Competence and Ideology

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September, 2008

Abstract

We develop a dynamic, 2-party citizen-candidate model in which candidates are distinguished by both their ideology and their valence. We provide sufficient conditions under which (i) the median voter is decisive and (ii) there exists a unique symmetric, stage-undominated, stationary perfect Bayesian equilibrium.

In our dynamic setting, reputational concerns endogenize incentives to compromise. In equilibrium, we prove that higher valence incumbents compromise more, compromise to more extreme policies, and are re-elected more. We find that the correlation between valence and extremism *varies* across different cohorts: the correlation is *negative* for first-term representatives, but *positive* in the long-run stationary distribution of office-holders (large congress). This novel result might partially explain the conflicting findings in the empirical literature and theoretical single-election models.

We then find that a FOSD improvement in the distribution of valences benefits *all* voters. More heterogeneity in valence benefits the median voter, but may hurt a majority of voters when voters are sufficiently risk averse.

We expand the model to allow interest groups to invest in candidate valence and provide conditions under which the equilibrium expected policy choice of office holders is more extreme when interest groups have more extreme ideologies, reducing welfare of all voters.

JEL classification: D72.
Keywords: Valence, spacial model, repeated elections, incumbent, citizen-candidate.

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1 Introduction

Classic papers extend Hotelling (1929), Black (1948, 1958) and Downs (1957) to study how politician and voter ideologies affect policy choice and electoral outcome. However, politicians are also distinguished by other fundamental characteristics that voters value such as competence, character, or organizational efficiency. Since Stokes (1963), a vast literature has examined the role of this so-called *valence* dimension. Most of these models consider a single election. In one class of these models, voters see the valence of *all* candidates prior to the election (see Ansolabehere and Snyder (2000), Aragones and Palfrey (2000), or Groseclose (2001)). In equilibrium, these models derive a *negative* correlation between valence and extremism[1], and a *positive* correlation between valence and the probability of winning the election. In another class of these static models, voters do not see valence prior to the election (Kartik and McAfee (2007) and Callander and Wilkie (2007)). In this class, candidates with character/valence have an *exogenous* cost of compromising, while candidates without character can costlessly locate moderately to try to win the election. In equilibrium, this class generates a *positive* correlation between character and extremism, and a *negative* correlation between valence and the probability of winning the election. Empirically, researchers obtain conflicting results regarding the correlation between valence and extremism.[2]

In this paper, we develop a dynamic citizen-candidate model of repeated elections, in which candidates are distinguished by both their ideology and valence. Reputation/re-election concerns drive policy choices, and serve to endogenize the costs of locating extremely. In our dynamic framework, we answer a series of questions regarding the effects of a politician's valence on policy choices and electoral outcomes. Who compromises more? Who is more likely to be re-elected? Who chooses more extreme policies? We show that the dynamic nature of elections is important: we find that the correlation between valence and extremism *varies* across different cohorts of incumbents. This result might underlie the inconsistent findings of empirical studies. We continue to describe how the distribution of candidate valences affects the welfare of different voters and to endogenize the distribution of valences via investment by interest groups.

In our model, valence is a characteristic of the office-holder that benefits each citizen by the same amount, independently of ideologies. Ideology is uncorrelated with valence in the population of candidates. Ideology and valence are initially private information to a candi-

[1] We use the term "extremism" to denote policy positions distant from the median voter's preferred policy.
[2] See the debate over the marginality hypothesis in Fiorina (1973), Groseclose (2001) and Griffin (2008).

date, but an office-holder's actions in office naturally reveal her valence to voters and convey information about her future actions were she re-elected. Hence, valence in our model reflects characteristics such as competence and organizational efficiency (e.g., ability to handle day-to-day non-ideological concerns of constituents, such as immigration issues or cutting red tape for firms), that are observed only after the politician holds office. Although our results also hold when political candidates are selected from the entire population at large, in most of our analysis we maintain the more realistic premise that citizens are divided according to their ideologies into liberal (left wing) and conservative (right wing) parties. Each period, the election is either between an incumbent and a challenger drawn at random from the opposing party, or the election is between untried candidates from the two parties.

Our modeling framework captures the fact that voters typically know more about an incumbent running for re-election than an untried challenger[3]. Voters observe incumbent performance in office and update their beliefs about her future policy choices and ability; while they may know little more about a challenger than the information contained in her party affiliation. Of course, this informational asymmetry between the incumbent and "risky challenger" need not translate into an electoral advantage for the incumbent. When voters learn that an incumbent is incompetent (has low valence) or adopts extreme policies, this informational asymmetry favors the untried challenger whose attributes have not been revealed. Existing single-election models with valence do not incorporate this central asymmetry between candidates—voters either know the valence of all candidates or none.

We provide sufficient conditions under which there exists a unique symmetric, stage-undominated, stationary perfect Bayesian equilibrium. The median voter is decisive and the equilibrium is completely summarized by thresholds that divide office-holders in 3 groups: centrists who adopt their preferred platforms and win re-election, moderates who compromise to win re-election, and extremists who adopt their extreme platforms and then lose re-election—as in Duggan (2000). Here, however, *thresholds vary with a politician's valence.* This result reflects that the decisive median voter is willing to trade-off valence for policy, so that incumbents with higher valence can win re-election by adopting more extreme policies that lower valence incumbents would lose with.

Since higher valence politicians have this advantage, one might conjecture that (a) they would be *less* willing to compromise, and (b) there would be a *positive* correlation between valence and extremism. We find that, for incumbents in their first-term in office, both

[3]Bernhardt and Ingberman (1985) is the first paper to consider the consequences of informational differences between incumbents and challengers.

of these conjectures are wrong. In our dynamic setting where reputational concerns drive policy choices, we prove that higher valence incumbents compromise more, compromise to more extreme policies, and are re-elected more. A high quality incumbent internalizes the fact that it is less costly to compromise—she can compromise to a more extreme position and be re-elected. More importantly, for a high valence office holder, it is more costly to be replaced by an untried candidate from the opposite party: (a) the new office-holder could have a lower valence, and (b) the ideology of a high valence incumbent who is indifferent to compromising is located further away from the expected policy of the challenger. Hence, both the compromise set and probability of winning re-election strictly increase in valence.

We then prove that valence is *negatively* correlated with the degree of extremism in the policy choice of newly-elected representatives. What drives this result are (a) higher valence politicians are more willing to compromise, and (b) the extreme platform choices taken by lemons—newly-elected representatives with both low valence and extreme ideologies who choose to take extreme losing positions that reflect their underlying ideological preferences.

Still, the dynamic nature of the economy means that one must also account for the re-election of good candidates and the replacement of bad ones: from a long-run perspective, the relevant distribution is the stationary distribution of office-holders. Over time, the likelihood of having an extremist in office falls because extremists are more likely to lose re-election. Combining this result with the fact that re-elected high valence incumbents can win by adopting more extreme policies, we obtain a *positive* correlation between valence and extremism in the stationary distribution of office-holders—equivalently, a positive correlation between valence and policy in the cross-section of a large congress.

We then investigate how the welfare of different voters is affected by the distribution of candidate valences. In particular, we investigate which voter ideologies benefit from a given change in the distribution of valences. We consider the welfare impacts of both first- and second- order stochastic dominance (FOSD, SOSD) shifts in the distribution of candidate valences. Because the median voter is decisive and trades off valence and policy differently from more extreme voters, the central questions are: does a stochastic shift in the distribution of valences affect voters with more extreme ideologies in the same way as the median voter? and how are the welfare gains or losses of different voters affected by the degree of voter risk aversion over policies? We consider two notions of voter welfare: (a) the ex ante expected lifetime payoff from electing an untried, first-term representative, and (b) the expected period payoff realized in the long-run stationary distribution of office-holders. These notions arise from the dynamic nature of our model, and correspond to the frameworks we use to analyze the correlation between valence and extremism.

We first prove that a first-order stochastic dominance improvement in the distribution of valences raises the median voter expected payoff from an untried candidate. This result reflects that an untried candidate is expected to have higher valence; and first-term representatives with higher valence tend to implement policies closer to the median voter.

In the long-run stationary distribution, however, higher valence incumbents tend to implement more extreme policies, which could hurt the median voter. There are opposing effects. On the one hand, because the untried challenger becomes more attractive after the FOSD improvement, to win re-election, an incumbent of *any* given valence level must compromise by more, locating closer to the median voter. On the other hand, the improvement in the quality of the untried candidate and the tighter re-election standards reduce the incentives to compromise, increasing the number of politicians implementing extreme policies. We prove that if incumbents are sufficiently likely to run for re-election, then the first effect dominates and a FOSD improvement strictly increases median voter's expected utility in the stationary distribution of office holders. We then show that even though the median voter trades off differently between valence and policy than voters with more extreme ideologies, that this welfare gain extends to *all* voters. In essence, we find that all voters gain because high valence candidates are *more* willing to compromise.

We next explore how a SOSD shift in the distribution of valences affects the welfare of voters with different ideologies. We prove that the median voter gains from increased heterogeneity in valences—the median voter values the option of electing an untried challenger who might have a high valence. We then retrieve the intuition that because the median voter trades off differently between valence and ideology, voters with more extreme ideologies might be hurt by greater dispersion in valences. In particular, we find that if voters are sufficiently risk averse, then a majority of voters (those with extreme ideologies) may prefer an economy of "average" politicians whose unique valence corresponds to the average valence in the economy with heterogeneity in valences.

Finally, we extend the model to allow interest groups to invest in valence of candidates, and explore how the ideology of interest groups affects their investments and equilibrium expected valence and policy. For example, the interest groups can provide resources that stochastically improve the professionalism of a representative's staff. We then derive conditions under which interest groups with more extreme ideologies invest *less* in valence, decreasing the expected utility of *all* voters. In essence, this result reflects that extreme interest groups are hurt less by a low valence candidate who also has an extreme ideology, and who locates extremely as a result. This reduced investment causes the median voter to set slacker re-election standards, thereby increasing expected extremism in the policies of elected officials; but it also induces more incumbents to compromise, reducing extremism. We define conditions under which the first effect dominates, so that investment by more extreme interest groups endogenously induces greater expected extremism in the policies of elected officials.

The paper is organized as follows. Section 2 relates our paper to the literature. Section 3 presents the model. Section 4 characterizes the equilibrium and the main results. Section 5 presents the welfare implications of exogenous changes in the valence distribution. Section 6 endogenizes valence via investments by interest groups. All the proofs are in the appendix.

2 Related Literature

Since Stokes (1963), a vast literature has examined the role of valence in politics, primarily in a single-election framework. In one class of models, candidate valences are known before the election and campaign policies are binding. Ansolabehere and Snyder (2000) consider a setting with purely-office motivated candidates where the identity of the median voter is public information. They show that when the valence advantage is not too large, then in the pure-strategy Nash equilibrium, the candidate with valence advantage chooses a moderate policy and wins the election with probability one. Aragones and Palfrey (2000) show that in the modified setting where the median voter position is unknown, the candidate with valence advantage adopts a mixed strategy with a distribution of policies closer to the expected median voter, and is more likely to win the election. Groseclose (2001) allows each candidate to have a known policy preference, symmetric around the median voter, and finds an analogous result: the candidate with valence advantage chooses a pure-strategy policy that is closer to the expected median voter and is more likely to win. Iaryczower and Mattozzi (2008) introduce an endogenous number of candidates and endogenous investments in valence to this static framework, where policy announcements are binding and valence is known before the election.

More recent papers maintain the single-election framework, but find opposite results when the candidate's type is private information. In Kartik and McAfee (2007), candidates with "character" are by definition unable to compromise—their platform/policy is always their ideology—but such "character" is also assumed to raise the utility of all voters. Candidates without character are purely office motivated, and can costlessly locate moderately. As a result, Kartik and McAfee generate a positive correlation between character and extremism, and find that candidates without character are more likely to win. Callander and Wilkie (2007) generalize Kartik and McAfee by supposing that campaign platforms are not binding and candidates with character face a convex, but not infinite, cost of making campaign promises further from their preferred, intended policy, and generate similar results. Callander (2008) investigates a model where candidates have private information about their motivation. Policy-motivated candidates have a higher cost of compromising. In equilibrium, office-motivated candidates locate closer to the median voter and are more likely to win.

In sum, there is no consensus about the theoretical correlation between valance and extremism in single-election models. When valence is known by the electorate, there is a *negative* correlation between valence and extremism. Higher valence candidates exploit this advantage by moving closer to the median voter to increase the probability of winning. When valence is unknown, the assumed *exogenous* correlation between valence and the cost of compromising generates a *positive* correlation between valence and extremism, and a consequent lower probability that high valence candidates win the election.

There is also no consensus in the empirical literature regarding the correlation between valence and extremism. It is a challenge both to define valence and to measure it. For example, Groseclose (2001) assumes that marginal incumbents, i.e., representatives with narrow margin of victory, have low valence. Hence, if empirically the marginality hypothesis holds—if marginal incumbents tend to moderate more—then it would suggest a *positive* correlation between valence and extremism. Groseclose refers to Fiorina (1973) to reject the marginality hypothesis and to argue that there is empirical evidence of a *negative* correlation between valence and extremism. However, Griffin (2008) provides a recent empirical defense of the marginality hypothesis. Our dynamic model suggests that the implications of valence for extremism are more subtle; and that the design of empirical investigations should account for the dynamic considerations that we identify.

Our model integrates valence into a repeated election framework along the lines of Duggan (2000), Bernhardt, Dubey and Hughson (2004) and Bernhardt et al. (2008). As in our paper, in that literature, voters observe an incumbent's policy choice in office and can forecast likely future actions, but have less information about challengers; and this gives rise to cutoff rules that characterize how the median voter selects between candidates, and the platforms that incumbents with different ideologies adopt. By integrating valence to these models, we show how the endogenous cost of compromising and the re-election standard varies across the different valence levels, and derive the consequences for voter welfare.

Meirowitz (2007) examines valence in a very different repeated election two party model, in which each period one party draws an independent and identically distributed net valence advantage. Policy preferences and valence advantage are known before election. When in office, a party has private information about the feasible set of policies. Meirowitz finds that a party with net valence advantage can select policies closer to its ideal point.

3 The Model

There is an interval $[-a, +a]$ of citizen candidates, each indexed by her private ideology x, distributed across society according to the c.d.f. F, with an associated single-peaked density f that is symmetric about the median voter's ideology, $x = 0$. Ideologies are private information to candidates. Each citizen candidate is also characterized by a valence $v \in V \subseteq [v_L, v_H]$, where $0 \leq v_L \leq v_H$. Valence is uncorrelated with candidate ideology, and is distributed in the population according to the c.d.f. G with support V. Valence is uncorrelated with candidate ideology. Valence is initially private information of a candidate before she holds office, but her performance in office reveals her valence to the electorate.

At any date t, an office holder with ideology x and valence v selects a policy $p(x, v) \equiv y$. The time-t utility of a citizen x depends on the implemented policy y, according to $u_x(y, v) = L_x(y) + v$, where $L_x(y) \equiv l(|x - y|)$ is a symmetric, single-peaked loss function that is $\mathcal{C}^2$, with $l' < 0$ and $l'' \leq 0$. We normalize $l(0) = 0$ without loss of generality. Note that u satisfies the single-crossing property: $\partial u_x / \partial y$ is increasing in x. Period utilities are discounted by factor $\delta \in (0, 1)$. In addition to the period utility $u_x(y, v)$, an office-holder also receives an ego rent of $\rho \geq 0$ each period in office. Each period, after taking her position in office, with probability $q \in [0, 1)$ an incumbent receives an exogenous shock and cannot run for re-election. One can interpret this re-election shock as an unanticipated retirement of the politician for health or family issues.

Although our results also hold when political candidates are selected from the entire population at large, in most of our analysis we assume for the sake of realism that citizens are divided into two parties, a left-wing party L, and a right-wing party R. Party L consists of all citizen-candidates with ideologies $x < 0$, and party R has all possible candidates with ideologies $x > 0$. At date 0, an office holder is randomly determined. In any subsequent date-t majority rule election, an incumbent who runs for re-election faces a challenger from the opposing party. The valence of an untested challenger is unknown to the voters, but its distribution G is common knowledge. If the incumbent receives a re-election shock and does not run for re-election, then both parties compete with untried candidates.

We assume that citizens adopt the weakly dominant strategy of voting for the candidate whom they believe will provide them strictly higher discounted lifetime utility if elected—citizens vote sincerely. We assume that a voter who is indifferent between an incumbent and an untried challenger selects the incumbent. We will identify conditions under which the median voter is decisive in equilibrium. Focusing on symmetric equilibria, we assume that in elections between two untried candidates, the indifferent median voter randomizes, selecting each candidate with equal probability. Importantly, our qualitative findings are unaffected if the outcome of an election between two untried candidates is determined by the actions of the departing incumbent, i.e., if the untried candidate from the party of the incumbent who is stepping down wins if and only if the incumbent would have won, had she run for re-election.

In summary, the sequence of events at any period t is:

1. An office holder with valence v and ideology x implements her policy choice $y = p(x, v)$.
2. The incumbent realizes a re-election shock
   (a) With probability $1 - q$, the incumbent runs for re-election.
   (b) With probability q, the incumbent cannot run for re-election, so that her party draws an untried candidate.
3. Opposing party draws an untried candidate.
4. Given the information about candidates (party affiliation for challengers; party affiliation, valence and past policy choices for incumbents), citizens vote for their preferred candidate.
5. The winning politician assumes office.

4 Equilibrium

We focus on symmetric, stationary and stage-undominated perfect Bayesian equilibrium (PBE). We view symmetry and stage undomination as natural equilibrium requirements. Stationarity permits a tractable representation of equilibrium that highlights the features of the trade off between valence and ideology, and the equilibrium behavior of incumbents of different valence levels. A stationary policy strategy p for an office holder prescribes that at any time t, she selects a policy that depends only on her ideology x and valence v. The policy strategy is symmetric if $p(x, v) = -p(-x, v)$.

Under the four sufficient conditions of Theorem 1 that we state momentarily, there is a unique symmetric, stage-undominated, stationary perfect Bayesian equilibrium. Before we present the theorem, we describe the roles that each of these sufficient conditions serves.

The first sufficient condition says that voters are not too risk averse. This assumption ensures that an equilibrium is completely summarized by threshold functions $w, c : V \to [0, a]$, where for each $v \in V$, $0 \leq w(v) < c(v) \leq a$ for party R, and $-a \leq -c(v) < -w(v) \leq 0$ for party L. Incumbents from party R with valence v and centrist ideology $x \in [0, w(v)]$ and extremist incumbents $x \in (c(v), a]$ adopt their preferred policy $y = x$ when in office. Moderate politicians $x \in (w(v), c(v)]$ do not adopt their preferred policy, as they would then lose office. Instead, they compromise and adopt the most extreme ideology that still allows them to win re-election, i.e., they locate at $w(v)$. In the next election, centrist and moderate incumbents are re-elected, while extremists are ousted from office. The characterization is symmetric for party L. Figure (1) depicts the thresholds for an office-holder with valence v.

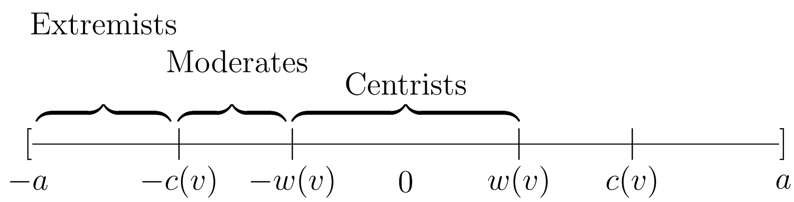

Figure 1: Thresholds for office-holders with valence v

If this sufficient condition is violated and voters are too risk averse, the compromise set might not be connected: some incumbents with less extreme ideologies and some with very extreme ideologies might compromise, while a group of incumbents with intermediate ideologies choose not to compromise. Analytically, our sufficient condition holds for Euclidean and quadratic loss functions. Numerically, we solved the model for two valences, uniform and truncated normal distributions for ideologies, and loss function $L_x(y) = -|x - y|^z$. We were unable to construct counterexamples even with high levels of risk aversion, with $z = 3$ or 4.

To guarantee that equilibrium threshold functions are interior, $0 < w(v) < c(v) < a$, we also require that ego rents are not so high that a high valence incumbent with the most extreme ideology a would compromise to win re-election, and that valences are not so dispersed that low valence candidates cannot win re-election, even if they adopt the median voter's preferred policy, $y = 0$. These are natural requirements to avoid an uninteresting equilibrium where low valence politicians always lose re-election and high valence politicians always win.

We prove uniqueness across equilibria that are ordered according to a natural monotonicity property. We then provide sufficient conditions for this monotonicity property to hold.

Definition: *Fix the parameters of the model, and let* (w, c) *and* (w', c') *be equilibrium thresholds given valence distributions* G *and* G'.

(a) *Changes in the threshold function* w *are* ***strictly monotone*** *if for every equilibria* (w, c) *and* (w', c')*, and every pair of valences* $v, \tilde{v} \in V$

$$w'(v) > w(v) \Rightarrow w'(\tilde{v}) > w(\tilde{v}).$$

(b) *Changes in the threshold function* c *are* ***weakly monotone*** *if for every equilibria* (w, c) *and* (w', c'), and valence $v \in V$

$$w'(v) \geq w(v) \Rightarrow c'(v) \geq c(v).$$

Lemma A.8 in the appendix proves that changes in the threshold function w are always strictly monotone. Lemma A.9 provides sufficient conditions under which changes in c are weakly monotone. When $q = 0$, the sufficient conditions simply imply that changes in (w, c) that reduce the payoff that the *median* voter expects from a *left wing* challenger, do not raise by too much the payoff that a *right wing* citizen with $x > 0$ expects from that challenger. Indeed, one might expect the opposite: changes in the left wing party that hurt the median voter, should hurt a right wing voter by more. We can prove that changes in the threshold function c are weakly monotone if the loss function is quadratic and selection of untried candidates is at-large. Numerically, with party selection, we always obtained monotonicity for uniform or truncated ideologies, loss function $L_x(y) = -|x - y|^z$ for $z \in [1, 4]$, and two valences.

Theorem 1 *Consider the class of symmetric, stationary, stage-undominated perfect Bayesian equilibrium (PBE). There exist uniform bounds* $M'' < 0$, $0 < M'''$, $0 < \overline{\rho}$ *and* $0 < \overline{v}$ *such that if*

- ***C1.*** *voters are not too risk averse,* $M'' \leq l'' \leq 0$ *and* $|l'''| \leq M'''$;
- ***C2.*** *ego rent is not too high,* $\rho \leq \overline{\rho}$;
- ***C3.*** *valence set is not too large,* $v_H - v_L \leq \overline{v}$,

then an equilibrium exists. The median voter is decisive, and every equilibrium completely summarized by threshold functions $w, c : V \to (0, a)$*, where for each* $v \in V$, $0 < w(v) <$

$c(v) < a$ for party R, and symmetric thresholds $-w(v)$ and $-c(v)$ for party L. Furthermore, the equilibrium is unique if

> ***C4.*** *changes in the threshold function c are weakly monotone.*

For the remaining of the paper we assume that conditions **C1–C4** hold. To simplify presentation we write $w_v \equiv w(v)$ and $c_v \equiv c(v)$.

Voter optimization. Let $U_x(y, v|w, c)$ denote the equilibrium continuation utility that a voter with ideology x expects to derive from a date-t office-holder with valence v who adopts platform y, if the incumbent is reelected every time she runs for office. Define U_x^j to be the equilibrium continuation utility that x expects to derive from selecting an untried representative from party $j = L, R$, and let $\overline{U}_x(w, c) \equiv (U_x^R(w, c) + U_x^L(w, c))/2$ represent the payoff x expects from an untried challenger drawn from at large. Integrating over the possibility of an election shock, the continuation payoff that x expects from an incumbent is

$$
\begin{aligned}
U_x(y, v|w, c) &= u_x(y, v)(1-\delta) + \delta\left[q\frac{U_x^L(w, c) + U_x^R(w, c)}{2} + (1-q)\,U_x(y, v|w, c)\right] &(1)\\
&= k\, u_x(y, v) + k\frac{\delta q}{(1-\delta)}\overline{U}_x(w, c), &(2)
\end{aligned}
$$

where $k \equiv \frac{(1-\delta)}{[1-\delta+\delta q]}$. If the date-$t$ incumbent from party L with valence v adopts platform y, then a voter with ideology x votes for incumbent if and only if $U_x(y, v|w, c) \geq U_x^R(w, c)$. Similarly, voter x selects an incumbent from party R if and only if $U_x(y, v|w, c) \geq U_x^L(w, c)$. The median voter is decisive whenever an incumbent is re-elected if and only if the median voter prefers the incumbent to the challenger. That is, an incumbent from party L with valence v who adopts policy y is re-elected if and only if $U_0(y, v|w, c) \geq U_0^R(w, c)$, and an incumbent from party R is re-elected if and only if $U_0(y, v|w, c) \geq U_0^L(w, c)$.

The equilibrium functions w, c obey the following recursive equations. First, for any $v \in V$,

$$U_0(w_v, v|w, c) = U_0^L(w, c) = U_0^R(w, c) = \overline{U}_0(w, c). \tag{3}$$

This recursive condition describes the voting rule for the decisive median voter. In particular, an incumbent with valence v who implements policy w_v leaves the median voter indifferent between the incumbent and a random challenger from the opposite party. In light of symmetry, the median voter is indifferent between random challengers from either party.

The second recursive equation describes the compromise decision for the marginal incumbent with valence v and ideology c_v. For any $v \in V$,

$$U_{c_v}(w_v, v|w, c) + \rho k = (v + \rho)(1-\delta) + \delta q \overline{U}_{c_v}(w, c) + \delta(1-q)\, U_{c_v}^L(w, c). \tag{4}$$

An incumbent from party R with valence v and ideology c_v is indifferent between (i) compromising to policy w_v to win if she runs for re-election, and (ii) adopting her own ideology c_v as a policy and losing to a challenger from the opposite party if she runs for reelection—which happens with probability $(1-q)$. An analogous recursive equation characterizes a party L incumbent with valence v and ideology $-c_v$.

For any citizen with ideology x, the PBE continuation expected value from electing a challenger from party L is:

$$\begin{aligned} U_x^L(w,c) &= 2k\int_V\left\{\int_{-w_v}^{0}\left[u_x(y,v)+\frac{\delta q}{1-\delta}\overline{U}_x(w,c)\right]dF(y)\right. \\ &+ \int_{-c_v}^{-w_v}\left[u_x(-w_v,v)+\frac{\delta q}{1-\delta}\overline{U}_x(w,c)\right]dF(y) \\ &+ \left.[1-\delta(1-q)]\int_{-a}^{-c_v}\left[u_x(y,v)+\frac{\delta q}{1-\delta}\overline{U}_x(w,c)+\frac{\delta(1-q)}{1-\delta}U_x^R(w,c)\right]dF(y)\right\}dG(v)\,. \end{aligned} \tag{5}$$

To understand this expression, recognize that the challenger's valence v is drawn from the set V. For each v, the challenger's ideology y will turn out to be either (a) centrist, $y\in[-w_v,0]$; (b) moderate, $y\in[-c_v,-w_v)$; or (c) extremist, $y\in[-a,-c_v)$. A centrist candidate adopts her own ideology as policy and is re-elected every time she runs for office, which provides an expected continuation payoff of $U_x(y,v|w,c)=ku_x(y,v)+k\frac{\delta q}{1-\delta}\overline{U}_x(w,c)$ to a voter with ideology x. A moderate candidate compromises to $-w_v$ and also wins re-election so that $U_x(-w_v,v|w,c)=ku_x(-w_v,v)+k\frac{\delta q}{1-\delta}\overline{U}_x(w,c)$. Finally, an extremist candidate adopts her own ideology and loses to an untried candidate from party R when she runs for re-election. Hence, voter x derives an expected continuation payoff from an extremist politician of

$$(1-\delta)u_x(y,v)+\delta q\overline{U}_x(w,c)+\delta(1-q)U_x^R(w,c),$$

which we rewrite as $k[1-\delta(1-q)][u_x(y,v)+\frac{\delta q}{1-\delta}\overline{U}_x(w,c)+\frac{\delta(1-q)}{1-\delta}U_x^R(w,c)]$.

Analogously, the payoff that voter x expects to derive from a challenger from party R is

$$\begin{aligned} U_x^R(w,c) &= 2k\int_V\left\{\int_{0}^{w_v}\left[u_x(y,v)+\frac{\delta q}{1-\delta}\overline{U}_x(w,c)\right]dF(y)\right. \\ &+ \int_{w_v}^{c_v}\left[u_x(w_v,v)+\frac{\delta q}{1-\delta}\overline{U}_x(w,c)\right]dF(y) \\ &+ \left.[1-\delta(1-q)]\int_{c_v}^{a}\left[u_x(y,v)+\frac{\delta q}{1-\delta}\overline{U}_x(w,c)+\frac{\delta(1-q)}{1-\delta}U_x^L(w,c)\right]dF(y)\right\}dG(v)\,. \end{aligned} \tag{6}$$

4.1 Equilibrium Characterization

From equation (2) for the median voter, reelecting an incumbent with valence v who adopts policy w_v results in an expected discounted lifetime payoff of

$$U_0(w_v, v|w, c) = k(v + L_0(w_v)) + k\frac{\delta q}{1-\delta}\overline{U}_0(w, c). \tag{7}$$

From equilibrium condition (3) we have $U_0(w_v, v|w, c) = \overline{U}_0(w, c)$, so simplifying (7) yields

$$U_0(w_v, v|w, c) = v + L_0(w_v). \tag{8}$$

For an office holder with valence v and ideology $x = c_v$, we use equation (2) to rewrite equilibrium condition (4) as

$$k\left[v + \rho + L_{c_v}(w_v) + \frac{\delta q}{1-\delta}\overline{U}_{c_v}(w, c)\right] = (v+\rho)(1-\delta) + \delta q\overline{U}_{c_v}(w, c) + \delta(1-q)\,U^L_{c_v}(w, c). \tag{9}$$

Proposition 1 *Take any equilibrium (w, c). For any $v_H, v_L \in V$,*

1. *High valence office-holders can take more extreme policy positions and be reelected,*

$$v_H > v_L \Rightarrow w_H > w_L;$$

2. *Valence is positively correlated with probability of being reelected,*

$$v_H > v_L \Rightarrow c_H > c_L;$$

3. *The compromise set is strictly increasing in valence,*

$$v_H > v_L \Rightarrow c_H - w_H > c_L - w_L.$$

The first result reflects that the decisive median voter is prepared to trade off valence for policy—she values valence and hence is willing to tolerate more extreme policies from higher valence incumbents. The second result reflects that an office-holder with higher valence v_H and ideology c_L is more willing to compromise than a lower valence v_L politician with the same ideology. This is because (a) her higher valence generates a higher payoff when in office, and (b) it is less costly for her to compromise since she can do so to a more extreme position, $w_H > w_L$.

The third result says that if $v_H > v_L$ then $c_H - c_L > w_H - w_L$. To understand this stronger result, consider a low valence incumbent with ideology c_L and a high valence incumbent with ideology $x_H = c_L + (w_H - w_L)$. In terms of the distance between incumbent's ideology and re-election standard w_v, both incumbents face the same cost of compromising to win re-election. However, incumbent x_H faces a higher cost than incumbent c_L of *not* compromising and then being replaced by an untried candidate from the opposing party—incumbent x_H is further from any untried challenger of the opposing party than c_L, and Lemma A.2 shows that as a result x_H faces a higher cost of being replaced. Moreover, the higher valence v_H generates more utility than v_L when incumbent x_H is in office. Therefore, the higher benefit from compromising together with the higher cost of not compromising makes the higher valence office-holder x_H more willing to compromise, which results in a larger compromise set.

These results imply that valence is *positively* correlated with extremism for re-elected incumbents and for losing office holders. That is, on average, re-elected high valence office holders take more extreme policies than re-elected low valence office-holders; and losing office holders with high valence take more extreme positions than losing office holders with low valence. The positive correlation between valence and extremism for re-elected office-holders emerges because the median voter sets slacker re-election standards for higher valence candidates that allow them to adopt more extreme positions and be re-elected. Among losing candidates, the set of extremist incumbents $(c_v, a]$ is decreasing in valence because higher valence candidates are more willing to compromise, which implies that the losing higher valence candidates on average locate more extremely.

However, these results do *not* imply that valence is positively correlated with extremism in the population. This is because for any fixed valence level, losing incumbents adopt on average more extreme policies than re-elected officials; and since the number of extreme politicians falls with valence, so does the ratio of losing to re-elected officials. The next proposition shows that for politicians in their first term in office, the "lemons effect" dominates when ideologies are uniformly-distributed[4]—there are sufficiently more low valence candidates with extreme ideologies that the correlation between valence and extremism of first term office holders is *negative*.

Proposition 2 (Valence & Extremism: First-Term) *Valence is **negatively** correlated with extremism for first-term office holders when ideologies are uniformly distributed.*

[4]Numerically, we find the same result for truncated normal distributions.

Proposition 2 only addresses a subset of representatives—those in their first term in office. Our model is intrinsically dynamic so that we must also account for the re-election of good candidates and the replacement of bad ones—over time, the likelihood of having an extremist in office falls because extremists are more likely to lose re-election. From a long-run perspective, the relevant distribution is the stationary distribution of office-holders, or equivalently the cross-sectional distribution of policies and valence in a large congress.[5]

While Proposition 2 established a *negative* correlation between valence and extremism, Proposition 3 shows that this relationship is reversed in the steady-state distribution of a large congress. Analytically, Proposition 3 below establishes a positive correlation between valence and extremism whenever the probability q that an incumbent quits for exogenous reasons is either sufficiently small or large. Numerically, this correlation holds for all q.

Proposition 3 (Valence & Extremism: Large Congress) *There are bounds $\underline{q} > 0$ and $\overline{q} < 1$ such that if $q \in [0, \underline{q}]$ or $q \in [\overline{q}, 1)$, then in the long-run stationary distribution of office holders, valence is* ***positively*** *correlated with extremism.*

Proposition 3 shows that, in a large congress, higher valence office holders are more likely to implement more extreme policies, even though valence and ideology are ex ante uncorrelated in the population, and we do not impose exogenous costs of compromising. In fact, this result emerges despite the fact that high valence candidates compromise more (Proposition 1.3). The result is driven by the median voter's willingness to re-elect high valence office holders with more extreme policies (Proposition 1.1).

Propositions 2 and 3 show how important it is to consider the implications of incentives in a dynamic framework, when investigating the correlation between valence and extremism. They show that the sign of the correlation *varies* across different cohorts of office holders.

5 Ex-Ante Welfare

We consider two notions of voter welfare: (a) the ex ante expected discounted lifetime payoff from electing an untried challenger drawn from either party with equal probability to serve as a first-term representative, and (b) the expected period payoff integrating over valences and policy choices using the long-run stationary distribution of office-holders. These notions

[5] As in Bernhardt et al. (2004), we ignore the issue of how aggregation of ideologies in Congress affects policy outcomes. We simply assume that, at each election, voters behave as if only the ideology of their representative determines policy outcomes.

arise from the dynamic nature of our model, and correspond to the frameworks we used to analyze the correlation between valence and extremism in Propositions 2 and 3. We focus on how exogenous changes in the distribution of valences affect equilibrium strategies and the welfare of voters with different ideologies. As proved in Proposition A.1, a location shift of the valence distribution does not affect voter choices between candidates: from a strategic standpoint the mean of the valence distribution is simply an irrelevant lump sum transfer to all agents; what matters is the distribution of valences around the mean. Hence, without loss of generality, we can normalize the lowest valence to zero.

A more intriguing question is: how are voters affected by a first-order stochastic dominance improvement in the distribution of valences that raises the probability of high valence candidates? Our previous results revealed that incumbents with higher valences compromise to more extreme positions, and in the stationary distribution of office holders, they adopt more extreme positions. As a result, one might conjecture that the median voter might be hurt by an increase in the probability of high valence candidates. The next results show that this conjecture is false. While Proposition 4 only establishes analytically that an improvement in the distribution of valences benefits the median voter when q is sufficiently small or large, we find numerically that the result extends for all q.

Proposition 4 (Valence Distribution Improvement) *A First-Order Stochastic Dominance improvement in the distribution of valences* ***strictly increases*** *the median voter's*

- *Ex ante expected payoff from an untried, first-term representative.*
- *Expected period payoff in the long-run stationary distribution of office holders when* $q \in [0, \underline{q}]$*, with* $\underline{q} > 0$ *sufficiently small, or* $q \in [\overline{q}, 1)$*, with* $\overline{q} < 1$ *sufficiently large.*

Valence is valued and, for untried candidates, is negatively correlated with extremism. Hence, an improvement in the valence distribution raises the payoff that the median voter expects to derive from an untried candidate. The untried candidate becomes more attractive, inducing the decisive median voter to set tighter re-election standards for *all* valence levels: re-election cutoffs w_v move closer to the median voter. However, there is an indirect offsetting effect—the decline in w_v is accompanied by a decline in c_v, making this proposition far from trivial to establish. In particular, a politician with valence v and ideology c_v has (a) a higher cost of compromising, since w_v is now closer to the median voter, and (b) a lower cost of being replaced by a challenger, who now has a higher expected valence and faces tighter re-election standards.

As a result, more politicians choose to locate extremely and lose, and this hurts the median voter. However, we prove that the direct positive effect dominates—if not, the median voter would be worse off and hence set looser re-election standards, which would increase incentives of extremist incumbents to compromise, raising median voter welfare, a contradiction.

It is even more challenging to establish the second welfare result, because in the stationary distribution of office holders, valence is positively correlated with extremism, and this hurts the median voter. All re-elected officials who compromise locate closer to the median voter, but there are more extreme incumbents. When incumbents are likely to run for re-election (q is small), enough representatives in the large congress are returning centrist/compromising incumbents that the first effect dominates and the valence improvement benefits the median voter. When q is close to one, the difference in expected policy across valence levels is small, and tighter re-election standards together with higher expected valence benefits the median voter. Numerically these results extend to intermediate q.

Although the median voter is better off, voters with different ideologies trade off differently between valence and policy. The decisive median voter is more willing to accept a more extreme position from a high valence incumbent from party R than *any* voter in party L: voters in party L are further from the incumbent, and due to the concavity of the loss function, are less willing to trade off extremism for valence. Therefore, one might conjecture that some voters could be hurt by an increase in the probability of high valence candidates—and the consequent increase in the equilibrium number of extreme incumbents of *all* valence levels. This conjecture is also wrong.

Proposition 5 *If the loss function is quadratic, then all voters benefit by the same amount as the median voter from a First-Order Stochastic Dominance improvement in the distribution of valences.*

This analytical result is difficult to extend to other loss functions, due to the implications of changes in the compromise standards c_v of incumbents with different valences. Numerically, we find that all voters benefit from a stochastic improvement in the valence distribution when ideologies are drawn from uniform or truncated normal distribution, loss functions take the form $L_x(y) = -|x-y|^z$ for $z \in [1, 4]$, and there are two valences. What drives these numerical findings is that higher valence candidates are more willing to compromise (Proposition 1.3). Moreover, the higher expected valence of challengers induces the median voter to set more demanding re-election standards. Hence, incumbents of *all* valence levels must compromise to more moderate policies to win re-election, and this increases the ex ante welfare of all voters.

Now consider a two-type valence setting: with probability $p \in (0,1)$ an untried candidate has valence v_H, and with probability $(1-p)$ her valence is v_L, where $v_H > v_L$. How does the stationary policy in a large congress changes with a marginal increase in the probability p of drawing a high valence untried candidate? The direct effect of an increasing p is to increase the likelihood of a high valence candidate, who is expected to adopt a more extreme policy in the large congress. However, there are secondary effects on the equilibrium thresholds w_v and c_v. The better-off median voter sets stricter standards for re-election: w_v's decrease, decreasing expected extremism in the policies of re-elected officials. The stricter re-election standard and the increased expected utility from untried candidates decrease compromising: c_v's decrease, increasing expected extremism. Whenever the first and third effects dominate, a marginal increase in p induces greater expected extremism in the large congress equilibrium policy. When $p = 0$ and $p = 1$ we know from Proposition A.1 that the equilibrium policy is the same—indicating that whether an increase in p gives rise to more extreme expected policies must depend on parameters. Consider a uniform distribution of ideologies, and assume away ego rents and re-election shocks, $\rho = q = 0$. Numerically we verified that

Result 1: The expected stationary policy in a large congress is a strictly concave, single-peaked function of p.

5.1 Valence Heterogeneity

We now investigate whether and when voters prefer an environment with more heterogeneity in valences. In particular, given two valence distributions with the same mean, G and G', such that distribution G second-order stochastically dominates G', are some voters hurt by the "riskier" distribution G'? and if so, who? Although our results extend to more general distributions, for simplicity, we compare a single valence setting with a two-type valence setting.

In a single valence setting, candidates have valence $v \equiv \bar{v}$ with probability one. Proposition A.1 reveals that the equilibrium thresholds $\{\bar{w}, \bar{c}\}$ are independent of valence $\bar{v}$. A Δv increase in valence increases the expected utility of each voter by Δv. Further, Proposition A.2 shows that $\bar{w}$ and $\bar{c}$ are linear functions of a.

In a two-type valence setting, with probability $p \in (0,1)$ an untried candidate has valence v_H, and with probability $(1-p)$ her valence is v_L, where $v_H > v_L$. Let $\{w_H, c_H, w_L, c_L\}$ be the associated equilibrium thresholds. Proposition 6 compares the two cases.

Proposition 6 *Fix the parameters a, δ and z. Let $\{w_H, c_H, w_L, c_L\}$ be the equilibrium* *thresholds when there are two valence types, $v_H > v_L$, v_H occurring with probability $p \in (0,1)$. Let $\{\bar{w}, \bar{c}\}$ be the equilibrium thresholds when there is a unique valence type $\bar{v}$. Then:*

1. $w_H > \bar{w} > w_L$,
2. $c_H > \bar{c} > c_L$,
3. $c_H - w_H > \bar{c} - \bar{w} > c_L - w_L$.

That is, when we compare the high valence incumbent in the heterogeneous valence setting with the incumbent in the homogeneous valence setting, we find that a high valence incumbent (1) can be reelected with a more extreme policy, (2) is more likely to be re-elected, and (3) has a larger compromise set. The opposite holds for a low valence incumbent.

To illustrate how the likelihood of the high valence type affects equilibrium cutoffs we consider an example where ideologies are uniformly distributed on $[-10, 10]$, Euclidean loss functions, $L_x(y) = -|x - y|$, a discount factor of $\delta = 0.35$, and we assume away ego rents and re-election shocks, $\rho = q = 0$. Table 1 shows how cutoffs vary with p. Note that the low valence thresholds for $p = 0$ and the high valence thresholds for $p = 1$ are exactly the same: Both are strategically equivalent to a unique valence setting, and from Proposition A.1, the higher expected valence is simply a lump-sum transfer. As we move from $p = 0$ to $p = 1$, the increase in the expected valence induces the median voter to set tighter re-election standards for both types, and both types are less willing to compromise, since compromising is more costly and the untried challenger is stochastically better. Figure 2 illustrates this, contrasting the one-valence and two-valence cases for $p = .1$ and $p = .9$.

p	w_L	w_H	c_L	c_H
0.0	**3.8553**	4.8553	**7.7142**	9.7766
0.1	3.7509	4.7509	7.4979	9.5589
0.2	3.6476	4.6476	7.2841	9.3437
0.3	3.5452	4.5452	7.0729	9.1312
0.4	3.4438	4.4438	6.8642	8.9213
0.5	3.3434	4.3434	6.6581	8.7139
0.6	3.2439	4.2439	6.4544	8.509
0.7	3.1453	4.1453	6.2531	8.3066
0.8	3.0477	4.0477	6.0543	8.1067
0.9	2.9510	3.9510	5.8578	7.9093
1.0	2.8553	**3.8553**	5.6637	**7.7142**

Table 1: Thresholds for different values of p, using $a = 10$, $\delta = 0.35$, $z = 1$, $v_H = 1$, $v_L = 0$.

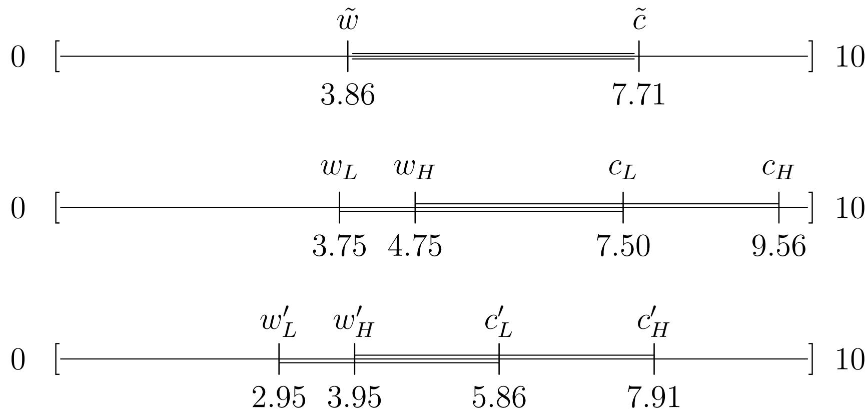

Figure 2: Thresholds for one and two-valence cases, with $p = 0.1$ and $p' = 0.9$.

Proposition 7 *For any given set of parameter values, there exists a $\underline{\delta} < 1$ such that, if $\delta \geq \underline{\delta}$—i.e., if voters are sufficiently patient—then the median voter's expected payoff from an untried challenger is **strictly higher** in a heterogeneous two-valence environment, with valences $v_H > v_L$ and probability $p \in (0,1)$ of v_H, than in a homogeneous environment where the unique valence is the average valence, $\bar{v} \equiv pv_H + (1-p)v_L$.*

The proposition says that a patient median voter prefers to have heterogeneity in valences.[6] This result could seem counterintuitive if one expects a risk-averse median voter to be hurt by (a) the gamble, (b) the more extreme equilibrium compromise cutoff by a high valence type ($w_H > \tilde{w}$), and (c) the reduced willingness of low valence types to compromise ($c_L < \tilde{c}$). However, those losses are more than compensated by the gains from the "competition" between good and bad candidates: (a) a low valence candidate has to take a more moderate position to be re-elected ($w_L < \tilde{w}$), (b) high valence candidates are more willing to compromise ($c_H > \tilde{c}$), and most importantly, (c) there is a positive option value associated with an untried challenger who may turn out to have a high valence—the decisive median voter has the option of voting extremist, low valence types out of office, in the hope of drawing a centrist/moderate high valence candidate. Since low valence incumbents are more likely to be ousted from office, in the long-run, heterogeneity raises the expected valence in the cross-section of office holders. When the median voter is patient the value of this future expected benefit exceeds the immediate costs associated with the reduced willingness of low valence candidates to compromise (a result that extends numerically to small δ).

But what about other voters? We now retrieve the intuition that because voters trade-off valence for policy differently, even though the median voter may gain from heterogeneity in

[6]Numerically we find that the result extends to small values of δ.

candidate qualities, voters with more extreme ideologies may be hurt. When we increase the heterogeneity in valence, it increases the long-run expected valence, which benefits all voters by the same amount. However, the relative impact of changes in equilibrium policies depends on the extent of voter risk aversion. To make this point, we consider loss functions $L_x(y) = -|x-y|^z$ with $z \geq 1$.

Euclidean loss function, $z = 1$**.** Analytically one can show that when voters have Euclidean loss function, changes that induce more extreme expected equilibrium policies hurt the median voter (and voters close to her) by more than extreme voters close to a. This is because extreme voters are "almost" risk neutral with respect to changes in the (symmetric) policy, and hence almost indifferent to mean zero shifts in policy. We know that the introduction of heterogeneity increases the expected equilibrium valence, which benefits all voters by the same amount. Therefore, if heterogeneity also yields more extreme policies, then voters with sufficiently extreme ideologies gain *more* than the median voter (and voters sufficiently close to her). Numerically we find that this is the case.

However, the fact that the median and extreme voters benefit from valence heterogeneity does **not** imply that all voters benefit. In fact, while most voters gain from valence heterogeneity (sufficiently extreme voters gaining more than voters close to the median), the welfare impact is not monotonic for ideologies close to the original cutoffs $\bar{w}$ and $\bar{c}$. These voters are the most affected by the changes to the two valence cutoffs $\{w_H, w_L\}$ and $\{c_H, c_L\}$, and voters close to $\bar{w}$ **lose**: both compromising candidates with high valence and low move away from $\bar{w}$ (low valence candidates compromise further, while high valence candidates compromise by less).

Quadratic loss function, $z = 2$**.** As Proposition 5 revealed, when voters have quadratic loss functions, $\overline{U}_x(w,c) = \overline{U}_0(w,c) - x^2$. Therefore, valence heterogeneity raises every voter's expected ex ante payoff from an untried challenger by the *same* amount as the median voter.

Cubic loss function, $z = 3$**.** When voters are more risk averse, with cubic loss functions, we find numerically that a shift from one-valence to a two-valence environment hurts all voters with sufficiently extreme ideologies: there exists an $\overline{x} > 0$ such that a voter with ideology x is hurt if and only if $|x| > \overline{x}$. For example, when ideologies are uniformly distributed in the interval $[-10, 10]$, $\delta = .15$, $v_L = 0$, $v_H = 1$, $p = 1/2$, $\rho = q = 0$, we find that $\overline{x} = 2.25$; i.e., even though the median voter benefits from valence heterogeneity, 77.5% of voters would prefer the economy of "average" politicians to that with heterogeneity in valences.

6 Investment in Valence

We now extend the model to endogenize the probability an untried candidate has high valence. To do this, we introduce two symmetric Interest Groups (IG) with ideologies $-i$ and $+i$. IG $-i$ supports party L while IG i supports party R. The interest groups have the same utility function as voters $\{-i, +i\}$. There are two possible valence levels, $v_H > v_L \geq 0$. In each election, an interest group can invest to increase the probability that an untried challenger from its supported party develops high valence. For example, the interest groups can provide resources that stochastically improve the professionalism of a representative's staff. At a cost of $c(p)$ for $p \in [0, 1]$, IGs can provide a probability p that the untried candidate has high valence. The cost function $c(p)$ is $\mathcal{C}^2$, with $c' > 0$, $c'' \geq 0$ and $c(0) = 0$. When an incumbent is running for reelection, her supporting IG cannot invest — an incumbent keeps her valence for her entire political career. The realized investment level is not observed by voters or the opposing IG, but agents correctly predict the equilibrium probability p^* that an untried candidate has high valence.

We focus on a setting where ideologies are uniformly distributed, the loss function is quadratic, $l(|x|) = -|x|^2$, all incumbents run for re-election ($q = 0$), and the IGs employ symmetric strategies. In equilibrium, the opposing IG never invests when the incumbent with valence v adopts a centrist policy $|y| \leq w_v$: the untried candidate is sure to lose. The opposing IG is only willing to invest if the incumbent chose an extreme policy $|y| > w_v$ and will not be re-elected. Now w_v leaves the median voter indifferent between reelecting an incumbent with valence v who adopts policy w_v and electing an untried candidate party who has the low valence v_L with probability one. The other equilibrium equations remain the same, but must now use the endogenous equilibrium probability p^*. The IG's investment is pinned down by the first-order condition

$$\begin{aligned} c'(p^*) \;&=\; \frac{2}{a}\Bigg\{\int_{c_H}^{a}[(1-\delta)(v_H-(i-y)^2)+\delta U_i^L(W,L)]dy+\int_{w_H}^{c_H}[v_H-(i-w_H)^2]dy \\ &+\; \int_{0}^{w_H}[v_H-(i-y)^2]dy-\int_{c_L}^{a}[(1-\delta)(v_L-(i-y)^2)+\delta U_i^L(W,L)]dy \\ &-\; \int_{w_L}^{c_L}[v_L-(i-w_L)^2]dy-\int_{0}^{w_L}[v_L-(i-y)^2]dy\Bigg\}. \end{aligned} \tag{10}$$

Equation (10) states that the marginal cost of the investment equals its marginal expected benefit, which is the expected payoff difference from drawing a high valence untried candidate versus a low valence one. For an IG whose ideology is close to the median voter's, there are three benefits from increasing the probability of a high valence candidate: (a) valence itself, (b) untried, high valence candidates are more likely to adopt policies closer to the median voter (Proposition 2), and (c) reduced turnover (Proposition 1.2). An IG with a more extreme ideology receives the same benefit from valence itself, but the other two factors move in opposite directions. The extreme right-wing IG prefers its supported candidate to adopt more extreme, right-wing policies—a moderate high valence candidate is not beneficial. However, turnover hurts more extreme interest groups, so they value the reduced turnover of high valence candidates. The next proposition shows that the preference for extreme policies dominates. Moreover, less investment[7] implies smaller p^*, and by Propositions 4 and 5, this implies that untried candidates yield lower payoffs to *all* voters.

Proposition 8 (Investment in Valence) *Investment in valence decreases with extremism of interest groups, which hurts all voters. It strictly decreases whenever* $p^* \in (0,1)$. *Moreover, sufficiently centrist interest groups always invest a strictly positive amount in valence,* $p^* > 0$.

Corollary 1 *Conditional on valence type, extremism of re-elected officials is positively correlated with extremism of interest groups.*

Figure 3 depicts the equilibrium probabilities of a high valence challenger for different values of IGs' ideologies when $c(p) = 20p^2$, and the model parameters are: $a = 100$, $v_H = 10$, $v_L = 0$, $\delta = .35$. When we move from a moderate IG to an extreme IG, the endogenous probability of drawing a high valence challenger drops from 83% to 74%.

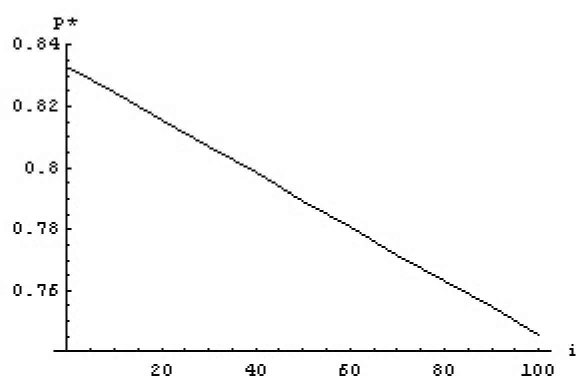

Figure 3: Equilibrium probability of high valence challenger. Parameters: $\alpha = 20$, $a = 100$, $v_H = 10$, $v_L = 0$, $\delta = .35$

[7] Our result only states that more extreme IG's invest less in the valence of their supported candidates, but we do not make any claims about total expenditures. We do not model advertisements or campaign expenditures—areas where empirical evidence suggests that more extreme IG's spend more money.

How Corollary 1 extends to the unconditional case? That is, how does the reduced investment by more extreme interest groups affect the expected stationary policy in a large congress? From Result 1, the expected stationary policy is a strictly concave, single peaked function of equilibrium probability $p*$. Therefore, there will be more extremism if the equilibrium probabilities $p*$ for each interest group are sufficiently high. That is,

Corollary 2 *If Result 1 holds, then extremism in a large congress is* ***positively*** *correlated with extremism of interest groups if the marginal cost of investment in valence is sufficiently low.*

For example, when $c(p) = \alpha p^2$, more extreme interest groups give rise to more polarized platforms if and only if α is sufficiently low (but not so low that $p = 1$).

7 Conclusion

In this paper, we develop a dynamic citizen-candidate model of repeated elections, in which candidates are distinguished by both their ideology and valence. Reputation/re-election concerns drive policy choices, and serve to endogenize the costs of locating extremely. We find that higher valence incumbents compromise more, compromise to more extreme policies, and are re-elected more. However, this does not imply that valence is negatively correlated with extremism: while the correlation is *negative* for first-term representatives, it is *positive* in the long-run stationary distribution of office-holders (large congress). That is, incorporating the dynamic nature of political competition is important. We then show that while a FOSD improvement in the distribution of valences benefits *all* voters, more heterogeneity in valence benefits the median voter, but may hurt most voters when they are sufficiently risk averse.

A maintained assumption of our model was that a politician's valence did not vary with her tenure. However, one might believe that valence may rise with tenure say due to greater pork provision by more senior incumbents, as in Bernhardt et al. (2004), or because, due to learning-by-doing, politicians become better at providing for their constituents with experience. When valence increases with tenure, it follows routinely that voters set slacker re-election standards for more senior incumbents. As a result, following any single re-elected politician along time, a researcher will uncover a positive correlation between extremism and tenure (*seniority effect*), as more senior incumbents need not moderate by as much to win re-election. However, if one compares the cohort of first-term representatives with the cohort of senior representatives, there is an opposing *group selection effect* because extremist first-term representatives are ousted from office. Disentangling and measuring these two effects, and their consequences for the relationship between extremism and valence, is an important, albeit complicating, task for empirical researchers.

8 Appendix

Proof: [**Theorem 1**] Define $W_v^L \subseteq [-a,0]$ as the party L win set for candidates with valence v. In equilibrium, an incumbent with ideology $x \in W_v^L$ and valence v implements her own ideology as policy and wins re-election. Define $C_v^L \subseteq [-a,0]$ as the party L compromise set for candidates with valence v. In equilibrium, an incumbent with ideology $x \in C_v^L$ and valence v does not adopt her own ideology as policy—she compromises to policy $p(x,v) = \arg\min_{w \in W_v^L} l(|x-w|)$, i.e., to the least costly policy that allows her to win re-election. Define the compromise function $c^L(y,v) = \arg\min_{w \in W_v^L} l(|x-w|)$. From symmetry, for $y<0$, $c^L(y,v) = -c^R(-y,v)$. Define $E_v^L \subseteq [-a,0]$ as the party L extremist set for candidates with valence v. In equilibrium, an incumbent with ideology $x \in E_v^L$ and valence v implements as policy her own ideology and loses re-election. Analogously define the symmetric sets W_v^R, C_v^R and E_v^R for party R. Notice that W_v^L, C_v^L, and E_v^L partition $[-a,0]$. Define the complete win set as $W = \{(y,v) \in [-a,a] \times V | y \in W_v^L \cup W_v^R\}$, and define C and E analogously.

For any voter x, the expected payoff from electing an untried candidate from party L is

$$\begin{aligned} U_x^L(W,C) &= k \int_V \Bigg\{ 2\int_{W_v^L} \left[u_x(y,v) + \frac{\delta q}{1-\delta} \overline{U}_x(W,C) \right] dF(y) \\ &+ 2\int_{C_v^L} \left[u_x\left(c^L(y,v),v\right) + \frac{\delta q}{1-\delta} \overline{U}_x(W,C) \right] dF(y) \\ &+ [1-\delta(1-q)]2\int_{E_v^L} \left[u_x(y,v) + \frac{\delta q}{1-\delta} \overline{U}_x(W,C) + \frac{\delta(1-q)}{1-\delta} U_x^R(W,C) \right] dF(y) \Bigg\} dG(v). \end{aligned}$$

Define $\beta \equiv \delta(1-q) \int_V 2\int_{E_v^L} dF(y)\ dG(v)$, which is $\delta(1-q)$ times the probability that a random candidate from party L belongs to the extremist set. Notice that $\beta \in [0,1)$ and

$$\int_V \left\{ 2\int_{W_v^L} dF(y) + 2\int_{C_v^L} dF(y) + [1-\delta(1-q)]2\int_{E_v^L} dF(y) \right\} dG(v) = 1-\beta. \tag{11}$$

With this notation in hand, we rewrite $U_x^L(W,C)$:

$$\begin{aligned} U_x^L(W,C) &= k \int_V \Bigg\{ 2\int_{W_v^L} u_x(y,v) dF(y) + 2\int_{C_v^L} u_x\left(c^L(y,v),v\right) dF(y) \\ &+ [1-\delta(1-q)]2\int_{E_v^L} u_x(y,v)\, dF(y) \Bigg\} dG(v) \\ &+ k(1-\beta)\frac{\delta q}{1-\delta} \overline{U}_x(W,C) + \beta U_x^R(W,C), \end{aligned}$$

where $\overline{U}_x(W,C)=\frac{U_x^L(W,C)+U_x^R(W,C)}{2}$. Analogously, we have

$$\begin{aligned}
U_x^R(W,C) &= k\int_V\Big\{2\int_{W_v^R}u_x(y,v)dF(y)+2\int_{C_v^R}u_x\left(c^R(y,v),v\right)dF(y)\\
&+ [1-\delta(1-q)]2\int_{E_v^R}u_x(y,v)\,dF(y)\Big\}dG(v)\\
&+ k(1-\beta)\frac{\delta q}{1-\delta}\overline{U}_x(W,C)+\beta U_x^L(W,C).
\end{aligned}$$

Substituting $U_x^R(W,C)$ into $U_x^L(W,C)$ and exploiting symmetry

$$\begin{aligned}
U_x^L(W,C) &= k\int_V\Big\{2\int_{W_v^L}[u_x(y,v)+\beta u_x(-y,v)]dF(y)\\
&+ 2\int_{C_v^L}[u_x\left(c^L(y,v),v\right)+\beta u_x\left(-c^L(y,v),v\right)]dF(y)\\
&+ [1-\delta(1-q)]2\int_{E_v^L}[u_x(y,v)+\beta u_x(-y,v)]dF(y)\Big\}dG(v)\\
&+ k[1-\beta][1+\beta]\frac{\delta q}{1-\delta}\overline{U}_x(W,C)+\beta^2U_x^L(W,C).
\end{aligned}$$

Subtracting $\beta^2U_x^L(W,C)$ from both sides and dividing both sides by $[1-\beta][1+\beta]$ yields

$$\begin{aligned}
U_x^L(W,C) &= \frac{k}{1-\beta}\int_V\Big\{2\int_{W_v^L}\frac{[u_x(y,v)+\beta u_x(-y,v)]}{1+\beta}dF(y)\\
&+ 2\int_{C_v^L}\frac{[u_x\left(c^L(y,v),v\right)+\beta u_x\left(-c^L(y,v),v\right)]}{1+\beta}dF(y)\\
&+ [1-\delta(1-q)]2\int_{E_v^L}\frac{[u_x(y,v)+\beta u_x(-y,v)]}{1+\beta}dF(y)\Big\}dG(v)+k\frac{\delta q}{1-\delta}\overline{U}_x(W,C).
\end{aligned}\tag{12}$$

For each pair valence v and ideology $y\le 0$, we take a weighted average between the instant utility of the (negative) ideology y and its symmetric positive counterpart $-y$, where more weight is given to the negative ideology. Symmetrically, we have

$$\begin{aligned}
U_x^R(W,C) &= \frac{k}{1-\beta}\int_V\Big\{2\int_{W_v^R}\frac{[u_x(y,v)+\beta u_x(-y,v)]}{1+\beta}dF(y)\\
&+2\int_{C_v^R}\frac{[u_x\left(c^R(y,v),v\right)+\beta u_x\left(-c^R(y,v),v\right)]}{1+\beta}dF(y)\\
&+[1-\delta(1-q)]2\int_{E_v^R}\frac{[u_x(y,v)+\beta u_x(-y,v)]}{1+\beta}dF(y)\Big\}dG(v)+k\frac{\delta q}{1-\delta}\overline{U}_x(W,C),
\end{aligned}$$

where most weight is given to the now positive y. Finally, the expected utility from a candidate drawn at large is

$$\begin{aligned}\overline{U}_x(W,C) \;&\equiv\; \frac{U_x^L\left(W,C\right)+U_x^R\left(W,C\right)}{2} \;=\; \frac{k}{1-\beta}\int_V \left\{ 2\int_{W_v^R} \frac{\left[u_x\left(y,v\right)+u_x\left(-y,v\right)\right]}{2} dF\left(y\right)\right. \\ &+2\int_{C_v^R} \frac{\left[u_x\left(c^R(y,v),v\right)+u_x\left(-c^R(y,v),v\right)\right]}{2} dF\left(y\right) \\ &\left.+[1-\delta(1-q)]2\int_{E_v^R} \frac{\left[u_x\left(y,v\right)+u_x\left(-y,v\right)\right]}{2} dF\left(y\right)\right\} dG\left(v\right) + k\frac{\delta q}{1-\delta}\overline{U}_x(W,C),\end{aligned}$$

where equal weight is given to both parties. Rearranging terms yields

$$\begin{aligned}\overline{U}_x(W,C) \;&=\; \frac{1}{1-\beta}\int_V \left\{ 2\int_{W_v^R} \frac{\left[u_x\left(y,v\right)+u_x\left(-y,v\right)\right]}{2} dF\left(y\right)\right. \\ &+\; 2\int_{C_v^R} \frac{\left[u_x\left(c^R(y,v),v\right)+u_x\left(-c^R(y,v),v\right)\right]}{2} dF\left(y\right) \\ &\left.+\; [1-\delta(1-q)]2\int_{E_v^R} \frac{\left[u_x\left(y,v\right)+u_x\left(-y,v\right)\right]}{2} dF\left(y\right)\right\} dG\left(v\right).\end{aligned} \tag{13}$$

We abuse notation and define $\beta(v)=\delta(1-q)2\int_{E_v^L}dF(y)$ which is $\delta(1-q)$ times the probability that a candidate from party L belongs to the extremist set *given* that the candidate has valence v. In equilibrium, the expected per period valence is defined as

$$E^*(v) \equiv \int_V \frac{v[1-\beta(v)]}{1-\beta} dG(v).$$

Notice that $v_L \leq E^*(v) \leq v_H$. Using this definition, we rewrite equation (12) as

$$\begin{aligned}U_x^L\left(W,C\right) \;&=\; kE^*(v)+\frac{k}{1-\beta}\int_V \left\{ 2\int_{W_v^L} \frac{\left[l(|x-y|)+\beta l(|x+y|)\right]}{1+\beta} dF\left(y\right)\right. \\ &+\; 2\int_{C_v^L} \frac{\left[l(|x-c^L(y,v)|)+\beta l(|x+c^L(y,v)|)\right]}{1+\beta} dF\left(y\right) \\ &\left.+\; [1-\delta(1-q)]2\int_{E_v^L} \frac{\left[l(|x-y|)+\beta l(|x+y|)\right]}{1+\beta} dF\left(y\right)\right\} dG\left(v\right) + k\frac{\delta q}{1-\delta}\overline{U}_x(W,C).\end{aligned} \tag{14}$$

For voter x, the equilibrium expected utility of re-electing an office-holder with valence v who adopts policy y is

$$U_x(y,v|W,C) = k[u_x(y,v)(1-\delta)+\delta q\overline{U}_x(W,C)]. \tag{15}$$

Define $\mathbf{S}_x^R$ as the retrospective $R-$set of voter with ideology x: the set of {implemented policy, valence} pairs of an incumbent from party R that x would re-elect over a random challenger from the opposite party (party L), and analogously define $\mathbf{S}_x^L$:

$$\begin{aligned}\mathbf{S}_x^R &= \left\{(y,v)|U_x(y,v|W,C)-U_x^L(W,C)\geq 0\right\}, \\ \mathbf{S}_x^L &= \left\{(y,v)|U_x(y,v|W,C)-U_x^R(W,C)\geq 0\right\}.\end{aligned}$$

Lemma A. 1 *There exists an upper bound $\overline{v}$, $0 < \overline{v}$, such that if $v_H - v_L \leq \overline{v}$, then for any $v \in V$*

1. $(0, v) \in \mathbf{S}_x^R$, $\forall x \in [0, a]$, *and*
2. $(0, v) \in \mathbf{S}_x^L$, $\forall x \in [-a, 0]$,

i.e., all voters prefer to re-elect an incumbent from their own party who adopts policy $y = 0$ over an untried candidate from the opposing party.

Proof: Take $x \in [0, a]$ and any $\tilde{v} \in V$. We must show that $U_x(0, \tilde{v}|W, C) - U_x^L(W, C) \geq 0$.

$$\begin{aligned}
U_x(0, \tilde{v}|W, C) - U_x^L(W, C) &= k\tilde{v} + kl(|x|) + k\frac{\delta q}{1-\delta}\overline{U}_x(W, C) - kE^*(v) \\
- \frac{2k}{1-\beta} \int_V \Bigg\{ &\int_{W_v^L} \frac{[l(|x-y|) + \beta l(|x+y|)]}{1+\beta} dF(y) + \int_{C_v^L} \frac{[l(|x - c^L(y, v)|) + \beta l(|x + c^L(y, v)|)]}{1+\beta} dF(y) \\
+ [1 - \delta(1-q)] &\int_{E_v^L} \frac{[l(|x-y|) + \beta l(|x+y|)]}{1+\beta} dF(y) \Bigg\} dG(v) - k\frac{\delta q}{1-\delta}\overline{U}_x(W, C).
\end{aligned}$$

Rewriting

$$\begin{aligned}
U_x(0, \tilde{v}|W, C) - U_x^L(W, C) &= k[\tilde{v} - E^*(v)] \\
&+ \frac{2k}{1-\beta} \int_V \Bigg\{ \int_{W_v^L} \left[l(|x|) - \frac{[l(|x-y|) + \beta l(|x+y|)]}{1+\beta} \right] dF(y) \\
&+ \int_{C_v^L} \left[l(|x|) - \frac{[l(|x - c^L(y, v)|) + \beta l(|x + c^L(y, v)|)]}{1+\beta} \right] dF(y) \\
&+ [1 - \delta(1-q)] \int_{E_v^L} \left[l(|x|) - \frac{[l(|x-y|) + \beta l(|x+y|)]}{1+\beta} \right] dF(y) \Bigg\} dG(v).
\end{aligned}$$

Concavity of the loss function implies that the term inside each integral is positive for any pair x and y, strictly positive for some. Therefore, if $\tilde{v} - E^*(v) \geq 0$, we are done—notice that this condition holds if there is a unique valence, $v_H = v_L$. If $\tilde{v} - E^*(v) < 0$, then we must show that

$$\begin{aligned}
E^*(v) - \tilde{v} \leq \; & \frac{1}{1-\beta} \int_V \Bigg\{ 2 \int_{W_v^L} \left[l(|x|) - \frac{[l(|x-y|) + \beta l(|x+y|)]}{1+\beta} \right] dF(y) \\
&+ 2 \int_{C_v^L} \left[l(|x|) - \frac{[l(|x - c^L(y, v)|) + \beta l(|x + c^L(y, v)|)]}{1+\beta} \right] dF(y) \\
&+ [1 - \delta(1-q)] 2 \int_{E_v^L} \left[l(|x|) - \frac{[l(|x-y|) + \beta l(|x+y|)]}{1+\beta} \right] dF(y) \Bigg\} dG(v).
\end{aligned} \tag{16}$$

Let $\overline{v} > 0$ be the LHS of equation (16). Since $E^*(v) - \tilde{v} \geq v_H - v_L$, a sufficient condition is that $\overline{v} \geq v_H - v_L$, which concludes the proof. Thus, for a fixed loss function $l(\cdot)$, the valence set V cannot be too dispersed, else the win set of some low valence politicians could be empty. More generally, higher concavity of the loss function would increase the upper bound $\overline{v}$.

Analogously, we can show that $(0, v) \in \mathbf{S}_x^L$. ■

Lemma A.1 implies that for any $v \in V$, $0 \in W_v^R$ and $0 \in W_v^L$. In particular, an incumbent with ideology $x \geq 0$ will not adopt a policy $y < 0$ since she could win by locating at zero.

Lemma A. 2 *The more moderate is a citizen's ideology, the higher is her expected utility from a challenger, whether selected from the opposing party or from a random party.*

In particular, for any pair $x', x \in [0, a]$ with $x' > x$,

$$U_x^L(W, C) > U_{x'}^L(W, C), \tag{17}$$

$$\overline{U}_x(W, C) > \overline{U}_{x'}(W, C), \tag{18}$$

$$U_x^L(W, C) - U_{x'}^L(W, C) > \overline{U}_x(W, C) - \overline{U}_{x'}(W, C). \tag{19}$$

Proof: Consider $x', x \in [0, a]$ with $x' > x$. From equation (13), using concavity of the loss function it is routine to show that $\overline{U}_x(W, C) > \overline{U}_{x'}(W, C)$. In particular, moderate citizen x loses less than extreme citizen x' from every candidate draw the opposite party, since the moderate is closer. While x' loses less for realizations of the same party that exceed $\frac{x'+x}{2}$, since $l'' \leq 0$, for every gain (smaller loss) that x' gets from extreme office-holders from the same party, x gains the same or more from the symmetric extreme office-holders from the other party.

This result and the same argument on equation (12) imply that $U_x^L(W, C) > U_{x'}^L(W, C)$.

To show that $U_x^L(W, C) - U_{x'}^L(W, C) > \overline{U}_x(W, C) - \overline{U}_{x'}(W, C)$, it is sufficient to show that $U_x^L(W, C) - U_{x'}^L(W, C) > U_x^R(W, C) - U_{x'}^R(W, C)$, which again follows from the concavity of $l(\cdot)$ and that fact that for any policy $y > (x' + x)/2$ voter x' loses less than voter x. ■

We now characterize the retrospective set of the median voter. From symmetry, $U_0^R(W, C) = U_0^L(W, C) = \overline{U}_0(W, C)$. An incumbent with valence $v \in V$ belongs to the retrospective set of the median voter if and only if she implements policy y such that

$$
\begin{aligned}
ku_0(y,v) + k\frac{\delta q}{1-\delta}\overline{U}_0(W,C) - \overline{U}_0(W,C) \geq 0 \quad &\Leftrightarrow \quad ku_0(y,v) - k\overline{U}_0(W,C) \geq 0 \\
&\Leftrightarrow \quad v + l(|y|) \geq \overline{U}_0(W,C).
\end{aligned}
$$

Define the threshold function $w : V \Rightarrow (0,a)$ by $w(v) = \{w_v \in (0,a) | v + l(w_v) = U_0(W,C)\}$, or simply $w(v) = |l^{-1}(U_0(W,C) - v)|$ where $l^{-1}(\cdot)$ denotes the inverse function of $l(\cdot)$. Again, the support of V cannot be too wide to have an interior solution $w_v \in (0,a)$. The retrospective set of the median voter is given by $R_0 = \{(y,v) | v \in V,\ y \in [-w(v), w(v)]\}$.

Lemma A. 3 *For each $v \in V$, the win set is connected, $W_v \equiv W_v^R \cup W_v^L = [-w_v, +w_v]$.*

Proof: Fix valence $v \in V$. From Lemma A.1, $0 \in W_v$. Suppose that $y > 0 \in W_v$, which implies that the incumbent is from party R. We will show that all citizens who vote for y also vote for any $y' \in [0,y]$. For each citizen $x \leq y'$ who votes for y, $U_x(y,v|W,C) \geq U_x^L(W,C)$ and since $U_x(y',v|W,C) \geq U_x(y,v|W,C)$, she also votes for y'. Every voter $x \geq y'$ also votes for y' since $U_x(y',v|W,C) \geq U_x(0,v|W,C) \geq U_x^L(W,C)$ where the last inequality follows from Lemma A.1. Therefore y' receives at least as many votes as y and $y' \in W_v$. The same argument applies to any $y < 0 \in W_v$. ■

Lemma A. 4 *The retrospective set of the median voter is contained in the win set:*

1. *If $(y,v) \in \mathbf{S}_0^R$ then $y \in W_v^R$;*
2. *If $(y,v) \in \mathbf{S}_0^L$ then $y \in W_v^L$.*

Proof: Let $(y,v) \in \mathbf{S}_0^R$ and $y \geq 0$. Every voter $x \geq y$ votes for y since $U_x(y,v|W,C) \geq U_x(0,v|W,C) \geq U_x^L(W,C)$ where the last inequality comes from Lemma A.1. Every voter $x \in [0,y]$ also votes for y since $U_x(y,v|W,C) \geq U_0(y,v|W,C) \geq U_0^L(W,C) \geq U_x^L(W,C)$ where the last inequality comes from Lemma A.2. Therefore x wins at least half of the votes and belongs to the win set. The same argument applies for $y \leq 0$. ■

Fix a $v \in V$. From Lemma A.3, every incumbent with valence v and ideology $x \in [0, w_v]$ adopts her own policy and is re-elected, and those incumbents with ideology $x > w_v$ that choose to compromise will adopt policy w_v since $w_v = \arg\min_{y \in W_v^R}(|x-y|)$. Similarly, incumbents $x < -w_v$ who compromise will adopt policy $-w_v$. For an incumbent with valence v and ideology $x > w_v$, the value of compromising to win if she runs for re-election is $U_x(w_v, v|W,C) + k\rho$, while the value of adopting her own ideology is $(1-\delta)(v+\rho) + \delta[(1-q)U_x^L(W,C) + q\overline{U}_x(W,C)]$. For an incumbent with valence v and ideology $x > w_v$, define $\Psi(x, v|W,C)$ as the net value of compromising,

$$\Psi(x,v|W,C) \equiv \delta(1-q)k(v+\rho) + k\, l(x-w_v) + \delta(1-q)k\frac{\delta q}{1-\delta}\overline{U}_x(W,C) - \delta(1-q)U_x^L(W,C).$$

The incumbent will compromise to w_v if and only if $\Psi(x,v|W,C) \geq 0$. For incumbent $x = w_v$, $\Psi(x,v|W,C) > 0$. Therefore, the necessary condition for the compromise set C_v^R to be connected is that $\Psi(x,v|W,C)$ crosses zero at most once for $x \in [w_v, a]$. A sufficient condition is that $\Psi(x,v|W,C)$ is concave in the range $x \in [w_v, a]$.

Lemma A. 5 *There exists a bound $0 < M'''$ such that if $|l'''| \leq M'''$ then $\Psi(x,v|W,C)$ is concave. Hence, for each valence $v \in V$, the compromise set consists of two symmetric, connected intervals around the win set, i.e., $C_v^L = [-c_v, -w_v]$ and $C_v^R = [w_v, c_v]$.*

Proof: Fix a $\tilde{v} \in V$. For $x > w_v$, after some algebra we can rewrite $\Psi(x,\tilde{v}|W,C)$ as

$$\begin{aligned}
\Psi(x,\tilde{v}|W,C) &= \delta(1-q)k[\tilde{v}+\rho-E^*(v)] \\
&+ \frac{k}{1-\beta}\int_V \Bigg\{ 2\int_{-w_v}^{0}\left[l(x-w_v) - \delta(1-q)\frac{[l(x-y)+\beta l(x+y)]}{1+\beta}\right] dF(y) \\
&+ 2\int_{C_v^L}\left[l(x-w_v) - \delta(1-q)\frac{[l(x+w_v)+\beta l(x-w_v)]}{1+\beta}\right] dF(y) \\
&+ [1-\delta(1-q)]2\int_{E_v^L}\left[l(x-w_v) - \delta(1-q)\frac{[l(x-y)+\beta l(|x+y|)]}{1+\beta}\right] dF(y)\Bigg\} dG(v).
\end{aligned}$$

The second derivative with respect to x is

$$\begin{aligned}
\frac{\partial^2}{\partial x^2}\Psi(x,\tilde{v} \quad | \quad W,C) &= \frac{k}{1-\beta}\int_V \Bigg\{ 2\int_{-w_v}^{0}\left[l''(x-w_v) - \delta(1-q)\frac{[l''(x-y)+\beta l(x''+y)]}{1+\beta}\right] dF(y) \\
&+ 2\int_{C_v^L}\left[l''(x-w_v) - \delta(1-q)\frac{[l''(x+w_v)+\beta l''(x-w_v)]}{1+\beta}\right] dF(y) \\
&+ [1-\delta(1-q)]2\int_{E_v^L}\left[l''(x-w_v) - \delta(1-q)\frac{[l''(x-y)+\beta l''(|x+y|)]}{1+\beta}\right] dF(y)\Bigg\} dG(v).
\end{aligned}$$

If $l''' = 0$, then l'' is a constant $l'' \leq 0$ and $\frac{\partial^2}{\partial x^2}\Psi(x,\tilde{v}|W,C) = l''(1-\delta(1-q)) \leq 0$. This implies that there exists a bound $0 < M'''$ such that if $|l'''| \leq M'''$ then $\Psi(x,v|W,C)$ is concave.

In particular, these conditions are satisfied by both Euclidean and quadratic loss functions. The condition requires that the risk aversion of citizens cannot grow too quickly (the second derivative cannot fall too fast), else compromise sets may not be connected—some representatives may prefer to lose the election rather than compromise, while representatives with more extreme ideologies may become so risk averse that they prefer to compromise. ■

Lemma A. 6 *If $U_x(0, v|W, C) - U_x^R(W, C)$ does not increase on x for any $x > 0$, then the win set is contained in the retrospective set of the median voter,*

1. *If $y \in W_v^R$, then $(y, v) \in \mathbf{S}_0^R$;*
2. *If $y \in W_v^L$, then $(y, v) \in \mathbf{S}_0^L$.*

Proof: First notice that if $U_x(0, v|W, C) - U_x^R(W, C)$ does not increase in x for any $x > 0$, then $U_x(y, v|W, C) - U_x^R(W, C)$ also does not increase in x for any $x > 0$ and $y < 0$, since $U_x(y, v|W, C)$ decreases at least as fast as $U_x(0, v|W, C)$ from concavity. Assuming that is the case, we will show that if $y \notin \mathbf{S}_0^L$, then $y \notin W_v^L$. Let $y \notin \mathbf{S}_0^L$ and $y < 0$. This implies that $0 > U_0(y, v|W, C) - U_0^R(W, C)$ and for every voter $x > 0$, the assumption states that $U_0(y, v|W, C) - U_0^R(W, C) \geq U_x(y, v|W, C) - U_x^R(W, C)$ which implies $U_x^R(W, C) > U_x(y, v|W, C)$. All voters with ideology $x \in [0, a]$ vote for the challenger and the incumbent will not be reelected, therefore $y \notin W_v^L$. Analogously we can show that any $y \notin \mathbf{S}_0^R$ and $y > 0$ will not belong to the win set. ■

Lemma A. 7 *There exists a lower bound $M'' < 0$ such that if $M'' \leq l'' \leq 0$ then $U_x(0, v|W, C) - U_x^R(W, C)$ does not increase in x for any $x > 0$.*

Proof: Fix a $\tilde{v} \in V$. For $x > 0$, after some algebra, one can solve for

$$
\begin{aligned}
U_x(0, \tilde{v}|W, C) - U_x^R(W, C) &= k[\tilde{v} - E^*(v)] \\
&+ \frac{k}{1-\beta} \int_V \left\{ 2 \int_0^{w_v} \left[l(x) - \frac{[l(|x-y|) + \beta l(x+y)]}{1+\beta} \right] dF(y) \right. \\
&+ 2 \int_{w_v}^{c_v} \left[l(x) - \frac{[l(|x-w_v|) + \beta l(x+w_v)]}{1+\beta} \right] dF(y) \\
&+ \left. [1 - \delta(1-q)] 2 \int_{c_v}^{a} \left[l(x) - \frac{[l(|x-y|) + \beta l(x+y)]}{1+\beta} \right] dF(y) \right\} dG(v).
\end{aligned}
$$

The first derivative with respect to x is

$$\begin{aligned}
& \frac{\partial}{\partial x}[U_x(0,\tilde{v}|W,C) - U_x^R(W,C)] \\
= \; & \frac{k}{1-\beta}\int_V \left\{ 2\int_0^{w_v} \left[\frac{\partial}{\partial x} l(x) - \frac{[\frac{\partial}{\partial x} l(|x-y|) + \beta \frac{\partial}{\partial x} l(x+y)]}{1+\beta} \right] dF(y) \right. \\
+ \; & 2\int_{w_v}^{c_v} \left[\frac{\partial}{\partial x} l(x) - \frac{[\frac{\partial}{\partial x} l(|x-w_v|) + \beta \frac{\partial}{\partial x} l(x+w_v)]}{1+\beta} \right] dF(y) \\
& \left. + [1-\delta(1-q)]2\int_{c_v}^{a} \left[\frac{\partial}{\partial x} l(x) - \frac{[\frac{\partial}{\partial x} l(|x-y|) + \beta \frac{\partial}{\partial x} l(x+y)]}{1+\beta} \right] dF(y) \right\} dG(v).
\end{aligned}$$

If $l'' = 0$, this quantity is indeed negative, because $|\frac{\partial}{\partial x} l(|x-y|)|$ is constant in x, y (negative for $y < x$ and positive for $y > x$). Therefore there is a uniform lower bound $M'' < 0$ such that if $M'' \leq l'' \leq 0$ then $U_x(0,\tilde{v}|W,C) - U_x^R(W,C)$ decreases in x.

The condition $\frac{\partial}{\partial x}\left[U_x(0,\tilde{v}|W,C) - U_x^R(W,C)\right] \leq 0$ is satisfied by Euclidean and quadratic loss function. ■

Therefore, the median voter is decisive and *every* equilibrium is fully characterized by a pair of functions $w, c : V \to [0,a]$ that satisfies the following equations for all $v \in V$

$$U_0(w_v, v|w,c) = U_0^R(w,c) = U_0^L(w,c) = \overline{U}_0(w,c), \tag{20}$$

$$U_{c_v}(w_v, v|w,c) + k\rho = (1-\delta)(v+\rho) + \delta q \overline{U}_{c_v}(w,c) + \delta(1-q)U_{c_v}^L(w,c). \tag{21}$$

Thus, Proposition 1 holds for *every* equilibrium.

Proof: **[Proposition 1]** Let $v_H, v_L \in V$ and $v_H > v_L$. From equation (3), $U_0(w_H, v_H|w,c) = U_0(w_L, v_L|w,c)$, thus from equation (8) $v_H + L_0(w_H) = v_L + L_0(w_L)$. Therefore $L_0(w_L) - L_0(w_H) = v_H - v_L$ and $L_0(w_L) > L_0(w_H)$. Since $l' < 0$, it implies $w_H > w_L$.

From our equilibrium characterization, $c_H > w_H$. Thus, trivially if $c_L \leq w_H$ then $c_H > c_L$. It remains to show that $c_H > c_L$ holds when $c_L > w_H$. Assume that is the case. In equilibrium, an office-holder with valence v_L and ideology c_L is indifferent between compromising to policy w_L and adopting her own ideology. From the indifference equation (9)

$$\begin{aligned}
& k(v_L + L_{c_L}(w_L)) + k\frac{\delta q}{1-\delta}\overline{U}_{c_L}(w,c) + \rho k \qquad (22) \\
= \; & (v_L+\rho)(1-\delta) + \delta q \overline{U}_{c_L}(w,c) + \delta(1-q)U_{c_L}^L(w,c).
\end{aligned}$$

The LHS of (22) represents the expected utility from compromising and the RHS the expected utility of adopting her own ideology. It suffices to show that an office-holder with ideology $x = c_L$ and valence v_H strictly prefers compromising to adopting her own ideology, i.e.,

$$\begin{aligned} & k(v_H + L_{c_L}(w_H)) + k\frac{\delta q}{1-\delta}\overline{U}_{c_L}(w,c) + \rho k & (23) \\ > \quad & (v_H + \rho)(1-\delta) + \delta q \overline{U}_{c_L}(w,c) + \delta(1-q) U^L_{c_L}(w,c). \end{aligned}$$

Subtract equation (22) from (23),

$$k[v_H - v_L + L_{c_L}(w_H) - L_{c_L}(w_L)] \quad > \quad (v_H - v_L)(1-\delta).$$

Rewrite,

$$(v_H - v_L)(k - 1 + \delta) + k[L_{c_L}(w_H) - L_{c_L}(w_L)] > 0. \tag{24}$$

Since $k > 1 - \delta$ the first term is strictly positive. Furthermore, $c_L > w_H > w_L$ implies that $(c_L - w_H) < (c_L - w_L)$. Hence, $L_{c_L}(w_H) > L_{c_L}(w_L)$ and the second term is also strictly positive. Therefore, the inequality holds for equations (24) and (23).

The indifference equation for a high valence office holder with ideology c_H is

$$\begin{aligned} & k(v_H + L_{c_H}(w_H)) + k\frac{\delta q}{1-\delta}\overline{U}_{c_H}(w,c) + \rho k & (25) \\ = \quad & (v_H + \rho)(1-\delta) + \delta q \overline{U}_{c_H}(w,c) + \delta(1-q) U^L_{c_H}(w,c). \end{aligned}$$

Subtracting the indifference equation (22) for a low valence candidate from (25) yields

$$\begin{aligned} & k(v_H - v_L + L_{c_H}(w_H) - L_{c_L}(w_L)) + k\frac{\delta q}{1-\delta}[\overline{U}_{c_H}(w,c) - \overline{U}_{c_L}(w,c)] & (26) \\ = \quad & (v_H - v_L)(1-\delta) + \delta q[\overline{U}_{c_H}(w,c) - \overline{U}_{c_L}(w,c)] + \delta(1-q)[U^L_{c_H}(w,c) - U^L_{c_L}(w,c)]. \end{aligned}$$

Rewriting

$$\begin{aligned} k(L_{c_H}(w_H) - L_{c_L}(w_L)) \quad = \quad & (1-\delta-k)(v_H - v_L) + \delta(1-q)[U^L_{c_H}(w,c) - U^L_{c_L}(w,c)] \\ & + \delta q(1 - \frac{k}{1-\delta})[\overline{U}_{c_H}(w,c) - \overline{U}_{c_L}(w,c)]. & (27) \end{aligned}$$

We must show that the RHS of equation (27) is strictly negative. The first term $(1 - \delta - k)(v_H - v_L)$ is strictly negative, so it remains to show that

$$(1-q)[U^L_{c_H}(w,c) - U^L_{c_L}(w,c)] + q(1 - \frac{k}{1-\delta})[\overline{U}_{c_H}(w,c) - \overline{U}_{c_L}(w,c)] < 0, \tag{28}$$

$$\Leftrightarrow (1-q)[U^L_{c_L}(w,c) - U^L_{c_H}(w,c)] > q(\frac{k}{1-\delta} - 1)[\overline{U}_{c_L}(w,c) - \overline{U}_{c_H}(w,c)]. \tag{29}$$

We know $c_H > c_L$. From lemma A.2, $U^L_{c_L}(w,c) - U^L_{c_H}(w,c) > \overline{U}_{c_L}(w,c) - \overline{U}_{c_H}(w,c)$. Furthermore, $1 > q \Rightarrow (1-\delta) > q(1-\delta) \Rightarrow 1 - \delta(1-q) > q \Rightarrow 1 > \frac{q}{1-\delta(1-q)} \Rightarrow 1 > \frac{qk}{1-\delta} \Rightarrow 1 - q > q(\frac{k}{1-\delta} - 1)$. Thus, equation (29) holds and the RHS of equation (27) is strictly negative. We know $k > 0$, thus $L_{c_H}(w_H) - L_{c_L}(w_L) < 0$, which implies $c_H - w_H > c_L - w_L$. ■

Lemma A. 8 *Any change in the threshold function w is strictly monotone.*

Proof: Fix the parameters of the model and let (w, c) and (w', c') be equilibrium thresholds given valence distributions G and G'. From median voter indifference condition, $U_0(w_v, v|w, c) = \overline{U}_0(w, c)$ can be simplified to $\overline{U}_0(w, c) = v + l(w_v)$. Hence, $v + l(w_v) = \tilde{v} + l(w_{\tilde{v}})$ and $v + l(w'_v) = \tilde{v} + l(w'_{\tilde{v}})$ for every $v, \tilde{v} \in V$. This implies

$$l(w'_v) - l(w_v) \quad = \quad l(w'_{\tilde{v}}) - l(w_{\tilde{v}})$$

for every $v, \tilde{v} \in V$. Since $l' < 0$, if for any $v \in V$ we have an increase from w_v to $w'_v > w_v$ then for all other valences $\tilde{v} \in V$ we must have $w'_{\tilde{v}} > w_{\tilde{v}}$. ■

Lemma A. 9 *Fix the parameters of the model and let (w, c) and (w', c') be equilibrium thresholds given valence distributions G and G'. Without loss of generality, let $w'_v \geq w_v$, for all $v \in V$. Define the changes in utility $\Delta U^L_x \equiv U^L_x(w', c') - U^L_x(w, c)$ and $\Delta \overline{U}_x \equiv \overline{U}_x(w', c') - \overline{U}_x(w, c)$. There exists an upper-bound function $B(c_v) \geq 0$ (strictly greater if $w'_v > w_v$) such that if for every incumbent c_v*

$$B(c_v) \geq \Delta U^L_{c_v} - k\frac{\delta q}{1-\delta}\Delta \overline{U}_{c_v}, \tag{30}$$

then the change in the threshold function c is weakly monotone.

Proof: Consider any two equilibria (w, c) and (w', c'). Exploiting lemma A.8 let $w'_v \geq w_v$, for all $v \in V$. Take any valence $v \in V$ and by contradiction suppose $c'_v < c_v$. Define the upper-bound function $B(c_v) \equiv \frac{k}{\delta(1-q)}[L_{c_v}(w'_v) - L_{c_v}(w_v)]$ and assume equation (30) holds. Notice that $B(c_v) \geq 0$, strictly greater if $w'_v > w_v$. Since (w, c) is an equilibrium, incumbent c_v is indifferent between compromising or not,

$$\begin{aligned} & k(v + L_{c_v}(w_v)) + k\frac{\delta q}{1-\delta}\overline{U}_{c_v}(w, c) + \rho k \\ = \quad & (v+\rho)(1-\delta) + \delta q \overline{U}_{c_v}(w, c) + \delta(1-q) U^L_{c_v}(w, c). \end{aligned} \tag{31}$$

For equilibrium (w', c'), incumbent c'_v is indifferent between compromising or not, which implies that incumbent c_v strictly prefers to not compromise,

$$\begin{aligned} & k(v + L_{c_v}(w'_v)) + k\frac{\delta q}{1-\delta}\overline{U}_{c_v}(w', c') + \rho k \\ < \quad & (v+\rho)(1-\delta) + \delta q \overline{U}_{c_v}(w', c') + \delta(1-q) U^L_{c_v}(w', c'). \end{aligned} \tag{32}$$

Subtract equation (31) from (32). After some algebra, we have

$$\frac{k}{\delta(1-q)}[L_{c_v}(w'_v) - L_{c_v}(w_v)] < \Delta U^L_{c_v} - k\frac{\delta q}{1-\delta}\Delta\overline{U}_{c_v},$$

a contradiction. Therefore, $c'_v \geq c_v$ for every $v \in V$. ■

Lemma A. 10 *If conditions C1 to C4 of theorem 1 are satisfied, then the system*

$$\begin{aligned} U_0(w_v, v|w, c) &= U^R_0(w, c) = U^L_0(w, c) = \overline{U}_0(w, c) \\ U_{c_v}(w_v, v|w, c) + k\rho &= (1-\delta)(v+\rho) + \delta q \overline{U}_{c_v} + \delta(1-q)U^L_{c_v}(w, c) \end{aligned}$$

has a unique solution (w, c).

Proof: Consider valence distribution $G = G'$. By contradiction, suppose (w, c) and (w', c') are both equilibria, $(w, c) \neq (w', c')$. Exploiting lemma A.8, without loss of generality let $w'_v \geq w_v$. If the threshold function c is weakly monotone, then $c'_v \geq c_v$ for every $v \in V$. We show that if incumbents do not become more extreme by reducing the thresholds c_v then the more extreme positions w'_v do not decrease sufficiently the expected utility of the median voter, violating her equilibrium condition $\overline{U}_0(w', c') = v + L_0(w_v)$. By definition,

$$\begin{aligned} \overline{U}_0(w', c') \quad - \quad \overline{U}_0(w, c) \;=\; & \int_V \Bigg\{ 2\int_0^{w_v} \left[k[0] + k\frac{\delta q}{1-\delta}[\overline{U}_0(w', c') - \overline{U}_0(w, c)] \right] dF(y) \\ + \quad & 2\int_{w_v}^{w'_v} \left[k[l(y) - l(w_v)] + k\frac{\delta q}{1-\delta}[\overline{U}_0(w', c') - \overline{U}_0(w, c)] \right] dF(y) \\ + \quad & 2\int_{w'_v}^{c_v} \left[k[l(w'_v) - l(w_v)] + k\frac{\delta q}{1-\delta}[\overline{U}_0(w', c') - \overline{U}_0(w, c)] \right] dF(y) \\ + \quad & 2\int_{c_v}^{c'_v} \left[k[v + l(w'_v)] + k\frac{\delta q}{1-\delta}\overline{U}_0(w', c') - (1-\delta)[v + l(y)] - \delta\overline{U}_0(w, c) \right] dF(y) \\ + \quad & 2\int_{c'_v}^{a} \left[(1-\delta)[0] + \delta\left[\overline{U}_0(w', c') - \overline{U}_0(w, c)\right] \right] dF(y) \Bigg\} dG(v). \end{aligned}$$

For each $v \in V$, substitute the expression inside the first two integrals by the smaller number $\left[k[l(w'_v) - l(w_v)] + k\frac{\delta q}{1-\delta}[\overline{U}_0(w',c') - \overline{U}_0(w,c)]\right]$. From equilibrium, $l(w'_v) - l(w_v) = \overline{U}_0(w',c') - \overline{U}_0(w,c)$ and since $k(1+\frac{\delta q}{1-\delta}) = 1$, the term inside each of the first three integrals simplifies to $\overline{U}_0(w',c') - \overline{U}_0(w,c)$. In the fourth integral, substitute the term $-(1-\delta)[v+l(y)]$ by the smaller number $-(1-\delta)[v+l(w_v)]$. From equilibrium condition, substitute $v+l(w'_v)$ by $\overline{U}_0(w',c')$ and $v+l(w'_v)$ by $\overline{U}_0(w,c)$. Again, the expression simplifies to $\overline{U}_0(w',c') - \overline{U}_0(w,c)$ and we have

$$\overline{U}_0(w',c') - \overline{U}_0(w,c) \quad > \quad [\overline{U}_0(w',c') - \overline{U}_0(w,c)] \int_V \left\{ 2\int_0^{c'_v} dF(y) + 2\delta \int_{c'_v}^{a} dF(y) \right\} dG(v).$$

Since $w'_v \geq w_v$, the median voter is not better off: $\overline{U}_0(w',c') - \overline{U}_0(w,c) \leq 0$. Moreover, $\int_V \left\{ 2\int_0^{c'_v} dF(y) + 2\delta \int_{c'_v}^{a} dF(y) \right\} dG(v) < 1$, which yields a contradiction. ∎

Proof: [**Proposition 2**] Assume ideologies are uniformly distributed in the interval $[-a, a]$. Define the expected policy (in absolute value) of an untried candidate with valence v,

$$EPol(v) = 2\left\{ \int_0^{w(v)} \frac{y}{2a} dy + \int_{w(v)}^{c(v)} \frac{w(v)}{2a} dy + \int_{c(v)}^{a} \frac{y}{2a} dy \right\}. \tag{33}$$

Fix any pair $v_H, v_L \in V$ with $v_H > v_L$. From Proposition 1, $w_H > w_L$ and $c_H > c_L$. Therefore,

$$\begin{aligned} EPol(v_L) - EPol(v_H) &= \frac{1}{a}\left\{ \int_{w_L}^{c_L} w_L \, dy + \int_{c_L}^{c_H} y \, dy - \int_{w_L}^{w_H} y \, dy - \int_{w_H}^{c_H} w_H \, dy \right\} \\ &= \frac{1}{a}\left\{ w_L(c_L - w_L) + \frac{c_H^2 - c_L^2}{2} - \frac{w_H^2 - w_L^2}{2} - w_H(c_H - w_H) \right\} \\ &= \frac{1}{2a}\left\{ c_H^2 - 2w_H c_H + w_H^2 - c_L^2 + 2w_L c_L - w_L^2 \right\} \\ &= \frac{1}{2a}\left\{ (c_H - w_H)^2 - (c_L - w_L)^2 \right\} > 0, \end{aligned}$$

where the inequality also comes from Proposition 1, $c_H - w_H > c_L - w_L$. ∎

Proof: [**Proposition 3**] From Proposition 1 we know $w_H > w_L$ and $c_H - w_H > c_L - w_L$ for any $v_H > v_L \in V$. This imply a strictly positive correlation between valence and extremism in the subset of re-elected officials. If $q = 0$, then in the stationary distribution, all office-holders are re-elected and the result holds. A small increase in q marginally changes thresholds

(w, c) and includes a small fraction of untried office holders in the stationary distribution, that is, politicians in their first term in office. If q is sufficiently small that the proportion of re-elected office holders in the stationary distribution is sufficiently large, the positive correlation still holds.

For $q = 1$, no office holder compromises since she never expects to run for re-election: the compromise set is empty, $w_v = c_v$ for all $v \in V$, and every incumbent adopts as policy her own ideology. Therefore, in the stationary distribution of office holders, the correlation between valence and extremism is zero. Furthermore, the result $w_H > w_L$ still holds—the median voter would accept more extremism and re-elect incumbents with higher valence, in the zero probability event that an incumbent runs for re-election. A small decrease in q marginally changes thresholds (w, c) and includes a small fraction of re-elected office holders in the stationary distribution. Since there is a strictly positive correlation in the subset of re-elected office holders, it follows that if q is sufficiently high but less than one, there is a positive correlation. ■

Proposition A. 1 (Valence Location Shift) *Let $\{w, c\}$ be the equilibrium threshold functions. If we change every valence $v \in V$ by the same amount γ, the new equilibrium threshold functions $\{w', c'\}$ are such that $w'(\gamma+v) = w(v)$ and $c'(\gamma+v) = c(v)$. Moreover, the expected utility of each citizen is changed by γ.*

Proof: Let $\{w, c\}$ be the solution of the model when the valence set is V and the distribution of valences has c.d.f. G. We now introduce a valence location shift: fix γ and construct the new set of valences $V' = \{v' \equiv \gamma + v | \forall v \in V\}$ and define the new distribution G' such that $G'(\gamma + v) = G(v)$, $\forall v \in V$. Conjecture that the new threshold functions $\{w', c'\}$ satisfy $w'(\gamma + v) = w(v)$, $c'(\gamma + v) = c(v)$, $\forall v \in V$ and that the expected utility of each citizen is increased by γ. We must show that these solve the equilibrium equations.

Since the original thresholds satisfy equation (8),

$$\begin{aligned} U_0(w(v), v|w, c) &= v + l(w(v)) \\ \gamma + U_0(w(v), v|w, c) &= \gamma + v + l(w(v)) \\ U_0(w'(v'), v'|w, c) &= v' + l(w'(v')), \end{aligned}$$

the new thresholds also satisfy this condition. To prove the same for equilibrium condition (9), first notice that $k[1 + \frac{\delta q}{1-\delta}] = 1$ and $1 - \delta + \delta q + \delta(1 - q) = 1$. Add $k[1 + \frac{\delta q}{1-\delta}]\gamma$ to the

LHS of (9) and $[1 - \delta + \delta q + \delta(1-q)]\gamma$ to the RHS, and rearrange terms to get the same result—the new equilibrium thresholds satisfy equation (9).

Finally, the LHS of equations (5) and (6) rises by γ, while on the RHS each period utility u_x and continuation utilities U_x^L and $\overline{U}_x$ rises by γ, so γ factors out and the equations still hold, concluding our proof. ■

Lemma A. 11 *In expectation, the median voter strictly prefers to draw an untested candidate with a higher valence to one with a lower valence.*

Proof: For any ideology x such that $|x| \in [w_H, c_L]$, both high and low valence politicians compromise. Therefore, the median voter is indifferent from equilibrium condition 8. For any other ideology, the median voter is strictly better if the candidate has higher valence. If both do not compromise, $|x| \in [0, w_L]$ or $|x| \in [c_H, a]$, policy choice is the same but valences are different. If the high valence candidate compromises and the low valence is an extremist, $|x| \in [c_L, c_H]$, the extremist with low valence yields a strictly lower payoff than the compromising high valence—which is why she is not re-elected. If the low valence compromises while the high valence is a centrist, $|x| \in [w_L, w_H]$, the high valence politician has the same valence but a more moderate position than a compromising high valence candidate, therefore yielding a higher payoff. ■

Proof: [**Proposition 4**] Suppose that the assumptions of theorem 1 hold. Let (w, c) be the unique equilibrium when valence is distributed accordingly to G, and (w', c') be the unique equilibrium when the distribution is G', where G' FOS dominates G. By contradiction and exploiting lemma A.8, suppose $w'_v \geq w_v$ for all $v \in V$. From weakly monotonicity of the threshold function c, $c'_v \geq c_v$ for all $v \in V$.

Equilibrium conditions imply that $\overline{U}_0(w', c') - \overline{U}_0(w, c) = L_0(w'_v) - L_0(w_v) \leq 0$. Define the expected utility $\overline{U}_0^*(w', c')$ as the expected utility of the median voter if we hold the distribution G constant and move from cutoffs (w, c) to (w', c'). Similarly to the proof of lemma A.10, the increase in $w's$ does not decrease sufficiently the utility of the median voter, so that $\overline{U}_0^*(w', c') - \overline{U}_0(w', c') > L_0(w'_v) - L_0(w_v)$.

Using lemma A.11, it is easy to show that $\overline{U}_0(w', c') > \overline{U}_0^*(w', c')$. Therefore, $\overline{U}_0(w', c') - \overline{U}_0(w', c') > L_0(w'_v) - L_0(w_v)$, a contradiction.

Hence, the median voter's expected payoff from an untried candidate strictly increases. Cutoffs w_v and c_v move closer to zero. We now prove that the median voter's expected period utility from the stationary distribution of office holders (large congress) also strictly increases for q sufficiently small or sufficiently big.

When q is close to one, the probability of running for re-election is close to zero. Hence, almost every incumbent adopts as policy her own ideology—the compromise set is almost empty—and the expected policy across different valence levels is almost the same. A FOSD improvement in valence distribution has a direct positive effect on the expected utility of the median voter—it increases expected valence—with a possibly negative effect due to the endogenous change in expected policy. If q is sufficiently close to one, this change in expected policy is sufficiently small that the valence improvement direct effect dominates, and the median voter is better off.

When q is close to zero, then in the stationary distribution of office holders, almost every representative in congress is a returning centrist or moderate incumbent. From the first part of this proof, the untried candidate is more attractive and the cutoffs w_v's strictly decrease. Therefore, in the stationary distribution, returning incumbents locate closer to the median voter. The improvement in valence decreases the cutoffs c_v, which also strictly increases the expected utility of the median voter in the stationary distribution. An incumbent with valence v and ideology c_v who was compromising to w_v becomes extreme, loses re-election and is (eventually) replaced by a centrist incumbent or a moderate incumbent. Both cases are strictly better than (v, w_v) by the median voter indifference condition and the fact that w_v is now closer to zero. ■

Proof: [**Proposition 5**] If the loss function is quadratic, one can show that for any equilibrium (w, c), we can write the expected utility of any voter x as function of the utility of the median voter, $\overline{U}_x(w, c) = \overline{U}_0(w, c) - x^2$. Therefore, when we change to any other equilibrium (w', c'), the expected utility of all voters will change by the same amount as the median voter. Since a FOSD shift strictly benefits the median voter, it strictly benefits all voters by the same amount. ■

Proposition A. 2 (Trade Off) *Assume ideologies are uniformly distributed, the loss function is given by $l(|x|) = |x|^z$ with $z \geq 1$, and fix parameter $\gamma > 0$. If we change*

- *the support of ideologies to* $[-\gamma a, +\gamma a]$*,*

- *every valence* $v \in V$ *to* $v' \equiv \gamma^z v$*,*

- *ego rents to* $\rho' = \gamma^z \rho$*,*

then equilibrium cutoff functions $\{w, c\}$ *become* $w'(v') = \gamma w(v)$*,* $c'(v') = \gamma c(v)$ *and the expected utility of the median voter is changed to* $U_0'(w', c') = \gamma^z U_0(w, c)$*. More generally, for each voter with ideology* $x' \equiv \gamma x$*, we have* $U_{x'}'(w', c') = \gamma^z U_x(w, c)$*.*

Proof: This proof is very similar to that of Proposition A.1. Conjecture that the new threshold functions and utilities satisfy the conditions in the proposition and verify that all equilibrium conditions hold.

For $\gamma > 1$, a larger set $[-a\gamma, +a\gamma]$ introduces more extreme candidates, so it is necessary to increase each valence v to $\gamma^z v$ in order to balance the choice of the median voter and office-holders. ■

Proof: [**Proposition 6**] If $p = 0$, then a candidate has low valence with probability one, and from Proposition A.1, $(w_L, c_L) = (\tilde{w}, \tilde{c})$. Using Proposition 1, we have $\tilde{w} < w_H$, $\tilde{c} < c_H$ and $\tilde{c} - \tilde{w} < c_H - w_H$. As we increase the probability p, we know from Proposition 4 that all cutoffs w_H, w_L, c_H and c_L strictly decline. When p reaches its maximum value of $p = 1$, using the same argument we have $(w_H, c_H) = (\tilde{w}, \tilde{c})$ and $w_L < \tilde{w}$, $w_L < \tilde{c}$ and $c_L - w_L < \tilde{c} - \tilde{w}$. ■

Proof: [**Proposition 7**] Let U_0^{Hom} and U_0^{Het} be the median voter's expected payoff from an untried challenger, in the homogeneous (one-valence) and in the heterogeneous (two-valence) cases, respectively. In equilibrium, we know that $U_0^{Hom} = \bar{v} + l(\bar{w})$ and $U_0^{Het} = v_H + l(w_H) = v_L + l(w_L)$. Write the difference in utilities as

$$U_0^{Het} - U_0^{Hom} = p[v_H + l(w_H)] + (1-p)[v_L + l(w_L)] - \bar{v} - l(\bar{w}) = pl(w_H) + (1-p)l(w_L) - l(\bar{w}) \equiv \epsilon.$$

We need to show that $\epsilon > 0$. By contradiction, suppose $\epsilon \leq 0$ for some $p \in (0, 1)$. Using the equilibrium conditions,

$$U_0^{Hom} = \bar{v} + 2\int_0^{\bar{w}} l(y)dF(y) + 2\int_{\bar{w}}^{\bar{c}} l(\bar{w})dF(y) + 2\int_{\bar{c}}^{a} [(1-\delta)l(y) + \delta l(\bar{w})]\, dF(y), \quad (34)$$

$$
\begin{aligned}
U_0^{Het} &= p\Big\{v_H + 2\int_0^{w_H} l(y)dF(y) + 2\int_{w_H}^{c_H} l(w_H)dF(y) + 2\int_{c_H}^{a} [(1-\delta)l(y)+\delta l(w_H)]\, dF(y)\Big\} \quad (35)\\
&+ (1-p)\Big\{v_L + 2\int_0^{w_L} l(y)dF(y) + 2\int_{w_L}^{c_L} l(w_L)dF(y) + 2\int_{c_L}^{a} [(1-\delta)l(y)+\delta l(w_L)]\, dF(y)\Big\}.
\end{aligned}
$$

Subtract equation (34) from (35). After some algebra

$$
\begin{aligned}
\epsilon &= 2\int_0^{w_L} 0dF(y) + 2\int_{w_L}^{\bar{w}} (1-\delta)[l(w_L)-l(y)]dF(y) + 2\int_{\bar{w}}^{w_H} [\epsilon - pl(w_H)]dF(y)\\
&+ 2\int_{w_H}^{c_L} \epsilon dF(y) + 2\int_{c_L}^{\bar{c}} [\epsilon + (1-p)(1-\delta)[l(y)-l(w_L)]]dF(y)\\
&+ 2\int_{\bar{c}}^{c_H} [\delta\epsilon + (1-\delta)p[l(w_H)-l(y)]]dF(y) + 2\int_{c_H}^{a} \delta\epsilon dF(y). \qquad (36)
\end{aligned}
$$

Rearranging terms,

$$
\begin{aligned}
\epsilon &\ \Big\{1 - 2\int_{\bar{w}}^{\bar{c}} dF(y) - 2\delta\int_{\bar{c}}^{a} dF(y)\Big\}\\
&= 2\int_{w_L}^{\bar{w}} (1-p)[l(w_L)-l(y)]dF(y) + 2\int_{\bar{w}}^{w_H} -pl(w_H)dF(y)\\
&+ 2\int_{c_L}^{\bar{c}} (1-p)(1-\delta)[l(y)-l(w_L)]dF(y) + 2\int_{\bar{c}}^{c_H} (1-\delta)p[l(w_H)-l(y)]dF(y). \qquad (37)
\end{aligned}
$$

The term $\Big\{1 - 2\int_{\bar{w}}^{\bar{c}} dF(y) - 2\delta\int_{\bar{c}}^{a} dF(y)\Big\}$ is strictly positive. All terms in the RHS of equation (37) are strictly positive, with the exception of $2\int_{c_L}^{\bar{c}}(1-p)(1-\delta)[l(y)-l(w_L)]dF(y)$. Therefore, there exists a $\underline{\delta} < 1$ such that, if $\delta \geq \underline{\delta}$, then the negative term is sufficiently close to zero and the RHS is strictly positive, a contradiction to $\epsilon \leq 0$. ■

Proof: [**Proposition 8**] In equilibrium, IGs correctly forecast future investments, the corresponding probability p^* that an untried candidate has high valence, and cutoffs $\{W, C\}$. If an IG invests, it chooses the p that equates marginal cost to marginal benefit. The marginal benefit is the difference between the expected utility of drawing a high valence candidate and the expected utility of drawing a low valence candidate, both from its own supported party. We will show that, given *any* equilibrium probability p^* and corresponding cutoffs $\{W, C\}$, the marginal benefit of drawing a high valence candidate strictly decreases with IG's ideology $i \geq 0$. Given the assumptions on the cost function, this implies that more extreme IG's invest less in valence.

When the loss function is quadratic, the following relationship holds between the expected utility of the median voter and the expected utility of the voter with ideology x:

$$\begin{aligned}
\overline{U}_x(W,C) &= \overline{U}_0(W,C) - x^2, \\
U_x^L(W,C) &= \overline{U}_0(W,C) - x^2 + x\overline{Y}^L(W,C).
\end{aligned}$$

The (negative) term $\overline{Y}^L(W,C)$ is the expected, discounted stream of policy choices conditional on electing an untried challenger from party L. The equilibrium indifference condition for incumbent c_H states that $v_H + \rho - (c_H - w_H)^2 = (1-\delta)(v_H+\rho) + \delta U_{c_H}^L$. For the median voter, $\overline{U}_0(W,C) = v_H - w_H^2$. Hence, we have

$$-(c_H - w_H)^2 = \delta[-\rho - w_H^2 - c_H^2 + c_H\overline{Y}^L(W,C)]. \tag{38}$$

Similarly,

$$(c_L - w_L)^2 = \delta[\rho + w_L^2 + c_L^2 - c_L\overline{Y}^L(W,C)]. \tag{39}$$

Consider an IG with $i \geq 0$ who supports party R. For any given equilibrium probability of a high valence challenger and corresponding cutoffs $\{W,C\}$, the IG's marginal benefit from investing in valence is:

$$\begin{aligned}
\mathrm{MB}_i(W,C) &= \frac{1}{a}\Bigg\{\int_{c_H}^{a}[(1-\delta)(v_H-(i-y)^2)+\delta U_i^L(W,L)]dy + \int_{w_H}^{c_H}[v_H-(i-w_H)^2]dy \\
&+ \int_0^{w_H}[v_H-(i-y)^2]dy - \int_{c_L}^{a}[(1-\delta)(v_L-(i-y)^2)+\delta U_i^L(W,L)]dy \\
&- \int_{w_L}^{c_L}[v_L-(i-w_L)^2]dy - \int_0^{w_L}[v_L-(i-y)^2]dy\Bigg\}.
\end{aligned}$$

In the first integral, substitute $U_i^L(W,L)$ by $(v_H - w_H^2) - i^2 + i\overline{Y}^L(W,C)$. In the fourth integral, substitute $U_i^L(W,L)$ by $(v_L - w_L^2) - i^2 + i\overline{Y}^L(W,C)$. After some algebra,

$$\begin{aligned}
\mathrm{MB}_i(W,C) &= \frac{1}{a}\Bigg\{\int_0^{w_L}[v_H-v_L]dy + \int_{w_L}^{w_H}[v_H-v_L+2i(y-w_L)-y^2+w_L^2]dy \\
&+ \int_{w_H}^{c_L}[v_H-v_L+2i(w_H-w_L)-w_H^2+w_L^2]dy \\
&+ \int_{c_L}^{c_H}[v_H-v_L+2i(w_H-y)-w_H^2+\delta w_L^2+(1-\delta)y^2+i\delta(2y-\overline{Y}^L(W,C))]dy \\
&+ \int_{c_H}^{a}(1-\delta)[v_H-v_L]dy\Bigg\}.
\end{aligned}$$

Taking the derivative with respect to i

$$\begin{aligned}\frac{\partial \mathrm{MB}_i(W,C)}{\partial i} &= \frac{1}{a}\Big\{\int_{w_L}^{w_H} 2(y-w_L)dy + \int_{w_H}^{c_L} 2(w_H-w_L)dy \\ &+ \int_{c_L}^{c_H}[2(w_H-y)+\delta(2y-\overline{Y}^L(W,C))]dy\Big\}.\end{aligned}$$

Integrating and rearranging terms yields

$$\frac{\partial \mathrm{MB}_i(W,C)}{\partial i} = \frac{1}{a}\Big\{-(c_H-w_H)^2+(c_L-w_L)^2+\delta[c_H^2-c_L^2-(c_H-c_L)\overline{Y}^L(W,C)]\Big\}.$$

Using equations (38) and (39), we have

$$\frac{\partial \mathrm{MB}_i(W,C)}{\partial i} = \frac{-\delta}{a}(w_H^2-w_L^2) < 0.$$

Therefore, the marginal benefit of the investment strictly decreases with extremism and more extreme IGs invest less in valence. They invest strictly less if the equilibrium probability is not zero or one. A reduced investment implies a smaller equilibrium probability p^* of high valence candidates, and by Propositions (4) and (5), untried candidates yield a lower payoff to *all* voters.

When an IG's ideology is sufficiently close to the median, Lemma A.11 implies that the IG strictly prefers the high valence candidate. Since $c(0)=0$ and $c(\cdot)$ is continuous, sufficiently centrist IGs always invest a strictly positive amount in valence. ■

References

[1] **Ansolabehere, Stephen and James M. Snyder, Jr.** "Valence Politics and Equilibrium in Spatial Election Models," *Public Choice*, June 2000, *103*, pp. 327-336.

[2] **Aragones, Enriqueta and Thomas R. Palfrey** "Mixed Equilibrium in a Downsian Model with a Favored Candidate," *Journal of Economic Theory*, March 2002, *103*(1): pp. 131-161.

[3] **Bernhardt, Dan and Daniel E. Ingberman** "Candidate Reputations and the Incumbency Effect, *Journal of Public Economics*, June 1985, *27*(1): pp. 47-67.

[4] **Bernhardt, Dan, Sangita Dubey, and Eric Hughson** "Term Limits and Pork Barrel Politics," *Journal of Public Economics*, December 2004, *88*(12),pp 2383-2422.

[5] **Bernhardt, Dan, Larissa Campuzano, Francesco Squintani and Odilon Câmara** "On the Benefits of Party Competition," *Working Paper*, 2008.

[6] **Black, Duncan** "On the Rationale of Group Decision-making," *Journal of Political Economy*, February 1948, *56*(1), pp. 23-34.

[7] **Black, Duncan** "The Theory of Committees and Elections," *Cambridge University Press*, 1958.

[8] **Callander, Steven** "Political Motivations", *Forthcoming, Review of Economic Studies*, 2008.

[9] **Callander, Steven and Simon Wilkie** "Lies, damned lies, and political campaigns," *Games and Economic Behavior* , August 2007, *60*(2), pp. 262-286.

[10] **Downs, Anthony** "An Economic Theory of Democracy," *New York: Harper and Row*, 1957.

[11] **Duggan, John** " Repeated Elections with Asymmetric Information," *Economics and Politics*, July 2000, *12*(2), pp. 109-135.

[12] **Enelow, James M. and Melvin J. Hinich (Eds.)** "Advances in the Spatial Theory of Voting," *Cambridge: Cambridge University Press*, 1990.

[13] **Fiorina, Morris P.** "Electoral Margins, Constituency Influence, and Policy Moderation: A Critical Assessment," *American Politics Quarterly*, October 1973, *1*(4), pp. 479-498.

[14] **Griffin, John D.** "Electoral Competition and Democratic Responsiveness: A Defense of the Marginality Hypothesis," *Forthcoming, Journal of Politics*, 2008.

[15] **Groseclose, Tim** "A Model of Candidate Location When One Candidate Has a Valence Advantage," *American Journal of Political Science*, October 2001, *45*(4), pp. 862-886.

[16] **Hotelling, Harold** "Stability in Competition," *The Economic Journal*, March 1929, *39*(153), pp. 41-57.

[17] **Iaryczower, Matias and Andrea Mattozzi** "Ideology and Competence in Alternative Electoral Systems," *Working Paper*, 2008.

[18] **Kartik, Navin and R. Preston McAfee** "Signaling Character in Electoral Competition," *American Economic Review*, June 2007, *97*(3), pp. 852-870.

[19] **Meirowitz, Adam** "Probabilistic Voting and Accountability in Elections with Uncertain Policy Constraints," *Journal of Public Economic Theory*, February 2007, *9*(1), pp. 41-68.

[20] **Stokes, Donald E.** "Spatial Models of Party Competition," *American Political Science Review*, June 1963, *57*(2), pp. 368-377.