Abstention, ideology and information acquisition[1]

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Abstract

Roll-off is usually explained as an informational phenomenon but in all models of abstention voters receive information exogenously. In this paper, however, we consider a committee where each member can collect information of different precision. Voters have asymmetric information and diverse preferences. Individual preferences are two dimensional and describe their ideological bias and the level of concern for the outcome of the election. We show that information and abstention are not necessarily negatively correlated at the individual level. In equilibrium, voters collect different qualities of information, and there are sometimes informed voters that abstain although they would have voted had they not collected information. The larger the electorate, the less information a voter collects and the higher the turnout is. In the limit, there is no abstention and no information acquisition. We also discuss how incentives to acquire information are non-monotonic regarding concern and ideology.

Keywords: Abstention, Information Acquisition, Heterogeneity.

JEL Codes: D71, D72, D82.

1 Introduction

Very few papers study equilibrium models of endogenous information in committees (Persico (2004), Gerardi and Yariv (2008), Gershkov and Szentes (2009), Feddersen and Sandroni (2006), Martinelli (2007), Cai (2009) and Li (2001)). None of them study abstention or *roll-off*: selective abstention when there are multiple elections in the same ballot. Considering that roll off is usually explained as an informational phenomenon, (Feddersen and Pesendorfer (1996)) a nexus between information acquisition and abstention seems appropriate. In this paper we study that nexus.

We study a committee making a binary decision by plurality rule where each member can collect information of different precision. Preferences of each member are described by their ideological bias and the level of concern for the outcome of the election. Preferences are diverse and each voter's preferences are private information. In this set up we answer the question, *who abstains in equilibrium?*

There is significant evidence that voter turnout and education are positively correlated (Matsusaka and Palda (1999), Milligan et al. (2004), Blais (2006)). Brady et al. (1995) point out that socioeconomic variables (including education) are correlated with the skills and resources a voter develops over time (Verba and Nie (1972)). Since these resources "explain" the voter decision to vote and who to vote for, they also explain the correlation between political activity and education. Hence education does not generate the turnout but is correlated with the ability to decide how to vote and when to vote. Matsusaka (1995) argues that one of these resources is the information a voter collects when deciding who is the appropriate candidate. He develops a decision theoretic model in which "knowledge" and "information" are strategic complements, so the more knowledgeable a person is, the stronger the effect of information on voting. Conversely, the more information available, the higher the impact knowledge has on the probability of voting.

Matsusaka (1995) uses a costly voting setup (Riker and Ordeshook (1968), Palfrey and Rosenthal (1983)) and assumes that the stronger a person feels about her choice the higher the utility this person receives from voting. Using a pure consumption model with agents that are not strategic, he shows that more knowledge as well as cheaper information lead to a higher probability of voting, .

Arguments based on the cost of voting cannot be applied to explain roll-off (Feddersen (2004)) since the voter is already in the booth and the "cost of voting" is sunk. Feddersen and Pesendorfer (1996) is the first paper providing an explanation for roll-off based on the level of information that a voter exogenously receives. They argue that uninformed voters rely on their peers for decisions since, on average, their peers are better informed. In essence, abstention is a form of delegation when a voter is poorly informed. This is the traditional swing voter's curse.[1]

Feddersen and Pesendorfer (1996) is extended in Feddersen and Pesendorfer (1999) by introducing preference and heterogeneity in the quality of information. They provide examples where "individuals with better information are more likely to participate than individuals with worse information..."[2] Their examples show that the probability of someone voting with some information is higher than the probability of someone voting with no information at all. Feddersen and Pesendorfer (1999) conclude that, "because uninformed independents abstain and informed independents vote, the model provides an informational explanation for why better educated individuals are more likely to vote" (Feddersen (2004), page 104).

Both Feddersen and Pesendorfer (1996) and Feddersen and Pesendorfer (1999) place the emphasis on information. This points out to the incentives voters have to acquire this information and to fully understand abstention we need to understand how voter's preferences, incentives to collect information and use of this information interact. In this paper we endogenize the decision to acquire information that voters end up using and provide a first analysis of that interaction..

[1] Abstention has been also studied in other decision theoretic models as in Ghirardato and Katz (2006) and Larcinese (2007)). Davis et al. (1970) assume that voters abstain because they do not gain much by switching the winner (indifference) or they do not win much by selecting any winner (alienation) and study elections when voters behave in that particular way. Shotts (2006) allows voters to signal by abstaining in order to affect the outcome of a second election.

[2] Feddersen and Pesendorfer (1999), page 382.

We present a traditional model of costless voting where voters have asymmetric information and diverse preferences, but we allow for voters to endogenously select the quality of information they will use to decide their vote. Our set up is based on Austen-Smith and Banks (1996): a two state of nature, two candidate election, where one candidate is preferred in one state while the other candidate is preferred in the remaining state. Hence, preferences show a common value component. Voters suffer no utility losses for electing the "correct" candidate, but differ on the utility losses they suffer for mistaken decisions. These losses are private information and reflect the private value component in preferences. Voters can collect information by selecting the precision of a binary signal that is correlated with the true state of the world.

Our model not only endogenize information but also introduces a richer set of preferences. Traditionally, preferences in committees are modeled with a single parameter that captures the ideological bias. There is no loss of generality when information is exogenous since all the incentives to vote can be captured with a relative ranking of alternatives. This assumption about preferences captures the relevant heterogeneity at the voting stage. Since the incentives to acquire information depend on the absolute level of utility losses, this restricted heterogeneity assumption matters to understand the link between costly information acquisition and abstention. To properly study information acquisition and to explain roll-off as a fully informational phenomenon, we must extend the model to unleash these incentives: in our model voters not only differ on the ideological level but also on the intensity of utility losses. There are voters with the same ideological bias that collect information of different quality depending on how much they care about possible mistakes. In contrast to other models of endogenous information,[3] in equilibrium voters collect information of different

[3] Persico (2004), Gerardi and Yariv (2008), Gershkov and Szentes (2009), Feddersen and Sandroni (2006) and Martinelli (2007) assume that voters are homogenous (at least those willing to collect information) and/or that each voter can receive an independent draw from a common distribution; Cai (2009) assumes that voters collect information before knowing their preferences and -therefore- they are homogenous at the information acquisition stage; Li (2001) assumes homogeneity at least on those that are willing to collect information; Martinelli (2006) allows for heterogeneity and different quality of information, but restricts the environment so in equilibrium every informed voter has the same incentives to collect information. The only exception is an example in Li (2001) with a very particular type of heterogeneity in a two-member

quality.

The existence of an equilibrium with voters endogenously collecting information of different qualities does not follow from a straightforward application of fixed point arguments. Since voters with different types (preferences) can and will select different qualities of information, the optimal information acquisition rule is a function from the space of preferences to the desired quality of information. Finding an equilibrium among all possible information acquisition rules requires the use of fixed point arguments in functional spaces. Compactness in functional spaces is not easy to achieve unless we severely restrict the information technology.[4] We solve this problem by transforming the existence of equilibrium problem in the space of best responses to a fixed point problem in the space of "pivotal" probabilities.

After showing existence we proceed to study the voter's behavior and the connection between information and abstention. We show that rational ignorance (making decisions by consciously not acquiring information) is driven by two different forces: 1) extreme (ex-ante) ideology and 2) balanced preferences combined with low intensity. We also show that there are some voters that *vote* the more informed they are and some voters that *abstain* the more informed they are. These behaviors are directly related to the voter's ideological bias and the fact that information relates to the underlying state of nature. In essence, these voters that collect information vote if this information reinforces their bias, but abstain if the information goes against their bias.

Abstention takes two different forms in our model. Both, though, are driven by the fact that interim preferences (the composition between ideological bias and information) are balanced. In a sense, the swing voter's curse happens because a voter does not have information (Feddersen and Pesendorfer (1996)) and this leaves him fairly indifferent between candidates

committee. Gerling et al. (2003) surveys models with information acquisition in committees.

[4]More technically, the quality of information may be a discontinuous mapping of the preference parameters, even among voters who decide to collect information. The best response function is only a C^0 function almost everywhere which precludes the application of fixed point arguments for infinite dimensional spaces (see Rudin (1973), in particular, the equicontinutity requirement in Schauder's Fixed Point Theorem). Second, because a particular behavior might not be optimal in a class of equilibria but it might in another class, the equilibrium takes on very different forms and fixed point arguments need to keep track of all these forms.

(Matsusaka (1995)) or the collected information goes against his original ideological bias thus creating indifference about the candidates at the interim level (Davis et al. (1970)).

These results beg the question whether it is lack of information what drives abstention. At an empirical level there is some evidence that information and turnout are in fact positively correlated. Wattenberg et al. (2000) uses survey data and aggregate data on Presidential and House races on the same ballot to show that information and abstention are negatively correlated. Coupé and Noury (2004) argue that there are some omitted variables in the previous study and that survey data suffers measurement error. They use data from the National Research Council regarding the quality of different research programs and find that roll off can be explained by lack of information. Larcinese (2007) and Lassen (2005) argue that information is endogenous and using an instrumental variable approach provide evidence that information and turnout are positively correlated.[5]

As pointed out by Matsusaka (1995) indifference at the interim stage is what makes voters abstain. This indifference arises in two different ways as our results suggest. We need to understand then why this was not found empirically. The reason is that most studies compare aggregate measures without conditioning for ideology (as we show matters) or define information in a coarse way. All these strategies lead to testing the composition of the electorate as a whole and not the voters behavior.[6]

At the aggregate level, these tests report a positive effect of information on turnout but, when looked at on individual levels, we argue that the effect is more complex and depends on the particular voter's ideology. While for some voters more information confirms their

[5]But Gentzkow (2006) finds that more TV exposure reduces turnout. He argues that the correlation between information and turnout is positive given that voters have substituted away other sources of information (newspapers and magazines). Gentzkow (2006) *assumes* that information and turnout are positively correlated and therefore need to explain why this correlation does not appear.

[6]Wattenberg et al. (2000), Larcinese (2007) and Lassen (2005) compare informed voters against uninformed voters. Coupé and Noury (2004) use three different levels of information quality to classify between informed and uninformed. To our knowledge the closest test regarding the effect of marginal information is Palfrey and Poole (1987). They found that "[in the distance utility model]...the probability of voting for Reagan increases with information level. The opposite is true for Carter." (Palfrey and Poole (1987), pp. 526). They also found that the effect of information on turnout is positive as expected. They decided to separate the decision "to vote" from the decision of "who to vote for" so they cannot properly analyze the effect of ideology on information acquisition and the overall effect on turnout.

bias and makes them more certain about their choice, for voter's with the opposite ideology more information contradicts their bias and makes them more uncertain about their choice. Eventually, this translates into a higher and lower probability of voting respectively. The aggregate tests then compare the relative sizes of different groups of voters. The mere existence of a large group of voters that collects enough information so they can rely only on the signal received (who we call independents) will generate the positive relation.

In the limit, our model predicts that voters collect very little information and, contrary to Feddersen and Pesendorfer (1996) and Feddersen and Pesendorfer (1999), the proportion of voters abstaining approaches 0 when the electorate gets large.[7] We show that restricting preferences to be one dimensional is not insignificant when information is endogenous and abstention is possible. Some strategies that are used by some voters in the model with richer preferences are strictly dominated for all members when restrictions on preferences are assumed. Since one dimensional preferences do not allow for intensity, strategies that depend on different intensity may or may not arise in equilibrium when preferences are restricted.[8] Moreover, if those strategies that are dominated in the model without intensity use abstention as part of an optimal voting strategy, restricted models fail to capture abstention as an equilibrium behavior. Therefore, restricting preferences may give misleading characterizations of abstention.

This paper will also show that information acquisition may not be a monotonic function of ideology and intensity: voters that have more at stake in an election may decide to collect less information. The optimal information acquisition function is discontinuous even among voters that collect some information since infinitessimal changes in preferences can lead to sharp changes in information acquisition. This happens when voters, endogenously, decide to use a different voting strategy (i.e. from following the information received to abstaining

[7]We simplify the set up by allowing voters to collect information from only one source while they allow voters to receive signals from different sources. On the other hand we do not assume a Poisson environment (where the number of voters is random).

[8]Larcinese (2009) ommits this dimension and concludes that "high incentives to be informed can be found at intermediate levels of partisanship." We show that this result is not generally true when preferences show intensity. In fact our results show that this relation is non monotonic.

if the information goes against their initial bias and vote if it confirms its bias). When voters use different voting strategies the value of information changes discontinuously, in turn, changing voters' demand for the quality of information they collect.

Our model allows us to study the correlation between information and abstention in detail. Because voters decide the precision of the information they use to decide their vote we can answer the question, do marginally better informed subjects vote with higher probability? We demonstrate that the answer depends on the ideology of the voter. While the question of whether informed voters show up more often than uninformed voters may be answered positively, the effect of marginally more information is still unclear (unless we fixed the ideological level of the voter). As pointed out by Downs (1957): "The knowledge (a person) requires is contextual knowledge as well as information" which we interpret to mean the decision to vote depends jointly on ideology and information.[9]

The rest of the paper is organized as follows. Our model is presented in Section 2 and Section 3 presents the main characterization and existence results. In Section 4 we focus on the plurality rule and discuss the incentives to abstain and the importance of our assumption about preferences. The main findings are provided in this section. Conclusions are provided in the last section and all proofs are provided in an Appendix.

2 The model

There is a set of potential voters $\mathcal{N}$ with $|\mathcal{N}| = n$ that must decide between two options A and Q; there are two equally likely states of nature $\omega \in \{a, q\}$. The winner is selected according to plurality rule.[10] The set of possible actions for a voter is $\{Q, \varnothing, A\}$ where Q (A) is a vote for candidate Q (A) and $\varnothing$ stands for abstention.

There are two classes of voters: **non partisan** and **partisan**. Partisans voters are described in terms of their behavior: with probability $\xi_x \in (0, 1)$, a partisan voter is type

[9] See the discussion in Matsusaka (1995).

[10] The existence and characterization results are robust to different rules and asymmetry across states as long as they verify some regularity conditions. Details can be provided upon request.

$x \in \{Q, \varnothing, A\}$ in which case she cast a ballot x, where $\sum_{x\in\{Q,\varnothing,A\}} \xi_x = 1$. Non partisan voters have contingent preferences described by $\theta = \{\theta_q, \theta_a\} \in [0,1]^2$: if A (Q) is selected in state q (a) then the voter type $\theta = \{\theta_q, \theta_a\}$ suffers a utility loss of θ_q (θ_a) and there is no utility loss for selecting A (Q) in state a (q). We refer to non partisan voter i's preferences as her type, and to a "non partisan voter type θ" simply as a "type θ". Voter's preferences are private information. With probability $\alpha \in (0,1)$ a voter i is partisan. If the voter is non partisan her preferences are drawn independently from a distribution with cumulative distribution function F on $[0,1]^2$ with no mass points. We assume further that no hyperplane of F has positive measure (hyperdiffuse distribution) so if we let $g(\theta_a)$ be any function we have that $\int dF(\theta_a, g(\theta_a)) = 0$.[11] We assume that F and α are common knowledge.

After knowing their types, each voter i can select the precision of the information they will receive: $p \in \left[\frac{1}{2}, 1\right]$ where p is the parameter of a Bernoulli random variable S that takes values on the set $\{s_q, s_a\}$. We assume that $\Pr(s_\omega \mid p, \omega) = p$ for $\omega \in \{a, q\}$ so the signal is correlated with the state and the precision is the same for both states. Information is costly and the precision cost is given by $C : \left[\frac{1}{2}, 1\right] \rightarrow \mathcal{R}_+$ where we assume that:

Assumption 1 *The cost function C is twice continuously differentiable everywhere in $\left[\frac{1}{2}, 1\right]$ and satisfies 1) $C'(p) > 0$ and $C''(p) > 0$ for all $p > \frac{1}{2}$, 2) $C''\left(\frac{1}{2}\right) \geq C\left(\frac{1}{2}\right) = C'\left(\frac{1}{2}\right) = 0$, 3) $\lim_{p\rightarrow 1} C'(p) \rightarrow \infty$.*

The set of voters $(\mathcal{N})$, the (common) distribution that characterize voters' $(\alpha, \xi_A, \xi_Q, F)$ preferences and the cost of information function (C), constitute a committee. We are going to say that a committee is **symmetric** if 1) $\xi_A = \xi_Q < \frac{1}{2}$, and 2) $F(x,y) = F(y,x)$ for all $(x,y) \in [0,1]^2$.

Since voters decide the precision of the signal and how they vote after receiving the signal a **pure strategy** of non partisan voter i is an investment function $P^i : [0,1]^2 \rightarrow \left[\frac{1}{2}, 1\right]$ and a voting function $V^i : [0,1]^2 \times \{s_q, s_a\} \rightarrow \{Q, \varnothing, A\}$, such that $P^i(\theta)$ is the investment level of

[11] We can ignore voters that are indifferent between strategies as in Caplin and Nalebuff (1991).

non partisan voter i with type θ, and $V^i(\theta, S) = (V^i(\theta, s_q), V^i(\theta, s_a))$ is the vote cast by non partisan voter i with type θ who receives the signal $s \in \{s_q, s_a\}$.[12] When we refer to a generic voting function, investment function or strategy, we omit the superscript indicating types. The voting function $V(\theta, S)$ is an ordered pair, where the first (second) element describes how the player votes after receiving s_q (s_a). [13] We will refer to a profile of strategies as $\left(\widetilde{P}, \widetilde{V}\right)$ where $\widetilde{P} = (P^1, ...P^n)$ and $\widetilde{V} = (V^1, ...V^n)$ are the profile of investment functions and voting functions for the whole committee. Analogously $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$ is the profile of strategies for all players but player i. We will say that, if $V^i(\theta, s) = v$ for all $s \in \{s_q, s_a\}$ player i of type θ uses an **uninformed** voting function, and if $V^i(\theta, s_q) \neq V^i(\theta, s_a)$ player i of type θ uses an **informed** voting function. We will identify strategies by the voting function they employ. We focus on strategies that do not depend on the identity of the voter but just on the type so we focus on equilibria in which the profile of strategies is the same for every voter: a **symmetric profile** of strategies $\left(\widetilde{P}, \widetilde{V}\right)$ is characterized by $(P^i(\theta), V^i(\theta)) = (P(\theta), V(\theta))$ for all $i = 1, ...n$.

The timing of the game is as follows: 1) Nature draws the profile of types and the state, 2) Each player i observes her own preferences, 3) non partisan player i privately decides whether or not to acquire information by selecting $p^i \in \left[\frac{1}{2}, 1\right]$, 4) each player draws a private signal from the selected distribution parameterized by p^i, 5) players vote simultaneously after signals are observed and, 6) the winner is elected according to simple majority rule.

Conditional on the profile of strategies of all voters but i, we define the probability that the winner is x in state ω, when voter i votes v, as

$$\Pr\left(x \mid \omega, v, \left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)\right) \tag{1}$$

[12] The reader may argue that voting rules should be contingent on the level of investment performed by each voter so $V^i : [0,1]^2 \times \left[\frac{1}{2}, 1\right] \times \{s_q, s_a\} \to \{Q, \varnothing, A\}$. Results are unaffected since no other public information is revealed to the voters between the investment decision and the voting decision.

[13] $V(\theta, S)$ describes the voter's behavior and $(v_q, v_a) \in \mathbf{X}^2$ is notation to describe arbitrary strategies (vote v_q after receiving s_q and vote v_a after receiving s_a) . When we want to refer to a particular vote we use just v.

The expected utility of player i of type θ when she votes v , and the state is ω, is

$$u^{i}\left(v \mid \theta, \omega\right) \equiv -\theta_{\omega} \Pr\left(\left(-\omega\right) \mid \omega, v, \left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)\right) \tag{2}$$

where we let $(-\omega) = Q(A)$ if $\omega = a(q)$. Expression (2) is just the product of the disutility of a mistake $(-\theta_{\omega})$ and the probability of a mistake in the state ω, given vote v. We define the expected utility of player i of type θ and investment choice p, when she votes v after receiving the signal s as

$$U^{i}\left(p, v \mid \theta, s\right) \equiv \sum_{\omega \in \{q,a\}} u^{i}\left(v \mid \theta, \omega\right) \Pr\left(\omega \mid s, p\right) \tag{3}$$

Using (3), the gross expected utility of player i of type θ and investment choice p, for a voting strategy (v_q, v_a) is

$$\mathcal{U}^{i}\left(p, \left(v_q, v_a\right)\right) \mid \theta) \equiv \sum_{x \in \{q,a\}} \frac{U^{i}\left(p, v_x \mid \theta, s_x\right)}{2} \tag{4}$$

where we used Bayes rule and the fact that both states are equally likely. We study Bayesian equilibria in symmetric profiles of pure strategies. Although we omit other players' strategies in definitions (3) and (4), the reader should understand that player i's payoffs depend on $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$.

Definition 1 *A symmetric Bayesian equilibrium for the voting game is a strategy* $\left(P^{*}(\theta), V^{*}(\theta, S)\right)$ *such that:* ***1)*** *for all* $j = 1, ...n$, $V^{j}(\theta, S) = V^{*}(\theta, S)$ *and* $P^{j}(\theta) = P^{*}(\theta)$ *for every type* θ*,* ***2)*** *for every type* θ*, for all signal* s*, and for any other feasible vote* v'*, the strategy* $\left(P^{*}(\theta), V^{*}(\theta, S)\right)$ *satisfies*

$$U^{i}\left(P^{*}(\theta), V^{*}(\theta, s) \mid \theta, s\right) \geq U^{i}\left(P^{*}(\theta), v' \mid \theta, s\right) \tag{5}$$

and ***3)*** *for every type* θ*, and for any other feasible votes* (v_q, v_a) *and* p*, the strategy* $\left(P^{*}(\theta), V^{*}(\theta, S)\right)$

satisfies

$$\mathcal{U}^i\left(P^*\left(\theta\right), V^*\left(\theta, S\right) \mid \theta\right) - C\left(P^*\left(\theta\right)\right) \geq \mathcal{U}^i\left(p, \left(v_q, v_a\right) \mid \theta\right) - C\left(p\right) \tag{6}$$

The probability that an arbitrary voter $j \neq i$ votes v, in state ω, when all other players but i are using the strategy $\left(P\left(\theta\right), V\left(\theta, S\right)\right)$ is

$$\Pr\left(v \mid \omega\right) = \left(1-\alpha\right) \int_{\theta\in[0,1]^2} \sum_{s\in\{s_q,s_a\}} \mathbf{I}\left(V\left(\theta, s\right) = v\right) \Pr\left(s \mid P\left(\theta\right), \omega\right) dF\left(\theta\right) + \alpha\xi_v \tag{7}$$

where $\mathbf{I}\left(x = y\right) = 1$ iff $x = y$ and 0 otherwise.[14] This expression aggregates over the two sources of private information present in the model: the voter's type and the signal received after investment.

3 Solving the Model

3.1 Voting Incentives

We omit the other player's strategies in (1) and let $\Pr\left(x \mid \omega, v\right)$ be the probability of a particular outcome $x \in \{Q, A\}$, in state ω, after player i votes v. Define the change in the probability of A winning when voter i switches her vote from $X \in \{Q, \varnothing\}$ to A in state ω as

$$\Delta \Pr\left(\omega, X\right) \equiv \Pr\left(A \mid \omega, A\right) - \Pr\left(A \mid \omega, X\right) \tag{8}$$

Note that $\Delta \Pr\left(\omega, Q\right)$ and $\Delta \Pr\left(\omega, \varnothing\right)$ are not the only expressions that reflect how chances of A winning change when a voter switches. Indeed, if the voter switches her vote from Q to $\varnothing$, A's chances of winning will also increase. That term can be described by $\Delta \Pr\left(\omega, Q\right) - \Delta \Pr\left(\omega, \varnothing\right)$, for $\omega \in \{q, a\}$.[15] The existence of partisan voters makes every outcome possible

[14] The first part of the right side is just the probability that a voter is non partisan multiplied by the probability that a non partisan votes v. The second part is the probability that a voter is partisan, multiplied by the probability that a partisan votes v.

[15] Note that $\Delta \Pr\left(\omega, X\right)$ is not the traditional expression of the probability of a particular state conditional on being pivotal and a particular signal ($\Pr\left(\omega \mid piv, s\right)$). Although these expressions are intimately related

in equilibrium and therefore:[16]

Lemma 1 *In any committee,* $\Delta \Pr(\omega, Q)$*,* $\Delta \Pr(\omega, \varnothing)$ *and* $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \varnothing)$ *are positive for each* $\omega \in \{q, a\}$*.*

Using the definition of expected utility in (4) and equation (5), a necessary condition for a non partisan voter type θ to vote for A after receiving the signal s is

$$\frac{\theta_q}{\theta_a} \frac{\Pr(q \mid s, p)}{\Pr(a \mid s, p)} \leq \min\left\{ \frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)}, \frac{\Delta \Pr(a, \varnothing)}{\Delta \Pr(q, \varnothing)} \right\} \tag{9}$$

and a necessary condition for her to vote for Q is

$$\frac{\theta_q}{\theta_a} \frac{\Pr(q \mid s, p)}{\Pr(a \mid s, p)} \geq \max\left\{ \frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)}, \frac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{\Delta \Pr(q, Q) - \Delta \Pr(q, \varnothing)} \right\} \tag{10}$$

Strict inequalities give sufficient conditions.

It is immediate to see that the set of uninformed voters (a voter is uninformed if $p = \frac{1}{2}$ which implies $\Pr\left(q \mid s, \frac{1}{2}\right) = \Pr\left(a \mid s, \frac{1}{2}\right)$) with type θ using $V(\theta, s_a) \neq V(\theta, s_q)$ has no mass. Therefore, only uninformed strategies with $V(\theta, s_a) = V(\theta, s_q)$ and informed strategies with $P(\theta) > \frac{1}{2}$ and $V(\theta, s_a) \neq V(\theta, s_q)$, need to be studied. Under which conditions is abstention an optimal action for a non partisan voter?

Lemma 2 *A necessary condition for abstention to be part of an optimal strategy for some non partisan voter* θ *in any committee is*

$$\frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)} \geq \frac{\Delta \Pr(a, \varnothing)}{\Delta \Pr(q, \varnothing)} \tag{11}$$

Proof. See Appendix (A.2). ■

Recalling that a voting strategy is a pair $(v_q, v_a) \in \{Q, A, \varnothing\}^2$, there are 9 possible voting strategies. Six of them may be part of an informed strategy: QA, $Q\varnothing$, AQ, $A\varnothing$, $\varnothing Q$, and

our presentation simplifies enormously the analysis of the incentives to vote and to collect information.

[16]For more general rules some care is needed. Details can be provided upon request.

$\varnothing A$. Some of them cannot be optimal with positive probability. Indeed, those that involve information being use in the wrong way are not optimal for a positive mass of players.

Lemma 3 *The voting strategies AQ, $A\varnothing$ or $\varnothing Q$ are not optimal for almost all types.*

Proof. See Appendix (A.2). ■

Now we need to consider only six voting strategies that may occur in equilibrium with positive probability. In equilibrium, voters can be separated in six different groups: **strong supporters for each candidate** ($\mathcal{SS}^A$ for A and $\mathcal{SS}^Q$ for Q), **weak supporters for each candidate** ($\mathcal{WS}^A$ for A and $\mathcal{WS}^Q$ for Q), **abstainers** ($\mathcal{A}$) and **independents** ($\mathcal{I}$). Weak supporters for A (Q) vote for A (Q) if $s = s_a$ ($s = s_q$) and abstain if $s = s_q$ ($s = s_a$) while strong supporters for A (Q) vote for A (Q) without collecting information. Abstainers do not collect information and abstain no matter the signal received and independents collect information and follow the signal they receive.

3.2 Information acquisition

It is straightforward to see that abstainers and strong supporters do not invest, while the probability that a type uses a weak supporter's strategy without performing any investment is 0. Now there are three relevant investment functions: one for each group that collects information (independents and weak supporters for A and Q). We define

Definition 2 *Let $P^x : [0,1]^2 \to \left[\frac{1}{2}, 1\right]$ for $x \in \{QA, \varnothing A, Q\varnothing\}$ be such that $P^{\varnothing A}(\theta)$, $P^{Q\varnothing}(\theta)$ and $P^{QA}(\theta)$ are the investment strategy of weak supporters for A, weak supporters for Q, and independents, respectively.*

Using (4) for each of the possible optimal strategies with investment and the information technology, we derive the optimal investment function implicitly as:

$$\begin{aligned} C'\left(P^{XA}(\theta)\right) &= \sum_{\omega\in\{q,a\}} \theta_\omega \frac{\Delta \Pr(\omega, X)}{2}, X \in \{Q, \varnothing\} \qquad (12) \\ C'\left(P^{Q\varnothing}(\theta)\right) &= C'\left(P^{QA}(\theta)\right) - C'\left(P^{\varnothing A}(\theta)\right) \end{aligned}$$

Since $\lim_{p\to 1} C'(p) \to \infty$, there is some $\eta < 1$ such that $P^x(\theta) \leq \eta$ for all informed voting strategies with $x \in \{QA, \varnothing A, Q\varnothing\}$.[17] The second equation in (12) illustrates that a player type θ using the strategy QA collects more information than she would have collected if she were a weak supporter. Why is this the case? Imagine a voter that is considering voting for A after signal s and compares the benefit of switching her vote to Q. That switch will change the outcome when there is a tie (making Q the winner instead of A), when A is winning by one vote (creating a tie instead of creating a wider margin for A), and when Q is winning by one vote (increasing the margin for Q instead of creating a tie). Now let's make that comparison with $\varnothing$. That switch will change the outcome when there is a tie (validating the tie instead of making A the winner), and when Q is winning by one vote (Q wins instead of creating a tie). Note that the situation where A was winning by one vote is not relevant for comparing A and $\varnothing$. In a sense, abstaining reduces the marginal value of the information and that is the reason why weak partisans collect less information than independents even though preferences might be similar.

For the independent behavior to be optimal, the level of investment required must be high. The next lemma states formally that whenever there are incentives to abstain, independents must invest a strictly positive amount so the precision of information must be strictly bigger than $\frac{1}{2}$.

Lemma 4 *A necessary condition for the independent behavior to be optimal with investment level p, is*

$$\left(\frac{p}{1-p}\right)^2 \geq \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(a,\varnothing)} \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} \tag{13}$$

Moreover, if there is endogenous abstention with positive probability ((11) holds with strict inequality) independents must invest a strictly positive amount.

Proof. See Appendix (A.2). ■

[17] It is worth noticing that the restriction of P to the domain $[0,1]^2$ is not needed. This will play an important role when we show that an equilibrium exists.

Assume that θ_a and θ_q are low so there is little investment in information acquisition. If they are about equal, the risk of introducing noise in the electorate plus the cost of investment entails a high cost of utility (direct and indirect). Since preferences are balanced (θ_a and θ_q are close), the non partisan voter prefers delegating to the electorate rather than voting for one or the other candidate with very weak evidence: being an abstainer is a better strategy than being independent because it saves on investment. This is the traditional non convexity in the value of information (Stiglitz and Radner (1984) and Chade and Schlee (2002)); in order for information to be useful when there a particular action depends on information and preferences, information should be enough to overpower the preferences.

When θ_a and θ_q are further apart, the argument is valid for the signal that favors the candidate the voter is biased against: abstention when that signal is received must be preferred to any positive vote. Basically the signal does not convey enough evidence to overturn the bias. Therefore, behaving as a weak supporter is better than being an independent. The fact that there are no independents close to the type $(0, 0)$ creates some technical problems when we prove existence of equilibrium: there can be very different classes of equilibria and the characterization depends on "how many" independents are.

3.3 Existence and Characterization

It is common to see in the literature existence results before characterization results. In order for us to be able to follow that strategy, our best responses must behave well enough. In particular our investment functions should belong to an equicontinuous family of real functions in order for the candidate space of best responses to be compact (see Rudin (1973)). We know that the investment functions are not continuous so we are forced to develop a new strategy in order to show existence.[18] We first characterize the equilibrium and then use its geometric properties to actually show that there is one.

In order to formally describe the equilibrium we need to define cutoff functions that

[18] Results that deal with discontinuous games usually require some sort of compactness (see Reny (1999)).

separate types according to the strategy they use. There are six possibly optimal strategies which implies that a particular type θ must perform 15 comparisons in order to decide which strategy to use. Fortunately, there are some cut off functions that do not intersect in the type space. For example, condition (6) makes the strategies AA and QQ jointly incompatible: if a voter is considering AA so (9) holds for $s \in \{s_a, s_q\}$ then (10) does not hold for $s \in \{s_a, s_q\}$. This reduces the number of comparisons to 10.

Each cutoff function will de described by a superscript. Let $(v_q v_a) \in \{A, Q, \varnothing\}^2$ and $\left(v_q' v_a'\right) \in \{A, Q, \varnothing\}^2$ be a pair of voting functions. Using the expression for expected utilities (4), an uninformed strategy that always uses $v_q = v_a = v$ for $v \in \{Q, A, \varnothing\}$ gives expected utility

$$\mathcal{U}^i\left(\frac{1}{2}, (vv) \mid \theta\right) = -\frac{\theta_a \Pr(Q \mid a, v) + \theta_q \Pr(A \mid q, v)}{2} \tag{14}$$

while an informed strategy with $v_q \neq v_a$ gives expected utility

$$\begin{aligned} \mathcal{U}^i\left(P^{v_q v_a}(\theta), (v_q v_a) \mid \theta\right) &= C'\left(P^{v_q v_a}(\theta)\right) P^{v_q v_a}(\theta) \\ &\quad - \frac{\theta_a \Pr(Q \mid a, v_q) + \theta_q \Pr(A \mid q, v_a)}{2} \end{aligned} \tag{15}$$

Using this expression for every pair $v_q v_a$ and $v_q' v_a'$ we can define the function $g^j(\theta_a)$ implicitly by

$$\begin{aligned} & \mathcal{U}^i\left(P^{v_q v_a}\left(g^j(\theta_a), \theta_a\right), (v_q v_a) \mid g^j(\theta_a), \theta_a\right) - C\left(P^{v_q v_a}\left(g^j(\theta_a), \theta_a\right)\right) \\ = \; & \mathcal{U}^i\left(P^{v_q' v_a'}\left(g^j(\theta_a), \theta_a\right), \left(v_q' v_a'\right) \mid g^j(\theta_a), \theta_a\right) - C\left(P^{v_q' v_a'}\left(g^j(\theta_a), \theta_a\right)\right) \end{aligned} \tag{16}$$

where j corresponds to the cutoff function for the strategies that use the voting strategy $v_q v_a$ and $v_q' v_a'$. Figure (1) shows which numbers correspond to which pair of strategies. In Appendix A.1 we present relations between g^i, $i \in \{1, 2, ...10\}$ that are used in the characterization.

Three important comments are in order. First, these functions are defined beyond $[0, 1]^2$.

Second, we cannot show that, $g_1^{10}(\theta_a)$ (a function that maps $\theta_a \in [0,1]$ into $\theta_q \in [0,1]$) or $g_2^{10}(\theta_q)$ (a function that maps $\theta_q \in [0,1]$ into $\theta_a \in [0,1]$) always exist. Nevertheless, we can show that, at least one of them exists and, when both are properly defined, they are each other's inverse: $g_2^{10}(g_1^{10}(x)) = x$. Third, contrary to all other cases, it may be that $g_1^{10}(\theta_a) > 1$ (or $g_2^{10}(\theta_q) > 1$) for all $\theta_a \in [0,1]$ (or $\theta_q \in [0,1]$). In that case, being an abstainer is always better than following an independent behavior.

Number	Strategy 1	Strategy 2
1	*QQ*	*QØ*
2	*QQ*	*QA*
3	*QØ*	*QA*
4	*QØ*	*ØØ*
5	*QØ*	*ØA*
6	*ØA*	*ØØ*
7	*ØA*	*QA*
8	*AA*	*QA*
9	*AA*	*ØA*
10	*ØØ*	*QA*

Figure 1: Number assigned to cut off functions according to the strategies that yield the same expected utilities.

Using the cutoff functions described previously, we can define the set of strong supporters as[19]

$$\mathcal{SS}^A \equiv \left\{\theta \in [0,1]^2 : \theta_q \leq \min\left\{g^9(\theta_a), g^8(\theta_a)\right\}\right\}$$
$$\mathcal{SS}^Q \equiv \left\{\theta \in [0,1]^2 : \theta_q \geq \max\left\{g^1(\theta_a), g^2(\theta_a)\right\}\right\}$$

Strong supporters are located where $\frac{\theta_a}{\theta_q}$ is extremely low or extremely high. The sets of weak

[19]Since its measure is zero we can assign types that are indifferent to any of the groups that provides the same expected utility.

supporters are defined as:

$$\begin{aligned}
\mathcal{WS}^A &\equiv \left\{\theta \in [0,1]^2 : \min\left\{g^7(\theta_a), g^6(\theta_a)\right\} \geq \theta_q, \theta_q > g^9(\theta_a)\right\} \\
\mathcal{WS}^Q &\equiv \left\{\theta \in [0,1]^2 : g^4(\theta_a) \leq \theta_q < g^1(\theta_a), \theta_a \leq g^3(\theta_q)\right\}
\end{aligned}$$

Weak supporters for A (Q) are located exactly above (below) strong supporters for A (Q). The case of independents and abstainers is more delicate because they are separated by the function $g_1^{10}(\theta_a)$ or $g_2^{10}(\theta_q)$ depending on which one is properly defined. We define the set of abstainers $\mathcal{A}$, when $1 \geq \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} + \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}$ (so $g_1^{10}(\theta_a)$ is well defined) as

$$\mathcal{A} \equiv \left\{\theta \in [0,1]^2 : g^6(\theta_a) < \theta_q < g^4(\theta_a), \theta_q \leq g_1^{10}(\theta_a)\right\}$$

while if $1 < \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} + \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}$ (so $g_2^{10}(\theta_q)$ is well defined) the set of abstainers $\mathcal{A}$ is defined by

$$\mathcal{A} \equiv \left\{(\theta_q, \theta_a) \in [0,1]^2 : g^6(\theta_a) < \theta_q < g^4(\theta_a), \theta_a \leq g_2^{10}(\theta_q)\right\}$$

Independents are defined as the complement of all these groups in $[0,1]^2$. If $1 \geq \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} + \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}$, independents are

$$\mathcal{I} \equiv \left\{\begin{array}{c} \theta \in [0,1]^2 : \theta_q > \max\{g^7(\theta_a), g^8(\theta_a)\} \\ g^2(\theta_a) > \theta_q > g_1^{10}(\theta_a), \theta_a > g^3(\theta_q) \end{array}\right\}$$

while if $1 < \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} + \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}$, independents are

$$\mathcal{I} \equiv \left\{\begin{array}{c} \theta \in [0,1]^2 : \theta_q > \max\{g^7(\theta_a), g^8(\theta_a)\}, g^2(\theta_a) > \theta_q \\ , \theta_a > \max\{g^3(\theta_q), g_2^{10}(\theta_q)\} \end{array}\right\}$$

Proposition 1 *Let $P^{\varnothing A}(\theta)$, $P^{Q\varnothing}(\theta)$ and $P^{QA}(\theta)$ be defined as in (12) and the sets $\mathcal{WS}^A$, $\mathcal{WS}^Q$, $\mathcal{SS}^A$, $\mathcal{SS}^Q$, $\mathcal{A}$ and $\mathcal{I}$ defined as above. In any committee the strategy $(P^*(\theta), V^*(\theta, S))$*

with

1. $P^*(\theta)$ *that prescribes* $P^{\varnothing A}(\theta)$ *for* $\theta \in \mathcal{WS}^A$, $P^{Q\varnothing}(\theta)$ *for* $\theta \in \mathcal{WS}^Q$, $P^{QA}(\theta)$ *for* $\theta \in \mathcal{I}$, *and* $P^*(\theta) = \frac{1}{2}$ *otherwise,*

2. $V^*(\theta, S)$ *that prescribes the uninformative behavior* $\varnothing\varnothing$ *for* $\theta \in \mathcal{A}$, XX *for* $\theta \in \mathcal{SS}^X$ *with* $X \in \{Q, A\}$, *and the informative behavior* $\varnothing A$ *for* $\theta \in \mathcal{WS}^A$, $Q\varnothing$ *for* $\theta \in \mathcal{WS}^Q$, *and* QA *for* $\theta \in \mathcal{I}$,

is a symmetric Bayesian equilibrium.

Proof. See Appendix (A.2). ■

Again, although we cannot prove uniqueness of equilibrium, our *characterization* describes *all* symmetric Bayesian equilibria.

It is important to note that, for low values of θ_a and θ_q, we know that the investment condition (13) does not hold so the only restriction for abstainers to exists in equilibrium is that there is a pair $(\theta_q, \theta_a) \in [0,1]^2$ such that $\theta_q \in (g^6(\theta_a), g^4(\theta_a))$. If (11) holds with strict inequality, $g^6(\theta_a) < g^4(\theta_a)$ for low values of θ_a, so

Lemma 5 *A sufficient condition for some non partisan voters to strictly prefer abstention rather than any other voting option after some signal is that (11) holds with strict inequality.*

Once the characterization is complete we are ready to prove existence. We have to consider that there are two possible configurations of equilibria. On one hand, if $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} > \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$, the equilibrium involves some non partisan voters that strictly prefer to abstain in equilibrium after some signal (endogenous abstention). On the other hand, if $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \leq \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$ the equilibrium involves abstention only by partisan voters (exogenous abstention).

We first need to show that the equilibrium with endogenous abstention "approaches" smoothly the equilibrium with only exogenous abstention when $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \searrow \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$. Here is where the transformation that uses all best responses as arguments plays a crucial role. The result will follow by considering that the set of abstainers and weak supporters disappear as soon as abstention is not part of an optimal voting strategy so the "pivotal" probabilities are close to each other. In a sense, all cutoff functions and investment behavior change smoothly when we move slowly from an equilibrium with endogenous abstention to an equilibrium without endogenous abstention.

Proposition 2 *There exists a symmetric Bayesian equilibrium. Moreover, this equilibrium is characterized by the strategy* $(P^*(\theta), V^*(\theta, S))$ *in Proposition (1).*

Proof. See Appendix (A.2). ■

4 Applications

4.1 Abstention under plurality rule

We can show that the plurality rule induces optimal abstention by exploiting the fact that the equilibrium verifies the following (symmetric) condition

Condition 1 ***a)*** $\Pr(\varnothing \mid a) = \Pr(\varnothing \mid q)$, ***b)*** $\Pr(A \mid a) = \Pr(Q \mid q)$

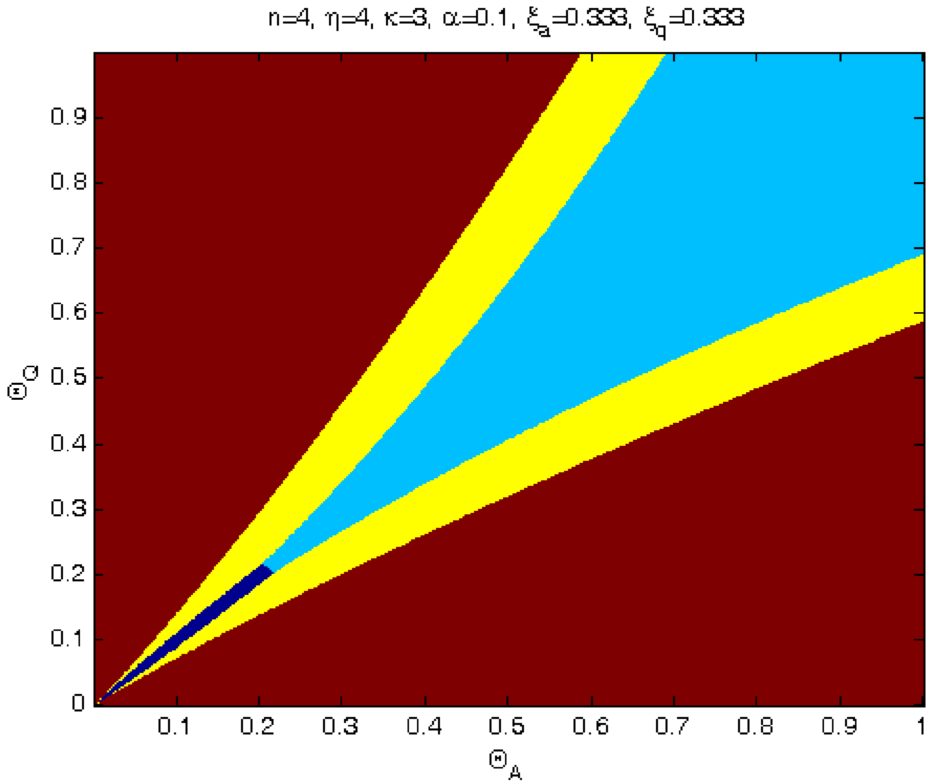
Note that these imply that $\Pr(Q \mid a) = \Pr(A \mid q)$ so the ex ante probability of voting for the right candidate (making a mistake) is the same in both states.

Proposition 3 *There exists an equilibrium in which non partisan voters abstain with positive probability.*

Proof. See Appendix (A.2). ■

The characterization of equilibrium is fairly intuitive and Figure (2) depicts one such possible equilibria. The figure confirms the symmetric structure of the equilibrium: if we divide the unit square in two using the 45^o degree line, one side is the mirror of the other

1



2.pdf

Figure 2: Strong partisans are in red, weak partisans are in yellow, independents are in light blue and abstainers are in dark blue. The distribution of θ_ω is beta with parameters $(2,2)$ and the committee consists of 4 ($n=4$) members that are partisan with 10% probability ($\alpha = 0.1$) and are evenly splited between the voting options ($\xi_a = \xi_q = \xi_\varnothing = \frac{1}{3}$). The cost function is $C(p) = 4\left(p - \frac{1}{2}\right)^3$.

one. Independents and abstainers are centered around the 45^o degree line and are distributed evenly around this line.

For low values of θ_a and θ_q, since independents require high levels of investment, the separation of types close to the origin is given by the functions $g^1(\theta_a)$ ($\mathcal{SS}^Q$ from $\mathcal{WS}^Q$) $g^4(\theta_a)$ ($\mathcal{A}$ from $\mathcal{WS}^Q$), $g^6(\theta_a)$ ($\mathcal{A}$ from $\mathcal{WS}^A$), and $g^9(\theta_a)$ ($\mathcal{SS}^A$ from $\mathcal{WS}^A$). Using the Appendix A.1 we show that $g^6(\theta_a) \geq g^9(\theta_a)$ and $g^1(\theta_a) \geq g^4(\theta_a)$ and also, if abstention is possible, $g^6(\theta_a) < g^4(\theta_a)$. Moreover, using results (1) and (4) in Appendix A.1 we get that $g^1(\theta_a) > g^4(\theta_a)$, and using results (5) and (8) in the Appendix A.1 we get $g^6(\theta_a) > g^9(\theta_a)$ which gives that, close to the origin groups of voters are always ordered clockwise as described.

First of all, fixed the level of intensity $\theta_a + \theta_q = \beta$ and assume that β is sufficiently low. Consider the case in Figure (2) starting from $\theta_a = 0$ and $\theta_q = \beta$ and walking down the line $\theta_a + \theta_q = \beta$ by increasing θ_a. Information is nil first (when $\theta \in \mathcal{SS}^Q$), grows when $\theta \in \mathcal{WS}^Q$ to be nil again when $\theta \in \mathcal{A}$; then information is positive when $\theta \in \mathcal{WS}^A$ to be nil again when $\theta \in \mathcal{SS}^A$. Clearly information is non monotonic on the ideological level. On the other side if β is sufficiently large from $\mathcal{WS}^Q$ we move to $\mathcal{I}$ and then to $\mathcal{WS}^A$. In this case information is not monotonic either but it could be argued that more centrists voters will collect more information. The relation between ideology, information and abstention is more complex. In particular, we cannot rule out that $\mathcal{I}$ and $\mathcal{SS}^A$ (or $\mathcal{SS}^Q$) are next to each other. That is, we cannot rule out that the functions $g^2(\theta_a)$ and $g^8(\theta_a)$ are necessary to describe the equilibrium as presented in Figure (3).

Unlike Feddersen and Pesendorfer (1996) and Feddersen and Pesendorfer (1999) in the limit nobody abstains in our model. The intuition hinges on the fact that investment is 0 in the limit. This directly implies that weak supporters disappear in the limit. The smaller it is the information collected by the average player the more a player relies on her own private ideological bias and the more likely it is a player would rather follow her bias than abstain and delegate the decision to the rest of the committee.

Proposition 4 *When $n \to \infty$ investment goes to* 0 *and the probability of a voter abstaining goes to* 0*.*

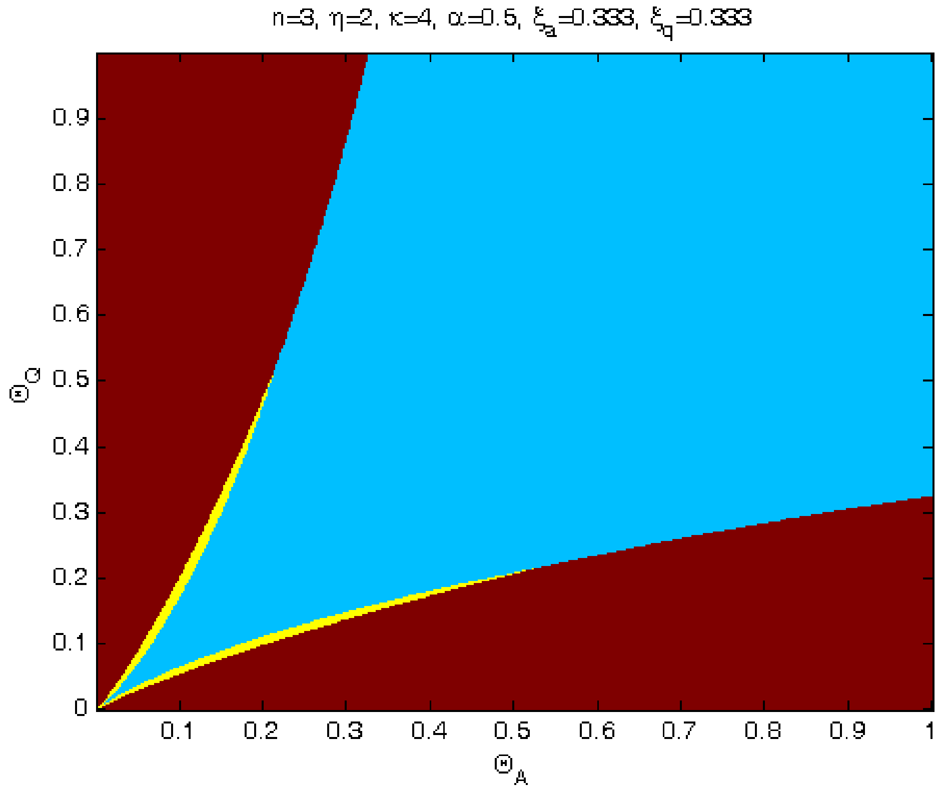
Proof. See Appendix (A.2). ■

4.2 The role of flexible preferences

In the model presented here, preferences are described by two parameters. It is traditional in voting models to assume that utility losses are perfectly and inversely correlated ($\theta_q + \theta_a = \delta_1$).[20] This assumption is sufficient to describe the voting strategy (see expressions (9)

[20] Assumptions presenting heterogeneity as $\theta_q - \theta_a = \phi$ or $\frac{\theta_q}{\theta_a} = \phi$ suffer the same drawback presented here.

2



3.pdf

Figure 3: Strong partisans are in red, weak partisans are in yellow, independents are in light blue and abstainers are in dark blue. The distribution of θ_ω is beta with parameters $(1, 2)$ and the committee consists of 3 ($n = 3$) members that are partisan with 10% probability ($\alpha = 0.1$) and are evenly splited between the voting options ($\xi_a = \xi_q = \xi_\varnothing = \frac{1}{3}$). The cost function is $C(p) = 2\left(p - \frac{1}{2}\right)^4$. The size of abstainers is significantly small.

and (10)), but the levels of these losses are relevant in terms of information acquisition (see expression (12)). We have already discussed the behavioral motivations for θ_q and θ_a to be imperfectly correlated: introducing voters that care about both types of mistakes (false positives and true negatives) and care differently about them. We now illustrate why allowing for flexible preferences matters theoretically, and in the next subsection we show why restricting preferences may lead to undesirable conclusions and predictions about information acquisition and abstention in committees.[21]

Let $\tau_1(\omega)$ and $\tau_2(k, \omega)$ be defined as in the proof of Proposition (3) provided in Appendix

[21] We do not provide formal statements about these claims but illustrate the potential problems that might arise when we restrict attention to a particular level of intensity.

(A.2):

$$
\begin{aligned}
\Delta \Pr(\omega, Q) &= \Delta \Pr(\omega, \varnothing) + \frac{\tau_1(\omega) + \tau_2(k+1, \omega)}{2} \\
\Delta \Pr(\omega, \varnothing) &= \frac{\tau_2(k, \omega) + \tau_1(\omega)}{2}
\end{aligned}
$$

Using the symmetric properties of the equilibrium (Condition 2) we have that condition (13) turns into $P^{QA}(\theta) \geq \frac{\tau_2(k,q)+\tau_1(q)}{\tau_2(k,a)+\tau_1(a)+\tau_2(k,q)+\tau_1(q)}$ when using (35). Note that $\frac{\tau_2(k,q)+\tau_1(q)}{\tau_2(k,a)+\tau_1(a)+\tau_2(k,q)+\tau_1(q)} > \frac{1}{2}$ iff $\tau_2(k,q) > \tau_2(k,a)$ which is true. Let $\Theta_\xi = \left\{\theta \in [0,1]^2 : |\theta_a + \theta_q - 1| < \xi\right\}$ and assume that $\widetilde{F}$ is such that $\widetilde{F}(\theta \in \Theta_\xi) = 1 = 1 - \widetilde{F}\left(\theta \in \Theta_\xi^C\right)$ for every $\xi > 0$ so all the mass is concentrated around the counter diagonal.[22] Imagine also that in any equilibrium for every $\theta \in \Theta_\xi$ we have that $P^{QA}(\theta) < \frac{\tau_2(k,q)+\tau_1(q)}{\tau_2(k,a)+\tau_1(a)+\tau_2(k,q)+\tau_1(q)}$. Independents will not be part of any equilibrium and every centrist would be an abstainer and we will conclude that only "intermediate levels" of ideology collect information (Larcinese (2009)).

Alternatively if $P^{QA}(\theta) > \frac{\tau_2(k,q)+\tau_1(q)}{\tau_2(k,a)+\tau_1(a)+\tau_2(k,q)+\tau_1(q)}$ abstainers will not be part of the equilibrium and every centrist would be an independent. Moreover, if some extra conditions hold,[23] it is possible that there is no equilibrium with abstention by non partisan voters. If $\widetilde{F}$, α or $\left(\xi_A, \xi_Q, \xi_\varnothing\right)$ are such that the equilibrium is described in Figure (3) weak supporters are driven away and only partisan voters abstain when we use restricted preferences. This restriction leads us to conclude that abstention is not an equilibrium phenomenon: non partisan voters never abstain. Restricting preferences diminishes the model's capacity of properly capturing optimal abstention as a social phenomenon. Restricting preferences is not innocuous when information is endogenous.

Note that even when 1) the priors between states are different or, 2) there is some asymmetry between the options, it might be that the "line" separating abstainers and independents is not parallel to the counter diagonal and for some particular configurations independents

[22] Although our assumptions prevent this situation when $\xi \to 0$ (the hyperdiffuse requirement on F), it is easy to show that the existence and characterization results hold when we reduce the dimension of the preference parameters.

[23] In particular, the set of weak supporters must be small and close to the origin.

and abstainers coexists. Again, we conjecture that modifying the level of intensity will make that coexistence disappear.[24]

4.3 The correlation between information and abstention

Let $\Pr(v \neq \varnothing \mid P, \omega)$ be the probability of voting conditional on the precision of signal P and the state ω. It is obvious that $\frac{d\Pr(v \neq \varnothing|P,\omega)}{dP} = 0$ for all those that strictly prefer not to collect information ($\mathcal{SS}^A$, $\mathcal{SS}^Q$ and $\mathcal{A}$) and those that strictly prefer to be independent voters ($\mathcal{I}$). On the other hand in state a (q), $\mathcal{WS}^A$ $(\mathcal{WS}^Q)$ present a negative correlation between information and abstention while $\mathcal{WS}^Q$ $(\mathcal{WS}^A)$ present a positive correlation between information and abstention. At the aggregate level the correlation between information and abstention depends on the relative size of the weak supporters for one or the other candidate. In our particular case (symmetry) we have that both groups cancel out in expectation and we should get no marginal correlation at all.

A different question is the difference between the probability of voting with and without information: $\Pr(v \neq \varnothing \mid P > 0, \omega) - \Pr(v \neq \varnothing \mid P = 0, \omega)$. In this case we have that only independents and strong supporters vote always, weak supporters abstain with some probability and abstainers do not vote. Let $(\omega) = A$ (Q) if $\omega = a$ (q) and we have

[24]It is easy to see that our model is isomorphic to a model in which agents differ only on the ideology dimension and on a cost parameter (see Triossi (2008).). Let $\widetilde{\theta}_i \in [0,1]$, $\kappa_i \in [1,\infty)$ and the cost function be $C_i(\kappa_i, P) = \kappa_i C(P)$ for a quality of information given by $P \in \left[\frac{1}{2}, 1\right]$. Defining preferences for voter i are such that $\theta_a = \frac{\widetilde{\theta}_i}{\kappa_i}$ and $\theta_q = \frac{1-\widetilde{\theta}_i}{\kappa_i}$ gives the equivalence. The discussion hence also translates to heterogenous cost of information vis a vis homogeneous cost of information.

$$
\begin{aligned}
& \Pr\left(v \neq \varnothing \mid P > 0, \omega\right) - \Pr\left(v \neq \varnothing \mid P = 0, \omega\right) \\
= & \frac{\Pr\left(\theta \in \mathcal{I}\right) + I\left(\left(\omega\right) = A\right)\left(\int\limits_{\theta \in \mathcal{WS}^A} P^{\varnothing A}\left(\theta\right) dF\left(\theta\right) + \int\limits_{\theta \in \mathcal{WS}^Q} \left(1 - P^{Q\varnothing}\left(\theta\right)\right) dF\left(\theta\right)\right)}{\Pr\left(\theta \in \mathcal{I}\right) + \Pr\left(\theta \in \mathcal{WS}^A\right) + \Pr\left(\theta \in \mathcal{WS}^Q\right)} \\
& + \frac{I\left(\left(\omega\right) = Q\right)\left(\int\limits_{\theta \in \mathcal{WS}^A} \left(1 - P^{\varnothing A}\left(\theta\right)\right) dF\left(\theta\right) + \int\limits_{\theta \in \mathcal{WS}^Q} P^{Q\varnothing}\left(\theta\right) dF\left(\theta\right)\right)}{\Pr\left(\theta \in \mathcal{I}\right) + \Pr\left(\theta \in \mathcal{WS}^A\right) + \Pr\left(\theta \in \mathcal{WS}^Q\right)} \\
& - \frac{\Pr\left(\theta \in \mathcal{SS}^A\right) + \Pr\left(\theta \in \mathcal{WS}^Q\right)}{\Pr\left(\theta \in \mathcal{A}\right) + \Pr\left(\theta \in \mathcal{WS}^A\right) + \Pr\left(\theta \in \mathcal{WS}^Q\right)}
\end{aligned}
$$

Clearly, this term measures the proportion of voters in each camp and therefore captures the structure of the electorate more than the actual correlation between information and abstention. Moreover, depending on which is the actual state the measure can yield stronger or weaker results. It is immediate to see that this measure is equivalent to the probability of abstaining holding no information minus the probability of abstaining holding some information: $\Pr\left(v = \varnothing \mid P = 0, \omega\right) - \Pr\left(v = \varnothing \mid P > 0, \omega\right)$.

Another interesting measure between information and abstention is just the correlation

$$
\begin{aligned}
& \Pr\left(v \neq \varnothing, P > 0 \mid \omega\right) \\
= & \Pr\left(\theta \in \mathcal{I}\right) + I\left(\left(\omega\right) = A\right)\left(\int\limits_{\theta \in \mathcal{WS}^A} P^{\varnothing A}\left(\theta\right) dF\left(\theta\right) + \int\limits_{\theta \in \mathcal{WS}^Q} \left(1 - P^{Q\varnothing}\left(\theta\right)\right) dF\left(\theta\right)\right) \\
& + I\left(\left(\omega\right) = Q\right)\left(\int\limits_{\theta \in \mathcal{WS}^A} \left(1 - P^{\varnothing A}\left(\theta\right)\right) dF\left(\theta\right) + \int\limits_{\theta \in \mathcal{WS}^Q} P^{Q\varnothing}\left(\theta\right) dF\left(\theta\right)\right)
\end{aligned}
$$

Clearly all these measures are considering the composition of the electorate and aggregating individual effects that might get hidden once the aggregation is used.

5 Conclusions

Few papers study abstention as optimal behavior and none of them allow for information acquisition. This contrasts with the result that roll off is an informational phenomenon. Following this idea, we presented a model of committees with *abstention* and *endogenous information acquisition* using two interdependent innovations: we allowed voters to *select the precision of the signal* they receive and committee members' preferences incorporate differences on the levels of *both ideology* and *concern*.

In equilibrium, there are three classes of uninformed voters: balance preferences and low intensity **abstainers**, and very biased **strong supporters** for each one of the candidates. Rational ignorance takes on two different forms. On one side, abstainers decide not to collect information and delegate on the other members by abstaining. On the other side, strong supporters always vote although their votes are not based on any information. There are also two classes of informed voters: **weak supporters** for each candidate with a relatively low ideological bias, and **independents** with balanced preferences and high intensity. The level of information acquisition changes discontinuously even among informed voters. Indeed, small changes that make a voter change his behavior from an independent to a weak supporter create jumps in the level of investment in information.

Empirical models that study abstention and information either test $\Pr(v = \varnothing)$ across different electorates or try to determine whether $\Pr(v \neq \varnothing \mid P > 0)$ is bigger than $\Pr(v \neq \varnothing \mid P = 0)$ (see Coupé and Noury (2004), Larcinese (2007) and Lassen (2005)). These tests only capture the relative size of the different groups that emerge in equilibrium. In essence, the strength of the test depends on which is the actual equilibrium represented in the data. Our model suggests that this is not the whole story. Empirical tests need to consider the ideological dimension to capture the differential effect of information acquisition on voting. For example, Palfrey and Poole (1987) use voters that actually voted while our model suggests that a more direct test of information and turnout must condition on ideology among those that did not vote: i.e. weak supporters that abstained.

In our set up, the plurality rule generates abstention as an equilibrium behavior. Our model predicts that voters abstain without assuming a random number of voters as in Poisson games (see Feddersen and Pesendorfer (1999)). Some voters abstain even if they have much at stake in the election and had strong evidence in favor of one candidate. Abstention is not simply the result of poor information but a more complex interaction between preferences and information. In our model some well informed voters may abstain precluding this good information to reach the electorate. Unlike Feddersen and Pesendorfer (1999) in the limit there is no abstention by non-partisans.

Although we base all of our analysis on roll off our model gives insightful results about absence. Indeed, if voters collect information before they approach the booth, we would predict absence even though voting is not costly. Therefore, our model can also provide links between information and turnout. We show that correlation patterns between information and turnout are present as long as we condition these patterns on particular groups of voters: some voters are *more likely to vote* the more informed they are, while some other voters are *more likely to abstain* the more informed they are.

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A Appendix

A.1 Cutoff functions

Let $L\left(P^X(x,y)\right) \equiv C'\left(P^X(x,y)\right) P^X(x,y) - C\left(P^X(x,y)\right)$ so

$$g^1(\theta_a)\frac{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)}{2} = L\left(P^{Q\varnothing}\left(g^1(\theta_a),\theta_a\right)\right) \tag{17}$$

$$g^2(\theta_a)\frac{\Delta \Pr(q,Q)}{2} = L\left(P^{QA}\left(g^2(\theta_a),\theta_a\right)\right) \tag{18}$$

$$L\left(P^{QA}\left(\theta_q, g^3(\theta_q)\right)\right) = L\left(P^{Q\varnothing}\left(\theta_q, g^3(\theta_q)\right)\right) + \frac{\theta_q \Delta \Pr(q,\varnothing)}{2} \tag{19}$$

$$\theta_a\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{2} = L\left(P^{Q\varnothing}\left(g^4(\theta_a),\theta_a\right)\right) \tag{20}$$

$$L\left(P^{(Q,\varnothing)}\left(g^5(\theta_a),\theta_a\right)\right) - \theta_a\frac{\left(\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)\right)}{2} = L\left(P^{(\varnothing,A)}\left(g^5(\theta_a),\theta_a\right)\right) - \frac{g^5(\theta_a)\Delta \Pr(q,\varnothing)}{2} \tag{21}$$

$$g^6(\theta_a)\frac{\Delta \Pr(q,\varnothing)}{2} = L\left(P^{\varnothing A}\left(g^6(\theta_a),\theta_a\right)\right) \tag{22}$$

$$0 = \theta_a\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{2} - L\left(P^{QA}\left(g^7(\theta_a),\theta_a\right)\right) + L\left(P^{\varnothing A}\left(g^7(\theta_a),\theta_a\right)\right) \tag{23}$$

$$\theta_a\frac{\Delta \Pr(a,Q)}{2} = L\left(P^{QA}\left(g^8(\theta_a),\theta_a\right)\right) \tag{24}$$

$$\theta_a\frac{\Delta \Pr(a,\varnothing)}{2} = L\left(P^{\varnothing A}\left(g^9(\theta_a),\theta_a\right)\right) \tag{25}$$

$$\begin{aligned}
L\left(P^{QA}\left(g^{10}_{(1)}(\theta_a),\theta_a\right)\right) - g^{10}_{(1)}(\theta_a)\frac{\Delta \Pr(q,\varnothing)}{2} &= \theta_a\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{2} \\
\text{if } \frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(q,Q)} &\neq P^{QA}\left(g^{10}_{(1)}(\theta_a),\theta_a\right) \\
L\left(P^{QA}\left(\theta_q, g^{10}_{(2)}(\theta_q)\right)\right) - \theta_q\frac{\Delta \Pr(q,\varnothing)}{2} &= g^{10}_{(2)}(\theta_q)\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{2} \\
\text{if } \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)} &\neq \left(1 - P^{QA}\left(\theta_q, g^{10}_{(2)}(\theta_q)\right)\right)
\end{aligned} \tag{26}$$

Here we summarize some useful properties of cutoff functions that are obtained by repeatedly applying the implicit function theorem.

Fact 1 $g^1 : \mathcal{R}_+ \to \mathcal{R}_+$, *is strictly convex and* $\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} < \frac{g^1(\theta_a)}{\theta_a} < \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} \frac{P^{Q\varnothing}\left(g^1(\theta_a),\theta_a\right)}{1 - P^{Q\varnothing}\left(\theta^5_q(\theta_a),\theta_a\right)}$

Fact 2 $g^2 : \mathcal{R}_+ \to \mathcal{R}_+$, *is strictly convex and* $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} < \frac{g^2(\theta_a)}{\theta_a} < \frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \frac{P^{QA}\left(g^2(\theta_a),\theta_a\right)}{(1 - P^{QA}(g^2(\theta_a),\theta_a))}$

Fact 3 $g^3 : \mathcal{R}_+ \to \mathcal{R}_+$, *verifies* $\frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(a,\varnothing)} \frac{1-P^{QA}\left(\theta_q, g^3\left(\widetilde{\theta}_q\right)\right)}{P^{QA}\left(\theta_q, g^3\left(\widetilde{\theta}_q\right)\right)} < \frac{g^3\left(\widetilde{\theta}_q\right)}{\theta_q} < \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(a,\varnothing)} \frac{1-P^{Q\varnothing}\left(\theta_q, g^3\left(\widetilde{\theta}_q\right)\right)}{P^{Q\varnothing}\left(\theta_q, g^3\left(\widetilde{\theta}_q\right)\right)}$

Fact 4 $g^4 : \mathcal{R}_+ \to \mathcal{R}_+$, *is strictly concave and verifies* $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} \frac{1-P^{Q\varnothing}\left(g^4(\theta_a),\theta_a\right)}{P^{Q\varnothing}\left(g^4(\theta_a),\theta_a\right)} < \frac{g^4(\theta_a)}{\theta_a} < \frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}$

Fact 5 $g^6 : \mathcal{R}_+ \to \mathcal{R}_+$, *is strictly convex and verifies* $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} < \frac{g^6(\theta_a)}{\theta_a} < \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} \frac{P^{\varnothing A}\left(g^6(\theta_a),\theta_a\right)}{1-P^{\varnothing A}\left(g^6(\theta_a),\theta_a\right)}$

Fact 6 $g^7 : \mathcal{R}_+ \to \mathcal{R}_+$, *verifies* $\frac{1-P^{QA}\left(g^7(\theta_a),\theta_a\right)}{P^{QA}\left(g^7(\theta_a),\theta_a\right)} \frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} < \frac{g^7(\theta_a)}{\theta_a} < \frac{1-P^{\varnothing A}\left(g^7(\theta_a),\theta_a\right)}{P^{\varnothing A}\left(g^7(\theta_a),\theta_a\right)} \frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}$

Fact 7 $g^8 : \mathcal{R}_+ \to \mathcal{R}_+$, *is strictly concave and verifies* $\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} \frac{1-P^{QA}\left(g^8(\theta_a),\theta_a\right)}{P^{QA}\left(g^8(\theta_a),\theta_a\right)} < \frac{g^8(\theta_a)}{\theta_a} < \frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}$

Fact 8 $g^9 : \mathcal{R}_+ \to \mathcal{R}_+$, *is strictly concave and verifies* $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} \frac{1-P^{\varnothing A}\left(g^9(\theta_a),\theta_a\right)}{P^{\varnothing A}\left(g^9(\theta_a),\theta_a\right)} < \frac{g^9(\theta_a)}{\theta_a} < \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$

Here we prove some properties of the $g^{10}_{(1)}(\theta_a)$ and $g^{10}_{(2)}(\theta_q)$ for the cases in which it is necessary to define these functions.

Claim 1 *If there is a type that it is indifferent between* $(\varnothing, \varnothing)$ *and* (Q, A)*, then if* $1 \leq (\geq) \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ *then* $\left(1 - P^{QA}(\theta)\right) < \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ *(*$P^{QA}(\theta) > \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)}$*).*

Proof. Take any type that it is indifferent between $(\varnothing, \varnothing)$ and (Q, A); this type verifies (26) and it must also verify

$$\theta_q\left(\Delta\Pr(q,Q)\,P^{QA}(\theta_q,\theta_a) - \Delta\Pr(q,\varnothing)\right) > \theta_a\left(\Delta\Pr(a,Q)\left(1 - P^{QA}(\theta_q,\theta_a)\right) - \Delta\Pr(a,\varnothing)\right) \tag{27}$$

since investment is positive for this type when using (Q, A). Using (9) and (10) we have that the strategy with $(\varnothing, \varnothing)$ is consistent whenever $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} \geq \frac{\theta_q}{\theta_a} \geq \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$.

Assume now that $\left(1 - P^{QA}(\theta)\right) \geq \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ and using that $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} \geq \frac{\theta_q}{\theta_a}$ for this type we have that condition (27) is now

$$\frac{\frac{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)}}{\frac{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}} > 1 - P^{QA}(\theta)$$

Multiplying both sides by $\Delta\Pr(a,Q)$ and subtracting to both sides $\Delta\Pr(a,\varnothing)$, and using $\left(1 - P^{QA}(\theta)\right) \geq \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$, some algebra gives $1 > \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$. Therefore, if $1 \leq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ we must have that $\left(1 - P^{QA}(\theta)\right) < \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$.

Now assume that $P^{QA}(\theta) \leq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)}$ and using $\frac{\theta_q}{\theta_a} \geq \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$, condition (27) implies $\frac{\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}}{\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}} < P^{QA}(\theta)$. Multiplying by $\Delta\Pr(q,Q)$ and subtracting $\Delta\Pr(q,\varnothing)$, some algebra and the assumption $P^{QA}(\theta) \leq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)}$ gives that $1 < \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$. Therefore, if $1 \geq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ we must have $P^{QA}(\theta) > \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)}$. ■

Recall that the strategy with $(\varnothing,\varnothing)$ is optimal only when $g^6(\theta_a) \leq g^4(\theta_a)$; we have already proved that $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} < \frac{g^6(\theta_a)}{\theta_a}$ and $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} > \frac{g^4(\theta_a)}{\theta_a}$, therefore, any type $\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ that satisfies $\widetilde{\theta}_q \in \left[g^6\left(\widetilde{\theta}_a\right), g^4\left(\widetilde{\theta}_a\right)\right]$ can play the strategy with $(\varnothing,\varnothing)$ consistently. As a conclusion

Fact 9 *For any type that is indifferent between $(\varnothing,\varnothing)$ and (Q,A), if $1 \leq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ the function $g^{10}_{(2)}(\theta_q)$ is well defined for every $\theta_a \in \left[\left(g^6\right)^{-1}(\theta_q), \left(g^4\right)^{-1}(\theta_q)\right]$ and is strictly concave, and if $1 \geq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ the function $g^{10}_{(1)}(\theta_a)$ is well defined for every $\theta_q \in [g^6(\theta_a), g^4(\theta_a)]$ and strictly concave.*

The following definition is straight forward when considering (26)

Definition 3 *If $1 \geq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$, every type $\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ with $\widetilde{\theta}_q > g^{10}_{(1)}\left(\widetilde{\theta}_a\right)$ prefers the informed strategy with (Q,A) to the uninformed strategy with $(\varnothing,\varnothing)$. If $1 \leq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$, every type $\left(\widetilde{\theta}_a,\widetilde{\theta}_q\right)$ that satisfies $\widetilde{\theta}_a > g^{10}_{(2)}\left(\widetilde{\theta}_q\right)$ prefers the informed strategy with (Q,A) to the uninformed strategy with $(\varnothing,\varnothing)$.*

Here we summarize some useful relations between different cutoff functions.

Fact 10 *For every pair $\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ satisfying $\widetilde{\theta}_q = g^6\left(\widetilde{\theta}_a\right)$ we have $\widetilde{\theta}_a \geq g^3\left(\widetilde{\theta}_q\right)$ and $g^6(\theta_a) \geq g^9(\theta_a)$ for all θ_a.*

Proof. Let $\widetilde{\theta}_q = g^6\left(\widetilde{\theta}_a\right)$ and replacing in the right hand side of (19) we have that the strategy with (Q,A) is preferred to the strategy with $(Q,\varnothing)$ whenever

$$\begin{aligned}
& C'\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
\geq\ & C\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
& +C'\left(P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)
\end{aligned}$$

If $P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) \leq P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ the result hold using that C is strictly convex and $P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) > P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$. Therefore, assume that $P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) > P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ and using the second line of (12) we can express (19) as

$$\begin{aligned}
& C'\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
\geq\ & C\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
& -C'\left(P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)
\end{aligned}$$

Because C is strictly convex and $P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) > P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ and we assume that $P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) > P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ the result holds. The last part follows directly by the fact that $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} < \frac{g^6(\theta_a)}{\theta_a}$ and $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} > \frac{g^9(\theta_a)}{\theta_a}$. ■

Fact 11 $g^1(\theta_a) \geq g^4(\theta_a) \geq g^7(\theta_a)$ *for all* θ_a.

Proof. The first result follows by the fact that $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} < \frac{g^1(\theta_a)}{\theta_a}$ and $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} > \frac{g^4(\theta_a)}{\theta_a}$. Let $\left(\widetilde{\theta}_a,\widetilde{\theta}_q\right)$ be such that $\widetilde{\theta}_q = g^4\left(\widetilde{\theta}_a\right)$ so (20) holds with equality; replacing $\widetilde{\theta}_a\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{2}$ in (23) and using the the second line of (12) we have that the strategy with (Q,A) is preferred to the strategy with $(\varnothing,A)$ whenever any of this inequalities hold

$$
\begin{aligned}
& C'\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
\geq\ & C\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
& + C'\left(P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)
\end{aligned} \tag{28}
$$

or

$$
\begin{aligned}
& C'\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
\geq\ & C\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
& + C'\left(P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)
\end{aligned} \tag{29}
$$

Using strict convexity of C (so $f(y) < f(x) + f'(y)(y-x)$ for $y > x$), if $P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) < P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ the condition (29) holds, and if $P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) \geq P^{\varnothing A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ the condition (28) holds. ■

Fact 12 $g^6(\theta_a) < g^5(\theta_a)$ *iff* $g^5(\theta_a) < g^4(\theta_a)$ *and* $g^6(\theta_a) > g^5(\theta_a)$ *iff* $g^5(\theta_a) > g^4(\theta_a)$. *Moreover, there is some* $\overline{\theta}_a \in (0,1]$ *such that, for all* $\theta_a \in \left(0,\overline{\theta}_a\right)$, *the relation* $g^4(\theta_a) > g^6(\theta_a)$ *holds.*

Proof. Assuming that $\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ satisfies $\widetilde{\theta}_q = g^5\left(\widetilde{\theta}_a\right)$. Note that the left hand side of (21) is just condition (20) rearranged (which defines $g^4(\theta_a)$) while the right hand side is condition (22) rearranged (which defines $g^8(\theta_a)$). Now assume that $\widetilde{\theta}_q > g^6\left(\widetilde{\theta}_a\right)$ (the uninformed strategy with $(\varnothing,\varnothing)$ is preferred to the informed strategy with $(\varnothing,A)$). By definition of $g^6(\theta_a)$ and (22) we have that the right hand side of (21) is positive; therefore

$$
\begin{aligned}
\widetilde{\theta}_a\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{2} \ >\ & C'\left(P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) \\
& - C\left(P^{Q\varnothing}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)
\end{aligned}
$$

and, using (20), we have that the uninformed strategy with $(\varnothing, \varnothing)$ is preferred to the informed strategy with $(Q, \varnothing)$. By definition of $g^4(\theta_a)$ it must be that $\widetilde{\theta}_q < g^4\left(\widetilde{\theta}_a\right)$. Assume that $\widetilde{\theta}_q = g^6\left(\widetilde{\theta}_a\right)$ and following the same steps the second result holds.

Finally, let $H(\theta_a) = g^4(\theta_a) - g^6(\theta_a)$. Because the function $g^4(\theta_a)$ is strictly concave and $g^6(\theta_a)$ is strictly convex, we have that $H(\theta_a)$ is strictly concave. Note that

$$\begin{aligned} H'(\theta_a) &= \frac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{\Delta \Pr(q, Q) - \Delta \Pr(q, \varnothing)} \frac{1 - P^{Q\varnothing}(g^4(\theta_a), \theta_a)}{P^{Q\varnothing}(g^4(\theta_a), \theta_a)} \\ &\quad - \frac{\Delta \Pr(a, \varnothing)}{\Delta \Pr(q, \varnothing)} \frac{P^{\varnothing A}(g^6(\theta_a), \theta_a)}{1 - p^{\varnothing A}(g^6(\theta_a), \theta_a)} \end{aligned}$$

Note that $\lim_{\theta_a \to 0} H'(\theta_a) = \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\varnothing)} - \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)} > 0$ so $H(\theta_a)$ is increasing for small values of θ_a. By strict concavity of $H(\theta_a)$ the result follows. ■

Fact 13 *From the previous results, the uninformed strategy that calls for abstention and no collection of information is optimal only for types such that* $g^4(\theta_a) \geq g^6(\theta_a)$.

Proof. Recalling that every type $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)$ satisfying $g^4\left(\widetilde{\theta}_a\right) < \widetilde{\theta}_q$ prefers the strategy with $(Q, \varnothing)$ to the strategy with $(\varnothing, \varnothing)$ and every type type $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)$ satisfying $\widetilde{\theta}_q < g^6\left(\widetilde{\theta}_a\right)$ prefers the strategy with $(\varnothing, A)$ to the strategy with $(\varnothing, \varnothing)$, we have that, if $g^4\left(\widetilde{\theta}_a\right) < g^6\left(\widetilde{\theta}_a\right)$, every type with $\widetilde{\theta}_q \leq g^4\left(\widetilde{\theta}_a\right)$ prefers the strategy with $(\varnothing, A)$ to the strategy with $(\varnothing, \varnothing)$ and every type with $\widetilde{\theta}_q > g^4\left(\widetilde{\theta}_a\right)$ prefers the strategy with $(Q, \varnothing)$ to the strategy with $(\varnothing, \varnothing)$. ■

Fact 14 $g^8(\theta_a) < g^9(\theta_a)$ *iff* $g^7(\theta_a) < g^8(\theta_a)$ *and* $\theta_q^9(\theta_a) < g^8(\theta_a)$ *iff* $g^8(\theta_a) < g^7(\theta_a)$.

Proof. Note that we can express (23) as

$$\begin{aligned} & -\theta_a \frac{\Delta \Pr(a, Q)}{2} \\ & + C'\left(P^{QA}\left(g^7(\theta_a), \theta_a\right)\right) P^{QA}\left(g^7(\theta_a), \theta_a\right) - C\left(P^{QA}\left(g^7(\theta_a), \theta_a\right)\right) \\ = \; & C'\left(P^{\varnothing A}\left(g^7(\theta_a), \theta_a\right)\right) P^{\varnothing A}\left(g^7(\theta_a), \theta_a\right) - C\left(P^{\varnothing A}\left(g^7(\theta_a), \theta_a\right)\right) \\ & -\theta_a \frac{\Delta \Pr(a, \varnothing)}{2} \end{aligned} \tag{30}$$

Note that if the left hand side of (30) is positive, the left hand side of (24) is bigger than the right hand side of (24) and therefore the strategy with (Q, A) is preferred to the strategy with (A, A); at the same time, the right hand side of (30) being positive implies that the left hand side of (25) is bigger than the right hand side of (25) and, therefore, the informed strategy with $(\varnothing, A)$ is preferred to the uninformed strategy with (A, A). This implies that there are only two possible cases: $g^7(\theta_a) \geq \max\{g^9(\theta_a), g^8(\theta_a)\}$ or $g^7(\theta_a) \leq \min\{g^9(\theta_a), g^8(\theta_a)\}$.

Assume the first case and suppose that $g^9(\theta_a) > g^8(\theta_a)$. Take some type $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right) \in [0,1]^2$ with $g^9\left(\widetilde{\theta}_a\right) > \widetilde{\theta}_q > g^8\left(\widetilde{\theta}_a\right)$. This type prefers the strategy with (Q, A) to the strategy with (A, A) (that is $\widetilde{\theta}_q > g^8\left(\widetilde{\theta}_a\right)$), the strategy with (A, A) to the strategy with $(\varnothing, A)$ (that is $g^9\left(\widetilde{\theta}_a\right) > \widetilde{\theta}_q$) and the strategy with $(\varnothing, A)$ to the strategy with (Q, A) ($g^7\left(\widetilde{\theta}_a\right) \geq g^9\left(\widetilde{\theta}_a\right) > \widetilde{\theta}_q$). This is a contradiction.

In the second case assume that $g^9(\theta_a) < g^8(\theta_a)$; let type $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right) \in [0,1]^2$ be such that $g^8\left(\widetilde{\theta}_a\right) > \widetilde{\theta}_q > g^9\left(\widetilde{\theta}_a\right)$. Therefore $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)$ prefers the strategy with (A, A) to the strategy with (Q, A) (that is $g^8\left(\widetilde{\theta}_a\right) > \widetilde{\theta}_q$), the strategy with $(\varnothing, A)$ to the strategy with (A, A) ($\widetilde{\theta}_q > g^9\left(\widetilde{\theta}_a\right)$) and the strategy with (Q, A) to the strategy with $(\varnothing, A)$ ($\widetilde{\theta}_q > g^9\left(\widetilde{\theta}_a\right) \geq g^7\left(\widetilde{\theta}_a\right)$). This is another contradiction. ■

Fact 15 *for every $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)$ that satisfies $\widetilde{\theta}_a = g^3\left(\widetilde{\theta}_q\right)$, it also holds that $\theta_q^1\left(\widetilde{\theta}_a\right) > g^2\left(\widetilde{\theta}_a\right)$ iff $g^2\left(\widetilde{\theta}_a\right) > \widetilde{\theta}_q$ and $g^1\left(\widetilde{\theta}_a\right) < g^2\left(\widetilde{\theta}_a\right)$ iff $g^2\left(\widetilde{\theta}_a\right) < \widetilde{\theta}_q$.*

Proof. Assume that the type $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right) \in [0,1]^2$ satisfies $\widetilde{\theta}_a = \theta_q^3\left(\widetilde{\theta}_q\right)$, therefore the condition (19) must hold with equality and rearranging we have

$$\begin{aligned} & -\widetilde{\theta}_q \frac{\Delta \Pr(q, Q)}{2} \\ & +C'\left(P^{QA}\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)\right) P^{QA}\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right) - C\left(P^{QA}\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)\right) \\ = \; & C'\left(P^{Q\varnothing}\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)\right) P^{Q\varnothing}\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right) - C\left(P^{Q\varnothing}\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)\right) \\ & -\widetilde{\theta}_q \frac{\Delta \Pr(q, Q) - \Delta \Pr(q, \varnothing)}{2} \end{aligned} \tag{31}$$

Note that if the left hand side of (31) is positive (and also the right hand side of (31) is positive), we must have that the left hand side of (18) is bigger than the right hand side of (18) and therefore the strategy with (Q, A) is preferred to the strategy with (Q, Q); at the same time the left hand side of (17) is bigger than the right hand side and the strategy with $(Q, \varnothing)$ is better than the strategy with (Q, Q). We are left with two cases: $\widetilde{\theta}_q \geq \max\left\{g^1\left(\widetilde{\theta}_a\right), g^2\left(\widetilde{\theta}_a\right)\right\}$ or $\widetilde{\theta}_q \leq \min\left\{g^1\left(\widetilde{\theta}_a\right), g^2\left(\widetilde{\theta}_a\right)\right\}$.

For the first case assume that $g^1(\theta_a) > g^2(\theta_a)$ and let $\left(\widehat{\theta}_q, \widehat{\theta}_a\right) \in [0,1]^2$ be such that $g^1\left(\widehat{\theta}_a\right) > \widehat{\theta}_q > g^2\left(\widehat{\theta}_a\right)$. Since the type $\left(\widehat{\theta}_q, \widehat{\theta}_a\right)$ that it is indifferent between the strategy with (Q, A) and the strategy with $(Q, \varnothing)$ satisfies $\widetilde{\theta}_q \geq g^1\left(\widetilde{\theta}_a\right)$ and $\widetilde{\theta}_q \geq g^2\left(\widetilde{\theta}_a\right)$, we must have that $\widetilde{\theta}_q > \widehat{\theta}_q$ and the type $\left(\widehat{\theta}_q, \widehat{\theta}_a\right)$ prefers the strategy with (Q, A) to the strategy with $(Q, \varnothing)$. At the same time, the type $\left(\widehat{\theta}_q, \widehat{\theta}_a\right)$ prefers the strategy with $(Q, \varnothing)$ to the strategy with (Q,Q) ($g^1\left(\widehat{\theta}_a\right) > \widehat{\theta}_q$) and the strategy with (Q,Q) to the strategy with (Q,A) ($\widehat{\theta}_q > g^2\left(\widehat{\theta}_a\right)$). This is a contradiction.

For the second case, assume that $g^1(\theta_a) < g^2(\theta_a)$ and let let $\left(\widehat{\theta}_q, \widehat{\theta}_a\right) \in [0,1]^2$ be such that $g^1\left(\widehat{\theta}_a\right) < \widehat{\theta}_q < g^2\left(\widehat{\theta}_a\right)$. Again, if the type $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)$ is indifferent between the strategy with (Q,A) and the strategy with $(Q,\varnothing)$ we have that $\widetilde{\theta}_q < \widehat{\theta}_q$ and therefore the strategy with $(Q,\varnothing)$ is preferred to the strategy with (Q,A) for the type $\left(\widehat{\theta}_q, \widehat{\theta}_a\right)$. At the same time the type $\left(\widetilde{\theta}_q, \widetilde{\theta}_a\right)$ prefers the strategy with (Q,Q) to the strategy with $(Q,\varnothing)$ (recall that $g^1\left(\widetilde{\theta}_a\right) < \widetilde{\theta}_q$) and the strategy with (Q,A) to the strategy with (Q,Q) ($\widetilde{\theta}_q < g^2\left(\widetilde{\theta}_a\right)$). Another contradiction. ■

A.2 Proofs

Proof of Lemma (2). The condition $\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} \geq \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$ is equivalent to $\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} \leq \frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}$. Assume then that inequality (11) does not hold. Then (9) and (10) become

$$\frac{\Pr(q \mid s,p)}{\Pr(a \mid s,p)} \leq \frac{\theta_a}{\theta_q}\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} \leq \frac{\Pr(q \mid s,p)}{\Pr(a \mid s,p)}$$

which implies for almost all types that, a positive vote, either for A or Q, is preferred to abstaining. ■

Proof of Lemma (3). We will show the proof for the case $A\varnothing$; the cases $\varnothing Q$ and AQ are analogous. If a non partisan voter uses $A\varnothing$, (9) gives

$$\frac{\Pr(q \mid s_q,p)}{\Pr(a \mid s_q,p)} \leq \frac{\theta_a}{\theta_q}\min\left\{\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}, \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}\right\} \leq \frac{\Pr(q \mid s_a,p)}{\Pr(a \mid s_a,p)}$$

which is a contradiction since $\Pr(\omega \mid s_\omega,p) > \Pr(\omega \mid s_{-\omega},p)$ for $p > \frac{1}{2}$. If $p = \frac{1}{2}$, it is optimal only for types that satisfy $\frac{\theta_q}{\theta_a} = \min\left\{\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}, \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}\right\}$. ■

Proof of Lemma (4). Using the optimal conditions for voting, (9) and (10), we have that it is necessary for independents that $\frac{\Pr(a|s_q,p)}{\Pr(q|s_q,p)}\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} \leq \frac{\theta_q}{\theta_a} \leq \frac{\Pr(a|s_a,p)}{\Pr(q|s_a,p)}\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$. Using that $\frac{\Pr(q|s_q,p)}{\Pr(a|s_q,p)} = \frac{\Pr(a|s_a,p)}{\Pr(q|s_a,p)} = \frac{p}{1-p}$, it is necessary that $\frac{1-p}{p}\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} \leq \frac{\theta_q}{\theta_a} \leq \frac{p}{1-p}\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$ which gives (13). Now assume that there is endogenous abstention with positive probability. Lemma (2) gives that (11) holds with strict inequality, and therefore $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} > \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$. Using (13) gives that $\left(\frac{p}{1-p}\right)^2 > 1$ and, $p > \frac{1}{2}$ is necessary. ■

Proof of Proposition 1. Strategies that do not verify (5) cannot be optimal. First we are going to prove that the strategies proposed verify conditions (5) and (6). Then we are going to show that they actually cover all the space of types and that the set of types using each strategy do not intersect with each other.

Strong supporters

Since every pair with $\theta \in \mathcal{SS}^A$ satisfies $\theta_q \leq \min\{g^9(\theta_a), g^8(\theta_a)\}$ we must have that

$\varnothing A$ and QA do not verify (6) by definition of $g^9(\theta_a)$ and $g^8(\theta_a)$. Using that $g^9(\theta_a) < \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}\theta_a$, the strategies that involve QQ (inequality (10)) and $\varnothing\varnothing$ (converse of inequality (9)) do not verify (5) for $\theta \in \mathcal{SS}^A$.Recalling (10), condition (5) for $Q\varnothing$ requires $\frac{\theta_q}{\theta_a} \geq \frac{P^{Q\varnothing}(\theta_q,\theta_a)}{1-P^{Q\varnothing}(\theta_q,\theta_a)}\frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)} \geq \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}$ which does not hold since $g^9(\theta_a) < \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(a,Q)}\theta_a$.

For $\theta \in \mathcal{SS}^Q$, it holds that $\theta_q \geq \max\{g^1(\theta_a), g^2(\theta_a)\}$ which implies that QA and $Q\varnothing$ do not verify (6) by definition of $g^1(\theta_a)$ and $g^2(\theta_a)$. Using $g^1(\theta_a) > \theta_a\frac{\Delta \Pr(a,Q)-\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q)-\Delta \Pr(q,\varnothing)}$, the converse of inequality (10) gives that $\varnothing\varnothing$ does not verify (5) for $\theta \in \mathcal{SS}^Q$ and $g^2(\theta_a) > \theta_a\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)}$ with (9) gives that AA does not verify (5) for $\theta \in \mathcal{SS}^Q$. Now recalling that condition (6) for $\varnothing A$ requires $\frac{\theta_q}{\theta_a} \leq \frac{\Delta \Pr(a,Q)-\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q)-\Delta \Pr(q,\varnothing)}\frac{1-P^{\varnothing A}(\theta_q,\theta_a)}{P^{\varnothing A}(\theta_q,\theta_a)}$ which does not hold since $\theta_q > g^1(\theta_a)$ and $g^1(\theta_a) > \theta_a\frac{\Delta \Pr(a,Q)-\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q)-\Delta \Pr(q,\varnothing)}$ rules out $\varnothing A$.

It remains to see if $\mathcal{SS}^A$ and $\mathcal{SS}^Q$ are using strategies that verify (5). Using that $g^9(\theta_a) < \frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}\theta_a$ and $g^8(\theta_a) < \frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)}\theta_a$ we get the result for $\mathcal{SS}^A$; $g^1(\theta_a) > \theta_a\frac{\Delta \Pr(a,Q)-\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q)-\Delta \Pr(q,\varnothing)}$ and $g^2(\theta_a) > \theta_a\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)}$ give the result for $\mathcal{SS}^Q$.

Weak supporters.

Let $\theta \in \mathcal{WS}^A$ which implies that $\min\{g^7(\theta_a), g^6(\theta_a)\} \geq \theta_q$. By definition of $g^7(\theta_a)$ we have that QA does not verify (6) and by definition of $g^6(\theta_a)$ we have that $\varnothing\varnothing$ does not verify (6) either. Since $g^7(\theta_a) < \frac{\Delta \Pr(a,Q)-\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q)-\Delta \Pr(q,\varnothing)}$, (10) gives that QQ does not verify (5). Using (11), $g^4(\theta_a) \geq g^7(\theta_a)$, it must be that $g^4(\theta_a) > \theta_q$ so $Q\varnothing$ is worse than $\varnothing\varnothing$ by definition of $g^4(\theta_a)$ and since $\varnothing A$ is better than $\varnothing\varnothing$, we have that $\varnothing A$ is preferred to $Q\varnothing$. By definition of $g^9(\theta_a)$, $\varnothing A$ is preferred to AA.

Let $\theta \in \mathcal{WS}^Q$ so $g^4(\theta_a) \leq \theta_q$ and it follows directly that $Q\varnothing$ is preferred to $\varnothing\varnothing$ by definition of $g^4(\theta_a)$. At the same time, $\theta_a \leq g^3(\theta_q)$ gives directly that it is also better than QA by definition of $g^3(\theta_q)$. Since $\frac{\Delta \Pr(q,\varnothing)}{\Delta \Pr(a,\varnothing)} > \frac{g^3(\theta_q)}{\theta_q}$, the uninformative strategy AA does not verify (5) (see expression (9)).

Using that $\theta_a \leq g^3(\theta_q)$ implies that $\theta_q > g^6(\theta_a)$ (by relation (10)) we have that $\varnothing\varnothing$ is preferred to $\varnothing A$ (by definition of $g^6(\theta_a)$) and since $Q\varnothing$ is preferred to $\varnothing\varnothing$ (by definition of $g^4(\theta_a)$), it must be that $Q\varnothing$ is also preferred to $\varnothing A$. By definition of $g^1(\theta_a)$ we get that $Q\varnothing$ is preferred to QQ.

Condition (6) for the voting strategy $\varnothing A$ is verified by the properties

$$\begin{aligned}
\frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}\frac{1-P^{\varnothing A}(\theta_a, g^9(\theta_a))}{P^{\varnothing A}(\theta_a, g^9(\theta_a))} &< \frac{g^9(\theta_a)}{\theta_a} \\
\frac{\Delta \Pr(a,Q)-\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,Q)-\Delta \Pr(q,\varnothing)}\frac{1-P^{\varnothing A}(\theta_a, g^7(\theta_a))}{P^{\varnothing A}(\theta_a, g^7(\theta_a))} &> \frac{g^7(\theta_a)}{\theta_a} \\
\frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)}\frac{P^{\varnothing A}(\theta_a, g^6(\theta_a))}{1-P^{\varnothing A}(\theta_a, g^6(\theta_a))} &> \frac{g^6(\theta_a)}{\theta_a}
\end{aligned}$$

and condition (5) holds for $Q\varnothing$ by the properties

$$
\begin{aligned}
\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}\frac{1-P^{Q\varnothing}(\theta_a,g^4(\theta_a))}{p^{Q\varnothing}(\theta_a,g^4(\theta_a))} &< \frac{g^4(\theta_a)}{\theta_a}\\
\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}\frac{P^{Q\varnothing}(\theta_a,g^1(\theta_a))}{1-P^{Q\varnothing}(\theta_a,g^1(\theta_a))} &> \frac{g^1(\theta_a)}{\theta_a}\\
\frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(a,\varnothing)}\frac{1-P^{Q\varnothing}(g^3(\theta_q),\theta_q)}{P^{Q\varnothing}(g^3(\theta_q),\theta_q)} &> \frac{g^3(\theta_q)}{\theta_q}
\end{aligned}
$$

Abstainers.

The constraint that $\theta_q \in (g^6(\theta_a), g^4(\theta_a))$ ensures that either $g_1^{10}(\theta_a)$ or $g_2^{10}(\theta_q)$ is well defined as proven in the companion Appendix (A.1). Using $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} > \frac{g^4(\theta_a)}{\theta_a}$ and $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} < \frac{g^6(\theta_a)}{\theta_a}$, we have that AA and QQ do not verify (5) by (9) and (10), respectively. By definition of $g^6(\theta_a)$, the relation $g^6(\theta_a) < \theta_q$ implies that $\varnothing\varnothing$ is preferred to $\varnothing A$; the same argument applies for $\theta_q < g^4(\theta_a)$ which ensures that $\varnothing\varnothing$ is preferred to $Q\varnothing$. Now assume that $1 \geq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ and recall that $\theta_q \leq g_1^{10}(\theta_a)$ which implies that $\varnothing\varnothing$ is preferred to QA by definition of $g_1^{10}(\theta_a)$. On the other hand, if $1 \leq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ the definition of $g_2^{10}(\theta_q)$ gives that all types that satisfy $\theta_a \leq g_2^{10}(\theta_q)$ prefer the uninformed strategy with $\varnothing\varnothing$ to the informed strategy with QA.

Condition (5) for $\varnothing\varnothing$ follows by $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} > \frac{g^4(\theta_a)}{\theta_a}$ and $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} < \frac{g^6(\theta_a)}{\theta_a}$ which reverse the inequalities (9) and (10).

Independents.

If there are no independents we are done, so let $g_1^{10}(\theta_a) < 1$ for some $\theta_a \leq 1$ or $g_2^{10}(\theta_q) < 1$ for some $\theta_q \leq 1$ when appropriate. The condition $\theta_q > \max\{g^7(\theta_a), g^8(\theta_a)\}$ gives that QA is preferred to $\varnothing A$ and AA by definition of $g^7(\theta_a)$ and $g^8(\theta_a)$ respectively. By definition of $g^3(\theta_q)$ and $g^2(\theta_a)$, if $\theta_a > g^3(\theta_q)$ we have that QA is preferred to $Q\varnothing$ and if $g^2(\theta_a) > \theta_q$ we have that QA is preferred to QQ. If $1 \geq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ by definition of $g_1^{10}(\theta_a)$ we have that QA is preferred to $\varnothing\varnothing$. The case $1 \leq \frac{\Delta\Pr(q,\varnothing)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(a,Q)}$ follows by the same arguments.

Condition (5) for QA follows because (10) for $s = s_q$ is verified by $\theta \in \mathcal{I}$ since

$$
\begin{aligned}
\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} &< \frac{P^{QA}(g^7(\theta_a),\theta_a)}{1-P^{QA}(g^7(\theta_a),\theta_a)}\frac{g^7(\theta_a)}{\theta_a}\\
\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} &< \frac{P^{QA}(g^8(\theta_a),\theta_a)}{1-P^{QA}(g^8(\theta_a),\theta_a)}\frac{g^8(\theta_a)}{\theta_a}
\end{aligned}
$$

while (9) for $s = s_a$ is verified by $\theta \in \mathcal{I}$ since

$$
\begin{aligned}
\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} &> \frac{\theta_q}{g^3(\theta_q)}\frac{1-P^{QA}(\theta_q,g^3(\theta_q))}{P^{QA}(\theta_q,g^3(\theta_q))}\\
\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} &> \frac{1-P^{QA}(g^2(\theta_a),\theta_a)}{P^{QA}(g^2(\theta_a),\theta_a)}\frac{g^2(\theta_a)}{\theta_a}
\end{aligned}
$$

It is easy to verify that the sets $\mathcal{SS}^A$, $\mathcal{SS}^Q$, $\mathcal{WS}^A$, $\mathcal{WS}^Q$, $\mathcal{A}$ and $\mathcal{I}$ cover all types in $[0,1]^2$ without intersecting each other. ■

Proof of Proposition 2. Let $\phi = \frac{\left(1-\left(\xi_A+\xi_Q\right)\alpha\right)^{n-1}}{2}$ and define the spaces

$$\begin{aligned}
X_1 &\equiv \left\{(x,y) \in \left[\xi_A\alpha, 1-\left(\xi_\varnothing+\xi_Q\right)\alpha\right] \times \left[\xi_Q\alpha, 1-\left(\xi_\varnothing+\xi_A\right)\alpha\right]\right\} \\
X_2(\phi) &\equiv \left\{(x,y,v,z) \in [\phi,1]^2 \times [\phi,1]^2 : x+\phi \le v, y+\phi \le z\right\} \\
X_3(\phi) &\equiv \left\{(x,y) \in \left\{\phi, \frac{1}{\phi}\right\}^2\right\}
\end{aligned}$$

Let $\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right)$ by a generic element of the space $X_2(\phi)$ and $\left(\Pi^{Q-\varnothing}, \Pi^\varnothing\right)$ a generic element of the space $X_3(\phi)$. Note that $y^\omega_\varnothing$ plays the role of $\Delta\Pr(\omega,\varnothing)$ and y^ω_Q plays the role of $\Delta\Pr(\omega,Q)$ for $\omega \in \{a,q\}$. On the other hand, $\Pi^{Q-\varnothing}$ plays the role of $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}$ and $\Pi^\varnothing$ plays the role of $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$.

Let $p^i : [0,1]^2 \times X_2(\phi) \to \left[\frac{1}{2}, 1-\eta\right], i = 1,2,3$ be implicitly defined by $C'(p^1) = \frac{\theta_a y^a_\varnothing + \theta_q y^q_\varnothing}{2}$, $C'(p^2) = \frac{\theta_a y^a_Q + \theta_q y^q_Q}{2}$, and $C'(p^3) = \frac{\theta_a\left(y^a_Q - y^a_\varnothing\right) + \theta_q\left(y^q_Q - y^q_\varnothing\right)}{2}$, and let η be such that $C'(1-\eta) > 1$. So p^1 plays the role of $P^{\varnothing A}$, p^2 plays the role of P^{QA} and p^3 plays the role of $P^{Q\varnothing}$.

Now consider an element $\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right) \in X_2(\phi)$ and using (p^1, p^2, p^3), we can define the cutoff functions used in the characterization of equilibrium . Therefore, the sets of strong and weak supporters, independents and abstainers are well defined. Using Proposition (1) we have that $P(X^\omega)$, the probability of a vote for $X \in \{Q,A\}$ in state $\omega \in \{q,a\}$, is

$$\begin{aligned}
\Pr(A^a) &\equiv \int_{\theta\in\mathcal{WS}^A} p^1(\theta)\,dF(\theta) + \int_{\theta\in\mathcal{SS}^A} dF(\theta) + \int_{\theta\in\mathcal{I}} p^2(\theta)\,dF(\theta) \qquad (32)\\
\Pr(A^q) &\equiv \int_{\theta\in\mathcal{WS}^A} \left(1-p^1(\theta)\right)dF(\theta) + \int_{\theta\in\mathcal{SS}^A} dF(\theta) + \int_{\theta\in\mathcal{I}} \left(1-p^2(\theta)\right)dF(\theta)
\end{aligned}$$

$$\begin{aligned}
\Pr(Q^q) &\equiv \int_{\theta\in\mathcal{WS}^Q} p^3(\theta)\,dF(\theta) + \int_{\theta\in\mathcal{SS}^Q} dF(\theta) + \int_{\theta\in\mathcal{I}} p^2(\theta)\,dF(\theta) \qquad (33)\\
\Pr(Q^a) &\equiv \int_{\theta\in\mathcal{WS}^Q} \left(1-p^3(\theta)\right)dF(\theta) + \int_{\theta\in\mathcal{SS}^Q} dF(\theta) + \int_{\theta\in\mathcal{I}} \left(1-p^2(\theta)\right)dF(\theta)
\end{aligned}$$

For functions (p^1, p^2, p^3) and $\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right) \in X_2(\phi)$ and $\left(\Pi^{Q-\varnothing}, \Pi^\varnothing\right) \in X_3(\phi)$, we define the functions $G_X^\omega : X_2(\phi) \times X_3(\phi) \to X_1$ for $X = A, Q$ such that

$$\begin{aligned}
G_A^\omega \left(y_\varnothing^a, y_\varnothing^q, y_Q^a, y_Q^q, \Pi^{Q-\varnothing}, \Pi^\varnothing\right) &\equiv \xi_A \alpha + (1-\alpha)\Pr(A^\omega) I\left(\Pi^{Q-\varnothing} > \Pi^\varnothing\right) \\
&\quad + (1-\alpha)\Pr(A \mid \omega) I\left(\Pi^{Q-\varnothing} \leq \Pi^\varnothing\right) \\
G_Q^\omega \left(y_\varnothing^a, y_\varnothing^q, y_Q^a, y_Q^q, \Pi^{Q-\varnothing}, \Pi^\varnothing\right) &\equiv \xi_Q \alpha + (1-\alpha)\Pr(Q^\omega) I\left(\Pi^{Q-\varnothing} > \Pi^\varnothing\right) \\
&\quad + (1-\alpha)\Pr(Q \mid \omega) I\left(\Pi^{Q-\varnothing} \leq \Pi^\varnothing\right)
\end{aligned}$$

where $\Pr(A \mid \omega)$ and $\Pr(Q \mid \omega)$ are defined for the case where a non partisan voter never abstains. That is

$$\begin{aligned}
\Pr(A \mid a) &\equiv \int_0^1 \int_0^{\min\left\{1, g^8(\theta_a)\right\}} dF(\theta) + \int_0^1 \int_{\min\{1, g^8(\theta_a)\}}^{\min\left\{1, g^2(\theta_a)\right\}} P^{QA}(\theta)\, dF(\theta) \\
\Pr(A \mid q) &\equiv \int_0^1 \int_0^{\min\left\{1, g^8(\theta_a)\right\}} dF(\theta) + \int_0^1 \int_{\min\{1, g^8(\theta_a)\}}^{\min\left\{1, g^2(\theta_a)\right\}} \left(1 - P^{QA}(\theta)\right) dF(\theta)
\end{aligned}$$

and $\Pr(Q \mid \omega) + \Pr(A \mid \omega) = 1$. Now, for a pair $(x_1^\omega, x_2^\omega) \in X_1$ we can define $\Pr\left(T_{n-1}^m = l \mid \omega\right)$ in terms of (x_1^ω, x_2^ω) as $\Pr\left(T_{n-1}^m = l \mid (x_1^\omega, x_2^\omega), \omega\right) \equiv \frac{(n-1)!}{l!(m-l)!(n-1-m)} (x_1^\omega)^l (x_2^\omega)^{m-l} (1 - (x_1^\omega + x_2^\omega))^{n-1-m}$. Recalling the expressions for $\Delta\Pr(\omega, \varnothing)$ and $\Delta\Pr(\omega, Q)$, we define the function $K_i : X_1 \times X_1 \to X_2(\phi)$, such that for $i \in \{1, 2\}$ we let $K_i(x_1^a, x_2^a, x_1^q, x_2^q) = \frac{\tau_2(k,\omega) + \tau_1(\omega)}{2}$ and for $i \in \{3, 4\}$, we let $K_i(x_1^a, x_2^a, x_1^q, x_2^q) = \tau_1(\omega) + \frac{\tau_2(k,\omega) + \tau_2(k+1,\omega)}{2}$ where

$$\tau_2(l, \omega) \equiv \sum_{k=0}^{\left\lfloor \frac{n}{2} - 1 \right\rfloor} \frac{(n-1)!}{k!(k+1)!(n-2k-2)} \left(\frac{x_1^\omega}{(1 - (x_1^\omega + x_2^\omega))}\right)^l \left(\frac{x_2^\omega}{(1 - (x_1^\omega + x_2^\omega))}\right)^{2k+1-l} \tag{34}$$

$$\tau_1(\omega) \equiv (1 - (x_1^\omega + x_2^\omega))^{n-1} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{(n-1)!}{k!k!(n-1-2k)} \left(\frac{x_1^\omega}{(1 - (x_1^\omega + x_2^\omega))} \frac{x_2^\omega}{(1 - (x_1^\omega + x_2^\omega))}\right)^k$$

So, if we let $\omega = a$ for $i \in \{1, 3\}$ and $\omega = q$ for $i \in \{2, 4\}$, $(x_1^a, x_2^a, x_1^q, x_2^q)$ are the probabilities of voting for A or Q in different states, and K_1 gives $\Delta\Pr(a, \varnothing)$, K_2 gives $\Delta\Pr(q, \varnothing)$, K_3 gives $\Delta\Pr(a, Q)$, and K_4 gives $\Delta\Pr(q, Q)$.

We also define the function $L : X_2(\phi) \to X_3(\phi)$ such that $L\left(y_\varnothing^a, y_\varnothing^q, y_Q^a, y_Q^q\right) \equiv \left(\frac{y_\varnothing^a}{y_\varnothing^q}, \frac{y_Q^a - y_\varnothing^a}{y_Q^q - y_\varnothing^q}\right)$, which maps the probabilities of changing the election according to the change in the vote ($\Delta\Pr(a, \varnothing)$, $\Delta\Pr(q, \varnothing)$, $\Delta\Pr(a, Q)$, and $\Delta\Pr(q, Q)$), into the ratios that gives the incentives to abstain: $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$ and $\frac{\Delta\Pr(a,Q) - \Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q) - \Delta\Pr(q,\varnothing)}$. Now we have all the elements to show that an equilibrium actually exists.

Take an arbitrary element of $\mathcal{S} \equiv (X_1)^2 \times X_2(\phi) \times X_3(\phi)$, define the function $\Gamma : \mathcal{S} \to \mathcal{S}$ such that $\Gamma \equiv \left\{G_A^a, G_Q^a, G_A^q, G_Q^q, K, L\right\}$, where the components are defined above.

We are going to show first that actually Γ is a continuous function.

For continuity of $\left(G_A^a, G_Q^a, G_A^q, G_Q^q\right)$ we first observe that all the cutoff functions that determine the types (weak and strong supporters, abstainers and independents), are well defined and continuous for $\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right)$ and (p^1, p^2, p^3) as defined above. Therefore $\Pr(A^\omega)$ and $\Pr(Q^\omega)$, are continuous on $\left(y^a_\varnothing, y^q_\varnothing, y^a_Q, y^q_Q\right)$ when we consider that (p^1, p^2, p^3) are also continuous and well defined for $y^\omega_\varnothing \in [\phi, 1]$, $y^\omega_Q \in [\phi, 1]$. We only need to prove that $\Pr(X^\omega) \to \Pr(X \mid \omega), X \in \{A, Q\}$ when $\Pi^{Q-\varnothing} \to \Pi^\varnothing$, so the probability that $\theta \in \mathcal{WS}^X$ approaches 0 for $X \in \{A, Q\}$. We show the case of $X = A$. Since

$$\begin{aligned}\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}\frac{1-P^{\varnothing A}\left(g^9(\theta_a),\theta_a\right)}{P^{\varnothing A}\left(g^9(\theta_a),\theta_a\right)} &< \frac{g^9(\theta_a)}{\theta_a}\\ \frac{1-P^{\varnothing A}\left(g^7(\theta_a),\theta_a\right)}{P^{\varnothing A}\left(g^7(\theta_a),\theta_a\right)}\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} &> \frac{g^7(\theta_a)}{\theta_a}\end{aligned}$$

and recalling that $\theta \in \mathcal{WS}^A$ verifies $g^7(\theta_a) \geq \theta_q > g^9(\theta_a)$, it must hold that

$$\frac{1-P^{\varnothing A}\left(g^7(\theta_a),\theta_a\right)}{P^{\varnothing A}\left(g^7(\theta_a),\theta_a\right)}\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} > \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}\frac{1-P^{\varnothing A}\left(g^9(\theta_a),\theta_a\right)}{P^{\varnothing A}\left(g^9(\theta_a),\theta_a\right)}$$

When $\Pi^{Q-\varnothing} \to \Pi^\varnothing$ the previous inequality is just $\frac{1-P^{\varnothing A}\left(g^7(\theta_a),\theta_a\right)}{P^{\varnothing A}\left(g^7(\theta_a),\theta_a\right)} \geq \frac{1-P^{\varnothing A}\left(g^9(\theta_a),\theta_a\right)}{P^{\varnothing A}\left(g^9(\theta_a),\theta_a\right)}$, which implies that $g^7(\theta_a) \leq g^9(\theta_a)$ and therefore $\mathcal{WS}^A = \varnothing$. Using that abstainers must satisfy that $\theta_q \in \left(g^6(\theta_a), g^4(\theta_a)\right)$, $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} < \frac{g^6(\theta_a)}{\theta_a}$, and $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} > \frac{g^4(\theta_a)}{\theta_a}$, it must be that $\frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)} < \frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)}$ which implies that if $\frac{\Delta\Pr(a,Q)-\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,Q)-\Delta\Pr(q,\varnothing)} \to \frac{\Delta\Pr(a,\varnothing)}{\Delta\Pr(q,\varnothing)}$ then the probability that $\theta \in \mathcal{A}$ approaches 0. Therefore only strong supporters and independents survive and $\mathcal{I} \to \left\{(\theta_q, \theta_a) \in [0,1]^2 : g^8(\theta_a) < \theta_q < g^2(\theta_a)\right\}$ which implies the desire result.

The fact that K is continuous in $(x^a_1, x^a_2, x^q_1, x^q_2)$ follows trivially by continuity of $\Pr\left(T^m_{n-1} = l \mid (x^\omega_1, x^\omega_2), \omega\right.$ in $(x^a_1, x^a_2, x^q_1, x^q_2)$. The same applies for continuity of L when we consider that $y^q_Q - y^q_\varnothing \geq \phi$ and $y^q_\varnothing \geq \phi$.

X_1, $X_2(\phi)$ and $X_3(\phi)$ are convex and compact, so Brouwer's fixed point theorem holds (Border (1985)) and there is some $x \in \mathcal{S}$ such that $\Gamma(x) = x$. ■

Proof of Proposition 3. We are going to show first that the following condition

Condition 2 ***a)*** $\Delta\Pr(a,Q) = \Delta\Pr(q,Q)$, ***b)*** $\Delta\Pr(\omega,Q) - \Delta\Pr(\omega,\varnothing) = \Delta\Pr(-\omega,\varnothing)$ *for* $(\omega, -\omega) \in \{(q,a),(a,q)\}$

is necessary and sufficient for condition 1. We are going to use that (34) gives:

$$\begin{aligned}\Delta\Pr(\omega,Q) &= \Delta\Pr(\omega,\varnothing) + \frac{\tau_1(\omega)+\tau_2(k+1,\omega)}{2} \qquad (35)\\ \Delta\Pr(\omega,\varnothing) &= \frac{\tau_2(k,\omega)+\tau_1(\omega)}{2}\end{aligned}$$

Necessity

Assume first that condition (1) holds. Using expression (34), it is straightforward to see that $\tau_1(a) = \tau_1(q)$, $\tau_2(k,a) = \tau_2(k+1,q)$ and $\tau_2(k+1,a) = \tau_2(k,q)$. Using these equalities in (35) we have $\Delta\Pr(\omega,Q) - \Delta\Pr(\omega,\varnothing) = \Delta\Pr(-\omega,\varnothing)$ and $\Delta\Pr(\omega,Q) = \Delta\Pr(-\omega,Q)$ where $-\omega = a$ if $\omega = q$ and $-\omega = q$ if $\omega = a$.

Sufficiency

Assume now that condition (2) holds.

Take any type $(\theta_q = y, \theta_a = x)$. Using (12) and $\Delta \Pr(a, Q) = \Delta \Pr(q, Q)$, for $\omega \in \{q, a\}$, it follows that $P^{QA}(x, y) = P^{QA}(y, x)$. Using (12) and $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \varnothing) = \Delta \Pr(-\omega, \varnothing)$, it follows that $P^{\varnothing A}(x, y) = P^{Q\varnothing}(y, x)$.

We are going to use the definition of cut cutoff functions (17)-(26). Using $P^{Q\varnothing}(x, y) = P^{\varnothing A}(y, x)$ we get that $L\left(P^{Q\varnothing}(x, y)\right) = L\left(P^{\varnothing A}(y, x)\right)$; recalling that $\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{2} = \frac{\Delta \Pr(q,\varnothing)}{2}$, if $g^4(x) = y$ it must be true that $g^6(y) = x$. Using the same argument we have 1) $g^9(x) = y$ iff $g^1(y) = x$, 2) $g^8(x) = y$ iff $g^2(y) = x$, and 3) $g^7(x) = y$ iff $g^3(y) = x$. Take now a type (x, y) such that $x > g^4(y)$, so $Q\varnothing$ is preferred to $\varnothing\varnothing$, and we must have that $g^6(x) > y$, so the type (y, x) must prefer the strategy with $\varnothing A$ to the one with $\varnothing\varnothing$. This result extends to all functions presented above 1) $g^9(x) > y$ iff $x > g^1(y)$, 2) $g^8(x) > y$ iff $x > g^2(y)$, and 3) $g^7(x) > y$ iff $x > g^3(y)$, so $(x, y) \in \mathcal{SS}^A$ iff $(y, x) \in \mathcal{SS}^Q$ and $(x, y) \in \mathcal{WS}^A$ iff $(y, x) \in \mathcal{WS}^Q$. The problem arises when we want to compare independents and abstainers, since we do not have a "mirror" function. In this case, (x, y) prefer being independent rather than being abstainer if:

$$L\left(P^{QA}(x, y)\right) \geq x \frac{\Delta \Pr(q, \varnothing)}{2} + y \frac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{2}$$

Using that $L\left(P^{QA}(y, x)\right) = L\left(P^{QA}(x, y)\right)$, $\frac{\Delta \Pr(q,\varnothing)}{2} = \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\varnothing)}{2}$ we must have that

$$L\left(P^{QA}(y, x)\right) \geq x \frac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{2} + y \frac{\Delta \Pr(q, \varnothing)}{2}$$

so the type (y, x) also prefers the informed strategy with QA to the uninformed strategy with $\varnothing\varnothing$: $(x, y) \in \mathcal{I}$ iff $(y, x) \in \mathcal{I}$. Given that $\mathcal{A}$ is the complement of the previous groups (weak supporters, strong supporters and independents) in $[0, 1]^2$, it is true that $(x, y) \in \mathcal{A}$ iff $(y, x) \in \mathcal{A}$.

Using the symmetry of F, and the previous results we get $\int\limits_{\theta \in \mathcal{WS}^A} dF(\theta) = \int\limits_{\theta \in \mathcal{WS}^Q} dF(\theta)$ and $\int\limits_{\theta \in \mathcal{SS}^Q} dF(\theta) = \int\limits_{\theta \in \mathcal{SS}^A} dF(\theta)$. Now, with the symmetry of F, the result that $(x, y) \in \mathcal{WS}^A$ iff $(y, x) \in \mathcal{WS}^Q$ and $p^{\varnothing A}(x, y) = p^{Q\varnothing}(x, y)$, implies that $\int\limits_{\theta \in \mathcal{WS}^Q} P^{Q\varnothing}(\theta)\, dF(\theta) = \int\limits_{\theta \in \mathcal{WS}^A} P^{\varnothing A}(\theta)\, dF(\theta)$.

Using that $(x, y) \in \mathcal{I}$ iff $(y, x) \in \mathcal{I}$ and that $P^{QA}(x, y) = P^{QA}(x, y)$, and recalling the characterization when abstention occurs in equilibrium of $\Pr(Q^\omega)$ and $\Pr(A^\omega)$ in (32) and (33), the condition (1) follows as desired.

To finish the existence proof, note that condition 2 and condition 1 define a closed and convex subset of $\mathcal{S} = (X_1)^2 \times X_2(\phi) \times X_3(\phi)$, as defined in Proposition (2) so we can apply

Brouwer's fixed point theorem (Border (1985)) and there is some $x \in \mathcal{S}$ such that $\Gamma(x) = x$ where x verifies both set of condition 2 and condition 1, where Γ is defined as in the previous Proposition (2).

We prove now that there is a abstention. Because $\Delta \Pr(\omega, \varnothing) > 0$ and $\Delta \Pr(\omega, Q) > 0$ it must be that information is collected. This implies that $\Pr(A \mid a) > \Pr(Q \mid a)$ so $\tau_2\left(\frac{m+1}{2}, a\right) > \tau_2\left(\frac{m-1}{2}, a\right)$. It follows that $\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing) > \Delta \Pr(a, \varnothing)$, while using $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \varnothing) = \Delta \Pr(-\omega, \varnothing)$

$$\frac{\Delta \Pr(a, Q) - \Delta \Pr(a, \varnothing)}{\Delta \Pr(a, \varnothing)} > 1 > \frac{\Delta \Pr(q, Q) - \Delta \Pr(q, \varnothing)}{\Delta \Pr(q, \varnothing)}$$

so non partisan voters abstain in equilibrium with positive probability (see (11)). ■

Proof of Proposition (4). The first part follows by noting that $\Delta \Pr(\omega, Q) \to 0$ when $n \to \infty$. For the second part recall that under symmetry (see proof of Proposition (3)) we have that $\tau_1(a) = \tau_1(q)$, $\tau_2(k, a) = \tau_2(k+1, q)$ and $\tau_2(k+1, a) = \tau_2(k, q)$ it follows that $\Delta \Pr(a, \varnothing) = \frac{\tau_2(k+1,q)+\tau_1(q)}{2}$. Using now that $\tau_2(k+1, q) \equiv \left(\frac{\left(1-\left(x_1^q+x_2^q\right)\right)}{\left(1-\left(x_1^q+x_2^q\right)\right)} \frac{x_1^q}{x_2^q}\right) \tau_2(k, q)$ we have $\Delta \Pr(a, \varnothing) = \frac{\left(\frac{\left(1-\left(x_1^q+x_2^q\right)\right)}{\left(1-\left(x_1^q+x_2^q\right)\right)} \frac{x_1^q}{x_2^q}\right)\tau_2(k,q)+\tau_1(q)}{2}$ which implies that

$$\begin{aligned}
\frac{\Delta \Pr(a, \varnothing)}{\Delta \Pr(q, \varnothing)} &= \frac{\left(\frac{\left(1-\left(x_1^q+x_2^q\right)\right)}{\left(1-\left(x_1^q+x_2^q\right)\right)} \frac{x_1^q}{x_2^q}\right) \tau_2(k, q) + \tau_1(q)}{\tau_2(k, q) + \tau_1(q)} \\
&= 1 - \left(x_2^q - x_1^q\right) \frac{\tau_2(k, q)}{\tau_2(k, q) + \tau_1(q)} \\
&= 1 - \left(x_2^q - x_1^q\right) \frac{1}{1 + \frac{\tau_1(q)}{\tau_2(k,q)}} < 1 + \left|x_1^q - x_2^q\right|
\end{aligned}$$

where $(x_1^a, x_2^a, x_1^q, x_2^q)$ are the probabilities of voting for A or Q in different states as defined in Proposition (2). Using again the notation in the proof of Proposition (2) we get

$$x_1^q - x_2^q = \left\{ \begin{array}{c} \int\limits_{\theta \in \mathcal{WS}^A} \left(1 - p^1(\theta)\right) dF(\theta) - \int\limits_{\theta \in \mathcal{WS}^Q} p^3(\theta)\, dF(\theta) + 2 \int\limits_{\theta \in \mathcal{I}} \left(\frac{1}{2} - p^2(\theta)\right) dF(\theta) \\ 2\left(\int\limits_0^1 \int\limits_0^{\min\left\{1, g^8(\theta_a)\right\}} dF(\theta) + \int\limits_0^1 \int\limits_{\min\left\{1, g^8(\theta_a)\right\}}^{\min\left\{1, g^2(\theta_a)\right\}} \left(1 - P^{QA}(\theta)\right) dF(\theta) - \frac{1}{2} \right) \end{array} \right\} \tag{36}$$

Since $\Delta \Pr(a, Q)$ and $\Delta \Pr(a, \varnothing)$ approach 0 as the number of voters grow, using (12) we get that investment in information goes to 0. Using symmetry, we get that $\int\limits_{\theta \in \mathcal{WS}^A} dF(\theta) -$

$\int\limits_{\theta \in \mathcal{WS}^Q} dF(\theta) \to 0$ and since $p^2(\theta) \to \frac{1}{2}$, the first line in (36) is equal to 0 when the number of voters is large. Using Remark (2) and (7) in Appendix (A.1) together with the fact that symmetry implies $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} = 1$ we get $1 < \frac{g^2(\theta_a)}{\theta_a} < \frac{P^{QA}\left(g^2(\theta_a),\theta_a\right)}{(1-P^{QA}(g^2(\theta_a),\theta_a))}$ and $\frac{1-P^{QA}\left(g^8(\theta_a),\theta_a\right)}{P^{QA}(g^8(\theta_a),\theta_a)} < \frac{g^8(\theta_a)}{\theta_a} < 1$. Since investment goes to 0 in the limit we must have that $P^{QA} \to \frac{1}{2}$ and therefore $g^2(\theta_a) \to g^8(\theta_a)$ when the number of voters is large. Note that this also implies that $g^8(\theta_a) \to \theta_a$ so the second line in (36) is just $x_1^q - x_2^q \approx 2\left(\int\limits_0^1 \int\limits_0^{\theta_a} dF(\theta) - \frac{1}{2}\right)$ and symmetry of F gives that $x_1^q - x_2^q \to 0$ when the number of voters is large. Therefore $\frac{\Delta \Pr(a,\varnothing)}{\Delta \Pr(q,\varnothing)} \to 1$.

Looking at all the remarks in Appendix (A.1) we have that all $g^j(\theta_a) \to \theta_a$ and only strong supporters for A and Q survive. ■