we can always monotonically transform preferences so that it holds. Condition (48) is stronger, and says that if the proposer makes the best offer possible, $x = 1$, and $\omega = H$, then a responder with the *average* private valuation $E[\sigma_i|H]$ prefers the offer to the status quo. When both conditions are satisfied we obtain:

Proposition 8 ***(Unanimity worse for private values creditors)***
Suppose that conditions (48) and (49) hold. Then there exists $\Lambda < 1$ such that whenever $\lambda \geq \Lambda$, i.e., responders' preferences are sufficiently close to private values, then there exists a majority voting rule $\hat{\alpha} < 1$ such that the equilibrium outcome under $\hat{\alpha}$ Pareto dominates the equilibrium outcome under unanimity rule.

Proof: Fix a preference parameter λ sufficiently small such that $\Delta^H (x = 1, \underline{\sigma}, \lambda) < 0$ (this is possible by Assumption 8), and choose $\varepsilon > 0$. By Assumption 3, $\Delta^H (x = 1, \bar{\sigma}, \lambda) > 0$. Recall that $\sigma_H(\alpha)$ is decreasing α. Provided λ and ε are small enough, there exists a majority voting rule $\hat{\alpha} < 1$ such

$$\Delta^H (1, \sigma_H(\hat{\alpha}), \lambda) > 0 > \Delta^L (1, \sigma_L(\hat{\alpha}), \lambda) \tag{50}$$

and

$$\sigma_H(\hat{\alpha}) + \varepsilon < E[\sigma_i|H]. \tag{51}$$

From (50), $x_H(\lambda, \hat{\alpha}) \neq \infty = x_L(\lambda, \hat{\alpha})$, and so by Proposition 1 the responders' payoff has the following limit as the number of responders grows large:

$$\Pi_n^{*R}(\lambda, \alpha) \to E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i)\right] + \Pr(H)\, E\left[\Delta^H (x_H(\lambda, \hat{\alpha}), \sigma_i, \lambda)\,|H\right].$$

By Jensen's inequality, the above limiting expression is weakly greater than

$$
\begin{aligned}
& E_{\sigma_i,\omega}\left[\bar{U}^{\omega}\left(\sigma_i\right)\right]+\Pr\left(H\right)\Delta^{H}\left(x_H\left(\lambda,\hat{\alpha}\right),E\left[\sigma_i|H\right],\lambda\right) \\
\geq\; & E_{\sigma_i,\omega}\left[\bar{U}^{\omega}\left(\sigma_i\right)\right]+\Pr\left(H\right)\Delta^{H}\left(x_H\left(\lambda,\hat{\alpha}\right),\sigma_H\left(\hat{\alpha}\right)+\varepsilon,\lambda\right) \\
\geq\; & E_{\sigma_i,\omega}\left[\bar{U}^{\omega}\left(\sigma_i\right)\right]+\Pr\left(H\right)\varepsilon\min_{\sigma\in\left[\sigma_H\left(\hat{\alpha}\right),\sigma_H\left(\hat{\alpha}\right)+\varepsilon\right]}\frac{\partial}{\partial\sigma}\Delta^{H}\left(x_H\left(\lambda,\hat{\alpha}\right),\sigma,\lambda\right),
\end{aligned}
$$

where the first inequality follows from (51). In contrast, from Lemma 11, under the unanimity rule the responders' payoff converges to $E\left[\bar{U}^{\omega}\left(\sigma_i,\lambda\right)\right]$. Thus when the number of responders n is sufficiently large, the responders' payoff is higher under the majority rule $\hat{\alpha}$. For the proposer, simply observe that the acceptance probability under $\hat{\alpha}$ converges to $\Pr\left(H\right)$ while the acceptance probability under unanimity rule converges to 0. From (50), $x_H\left(\lambda,\hat{\alpha}\right)<1$, and so the proposer is strictly better off when his offer is accepted. This completes the proof. ∎