Ideology as Opinion: A Spatial Model of Common-value Elections

Joseph McMurray[*,†]

September 2011

Abstract

This paper analyzes a spatial model of common-value elections. As in Downs' (1957) classic model, citizens vote for candidates who choose policies from a one-dimensional spectrum. As in Condorcet's (1785) classic model, voters' opinions reflect attempts to identify an optimal policy, that is ultimately superior to all others. Competitive pressure drives office-motivated candidates to the political center, as in standard models, but this is no longer in voters' interest. Policy-motivated candidates instead offer divergent platforms, and may even respond to electoral "mandates" from voters, conveyed through the margin of victory. Voting then plays a signaling role, providing a possible explanation for supporting minor party candidates. The swing voter's curse does not apply in that case, but an analogous "signaling voter's curse" nevertheless leads poorly informed citizens to abstain, thereby explaining various empirical patterns. The model extends to multiple dimensions, but two-candidate competition remains inherently unidimensional in equilibrium.

JEL Classification Number D72, D82

Keywords: Voting, Elections, Median Voter, Information, Platform Convergence, Jury Theorem, Swing Voter's Curse, Turnout, Abstention, Roll-off

1 Introduction

There are two fundamental paradigms by which elections and other democratic institutions can be understood. The first of these is the lens of preference aggregation, especially

[*]Brigham Young University Economics Department. Email jmcmurray@byu.edu.

[†]This paper is part of my PhD dissertation at the University of Rochester. Special thanks to my advisor, Mark Fey, for his patient encouragement and extremely helpful advice. Thanks also to John Duggan, Gabor Virag, and Brennan Platt for their interest and suggestions.

the spatial models of Hotelling (1929) and Downs (1957) and their many extensions. In these models, voter preferences are single-peaked along a one-dimensional policy space, and competition for office drives candidates toward the political center (e.g. to the ideal policy of the median voter or the mean voter, depending on model specifications); this outcome reflects a compromise between competing interests, which is good because it minimizes the disutility voters must suffer from a policy that is far from their ideal.[1][2] The second paradigm, which is less common but actually much older, is Condorcet's (1785) classic model of information aggregation: if one of two policy alternatives is better for society in some objective sense, and voters seek independently to identify that policy, with even minimal success, the "jury theorem" states that majority opinion is likely—in fact, almost certain, in a large electorate—to correctly identify the superior policy (see Young, 1988).

The political tug-of-war depicted in spatial models fails to recognize that the broad goals of the public policies voters care about most—such as national defense, economic and environmental stability, and eliminating crime, poverty, and corruption—have essentially unanimous appeal; the sheer complexity of public policy inevitably breeds disagreements over how these goals can be achieved, but the desirability of the goals themselves is rarely challenged. On the other hand, spatial models are far less primitive than information models, which typically include only two policy alternatives, and so provide no insight into the role of candidates or parties in determining policy outcomes; indeed, the geometry of spatial models accommodates the empirical notion of voter *ideology*, which clearly plays a central role in political conflicts, and which according to the standard parlance is one-dimensional, ranging from liberal to conservative (or left to right).

In an effort to synthesize these two disparate paradigms in a meaningful way, this paper models ideology as an informational phenomenon. Like standard spatial models, the model below considers a continuum of policy alternatives. As in Condorcet (1785), however, one of these policies is ultimately optimal for society. Voters unanimously prefer the optimal policy, but disagree regarding its location: specifically, each forms beliefs on the basis of a private signal that is correlated with the unknown state of the world. In a later specification of the model, any policy may be optimal. The first specification considers the simpler case in which the optimal policy is known *ex ante* to lie at one of the two extremes of the policy space; even in that case, risk-averse voters prefer policies in the interior of the policy interval, as a hedge against error.

[1] See Davis and Hinich (1968). If utility functions are tent-shaped or quadratic, for example, then total voter utility is maximized at the median or mean, respectively. Necessarily, the utilitarian optimum lies in the interior of the policy space.

[2] The median voter's ideal policy is also endorsed by May's (1952) axioms, since it is majority-preferred to any other policy, as Black (1948) demonstrates.

As an example of a policy issue that is inherently discrete, consider a society of individuals who all wish to end an economic recession: according to Keynesian macroeconomic theory, a large "stimulus" policy of increased fiscal spending can spur economic growth, but more classical theory views stimulus spending as ineffective and wasteful; a moderate-sized stimulus might minimize the damage from adopting the wrong economic model, but is not optimal in either case. Alternatively, consider a decision between funding two programs for improving school quality, such as increasing teacher salaries or reducing class sizes: partially funding both programs may be optimal in the face of uncertainty, but an individual who believes one of the two programs to have the greater impact on education will want that program to receive all of the available funding.[3]

These settings illustrate a very common phenomenon, which is that individuals sort themselves into philosophical "camps", according to their opinions on particular issues. In the case of political ideologies, the relevant question is a holistic view of politics, but the structure is the same: citizens may adopt "liberal" or "conservative" philosophies, or may remain politically moderate, to avoid pursuing the wrong extreme. In any of the above contexts, a simple but important observation is that voters' underlying beliefs are continuous, even if division into camps is inherently discrete: the strength of an individual's conviction in favor of one particular camp can be described as a probability, which may range from zero to one. Thus, the familiar one-dimensional geometry of spatial models arises quite naturally in this information environment, as a description of beliefs.

In a later specification of the model, the optimal policy may lie anywhere in the policy interval. This may describe funding decisions similar to the education setting described above, but in which alternative programs have complimentary effects. The optimal allocation of defense spending, for example, likely includes a combination of air and naval forces, rather than either one in isolation. In still other situations, the nature of uncertainty is inherently continuous, such as the unknown strength of an enemy, which may determine the optimal level of defense spending. In the context of ideologies, a continuous state variable admits the possibility that moderate policies are truly optimal, not merely in the face of uncertainty, as Hill (2009) argues vigorously.

With a continuum of policy alternatives, this model provides a framework for the analysis of candidate behavior. The vast literature on spatial competition suggests a number of plausible assumptions, which the model treats in turn, with some results that are familiar, and other insights that are new. For example, if candidates are office motivated and campaign platform commitments are binding, as in Downs (1957), then there is a unique equilibrium,

[3] Identical logic applies, of course, for programs to reduce pollution, crime, corruption, or poverty—or, in a business setting, to increase sales or profits.

in which candidates adopt identical platforms at the center of the policy interval. This resembles that of the canonical median voter theorem of Hotelling (1929), Black (1948), and Downs (1957)—and arises for the same reason, since moving away from the center merely concedes votes to a candidate's opponent—but has a dramatically different welfare implication, since the policy at the center, like a moderate-sized stimulus in the macroeconomic example above, is known *ex ante* not to be optimal. Voters are better off if candidates are policy motivated, in which case equilibrium platforms diverge. As in Condorcet's (1785) jury theorem, the candidate with the better platform is quite likely to win the election. A vote for this candidate is less likely to be pivotal than a vote for his[4] opponent, however, so poorly informed citizens abstain from voting, as in Feddersen and Pesendorfer (1996), to avoid the "swing voter's curse".

If candidates are policy motivated and platform commitments are not binding, the winning candidate can infer information about the state of the world from his margin of victory. If he responds to this information, voting takes on a signaling role, pushing the candidate's beliefs in one direction or another. The standard pivotal voting calculus is irrelevant in that case, so the swing voter's curse no longer arises, but poorly informed citizens again abstain, to avoid the "signaling voter's curse" of pushing policy in the wrong direction. Contrary to the intuition that decisions are best made on the basis of as much information as possible, this abstention actually enhances voter welfare. The model can also be generalized to multiple dimensions, where equilibrium behavior simply ignores all but one dimension.

This model has a variety of applications. For example, logical connections can cause issues to be correlated across dimensions, providing an explanation for the apparent unidimensionality of ideological views. The relationship between information, ideology, and voter participation explains the empirical correlations between these variables. Divergent platforms explain large empirical margins of victories, and the popular notion of electoral "mandates" from voters. Signaling considerations also provide a rationale for voting for minor party candidates, who are unlikely to win the election.

This paper relates to multiple literatures. References to the most relevant models of spatial competition, information aggregation, and voter participation are provided in section 6, where direct comparisons are made. Feddersen and Pesendorfer (1999) add a private-value dimension to their earlier common-values model, which they interpret as voter ideology, and find that in large electorates, poorly informed citizens no longer abstain; instead, they vote on the basis of private values alone. That model does not analyze candidate behavior, however. Bernhardt, Duggan, and Squintani (2009) consider a spatial model with a commonly-valued shock, and demonstrate that some policy divergence is optimal. Maskin and Tirole (2004)

[4]Throughout this paper, feminine pronouns refer to voters and masculine pronouns refer to candidates.

consider incentives for candidates to "pander" to voter opinion in an agency model, even when these opinions are incorrect.

In addition to the vast literatures of spatial competition and information aggregation, there are a handful of models that ascribe a signaling role to the act of voting. Lohmann (1993) and Razin (2003) demonstrate a signaling incentive in common-value environments, but candidate behavior is specified exogenously. Castanheira (2003), Shotts (2006), and Meirowitz and Shotts (2009) consider two-period models in which election results from the first period signal the location of the median voter's ideal point to candidates in the second stage, but none considers candidate convergence or non-convergence, and none includes components of common value, or discusses information. Among these models, only Castanheira (2003) considers multiple candidates, and only Lohmann (1993) and Shotts (2006) allow abstention. In every case, private signals are drawn from identical distributions, so none of these models highlights the relationships between beliefs, ideology, and participation. Also, none highlights the welfare impact of convergent candidate platforms, and none considers multiple dimensions.

The remainder of this paper is organized as follows. Section 2 introduces the model, and sections 3 and 4 analyze electoral competition under the assumptions that platform commitments are binding or not. Section 5 repeats this analysis for the case of a convex state space, and considers the possibility of multiple dimensions. Section 6 describes the empirical applications, and section 7 concludes. Proofs of analytical results are presented in the Appendix.

2 The Model

A society consists of N citizens and two candidates, A and B. For technical convenience, I adopt Myerson's (1998, 2000) assumption that the precise number N of citizens is unknown, but is drawn from a Poisson distribution with mean n.[5] Together, this electorate must choose and implement a policy from the interval $[-1, 1]$ of alternatives, which will provide a common benefit to every citizen. Let Z denote an unknown state of the world, which designates the policy that in truth is best for society. Section 5.1 treats the case in which Z is distributed continuously on $[-1, 1]$, but for now assume for simplicity that $Z \in \{-1, 1\}$ lies at one of the extreme ends of the policy space, with equal probability: $\Pr(Z = 1) = \Pr(Z = -1) = \frac{1}{2}$.

If policy $x \in [-1, 1]$ is implemented in state Z then each citizen receives utility $u(x, Z)$,[6]

[5] In the numerical examples in Sections 4 and 4.3, N is instead fixed and known.

[6] The assumption of identical preferences is admittedly quite strong. As McMurray (2010) argues, however, it is sufficient to assume that the optimal policies for different citizens are positively correlated, since

which simply declines quadratically with the distance between x and Z:[7]

$$u(x, Z) = -(x - Z)^2. \tag{1}$$

Thus, $Z = -1$ implies that the best policies lie at the lower end of the policy space, while $Z = 1$ implies that higher policies are better. Note, however, that $u(x, Z)$ is concave in x, implying that citizens are risk-averse, and may therefore prefer policies in the interior of the policy space, as a hedge against error. Specifically, conditional on information Ω, the expectation of (1) is maximized at the conditional expectation of the state, $E(Z|\Omega)$. *Ex ante*, then, the optimal policy lies at the very center $E(Z) = 0$ of the interval.

A citizen's private opinion regarding the optimal policy is represented by a private signal $S_i \in \{-1, 1\}$ which is positively correlated with Z. Because citizens differ in expertise, however, these signals are of heterogeneous quality. Specifically, each citizen is first endowed with information quality $Q_i \in [0, 1]$, drawn independently (and independent of Z) from a common distribution F. For technical convenience, assume that F has a differentiable and strictly positive density f.[8] Conditional on Q_i and Z, the distribution of S_i is given as follows,

$$\Pr(S_i = s|Q_i = q, Z = z) = \frac{1}{2}(1 + zsq), \tag{2}$$

for any $s, z \in \{-1, 1\}$ and $q \in [0, 1]$. With this distribution, Q_i is equivalent to the correlation coefficient between S_i and Z, and thus measures the strength of a citizen's conviction that her private opinion of the optimal policy is correct. To a perfectly informed (i.e. $Q_i = 1$) citizen, for instance, S_i reveals Z perfectly; to a perfectly uninformed (i.e. $Q_i = 0$) citizen, S_i reveals nothing.

The distribution F of expertise within the population is common knowledge, but Q_i and S_i are observed only privately. Conditional on private information, the posterior distribution of Z is given by the same expression as (2),

$$\Pr(Z = z|Q_i = q_i, S = s) = \frac{1}{2}(1 + zsq). \tag{3}$$

A citizen's private expectation of the optimal policy, then, is simply $E(Z|Q_i, S_i) = Q_i S_i$, and the utility she expects from any policy declines quadratically with the distance from this expectation. Thus, as in traditional models, private policy preferences are quadratic and

in that case one citizen can still rely on another's expertise.

[7] The functional form of (1) dramatically simplifies the analysis below, but is clearly not necessary for the intuition of the substantive results. It seems reasonable to conjecture that all of the results below would hold for any concave function of the distance between x and Z.

[8] The distribution F of information quality is assumed here to be exogenous. For information acquisition games that lead in equilibrium to heterogeneous expertise, see Martinelli (2006, 2007) and Oliveros (2011).

single-peaked, and the strength of a citizen's *ideology* can be measured by the location of her ideal policy. Furthermore, since f is strictly positive, $Q_i S_i$ has full support on the policy interval. Thus, while preferences are ultimately identical, asymmetric information induces ideological differences that range across the political spectrum.

With individual citizens thus informed, candidates propose policy platforms $x_A, x_B \in [0,1]$.[9] Observing these platforms, each citizen then votes (at no cost) for one of the two candidates. A voting strategy $\sigma : [0,1] \times \{-1,1\} \times [0,1]^2 \rightarrow \{A, B\}$ specifies behavior for every possible combination of private information $(q, s) \in [0,1] \times \{-1,1\}$ and candidate platforms $(x_A, x_B) \in [0,1]^2$;[10] for the voting subgame associated with a particular platform pair (x_A, x_B), the projection of σ onto the set of information types defines a subgame strategy $\bar{\sigma} : [0,1] \times \{-1,1\} \longrightarrow \{A, B\}$. Let Σ and $\bar{\Sigma}$ denote the sets of strategies in the complete game and in the voting subgame, respectively.[11]

Votes are cast simultaneously, and an election winner $W \in \{A, B\}$ is determined by simple majority rule, breaking a tie if necessary by a fair coin toss. If elected, candidate j implements policy $y_j \in [-1,1]$. A *committed* candidate must implement his pre-election platform policy $y_j = x_j$; a *responsive* candidate may implement any policy, and his policy choice may be a function $y_j : \mathbb{Z}_+^2 \longrightarrow [-1,1]$ of realized vote totals $a, b \in \mathbb{Z}_+$.[12] Let Υ denote the set of all such policy functions. The ultimate policy outcome $Y \in [-1,1]$ therefore depends on the strategy choices of both voters and candidates, and on the realizations of the election winner W and the numbers N_A and N_B of votes for each candidate, which in turn depend on the private information (Q_i, S_i) of each citizen, and therefore on the state Z.

In choosing strategies, citizens and *policy-motivated* candidates seek to maximize their expectations of $u(Y, Z)$, while *office-motivated* candidates maximize the probability of being elected. The analysis below characterizes *perfect Bayesian equilibria*—defined separately below for each of the various versions of the model. Given the assumption of Poisson

[9] Informally, candidates can be thought of as citizens who hold private opinions, as in the citizen candidate frameworks of Osborne and Slavinski (1996) and Besley and Coate (1997). Formally, however, candidates do not receive private signals of their own. Alternatively, the information structure of this model can be reinterpreted as describing updated beliefs after both candidates announce their private signals.

[10] Mixed strategies could be allowed, but would be used with zero probability in equilibrium, as the analysis below makes clear.

[11] In Sections 3.1 through 3.3 and in Sections 4.1 and 4.3, abstention from voting is not allowed, but Sections 3.3 and 4.2 introduce voter abstention, so the action set expands to $\{A, B, 0\}$ and the sets of strategies and subgame strategies are redefined as Σ' and $\bar{\Sigma}'$, respectively. Section 4.3 considers the possibility of additional candidates; in that case, the action set again expands to $\{A, B, C, D, 0\}$, and strategy and subgame strategy sets are redfined as Σ'' and $\bar{\Sigma}''$.

[12] In principle, y_j could be allowed to vary in response to platform policies x_A and x_B. This would never occur in equilibrium, however, and so is omitted from the analysis.

population uncertainty, such equilibria are necessarily also *symmetric* with respect to voter strategies, meaning that every voter plays the same strategy in equilibrium.[13]

3 Committed Candidates

This section assumes that candidates' campaign platform commitments are binding, so that the winning candidate must implement his platform policy $y_j = x_j$. section 3.1 begins by analyzing voter incentives in the subgame associated with a particular pair (x_A, x_B) of candidate strategies. Sections 3.2 and 3.3 then proceed by backward induction to analyze candidates' incentives for platform selection, under the assumptions that candidates are office- or policy-motivated, respectively. Section 3.3 then introduces the option of voter abstention into the model, and evaluates citizens' participation incentives.

3.1 Voting

In the subgame associated with any particular pair (x_A, x_B) of candidate platforms, the voting strategy $\bar{\sigma}^{br} \in \bar{\Sigma}$ is a *best response* to $\bar{\sigma} \in \bar{\Sigma}$ if it maximizes $Eu\,(Y, Z|q, s; \bar{\sigma}_i, \bar{\sigma})$ for every (q, s) pair, and a Bayesian equilibrium $\bar{\sigma}^* \in \bar{\Sigma}$ is its own best response. A citizen's own vote for candidate j only influences her utility if it is *pivotal*, reversing the election outcome by either making or breaking a tie. The probabilities of these events are best described by some additional notation, described below.

First, let $\phi_z\,(j)$ denote the expected fraction of citizens who vote for candidate $j \in \{A, B\}$ in state $z \in \{-1, 1\}$.

$$\begin{aligned} \phi_z\,(j) &= \Pr\left[\bar{\sigma}\,(Q_i, S_i) = j | Z = z\right] \\ &= \sum_{s=1,-1} \int_{q:\bar{\sigma}(q,s)=j} \frac{1}{2}\,(1 + zsq)\, dF\,(q)\,. \end{aligned} \tag{4}$$

By the decomposition property of Poisson random variables (see Myerson, 1998), the numbers N_A and N_B of A and B votes in state z are independent Poisson random variables with means $n\phi_z\,(A)$ and $n\phi_z\,(B)$. Candidate J therefore receives j votes with probability $\psi_z\,(j) = \frac{e^{-n\phi_z(J)}}{a!}\,[n\phi_z\,(J)]^j$ in state z, and the probability $\psi_z\,(a, b)$ of simultaneous vote totals $N_A = a$ and $N_B = b$ is simply the product

$$\psi_z\,(a, b) = \psi_z\,(a)\,\psi_z\,(b)\,. \tag{5}$$

[13] In games of Poisson population uncertainty, the finite set of citizens who actually play the game is a random draw from an infinite set of *potential* citizens, for whom strategies are defined. The distribution of opponent behavior is therefore the same for any two individuals within the game (unlike a game between a finite set of players), implying that the best response for one citizen is a best response for all.

These determine the probabilities $\pi_z^A(m)$ and $\pi_z^B(m)$ with which either candidate wins the election by a margin of exactly $m \in \mathbb{Z}$ votes (where $m < 0$ denotes losing).

$$\begin{array}{l} \pi_z^A(m) = \sum_{k=\min(0,-m)}^{\infty} \psi_z(k+m, k) \\ \pi_z^B(m) = \sum_{k=\min(0,-m)}^{\infty} \psi_z(k, k+m). \end{array} \tag{6}$$

By definition, of course, $\pi_z^A(m) = \pi_z^B(-m)$. With this notation, candidate j wins the election in state z with probability $\Pr_z(W = j) \equiv \Pr(W = j|Z = z)$:

$$\Pr_z(W = j) = \sum_{m=1}^{\infty} \pi_z^j(m) + \frac{1}{2}\pi_z^j(0). \tag{7}$$

By the environmental equivalence property of Poisson games (see Myerson, 1998), an individual citizen from within the game reinterprets N_A and N_B as the numbers of A and B votes cast by her peers; by voting herself, she can add one to either total. Her vote for candidate j is pivotal in (event piv_j) if the candidates exactly tie and j loses the tie-breaking coin toss, or if j wins the coin toss but loses the election by exactly one vote; in state z, the probability of this is

$$\Pr_z(piv_j) = \frac{1}{2}\pi_z^j(0) + \frac{1}{2}\pi_z^j(-1). \tag{8}$$

A vote for one of the two candidates is pivotal in state z with probability

$$\Pr_z(piv) = \Pr_z(piv_A) + \Pr_z(piv_B), \tag{9}$$

and in general with probability

$$\Pr(piv) \equiv \Pr(piv_{-1}) + \Pr(piv_1). \tag{10}$$

In terms of these pivot probabilities and candidate platforms x_A and x_B, the expected benefit $\Delta_{AB}(q, s)$ to a citizen with private information (q, s) from switching her vote from A to B can be written as follows,

$$\begin{aligned} \Delta_{AB}(q,s) &= \sum_{z=-1,1} [u(x_B, z) - u(x_A, z)] \Pr_z(piv_B) \frac{1}{2}(1 + qsz) \\ &\quad - \sum_{z=-1,1} [u(x_A, z) - u(x_B, z)] \Pr_z(piv_A) \frac{1}{2}(1 + qsz) \\ &= \sum_{z=-1,1} 2(x_B - x_A)(z - \bar{x}) \left[\Pr_z(piv_A) + \Pr_z(piv_B)\right] \frac{1}{2}(1 + qsz) \\ &= 2(x_B - x_A) \sum_{z=-1,1} (z - \bar{x})\, 2 \Pr(z, piv, q, s) \\ &= 4(x_B - x_A)[E(z|piv, q, s) - \bar{x}] \Pr(piv, q, s). \end{aligned} \tag{11}$$

It is straightforward to show that $E(z|piv,q,s)=\frac{\sum_{z=-1,1} z\Pr_z(piv)\frac{1}{4}(1+qsz)}{\sum_{z=-1,1}\Pr_z(piv)\frac{1}{4}(1+qsz)}$ is increasing in qs, and exceeds $\bar{x}$ if and only if qs exceeds the threshold T_{AB}^{br}, defined as follows.

$$T_{AB}^{br}=\frac{\bar{x}-E(Z|piv)}{E(Z^2|piv)-\bar{x}E(Z|piv)}. \tag{12}$$

$\Delta_{AB}(q,s)$ is positive, therefore, either if $x_A<x_B$ and $qs\geq T_{AB}^{br}$ or if both inequalities are reversed. In other words, the best response to $(x_A,x_B,\bar{\sigma})$ is a *belief threshold (subgame) strategy*: a citizen votes for the candidate on the right if her private expectation qs of the state exceeds a *belief threshold* $T\in[-1,1]$, and for the candidate on the left if $qs<T$.[14]

Lemma 1 now states formally the existence of a belief threshold strategy that is an equilibrium in the voting subgame. If (and only if) candidate platforms are symmetric around zero, equilibrium voting consists of honestly reporting private information (i.e. voting A if $s=-1$ and B if $s=1$).

Lemma 1 *If candidates are committed to implement platforms $x_A,x_B\in[-1,1]$ then there exists an equilibrium strategy $\bar{\sigma}^*\in\bar{\Sigma}$ in the voting subgame. If $x_A\neq x_B$ then $\bar{\sigma}^*$ is a belief threshold strategy, with $T^*=0$ if and only if $x_A=-x_B$.*

The equilibrium voting behavior described in lemma 1 is informative: citizens with positive signals vote for the candidate on the right, and those with negative signals vote for the candidate on the left. Since positive and negative signals are most likely in states 1 and -1 respectively, this implies that the candidate whose platform is closest to the true state of the world is likely to win the election. This result is essentially Condorcet's (1785) original jury theorem, but is now so familiar and unsurprising in this context that it is stated here as a proposition, rather than a theorem.

Proposition 1 (Jury Theorem) *If candidates are committed to implement platforms $x_A,x_B\in[-1,1]$ and the subgame strategy $\bar{\sigma}_n^*\in\bar{\Sigma}$ maximizes welfare $Eu(Y,Z;n)$ for a particular population size parameter n, then*

1. $\bar{\sigma}_n^*$ is a belief threshold strategy, and constitutes an equilibrium in the voting subgame, and

2. The associated sequence $\bar{Y}_n^*$ of equilibrium policy outcomes approaches $p\lim_{k\to\infty}\left(\bar{Y}_n^*|Z\right)=\begin{cases}\min\{x_A,x_B\} & \textit{if } Z=-1\\ \max\{x_A,x_B\} & \textit{if } Z=1\end{cases}.$

[14] If $qs=T$, of course, then a citizen may take either action.

3.2 Office Motivation

An *office-motivated* candidate j receives utility 1 if he wins the election, and 0 otherwise; his expected utility is therefore simply his probability $\Pr(W = j; x_A, x_B, \sigma)$ of winning the election, which depends on the voting strategy σ as well as both his own and his opponent's campaign platforms. A *perfect Bayesian equilibrium* (σ^*, x_A^*, x_B^*) thus consists of a voting strategy σ^* that induces an equilibrium subgame strategy in response to any pair (x_A, x_B) of candidate platforms, together with candidate platform strategies x_A^* and x_B^* that maximize $\Pr(W = A; x_A, x_B^*, \sigma^*)$ and $\Pr(W = B; x_A^*, x_B, \sigma^*)$, respectively. Theorem 1 states that such an equilibrium exists, and that σ^* is a *belief threshold strategy*, meaning that it induces a belief threshold subgame strategy for every platform pair $(x_A, x_B) \in [-1, 1]^2$, with belief thresholds $T(x_A, x_B)$ designated by a *belief threshold function* $T : [-1, 1]^2 \longrightarrow [-1, 1]$.

Under a belief threshold strategy, each voter supports the candidate whose platform is closest to the policy that seems optimal from her perspective. This implies that the candidate whose platform is closest to the most voters' expectations of the optimal policy will win the election. Given the symmetry of the model, this leads to the familiar result that candidate platforms converge to the center of the policy space (i.e. the zero policy, which is the *ex ante* median of citizens' expectations).

Theorem 1 (Median Voter Theorem) *If candidates are committed and office-motivated then (x_A^*, x_B^*, σ^*) is a perfect Bayesian equilibrium only if $x_A^* = x_B^* = 0$ and σ^* is almost everywhere equivalent to a belief threshold strategy, with threshold function T^* such that $T^*(0, 0) = 0$. Furthermore, such an equilibrium exists.*

Theorem 1 is clearly reminiscent of the canonical median voter theorem introduced by Hotelling (1929), Black (1948), and Downs (1952), and stems from identical logic: whichever candidate is closer to the median voter's ideal point expects to win the election, so competition for votes drives the candidates' platforms toward one another, and toward the median voter's preferred policy. As emphasized in section 1, however, the welfare implications of theorem 1 differ starkly from standard models, in which centrist policy outcomes such as the median voter's ideal point reflect desirable compromises between competing extremes, thereby improving welfare by minimizing the disutility that any citizen must suffer from a policy outcome that is far from her ideal. Here, however, voters' current opinions do not reflect their true preferences; in reality, each wishes to implement policy Z, which by assumption lies at one of the extreme ends of the policy interval. The zero policy is optimal on the basis of prior information alone, but any additional information would induce a preference for a more extreme policy. In essence, then, the result that competition drives candidates to the political center represents a dramatic political failure: even choosing a "dictator" at random from the electorate would provide better policy outcomes than relying on the platforms of office-motivated candidates.

3.3 Policy Motivation

Section 3.2 adopts Downs' (1952) assumption that candidates choose policy platforms as a means to winning office. This section instead adopts Wittman's (1977) assumption that candidates seek office as a means to implementing desirable policies. In the context of the model, the natural assumption is that candidates, themselves citizens, prefer policies as close as possible to Z. Specifically, each seeks to maximize the expectation of (1). At the platform stage, this requires anticipating voter behavior, and choosing a policy $\hat{z}_j \equiv E(Z|W=j)$ that reflects the expectation of Z, conditional on winning the election. When voting follows a belief threshold strategy, the candidate on the left tends to win when the state is -1 and the candidate on the right tends to win in state 1. Thus, condition on winning, candidates' expectations of the state, and therefore their policy platforms, diverge in equilibrium.

Theorem 2 (Policy divergence) *If candidates are committed and policy-motivated then (x_A^*, x_B^*, σ^*) is a perfect Bayesian equilibrium only if candidate platforms are given by $x_j^* = E(Z|W=j)$ for $j = A, B$, with $x_A^* < 0 < x_B^*$, and the voting strategy σ^* is almost everywhere a belief threshold strategy. Furthermore, such an equilibrium exists, with platforms $x_A^* = -x_B^*$ symmetric around zero and $T^*(x_A^*, x_B^*) = 0$.*

Theorem 2 is useful for explaining why candidates in real world elections do not offer identical platforms, as the standard median voter theorem predicts. This is discussed more in section 6.3. The following proposition is an extension of proposition 1, stating that the optimal combination of candidate and voting behavior constitutes an equilibrium. In that case, not only will the candidate with the superior policy platform be elected, but his platform will converge to the optimal policy, as the electorate grows large. This outcome is superior, of course, to the outcome in theorem 1; thus, proposition 2 demonstrates that voters are better off when candidates are policy motivated than when they are office motivated.

Proposition 2 (Jury Theorem 2) *If candidates are committed and policy-motivated and the strategy combination $\left[\left(x_j^*\right)_{j=A,B}, \sigma^*\right]_n \in [-1,1]^2 \times \Sigma$ maximizes welfare $Eu(Y, Z; n)$ for a particular population size parameter n, then*

1. *$\left[\left(x_j^*\right)_{j=A,B}, \sigma^*\right]_n$ constitutes a perfect Bayesian equilibrium, and*
2. *The associated sequence Y_n^* of equilibrium policy outcomes approaches $p\lim_{n\to\infty}(Y_n^*|Z) = Z$.*

Abstention

The analysis above assumes that every citizen must vote. In most real-world voting environments, however, voters are allowed to abstain; indeed, in most democracies, abstention rates tend to be fairly high. In this section, abstention is allowed, and denoted by a vote for candidate 0, so that the set of actions expands to $\{A, B, 0\}$. Let Σ' and $\bar{\Sigma}'$ denote the set of strategies and the set of induced subgame strategies, respectively, in the new environment. With this modification, lemma 2 repeats the voting subgame analysis of section 3.1 for a given pair of policy platforms. As before, citizens who strongly believe the state to be high or low have the strongest preferences to vote B or A, respectively, but now a *belief threshold (subgame) strategy* must be redefined using two thresholds instead of one, to allow for the possibility that some citizens abstain altogether from voting. Specifically, if $x_A < x_B$ then there are two belief thresholds, $T_1 \leq T_2$, such that citizens vote A if $qs < T_1$, abstain if $T_1 < qs < T_2$, and vote B if $qs > T_2$.[15]

Since voting is costless, and since each citizen's private signal induces a strict preference ordering over the two policy outcomes, it may seem that all citizens should vote, even given the option to abstain. Lemma 1 states, however, that $T_1 < T_2$ in any voting equilibrium, implying positive abstention. The logic behind this result is Feddersen and Pesendorfer's (1996) swing voter's curse: since votes reflect private opinions which are correlated with the truth, the candidate with the truly superior policy is more likely to win the election by one vote than to lose by one vote. This makes a vote for the inferior candidate more likely to be pivotal than a vote for the superior candidate, so a citizen who is privately indifferent between voting for the two candidates—or, by continuity, almost indifferent—strictly prefers to abstain. The last part of Lemma 2 points out that if policy outcomes are symmetric around the zero policy then equilibrium voting behavior exhibits the same symmetry. In that case, as in McMurray (2010), whether a citizen votes or not depends solely on her information quality Q_i.

Lemma 2 (Swing voter's curse) *If candidates are committed and abstention is allowed then, for any pair (x_A, x_B) of platform policies, there exists a subgame voting strategy $\bar{\sigma}^* \in \bar{\Sigma}'$ that constitutes an equilibrium in the voting subgame. If $x_A \neq x_B$ then $\bar{\sigma}^*$ is a belief threshold strategy, with belief thresholds $T_1^* < T_2^*$. Also, $T_1^* = -T_2^*$ if and only if $x_A = -x_B$.*

Like lemma 1, lemma 2 characterizes equilibrium responses to exogenous policy platforms. Proposition 3 now treats the case in which campaign platforms are chosen by

[15] Again, if qs is right at a threshold then a citizen may take either action. Also, if $x_A > x_B$ then a belief threshold strategy is defined as in the text, but with inequalities reversed.

policy-motivated (i.e. citizen) candidates. For any pair of distinct candidate platforms, of course, lemma 1 gives the equilibrium response from voters as a belief threshold subgame strategy, with a positive fraction of the electorate abstaining. Combining all such subgames, these constitute a *belief threshold strategy*, redefined for this section to allow abstention, with a bivariate *belief threshold function* $T : [-1,1]^2 \longrightarrow [-1,1]^2$ that designates belief thresholds $T_1(x_A, x_B) \leq T_2(x_A, x_B)$. As in theorem 2, candidates infer different information from voters, conditional on winning, so equilibrium campaign platforms diverge. In particular, the equilibrium may be symmetric, so that voting is sincere.

Proposition 3 *If candidates are committed and policy-motivated then (x_A^*, x_B^*, σ^*) is a perfect Bayesian equilibrium only if σ^* is almost everywhere equivalent to a belief threshold strategy, with $T_1^* < T_2^*$, and candidate platforms are given by $x_j^* = E(Z|W=j)$ for $j = A, B$, with $x_A^* \neq x_B^*$. Furthermore, such an equilibrium exists, with platforms $x_A^* = -x_B^*$ and belief thresholds $T_1^*(x_A^*, x_B^*) = -T_2^*(x_A^*, x_B^*)$ symmetric around zero.*

The result that citizens receive informative signals but abstain in equilibrium implies that the election mechanism fails to aggregate all available information. This may seem to endorse mandatory voting rules, which are prevalent in many committee settings, as well as in several democracies throughout the world. Proposition 4 states, however, that the optimal combination of voter and candidate strategies constitutes an equilibrium; in light of Proposition 3, this implies that optimal voting necessarily involves some abstention. As McMurray (2010) explains, this is because the optimal use of information would weight signals according to their underlying quality, but high- and low-quality votes cannot be distinguished. Abstention by those with relatively less information provides a crude mechanism, however, of transferring weight to votes reflecting higher quality signals. Like proposition 2, proposition 4 also states a strengthened version of the Condorcet jury theorem, which is that the policy outcome converges to the optimal policy as the electorate grows large.

Proposition 4 (Jury Theorem 3) *If candidates are committed and policy-motivated and the strategy combination $\left[\left(x_j^*\right)_{j=A,B}, \sigma^*\right]_n \in [-1,1]^2 \times \Sigma'$ maximizes welfare $Eu(Y, Z; n)$ for a particular population size parameter n, then*

Theorem 3 *1. $\left[\left(x_j^*\right)_{j=A,B}, \sigma^*\right]_n$ constitutes a perfect Bayesian equilibrium, and*

2. The associated sequence Y_n^* of equilibrium policy outcomes approaches $p\lim_{n\to\infty}(Y_n^*|Z) = Z$.

3.4 Mixed Motivations

In analyzing the platform decisions of committed candidates, section 3.2 assumes that candidates are willing to adopt any policy, and desire only to win office, while section 3.3

assumes that candidates seek office purely out of a desire to implement good policies. This section considers the case of *mixed motivation*, meaning that a candidate's expected utility

$$EU_j = \gamma P(W = j) + (1 - \gamma) Eu(Y, Z) \tag{13}$$

places weight $\gamma \in (0, 1)$ on gaining office and weight $(1 - \gamma)$ on achieving desirable policy outcomes. As in section 3.3, a candidate's preferred policy (conditional on winning) is $E(Z|W = j)$, but as in section 3.2, moving toward the center improves his odds of winning the election.

As corollary 1 now states, if office motivation is sufficiently strong then candidate platforms converge in equilibrium, as in theorem 1. Otherwise, platforms do not converge completely, but some policy concessions are nevertheless inevitable, relative to the behavior characterized in proposition 3 (and theorem 2). As γ increases, candidates become more willing to make such policy concessions, and equilibrium platforms move toward the center.

Corollary 1 *If candidates are committed and have mixed motivation then there exists a γ^* such that (x_A^*, x_B^*, σ^*) is a perfect Bayesian equilibrium only if σ^* is almost everywhere equivalent to a belief threshold strategy and either $\gamma \geq \gamma^*$ and $x_A^* = x_B^* = 0$ or $\gamma < \gamma^*$ and $E(Z|W = A) < x_A^* < x_B^* < E(Z|W = B)$. In the latter case, x_A^* increases and x_B^* decreases (and $Eu(Y, Z)$ decreases) as γ increases. Furthermore, an equilibrium exists, with platforms $x_A^* = -x_B^*$ and belief thresholds $T_1^*(x_A^*, x_B^*) = -T_2^*(x_A^*, x_B^*)$ symmetric around zero.*

Proposition 4 states that the platform pair that is optimal from voters' perspective satisfies $x_j = E(Z|W = j)$; when $\gamma = 0$, candidates offer this pair in equilibrium. Corollary 1 points out, however, that platforms move away from this optimum as office motivation grows stronger, implying that voter welfare declines.

4 Responsive Candidates

In theorem 2 and proposition 3, campaign platforms diverge because policy-motivated candidates learn different information upon winning the election. Specifically, each learns that a majority of voters prefers his own platform over his opponent's. A winning candidate may learn additional information, however, from his margin of victory: if he wins by a landslide, for example, then he can be quite certain that his own platform was actually superior to his opponent's; if he wins only narrowly, he will be less confident. Given the assumption of risk aversion, this additional information may give the winning candidate an incentive to implement some policy other than that which he had previously committed to.

As Alesina (1988) points out, enforcing candidates' commitments to campaign platforms is difficult. Accordingly, this section assumes that candidates may implement any policy in the policy space. In particular, this allows them to be *responsive* to vote totals, which are unobserved at the platform selection stage. As in section 3.3, candidates are policy motivated, just like ordinary citizens.[16] In addition to platform policies and a voting strategy, therefore, a perfect Bayesian equilibrium $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ now includes a pair $(y_A^*, y_B^*) \in \Upsilon^2$ of policy functions that maximize the expectation of (1), taking opponent strategies as given. In section 4.1, as in sections 3.1 through 3.3, voting is mandatory. Like section 3.3, section 4.2 allows abstention. Section 4.3 then considers the possibility of multiple candidates.

4.1 Signaling

In the model analyzed in section 3, candidates announce platform policies and then citizens vote. The addition of responsive candidates adds a third subgame, in which the winning candidate chooses which policy to implement. As discussed in section 2, expected utility the expectation of (1) is maximized at the expectation of Z, conditional on any information. In particular, the optimal policy $\hat{z}_{a,b} \equiv E\left(Z | N_A = a, N_B = b\right)$ for a candidate who observes vote totals N_A and N_B is given by

$$\begin{aligned} \hat{z}_{a,b} &= \frac{\sum_{z=-1,1} z\psi_z\left(a,b\right)\Pr\left(Z=z\right)}{\sum_{z=-1,1}\psi_z\left(a,b\right)\Pr\left(Z=z\right)} \\ &= \frac{\psi_z\left(a,b\right)-\psi_{-z}\left(a,b\right)}{\psi_z\left(a,b\right)+\psi_{-z}\left(a,b\right)}. \end{aligned} \tag{14}$$

Since vote totals shape beliefs and influence policy, each citizen must consider the impact of her own behavior on the winning candidate's expectations. Specifically, the expected benefit $\Delta_{AB}\left(q,s\right)$ to a citizen with private information (q,s) of changing her vote from A to B becomes

$$\begin{aligned} \Delta_{AB}\left(q,s\right) &= E_{Z,N_A,N_B}\left[u\left(\hat{z}_{N_A,N_B+1},Z\right)-u\left(\hat{z}_{N_A+1,N_B},Z\right)|q,s\right] \\ &= E_{Z,N_A,N_B}\left[-\left(\hat{z}_{N_A,N_B+1}-Z\right)^2+\left(\hat{z}_{N_A+1,N_B}-Z\right)^2|q,s\right] \\ &= E_{Z,N_A,N_B}\left[\left(\hat{z}_{N_A,N_B+1}-\hat{z}_{N_A+1,N_B}\right)\left(Z-\frac{\hat{z}_{N_A+1,N_B}+\hat{z}_{N_A,N_B+1}}{2}\right)|q,s\right] \end{aligned} \tag{15}$$

[16] Downs (1957) vigorously supports the assumption of office-motivated candidates, but when platform commitments are non-binding, candidates have no way to influence their election prospects. Furthermore, once a candidate gets elected, the assumption of office motivation provides no guidance as to how policy should be chosen; presumably, even if policy preferences are of second order, they are all that is relevant after the election. Since a policy-motivated candidate's behavior is optimal from voters' perspective, an office-motivated should also do well to adopt the same strategy.

instead of (11), reflecting the choice between policies $\hat{z}_{N_A+1,N_B}$ and $\hat{z}_{N_A,N_B+1}$, which depend on the realized vote totals N_A and N_B among her peers.

Since votes are cast anonymously, a citizen's vote is interpreted according to the voting strategy used by the rest of the electorate. For example, define a subgame strategy $\bar{\sigma}$ as *informative* if $\frac{\phi_{-1}(B)}{\phi_{-1}(A)} < \frac{\phi_1(B)}{\phi_1(A)}$, meaning that B votes become more common, relative to A votes, as the state increases from -1 to 1, and define a voting strategy σ as *informative* if it induces informative subgame strategies for every pair of candidate platforms. An example of such a strategy is a belief threshold strategy. Lemma 3 now states that, when voting is informative, A and B votes lower and raise the winning candidate's expectation of Z, respectively.

Lemma 3 *If $\bar{\sigma}$ is an informative subgame strategy then $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ for all $a, b \in \mathbb{Z}_+$.*

Whether a citizen wishes to lower or raise the policy outcome depends on her private beliefs regarding the state of the world. Once again, if qs is low then a citizen prefers to push policy to the left; if qs is high she prefers to push policy to the right. In other words, she follows a belief threshold strategy, just as in section 3. Theorem 4 states this formally, and also notes that platforms, voting, and policy strategies may be symmetric around zero in equilibrium.

In addition to the informative equilibrium highlighted in theorem 4, there are other equilibria which are uninformative. For example, if citizens vote without regard to private information (e.g. vote randomly, or all vote A) then the winning candidate can infer nothing from realized vote totals, and so simply implements his *ex ante* expectation $E(Z) = 0$, regardless of vote totals; in that case, a citizen has no incentive to deviate, since her vote will be ignored. Uninformative equilibria such as this would not be robust, however, if for example candidate platforms were binding with some positive probability. In the campaign stage of the game, this would also make it optimal for candidates to promise the policies identified in theorem 2 as platform commitments. Accordingly, theorem 4 focuses on these platform strategies, along with informative voting strategies, even though non-binding platform commitments actually play no role in equilibrium.[17]

Theorem 4 (Signaling Equilibrium) *If candidates are responsive and policy motivated and σ^* is an informative voting strategy then $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ is a perfect Bayesian equilibrium only if*

[17] Another strategy that constitutes an equilibrium is to vote for the candidate who seems *worst*, instead of best. This actually conveys the same information as the equilibrium described in theorem 4, but like other uninformative equilibria is not robust to a positive probability of binding platform commitments.

1. $y_j^*(a,b) = \hat{z}_{a,b}$ *for all* $a, b \in \mathbb{Z}_+$ *and for* $j = A, B$*, and* $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ *for all* $a, b \in \mathbb{Z}_+$*.*

2. σ^* *is a belief threshold strategy.*

Furthermore, such an equilibrium exists, which further satisfies the following:

3. Platforms are given by $x_j^* = E(Z|W = j)$ *for* $j = A, B$*.*

4. Platforms $x_A^* = -x_B^*$ *and policy functions* $y_A^*(a,b) = -y_B^*(b,a)$ *are symmetric around zero, and* $T^*(x_A, x_B) = 0$ *for all* (x_A, x_B) *pairs.*

In sections 3.1 through 3.3, as in standard voting models, an individual vote has influence only in the extremely unlikely event that it is "pivotal", either making or breaking a tie. In the equilibrium of theorem 4, this is no longer the case; instead, *every* vote is pivotal, in the sense that every vote influences the ultimate policy outcome, by pushing the policy-maker's expectations one way or the other. In this setting, then, the popular mantra that "every vote counts" in public elections can be taken quite literally. Theorem 4 also validates the popular notion of electoral "mandates". That is, candidates who win by large margins feel emboldened to implement extreme policy changes, relative to those who win only narrowly. This arises in the model because candidate platforms $\hat{z}_j$ are weighted averages of the policy outcomes $\hat{z}_{a,b}$ associated with particular vote totals, so policy outcomes become more extreme than campaign platforms whenever a candidate wins by a larger margin than expected, and become less extreme when the margin is smaller than expected.

Proposition 5 next states another extension of propositions 1, 2, and 4. Once again, the optimal combination of voter and candidate behavior constitutes an equilibrium, and the policy outcome converges to the optimal policy as the electorate grows large. In particular, the optimality of candidates' policy responses implies that voters are better off when candidates are responsive than when their platform commitments are binding.

Proposition 5 (Jury Theorem 4) *If candidates are responsive and policy motivated, abstention is allowed, and the strategy combination* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]_n \in [-1,1]^2 \times \Upsilon^2 \times \Sigma$ *maximizes welfare* $Eu(Y, Z; n)$ *for a particular population size parameter* n*, then*

Theorem 5 *1.* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]_n$ *constitutes a perfect Bayesian equilibrium, and*

2. The associated sequence Y_n^* *of equilibrium policy outcomes approaches* $p\lim_{n\to\infty}(Y_n^*|Z) = Z$*.*

4.2 Abstention

Section 4.1 assumes that all citizens vote. In this section, citizens are instead allowed to abstain, so the relevant strategy space is Σ', as defined in section 3.3. A belief threshold strategy is also defined as in section 3.3. A strategy in the voting subgame is defined as *informative* if $\frac{\phi_1(A)}{\phi_{-1}(A)} < \frac{\phi_1(0)}{\phi_{-1}(0)} < \frac{\phi_1(B)}{\phi_{-1}(B)}$, and a voting strategy in the broader game is *informative* if it induces informative subgame strategies for every platform pair.

Recall from section 3.3 that, though voting is costless, and though intuition suggests that aggregating more signals should improve the election outcome, the least informed citizens nevertheless abstain in equilibrium. The motivation for abstention in that version of the model is Feddersen and Pesendorfer's (1996) swing voter's curse: a vote for the superior candidate is less likely to be pivotal than a vote for his opponent, so by casting a vote, an uninformed citizen is more likely to make the election outcome worse than to make it better, and strictly prefers to abstain. In this section, however, the pivotal voting calculus is no longer relevant: theorem 4 implies that *every* vote is pivotal, in the sense that the ultimate policy outcome would be different if any individual vote were eliminated. Furthermore, a vote that changes the identity of the election winner no longer matters, because in this simple version of the model, candidates interpret vote totals identically in equilibrium, and so adopt identical policy functions.

While the logic of the swing voter's curse no longer applies, theorem 6 states that poorly informed citizens nevertheless abstain in equilibrium (i.e. $T_1^* < T_2^*$). The rationale for this behavior is that, without her help, a citizen's peers are likely to lead the winning candidate toward the optimal policy decision; voting for either candidate would therefore only drag the policy decision away from this outcome, thereby creating a *signaling voter's curse*. An alternative intuition is that the winning candidate responds to each vote as if it were of average quality, thereby over-reacting to some citizens' vote. A moderately informed citizen prefers this over-reaction to the under-reaction that will occur if she abstains, but a completely uninformed citizen (or, by continuity, one with sufficiently low expertise) prefers to abstain.

Theorem 6 (Signaling voter's curse) *If candidates are responsive and policy motivated and voter abstention is allowed and* σ^* *is an informative voting strategy then* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ *is a perfect Bayesian equilibrium only if*

1. $y_j^*(a,b) = \hat{z}_{a,b}$ *for all* $a, b \in \mathbb{Z}_+$ *and for* $j = A, B$, *and* $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ *for all* $a, b \in \mathbb{Z}_+$.

2. σ^* *is a belief threshold strategy, with* $T_1^* < T_2^*$.

Furthermore, such an equilibrium exists, which further satisfies the following:

3. Platforms are given by $x_j^* = E(Z|W = j)$ *for* $j = A, B$.

4. Platforms $x_A^* = -x_B^*$, *policy functions* $y_A^*(a,b) = -y_B^*(b,a)$, *and belief thresholds* $T_1^* = -T_2^*$ *are all symmetric around zero.*

Proposition 6 now extends Condorcet's jury theorem once again. Like proposition 4, the implication is that allowing abstention improves the election outcome. While the mechanism of signaling differs from the mechanism of comparing pivot probabilities, the underlying logic of these two propositions is the same: if the winning candidate is responding optimally to the information of her peers, an uninformed citizen prefers not to intervene.

Proposition 6 (Jury Theorem 5) *If candidates are responsive and policy motivated, abstention is allowed, and the strategy combination* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]_n \in [-1,1]^2 \times \Upsilon^2 \times \Sigma'$ *maximizes welfare* $Eu(Y, Z; n)$ *for a particular population size parameter* n*, then*

Theorem 7 *1.* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]_n$ *constitutes a perfect Bayesian equilibrium, and*

2. The associated sequence Y_n^* *of equilibrium policy outcomes approaches* $p\lim_{n\to\infty}(Y_n^*|Z) = Z$.

One way to understand proposition 6 is that, as McMurray (2010) points out, an optimal election mechanism would place greater weight on the votes of citizens with high-quality information than on those of poorly informed citizens; allowing abstention is a crude way of accomplishing this. With responsive candidates, an alternative intuition comes from viewing voters and candidates as senders and receivers in a "cheap talk" game (a la Crawford and Sobel, 1982). Within that framework, allowing abstention amounts to expanding the size of the message space from two messages to three.

4.3 Multiple Candidates

This section analyzes an election with four candidates instead of two.[18] As in section 4.2, abstention is also allowed.[19] Thus, denote the set of voting strategies by $\Sigma'' = \left\{\sigma : [0,1] \times \{-1,1\} \times [0,1]^2 \to \{A, B, C, D, 0\}\right\}$ and the set of induced subgame strategies by $\bar{\Sigma}''$. In this setting, an *informative strategy* is redefined to mean that $\frac{\phi_{-1}(j')}{\phi_{-1}(j)} < \frac{\phi_1(j')}{\phi_1(j)}$ whenever j proceeds j' in the order $\{A, B, 0, C, D\}$. That is, votes for candidate j' become more common, relative to votes for candidate j, when the state increases from -1 to 1. The special case of a *belief threshold (subgame) strategy* $\bar{\sigma} \in \bar{\Sigma}''$ is redefined with four *belief*

[18] Results of this section can be obtained for any number of candidates, but the symmetry inherent with four candidates simplifies notation.

[19] The results of this section also hold when voting is mandatory.

thresholds $T_1 \le T_2 \le T_3 \le T_4$, as follows,

$$\bar{\sigma}(q,s) = \begin{cases} A \text{ if } qs \in (-1, T_1) \\ B \text{ if } qs \in (T_1, T_2) \\ 0 \text{ if } qs \in (T_2, T_3) \\ C \text{ if } qs \in (T_3, T_4) \\ D \text{ if } qs \in (T_4, 1) \end{cases},$$

and a *belief threshold strategy* $\sigma \in \Sigma''$ is redefined to induce belief threshold subgame strategies for every set $(x_j)_{j=A,B,C,D}$ of campaign platforms, according to the *belief threshold function* $T : [-1,1]^4 \longrightarrow [-1,1]^4$. Under such a strategy, citizens with the strongest private opinions vote for candidates A or D, citizens with moderate opinions vote for candidates B or C, and citizens with only weak opinions abstain.

With vote totals for four candidates outcome probabilities (5) naturally generalizes to

$$\psi_z(a,b,c,d) = \psi_z(a)\,\psi_z(b)\,\psi_z(c)\,\psi_z(d)\,. \tag{16}$$

Just as in section 4.1, the optimal policy choice for a winning candidate is his expectation $\hat{z}_{a,b,c,d} \equiv E(Z|N_A = a, N_B = b, N_C = c, N_D = d)$ of the state variable. Like lemma 3, lemma 4 describes the impact of informative voting on these beliefs: specifically, A and B votes push $\hat{z}_{a,b,c,d}$ to the left by large and small amounts, respectively, while C and D votes push $\hat{z}_{a,b,c,d}$ to the right, by small and large amounts, respectively.

Lemma 4 *If $\bar{\sigma}$ is an informative subgame strategy then $\hat{z}_{a+1,b,c,d} < \hat{z}_{a,b+1,c,d} < \hat{z}_{a,b,c,d} < \hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1}$ for all $a,b,c,d \in \mathbb{Z}_+$.*

Theorem 8 now states the existence of a perfect Bayesian equilibrium, characterized by belief threshold voting. As prescribed by lemma 3, the winning candidate implements his expectation of the state, conditional on vote totals; as in lemma 3, the effect of a single vote is to push the policy outcome in one direction or another. Because more extreme citizen types vote for candidates A and D than B and C, votes for these two candidates have a greater impact on the winning candidate's beliefs. Thus, voting for an extreme candidate pushes policy by more than voting for a moderate candidate.

Theorem 8 (Multiple candidates) *If candidates A, B, C, and D are responsive and policy motivated, voter abstention is allowed, and σ^* is an informative voting strategy then $\left[(x_j^*, y_j^*)_{j=A,B,C,D}, \sigma^*\right]$ is a perfect Bayesian equilibrium only if*

1. $y_j^*(a,b,c,d) = \hat{z}_{a,b,c,d}$ for all $a,b,c,d \in \mathbb{Z}_+$ and for $j = A,B,C,D$, and $\hat{z}_{a+1,b,c,d} < \hat{z}_{a,b+1,c,d} < \hat{z}_{a,b,c,d} < \hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1}$.

2. σ^* is a belief threshold strategy, with $T_1^* < T_2^* < T_3^* < T_4^*$.

Furthermore, such an equilibrium exists, which further satisfies the following:

3. Platforms are given by $x_j^* = E(Z|W=j)$ for $j = A, B, C, D$.

4. Platforms $x_A^* = -x_D^*$ and $x_B^* = -x_C^*$, policy functions $y_j^*(a,b,c,d) = -y_{j'}^*(d,c,b,a)$, and belief thresholds $T_1^* = -T_4^*$ and $T_2^* = -T_3^*$ are all symmetric around zero.

One noteworthy comparative static result from this section is that the intensity of a citizen's political preference is positively related to her information quality. That is, candidates with extreme positions are supported by voters with the strongest opinions. Relatively less informed citizens avoid these candidates, even though they recognize that the ideal policy lies (by assumption) at one of the extreme ends of the policy space. In essence, the same "signaling voter's curse" that in section 4.2 caused citizens with the poorest information to abstain altogether from voting now leads moderately informed citizens to vote for a moderate candidate, rather than an extremist.

Proposition 7 next extends Condorcet's jury theorem one more time. As before, the theorem states that the optimal combination of voter and candidate behavior constitutes an equilibrium, and that the policy outcome converges to the optimal policy as the electorate grows large. Optimality implies, in particular, that voters are better off with the additional candidates C and D than they are with candidates A and B alone.

Proposition 7 (Jury Theorem 6) *If candidates A, B, C, and D are responsive and policy motivated, abstention is allowed, and the strategy combination $\left[\left(x_j^*, y_j^*\right)_{j=A,B,C,D}, \sigma^*\right]_n \in [-1,1]^4 \times \Upsilon^4 \times \Sigma''$ maximizes welfare $Eu(Y,Z;n)$ for a particular population size parameter n, then*

Theorem 9 *1. $\left[\left(x_j^*, y_j^*\right)_{j=A,B,C,D}, \sigma^*\right]_n$ constitutes a perfect Bayesian equilibrium, and*

2. The associated sequence Y_n^* of equilibrium policy outcomes approaches $p\lim_{n\to\infty}(Y_n^*|Z) = Z$.

5 Extensions

5.1 Convex State Space

The analysis above has maintained the assumption that the optimal policy $Z \in \{-1, 1\}$ lies at one of the extremes of the policy space. In this section, $Z \in [-1,1]$ may also be an interior policy. In most instances, this is probably much more realistic. For example, funds budgeted for national defense can be allocated toward improving naval or air forces, or both.

Presumably, the optimal allocation includes some improvements to both. Viewing this decision as an investment evokes the quite general principle that non-diversified investment strategies are rarely optimal. Another setting in which a continuous state variable is more realistic is a traditional *valence* model, in which one candidate is superior to his opponent in dimensions not directly related to policy (e.g. leadership ability, negotiation skills, etc.): in that setting, $Z \in \{-1, 1\}$ reflects the assumption that either candidate may in truth be superior, but $Z \in [-1, 1]$ further allows the quality differential to be either large or small.

As before, assume that prior beliefs are diffuse, so that all states of the world are equally likely. In other words, the state variable Z is distributed uniformly on $[-1, 1]$. Assume that a citizen's private signal S_i is similarly uniform on $[-1, 1]$, and that conditional densities are given by

$$g(s|z, q) = g(z|s, q) = \frac{1}{2}(1 + zsq) \tag{17}$$

over the domain, conditional on signal quality $Q_i = q$, which again follows an arbitrary distribution F on $[0, 1]$. This formulation can be interpreted almost identically to the model described in section 2, except that now the correlation between S_i and Z is only $corr(S_i, Z|Q_i) = \frac{1}{3}Q_i$, implying that even the most expert citizens have very imperfect information, and that ideology is now given by $E(Z|Q_i, S_i) = \frac{1}{3}Q_iS_i$.

Conveniently, the density function $g(z) = \left\{ \begin{array}{ll} \frac{1}{2} & \text{if } z \in [-1, 1] \\ 0 & \text{otherwise} \end{array} \right.$ of the continuous uniform distribution on $[-1, 1]$ closely resembles the mass function $\Pr(z) = \left\{ \begin{array}{ll} \frac{1}{2} & \text{if } z \in \{-1, 1\} \\ 0 & \text{otherwise} \end{array} \right\}$ of the discrete uniform distribution analyzed in section 2, and the conditional density (17) closely resembles the conditional probability given in (2). Accordingly, the entire analysis of sections 3 and 4 can be repeated almost verbatim, with only minor adjustments to accommodate the newly specified model, such as replacing summations with integrals.

Theorem 10 *In the model described in this section, all of the formal results stated in sections 3 and 4 remain valid.*

Examples

This section presents a series of simple numerical examples, to illustrate the equilibrium characterization above. Each assumes responsive, policy-motivated candidates, and a convex states space, as in section 5.1, so that the optimal policy Z follows a continuous distribution. Unfortunately, constructing large examples is quite onerous, because the number of possible policy outcomes increases quickly with the number of voters. Nevertheless, all of the above intuition can be illustrated even for tiny electorates. With such small electorates, it is convenient to abandon the assumption of population uncertainty, and assume instead that the number N of voters is fixed and known.

Example 1 begins with the case of $N = 2$, where voter abstention is allowed, and for simplicity, the distribution of expertise is assumed to be uniform. In that case, the candidates can tie, or either can win by one or two votes. In equilibrium, candidates adopt platform positions at ± 0.165, and then move either to 0, ± 0.15, or ± 0.29, respectively, depending on the size of the plurality. Even with such a small electorate, then, electoral mandates are apparent: a candidate who wins only narrowly moderates his policy position, relative to his platform, while a candidate who receives a large plurality becomes more extreme. Example 1 also starkly illustrates the signaling voter's curse: in equilibrium, a citizen is more likely to abstain than to vote.

Example 1 *Assume that* $N = 2$*,* Q_i *is uniformly distributed on* $[0,1]$*, and a ballot consists of* $\{A, 0, B\}$*. Then* $\left((x_j, y_j)_{j=A,B}, \sigma^*\right)$ *is a perfect Bayesian equilibrium if platform policies are given by* $x_B = -x_A = 0.165$*;* σ^* *is a belief threshold voting strategy, with belief thresholds* $T_2^* = -T_1^* = 0.229$ *constant across platform pairs (implying* 43.3% *turnout); and policy functions are given by* $y_j(0,2) = -y_j(2,0) = 0.29$*,* $y_j(0,1) = -y_j(1,0) = 0.15$*, and* $y_j(0,0) = y_j(1,1) = 0$*, for* $j = A, B$*. Equilibrium welfare is* $Eu(Y, Z) = -0.314$*.*

One intuition [described in section 4.2] for the signaling voter's curse is that a citizen defers to the judgment of those with better information. As example 2 illustrates, however, abstention can also arise even in an "electorate" comprised of only a single voter. In fact, in this example, the voter is only slightly more likely to vote than in example 1 (43.4% instead of 43.3%). This illustrates the alternative intuition behind theorem 6[, explained in section 4.2,] which is that a voter knows the quality of her signal exactly, but the winning candidate knows only the distribution of possible expertise, and so interprets the vote as if it were of average quality. When her realized expertise is below average, the citizen knows that the candidate will over-react to her information. A sufficiently informed citizen prefers this over-reaction to the under-reaction that occurs if she abstains, but a sufficiently uninformed citizen prefers to abstain.

Example 2 *Assume that* $N = 1$*,* Q_i *is uniformly distributed on* $[0,1]$*, and a ballot consists of* $\{A, 0, B\}$*. Then* $\left((x_j, y_j)_{j=A,B}, \sigma^*\right)$ *is a perfect Bayesian equilibrium if platform policies are given by* $x_B = -x_A = 0.09$*;* σ^* *is a belief threshold voting strategy, with belief thresholds* $T_2^* = -T_1^* = 0.228$ *constant across platform pairs (implying* 43.4% *turnout); and policy functions are given by* $y_j(0,1) = -y_j(1,0) = 0.15$ *and* $y_j(0,0) = 0$*, for* $j = A, B$*. Equilibrium welfare is* $Eu(Y, Z) = -0.323$*.*

To illustrate the equilibrium effect of abstention, example 3 repeats example 2, but with mandatory voting. In that case, the average quality of a vote declines, because citizens with low-precision signals no longer abstain. Thus the policy outcomes are not as extreme as before. As proposition 6 predicts, however, welfare declines, despite the fact that a larger number of signals are being reported.

Example 3 *Assume that $N = 1$, Q_i is uniformly distributed on $[0,1]$, and a ballot consists of $\{A, B\}$. Then $\left((x_j, y_j)_{j=A,B}, \sigma^*\right)$ is a perfect Bayesian equilibrium if σ^* is a belief threshold voting strategy, with belief threshold $T^* = 0$ constant across platform pairs; and platforms and policy functions are given by $x_B = -x_A = y_{0,1} = -y_{1,0} = 0.08$ for $j = A, B$. Equilibrium welfare is $Eu(Y,Z) = -0.326$. Equilibrium welfare is $Eu(Y,Z) = -0.326$.*

The next example expands the number of candidates from two to four. Contrary to Duverger's law, but as theorem 8 predicts, each candidate wins the election with positive probability. Also, as proposition 7 predicts, the presence of additional candidates improves welfare.

Example 4 *Assume that $N = 1$, Q_i is uniformly distributed on $[0,1]$, and a ballot consists of $\{A, B, C, D\}$. Then $\left((x_j, y_j)_{j=A,B}, \sigma^*\right)$ is a perfect Bayesian equilibrium if σ^* is a belief threshold voting strategy, with belief thresholds $T_3^* = -T_1^* = 0.33$ and $T_2^* = 0$ constant across platform pairs (implying expected vote shares $.15$, $.35$, $.35$, and $.15$); and platforms and policy functions are given by $x_D = -x_A = y_{0,0,0,1} = -y_{1,0,0,0} = 0.18$ and $x_C = -x_B = y_{0,0,1,0} = -y_{0,1,0,0} = 0.04$. Equilibrium welfare is $Eu(Y,Z) = -0.322$.*

Example 5 replaces the uniform distribution of expertise assumed in the examples above with one that is skewed. This reflects the empirical reality that many voters know very little about politics and, as McMurray (2010) argues, produces more plausible margins of victory. Not surprisingly, this general decrease of expertise lowers equilibrium welfare.

Example 5 *Assume that $N = 1$, Q_i has density $f(q) = 2 - 2q$ on $[0,1]$, and a ballot consists of $\{A, B, C, D\}$. Then $\left((x_j, y_j)_{j=A,B}, \sigma^*\right)$ is a perfect Bayesian equilibrium if σ^* is a belief threshold voting strategy, with belief thresholds $T_3^* = -T_1^* = .25$ and $T_2^* = 0$ constant across platform pairs (implying expected vote shares $.12$, $.38$, $.38$, and $.12$); and platforms and policy functions are given by $x_D = -x_A = y_{0,0,0,1} = -y_{1,0,0,0} = 0.14$ and $x_C = -x_B = y_{0,0,1,0} = -y_{0,1,0,0} = 0.03$. Equilibrium welfare is $Eu(Y,Z) = -0.328$.*

It is interesting to note that the expected vote shares of the more moderate candidates B and C are over three times those of the extreme candidates, A and D. This occurs for purely informational reasons: citizens with moderate opinions simply outnumber those who hold more extreme beliefs. This pattern is amplified by the skewed distribution of expertise, but does also arise in example 4, where expertise is distributed uniformly. Restricting movements away from candidates' campaign platforms, of course, would reinforce this tendency, by inducing preferences over the identity of the election winner, and thus leading voters to coordinate behind the most electable candidates B and C, rather than using their votes to signal their extreme opinions.

5.2 Multiple Dimensions

In the model(s) described above, the policy space $[-1, 1]$ has only a single dimension. In the real world, of course, policies are very multi-faceted and complex. A natural question, therefore, is what happens when the policy space has multiple dimensions. A complete and rigorous answer to this question would require extensive additional notation, and is thus left for future investigation. A partial answer, however, is immediate from the model analyzed above. That is, it is an equilibrium in the multidimensional model for candidates to simply compete along a single dimension.

To see this, consider a policy space consisting of the square $[-1, 1]^2$, with optimal policy $Z = (Z_1, Z_2)$, and suppose that citizens now each receive a two-dimensional signal $S_i = (S_{1i}, S_{2i})$, where S_{1i} is correlated with Z_1 and S_{2i} is correlated with Z_2. If citizens vote on the basis of S_{1i} alone, ignoring S_{2i}, then candidates can infer no information about Z_2 from voters, and can only choose policy in that dimension on the basis of prior beliefs. Given this behavior by candidates, voters should not expect their behavior to influence policy in the second dimension, and so can vote according to S_{1i} alone. Having reduced the multidimensional model to a single dimension, of course, the equilibrium characterization above applies: citizens follow a belief threshold voting strategy, and the policy choice of a responsive candidate moves to the left or right with each A or B vote, respectively. Just as it is an equilibrium to ignore S_{2i} and compete along the first policy dimension alone, of course, it is an equilibrium to ignore S_1, and compete on the second dimension instead. In addition to these two possibilities, in fact, equilibrium could operate on any line through the *ex ante* optimal policy pair.

With no additional information about the nature of policy disagreements, it is difficult to say which of these infinite possibilities is most likely to occur. In many cases, however, the desirability of policies in multiple dimensions may ultimately depend on a smaller set of disputed facts. To extend the example of section 1, the question of whether a Keynesian model or a classical model more accurately reflects macroeconomic realities could have consequences for *both* monetary and fiscal policies. In a case such as this, the two policy components

Z_1 and Z_2 (and therefore S_{1i} and S_{2i}) are likely to be correlated. In that case, if S_{1i} and S_{2i} reflect a voter's opinions regarding the optimal fiscal and monetary policies, respectively, then S_{1i} conveys information about Z_2 (in addition to the information it conveys about Z_1) and similarly S_{2i} conveys information about Z_1. Therefore, the behavior described above would no longer constitute an equilibrium: even if citizens ignored S_{2i} and voted purely on the basis of S_{1i}, candidates could infer information about Z_2, and should react accordingly. As a conjecture, it seems likely that equilibrium in that case would involve political competition along the main diagonal of the policy space. Essentially, then, an election to jointly determine fiscal and monetary policy could be viewed simply as a referendum regarding the underlying macroeconomic model.

The reason that multidimensional disagreements collapse here to a single dimension in equilibrium is simply the linear nature of two-candidate competition: votes for A push beliefs in one direction, and votes for B push beliefs in the opposite direction. In other words, the margin of victory in a two-candidate election is inherently one-dimensional. This is not the case with multiple candidates. Returning to the case in which Z_1 and Z_2 are uncorrelated, if A and B choose platform policies $(-x, 0)$ and $(x, 0)$ and there are no other candidates then equilibrium voting will respond only to S_{1i}, and the policy outcome will respond to the margin of victory, but will only vary along the Z_1-axis. If a third candidate C enters the race and chooses some policy $(0, x)$, however, then voters who hold strong opinions in the second dimension, but not the first, may opt to signal this by voting for candidate C. If so, then even if candidate C loses the election, its votes allow the winning candidate (whether A or B) to infer the location of Z_2, in addition to Z_1. While this discussion moves several steps beyond the model of section 2 and is therefore somewhat informal by necessity, it suggests the possibility of a still stronger version of Condorcet's jury theorem: with three candidates and an arbitrarily large electorate, vote totals are presumably sufficient to identify the optimal policy pair (Z_1, Z_2) in the two-dimensional policy space $[-1, 1]^2$. More generally, vote totals for $k+1$ candidates would identify the optimal k-tuple in $[-1, 1]^k$.

6 Applications

6.1 Ideology

The notion of ideology is ubiquitous in all types of political discourse. The standard parlance is a left-right spectrum, ranging from liberal to moderate to conservative. This geometry has become a standard feature of workhorse models of political economy (Persson and Tabellini, 2000), and is also familiar outside academia: 96% of the 47,438 respondents in the American National Election Studies cumulative file (1948-2004) were able to locate themselves on a seven-point ideological scale. At least to a certain extent, ideological labels seem to be objective, not just subjective: in the United States, for example, few would disagree that the Republican party is more conservative than the Democratic party, or that President Barack Obama is more liberal than former President George W. Bush.

Standard spatial models typically derive ideology from wealth or income.[20] In Bergstrom and Goodman (1973), for example, wealth determines the demand for a public good; in Romer (1975), Roberts (1977), and Meltzer and Richard (1981), it determines the demand for redistribution. One challenge to this approach is that, while income and ideology are indeed correlated, the correlation is weaker than a pure income story might suggest: many wealthy Americans tend to be quite liberal, while many who are less affluent (e.g. in the rural south) tend to be strongly conservative.[21] A related weakness is that divisions on specific issues don't always fit the standard paradigm: national defense and environmental protection are both classic examples of public goods, for example, but are traditionally associated with conservative and liberal ideologies, respectively. A more fundamental problem is that standard spatial models are one-dimensional, and must therefore grossly simplify policy decisions (including public goods provision and redistribution) that in the real world are extremely multi-faceted and complex.

One reason for the pervasive use of one-dimensional models, despite this theoretical disconnect, is a lack of alternatives: extensive efforts to generalize standard models have been repeatedly frustrated by the result that in higher dimensions, equilibria often do not exist (e.g. Plott, 1967; Duggan and Fey, 2005; and Duggan, 2005). A second reason, which presents more of a puzzle, is that for some reason political positions on multiple issues appear to be strongly correlated, so that a simple taxonomy along the liberal-conservative ideological spectrum mirrors reality surprisingly adequately. Converse (1964, p. 207) points out, for example, that "...if a person is opposed to the expansion of social security, he is probably a conservative and is probably opposed as well to any nationalization of private industries, federal aid to education, sharply progressive income taxation, and so forth." Poole and Rosenthal (2001) also find that estimating a one-dimensional spatial model accounts for a large majority of the variation in roll call voting in the U.S. congress, especially in recent decades.

As section 1 points out, an information model of politics quite naturally produces a geometry reminiscent of standard ideological language: a complicated proposition relating to any

[20] Alternatively, ideology is often modeled simply as an arbitrary taste parameter.

[21] McCarty, Poole, and Rosenthal (2006, ch. 3) report evidence from the American National Election Studies that in recent decades citizens in the top income quintile were only twice as likely as those in the bottom quintile to vote or identify as Republicans; in previous decades, there was no difference at all.

particular issue is ultimately either true or not, but beliefs may vary continuously regarding the credibility of available evidence. Policy alternatives between the two extremes provide risk-averse decision makers an opportunity to hedge against error, but opinions regarding the optimal degree of hedging will vary with beliefs regarding the proposition itself. With multiple issues, equilibrium still exists, as section 5.2 discusses, and in fact may resemble unidimensional equilibrium, which may explain why such models are so adequate empirically. Multiple equilibria may arise in that case, but the informal discussion of section 5.2 suggests this is unlikely if opinions on multiple issues are correlated, which seems extremely likely in an informational context, because of logical relationships between propositions. Converse (1964, p. 209) gives one example of such a logical relationship: "one cannot believe that government expenditures should be increased, that government revenues should be decreased, and that a more favorable balance of the budget should be achieved all at the same time" (p. 209). Thus, voters' world views and opinions on fundamental principles may jointly determine their positions on a host of more specific policy issues.

It is worth mentioning that informational differences might also be correlated to variables such as income and wealth, because these determine what type of information is salient. For example, workers and managers, respectively, may be more aware of the pros and cons associated with minimum wage laws, which could drive political differences independently from the incentives inherent to their positions. In that sense, then, an information theory may be consistent with empirical correlations between income and ideology.

6.2 Information, Ideology, and Participation

Feddersen and Pesendorfer (1996) use strategic abstention both to explain why many citizens *roll off* by voting in some—but not all—races on a ballot, even after voting costs have been paid, and also to explain the growing body of empirical evidence that information makes citizens more likely to vote. For example, Banerjee et al. (2010) report an experiment in Delhi, India, where providing candidate "report cards" made citizens more likely to vote, especially when incumbent politicians had performed poorly. Lassen (2005) reports that randomly chosen participants in a government restructuring experiment in Copenhagen, Denmark, participated disproportionately in a referendum on implementing the program city-wide. McMurray (2011) cites various earlier evidence that voter participation correlates with information variables such as political knowledge, education, age, access to news media, and contact from campaign workers, and Wattenberg et al. (2000) find that best-informed citizens are the least likely to roll off. Voter turnout is also consistently higher in general than in primary elections, perhaps because of greater media exposure, or because party labels more conveniently summarize candidate positions.

McMurray (2010) and section 3.3 of this paper affirm Feddersen and Pesendorfer's (1996) explanation of the empirical facts above by extending their original model to include candidate behavior, and an entire continuum of voter expertise. In all three cases, however, the mechanism driving the "swing voter's curse" is the comparison of miniscule pivot probabilities, which some (e.g. Margolis, 2002) have criticized as demanding inordinate voter sophistication. On the other hand, section 4.2 demonstrates that abstention also occurs in a signaling model, where the pivotal voting calculus is completely irrelevant. Apparently, then, the incentive to strategically defer to others' information has more to do with voters' shared preferences and heterogeneous expertise, and less to do with the mechanics of pivot probabilities. In departing from the mechanical role of elections, this signaling model admits reinterpretation in other political contexts. Citizens who write letters to legislators or participate in political rallies or protests, for example, seek to push policy outcomes in one direction or another, but a legislator's reaction to such political activity is presumably continuous, rather than jumping discontinuously when the number of letters for or against a particular position differs by one. Similarly, respondents to public opinion surveys presumably anticipate continuous reactions to survey results. Empirically, information indeed makes individuals more likely to participate in surveys (Zaller, 1986; Coupé and Noury, 2004) and to express opinions on individual issues (Althaus, 1996), as this model predicts.

In addition to corroborating and extending the known link between information and voter participation, the model above relates both variables to ideology. Specifically, citizens only favor extreme policy positions when they believe their signals to be quite accurate. Empirically, this is precisely the finding of Palfrey and Poole (1987): citizens who have the best information regarding candidates' policy positions are more likely to express preferences for those with extreme positions, and are also more likely to vote. Though those authors do not consider roll-off, casual observation suggests that citizens with the strongest ideological positions are also those most likely to complete a ballot. Similarly, individuals with extreme policy opinions seem the most likely to participate in public rallies or protests, and even non-political consumer product surveys tend to be dominated by consumers who are extremely happy or dissatisfied with a product, rather than those with moderate opinions.

6.3 Policy Divergence and Electoral Margins

Beginning with Hotelling (1929) and Downs (1957), most standard spatial models of electoral competition predict that competition for votes should drive candidates' policy platforms to the center of the policy space. Surprisingly, this result often holds even when candidates prefer policies at opposite extremes of the political spectrum, and there is no benefit to holding office, other than the privilege of choosing policy (see Duggan, 2005, for a recent review). Empirically, however, the difference between candidates' policy stances seem rather large. In the U.S., Bafumi and Herron (2010) find that individual members of congress are generally more ideologically extreme than their constituents, and that the congress collectively is more ideologically extreme than the American electorate. That such policy distance remains, despite the competitive advantage that either candidate could obtain by moderating, remains somewhat of a puzzle.

An information model of politics provides a natural explanation for the non-convergence puzzle. Section 3.2 does demonstrate the moderating pressure of office motivation, but sections 3.3 and 3.4 show that the policy choices of sufficiently policy-motivated candidates diverge in equilibrium. It has long been recognized that policy motivation and electoral uncertainty together prevent complete policy convergence (e.g. Wittman, 1977; Calvert, 1985), but models with these elements often exhibit no equilibrium at all. Furthermore, standard models that assume candidates to have extreme policy preferences beg the question of why candidates with more moderate preferences—which would have an electoral advantage—don't enter the race. This is less problematic in an information model; while candidate entry is admittedly beyond the scope of this paper, the present model suggests the intuition that the candidates with the strongest incentive to run for office are precisely those with the strongest (and therefore the most extreme) political convictions.[22]

Related to the puzzle of non-convergence is another puzzle, which is the occurrence of large margins of victory. Mueller (2003, ch. 11) reports that state governors throughout U.S. history win reelection by an average margin of 23%, but in standard models, candidates typically tie (in expectation) in equilibrium.[23] This is most stark in spatial models with convergent campaign platforms, since indifferent voters presumably vote for either candidate with equal probability. Even when platforms diverge, however, they often split the electorate into equal halves. As a general rule, a candidate can obtain half of the votes in the electorate simply by adopting his opponent's platform, and so should not settle for anything less; if campaigning is costly, a candidate who expects fewer votes than his opponent may prefer not enter the race at all [e.g. cite citizen candidate models]. Even if candidates do not adjust to close the electoral gap, voters may do so: if voting is costly, the incentive to free-ride is stronger in the majority than in the minority group, producing an "underdog effect" by

[22] In the parlance of section 6.2, running for office is merely another mode of political participation, just like voting and participating in rallies and protests. The citizens with the strongest incentive to participate, therefore, are those who are most confident that they are pushing the eventual policy outcome in the proper direction.

[23] Average margins have been smaller in recent decades, but still over 10%. Coate, Conlin, and Moro (2008) reject standard rational voting models on the basis of large margins of victory in liquor referenda in Texas.

which candidates tie even though one is more popular (see Krasa and Polborn, 2009; Medina, 2011).

In this model, the precise margin of victory depends on the distribution of expertise, but (despite the model's symmetry) the expected election outcome is never a tie. Ultimately, the platform adopted by one of the candidates is superior to his opponent's, and this will be correctly identified by a majority of voters. As McMurray (2010) points out, large margins of victory are most likely on issues that are "obvious" in some sense, such as revising archaic government procedures or constitutional language.

6.4 Mandates and Multiple Candidates

The view of elections as a forum for citizens to communicate with their elected representatives is not new; popular media has long interpreted landslide electoral victories as "mandates" for winners to implement extreme policies. Peterson et al. (2003) catalogue references to electoral mandates in U.S. newspaper archives as early as [year], and show that U.S. congressional voting shifts after elections that are widely interpreted as mandates. Faravelli and Walsh (2011) also find that partisanship in individual congress members' voting records is correlated with the most recent electoral margin. Standard election models say little about electoral mandates, however, both because they focus on the identity of an election winner, whether he wins by a single vote or a million votes, and also because margins of victory are zero in expectation, as discussed above. If candidates are policy motivated and responsive, however, then this model predicts mandates as a natural feature of equilibrium: each vote pushes the winning candidate's beliefs, and therefore, the ultimate policy outcome, to the right or to the left.

Strictly speaking, the audience of voters' communication need not be limited to the candidates in the election at hand. In 2010, for example, Massachusetts voters elected Republican Scott Brown to fill the senate seat vacated by the passing of veteran Democrat Edward Kennedy; at the time, this was widely interpreted as a message of warning to President Barack Obama and leaders of the Democratic party, who had recently made dramatic changes to U.S. health care policy.[24] In fact, it is common in the U.S. for the president's party to lose legislative seats during midterm elections; in the context of the model, this could be viewed as an effort by voters to reduce or even reverse the mandate interpreted from the previous presidential election. While simultaneous races are not modeled here, the intuition of section 5.1 suggests that some citizens might find it optimal to vote for opposing

[24] See Cooper, Michael (2010, January 10). G.O.P. Victory Stuns Democrats. *The New York Times*, http://www.nytimes.com/2010/01/21/us/politics/21elect.html?scp=9&sq=Scott%20Brown&st=cse (accessed 6 February 2010).

parties to fill various offices within the government. With only a single race, for example, a moderate citizen (i.e. S_i close to zero) abstains from voting, even if her expertise is quite high, to avoid giving either candidate a more extreme mandate. In doing so, however, she mimics the behavior of citizens who simply have no information (i.e. Q_i close to zero), and so the informative content of her (in-)action may be lost. With multiple races, however, it seems reasonable to conjecture that poorly informed citizens would still abstain, while citizens with (strong) moderate beliefs would vote for opposing parties. Empirically, this practice of "split ticket" voting is indeed ubiquitous.[25]

Another possible application of the logic of electoral mandates is the well-known turnout paradox, which is that millions of citizens participate in costly elections even when the probability of casting a pivotal vote is miniscule. Electoral mandates provide a possible resolution, since *every* vote influences the margin of victory, and therefore influences policy.[26] The impact of a single vote on the margin of victory is also quite small, of course, and a formal comparison of the two incentives in this model is beyond the scope of this paper, but Meirowitz and Shotts (2009) show that signaling incentives dominate pivot incentives in large electorates in a model similar to this (see also Herrera and Morelli, 2010), and the analysis above suggests at least the possibility of vindicating the popular notion that voting secures benefits even when it does not change the identity of the election winner, so that "every vote counts" as in the popular mantra. If so, the incentive to contribute to margins of victory could also explain evidence from Farber (2010) that voter turnout remains high even when elections are not close.[27]

Still another puzzle for standard voting models is the persistence of minor party candidates. As Palfrey (1989) and Chen and Xia (2009) show, winner-take-all electoral systems generate an incentive for voters to coordinate behind the two strongest candidates, because a vote for minor parties is even less likely to be pivotal than a vote for a major candidate, and is therefore "wasted". Minor party candidates should therefore expect no votes and, if campaigns are costly, should not even run for office. As Duverger's (1954) law states, therefore, only two candidates should participate in elections. To the contrary, however, minor party candidates do run for office, and do receive votes: in the 2008 U.S. presidential election, for example, minor parties garnered over one and a half million votes; in 2000, they received nearly four million. By divorcing rationality from the pivotal voting calculus, the signaling

[25] See the references cited by Chari, Jones, and Marimon (1997).

[26] Several recent papers (e.g. Feddersen and Sandroni, 2006; Edlin, Gelman, and Kaplan, 2007; Evren, 2010; Faravelli and Walsh, 2011) point to ethical motivations for costly voting, such as altruism. McMurray (2010) argues that such motivations are most natural in a common-values environment such as this.

[27] In a meta-analysis of election studies, Geys (2006) reports that about two thirds of empirical tests find a positive correlation between closeness and turnout, but the remaining third do not.

equilibrium described in section 4.3 provides a straightforward explanation for minor party candidates and their supporters: voting for an extreme party signals a more extreme opinion, and therefore pushes the eventual policy outcome by a larger amount than voting for a major party. As noted above, this implies that citizens with the best information and/or the most extreme political views should vote support candidates from extreme parties, which is precisely the finding of Palfrey and Poole (1987). Alternatively, the informal discussion in section 5.2 suggests another role, which is that votes for a third-party candidate can push policy in a new direction. If Republicans and Democrats represent certain positions on the appropriate levels of taxation and government expenditures, for example, then the only way to signal preferences regarding environmental policy may be to vote for the Green party candidate, even if he has little chance of winning the election.

The analysis above treats the polar cases in which candidates cannot deviate from campaign platforms at all, and in which deviations are costless. An intermediate possibility is that deviations are costly, but not impossible. While not strictly modeled above, intuition suggests that in such a model, citizens would feel conflicted between voting for an extreme party to convey strong opinions, or voting for a major party to prevent an accidental victory by the major opposing party. If so, it seems likely that the major parties would receive much larger vote shares than the minor parties. In fact, examples 4 and 5 illustrate that this is the case even absent pivotal voting considerations, especially if the distribution of expertise is skewed, which seems likely. Thus, while admitting minor party candidates, the model also seems consistent with Cox's (1997) evidence (summarized by Mueller, 2003, ch. 13) that winner-take-all systems exhibit only two major parties, even if the actual number of parties is higher. In some jurisdictions, such as the state of New York, minor parties can endorse major party candidates, giving voters an opportunity to signal strong opinions (by voting for the major party candidate, but on the minor party's line) without risking upsetting the election.[28]

7 Conclusion

The political behavior of both voters and elected officials are driven to a great extent by ideology. For over half a century, this has been modeled exclusively as the result of conflicts of interest, for example because of differences in wealth. This paper considers the alternative

[28]In the 2006 governor's race, for example, New York residents could vote for Eliot Spitzer via the Democratic, Independence, or Working Families party lines, and for John Faso on the Republican or Conservative lines (New York State Board of Elections, 2006). Both candidates received more than 10% of their votes from minor party lines.

possibility, consistent with Condorcet's (1785) much older view of elections, that ideological disagreements stem from differences of *opinion* regarding the underlying facts that determine the optimality of various policy options.[29] A simple but important observation is that the standard geometry arises quite naturally in an information model, even if the truth about an underlying proposition is inherently binary, because individuals who possess different levels of expertise will form beliefs with different levels of conviction, and therefore favor different policies, as hedges against error.

Equilibrium outcomes in this information model in many ways resemble results from existing models, but with important differences. For example, competition drives office-motivated candidates to the political center, as in standard spatial models, but this is no longer in voters' interest; indeed, it occurs even when the optimal policy is restricted *ex ante* to lie at one of the extremes of the policy space. Similarly, a common view is that democracy is undermined by political polarization, broken campaign promises, extremist third parties, or voter nonparticipation, but the current model actually highlights the benefits of policy motivation, platform flexibility, abstention, and multiple candidates. That such similar models could produce such opposite welfare implications suggests the need for caution in structuring policy or political institutions on the basis of existing models.

In addition to its relevance for spatial models, this paper makes important contributions to the information literature, expanding to a convex set of policy alternatives, and introducing candidates and political competition. It extends both literatures by analyzing margins of victory, electoral mandates, multiple parties, and multiple dimensions. As described in section 6, doing so provides possible explanations for empirical phenomena, such as the unidimensionality of political competition; the three-way empirical correlation between information, ideology, and voter participation; large margins of victory; policy divergence; and support for minor parties.

As section 6 points out, participation in costless elections is akin to participation in political protests, surveys, or letter writing campaigns. By a similar token, an extension of this model might provide an information context through which to study proportional rule voting in parliamentary elections, where political parties receive legislative seats in proportion to their vote shares. Standard analysis is difficult in that setting, because there are many ways for a vote to be pivotal; essentially, the role of a vote is to push the composition of the electorate toward one party or another, just as it pushes the electoral mandate in this model, so a model similar to this may be applicable.

An important direction for future work is to enrich the information structure of this

[29] An information ideology also seems appropriate linguistically: the word "ideology" shares obvious etymological roots with words such as "ideas" and "logic".

model. In the current version, for example, private signals are uncorrelated. This seems unlikely, both because communication within the electorate should generate dependence in voters' beliefs, and because the current model makes the unrealistic recommendation that a citizen who learns that she belongs to the minority should immediately reverse her opinion. If a citizen's peers had correlated information, on the other hand, they could simultaneously reach an erroneous conclusion; this should make a citizen less willing to abandon her own opinion, though it may also undermine the jury theorem. Another important informational consideration is that the policy that is *ex ante* expected to be optimal lies at the center of the policy space, so that additional information can only make a voter more extreme; in some cases, it may be more realistic to assume divergent prior beliefs (e.g. reflecting cognitive limitations, perhaps), in which case private information may have a moderating effect. Indeed, it is often the case that political controversies seem one-sided and obvious at first, but upon closer investigation prove to be more complicated, requiring a more careful, and even more moderate, policy response.

A Appendix: Proofs

Lemma 1 *If candidates are committed to implement platforms $x_A, x_B \in [-1,1]$ then there exists an equilibrium strategy $\bar{\sigma}^* \in \bar{\Sigma}$ in the voting subgame. If $x_A \neq x_B$ then $\bar{\sigma}^*$ is a belief threshold strategy, with $T^* = 0$ if and only if $x_A = -x_B$.*

Proof. If $x_A = x_B$ then voters of all types are indifferent between election outcomes, so any subgame strategy constitutes a voting equilibrium. If $x_A \neq x_B$ then, as discussed above, the best response $\bar{\sigma}^{br}$ to any voting strategy $\bar{\sigma}$ is a belief threshold strategy. In particular, if $\bar{\sigma}$ is itself a belief threshold strategy then the threshold $T_{AB}^{br}(T)$ can be interpreted as a continuous function from the compact set $[-1,1]$ of thresholds into itself.[30] Brouwer's theorem then guarantees the existence of a fixed point $T^* = T_{AB}^{br}(T^*)$, which characterizes a belief threshold strategy $\bar{\sigma}^*$ that is its own best response—and therefore an equilibrium in the voting subgame.

If $T = 0$ then voting is symmetric with respect to signals, and therefore with respect to states, so (4) through (10) reduce such that $\phi_z(A) = \phi_{-z}(B)$, $\psi_z(a,b) = \psi_{-z}(b,a)$, $\pi_z^j(m) = \pi_{-z}^j(-m)$, $\Pr_z(piv_A) = \Pr_{-z}(piv_B)$, implying that $E(Z|piv) = E(Z) = 0$ and $E(Z^2|piv) = E(Z^2)$, and therefore that (12) reduces to $T_{AB}^{br} = \frac{\bar{x}}{E(Z^2)}$. Thus, a symmetric voting strategy is its own best response if and only if $\bar{x} = 0$. ■

Proposition 1 (Jury Theorem) *If candidates are committed to implement platforms $x_A, x_B \in$*

[30] Continuity follows from the continuity of component functions $\pi_z^j(m)$, $\psi_z(a,b)$, and $\phi_z(j)$.

$[-1,1]$ *and the subgame strategy* $\bar{\sigma}_n^* \in \bar{\Sigma}$ *maximizes welfare* $Eu(Y,Z;n)$ *for a particular population size parameter* n*, then*

Theorem 11 *1.* $\bar{\sigma}_n^*$ *is a belief threshold strategy, and constitutes an equilibrium in the voting subgame, and*

2. The associated sequence $\bar{Y}_n^*$ *of equilibrium policy outcomes approaches* $p\lim_{k\to\infty}\left(\bar{Y}_n^*|Z\right) = \begin{cases} \min\{x_A,x_B\} & \text{if } Z=-1 \\ \max\{x_A,x_B\} & \text{if } Z=1 \end{cases}.$

Proof. As a preliminary step, let $\bar{\sigma}_n'$ denote the (non-equilibrium) belief threshold strategy characterized by $T=0$ (for all n), and let $\bar{Y}_n'$ denote the resulting policy outcome. As the electorate grows large, this leads to the desired election outcome $p\lim_{k\to\infty}\left(\bar{Y}_n'|Z\right) = \begin{cases} \min(x_A,x_B) & \text{if } Z=-1 \\ \max(x_A,x_B) & \text{if } Z=1 \end{cases}$ almost surely, by the law of large numbers, thereby providing maximal expected utility in the limit (i.e. $\lim_{n\to\infty}\left[Eu\left(\bar{Y}_n',Z;n\right)-\max_j u(x_j,z;n)\right]=0$). The voting subgame associated with a particular pair (x_A,x_B) of platform policies is a symmetric common interest game, meaning that citizens have identical preferences and face identical strategy options. In settings such as this, McLennan (1998) demonstrates that the symmetric strategy profile $\bar{\sigma}_n^*$ that maximizes the common utility function constitutes an equilibrium. By definition, this optimal strategy produces at least as much utility as $\bar{\sigma}_n'$, implying that the two sequences of policy outcomes approach the same limit. ■

Lemma A1 is a useful precursor to theorems 1 and 2.

Lemma A1 *If* $\bar{\sigma}$ *is a belief threshold strategy then* $E(Z|piv)$ *has the same sign as the belief threshold* T*.*

Proof. Let $T>0$. (Symmetric arguments apply if $T\le 0$.) In this case, $\bar{\sigma}$ specifies that citizens vote A if $q<T$ or $s=1$, or both. Thus, expected vote shares ϕ_z reduce from (4) to the following.

$$\begin{array}{ll} \phi_{-1}(A)=F(T)+\int_T^1 \frac{1+q}{2}dF(q) & \phi_{-1}(B)=\int_T^1 \frac{1-q}{2}dF(q) \\ \phi_1(A)=F(T)+\int_T^1 \frac{1-q}{2}dF(q) & \phi_1(B)=\int_T^1 \frac{1+q}{2}dF(q). \end{array} \tag{18}$$

From these, it is straightforward to verify that $\phi_1(A)\phi_1(B)>\phi_{-1}(A)\phi_{-1}(B)$, which implies that a k-vote tie is more likely in state 1 than state -1,

$$\begin{aligned} \psi_1(k,k)-\psi_{-1}(k,k) &= \frac{e^{-\phi_1(A)n-\phi_1(B)n}\left[\phi_1(A)n\right]^k\left[\phi_1(B)n\right]^k}{k!k!}-\frac{e^{-\phi_{-1}(A)n-\phi_{-1}(B)n}\left[\phi_{-1}(A)n\right]^k\left[\phi_{-1}(B)n\right]^k}{k!k!} \\ &= \frac{e^{-n}n^{2k}}{k!k!}\left\{\left[\phi_1(A)\phi_1(B)\right]^k-\left[\phi_{-1}(A)\phi_{-1}(B)\right]^k\right\}>0, \end{aligned}$$

implying in turn that a vote is more likely to be pivotal in state 1 than state -1,

$$\begin{aligned}
\Pr_1(piv) - \Pr_{-1}(piv) &= \left\{\frac{1}{2}\left[\frac{1}{2}\pi_1^A(0) + \frac{1}{2}\pi_1^A(1)\right] + \frac{1}{2}\left[\frac{1}{2}\pi_1^B(0) + \frac{1}{2}\pi_1^B(1)\right]\right\} \\
&\quad - \left\{\frac{1}{2}\left[\frac{1}{2}\pi_{-1}^A(0) + \frac{1}{2}\pi_{-1}^A(1)\right] + \frac{1}{2}\left[\frac{1}{2}\pi_{-1}^B(0) + \frac{1}{2}\pi_{-1}^B(1)\right]\right\} \\
&= \frac{1}{4}\sum_{k=0}^{\infty}\left\{\begin{array}{c} [\psi_1(k,k) + \psi_1(k+1,k) + \psi_1(k,k) + \psi_1(k,k+1)] \\ -[\psi_{-1}(k,k) + \psi_{-1}(k+1,k) + \psi_{-1}(k,k) + \psi_{-1}(k,k+1)] \end{array}\right\} \\
&= \frac{1}{4}\sum_{k=0}^{\infty}\left\{\begin{array}{c} \psi_1(k,k)\left[1 + \frac{p_1(A)}{k+1} + 1 + \frac{p_1(B)}{k+1}\right] \\ -\psi_{-1}(k,k)\left[1 + \frac{p_{-1}(A)}{k+1} + 1 + \frac{p_{-1}(B)}{k+1}\right] \end{array}\right\} \\
&= \frac{1}{4}\sum_{k=0}^{\infty}\left\{\psi_1(k,k)\left(2 + \frac{1}{k+1}\right) - \psi_{-1}(k,k)\left(2 + \frac{1}{k+1}\right)\right\} \\
&= \frac{1}{4}\sum_{k=0}^{\infty}\left[\psi_1(k,k) - \psi_{-1}(k,k)\right]\left(2 + \frac{1}{k+1}\right) > 0.
\end{aligned}$$

and therefore that the conditional expectation of Z is positive:

$$E(Z|piv) = \frac{(1)\Pr_z(piv) + (-1)\Pr_z(piv)}{\Pr_z(piv) + \Pr_z(piv)} > 0.$$

■

Theorem 1 (Median Voter Theorem) *If candidates are committed and office-motivated then (x_A^*, x_B^*, σ^*) is a perfect Bayesian equilibrium only if $x_A^* = x_B^* = 0$ and σ^* is almost everywhere equivalent to a belief threshold strategy, with threshold function T^* such that $T^*(0,0) = 0$. Furthermore, such an equilibrium exists.*

Proof. The "almost everywhere" caveat acknowledges that any voting behavior is consistent with equilibrium in subgames for which $x_A = x_B$, but that these represent a set of measure zero in the space of (x_A, x_B) pairs. If $x_A \neq x_B$ then, as lemma 1 shows, the best response to a belief threshold strategy $\bar{\sigma}$ with threshold T is another belief threshold strategy, with threshold $T_{AB}^{br}(T)$ defined in (12). If $\bar{x} = 0$ then this reduces to $T_{AB}^{br}(T) = \frac{-E(Z|piv)}{E(Z^2|piv)}$. Since lemma A1 states that T and $E(Z|piv)$ have the same sign, this implies that T and $T_{AB}^{br}(T)$ have opposite signs. The next step is to note that (12) is increasing in $\bar{x}$ (for any value of T): by the product rule, the derivative $\frac{\partial T_{AB}^{br}(T)}{\partial \bar{x}}$ has the same sign as

$$\begin{aligned}
&\left[E\left(Z^2|piv\right) - \bar{x}E(Z|piv)\right] + E(Z|piv)\left[\bar{x} - E(Z|piv)\right] \\
=\ & E\left(Z^2|piv\right) + \left[E(Z|piv)\right]^2 \\
=\ & Var(Z|piv) > 0.
\end{aligned}$$

The result that 0 lies between T and $T_{AB}^{br}(T)$ whenever $\bar{x}=0$, together with the result that $T_{AB}^{br}(T)$ increases with $\bar{x}$, implies that $T_{AB}^{br}(T)$ is positive whenever $T<0<\bar{x}$, and negative whenever $\bar{x}<0<T$. A fixed point $T^*=T_{AB}^{br}(T^*)$, therefore, cannot be negative when $\bar{x}>0$ or positive when $\bar{x}<0$; in other words, T^* and $\bar{x}$ must have the same sign.

If $\bar{x}>0$ and $T^*>0$ then (4) reduces to (18), as in the proof of lemma A1, implying that $\phi_{-1}(A)>F(T)+\phi_{-1}(B)$ and $F(T)+\phi_1(B)>\phi_1(A)$, and therefore that candidate A's expected vote share exceeds that of candidate B,

$$\frac{1}{2}\phi_{-1}(A)+\frac{1}{2}\phi_1(A)>\frac{1}{2}\phi_{-1}(B)+\frac{1}{2}\phi_1(B),$$

and therefore exceeds $\frac{1}{2}$. This implies that A is more likely to win by a margin of m votes than to lose by a margin of m votes,

$$\begin{aligned}
& \left[\frac{1}{2}\pi_{-1}^A(m)+\frac{1}{2}\pi_1^A(m)\right]-\left[\frac{1}{2}\pi_{-1}^A(-m)+\frac{1}{2}\pi_1^A(-m)\right] \\
= & \frac{1}{2}\sum_{k=0}^{\infty}\left[\psi_{-1}(k+m,k)+\psi_1(k+m,k)-\psi_{-1}(k,k+m)-\psi_1(k,k+m)\right] \\
= & \frac{1}{2}\sum_{k=0}^{\infty}\frac{e^{-n}}{k!\,(k+m)!}\begin{bmatrix}\phi_{-1}^{k+m}(A)\,\phi_{-1}^{k}(B)+\phi_1^{k+m}(A)\,\phi_1^{k}(B) \\ -\phi_{-1}^{k}(A)\,\phi_{-1}^{k+m}(B)-\phi_1^{k}(k+m)\,\phi_1^{k+m}(B)\end{bmatrix} \\
> & \frac{1}{2}\sum_{k=0}^{\infty}\frac{e^{-n}}{k!\,(k+m)!}\begin{bmatrix}\phi_1^{k+m}(B)\,\phi_{-1}^{k}(B)+\phi_1^{k+m}(A)\,\phi_1^{k}(B) \\ -\phi_{-1}^{k}(A)\,\phi_{-1}^{k+m}(B)-\phi_{-1}^{k}(B)\,\phi_1^{k+m}(B)\end{bmatrix} \\
= & \frac{1}{2}\sum_{k=0}^{\infty}\frac{e^{-n}}{k!\,(k+m)!}\left[\psi_1(k,k)\,\phi_1^m(A)-\psi_{-1}(k,k)\,\phi_{-1}^m(B)\right]>0,
\end{aligned}$$

and is therefore more likely than candidate B to win the election:

$$\begin{aligned}
& \Pr(W=A)-\Pr(W=B) \\
= & \frac{1}{2}\left[\sum_{m=0}^{\infty}\pi_{-1}^A(m)+\frac{1}{2}\pi_{-1}^A(0)\right]+\frac{1}{2}\left[\sum_{m=0}^{\infty}\pi_1^A(m)+\frac{1}{2}\pi_1^A(0)\right] \\
& -\frac{1}{2}\left[\sum_{m=0}^{\infty}\pi_{-1}^B(m)+\frac{1}{2}\pi_{-1}^B(0)\right]+\frac{1}{2}\left[\sum_{m=0}^{\infty}\pi_1^B(m)+\frac{1}{2}\pi_1^B(0)\right] \\
= & \frac{1}{2}\sum_{m=0}^{\infty}\left[\pi_{-1}^A(m)-\pi_{-1}^B(m)\right]+\frac{1}{2}\sum_{m=0}^{\infty}\left[\pi_1^A(m)-\pi_1^B(m)\right] \\
= & \sum_{m=0}^{\infty}\left\{\left[\frac{1}{2}\pi_{-1}^A(m)+\frac{1}{2}\pi_1^A(m)\right]-\left[\frac{1}{2}\pi_{-1}^A(-m)+\frac{1}{2}\pi_1^A(-m)\right]\right\} \\
> & 0.
\end{aligned}$$

In other words, $\Pr(W=A)>\frac{1}{2}$. Similarly, B wins with probability greater than $\frac{1}{2}$ if $\bar{x}<0$ and $T^*<0$.

Together, the above results imply that the candidate whose platform is closer to zero wins the election with greater than $\frac{1}{2}$ probability. In equilibrium, therefore, both candidates must adopt the zero platform, resulting in an expected tie. As long as σ^* induces equilibrium strategies in each subgame (which exist by lemma 1), therefore, $(0, 0, \sigma^*)$ is a perfect Bayesian equilibrium. ■

Theorem 2 (Policy Divergence) *If candidates are committed and policy-motivated then* (x_A^*, x_B^*, σ^*) *is a perfect Bayesian equilibrium only if candidate platforms are given by* $x_j^* = E(Z|W = j)$ *for* $j = A, B$*, with* $x_A^* < 0 < x_B^*$*, and the voting strategy* σ^* *is almost everywhere a belief threshold strategy. Furthermore, such an equilibrium exists, with platforms* $x_A^* = -x_B^*$ *symmetric around zero and* $T^*(x_A^*, x_B^*) = 0$.

Proof. Lemma 1 shows that σ^* is inconsistent with equilibrium unless it induces a belief threshold strategy in every subgame for which $x_A \neq x_B$ (which is almost everywhere). That $E(Z|W = j)$ maximizes $Eu(x_j, Z|W = j)$ is demonstrated in section 2. To see that $x_A^* < 0 < x_B^*$, take σ^* to be a as given and consider instead platforms $0 \leq x_A \leq x_B$. (For reversed candidate labels or inequalities, of course, a symmetric argument applies.) As the proof of theorem 1 explains, any equilibrium belief threshold T^* has the same sign as the midpoint $\bar{x} = \frac{x_A + x_B}{2}$ between the two platforms, which in this case is positive. Accordingly, vote shares reduce from (4) to (18), as in the proof of lemma A1, implying that candidate A expects a higher vote share in state -1 than in state 1 (i.e. $\phi_{-1}(A) > \phi_1(A)$), and is therefore more likely to win in state -1 than state 1 (i.e. $\Pr_{-1}(W = A) > \Pr_1(W = A)$).[31] Thus, conditional on winning, A expects Z to be negative (i.e. $\hat{z}_A < 0$), and therefore $0 \leq x_A \leq x_B$ is inconsistent with equilibrium.

When candidate platforms $x_A = -x_B$ are symmetric around zero, lemma 1 states that the unique equilibrium in the voting subgame is a belief threshold strategy with $T^* = 0$. This implies that (4), (5), (6), and (7) simplify such that $\phi_{-1}(A) = \phi_1(B)$, $\phi_1(A) = \phi_{-1}(B)$, $\psi_{-1}(a, b) = \psi_1(b, a)$, $\pi_z^A(m) = \pi_{-z}^B(m)$, and $\Pr_z(W = A) = \Pr_{-z}(W = B)$, so that $x_B^{br} = -x_B^{br}$. Such symmetric platform pairs can be completely characterized by candidate B's platform, so the continuous function $x_B^{br}(x_B)$ from $[0, 1]$ into itself can be interpreted as mapping platform pairs $(-x_B, x_B)$ that are symmetric around zero into best-response pairs $\left(-x_B^{br}, x_B^{br}\right)$ that are also symmetric around zero. By Brouwer's theorem, a fixed point $x_B^* = x_B^{br}(x_B^*) > 0$ exists, defining a platform pair $(-x_B^*, x_B^*)$ that, together with the voting

[31] For any population size $N = k$, the number of A votes follows a binomial distribution with parameters k and $\phi_z(A)$. Therefore, A and B win the election with probabilities $\Pr_z\left(N_A > \frac{k}{2}|N = k\right) + \frac{1}{2}\Pr\left(N_A = \frac{k}{2}|N = k\right)$ and $\Pr_z\left(N_A < \frac{k}{2}|N = k\right) + \frac{1}{2}\Pr\left(N_A = \frac{k}{2}|N = k\right)$, the difference $\Pr_z\left(N_A > \frac{k}{2}|N = k\right) - \Pr_z\left(N_A < \frac{k}{2}|N = k\right)$ between which is increasing in $\phi_z(A)$, and equal to zero if $\phi_z(A) = \frac{1}{2}$. Thus, $\Pr_z(W = A|N = k) > \frac{1}{2}$ and therefore $\Pr_z(W = A) = E_N[\Pr(W = A|N = k)] > \frac{1}{2}$.

strategy identified in lemma 1, constitutes a perfect Bayesian equilibrium. As lemma 1 shows, the symmetry of candidate platforms leads to symmetric voting (i.e. $T^*(x_A^*, x_B^*) = 0$). ■

Proposition 2 (Jury Theorem 2) *If candidates are committed and policy-motivated and the strategy combination* $\left[\left(x_j^*\right)_{j=A,B}, \sigma^*\right]_n \in [-1,1]^2 \times \Sigma$ *maximizes welfare* $Eu(Y, Z; n)$ *for a particular population size parameter* n*, then*

1. $\left[\left(x_j^*\right)_{j=A,B}, \sigma^*\right]_n$ *constitutes a perfect Bayesian equilibrium, and*
2. *The associated sequence* Y_n^* *of equilibrium policy outcomes approaches* $p\lim_{n\to\infty}(Y_n^*|Z) = Z$.

Proof. Proposition 1 states that the voting response to any platform pair that maximizes $Eu(Y, Z; n)$ constitutes an equilibrium in the voting subgame, and when candidates are policy-motivated, equilibrium candidate strategies maximize $Eu(Y, Z; n)$ in response to equilibrium voting. Together, these observations establish part 1. Since candidates A and B win the election almost surely in states -1 and 1, respectively, equilibrium candidate platforms approach $\lim_{n\to\infty}\left(x_j^*\right)_n = \lim_{n\to\infty} E\left(Z|W = j; \sigma_n'\right) = \begin{cases} -1 \text{ if } j = A \\ \ \ 1 \text{ if } j = B \end{cases}$, and the equilibrium policy outcome therefore approaches $p\lim_{n\to\infty}(Y_n^*|Z) = \begin{cases} -1 & \text{if } Z = -1 \\ 1 & \text{if } Z = 1 \end{cases}$. ■

Lemma 2 (Swing Voter's Curse) *If candidates are committed and abstention is allowed then, for any pair* (x_A, x_B) *of platform policies, there exists a subgame voting strategy* $\bar{\sigma}^* \in \bar{\Sigma}'$ *that constitutes an equilibrium in the voting subgame. If* $x_A \neq x_B$ *then* $\bar{\sigma}^*$ *is a belief threshold strategy, with belief thresholds* $T_1^* < T_2^*$*. Also,* $T_1^* = -T_2^*$ *if and only if* $x_A = -x_B$.

Proof. If $x_A = x_B$ then voters of all types are indifferent between election outcomes, so any strategy constitutes an equilibrium in the voting subgame. If $x_A < x_B$ then, in response to opponent voting strategy $\bar{\sigma}$, the expected benefit $\Delta_{0B}(q, s)$ to a citizen with private information (q, s) from switching her vote from A to B is given by (11), and is positive if and only if qs exceeds (12). Similarly, the benefit of voting B (instead of abstaining) is given by,

$$\begin{aligned} \Delta_{0B}(q, s) &= \sum_{z=1,-1} \left[u(x_B, z) - u(x_A, z)\right] \Pr_z(piv_B) \frac{1}{2}(1 + zqs) \\ &= 4(x_B - x_A)\left[E(z|piv_B, q, s) - \bar{x}\right] \Pr(piv_B, q, s), \end{aligned} \tag{19}$$

which is positive if and only if qs exceeds

$$T_{0B}^{br} = \frac{\bar{x} - E(Z|piv_B)}{E(Z^2|piv_B) - \bar{x}E(Z|piv_B)}, \tag{20}$$

and the benefit

$$\begin{aligned}\Delta_{A0}(q,s) &= \sum_{z=1,-1}\left[u(x_B,z)-u(x_A,z)\right]\Pr_z(piv_A)\frac{1}{2}(1+zqs)\\ &= 4(x_B-x_A)\left[E(z|piv_A,q,s)-\bar{x}\right]\Pr(piv_A,q,s) \qquad (21)\end{aligned}$$

of abstaining instead of voting A is positive if and only if qs exceeds

$$T_{A0}^{br}=\frac{\bar{x}-E(Z|piv_A)}{E(Z^2|piv_A)-\bar{x}E(Z|piv_A)}. \qquad (22)$$

Thus, the best response to $\bar{\sigma}$ is a belief threshold strategy, with thresholds $T_1^{br}=\min\left\{T_{A0},T_{AB}^{br}\right\}$ and $T_2^{br}\equiv\max\left\{T_{0B}^{br},T_{AB}^{br}\right\}$.[32]

If $\bar{\sigma}$ is itself a belief threshold strategy, with thresholds T_1 and T_2, then the best-response belief thresholds T_1^{br} and T_2^{br} can together be viewed as a single continuous function, from the compact set $\{(T_1,T_2):-1\leq T_1\leq T_2\leq 1\}$ of possible belief threshold pairs into itself. Brouwer's theorem guarantees the existence of a fixed point pair (T_1^*,T_2^*) of belief thresholds that define a belief threshold strategy that is its own best response, and thus an equilibrium in the voting subgame.[33]

To see that $T_1^*<T_2^*$ in equilibrium, consider a belief threshold strategy in which no one abstains: $T_1=T_2\equiv T$. In this case, the expected state is higher when an A vote is pivotal than when a B vote is pivotal. This is because, by Lemma 3 below, $E(Z|N_A=k,N_B=k+1)>E(Z|N_A=N_B=k)$ for any number k of A votes, implying that $E(Z|N_B-N_A=1)>E(Z|N_A=N_B)$, and therefore that

$$\begin{aligned}E(Z|piv_A,q,s) &= \omega E(Z|N_B-N_A=1,q,s)+(1-\omega)E(Z|N_A=N_B,q,s)\\ &< E(Z|N_A=N_B,q,s),\end{aligned}$$

for any private information (q,s), where $\omega=\frac{\Pr(N_A+1=N_B)}{\Pr(N_A+1=N_B)+\Pr(N_A=N_B)}$. Similarly, $E(Z|piv_B,q,s)<E(Z|N_A=N_B,q,s)$, and by transitivity $E(Z|piv_B,q,s)<E(Z|piv_A,q,s)$. If a voter is indifferent between voting A and B, it must be that $\Delta_{AB}=\Delta_{A0}+\Delta_{0B}=0$, implying that Δ_{A0} and Δ_{0B} have opposite signs. Specifically, $E(Z|piv_B,q,s)<E(Z|piv_A,q,s)$ implies that $\Delta_{A0}>0>\Delta_{0B}$, as (19) and (21) make clear. But this is equivalent to $T_{A0}^{br}<qs<T_{0B}^{br}$, implying that the threshold pair (T,T) does not characterize its own best response.

Symmetric belief thresholds $T_1=-T_2$ induce symmetric voting behavior with respect both to candidates and to the state variable, so $\phi_z(A)=\phi_{-z}(B)$, $\psi_z(a,b)=\psi_{-z}(b,a)$, $\pi_z^j(m)=\pi_{-z}^j(-m)$, and $\Pr_z(piv_A)=\Pr_{-z}(piv_B)$. Also, $E(Z|piv_A)=-E(Z|piv_B)$ and $E(Z^2|piv_A)=E(Z^2|piv_B)$. In that case, however, it is clear from (19) and (21) that $T_{0B}^{br}=T_{A0}^{br}$ if and only if $\bar{x}=0$. ■

[32] T_1^{br} and T_2^{br} can be defined similarly, of course, if $x_A>x_B$.

[33] Continuity again follows from the continuity of component functions $\pi_z^j(m)$, $\psi_z(a,b)$, and $\phi_z(j)$.

Proposition 3 *If candidates are committed and policy-motivated then* (x_A^*, x_B^*, σ^*) *is a perfect Bayesian equilibrium only if* σ^* *is almost everywhere equivalent to a belief threshold strategy, with* $T_1^* < T_2^*$*, and candidate platforms are given by* $x_j^* = \hat{z}_j$ *for* $j = A, B$*, with* $x_A^* \neq x_B^*$*. Furthermore, such an equilibrium exists, with platforms* $x_A^* = -x_B^*$ *and belief thresholds* $T_1^*(x_A^*, x_B^*) = -T_2^*(x_A^*, x_B^*)$ *symmetric around zero.*

Proof. The logic of this proof is identical to that of theorem 2, merely utilizing lemma 2 instead of lemma 1. ■

Proposition 4 (Jury Theorem 3) *If candidates are committed and policy-motivated and the strategy combination* $\left[(x_j^*)_{j=A,B}, \sigma^*\right]_n \in [-1,1]^2 \times \Sigma'$ *maximizes welfare* $Eu(Y, Z; n)$ *for a particular population size parameter* n*, then*

Theorem 2 *1.* $\left[(x_j^*)_{j=A,B}, \sigma^*\right]_n$ *constitutes a perfect Bayesian equilibrium, and*

2. The associated sequence Y_n^* *of equilibrium policy outcomes approaches* $p\lim_{n\to\infty}(Y_n^*|Z) = Z$*.*

Proof. The logic of this proof is identical to that of theorem 2, merely replacing Σ with Σ'. ■

Corollary 1 *If candidates are committed and have mixed motivation then there exists a* γ^* *such that* (x_A^*, x_B^*, σ^*) *is a perfect Bayesian equilibrium only if* σ^* *is almost everywhere equivalent to a belief threshold strategy and either* $\gamma \geq \gamma^*$ *and* $x_A^* = x_B^* = 0$ *or* $\gamma < \gamma^*$ *and* $E(Z|W = A) < x_A^* < x_B^* < E(Z|W = B)$*. In the latter case,* x_A^* *increases and* x_B^* *decreases (and welfare decreases) as* γ *increases. Furthermore, an equilibrium exists, with platforms* $x_A^* = -x_B^*$ *and belief thresholds* $T_1^*(x_A^*, x_B^*) = -T_2^*(x_A^*, x_B^*)$ *symmetric around zero.*

Proof. Lemma 2 shows that σ^* is inconsistent with equilibrium unless it induces a belief threshold strategy in every subgame for which $x_A \neq x_B$ (which is almost everywhere). Fixing voting behavior, $x_A^* = x_B^* < 0$ is inconsistent with equilibrium (and, by symmetric arguments, so is $x_A^* = x_B^* > 0$) because a candidate who loses with probability $\frac{1}{2}$ can improve his probability of winning above $\frac{1}{2}$ by moving toward zero. In that case, the platform $E(Z|W = j)$ that maximizes the policy portion of his utility is positive (as the proof of theorem 2 shows), implying that the move toward zero improves both utility components.

If $x_A < x_B$ and $x_B \geq E(Z|W = B)$ then candidate B can increase (13) by lowering his platform policy, which increases $\Pr(W = j)$ by moving toward his opponent (as the proof of theorem 1 shows), and also increases $Eu(x, Z)$ by moving toward $E(Z|W = B)$. The same logic holds for candidate A, so $E(Z|W=A) < x_A^* < x_B^* < E(Z|W=B)$ as long as $\gamma > 0$. In equilibrium, it must be that the marginal decrease in $\gamma P(W=B)$ from increasing x_B equals the marginal increase in $(1-\gamma) Eu(Y,Z)$. If γ increases, however, $\frac{\partial}{\partial x_j}\gamma P(W=B)$ becomes more negative, and $\frac{\partial}{\partial x_j}(1-\gamma) Eu(Y,Z)$ becomes less positive, implying that B's best response to x_A decreases. By symmetric reasoning, of course, candidate A's best response to x_B increases with γ. Thus, as γ increases, the distance between x_A and x_B shrinks, until they converge for $\gamma \geq \gamma^*$. Since proposition 4 implies that the optimal policy outcome is $E(Z|W=j)$, moving platforms away from this reduces expected welfare.

Restricting attention to symmetric platform pairs $x_A = -x_B$, the symmetry of the model guarantees symmetric best-response platforms $x_A^{br} = -x_B^{br}$. Therefore, $x_B^{br}(x_B)$ can be viewed as a continuous function that maps platform pairs $(-x_B, x_B)$ into best-response platform pairs $\left(-x_B^{br}, x_B^{br}\right)$. Interpreted this way, x_B^{br} is a continuous function from the compact space $[0,1]$ of positive platforms into itself, so by Brouwer's theorem there exists a fixed point $x_B^* = x_B^{br}(x_B^*)$ such that $(-x_B^*, x_B^*, \sigma^*)$ constitutes a perfect Bayesian equilibrium. As in proposition 3, these symmetric platforms induce symmetric belief thresholds $T_1^*(x_A^*, x_B^*) = -T_2^*(x_A^*, x_B^*)$. ■

Lemma 3 *If $\bar{\sigma}$ is an informative voting subgame strategy then $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ for all $a, b \in \mathbb{Z}_+$.*

Proof. If $\bar{\sigma}$ is informative then it is straightforward to confirm from (4) that $\phi_z(A)$ and $\phi_z(B)$ are decreasing and increasing in z, respectively. Note from (5) that $\psi_z(a, b+1) = \frac{n\phi_z(B)}{b+1}\psi_z(a,b)$, which implies that the expectation $\hat{z}_{a,b}$ from (14) increases with an additional B vote,

$$\begin{aligned}
\hat{z}_{a,b+1} &= \frac{(-1)\,\psi_{-1}(a,b+1) + (1)\,\psi_1(a,b+1)}{\psi_{-1}(a,b+1) + \psi_1(a,b+1)} \\
&= \frac{(-1)\,\phi_{-1}(B)\,\psi_{-1}(a,b) + (1)\,\phi_1(B)\,\psi_1(a,b)}{\phi_{-1}(B)\,\psi_{-1}(a,b) + \phi_1(B)\,\psi_1(a,b)} \\
&> \frac{(-1)\,\phi_1(B)\,\psi_{-1}(a,b) + (1)\,\phi_1(B)\,\psi_1(a,b)}{\phi_1(B)\,\psi_{-1}(a,b) + \phi_1(B)\,\psi_1(a,b)} \\
&= \hat{z}_{a,b}.
\end{aligned}$$

Similarly, $\psi_z(a+1,b) = \frac{n\phi_z(A)}{b+1}\psi_z(a,b)$, which implies that $\hat{z}_{a,b}$ decreases with each additional A vote (i.e. $\hat{z}_{a+1,b} < \hat{z}_{a,b}$). ■

Theorem 4 (Signaling Equilibrium) *If candidates are responsive and policy motivated and σ^* is an informative voting strategy then $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ is a perfect Bayesian equilibrium only if*

1. $y_j^*(a,b) = \hat{z}_{a,b}$ for all $a,b \in \mathbb{Z}_+$ and for $j = A,B$, and $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ for all $a,b \in \mathbb{Z}_+$.

2. σ^* is a belief threshold strategy.

Furthermore, such an equilibrium exists, which further satisfies the following:

3. Platforms are given by $x_j^* = E(Z|W=j)$ for $j = A,B$.

4. Platforms $x_A^* = -x_B^*$ and policy functions $y_A^*(a,b) = -y_B^*(b,a)$ are symmetric around zero, and $T^*(x_A,x_B) = 0$ for all (x_A,x_B) pairs.

Proof. $\hat{z}_{a,b}$ maximizes the expectation of (1), conditional on vote totals a and b, and the inequalities in Part 1 are stated in lemma 3, since σ^* is informative. To see part 2, first let $\hat{z}_{a,b,q,s} \equiv E(Z|N_A = a, N_B = b, Q = q, Q = s)$ denote a citizen's expectation of the state, conditional on her own private information (q,s) and on vote totals $a,b \in \mathbb{Z}_+$:

$$\begin{aligned}
\hat{z}_{a,b,q,s} &= \frac{\sum_{z=-1,1} z\psi_z(a,b)\Pr(z|q,s)}{\sum_{z=-1,1} \psi_z(a,b)\Pr(z|q,s)} \\
&= \frac{\sum_{z=-1,1} z\psi_z(a,b)\frac{1+qsz}{2}}{\sum_{z=-1,1} \psi_z(a,b)\frac{1+qsz}{2}}.
\end{aligned} \tag{23}$$

The derivative $\frac{\partial}{\partial qs}\hat{z}_{a,b,q,s}$ has the same sign as

$$\begin{aligned}
& \sum_{z=-1,1} z^2\psi_z(a,b)\left[\sum_{z=-1,1}\psi_z(a,b) + \sum_{z=-1,1} z\psi_z(a,b)\right] \\
& -\left[\sum_{z=-1,1} z\psi_z(a,b) + \sum_{z=-1,1} z^2\psi_z(a,b)\right]\sum_{z=-1,1} z\psi_z(a,b) \\
=& \sum_{z=-1,1} z^2\psi_z(a,b)\sum_{z=-1,1}\psi_z(a,b) - \sum_{z=-1,1} z\psi_z(a,b)\sum_{z=-1,1} z\psi_z(a,b) \\
=& [\Pr(a,b)]^2\left\{E\left(Z^2|a,b\right) - [E(Z|a,b)]^2\right\} \\
=& [\Pr(a,b)]^2 V(Z|a,b) > 0,
\end{aligned}$$

implying that (23) increases with qs for any $a,b \in \mathbb{Z}_+$. The expected benefit $\Delta_{AB}(q,s)$ of voting B instead of A therefore increases with qs as well, as (15) can be rewritten as a weighted average (across (N_A,N_B) pairs) of an increasing function of (23):

$$\begin{aligned}
\Delta_{AB}(q,s) &= E_{Z,N_A,N_B}\left[-\left(\hat{z}_{N_A,N_B+1} - Z\right)^2 + \left(\hat{z}_{N_A+1,N_B} - Z\right)^2 |Q=q,Q=s\right] \\
&= E_{N_A,N_B}\left\{E_Z\left[\left(\hat{z}_{N_A,N_B+1} - \hat{z}_{N_A+1,N_B}\right)\left(Z - \frac{\hat{z}_{N_A+1,N_B} + \hat{z}_{N_A,N_B+1}}{2}\right)|N_A,N_B,Q=q,Q=s\right]\right\} \\
&= E_{N_A,N_B}\left[\left(\hat{z}_{N_A,N_B+1} - \hat{z}_{N_A+1,N_B}\right)\left(\hat{z}_{N_A,N_B,q,s} - \frac{\hat{z}_{N_A+1,N_B} + \hat{z}_{N_A,N_B+1}}{2}\right)\right].
\end{aligned} \tag{24}$$

This implies the existence of a belief threshold $T \in [-1, 1]$ such that $\Delta_{AB}(q, s)$ is positive if and only if $qs \geq T$. In other words, σ^* is a best response to $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ only if it is a belief threshold strategy.

In the voting subgame associated with any platform pair, the belief threshold strategy $\bar{\sigma}^*$ for which $T = 0$ constitutes an equilibrium. To see this, simply note that, under $\bar{\sigma}^*$, (4) reduces such that $\phi_{-1}(A) = \phi_1(B) = \int_0^1 q dF(q) = E(Q)$ and $\phi_1(A) = \phi_{-1}(B) = \int_0^1 (1-q) dF(q) = 1 - E(Q)$, making clear that $\frac{\phi_{-1}(B)}{\phi_{-1}(A)} < \frac{\phi_1(B)}{\phi_1(A)}$, and therefore $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ by lemma 3. The best response to σ^* is therefore a belief threshold strategy, and symmetry further implies that (5) and (14) reduce such that $\psi_z(a, b) = \psi_{-z}(b, a)$ and $\hat{z}_{a,b} = -\hat{z}_{b,a}$. A citizen for whom $qs = 0$ is then indifferent between voting A and voting B:

$$\begin{aligned}
\Delta_{AB}(0, s) &= \sum_{z=1,-1} \sum_{a=0}^{\infty} \sum_{b=a+1}^{\infty} \left(\hat{z}_{a,b+1} - \hat{z}_{a+1,b}\right) \left(z - \frac{\hat{z}_{a+1,b} + \hat{z}_{a,b+1}}{2}\right) \psi_z(a, b) \\
&\quad + \sum_{z=1,-1} \sum_{b=0}^{\infty} \sum_{a=b+1}^{\infty} \left(\hat{z}_{a,b+1} - \hat{z}_{a+1,b}\right) \left(z - \frac{\hat{z}_{a+1,b} + \hat{z}_{a,b+1}}{2}\right) \psi_z(a, b) \\
&= \sum_{z=1,-1} \sum_{a=0}^{\infty} \sum_{b=a+1}^{\infty} \left(\hat{z}_{a,b+1} - \hat{z}_{a+1,b}\right) \left(z - \frac{\hat{z}_{a+1,b} + \hat{z}_{a,b+1}}{2}\right) \psi_z(a, b) \\
&\quad + \sum_{z=1,-1} \sum_{b=0}^{\infty} \sum_{a=b+1}^{\infty} \left(-\hat{z}_{b+1,a} + \hat{z}_{b,a+1}\right) \left(z - \frac{-\hat{z}_{b,a+1} - \hat{z}_{b+1,a}}{2}\right) \psi_{-z}(b, a) \\
&= \sum_{z=1,-1} \sum_{a=0}^{\infty} \sum_{b=a+1}^{\infty} \left(\hat{z}_{a,b+1} - \hat{z}_{a+1,b}\right) \left(z - \frac{\hat{z}_{a+1,b} + \hat{z}_{a,b+1}}{2}\right) \psi_z(a, b) \\
&\quad - \sum_{\tilde{z}=-1,1} \sum_{\tilde{a}=0}^{\infty} \sum_{\tilde{b}=c+1}^{\infty} \left(-\hat{z}_{\tilde{a}+1,\tilde{b}} + \hat{z}_{\tilde{a},\tilde{b}+1}\right) \left(\tilde{z} - \frac{\hat{z}_{\tilde{a},\tilde{b}+1}\hat{z}_{\tilde{a}+1,\tilde{b}}}{2}\right) \psi_{\tilde{z}}\left(\tilde{a}, \tilde{b}\right) \\
&= 0.
\end{aligned}$$

Thus, $\bar{\sigma}^*$ is its own best response, and the strategy σ^* that induces $\bar{\sigma}^*$ in every voting subgame constitutes a best response to any platform pair.

Since the equilibrium conditions for voting and policy subgames do not vary with the campaign platform pair adopted by candidates at the beginning of the game, any choice of platform policies can be consistent with equilibrium, including $x_j^* = E(Z|W = j)$, as in part 3. The symmetry of σ^* and y_j^* imply symmetry in $\psi_z(a, b) = \psi_{-z}(b, a)$ and therefore $E(Z|W = A) = -E(Z|W = B)$, as claimed in part 4. ■

Proposition 5 (Jury Theorem 4) *If candidates are responsive and policy motivated, abstention is allowed, and the strategy combination* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]_n \in [-1, 1]^2 \times \Upsilon^2 \times \Sigma$ *maximizes welfare* $Eu(Y, Z; n)$ *for a particular population size parameter* n*, then*

Theorem 4 *1.* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]_n$ *constitutes a perfect Bayesian equilibrium, and*

2. The associated sequence Y_n^* *of equilibrium policy outcomes approaches* $p\lim_{n\to\infty}\left(Y_n^*|Z\right) = Z$.

Proof. The proof of this theorem is identical to the proof of theorem 4, with the addition of policy functions y_j^*. ∎

Theorem 6 (Signaling Voter's Curse) *If candidates are responsive and policy motivated and voter abstention is allowed and* σ^* *is an informative voting strategy then* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ *is a perfect Bayesian equilibrium only if*

1. $y_j^*(a,b) = \hat{z}_{a,b}$ *for all* $a, b \in \mathbb{Z}_+$ *and for* $j = A, B$, *and* $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ *for all* $a, b \in \mathbb{Z}_+$.

2. σ^* *is a belief threshold strategy, with* $T_1^* < T_2^*$.

Furthermore, such an equilibrium exists, which further satisfies the following:

3. Platforms are given by $x_j^* = E(Z|W = j)$ *for* $j = A, B$.

4. Platforms $x_A^* = -x_B^*$, *policy functions* $y_A^*(a,b) = -y_B^*(b,a)$, *and belief thresholds* $T_1^* = -T_2^*$ *are all symmetric around zero.*

Proof. $\hat{z}_{a,b}$ maximizes the expectation of (1), conditional on vote totals a and b, and the inequalities in Part 1 are stated in lemma 3, since σ^* is informative. Since $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$, the expected benefit $\Delta_{0B}(q,s)$ to an individual of type (q,s) of voting B instead of abstaining (in any voting subgame) can, like (15), be written as an increasing function

$$\Delta_{0B}(q,s) = E_{N_A,N_B}\left[\left(\hat{z}_{N_A,N_B+1} - \hat{z}_{N_A,N_B}\right)\left(\hat{z}_{N_A,N_B,q,s} - \frac{\hat{z}_{N_A,N_B} + \hat{z}_{N_A,N_B+1}}{2}\right)\right]$$

of the conditional expectation $\hat{z}_{a,b,q,s}$, defined in (23). The proof of theorem 4 shows that $\hat{z}_{a,b,q,s}$ increases in qs, implying that $\Delta_{0B}(q,s)$ increases in qs as well. This implies the existence of a belief threshold T_{0B}^{br} such that $\Delta_{0B}(q,s) \geq 0$ if and only if $qs \geq T_{0B}^{br}$. By similar reasoning, there exist thresholds T_{A0}^{br} and T_{AB}^{br} such that the benefits $\Delta_{A0}(q,s)$ or $\Delta_{AB}(q,s)$ of abstaining or voting B instead of voting A are positive if and only if $qs \geq T_{A0}^{br}$ or $qs \geq T_{AB}^{br}$, respectively. Setting $T_1^{br} = \min\left\{T_{AB}^{br}, T_{A0}\right\}$ and $T_2^{br} = \max\left\{T_{AB}^{br}, T_{0B}^{br}\right\}$ then defines a belief threshold subgame strategy that is the unique best response to the subgame strategy induced by σ^*. An analogous derivation for each subgame produces a belief threshold strategy that is the unique best response to σ^*.

Not all citizens vote (i.e. $T_1^* < T_2^*$), because perfectly uninformed (and, by continuity, sufficiently uninformed) citizens prefer to abstain. This is because such citizens possess no better information regarding the underlying state of the world than the candidate does. Conditional on a particular pair (a,b) of vote totals for either candidate, therefore, the policy that uniquely maximizes an uninformed citizen's utility function is precisely the policy $y^*(a,b) = \hat{z}_{a,b}$ that the candidate adopts in equilibrium.[34] By abstaining, therefore, she achieves her optimal policy no matter the electoral outcome. If she instead voted for candidate A or B, she would merely push the policy outcome to $\hat{z}_{a+1,b}$ or $\hat{z}_{a,b+1}$, away from her own optimum.

The proof of existence is almost identical to the that of theorem 4: if belief thresholds $T_1 = -T_2$ are symmetric around zero then (4) and (5) reduce such that $\phi_z(A) = \phi_{-z}(B)$ and $\psi_z(a,b) = \psi_{-z}(b,a)$, so that expectations $\hat{z}_{a,b} = -\hat{z}_{b,a}$ are symmetric as well. This implies symmetric differences $\Delta_{AB}(q,-s) = -\Delta_{AB}(q,s)$ and $\Delta_{0B}(q,-s) = \Delta_{A0}(q,s)$ in expected utility,

$$\begin{aligned}
\Delta_{0B}(q,-s) &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left[-\left(\hat{z}_{a,b+1}-z\right)^2+\left(\hat{z}_{a,b}-z\right)^2\right]\psi_z(a,b)\frac{1}{2}(1-zsq) \\
&= \sum_{\tilde{z}=-1,1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left[-\left(-y^*_{b+1,a}+\tilde{z}\right)^2+\left(-y^*_{b,a}+\tilde{z}\right)^2\right]\psi_{\tilde{z}}(a,b)\frac{1}{2}(1+\tilde{z}sq) \\
&= \sum_{\tilde{z}=-1,1}\sum_{b=0}^{\infty}\sum_{a=0}^{\infty}\left[-\left(\hat{z}_{a+1,b}-\tilde{z}\right)^2+\left(\hat{z}_{a,b}-\tilde{z}\right)^2\right]\psi_{\tilde{z}}(a,b)\frac{1}{2}(1+\tilde{z}sq) \\
&= \Delta_{A0}(q,s),
\end{aligned}$$

and therefore symmetric thresholds $T^{br}_{AB} = 0$ and $T_{A0} = -T^{br}_{0B}$, so that best-response belief thresholds $T^{br}_1 = -T^{br}_2$ are symmetric around zero as well. In other words, the thresholds $\left(-T^{br}_2, T^{br}_2\right)$ characterize the best response to the strategy characterized by $(-T,T)$; thus, $T^{br}_2(T)$ can be viewed as a continuous function from the compact set $[0,1]$ of thresholds into itself, and Brouwer's theorem guarantees the existence of a fixed point $(-T^*_2, T^*_2)$, characterizing a strategy that is its own best response, and thus an equilibrium in the voting subgame. If σ^* induces this strategy for every subgame and $\left(x^*_j, y^*_j\right)_{j=A,B}$ are defined as described above then $\left[\left(x^*_j, y^*_j\right)_{j=A,B}, \sigma^*\right]$ is a perfect Bayesian equilibrium of the larger election game. ■

Proposition 6 (Jury Theorem 5) *If candidates are responsive and policy motivated, abstention is allowed, and the strategy combination* $\left[\left(x^*_j, y^*_j\right)_{j=A,B}, \sigma^*\right]_n \in [-1,1]^2 \times \Upsilon^2 \times \Sigma'$ *maximizes welfare* $Eu(Y,Z;n)$ *for a particular population size parameter* n*, then*

Theorem 6 *1.* $\left[\left(x^*_j, y^*_j\right)_{j=A,B}, \sigma^*\right]_n$ *constitutes a perfect Bayesian equilibrium, and*

2. The associated sequence Y^*_n *of equilibrium policy outcomes approaches* $p\lim_{n\to\infty}(Y^*_n|Z) = Z$*.*

[34] Recall that, by the environmental equivalence property of Poisson games (see Myerson, 1998), a citizen perceives the same distribution of A and B votes cast by her peers as a candidate or other outside observer perceives.

Proof. The proof of this theorem is identical to the proof of theorem 5, except replacing Σ with Σ'. ■

Lemma 4 *If $\bar{\sigma}$ is an informative subgame strategy then $\hat{z}_{a+1,b,c,d} < \hat{z}_{a,b+1,c,d} < \hat{z}_{a,b,c,d} < \hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1}$ for all $a, b, c, d \in \mathbb{Z}_+$.*

Proof. First note from (16) that $\psi_z(a, b, c+1, d) = \frac{n\phi_z(C)}{c+1}\psi_z(a, b, c, d)$ and $\psi_z(a, b, c, d+1) = \frac{n\phi_z(D)}{d+1}\psi_z(a, b, c, d)$, where $\psi_z(a, b, c, d)$ denotes the join probability of observing vote totals $(N_A, N_B, N_C, N_D) = (a, b, c, d)$. The inequality $\frac{\phi_1(C)}{\phi_{-1}(C)} < \frac{\phi_1(D)}{\phi_{-1}(D)}$ then implies that a D vote increases the expectation $\hat{z}_{a,b,c,d}$ from (14) by more than an additional C vote:

$$\begin{aligned}
\hat{z}_{a,b,c,d+1} &= \frac{(-1)\,\psi_{-1}(a, b, c, d+1) + (1)\,\psi_1(a, b, c, d+1)}{\psi_{-1}(a, b, c, d+1) + \psi_1(a, b, c, d+1)} \\
&= \frac{-\phi_{-1}(D)\,\psi_{-1}(a, b, c, d) + \phi_1(D)\,\psi_1(a, b, c, d)}{\phi_{-1}(D)\,\psi_{-1}(a, b, c, d) + \phi_1(D)\,\psi_1(a, b, c, d)} \\
&> \frac{-\phi_{-1}(C)\,\psi_{-1}(a, b, c, d) + \phi_1(C)\,\psi_1(a, b, c, d)}{\phi_{-1}(C)\,\psi_{-1}(a, b, c, d) + \phi_1(C)\,\psi_1(a, b, c, d)} \\
&= \hat{z}_{a,b,c+1,d}.
\end{aligned}$$

Similar reasoning establishes the remaining inequalities. ■

Theorem 8 (Multiple candidates) *If candidates A, B, C, and D are responsive and policy motivated, voter abstention is allowed, and σ^* is an informative voting strategy then $\left[\left(x_j^*, y_j^*\right)_{j=A,B,C,D}, \sigma^*\right]$ is a perfect Bayesian equilibrium only if*

1. $y_j^*(a, b, c, d) = \hat{z}_{a,b,c,d}$ for all $a, b, c, d \in \mathbb{Z}_+$ and for $j = A, B, C, D$, and $\hat{z}_{a+1,b,c,d} < \hat{z}_{a,b+1,c,d} < \hat{z}_{a,b,c,d} < \hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1}$.

2. σ^* is a belief threshold strategy, with $T_1^* < T_2^* < T_3^* < T_4^*$.

Furthermore, such an equilibrium exists, which further satisfies the following:

3. Platforms are given by $x_j^* = E(Z|W = j)$ for $j = A, B, C, D$.

4. Platforms $x_A^* = -x_D^*$ and $x_B^* = -x_C^*$, policy functions $y_j^*(a, b, c, d) = -y_{j'}^*(d, c, b, a)$, and belief thresholds $T_1^* = -T_4^*$ and $T_2^* = -T_3^*$ are all symmetric around zero.

Proof. $\hat{z}_{a,b}$ maximizes the expectation of (1), conditional on vote totals a, b, c, and d, and the inequalities in Part 1 are stated in lemma 4, since σ^* is informative. Analogous to (24), the expected benefit $\Delta_{CD}(q, s)$ to a citizen of type (q, s) of voting for candidate D instead of candidate C can be written as an increasing function

$$\Delta_{CD}(q, s) = E_{N_A,N_B,N_C,N_D}\left[\begin{array}{c} \left(\hat{z}_{N_A,N_B,N_C,N_D+1} - \hat{z}_{N_A,N_B,N_C+1,N_D}\right)\times \\ \left(\hat{z}_{N_A,N_B,N_C,N_D,q,s} - \frac{\hat{z}_{N_A,N_B,N_C+1,N_D}+\hat{z}_{N_A,N_B,N_C,N_D+1}}{2}\right) \end{array}\right], \quad (25)$$

of a voter's expectation $\hat{z}_{N_A,N_B,N_C,N_D,q,s} \equiv E\left[Z|\left(N_A,N_B,N_C,N_D\right)=(a,b,c,d),Q_i=q,S_i=s\right]$ of the state, conditional on the voting outcome and her own private information. This expectation is increasing in qs (by reasoning identical to the argument in the proof of theorem 4, that (23) is increasing in qs), which implies that $\Delta_{CD}(q,s)$ is increasing in qs as well, in turn implying the existence of a belief threshold T_{CD} such that citizens prefer voting D to voting C if and only if $qs \geq T_{CD}$. By an analogous derivation, $\Delta_{jj'}(q,s)$ is increasing in qs whenever j precedes j' in the ordering $\{A,B,C,D\}$, implying the existence of thresholds $T^{br}_{jj'}$ such that a citizen prefers voting for j' instead of j if and only if $qs \geq T^{br}_{jj'}$.

The expected benefit $\Delta_{0D}=\Delta_{0C}+\Delta_{CD}$ of voting D instead of abstaining can be decomposed into the benefit of voting C instead of abstaining, and the benefit of voting D instead of voting C. When a citizen of type (q,s) is indifferent between abstaining and voting D, she strictly prefers to vote C. To see this, rewrite (25) as

$$\Delta_{CD}(q,s)=E_{N_A,N_B,N_C,N_D}\left[u_{q,s}\left(\hat{z}_{N_A,N_B,N_C,N_D+1}\right)-u_{q,s}\left(\hat{z}_{N_A,N_B,N_C+1,N_D}\right)\right],$$

in terms of the quadratic distance $u_{q,s}(x) \equiv -\left(x-\hat{z}_{a,b,c,d,q,s}\right)$ between policy x and the expectation $\hat{z}_{a,b,c,d,q,s}$. Rewriting Δ_{0C} similarly, the difference can be written as

$$\begin{aligned}
\Delta_{0C}(q,s)-\Delta_{CD}(q,s) &= E_{N_A,N_B,N_C,N_D}\left[\begin{array}{c} u_{q,s}\left(\hat{z}_{N_A,N_B,N_C+1,N_D}\right)-u_{q,s}\left(\hat{z}_{N_A,N_B,N_C,N_D}\right) \\ -u_{q,s}\left(\hat{z}_{N_A,N_B,N_C,N_D+1}\right)+u_{q,s}\left(\hat{z}_{N_A,N_B,N_C+1,N_D}\right)\end{array}\right] \\
&= 2E_{N_A,N_B,N_C,N_D}\left[u_{q,s}\left(\hat{z}_{N_A,N_B,N_C+1,N_D}\right)-\frac{u_{q,s}\left(\hat{z}_{N_A,N_B,N_C,N_D}\right)+u_{q,s}\left(\hat{z}_{N_A,N_B,N_C,N_D}\right.}{2}\right. \\
&> 0,
\end{aligned}$$

with inequality because $u_{q,s}$ is concave in x, and $\hat{z}_{N_A,N_B,N_C+1,N_D}$ lies between $\hat{z}_{N_A,N_B,N_C,N_D}$ and $\hat{z}_{N_A,N_B,N_C,N_D+1}$. Together, $\Delta_{0D}(q,s)=0$ and $\Delta_{0C}(q,s)>\Delta_{CD}(q,s)$ imply that $\Delta_{0C}(q,s)>0>\Delta_{CD}(q,s)$. Thus, $T^{br}_{0C}<T^{br}_{CD}$. Similarly, $T^{br}_{jj'}<T^{br}_{jj''}$ whenever j precedes j' precedes j'' in the order $\{A,B,0,C,D\}$. The best response to σ^* is therefore a belief threshold strategy with thresholds $T^{br}_{AB}<T^{br}_{B0}<T^{br}_{0C}<T^{br}_{CD}$, establishing part 2.

If the belief thresholds that characterize a voting subgame strategy are symmetric around zero (i.e. $T_1=-T_4$ and $T_2=-T_3$) then (4) and (16) reduce such that $\phi_z(A)=\phi_{-z}(D)$, $\phi_z(B)=\phi_{-z}(C)$, and $\psi_z(a,b,c,d)=\psi_{-z}(d,c,b,a)$, so that expectations $\hat{z}_{a,b,c,d}=-\hat{z}_{d,c,b,a}$

are symmetric as well. Furthermore, $\hat{z}_{a,b,c,d,q,s} = -\hat{z}_{d,c,b,a,q,-s}$:

$$\begin{aligned}
\hat{z}_{a,b,c,d,q,s} &= \frac{\int_{-1}^{1} z\psi_z(a,b,c,d)\frac{1+qsz}{2}}{\int_{-1}^{1}\psi_z(a,b,c,d)\frac{1+qsz}{2}} \\
&= \frac{\int_{1}^{-1}(-\tilde{z})\psi_{-\tilde{z}}(a,b,c,d)\frac{1-qs\tilde{z}}{2}}{\int_{1}^{-1}\psi_{-\tilde{z}}(a,b,c,d)\frac{1-qs\tilde{z}}{2}} \\
&= \frac{-\int_{-1}^{1}\tilde{z}\psi_{\tilde{z}}(d,c,b,a)\frac{1+q(-s)\tilde{z}}{2}}{\int_{-1}^{1}\psi_{\tilde{z}}(d,c,b,a)\frac{1+q(-s)\tilde{z}}{2}} \\
&= \hat{z}_{d,c,b,a,q,-s},
\end{aligned}$$

implying that

$$\begin{aligned}
\Delta_{CD}(q,s) &= \sum_{(a,b,c,d)\in\mathbb{Z}_+^4}\left(\hat{z}_{a,b,c,d+1}-\hat{z}_{a,b,c+1,d}\right)\left(\hat{z}_{a,b,c,d,q,s}-\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right) \\
&= \sum_{\left(\tilde{d},\tilde{c},\tilde{b},\tilde{a}\right)\in\mathbb{Z}_+^4}\left(\hat{z}_{\tilde{d},\tilde{c},\tilde{b},\tilde{a}+1}-\hat{z}_{\tilde{d},\tilde{c},\tilde{b}+1,\tilde{a}}\right)\left(\hat{z}_{\tilde{d},\tilde{c},\tilde{b},\tilde{a},q,s}-\frac{\hat{z}_{\tilde{d},\tilde{c},\tilde{b}+1,\tilde{a}}+\hat{z}_{\tilde{d},\tilde{c},\tilde{b},\tilde{a}+1}}{2}\right) \\
&= \sum_{\left(\tilde{d},\tilde{c},\tilde{b},\tilde{a}\right)\in\mathbb{Z}_+^4}\left(-\hat{z}_{\tilde{a}+1,\tilde{b},\tilde{c},\tilde{d}}+\hat{z}_{\tilde{a},\tilde{b}+1,\tilde{c},\tilde{d}}\right)\left(-\hat{z}_{\tilde{a},\tilde{b},\tilde{c},\tilde{d},q,-s}-\frac{-\hat{z}_{\tilde{a},\tilde{b}+1,\tilde{c},\tilde{d}}-\hat{z}_{\tilde{a}+1,\tilde{b},\tilde{c},\tilde{d}}}{2}\right) \\
&= -\sum_{\left(\tilde{d},\tilde{c},\tilde{b},\tilde{a}\right)\in\mathbb{Z}_+^4}\left(\hat{z}_{\tilde{a},\tilde{b}+1,\tilde{c},\tilde{d}}-\hat{z}_{\tilde{a}+1,\tilde{b},\tilde{c},\tilde{d}}\right)\left(\hat{z}_{\tilde{a},\tilde{b},\tilde{c},\tilde{d},q,-s}-\frac{\hat{z}_{\tilde{a},\tilde{b}+1,\tilde{c},\tilde{d}}+\hat{z}_{\tilde{a}+1,\tilde{b},\tilde{c},\tilde{d}}}{2}\right) \\
&= -\Delta_{AB}(q,-s).
\end{aligned}$$

Similarly, $\Delta_{BC}(q,s) = -\Delta_{BC}(q,-s)$. This symmetry implies that belief thresholds $T_{AB}^{br} = -T_{CD}$ and $T_{B0} = -T_{0C}$ are also symmetric around zero. In other words, the best response to the belief threshold strategy characterized by $(-T_4,-T_3,T_3,T_4)$ is the belief threshold strategy characterized by $(-T_{CD},-T_{0C},T_{0C},T_{CD})$. Therefore, (T_{0C},T_{CD}) can be viewed continuous function from the compact set $\{(T_3,T_4): 0 \leq T_3 \leq T_4 \leq 1\}$ of possible thresholds into itself. Interpreted this way, Brouwer's theorem guarantees the existence of a fixed point (T_3^*,T_4^*), such that the belief threshold strategy characterized by $(-T_4^*,-T_3^*,T_3^*,T_4^*)$ is its own best response, and therefore an equilibrium in the voting subgame. If σ^* induces this strategy for every subgame and $\left(x_j^*,y_j^*\right)_{j=A,B}$ are defined as described above then $\left[\left(x_j^*,y_j^*\right)_{j=A,B},\sigma^*\right]$ is a perfect Bayesian equilibrium of the larger election game. ■

Proposition 7 (Jury Theorem 6) *If candidates A, B, C, and D are responsive and policy motivated, abstention is allowed, and the strategy combination* $\left[\left(x_j^*,y_j^*\right)_{j=A,B,C,D},\sigma^*\right]_n \in [-1,1]^4\times\Upsilon^4\times\Sigma''$ *maximizes welfare $Eu(Y,Z;n)$ for a particular population size parameter n, then*

1. $\left[\left(x_j^*, y_j^*\right)_{j=A,B,C,D}, \sigma^*\right]_n$ *constitutes a perfect Bayesian equilibrium, and*
2. The associated sequence Y_n^* *of equilibrium policy outcomes approaches* $p\lim_{n\to\infty}\left(Y_n^*|Z\right) = Z$.

Proof. The proof of this theorem is identical to the proof of theorem 6, except that Σ'' replaces Σ'. ■

Theorem 10 *In the model described in this section, all of the formal results stated in sections 3 and 4 remain valid.*

Proof. The proof of each result follows the logic used previously, with only minor adjustments to accommodate the newly specified model. For example, the expected benefit 11 must be rewritten with an integral instead of a summation (and the conditional density $g(z|s)$ replacing the conditional probability $\Pr(z|s)$),

$$\begin{aligned} \Delta_{AB}(q,s) &= \int \left[u(x_B,z) - u(x_A,z)\right] \Pr_z(piv_B)\,\frac{1}{2}(1+qsz)\,dz \\ &\quad - \int \left[u(x_A,z) - u(x_B,z)\right] \Pr_z(piv_A)\,\frac{1}{2}(1+qsz)\,dz \\ \ldots &= 2(x_B - x_A)\left[E(Z|piv,q,s) - \bar{x}\right]\Pr(piv,q,s), \end{aligned}$$

but the expected value notation remains unchanged. Similarly, the expected vote shares in (18) of lemma A1 in the appendix associated with a belief threshold strategy with $T > 0$ must be obtained by integrating, rather than summing, over signal realizations,

$$\begin{aligned} \phi_z(A) &= F(T) + \textstyle\int_T^1 \int_{-1}^{T/q} \frac{1+qsz}{2} ds dF(q) \\ \phi_z(B) &= \textstyle\int_T^1 \int_{T/q}^1 \frac{1+qsz}{2} ds dF(q), \end{aligned}$$

but it is still the case that $\phi_z(A)$ and $\phi_z(B)$ decrease and increase with z, respectively. The weaker statement that $\frac{\phi_z(B)}{\phi_z(A)}$ increases with z is the natural generalization of the criterion in section 4.1 for a voting strategy to be *informative*; with these simple and natural extensions, the proofs of all of the results above proceed as before. ■

A.1 Revise here:

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