Dynamic Coalitions*

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Abstract

We present a theory of dynamic coalitions for a legislative bargaining game in which policies can be changed in each period but continue in effect in the absence of new legislation. We characterize a class of Markov perfect equilibria with dynamic coalitions, which are decisive sets of legislators whose members strictly prefer preserving the coalition to having it dissolve with a new coalition formation opportunity resulting. The equilibria can support minimal winning, surplus and universal dynamic coalitions as well as positive allocations to non-coalition members. Policies supported can be efficient or inefficient, hence dynamic coalitions can result in political failure. Vested interests can support equilibrium policies that no legislator would propose if forming a coalition. If uncertainty is associated with the implementation of a policy, there is a continuum of policies supported by coalition equilibria. These coalition equilibria have the same allocation in every period when the coalition persists, but with positive probability the coalition dissolves due to the uncertainty. Equilibria also exist in which members tolerate a degree of implementation uncertainty, resulting in coalition policies that can change from one period to the next. Dynamic coalitions are thus robust to implementation uncertainty. The predictions of the theory are compared to experiment results. JEL Classification: C73, D72

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1 Introduction

Most laws and government programs are continuing and remain in effect in the absence of new legislation. Rules promulgated by regulatory commissions also remain in effect until modified or rescinded. Social security, welfare, and other distributive programs are typically continuing, and distributions are governed by formulas that are changed only infrequently. Tax rates also continue in the absence of new legislation. At the state level, legislatures establish continuing policies including Medicaid eligibility and benefits as well as other state distributive programs. State regulatory commissions establish prices, rules governing lifeline and other cross-subsidization programs, and environmental and energy-efficiency policies. These programs have the property that the policy adopted or in place in the current period becomes the status quo for the next period. Policy choice thus can be viewed as a dynamic legislative bargaining game with an endogenous status quo in which a legislature has the opportunity to choose the policy in every period and agenda-setting control can change over time. Despite their dynamic nature and the opportunities for change, many policies are stable and are supported by coalitions that persists from period to period. This paper presents a theory of dynamic coalitions that identifies which coalitions can form in an equilibrium and which policies those coalitions support. Coalitions that persist over time are natural when the preferences of legislators are aligned, so to provide a strong test for dynamic coalitions, the policies considered are purely distributive with the preferences of legislators directly opposing.

In a voting game the term coalition is typically used for a decisive set of players. For example, in sequential legislative bargaining a coalition is a decisive set each of whose members receives a positive allocation, but in dynamic legislative bargaining a legislator with a positive allocation in every period in an equilibrium can strictly prefer that the coalition dissolve rather than continue. A dynamic coalition thus is defined as a decisive set of legislators whose members strictly prefer that the coalition be preserved from one period to the next to it being dissolved with a new coalition formation opportunity resulting. We show the existence of Markov perfect equilibria using a class of basic strategies and characterize the properties of dynamic coalitions and the policies they support. A dynamic coalition can be larger than minimal winning, and the set of legislators receiving a positive allocation can be strictly larger than the set of legislators in the dynamic coalition. Indeed, all legislators may receive positive allocations in every period, yet only the members of the dynamic coalition want the coalition preserved.

The theory provides explanations for empirical phenomena such as universalism and governments formed with surplus members. Studies of distributive politics typically find that benefits are allocated to more than a minimal majority of legislative districts even though a minimal winning decisive set could do strictly better.[1] Moreover, distributive programs, such as those providing pork, are often viewed as inefficient. Dynamic coalitions can support inefficient policies in addition to policies that provide benefits to all legislators. Dynamic coalitions may strictly prefer such policies to a new coalition formation opportunity. When inefficient

[1] See Primo and Snyder (2008), Stein and Bickers (1994), and Weingast (1994).

policies are supported dynamic coalitions result in political failure in the sense of Besley and Coate (1998).

In parliamentary systems governments often include more parties than required for a majority in parliament, and minority governments are frequently observed in a number of political systems.[2] The theory presented here shows that minimal winning, surplus, and consensus coalition governments as well as minority governments can arise in equilibrium even in a pure distribution setting. Moreover, both minimal winning and surplus dynamic coalitions can in equilibrium allocate benefits to out members. The surplus member prefers that the dynamic coalition continue, whereas the out member prefers that the coalition dissolve and a new coalition opportunity commence. A dynamic coalition can have senior and junior members, where the latter have strictly lower allocations than do senior members. What determines whether a legislator is a senior, junior, or out member in a dynamic coalition thus is the size of their allocation.

The dynamic legislative bargaining game considered is an extension of the sequential legislative bargaining game introduced by Baron and Ferejohn (1989). In the stationary equilibrium in that static game a bargain is reached with the first proposal, and the decisive set supporting the bargain is minimal-winning. The proposer captures what otherwise would be the allocation of those legislators excluded from the decisive set and does not share the gains with other members of the decisive set. In the dynamic game with an endogenous status quo, the equilibria exhibit some of the properties of the sequential game but not others. In the basic dynamic game with a sufficiently high discount factor, a dynamic coalition is formed in the first period and persists thereafter. In contrast to sequential legislative bargaining where coalitions are minimal winning, surplus coalitions can result in equilibrium for all decisiveness rules other than unanimity. For simple majority rule the sequential equilibrium policy cannot be supported by a coalition equilibrium. With a supermajority rule, however, it can be supported.

Coalition equilibria use basic strategies and symmetric sets of policies so that no legislator is ex ante advantaged in the legislative bargaining. A basic strategy calls for legislators to propose the status quo if it is in the equilibrium supported set, and if it is not, to randomize among all alternatives that provide the highest continuation value. Legislators vote for the proposal or the status quo depending on which yields the greater dynamic payoff and vote for the status quo when indifferent. Despite the dynamic nature of the game, the necessary and sufficient conditions for a coalition equilibrium can be stated in terms of static payoffs. The coalition equilibria are particularly simple, exhibit policy and coalition stability, and could be coordinated on through straightforward communication between the originator of the coalition and potential coalition partners.

Because strategies are Markov, player-specific, or selective, punishment strategies cannot be used to support equilibrium policies. With basic strategies, however, if a dynamic coalition dissolves, its members are collectively punished, since the new coalition formation opportunity gives all legislators, including legislators outside the coalition, the same payoff. This is less than the payoff from preserving the coalition, and this

[2] See Ansolabehere, Snyder, Strauss and Ting (2005), Laver and Shepsle (1996), and Strom (1990) for theory and evidence on government formation.

difference allows the coalition to be supported as an equilibrium.

Many policies have a degree of uncertainty associated with their implementation, and that uncertainty can affect the policies in a period and hence the status quo in the next period. The uncertainty could be due to exogenous factors or to endogenous factors associated with delegation to an administrative agency or regulatory commission or to choices made by those affected by the policy. Dynamic coalitions are shown to be robust to some implementation uncertainty. For a three-member legislature the basic model is extended to include implementation uncertainty that can cause policies to differ from those in the adopted policy. A class of specific-policy coalition equilibria is characterized in which a dynamic coalition persists as long as implementation uncertainty is not realized and dissolves when it is realized. The dynamic coalition, while it persists, implements the same policy in each period, and the originator of the dynamic coalition shares the gains from proposal power with the coalition partner but not necessarily equally.

A specific-policy coalition dissolves when implementation uncertainty changes the policy, but a coalition could tolerate some change due to uncertainty. Coalition equilibria exist that support a set of tolerated policies where the coalition persists if the allocation remains in the set and dissolves if it is outside the set. A tolerant coalition is more valuable to its members than is the corresponding specific-policy coalition. The originator of the coalition shares proposal power with the coalition partner, and because of the realized implementation uncertainty, the coalition partner could in some periods have a larger allocation than the originator. Tolerant coalitions provide an explanation for coalition governments that survive small shocks but fail in crises.

Epple and Riordan (1987) characterize subgame perfect equilibria in a model of distributive policy with a deterministic rule for selecting the proposer, whereas we characterize a set of coalition equilibria where the proposer is randomly selected. Baron (1996) considers a unidimensional policy and proves a dynamic median voter theorem. A number of recent papers have considered legislative bargaining games with an endogenous status quo, including Baron and Herron (2003), Bernheim, Rangel and Rayo (2006), Anesi (2010), Dziuda and Loeper (2010), Bowen (2011), Diermeier and Fong (2011), Zápal (2012), Nunnari (2012), Piguillem and Riboni (2012), Bowen, Eraslan and Chen (2012), and Nunnari and Zápal (2013). Policies in these papers are not purely distributive with the exception of Bernheim, Rangel and Rayo (2006), which has a finite horizon, and Nunnari (2012) which includes a veto player.

Kalandrakis (2004), Kalandrakis (2010), Bowen and Zahran (2012), Battaglini and Palfrey (2012), Richter (2013), and Anesi and Seidmann (2012) consider Markov perfect equilibrium in a distributive game with an endogenous status quo similar to the game considered here. Kalandrakis was the first to characterize Markov perfect equilibria in this setting, where in equilibrium a rotating dictator has all the bargaining power in a period. For a finite set of alternatives Battaglini and Palfrey (2012) also present an equilibrium that rotates among minimal winning coalitions. Bowen and Zahran (2012), Richter (2013), and Anesi and Seidmann (2012) identify equilibria that exhibit compromise where more than a minimal majority receive an allocation. Bowen and Zahran find compromise with risk-averse legislators, and Battaglini and Palfrey (2012) find compromise in a quantal response equilibrium with risk-averse players. When inefficient policies are possible, Richter (2013) identifies Markov perfect equilibria (MPE) in which all legislators share the benefits equally. Anesi and Seidmann (2012) characterize a class of MPE in which each player proposes a single policy, but all legislators sharing equally is not possible in their construction. In the basic model we characterize a class of equilibria with some similar properties, but players forming dynamic coalitions propose from symmetric sets of policies that may include equal sharing under some conditions. In the basic model we identify equilibria with outcomes as in Richter and properties as in Anesi and Seidmann.

Cooperation or policy moderation in a dynamic policy-making environment has been studied in Dixit, Grossman and Gul (2000), Lagunoff (2001), and Acemoglu, Golosov and Tsyvinski (2011). In contrast to these papers we consider Markov perfect equilibria in a game with a purely distributive policy, where there is no natural incentive to form coalitions or induce policy moderation. Besley and Coate (1998), Battaglini and Coate (2007), Battaglini and Coate (2008), Acemoglu, Egorov and Sonin (2012), and Baron, Diermeier and Fong (2012) show that dynamic incentives can lead to inefficiency. In the distributive policy setting we show that dynamic coalitions may support Pareto dominated policies because the members of the coalition prefer that the current policy persist rather than risk not being a part of a new coalition with a different policy.

Duggan and Kalandrakis (2012) provide a general existence result for dynamic legislative bargaining games with an endogenous status quo and uncertainty over legislators' preferences. In the environment considered here, legislators' preferences are fixed, but straightforward necessary and sufficient conditions for existence of coalition equilibria can be given.[3] Ray and Vohra (2013) embed the legislative bargaining framework in a more general framework that accommodates cooperative game solution concepts, and Anesi (2010) relates bargaining equilibria to the solution concept of stable sets.

Battaglini and Palfrey conducted experiments that implement the dynamic game considered here. Their findings provide some evidence of dynamic coalitions, although participants in the experiments exhibit behaviors that may not correspond to an equilibrium.

The basic model is introduced in the next section, and Section 3 presents the basic strategies used in the coalition equilibria. Section 4 presents necessary and sufficient conditions for a coalition equilibrium and characterizes all coalition equilibria for the basic model. Section 5 introduces implementation uncertainty, and specific-policy coalition equilibria are characterized in Section 6. Section 7 considers coalitions that tolerate a degree of implementation uncertainty. Section 8 compares the coalition equilibria to the results of the Battaglini and Palfrey experiments, and conclusions are provided in the final section.

[3] Duggan (2012) also proves a general existence result for MPE in noisy stochastic games that requires norm-continuity of state transition probabilities. Norm-continuity is violated with voting, however.

2 The Basic Model

The model represents a political process with an endogenous status quo where legislators can adopt a new policy or leave the status quo in place. Legislators in this model could also be thought of as party leaders in a parliamentary system forming a government or governing once in office or members of a commission bargaining each period over the division of a budget. In each period $t = 1, 2, \ldots$, legislator $i \in \{1, \ldots, n\}$ is recognized with probability $p = \frac{1}{n}$ to propose a policy, which is then voted against the status quo policy from the previous period according to an m-majority rule, where $\lceil \frac{n+1}{2} \rceil \leq m \leq n-1$. The winner becomes the policy in place in the current period and the status quo for the next period. In each period legislators allocate a dollar, possibly with waste, so the feasible set of policies in each period is $X = \{x \in \mathbb{R}^n_+ | \sum_{i=1}^n x_i \leq 1\}$. A proposal by a legislator in period t is a policy $y^t \in X$, and the status quo policy at the beginning of period t is denoted $q^{t-1} \in X$. The agenda on which legislators vote is $\{q^{t-1}, y^t\}$, and the implemented policy in period t is denoted by x^t and $q^t = x^t$. Legislator i derives utility $u(x_i^t)$ from the allocation of the dollar it receives in period t with $u(\cdot)$ increasing. Legislators maximize the expectation of the discounted, infinite stream of utilities $\sum_{t=1}^{\infty} \delta^{t-1} u(x_i^t)$, where $\delta \in [0, 1)$ is the discount factor for all legislators. An extension in which discount factors and selection probabilities can differ among legislators is presented in the Appendix. A generalization of the results for the basic model in Section 4 are given for the extended model.

A history of the game may include all proposals made, the identity of the proposer, votes cast and policies implemented. A stationary Markov perfect equilibrium is a subgame perfect equilibrium in which strategies depend only on the payoff-relevant history, which at the proposal stage is the status quo q^{t-1}, and does not depend on calendar time. A stationary Markov strategy for legislator i is a pair of functions (σ_i, ω_i), where $\sigma_i : X \to X$ is a proposal strategy and $\omega_i : X \times X \to \{0, 1\}$ is a voting strategy.[4] Legislator i's proposal strategy $\sigma_i(q^{t-1}) = y^t$ selects a proposal y^t conditional on the status quo. Legislator i's voting strategy $\omega_i(q^{t-1}, y^t)$ assigns a vote conditional on the proposal and the status quo, where $\omega_i(q^{t-1}, y^t) = 1$ denotes a vote for the proposal. The proposal is approved if and only if $\sum_{i=1}^n \omega_i(q^{t-1}, y^t) \geq m$. The status quo q^t in period $t+1$ is then

$$q^t = \begin{cases} q^{t-1} & \text{if } \sum_{i=1}^n \omega_i(q^{t-1}, y^t) < m \\ y^t & \text{if } \sum_{i=1}^n \omega_i(q^{t-1}, y^t) \geq m. \end{cases}$$

The state thus evolves as proposals are made and votes are cast.

Letting σ and ω denote a profile of strategies, the continuation value $v_i(\sigma, \omega | q^{t-1})$ for i depends on t only through the state and is defined by

$$v_i(\sigma, \omega | q^{t-1}) = E^t[u(q_i^t) + \delta v_i(\sigma, \omega | q^t)],$$

[4] We abuse notation slightly by writing proposal strategies as pure strategies. The coalition equilibria we characterize involve mixing, but the mixing among pure strategies is simple so for clarity we use pure strategy notation.

where E^t denotes expectation with respect to the selection of the proposer and any uncertainty affecting payoffs and transitions.

A perfect equilibrium requires that in every subgame for each t every legislator's dynamic payoff is optimal given the equilibrium strategies of the other legislators. That is, a stationary Markov strategy profile (σ^*, ω^*) is a perfect equilibrium if and only if

$$v_i(\sigma^*, \omega^*|q^{t-1}) \geq v_i((\sigma^*_{-i}, \hat{\sigma}_i), (\omega^*_{-i}, \hat{\omega}_i)|q^{t-1}), \text{ for all } (\hat{\sigma}_i, \hat{\omega}_i), i = 1, \ldots, n, \text{ and all } q^{t-1} \in X$$

where the strategies $(\hat{\sigma}_i, \hat{\omega}_i)$ may depend on any history of actions and states. We henceforth refer to a stationary Markov perfect equilibrium simply as an equilibrium.

The focus in this paper is on dynamic coalitions of legislators who are able to sustain favorable policies over time, and the policies they support. In the next section we introduce a class of *basic strategies* to facilitate the analysis of dynamic coalitions, and equilibria using basic strategies are referred to as *coalition equilibria*. Coalition equilibria support dynamic coalitions for all status quos $q^{t-1} \in X$. In Section 4 necessary and sufficient conditions for the existence of a coalition equilibrium are presented, and dynamic coalitions and the policies they support are characterized.

3 Basic Strategies and Coalition Equilibria

This section introduces a class of basic strategies that are simple, result in no-delay equilibria, and have proposals in a subset of X. The strategies are symmetric and hence do not favor any particular legislator in coalition formation. Equilibria employing basic strategies are coalition equilibria.

We focus on supporting a symmetric set $Z \subset X$ of policies in equilibrium. Let $S \subset X$ be a set of policies, and let $\mathcal{Z}(S)$ be the operator that returns the set of all permutations of the policies in S. The set $Z = \mathcal{Z}(S)$ thus is symmetric and provides the same opportunities to each legislator, so no legislator is advantaged. As an example, let $n = 3$ and $S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}$. Then $\mathcal{Z}(S) = \{(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2}), (0, \frac{1}{2}, \frac{1}{2})\}$, and $Z = \mathcal{Z}(S)$ is symmetric.

Let $Z_i \subseteq Z$ be the set of policies that are most advantageous for legislator i, and denote the maximum allocation in Z for any i as $z_{\max}$. Because of symmetry $z_{\max}$ is the same for all i. Then $Z_i = \{z \in Z : z_i = z_{\max}\}$. In the previous example $Z_1 \subset Z$ are all policies that give legislator 1 the allocation $z_{\max} = \frac{1}{2}$, or $Z_1 = \{(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2})\}$. To ensure basic strategies are well-defined, we assume Z is compact and is such that Z_i is finite.

Definition 1. A *basic strategy* profile is a (σ, ω) such that for all $i = 1, \ldots, n$ and for some set of policies Z:

(i) Legislators propose the status quo if it is in the set Z and otherwise randomize over favorable policies in Z

$$\sigma_i(q^{t-1}) = \begin{cases} q^{t-1} & \text{if } q^{t-1} \in Z \\ z \in Z_i \text{ with probability } \frac{1}{|Z_i|} & \text{if } q^{t-1} \notin Z, \end{cases}$$

(ii) voting strategies are stage undominated

$$\omega_i(q^{t-1}, y^t) = \begin{cases} 1 & \text{if } E^t[u(y_i^t) + \delta v_i(\sigma, \omega | y^t)] > E^t[u(q_i^{t-1}) + \delta v_i(\sigma, \omega | q^{t-1})] \\ 0 & \text{if } E^t[u(y_i^t) + \delta v_i(\sigma, \omega | y^t)] < E^t[u(q_i^{t-1}) + \delta v_i(\sigma, \omega | q^{t-1})], \end{cases}$$

(iii) legislator i votes for the status quo when indifferent between the status quo and the proposal, that is, $\omega_i(q^{t-1}, y^t) = 0$ if

$$E^t[u(y_i^t) + \delta v_i(\sigma, \omega | y^t)] \quad = \quad E^t[u(q_i^{t-1}) + \delta v_i(\sigma, \omega | q^{t-1})].$$

Basic strategies are stationary and Markov.

With a basic strategy the set Z includes all the policies implemented once a dynamic coalition is in place. We postpone a formal definition of a dynamic coalition to the next section, but it can be thought of as those legislators who benefit from the continuation of a policy in Z. If a dynamic coalition is not in place, that is, if $q^{t-1} \notin Z$, a new coalition formation opportunity results where the proposer randomizes over all policies that provide the highest allocation from the set Z of policies. In equilibrium, this is equivalent to choosing coalition partners among whom the proposer is indifferent. Once the status quo is in the set Z, legislators propose the status quo whenever they are the proposer, hence sustaining the coalition. They are willing to do so even if the status quo is disadvantageous for them because they know that any other policy will not receive a majority of votes.[5] Randomization among potential coalition partners occurs when a coalition is being formed, and once formed legislators use pure strategies on the equilibrium path. If there is a deviation from that path so that the status quo is not in Z, the next proposer j randomizes over the policies in Z_j, and this serves as a collective punishment to those legislators with high allocations under the status quo. Basic strategies require that legislators vote for the status quo when indifferent between the status quo and the new policy, which assures the stability of the coalition.[6] If the discount factor is sufficiently high and the set Z is appropriately restricted, legislators have no incentive to deviate, resulting in an equilibrium in basic strategies.

We define a coalition equilibrium as an equilibrium in which basic strategies are employed.

[5] The basic strategies could be modified such that legislator i proposes the status quo if $q^{t-1} \in Z_i$ and otherwise proposes at random a policy in Z_i. The same sets of policies are supported by a coalition equilibrium with these strategies, but the notation is a bit more cumbersome.

[6] We discuss the role of the indifference rule in the next section.

Definition 2. The strategy profile (σ, ω) is a *coalition equilibrium* if it is a Markov perfect equilibrium using basic strategies.

To analyze dynamic coalitions, we ask which Z are supported with coalition equilibria.

4 Dynamic Coalitions in the Basic Model

For any symmetric set Z the continuation values in a coalition equilibrium are straightforward to characterize. Basic strategies call for all legislators to propose the status quo if it is in Z, so any policy in Z is an absorbing state. For any status quo not in Z, legislator i, if selected, proposes with equal probability all elements of the set Z_i, and symmetry implies that $|Z_i| = |Z_j|$, so the probabilities of receiving particular allocations are identical across legislators. This gives Lemma 1, the proof of which is straightforward and hence is omitted. To simplify the notation, let $v_i(q^{t-1})$ denote $v_i(\sigma, \omega | q^{t-1})$.

Lemma 1. *In the basic model if (σ, ω) is a coalition equilibrium supporting a set Z:*

(*i*) *The continuation value for player i for $q^{t-1} \in Z$ is $v_i(q^{t-1}) = \frac{u(q_i^{t-1})}{1-\delta}$.*

(*ii*) *The continuation value for player i for $q^{t-1} \notin Z$ is $v_i(q^{t-1}) = v^* \equiv \frac{\overline{u}}{1-\delta}$, where*

$$\overline{u} \equiv \frac{1}{n|Z_i|} \sum_{z \in Z_i} \sum_{i=1}^{n} u(z_i).$$

With a symmetric set Z the continuation value for $q^{t-1} \notin Z$ is the same for all i and for all $q^{t-1} \notin Z$. The continuation value v^* is the discounted average utility $\bar{u}$ available in the proposal made in Z_i when $q^{t-1} \notin Z$. This highlights a feature of the basic strategies that simplifies the characterization of coalition equilibria – every possible deviation from the set Z is met by the same response (random formation of a new coalition in the next period). All such deviations thus result in the same continuation value v^* for all legislators.

A coalition equilibrium and the corresponding continuation values yield a natural definition of a dynamic coalition.

Definition 3. If (σ, ω) is a coalition equilibrium supporting a set Z, a *dynamic coalition* corresponding to $q^{t-1} \in Z$ is a decisive set each of whose members i has $v_i(q^{t-1}) > v^*$.

The condition $v_i(q^{t-1}) > v^*$ is equivalent to $u(q_i^{t-1}) > \bar{u}$, which is a static comparison between the utility from the coalition policy and the average utility if the coalition dissolves.

In a coalition equilibrium a dynamic coalition is a set of legislators all of whose members strictly prefer the coalition, and its policy, to continue to the next period rather than dissolve with a new coalition opportunity resulting in the next period. It is the threat of the coalition dissolving that provides the incentives to support a dynamic coalition and the corresponding coalition equilibrium. There is thus a dynamic coalition corresponding to each policy $z \in Z$ supported by an equilibrium in basic strategies. We provide a simple example of a coalition equilibrium to illustrate these points.

Coalition Equilibrium Example

Let $n = 3$, $m = 2$, $u(x_i) = x_i$, and $S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}$. By Lemma 1, if $q^{t-1} \in Z$, the continuation value for legislators i and j receiving $\frac{1}{2}$ under the status quo is $v_i(q^{t-1}) = v_j(q^{t-1}) = \frac{1}{2(1-\delta)}$, and $v_k(q^{t-1}) = 0$ for the other legislator k. For $q^{t-1} \notin Z$ the continuation value is $v^* = \frac{1}{3(1-\delta)}$ for all i. When $q^{t-1} \in Z$, the dynamic coalition is then naturally each legislator i with $q_i^{t-1} = \frac{1}{2}$.

Suppose the status quo is $q^{t-1} = (\frac{1}{2}, \frac{1}{2}, 0)$ so that legislators 1 and 2 constitute the dynamic coalition, and assume that legislator 3 is selected as the proposer. Suppose legislator 3 proposes $y^t = (0, \frac{1}{2}, \frac{1}{2})$, which gives $\frac{1}{2}$ to coalition member 2. If 2 votes for the proposal, legislators 2 and 3 would constitute a coalition thereafter. Legislator 2 is indifferent between voting for the status quo and the proposal, and, according to the indifference rule in basic strategies, votes against the proposal and for the present coalition $\{1, 2\}$. Legislator 1 also votes no, so q^{t-1} prevails for all $\delta > 0$. Note that if legislator 3 were to offer more than $\frac{1}{2}$ to legislator 2, the proposal would not be in Z, causing the dynamic coalition to dissolve with a new coalition formation opportunity commencing in the next period.

Now suppose legislator 3 proposes $y^t \notin Z$. Coalition members strictly prefer q^{t-1} to the proposal if the discount factor is large enough. The proposal $(1, 0, 0)$ gives legislator 1 the highest dynamic payoff for any proposal not in Z, because all proposals not in Z give the same continuation value v^*. Note that legislator 3 strictly prefers the policy $(1, 0, 0)$ to the status quo for all $\delta > 0$, since

$$0 + \delta \frac{1}{3(1-\delta)} > 0 + \delta 0.$$

Legislator 1 will reject the proposal $(1, 0, 0)$ if and only if

$$\tfrac{1}{2} + \delta \tfrac{1}{2(1-\delta)} \geq 1 + \delta \tfrac{1}{3(1-\delta)},$$

which is satisfied if and only if $\delta \geq \underline{\delta} \equiv \frac{3}{4}$.

If the status quo is $(1, 0, 0)$ and legislator 2 is the proposer, legislator 1 has a strict incentive to accept the equilibrium proposal $(\frac{1}{2}, \frac{1}{2}, 0)$ if and only if $\delta > \frac{3}{4}$. The set Z thus is supported by a coalition equilibrium if and only if $\delta > \frac{3}{4}$. Legislators 1 and 2 will form the dynamic coalition, and once formed strictly prefer to maintain the coalition rather than see it dissolve and face a new coalition formation opportunity.

Dynamic Coalitions Maintain the Status Quo when Indifferent

The indifference rule used in basic strategies specifies that a legislator votes for the status quo when indifferent. The role of the indifference rule is to preclude a coalition member from deviating to an apparently equivalent proposal by a non-coalition legislator. That is, if $q^{t-1} = (\frac{1}{2}, \frac{1}{2}, 0)$, legislator 3 could propose $\left(0, \frac{1}{2}, \frac{1}{2}\right)$, and legislator 2 would be indifferent between the status quo and the proposal. If legislators were to vote for the proposal when indifferent, however, basic strategies would not constitute an equilibrium, since legislator 2 would defect from the coalition with legislator 1. The indifference rule of voting against the proposal when indifferent thus is needed to support a coalition equilibrium in the dynamic legislative bargaining game.[7]

Indifference rules can therefore support at least two types of equilibria. If all legislators vote for the proposal when indifferent, Kalandrakis (2004) demonstrates that a rotating dictatorship is an equilibrium. Coalition equilibria have stable policies and coalitions, whereas the rotating equilibria have policies and majorities that change with the proposer selected in a period. Which is a better predictor of the play of this dynamic game is an empirical matter, and the experiments by Battaglini and Palfrey provide evidence about that play. They conclude that for a legislature with 3 members and efficient policies the experiment results do not support the rotating dictator equilibrium and do support a Markov Logistic Quantal Response Equilibrium (MLQRE) (McKelvey and Palfrey (1995) McKelvey and Palfrey (1998)) when players are sufficiently risk-averse. The Battaglini and Palfrey experiments also show evidence of dynamic coalitions as considered in Section 8, and we discuss the role of communication in coordinating behavior in a laboratory experiment on coalition equilibria.

4.1 A Class of Simple Coalition Equilibria

The following proposition identifies a class of coalition equilibria that includes the example in the previous section. All proofs are presented in the Appendix.

Proposition 1. *In the basic model Z is supported by a coalition equilibrium if $\delta > \underline{\delta} \equiv \frac{u(1)-u(z_{\max})}{u(1)-\overline{u}}$, and*

(a) $z_i = z_{\max}$ for at least m legislators, for all $z \in Z$.

(b) $u(z_j) < u(z_{\max})$ for some j and some $z \in Z$.

Part (a) of Proposition 1 states that at least a minimal majority of legislators receive allocations equal to the maximum allocation in Z, and part (b) states that at least one other allocation must give strictly lower utility. These equilibria are simple and include dynamic coalitions in which a surplus $(s \in (m, n-1))$ of

[7] The equilibria established by Kalandrakis, Bowen and Zahran, Duggan and Kalandrakis, and others specify that a legislator who is indifferent between the status quo and the proposal votes for the proposal. Battaglini and Palfrey assume that a legislator votes for the proposal with positive probability. Anesi and Seidmann use an indifference rule that is conditional on the status quo and the proposal. For example, consider the equilibrium policy (c, c, w) with $0 < w < 1 - 2c$, which can be a simple solution. If $q^0 = (0, 1, 0)$ and legislator 1 proposes (c, c, w), legislator 2 is assumed to vote for the proposal if δ is such that legislator 2 is indifferent. When $q^1 = (c, c, w)$ and legislator 3 proposes (c, w, c), legislator 1 is assumed to vote for the status quo when indifferent between the two coalitions each of which yields c in every period.

legislators have equal allocations in every period. The bound $\underline{\delta}$ corresponds to the set Z of policies, and there is a dynamic coalition for each $z \in Z$.[8]

To illustrate the argument underlying the proof of Proposition 1, note that with basic strategies any $z \in Z$ is an absorbing state, so any deviation to a $z' \in Z$ from the coalition equilibrium supporting Z involves for legislator i a *static* comparison between z_i and z'_i. Since at least a majority have $z_i = z_{\max}$, under the indifference rule no deviation in Z is supported by a majority. For a deviation to a policy $z'' \notin Z$ legislator i has a dynamic payoff no greater than $u(1) + \delta v^*$. On the equilibrium path i receives $v_i(z) = \frac{u(z_i)}{1-\delta}$, which is greater than the dynamic payoff from a deviation to a policy not in Z for $\delta \in (0,1)$ such that $\frac{u(z_i)}{1-\delta} > u(1) + \delta v^*$. By Lemma 1 such a δ exists if and only if $u(z_i) > \overline{u}$ for a minimal-winning coalition, or if and only if the utility for a minimal-winning coalition is at least as great as the average utility available in the policies proposed if the coalition dissolves. By (a) a member of the minimal-winning coalition obtains utility $u(z_{\max})$ and by (b) $u(z_{\max}) > \overline{u}$, since Z contains at least one element that gives a strictly lower utility than $z_{\max}$. That is, at least m legislators are strictly punished if any policy not in Z is approved, so no deviation outside of the set Z is attractive. Note that this comparison is also *static*.

The following Corollary identifies properties that a dynamic coalition formed in the equilibria identified in Proposition 1 could have.

Corollary 1. *The following are properties of policies supported by the dynamic coalitions in Proposition 1:*

(a) *Dynamic coalitions can maintain both efficient and inefficient policies.*

(b) *Dynamic coalitions can maintain positive allocations to legislators outside the dynamic coalition.*

(c) *Dynamic coalitions can have surplus members; that is, the size of the dynamic coalition can be strictly larger than minimal winning.*

Proposition 1 identifies the bound on the discount factor for this set of simple coalition equilibria, and the following Corollary presents the bound for a class of particularly simple coalition equilibria in which all coalition members receive equal shares.

Corollary 2. *For all $\delta > \underline{\delta}$ there exists a coalition equilibrium in which $s \in [m, n-1]$ legislators receive $z_{\max}$ and $n - s$ legislators receive 0. The bound $\underline{\delta}$ is*

$$\underline{\delta} = \frac{u(1) - u(z_{\max})}{u(1) - u(0) - \frac{s}{n}(u(z_{\max}) - u(0))},$$

which is strictly increasing in s.

For $u(x)$ concave enough, $\underline{\delta}$ can be arbitrarily small. That is, normalize $u(\cdot)$ so that $u(1) = 1$ and $u(0) = 0$, and note that as $u(\cdot)$ becomes more concave, $u(z_{\max})$ approaches 1 and $\underline{\delta}$ approaches 0. The

[8] The proof of Proposition 1 is presented as a special case of the extended model with heterogeneous selection probabilities and discount factors to demonstrate the robustness of coalition equilibria.

theory of coalition equilibria thus can be applied to a large class of political settings, even where political actors place very low value on the future.

4.2 Characterizing the Set of Coalition Equilibria

Proposition 1 identifies a class of coalition equilibria, and the following proposition provides necessary and sufficient conditions for coalition equilibria. Let M denote a set of m legislators, and let $\mathcal{M}$ denote the collection of all such M. Let $W(z)$ denote the set of policies in Z that are strictly preferred by an m-majority of legislators; i.e., $W(z) = \{z' \in Z | \exists M \in \mathcal{M} \ni u(z'_i) > u(z_i), i \in M\}$. Then, if $W(z) = \emptyset$, there is no policy $z' \in Z$ that defeats z. A deviation proposal must include the legislator with the m^{th} largest allocation for any $z \in Z$, and let the smallest of these be denoted by $z_m^{\min}$.

Proposition 2. *In the basic model there exists a* $\underline{\underline{\delta}} \leq \frac{u(1)-u(z_m^{\min})}{u(1)-\overline{u}}$ *such that the set* Z *can be supported by a coalition equilibrium if and only if* $\delta > \underline{\underline{\delta}}$ *and:*

(a) *No policy in* Z *can be defeated by another policy in* Z *in a pairwise comparison; i.e.,* $W(z) = \emptyset$ *for all* $z \in Z$.

(b) *At least a majority* M *of legislators strictly prefers the coalition to continue rather than dissolve; i.e.,* $v_i(z) > v^*$ *for* m *legislators, for all* $z \in Z$*, or equivalently,* $u(z_m^{\min}) > \overline{u}$.

Proposition 2 gives two intuitive conditions that ensure a set Z is supported by a coalition equilibrium. Condition (a) ensures that no deviation from an allocation in Z to another allocation in Z is attractive, and Condition (b) ensures that legislators have no incentive to deviate from a policy in Z to a policy not in Z. Condition (b) states that the utility of the m^{th} largest allocation in z must be strictly larger than the average utility in Z_i for all z. This is an intuitive property of a policy supportable by a coalition equilibrium given that a minimal-winning set of legislators must prefer to remain in the coalition rather than face a new coalition formation opportunity. From Condition (b) it is immediate that the lower bound on the discount factor is strictly less than 1. Conditions (a) and (b) only involve static comparisons. Because the comparisons in Conditions (a) and (b) are static, legislators' voting and proposal behavior is indistinguishable from behavior in a single-period model with utility $\bar{u}$ if a coalition dissolves.

Coalition equilibria have stable policies, providing perfect risk smoothing over time. For risk averse legislators this allows coalition equilibria to exist for lower discount factors compared to risk neutral preferences.

A generalization of Proposition 2 is presented in the Appendix for an extension of the model in which legislators have different discount factors δ_i and different selection probabilities p_i. In contrast to sequential legislative bargaining, the continuation values of the legislators are irrelevant to the selection of coalition partners even though when the discount factors are equal the values are ordered by $p_i > p_j$, i.e., $v_i(q^{t-1}) > v_j(q^{t-1})$ for $q^{t-1} \notin Z$. All that matters to proposers when $q^{t-1} \notin Z$ is that they receive $z_{\max}$ in the next period.

The following lemma gives a necessary condition for policies to satisfy Condition (a) in Proposition 2 if $u(\cdot)$ is strictly increasing.

Lemma 2. *If $u(\cdot)$ is strictly increasing, a set Z satisfies Condition (a) of Proposition 2, i.e., $W(z) = \emptyset$ for all $z \in Z$, only if there are m or fewer distinct allocations in z, for all $z \in Z$.*

To demonstrate why this is necessary, consider the example $n = 5$, $m = 3$, $u(x_i) = x_i$ and $z = (a, a, b, c, d) \in Z$ with $a > b > c > d$. If $q^{t-1} = z$, legislator 3 receiving b under the status quo can propose the policy $z' = (d, a, a, b, c) \in Z$. Since all elements of Z are absorbing states, legislators 3, 4 and 5 are strictly better off and will vote for z'. Hence, Z cannot be supported by a coalition equilibrium. Note that Lemma 2 rules out a number of policies that can be supported as coalition equilibria, and in the case of a three-member legislature with simple majority rule, restricts policies to be of the form (c, c, w).

The following two corollaries provide examples of policies that satisfy Condition (a) but fail Condition (b) in Proposition 2, and hence are not supported by coalition equilibria.

Corollary 3. *Coalition equilibria cannot support a dictator outcome $(1, 0, \ldots, 0)$ or any policy such that fewer than m legislators receive positive allocations.*

Corollary 4. *If $u(x_i) = x_i$ and all policies in S are efficient, no coalition equilibrium can support the universal coalition (with all legislators receiving equal allocations). If S is a singleton, no coalition equilibrium can support the universal coalition for any $u(\cdot)$.*

Although $S = \{(\frac{1}{n}, \ldots, \frac{1}{n})\}$, that is, the universal coalition, is not supported by a coalition equilibrium under the conditions of Corollary 4, if an inefficient policy is included in S, for example $S = \{(\frac{1}{n}, \ldots, \frac{1}{n}), (\frac{1}{n}, \ldots, \frac{1}{n}, 0, \ldots, 0)\}$ with m legislators receiving $\frac{1}{n}$ in the second policy, then $\mathcal{Z}(S)$ is supported by a coalition equilibrium if $u(\frac{1}{n}) > u(0)$. The inclusion of the policy $(\frac{1}{n}, \ldots, \frac{1}{n}, 0, \ldots, 0)$ in S provides a threat of collective punishment through randomization with basic strategies for coalition members.[9] Universal coalitions thus can form when a threat is present. Since with basic strategies the policy $(\frac{1}{n}, \ldots, \frac{1}{n}, 0, \ldots, 0)$ is proposed with positive probability, a Pareto dominated policy can result in a coalition equilibrium, representing a political failure in the sense of Besley and Coate.

The proposal strategy specified in basic strategies for $q^{t-1} \notin Z$ calls for the proposal of any policy that yields the proposer the maximum allocation in Z, which as indicated above can include inefficient policies. Alternative proposal strategies could be used in a theory of dynamic coalitions and the policies they support. For example, a lexicographic proposal strategy could specify proposals that yield the proposer the maximum allocation in Z, and among the those policies, select the most efficient one. With this proposal strategy neither the set $S = \{(\frac{1}{n}, \ldots, \frac{1}{n}), (\frac{1}{n}, \ldots, \frac{1}{n}, 0, \ldots, 0)\}$ nor any set S that includes the universal policy could be supported by a coalition equilibrium. The lexicographic proposal rule eliminates the threat from a policy

[9]This is similar to Richter (2011), who shows that equal division can be supported by allowing some of the dollar to be wasted.

more inefficient than the policy that yields the maximal allocation, resulting in a smaller set of policies that can be supported by a coalition equilibrium.

Identifying Dynamic Coalitions

To illustrate the identification of the members of a dynamic coalition, consider $n = 3$, $m = 2$, $u(x_i) = x_i$, $S = \{(c, c, w)\}$, and $c > w$. Suppose the status quo is $z = (c, c, w)$, so $v_1(z) = v_2(z) = \frac{c}{1-\delta}$ and $v_3(z) = \frac{w}{1-\delta}$. The continuation value when the coalition dissolves is $v^* = \frac{2c+w}{3(1-\delta)}$. Then, $v_i(z) > v^*$ for $i = 1, 2$, and $v_3(z) < v^*$. Legislator 3 benefits when the coalition dissolves and hence is not in the dynamic coalition, whereas legislators 1 and 2 are punished if the coalition dissolves. In a three-member legislature when S is a singleton, the dynamic coalition is always minimal-winning.

As another illustration consider $n = 5$, $m = 3$, $u(x_i) = x_i$, $S = \{(c, c, c, a, b)\}$, with $c > a \geq b$. If the status quo is (c, c, c, a, b), legislators 1, 2, and 3 strictly prefer that the coalition continue rather than dissolve, whereas legislator 4 strictly prefers that it continue if and only if $a > \frac{1}{4}(3c + b)$. If the policy is efficient, legislator 4 strictly prefers that the coalition continue if and only if $a > \frac{1}{5}$. Consequently, if a is sufficiently high, the dynamic coalition has four members (a surplus coalition, where legislator 4 can be thought of as a "junior" member). If $a \leq \frac{1}{4}(3c + b)$, the dynamic coalition has 3 members (a minimal-winning coalition).

Alternatively, suppose $S = \{(c, c, a, a, 0)\}$, $m = 4$, and $c > a > \frac{2}{3}c$. If the status quo is $(c, c, a, a, 0)$ then legislators (parties) 1 and 2 can be thought of as a minority government supported by parties 3 and 4. If $S = \{(c, a, a, a, 0)\}$, $m = 3$, and $c > a > \frac{c}{2}$, the party receiving c can be thought of as a single-party minority government supported by three other parties. Minority governments thus provide policies sufficiently beneficial to a set of other parties to maintain their support.

Vested Interests

Proposition 2 does not require the maximum allocation in each policy in S to be the same. This suggests that coalition equilibria can support a policy that is not proposed from status quos other than itself. To identify this feature of coalition equilibria, note that the set Z supported by a coalition equilibrium includes the policies in $\bigcup_i Z_i$ that are proposed by legislators when the status quo is not in Z. The set Z also can include policies that are supported by dynamic coalitions but would never be proposed by any legislator when forming a coalition. Let the latter set of policies be denoted by Q^0. That is $Q^0 = Z \setminus (\bigcup_i Z_i)$. The set Q^0 can be empty as when $S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}$ or more generally for $|S| = 1$. For $|S| > 1$ the set Q^0 can be nonempty. For example, if $u(x_i) = x_i$, $m = 3$, and $S = \{z', z''\}$ where $z' = (a, b, b, c, c)$ and $z'' = (b, b, b, d, d)$, $a > b \geq d > c$, and $\frac{1}{3}(a + 2c) < b$, legislators only propose a permutation of z' and never propose a permutation of z'' when a coalition is being formed. If the initial status quo is z'', however, all legislators propose that policy, and it persists thereafter. Hence $Q^0 = \mathcal{Z}(\{z''\})$.

The policies $z \in Q^0$ can be thought of as supported by interests vested in the initial status quo. Those interests constitute a dynamic coalition that supports z against alternative proposals. Vested interests thus preserve the initial status quo. This can be interpreted as a form of gridlock in dynamic legislative bargaining in the sense that every legislator has the opportunity to propose a different policy yet all proposals will be defeated by the vested interests. Note that this is driven by the indifference rule. No majority strictly prefers an alternative policy to z, so the will of the majority is not thwarted. To illustrate this notion of gridlock, suppose that a parliamentary system of government with policy z falls because of an event such as a scandal and a new government formation opportunity commences. If the policy of the fallen government remains in place, the interests vested in the fallen government can prevail resulting in no change. If the policy is subject to a shock, however, as in Sections 5-7, a different government can emerge with a policy $z' \notin Q^0$.

A variety of policies can be sustained by equilibria when there are vested interests. For example, if $c = 0$, z' is minimal winning, yet vested interests prevail. If $d = b = \frac{1}{5}$, z'' is the universal policy. A universal policy thus can be supported by a coalition equilibrium in two ways. First, it can be an element of Z_i and hence be proposed when a coalition is being formed as illustrated in the previous section, and second, it can be supported by vested interests when it would never be proposed when a coalition is being formed. The latter requires z' in the example to be inefficient, and if z' is efficient, the universal policy z'' cannot be supported by vested interests.

Vested interests can also support Pareto dominated policies, since the vested interests can prefer such a policy to a new coalition formation opportunity in the next period. For example, for $n = 5$, $m = 3$, and $u(x_i) = x_i$, consider $S = \{z', z''\}$, where $z' = (a, b, b, b, c)$, $z'' = (b, b, b, b, 0)$, $a > b \geq c \geq 0$, and $b > \frac{a+c}{3}$. The policy z' Pareto dominates z'', but $W(z'') = \emptyset$, and $Z = \mathcal{Z}(\mathcal{S})$ can be supported by a coalition equilibrium. Vested interests thus can result in a political failure.

Dynamic Coalitions and Sequential Bargaining

Coalition equilibria are symmetric, the ex ante values of the game are the same for all legislators, and all legislators have an equal probability of being in a coalition, which are also the predictions of sequential legislative bargaining theory. In sequential bargaining the proposer captures all the benefits of proposal power; that is, the equilibrium policies with $\delta = 1$ are $(1 - \frac{m-1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}, 0 \ldots, 0)$, with $m - 1$ legislators receiving $\frac{1}{n}$ and $n - m$ legislators receiving 0. If $u(x_i) = x_i$, the sequential bargaining outcome cannot be sustained since $\overline{u} = \frac{1}{n}$ and $z_m^{\min} \leq \frac{1}{n}$, so a minimal-winning coalition is not punished when the coalition dissolves. If $|S| \geq 2$, S can include policies that lower the average allocation, thereby providing a threat of collective punishment for the minimal-winning coalition. This ensures a punishment for a policy outside of Z, but we must still ensure that there is no incentive to deviate to a permutation of the policy. A supermajority rule is necessary for this purpose, and the following corollary gives necessary conditions for coalition equilibria to support the sequential bargaining outcome.

Corollary 5. *If $u(x_i) = x_i$, coalition equilibria admit the sequential bargaining outcome only if $|S| \geq 2$, $m > \frac{n+1}{2}$, and $\frac{1}{n} \geq z_m^{\min} > \overline{u}$.*

4.3 Dynamic Coalitions in a Three-Member Legislature

For $n = 3$ Proposition 1 indicates that any policy of the form (c, c, w) with $c > w$ is supportable by a coalition equilibrium. Conditions (a) and (b) in Proposition 1 are also necessary when $u(\cdot)$ is strictly increasing.

Proposition 3. *If $n = 3$, $m = 2$, and $u(\cdot)$ is strictly increasing, a coalition equilibrium supports the set Z if and only if $z = (c, c, w^z)$ for all $z \in Z$ and $c > w^z$ for some $z \in Z$.*

These equilibria all have equal sharing within the coalition of the gains from maintaining the coalition.

Figure 1 illustrates the efficient policies for a three-member legislature. If $u(x_i) = x_i$ or $|S| = 1$, the centroid (the universal coalition) is excluded from the set of efficient supportable policies by Corollary 4.

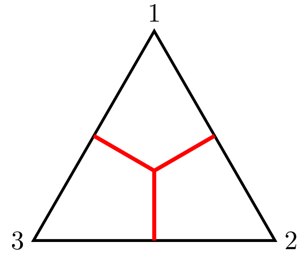


Figure 1: Efficient coalition policies for $n = 3$

The set of efficient policies supported by a coalition equilibrium includes the von Neumann Morgenstern stable set $\{(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2}), (0, \frac{1}{2}, \frac{1}{2})\}$. Even though a policy $z' = (\frac{1}{2}, \frac{1}{2}, 0)$ is in the stable set, if $z' \notin Z$ is proposed, it is defeated by all policies in Z.

Proposition 3 provides a benchmark for the case of implementation uncertainty beginning in Section 5, since only balanced allocations within a coalition are possible. If efficient policies with non-coalition members receiving a zero allocation are focal, this provides a unique benchmark $S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}$ for comparison. Furthermore, $(\frac{1}{2}, \frac{1}{2}, 0)$ is supportable by a coalition equilibrium only when $S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}$. In contrast to these equilibria in which the gains from proposal power are shared equally within the coalition, with implementation uncertainty dynamic coalitions with unbalanced allocations are possible, as considered in Sections 5-7.

5 Implementation Uncertainty

Uncertainty can be associated with the implementation of a policy, and that uncertainty can affect not only the payoff in the current period but also the status quo for the following period.[10] Since the uncertainty

[10] In their MLQRE Battaglini and Palfrey assume that players use behavioral strategies that place positive probability on every available action (on a grid). That probability is proportional to the continuation value, and as that proportion increases the limit points correspond to MPE. This uncertainty affects strategies and hence payoffs but does not affect state transitions other than through the strategies. Duggan and Kalandrakis (2012) show the existence of stationary MPE in pure strategies for a class of dynamic games that accommodate uncertainty in the current period payoffs and in the transitions from one state to another, but they do not provide an equilibrium construction.

affects the policy that is implemented, the status quo can move away from the coalition policy in which case the coalition could dissolve and a new coalition form as in Section 6 or the coalition members could tolerate the new policy as in Section 7. The analysis is presented for $n = 3$, $m = 2$, $u(x_i) = x_i$ and efficient policies, which is the case studied by Kalandrakis (2004) and Battaglini and Palfrey. In this case by Proposition 3 dynamic coalitions have equal division among a minimal majority of legislators, so in contrast to sequential legislative bargaining, the gains from proposal power are shared equally among the coalition members. Dynamic coalitions survive the possibility of implementation uncertainty, and can support unbalanced allocations among the coalition members. Implementation uncertainty is modeled to simplify the analysis of the incentive constraints and facilitate comparative statics analysis of the set of policies supported by dynamic coalitions.

Uncertainty resulting from implementation represents the observation that policy does not always work as intended. The implementation of legislation is typically delegated to administrative agencies or regulatory commissions that develop the details for the application of the legislation. A degree of uncertainty can be associated with that delegation, and legislators take that uncertainty into account in choosing a policy. The uncertainty could also be associated with the response to the enacted policy by those affected, and the realization of that uncertainty can affect the status quo and the strategies of legislators in the future. As an extreme example, with the support of the American Association of Retired Persons Congress overwhelmingly enacted the Medicare Catastrophic Coverage Act of 1988 which provided generous benefits for catastrophic care under Medicare and financed the benefits through increases in Medicare premiums. Before the change could be fully implemented, Medicare recipients began protesting the forthcoming premium increases, and facing uncertainty about the impact of the Act, Congress quickly repealed the Act it had passed in the previous session.

The likelihood of implementation uncertainty could depend on whether the legislature retains the current policy or chooses a new policy. When the legislature retains the current policy, implementation uncertainty is assumed to be present with probability η, and when the legislature chooses a new policy, the corresponding probability is $\gamma < 1$. With the complementary probabilities there is no uncertainty and hence the policy implemented equals that adopted by the legislature. When implementation uncertainty is realized, its magnitude is represented by a continuous, mean zero, random shock that is publicly observable. The former specification allows comparative statics analysis in terms of a single parameter, and the latter specification means that the probability is zero that the shocked policy equals the policy adopted by the legislature. A legislator cannot receive more that 1 or less than 0, so the shocked allocation may be truncated, in which case the truncated amount is assumed to be reallocated to other legislators to preserve the same aggregate allocation and ensure the policy remains in the feasible set. Details of the assumed implementation uncertainty are given in the Appendix, and we present here the substance of the assumption.

Let $Z(c) \equiv \mathcal{Z}(S(c))$, where $S(c) = \{(1 - c, c, 0)\}$ and $c \leq \frac{1}{2}$, so $z_{\max} = 1 - c$.

Assumption 1 (Substance). *Suppose $q^{t-1} \in Z(c)$ and $y^t \in Z(c)$. If $y^t = q^{t-1}$, with probability $1-\eta$, $q^t = q^{t-1}$ and with probability η, a shock $\tilde{\theta}^t$ distorts the policy as follows:*

$$q_i^t = \begin{cases} 1-c+\theta^t & \text{if } y_i^t = 1-c \\ c-\theta^t & \text{if } y_i^t = c, \\ 0 & \text{if } y_i^t = 0. \end{cases}$$

where $\tilde{\theta}^t$ is distributed uniformly on $[-\underline{\theta}, \underline{\theta}]$. If $y^t \neq q^{t-1}$ is approved, with probability $1-\gamma$, $q^t = y^t$, with probability γ a shock $\tilde{\varepsilon}^t$ distorts the policy as above but with with the realization ε^t replacing θ^t, where $\tilde{\varepsilon}^t$ is distributed uniformly on $[-\underline{\varepsilon}, \underline{\varepsilon}]$. If $q^{t-1} \notin Z(c)$ or $y^t \notin Z(c)$ is approved, allocations are similarly distorted.

The probability that implementation uncertainty is realized when a new policy is adopted could be greater than when the current policy is continued, and the uncertainty could also be greater.

Assumption 2. *Implementation of a new policy $y^t \neq q^{t-1}$ has a higher probability of a shock than implemention of the current (status quo) policy, i.e., $1 > \gamma \geq \eta$, and a stochastically larger shock, i.e., $\underline{\varepsilon} \geq \underline{\theta}$.*

6 Specific-Policy Equilibria with Implementation Uncertainty

This section shows by construction the existence of a set of coalition equilibria that support policies with unbalanced allocations to legislators in the dynamic coalition; i.e., in $Z(c)$. Since in these equilibria $S(c)$ consists of a single policy, the policy is specific to that c. The originator of a specific-policy coalition has proposal power and may not share the gains equally with the coalition partner. These coalitions persist with probability $1-\eta$ and dissolve with probability η when implementation uncertainty is realized. A new coalition then forms in the next period. The following bound on the shocks to the policy facilitates the exposition by simplifying the expressions for the continuation values.

Assumption 3. $\underline{\varepsilon} \leq \frac{1}{3}$.[11]

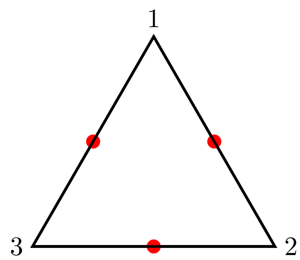
As before, basic strategies are used to establish the existence of *specific-policy equilibrium* supporting $Z(c)$.

Proposition 4. *With implementation uncertainty given in Assumptions 1-3, there exists a $c^+ \leq \frac{1}{2}$, such that for all $\delta > \delta^o \equiv \frac{3-\frac{3}{2}\eta\underline{\theta}}{4-\gamma-3\eta-\frac{3}{2}(1-\eta)\eta\underline{\theta}}$, all $c \in [c^+, \frac{1}{2}]$ and not too much uncertainty; i.e., $(\gamma, \eta) \in R(\underline{\theta}) \equiv \{(\gamma, \eta) | 1-\gamma-3\eta(1-\frac{\eta\underline{\theta}}{2}) > 0\}$, a coalition equilibrium exists supporting $Z(c)$.*

Proposition 4 identifies a class of specific-policy equilibria that are indexed by the allocation c to the coalition partner, where the originator of the coalition receives the larger allocation $1-c$. By Proposition 3

[11]The requirement that $\underline{\varepsilon} \leq \frac{1}{3}$ assures that on the equilibrium path the payoffs to coalition members are in $[0, 1]$ with probability 1 if $c > \frac{1}{3}$. We show in Lemma 4 that $c > \frac{1}{3}$.

the only c supportable in the absence of uncertainty, i.e., when $\gamma = \eta = 0$, is $c = \frac{1}{2}$, but with implementation uncertainty strictly unbalanced allocations as illustrated in Figure 2 are supported.



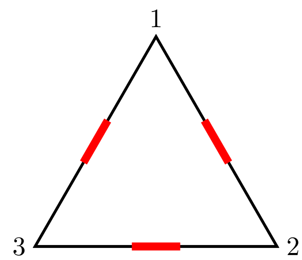


Figure 2: The supportable set $Z(c)$

Proposition 4 is proven in four steps. In Lemma 3 the continuation values corresponding to basic strategies are derived. In Lemma 4 bounds on c are identified such that legislators have no incentive to deviate from basic strategies. Lemma 5 derives δ^o such that for all $\delta > \delta^o$ there is a non-empty set of c satisfying these bounds. Lemma 6 establishes restrictions on the implementation uncertainty such that δ^o is strictly less than one, completing the proof.

Lemma 3. *With implementation uncertainty given in Assumptions 1-3, if (σ, ω) is a coalition equilibrium for a set $Z(c)$ with $c > \frac{1}{3}$:*

(*i*) *The continuation value for legislator i for $q^{t-1} \in Z(c)$ is*

$$v_i(q^{t-1}) = \frac{3(1-\delta)q_i^{t-1} + \eta\delta}{3(1-\delta)(1-\delta(1-\eta))} \ , \ q_i^{t-1} \in \{1-c, c, 0\}. \quad (1)$$

(*ii*) *The continuation value for legislator i for $q^{t-1} \notin Z(c)$ is*

$$v_i(q^{t-1}) = \hat{v} \equiv \frac{1}{3(1-\delta)}.$$

The originator of the coalition receives an allocation of $1 - c \geq \frac{1}{2}$ every period in which the coalition persists, and the coalition partner receives c. If selected as the proposer, the coalition partner can propose the status quo and receive c with probability $1-\eta$ or propose a policy that gives c to the legislator not in the coalition and obtain $1-c$ with probability $1-\gamma$. For $\gamma > \eta$ the coalition partner accepts c in every period if the discount factor is sufficiently high. The continuation value for the originating coalition member receiving $1 - c$ under the status quo is greater than the continuation value of the coalition partner who receives c, which is greater than when the coalition dissolves, provided that $c > \frac{1}{3}$. It is the difference between the dynamic payoffs when in the coalition persists and $\hat{v}$ when the coalition dissolves that provides the incentive to accept the lower payoff. The dynamic payoff if the coalition persists is strictly decreasing in η, so greater uncertainty means that sustaining the coalition is less likely.

The continuation values have the expected comparative statics properties. A higher probability η of implementation uncertainty reduces the probability of staying on the equilibrium path, reduces the continuation value to the coalition members and increases the continuation value to the legislator not in the coalition. The continuation values are increasing in δ and independent of γ.

When c is not too small, the coalition partner has an incentive both to accept the coalition proposal and maintain the status quo once the dynamic coalition has formed. Lemma 4 identifies the bounds on the allocation c and demonstrates that all feasible c are strictly greater than $\frac{1}{3}$, so the continuation values are as in Lemma 3.[12]

Lemma 4. *(i) No legislator has an incentive to deviate from the basic strategies if and only if*

$$\begin{aligned} c^* \leq \quad & c \quad \leq \tfrac{1}{2}, \text{ and} \\ c^o < \quad & c \quad < 1 - c^o, \end{aligned}$$

where

$$\begin{aligned} c^* &= \frac{3 - 2\delta(\gamma - \eta)}{3(2 - \delta(\gamma - \eta))} \\ c^o &= \frac{3 - \delta(2 + \gamma - 3\eta) - \frac{3}{4}\eta\underline{\theta}(1 - \delta(1 - \eta))}{3(1 - \delta(\gamma - \eta))}, \end{aligned} \tag{2}$$

(ii) $c^* > \frac{1}{3}$.

The lower bound c^* ensures that the coalition partner receiving c in the status quo has no incentive to propose (or accept) an allocation in which he receives $1 - c$, since changing the policy results in increased implementation uncertainty and a lower expected dynamic payoff. The bound c^* indicates the importance of the implementation uncertainty for sustaining unbalanced coalition allocations, since $c^* = \frac{1}{2}$ for $\gamma = \eta$, in which case the only allocation that can be supported has equal sharing among the coalition members.

The bound c^o ensures that the coalition partner being offered c accepts the coalition proposal for status quos not in $Z(c)$. Remaining at a status quo outside $Z(c)$ has a continuation value of $\hat{v} = \frac{1}{3(1-\delta)}$, and for $c \geq c^o$, the coalition partner votes for the proposal and obtains the coalition payoff with probability $1-\gamma$ and $\hat{v}$ with probability γ. The upper bound $1 - c^o$ is the coalition originator's analogue of c^o, and the intuition is similar.

The following lemma establishes that when $\delta > \delta^o$ there exists a c that satisfies the restrictions in Lemma 4.

Lemma 5. *For $\delta > \delta^o$ there exists a c such that $c^o < c < 1 - c^o$ and $c^* \leq c \leq \frac{1}{2}$. That is, $c^o < \frac{1}{2}$ and $c^* \leq \frac{1}{2}$, when $\delta > \delta^o$.*

[12] Lemma 4 indicates that Assumption 3 can be weakened to $\underline{\varepsilon} \leq \min\{c^*, c^o\}$ for $\delta > \delta^o$.

Lemma 5 establishes that if legislators are sufficiently patient, there is a non-empty set of policies that can be supported by specific-policy coalition equilibria. It remains to determine if $\delta^o < 1$. Consider a status quo policy $(0, 1, 0)$ for which legislator 1 makes the proposal $(1 - c, c, 0)$. The status quo is very attractive for legislator 2, and with γ high it is very likely that the coalition never materializes. Furthermore, with η sufficiently high once the coalition has formed it dissolves with high probability. So for γ and η sufficiently high legislator 2 could reject the proposal regardless of the discount factor. The following Lemma establishes conditions on γ and η such that $\delta^o < 1$, which completes the proof of Proposition 4.

Lemma 6. *For* $(\gamma, \eta) \in R(\underline{\theta}) \equiv \{(\gamma, \eta) | 1 - \gamma - 3\eta(1 - \frac{\eta\underline{\theta}}{2}) > 0\}$, $\delta^o \in (0, 1)$.

Both c^* and c^o are strictly decreasing in δ, which establishes the following corollary.

Corollary 6. *The set of policies supported by specific-policy coalition equilibria is strictly increasing in* δ *for* $\delta \in (\delta^o, 1)$.

The greater lower bound on the set of c for which the strategies constitute an equilibrium are characterized in Corollary 7 in terms of a cut-point on δ that establishes that $c^+ = c^*$ for sufficiently high discount factors and $\gamma \leq \frac{2}{3}$.[13]

Corollary 7. *For* $\gamma \leq \frac{2}{3}$, $c^+ = c^*$ *for* $\delta \geq \delta^+$ *and* $c^+ = c^o$ *for* $\delta^o < \delta < \delta^+$, *where*

$$\delta^+ \equiv \frac{(4(1-\eta) + \frac{3}{4}\eta\underline{\theta}(2 + \gamma - 3\eta))}{2(\gamma - \eta)(2 - \gamma - \eta - \frac{3}{4}\eta\underline{\theta}(1 - \eta))} - \frac{\sqrt{(4(1-\eta) + \frac{3}{4}\eta\underline{\theta}(2 + \gamma - 3\eta))^2 - 4(3 - \frac{3}{2}\eta\underline{\theta})(\gamma - \eta)(2 - \gamma - \eta - \frac{3}{4}\eta\underline{\theta}(1 - \eta))}}{2(\gamma - \eta)(2 - \gamma - \eta - \frac{3}{4}\eta\underline{\theta}(1 - \eta))}. \tag{3}$$

When the discount factor is high ($\delta \geq \delta^+$), the binding incentive constraint is for the coalition partner to stay on the equilibrium path; i.e., to accept the allocation c and not propose a policy in $Z(c)$ that would yield $1-c$. When $\delta \in (\delta^o, \delta^+)$, the binding incentive constraint is for the potential coalition partner to accept the coalition originator's proposal for any status quo. The binding incentive constraints are associated with the coalition member who receives the lower allocation.

In a specific-policy coalition equilibrium with $c < \frac{1}{2}$ the originator of the coalition does not share the gain from proposal power equally with the coalition partner. Both c^* and c^o are greater than $\frac{1}{3}$, however, so the coalition partner receives more in each period than in sequential legislative bargaining. The dynamic incentives arising from the coalition partner's opportunity to reverse roles and propose the coalition originator's policy require the proposer to take less than in sequential bargaining theory.

The following corollary presents a condition such that when $\gamma = \eta$ and η is not too large, the set of policies supported by a specific-policy coalition equilibrium is a singleton with equal division among the coalition members.

[13] In the Appendix, c^+ is also characterized in terms of a cut-off on γ.

Corollary 8. *For $\gamma = \eta$, the set of allocations c that can be supported as a coalition equilibrium is the singleton $\{\frac{1}{2}\}$ for $\eta < \frac{1}{3\underline{\theta}}[4 - 2(4 - \frac{3}{2}\underline{\theta})^{\frac{1}{2}}]$.*

The balanced coalition policy thus can be supported when the probability of implementation uncertainty is the same if the policy remains the same or changes. The equal division property in Proposition 3 is thus robust to implementation uncertainty provided that the probability that uncertainty is realized is the same when a new policy is adopted as when the policy remains the same.

7 Tolerant Coalitions

Specific-policy dynamic coalitions dissolve if implementation uncertainty is realized, since the shock moves the implemented policy away from the coalition policy. A dynamic coalition could, however, tolerate some change in policy due to implementation uncertainty. This section identifies tolerant dynamic coalitions in which coalition members tolerate a degree of variation in the coalition policy, i.e., the coalition persists if the policy remains in a tolerated set of policies and dissolves if it is outside the set. The coalition thus withstands small shocks but not large shocks.

We show that basic strategies support policies in a set $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ for some $\underline{c} \leq \frac{1}{2}$. Letting $\zeta \equiv [\underline{c}, 1-\underline{c}]$, the coalition persists when the realization θ^t of the implementation uncertainty satisfies $c - \theta^t \in \zeta$. In this case $z_{\max} = 1 - \underline{c}$, so if a status quo policy is not in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, with basic strategies the proposer randomizes over policies that give him $1 - \underline{c}$. If the status quo is in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ for any $c \in \zeta$, the status quo is proposed. The following assumption assures that the implementation uncertainty is sufficiently great that the coalition can dissolve for all $c \in \zeta$ from either a very high or a very low realization of $\tilde{\theta}^t$. This simplifies the expressions for the continuation values and facilitates the comparison between tolerant coalition and specific-policy coalition equilibria.

Assumption 4. $1 - 2\underline{c} \leq \underline{\theta} \leq \underline{\varepsilon} \leq \underline{c}$.

The bound $\underline{c}$ of the set ζ is obtained from the incentive constraints as in the specific policy coalition equilibrium. The incentive constraints identify a set of bounds $\underline{c} \in [\underline{c}^+, \frac{1}{2}]$ on the tolerated allocations, where $\underline{c}^+$ is the analogue of c^+ in Proposition 4 for a specific-policy equilibrium. The analogues c^{**} of c^* and c^{oo} of c^o in (2) are

$$c^{**} = c^* - \frac{\delta\left(\delta\eta(\gamma-\eta) - (1-\delta(1-\eta))\left(\gamma\frac{\underline{\theta}}{\underline{\varepsilon}} - \eta\right)\right)\nu(c^{**})}{2 - \delta(\gamma-\eta)}, \quad (4)$$

$$c^{oo} = c^o - \frac{\delta\left((1-\gamma)\eta\delta + \gamma(1-\delta(1-\eta))\frac{\underline{\theta}}{\underline{\varepsilon}}\right)\nu(c^{oo})}{1 - \delta(\gamma-\eta)}, \quad (5)$$

where

$$\nu(c) \equiv \frac{1-2c}{6[2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2c)]}. \tag{6}$$

For tolerant policies we also require $c \geq c^{\ell\ell} \equiv c^{\ell} - \eta\delta\nu(c^{\ell\ell})$, where c^{ℓ} is the specific-policy analogue of $c^{\ell\ell}$ defined in the Appendix. These bounds are derived from incentive constraints corresponding to deviations at the boundaries of ζ and X, but an additional deviation must be considered with tolerant equilibria. The status quo could be a policy $(1-a, a, 0)$ where $a \in (\underline{c}, 1)$, which has the property that the current period allocation $1-a+\theta^t$ is less likely to be truncated at 1 than $1-c+\theta^t$ and also has a higher probability that the allocation will be in ζ than a status quo $(1,0,0)$. A characterization of the a that maximizes the dynamic payoff is provided in the Appendix, and the corresponding bound is denoted $\hat{c}^{oo}$. In addition, starting from a status quo not in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, the proposing legislator may propose $(1-a, a, 0)$ rather than the equilibrium proposal in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$. To ensure no such deviation is attractive requires an upper bound $\hat{\hat{c}}_2$.

The following lemma identifies policies that can be supported by tolerant dynamic coalitions.

Lemma 7. *With implementation uncertainty given in Assumptions 1, 2, and 4 basic strategies are a coalition equilibrium supporting* $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ *for all* $\underline{c} \in [\underline{c}^{+}, \frac{1}{2}]$*, if* $\hat{\hat{c}}_2 \geq \frac{1}{2}$*, and*

$$\underline{c}^{+} \equiv \max\{c^{**}, c^{oo}, \hat{c}^{oo}, c^{\ell\ell}\}. \tag{7}$$

An equilibrium supporting $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c), \underline{c} \in [\underline{c}^{+}, \frac{1}{2}]$, is referred to as a *tolerant coalition equilibrium.* In tolerant coalition equilibria coalition members in period $t+1$ propose the status quo when the allocation to a coalition member in period t is in ζ. If the realized implementation uncertainty θ^t is such that $c-\theta^t$ is not in ζ, the coalition dissolves and the legislator i selected in the next period proposes a policy in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ that yields $1-\underline{c}$ to i. Tolerant coalitions thus form immediately with the composition of the coalition determined by the selection of a proposer and the random selection of a coalition partner. When a tolerant coalition forms, the originator of the coalition receives a strictly greater allocation than the coalition partner, but following a tolerated realization of the implementation uncertainty, the allocation to the other coalition member and the corresponding continuation value can be larger. When a tolerant coalition forms with $c = \underline{c}$, the probability that it dissolves due to implementation uncertainty is $\frac{1}{2}\eta$, since the distribution of $\tilde{\theta}^t$ is symmetric about 0. If the coalition persists beyond one period, the probability that it dissolves in the next period is smaller, since $c \geq \underline{c}$.

The bounds c^{oo} and $c^{\ell\ell}$ are less than their counterparts for specific-policy equilibria, and the following lemma gives sufficient conditions for $c^{oo} < \frac{1}{2}$, $c^{\ell\ell} < \frac{1}{2}$ and $c^{**} \leq \frac{1}{2}$.

Lemma 8. *For* $\delta > \delta^{o}$ *the bounds satisfy* $c^{**} \leq \frac{1}{2}$*,* $c^{oo} < \frac{1}{2}$*, and* $c^{\ell\ell} < \frac{1}{2}$*.*

The bound c^{**} can be greater than c^{*} because with a tolerant coalition equilibrium a coalition member receiving $\underline{c}$ in the status quo has a stronger temptation to propose $1-\underline{c}$ because the probability of preserving the coalition is higher than in the corresponding specific-policy proposal.

Analogous to $R(\underline{\theta})$ for specific-policy equilibria, restrictions are required so that the implementation uncertainty is not so great that a lower bound on the tolerated set of allocations is greater than one-half or an upper bound is less than one-half. Tolerant coalition equilibria exist, although identifying the set $R^T(\underline{\varepsilon}, \underline{\theta})$ of allowable implementation uncertainty and a bound on the discount factor is complex. Section 7.1 establishes existence of tolerant coalition equilibria for $\gamma > \eta = 0$, and by continuity tolerant coalition equilibria exist for at least small η.

Lemma 7 is proven by checking legislators incentives to deviate from the basic strategies. Lemma 9 identifies the continuation values corresponding to a tolerant coalition equilibrium, which are used to compare the payoffs in the specific policy coalition equilibrium to those in the corresponding tolerant coalition equilibrium.

Lemma 9. *With implementation uncertainty given in Assumptions 1, 2, and 4, if (σ, ω) is a coalition equilibrium for some set $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$:*

(i) The continuation value $\overline{v}_i(q^{t-1})$ for legislator i for $q^{t-1} \in \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ is

$$\overline{v}_i(q^{t-1}) = \frac{3(1-\delta)q_i^{t-1} + \eta\delta + 3(1-\delta)\eta\delta\nu(\underline{c})}{3(1-\delta)(1-\delta(1-\eta))} \text{ , } q_i^{t-1} \in \{1-c, c, 0\}, \tag{8}$$

(ii) the continuation value $\overline{v}_i(q^{t-1})$ for legislator i for $q^{t-1} \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ is

$$\overline{v}_i(q^{t-1}) = \hat{v} \equiv \frac{1}{3(1-\delta)},$$

(iii) $\nu(\underline{c}) \geq 0$ and $\nu(\underline{c}) > 0$ if $\underline{c} < \frac{1}{2}$.

The continuation values for a specific-policy coalition corresponding to c^+ can be compared to the continuation values for a tolerant coalition with $\underline{c} = c^+$.

Proposition 5. *Consider a $\underline{c} = c^+ < \frac{1}{2}$ such that both a specific-policy coalition equilibrium and a tolerant coalition equilibrium exist. If $\eta > (=)\ 0$, the continuation values in (1) for specific-policy coalition members are strictly less than (equal to) the continuation values in (8) for tolerant coalition members.*

With $\eta = 0$ in a specific-policy coalition equilibrium, once formed the coalition continues with probability one, as does a tolerant coalition. The continuation values on the equilibrium path thus are the same in the two equilibria. For $\eta > 0$, implementation uncertainty can be realized on the equilibrium path in which case the specific-policy coalition dissolves whereas the probability is positive that the allocation remains in ζ and the tolerant coalition continues. A tolerant coalition thus is more valuable to its members than is the corresponding specific-policy coalition when $\eta > 0$.

For $\gamma = \eta$ and $\underline{\varepsilon} = \underline{\theta}$ the unique tolerant coalition policy has equal allocations for the coalition members, so the coalition is no more tolerant than the specific-policy coalition in Proposition 4. This is formalized in the following corollary.

Corollary 9. *For $\gamma = \eta$ and $\underline{\varepsilon} = \underline{\theta}$, $c^{**} = c^* = \frac{1}{2}$, so coalition members receive equal allocations, and a tolerant coalition is no more tolerant than the specific-policy coalition.*

7.1 Implementation Uncertainty Only When Policy Changes

To provide a further characterization of tolerant coalition equilibria, consider the case in which there is implementation uncertainty ($\gamma > 0$) associated with a change in policy but no implementation uncertainty ($\eta = 0$) when the status quo policy is continued. This allows a complete characterization of the set of policies supported by a tolerant coalition equilibrium and the comparative statics properties on the bound on that set. When $\eta = 0$ once the coalition policy is on the equilibrium path it remains there, so a tolerant coalition is no more valuable than a specific-policy coalition, yet the equilibria are not the same. As shown in Proposition 8, the set of policies supported by a tolerant coalition equilibrium can be strictly smaller than the set of policies in Proposition 4 supported by a specific-policy equilibria.

The most tolerant coalition has $\underline{c}^+$ in (7) equal to c^{**}, c^{oo}, $\hat{c}^{oo}$ or $c^{\ell\ell}$. With $\eta = 0$, $\hat{c}^{oo} = c^{oo}$, $c^{\ell\ell} = c^{\ell}$ and solving (4) and (5) for c^{**} and c^{oo} yields

$$\begin{aligned} c^{**} &= \frac{3 - \delta\gamma(2 - \frac{1}{4\underline{\varepsilon}})}{3(2 - \delta\gamma(1 - \frac{1}{6\underline{\varepsilon}}))} \\ c^{oo} &= \frac{3 - 2\delta - \delta\gamma(1 + \frac{1}{4\underline{\varepsilon}})}{3(1 - \delta\gamma(1 + \frac{1}{6\underline{\varepsilon}}))}. \end{aligned}$$

The following Proposition states that for sufficiently high discount factors a tolerant coalition equilibrium exists with the equilibrium policy of the most tolerant coalition identified by c^{**}.

Proposition 6. *With implementation uncertainty given in Assumptions 1 and 4 and for $\eta = 0$, there exists a δ^{ς} satisfying $\delta^o = \frac{3}{4-\gamma} < \delta^{\varsigma} < 1$ such that for all $\delta > \delta^{\varsigma}$, a coalition equilibrium exists supporting $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ for all $\underline{c} \in [c^{**}, \frac{1}{2}]$.*

The policy proposed by the originator of the coalition is of the form $(1 - c^{**}, c^{**}, 0)$ for $\delta > \delta^{\varsigma}$. The following proposition characterizes the policy.

Proposition 7. *For $\delta > \delta^{\varsigma}$ and $\gamma > \eta = 0$, the allocation c^{**} to the coalition partner is (i) strictly less than $\frac{1}{2}$, (ii) strictly greater than $\frac{1}{3}$, (iii) strictly decreasing in γ, and (iv) strictly decreasing in δ.*

The originator of a tolerant coalition thus receives a strictly larger share in the first period of the coalition than does the coalition partner, but the originator's share is less than in sequential legislative bargaining theory. The proposer shares the gain from proposal power with the coalition partner, but the implementation uncertainty allows the originator to take a larger share. The allocation to the originator is greater the more important is the future and hence the more valuable is the dynamic coalition to the coalition partner. As with specific-policy coalition equilibria, the set of policies supported by the most tolerant coalition with c^{**} is increasing in the discount factor. Similarly, the coalition is more valuable the greater is the probability γ of implementation uncertainty when the policy changes.

Specific-policy equilibria, however, can support a larger divergence between the allocation to the coalition originator and the partner than in the most tolerant coalition equilibrium.

Proposition 8. *For $1 > \gamma > \eta = 0$ and $\delta > \max\{\delta^{\zeta}, \delta^{+}\}$, the set $[c^*, \frac{1}{2}]$ of coalition partner allocations supported by the class of specific-policy coalition equilibria strictly contains the set $[c^{**}, \frac{1}{2}]$ of coalition partner allocations supported by a tolerant coalition equilibrium.*

The intuition underlying Proposition 8 is as follows. When $\eta = 0$ the value of a specific-policy coalition corresponding to $\underline{c}$ equals the value of a tolerant coalition corresponding to $\underline{c}$, since once on the equilibrium path the policy does not change provided no coalition member deviates from the equilibrium strategies.[14] A deviation from the tolerant coalition equilibrium strategies, however, is not as costly to a coalition member as is a deviation from the specific-policy equilibrium strategies because the former deviation could still result in an allocation in the set ζ as a result of the realization of $\tilde{\varepsilon}^t$, whereas with a specific-policy coalition the coalition dissolves whenever the policy is shocked. The incentive constraint is thus tighter for a tolerant coalition equilibrium than for a specific-policy coalition equilibrium, and $c^{**} > c^*$.

When $\eta > 0$, the probability that a tolerant coalition persists is higher than the probability that a specific-policy coalition persists, since the shocked allocation can remain in the set ζ. The higher probability means that the continuation value for a tolerant coalition is higher as shown in Proposition 5. This effect is in the opposite direction of the effect characterized in Proposition 8, and the bound c^{**} can be lower than c^* for a specific-policy coalition if $\frac{\eta(1-\delta(1-\gamma))}{\gamma(1-\delta(1-\eta))} \geq \frac{\underline{\theta}}{\underline{\varepsilon}}$, which requires $\underline{\theta} < \underline{\varepsilon}$ when $\eta < \gamma$. A tolerant coalition equilibrium thus could have a greater difference between the allocations of the coalition members than in a specific-policy coalition.

8 Evidence from Experiments

8.1 The Battaglini-Palfrey Experiments

Battaglini and Palfrey conducted two experiments related to the basic model with policies restricted to be efficient. In the first, referred to as the no-Condorcet winner (NCW) experiment, the policy space consisted of four policies, and the second, referred to as the continuous experiment, was an approximation to the

[14]Note that $\overline{v}_\ell(q^{t-1}) = v_\ell(q^{t-1})$ for $\ell = i, j, k$ for $\eta = 0$.

efficient boundary of X.[15] The experiments provide evidence about how people play the game and provide a degree of support for dynamic coalition theory. In the experiment players have the opportunity to adopt equilibrium strategies, but there is nothing to identify an equilibrium a priori. Players may reason about the choice of a strategy or learn from the results of play, and in the absence of communication, coordination on an equilibrium is not a simple task. Also, players may not use Markov strategies or may not play consistent with any equilibrium. Despite these qualifications players frequently chose coalition-like policies and sustained coalitions over time.

8.1.1 NCW Discrete Allocation Space Experiment

The NCW experiment consisted of two sessions separated by 2 months, and the subjects were undergraduate students at Princeton University. The first session consisted of 10 matches in which 9 participants were randomly assigned to 3-person committees, and the second session consisted of 10 matches with 12 participants randomly assigned to three-person committees for a total of 70 committees. Each match continued to the next round (period) with probability 0.75, so $\delta = 0.75$, and matches lasted between 1 and 10 rounds for a total of 291 rounds. Each committee played the same game in every round with the status quo equal to the policy in place at the end of the previous round. The initial status quo was chosen randomly from among the alternatives. In every round each player chose a provisional proposal, and one proposal was selected randomly from the provisional proposals. In each round each committee had 60 units of experiment currency to allocate by majority rule.

In the NCW experiment the policies are $S = \{(30, 30, 0), (20, 20, 20)\}$ with all permutations $\mathcal{Z}(S)$ available. The NCW experiment is of interest because it provides evidence about the formation and persistence of coalitions in a simplified game. It also provides direct evidence about whether players behave in a manner consistent with the indifference rule; i.e., whether they voted for the status quo when indifferent between it and a proposal.

A Markov perfect equilibrium exists for all $\delta \in [0, 1)$, where the player selected proposes keeping 30 and allocates 30 to another player selected at random. Battaglini and Palfrey show that voting myopically for the alternative giving the strictly higher current round payoff and randomizing when the payoffs are the same is an equilibrium. With either far-sighted or myopic voting strategies the policies rotate probabilistically among the minimal winning coalitions, and each player has a continuation value of $\frac{20}{1-\delta}$.

A unique coalition equilibrium exists for all $\delta \in [0, 1)$ for $Z = \mathcal{Z}((30, 30, 0))$.[16] That is, players maintain the status quo if the allocation is of the form $(30, 30, 0)$ and otherwise propose keeping 30 and allocating 30 randomly to one of the other players. Policies do not change after the first round.

[15] Battaglini and Palfrey also conducted a discrete allocation experiment with one of the alternatives a Condorcet winner. That experiment is not considered here, since there is no Condorcet winner in X.

[16] The universal policy $(20, 20, 20)$ can be supported as a subgame perfect equilibrium for $\delta > \frac{2}{3}$ with deviations punished by the other two players.

The equilibria in the NCW game thus depend on the indifference rule. In the experiment there is nothing that tells players how to vote when indifferent between two alternatives in which they receive 30, so the players could be thought of as randomizing between the two alternatives. An allocation then would persist from one period to the next with probability one-third (since the universal allocation is not an equilibrium).[17] With an indifference rule of voting for the status quo when indifferent, the coalition equilibrium prediction is that once on the equilibrium path a policy persists from one period to the next with probability one.

The two indifference rules and the corresponding predictions can be assessed using the frequency with which policies persist from one period to the next. Any statistical test would reject the coalition equilibrium, since dynamic coalitions are not present in all rounds. Dynamic coalitions are present in 74.2% of the rounds, however, and if the rounds with a universal initial status quo are excluded, 78.5% of the rounds have coalitions that persist from one round to the next. Table 4 in Battaglini and Palfrey gives the frequency with which a particular coalition persist from one round to the next. The frequencies for coalitions $\{1, 2\}$, $\{1, 3\}$, and $\{2, 3\}$ are 0.77, 0.79, and 0.75, respectively. The frequency with which coalitions persist is closer to the coalition equilibrium prediction than to one-third, providing a degree of support for the coalition equilibrium prediction.

A stronger test for the presence of a dynamic coalition is for the players favored by the status quo to vote against any proposal that differs from the status quo. To examine the voting and the alternative indifference rules in more detail, the votes have been examined for those players who were indifferent between a proposal and the status quo when the two policies differed. In those rounds player one voted for the status quo in 23 of 35 rounds, player two in 27 of 53 rounds, and player three in 19 of 30 rounds. In the aggregate, the players voted for the status quo with probability 0.585. For those rounds in which a coalition was present, the corresponding numbers are 20 of 27, 24 of 45, and 16 of 23, respectively, so in the aggregate the players in a coalition voted for the status quo with probability 0.632. These data also provide a degree of support for the indifference rule of voting for the status quo when indifferent.

Table 1 presents data on the duration of dynamic coalitions. The 70 committees experienced 78 dynamic coalitions. Forty-seven coalitions lasted only 1 or two rounds, and 18 of those ended because the match ended. A dynamic coalition cannot be present in the first round if the initial status quo is (20,20,20), so the probability that a coalition could form in the first round is bounded above by three-quarters. The longest lasting coalitions were one of 9 rounds and 3 of 8 rounds. For 2 of the coalitions with 8 rounds the game ended after the 8th round. Twenty-two coalitions lasted throughout the match. The longest-lived coalition was formed in the second round and continued for 9 rounds until the match ended.

[17] Three-player coalitions were present in 5 of the 70 committees, and all but one resulted from an initial status quo of (20, 20, 20).

8.1.2 Continuous Allocation Space Experiment

The continuous allocation space experiment was implemented with a discrete grid with 60 available units allocable in increments of one, a common discount factor of $\delta = \frac{5}{6}$, and equal recognition probabilities.[18] The experiment consisted of 10 matches in which 12 participants were randomly assigned to three-person committees.[19] The theory presented in Section 4.3 provides a set of predictions for coalition equilibria in the continuous allocation experiment. The prediction from Proposition 3 is that dynamic coalitions support policies with equal allocations among the coalition members and a lower, possibly 0, allocation to the third player.

As in Battaglini and Palfrey, Table 2 presents the experiment results for majoritarian, universal, and dictatorial policies, where a universal policy is defined as each player receiving at least 15 units, a dictatorial policy has one player receiving at least 50, and for the policies that are neither universal nor dictatorial are majoritarian with M_{ij} denoting a policy in which i and j receive at least as much as k.[20] The numbers in parentheses in Table 2 are the number of policies in each category. Participants played a universal policy with frequency 0.370, a majoritarian policy with frequency 0.536, and a dictatorial policy with frequency 0.093.

The probability of transitioning to the same majoritarian policy set is 0.522 for M_{12} and 0.582 for M_{13}. The transition probability for M_{23} is only 0.313, and the probability of transitioning from M_{23} to the universal allocation set is 0.281.[21] Battaglini and Palfrey use somewhat larger sets M_{ij} and report transition probabilities of 0.55, 0.67, and 0.39 for M_{12}, M_{13}, and M_{23}, respectively. With the exception of M_{23}, the probabilities are substantially higher than with random transitions. Since not all initial status quos were in supportable sets, some first-period transitions should be random by the theory. Table 3 thus reports the transition probabilities for rounds after the first. Overall, the transition probabilities for majoritarian coalitions are slightly higher than in Table 2 for M_{12} and M_{13}, and slightly lower for M_{23}. In Tables 2 and 3 the transition probabilities for universal policies are 0.848 and 0.875, respectively, suggesting that a dynamic coalition may be present, although with policies restricted to be efficient, the universal policy is not supported by a coalition equilibrium. The universal policy, however, can be supported by a subgame perfect equilibrium with player-specific punishments. Overall, the transition probabilities provide modest support for the presence of dynamic coalitions. Battaglini and Palfrey show that a logistic quantal response equilibrium in which players' best response functions are subject to random shocks fits the experiment data when players are sufficiently risk averse.

[18]Battaglini and Palfrey also conducted an experiment with $\delta = \frac{3}{4}$, but $\delta^o = \frac{3}{4}$, so there is no coalition equilibrium with risk neutral players corresponding to the experiment.

[19]A programming error resulted in the loss of the initial status quo in 4 of the 10 matches, so the data discussed here include only the 6 matches for which the initial status quo is known.

[20]These definitions are slightly different from those used by Battaglini and Palfrey.

[21]The transition probabilities for M_{ij} are much lower for the $\delta = \frac{3}{4}$ experiment than for the $\delta = \frac{5}{6}$ experiment, as would be expected from the theory presented here.

8.2 Communication and Focal Points

Several experiments implementing the sequential legislative bargaining game introduced by Baron and Ferejohn find behavior that only weakly supports the theory.[22] These experiments do not allow communication among participants. Agranov and Tergiman (2012) show that allowing participants in a legislative bargaining experiment to communicate results in behavior that strongly supports the theory. The cheap talk communication allowed in the experiment takes place after a proposer has been selected, and any player can send a message to any subset of other players. Participants in the experiment used the communication opportunity to learn about the reservation values of other participants and to induce the proposer to include them in the majority.

As in the experiment by Agranov and Tergiman, communication among experiment participants could increase the frequency with which dynamic coalitions are formed, extend their duration, and select among multiple equilibria. As in their experiment the participant selected as the initial proposer could initiate communication before making a proposal, and that communication could explain the benefits of a coalition and invite the other participants to join in a coalition, understanding that the coalition partner would be selected at random. The other participants would then compete to be in the coalition, but at least for $n = 3$ the proposer would not bargain them down because the resulting coalition would not be durable. If policies are restricted to be efficient, the prediction of the theory for $n = 3$ is that dynamic coalitions are minimal winning but a non-coalition member can receive a positive allocation. If, however, policies can be inefficient, a variety of policies can be supported including the universal policy. Moreover, vested interests can support policies, including the universal policy, if participants envision a threat from an inefficient policy. Without further structure the principal prediction is that dynamic coalitions should be present and should persist and can support minimal winning, surplus, and universal policies. When there are multiple equilibria, an experiment can be informative both about the presence of coalitions and the selection among equilibria.

Communication could direct play away from inefficient policies and away from surplus coalitions and toward minimal winning, efficient coalitions. Once a proposer is selected, participants in an experiment could explain that the coalition members could receive a strictly higher allocation in every period by allocating the entire dollar rather than wasting a portion. The proposer could also explain that a surplus coalition provides an opportunity for a minimal majority to reallocate a portion of the dollar to themselves. Competition to be in the majority should drive behavior toward efficient and minimal winning coalitions. In this sense, efficient coalition equilibria with minimal winning dynamic coalitions can thought of as "communication proof," suggesting that they may be focal.

The experimental design of Battaglini and Palfrey could be combined with that of Agranov and Tergiman to produce an experiment that would directly test a focal point hypothesis of a minimal winning, efficient coalition equilibrium. An experiment could be extended by introducing implementation uncertainty to study

[22] See Frechette, Kagel and Lehrer (2003), Frechette, Kagel and Morelli (2005*a*), and Frechette, Kagel and Morelli (2005*b*).

whether coalition members can sustain unbalanced allocations and tolerate a degree of uncertainty.

9 Conclusions

Public policymaking is a dynamic process in which the opportunity to set the agenda gives legislators temporary power that can be used to set policy to their advantage. Distributive policy in particular could be prone to opportunistic behavior, and shifting agenda-setters could lead to policy instability. Yet most policies exhibit a measure of stability. This paper shows that dynamic coalitions can form beginning from any status quo and once formed support policies that are stable.

The originator of a dynamic coalition has agenda-setting or proposal power as in sequential bargaining theory, but in contrast to that theory the originator of a dynamic coalition shares proposal power with the other members of the coalition. Sharing is required to satisfy dynamic incentive constraints resulting from the opportunity of coalition members to propose alternative policies and to vote against the coalition policy when the status quo is favorable. In the basic model for a three member legislature the dynamic incentives require the originator to share proposal power equally with the coalition partner.

Uncertainty can be associated with the implementation of policies, and that uncertainty can be greater when the policy changes than when it remains unchanged. Specific-policy coalition equilibria exist with implementation uncertainty, and in the policies supported the originator of the coalition can receive more than the coalition partner. The coalition partner accepts the smaller allocation rather than break the coalition and face increased uncertainty when a new policy is chosen.

Coalitions in specific-policy equilibria dissolve when implementation uncertainty is realized, but a coalition could tolerate a degree of uncertainty and continue to the next period. A tolerant coalition continues when the implemented policy remains in a set of tolerated policies, but if the shock is large as in a crisis, the coalition dissolves. A tolerant coalition thus can persist over time, and policies have a degree of stability, provided that the realized implementation uncertainty is not too large.

Coalition equilibria have particularly simple strategies with legislators proposing the status quo if it is in the supported set of policies and otherwise proposing a new coalition with coalition partners selected randomly. When legislators are risk averse, coalition equilibria seem natural since they provide perfect risk smoothing over time. These equilibria could arise in the laboratory with the strategies of legislators coordinated through straightforward communication between the coalition originator and potential coalition partners.

The theory of dynamic coalitions can be applied to understand government formation in a multiparty parliamentary democracy. The theory predicts that governments can be formed even though party leaders are politically impatient. The theory explains that surplus and minority governments in addition to minimal-winning governments can form, and a consensus government can form if there is a threat. If there is uncertainty in implementing promised benefits, governments can survive small shocks. However, large shocks, which can be thought of as crises, can lead to the dissolution of a government. The theory thus provides an explanation for why and when governments fail. The theory also provides an explanation for failed government formation attempts when there are interests vested in the initial status quo that prevent a new, potentially Pareto improving, government to form.

The theory of dynamic coalitions can be extended in a number of directions. In the pure distribution game considered here, the preferences of legislators are directly opposing yet stable coalitions can form. With a policy space in which preferences are partially aligned, dynamic coalitions should also be present although their characteristics could differ. In particular, the extent to which proposal power is shared would depend on the preference alignment, as would coalition size and the set of policies supported. For example, in every period the legislature could allocate a budget between a public good and a distributive policy with legislators having quasi-linear preferences. The model could be extended to incorporate a richer model of government spending in which investment, debt and tax policies are chosen. In this setting, the focus of the analysis could be the role of dynamic coalitions in limiting the inefficiency resulting from the inability of government to commit to long-term policies. Another extension would be to enrich the model of politics by incorporating features of political institutions such as bicameralism, and committees with continuing agenda control. This might include incorporating periodic elections with the legislative bargaining model representing policy-making during the inter-election period. Elections could dislodge coalitions and create incentives for particular coalitions to form. The theory of dynamic coalitions could provide a foundation for a theory of endogenous political parties that could arise through legislative bargaining. A number of these extensions could be taken to the laboratory.

Appendix

Dynamic Coalitions in the Basic Model

We first present a more general *extended model* and prove Proposition 1e, the analogue to Proposition 1 in the basic model. The basic model is a special case of the extended model, so the proof of Proposition 1 is immediate.

The extended model

The extended model is the same as the basic model, but we allow heterogenous discount factors δ_i and heterogenous proposal probabilities $p_i < 1$ for legislator i.

Continuation values in the extended model are accordingly $v_i(\sigma, \omega | q^{t-1}) = E^t[u(q^t) + \delta_i v_i(\sigma, \omega | q^t)]$. All other definitions including the definition of basic strategies in the extended model are analogous to those in the basic model.

Lemma 1e gives continuation values for the extended model.

Lemma 1e. *In the extended model, if (σ, ω) is a coalition equilibrium supporting a symmetric set Z:*

(a) *The continuation value for legislator i for $q^{t-1} \in Z$ is $v_i(q^{t-1}) = \frac{u(q_i^{t-1})}{1-\delta_i}$.*

(b) *The continuation value for legislator i for $q^{t-1} \notin Z$ is*

$$v_i(q^{t-1}) = v_i^* \equiv \frac{p_i u(z_{\max}) + (1-p_i)\overline{\overline{u}}}{1-\delta_i}. \quad (9)$$

where

$$\overline{\overline{u}} \equiv \frac{1}{|Z_j|} \sum_{z \in Z_j} u(z_i), \text{ for } j \neq i.$$

Let $p_{\max} \equiv \max\{p_1, \ldots, p_n\}$, and $\delta_{\min} \equiv \min\{\delta_1, \ldots, \delta_n\}$.

Proposition 1e. *In the extended model a symmetric set Z is supported by a coalition equilibrium if $\delta_{\min} > \underline{\delta} \equiv \frac{u(1)-u(z_{\max})}{u(1)-p_{\max}u(z_{\max})-(1-p_{\max})\overline{\overline{u}}}$, and*

(a) *$z_i = z_{\max}$ for at least m legislators, for all $z \in Z$.*

(b) *$u(z_j) < u(z_{\max})$ for some j and some $z \in Z$.*

Proof. The proof proceeds by checking incentives to deviate from basic strategies, assuming (a) and (b) in Proposition 1e are true and $\delta_{\min} > \underline{\delta}$.

First consider $\mathbf{q^{t-1}} \in \mathbf{Z}$. Suppose that i is the proposer, and consider the votes of other legislators. The equilibrium proposal is the same as the status quo, so legislators vote for the status quo under the indifference rule.

Consider i's incentives to make a deviation proposal. The best potential deviation inside Z gives i a dynamic payoff of $\frac{u(z_{\max})}{1-\delta_i}$, and with the status quo i's dynamic payoff is $\frac{u(q_i^{t-1})}{1-\delta_i}$. If $u(q_i^{t-1}) < u(z_{\max})$, legislator i has an incentive to deviate to an allocation in Z unless a minimal-winning coalition will reject the proposal. A minimal-winning coalition will reject the proposal if (a) holds, because at least one member of the minimal-winning coalition can only be kept indifferent. Under the indifference rule that member rejects the deviation proposal.

The best potential deviation outside Z gives 1 to legislator i and nothing to the other legislators. If such a proposal passed, the continuation value for i would be v_i^*. If legislator i has the allocation $z_{\max}$ under the status quo, legislator i has no incentive to deviate if

$$\begin{aligned} u(1) + \delta_i v_i^* &\leq \frac{u(z_{\max})}{1-\delta_i} \\ \Leftrightarrow u(1) + \delta_i \frac{p_i u(z_{\max}) + (1-p_i)\overline{\overline{u}}}{1-\delta_i} &\leq \frac{u(z_{\max})}{1-\delta_i} \\ \Leftrightarrow \frac{u(1) - u(z_{\max})}{u(1) - p_i u(z_{\max}) - (1-p_i)\overline{\overline{u}}} &\leq \delta_i, \end{aligned} \tag{10}$$

The left hand side of (10) is increasing in p_i, so for $\delta_{\min} \geq \underline{\delta} = \frac{u(1)-u(z_{\max})}{u(1)-p_{\max}u(z_{\max})-(1-p_{\max})\overline{\overline{u}}}$ this holds for all i. Furthermore, if legislator i does not receive $z_{\max}$ under the status quo, at least a minimal-winning coalition does, so for $\delta_{\min} \geq \underline{\delta}$ a minimal-winning coalition rejects a policy outside of Z. By (b) $\overline{\overline{u}} < u(z_{\max})$, and since $p_{\max} < 1$, we have $\underline{\delta} < 1$.

Second, consider $\mathbf{q^{t-1}} \notin \mathbf{Z}$. Consider a legislator's incentives to accept a proposal $y^t \in Z$. A minimal-winning coalition obtains a dynamic payoff of $\frac{u(z_{\max})}{1-\delta_i}$ under the equilibrium strategies. The highest status quo payoff available gives a coalition parter an allocation of 1 and continuation payoff v_i^*. Legislators will reject the proposal if indifferent, hence legislators must have a strict incentive to accept the proposal. This is true if (10) holds strictly, or if $\delta_{\min} > \underline{\delta}$.

Consider a legislator's incentives to propose a policy $y^t \in Z$ that gives him an allocation $z_{\max}$. Suppose $q_i^{t-1} = 1$, then legislator i will prefer the equilibrium proposal to remaining at the status quo if $\delta_i > \underline{\delta}$. This is true for all i if $\delta_{\min} > \underline{\delta}$. Legislators do not have an incentive to propose any other policy in Z, since the dynamic payoff for any other allocation in Z is strictly less than the dynamic payoff from the equilibrium proposal $\frac{u(z_{\max})}{1-\delta_i}$.

Consider a proposal $y^t \notin Z$. Any proposer i receives at most 1 when $q_1^t = 1$ and as above for $\delta_i > \underline{\delta}$ prefers a proposal in Z_i that yields the dynamic payoff $\frac{z_{\max}}{1-\delta_i}$. A majority prefers the proposal to the status quo, so y^t is approved.

■

Proof of Proposition 1

The proof of Proposition 1 follows from Proposition 1e. It is straightforward to verify that for $p_i = \frac{1}{n}$ and $\delta_i = \delta$ for all i the lower bound on the discount factor simplifies to $\underline{\delta} \equiv \frac{u(1)-u(z_{\max})}{u(1)-\overline{u}}$.

Proof of Proposition 2

Sufficiency: The proof of sufficiency proceeds by checking incentives to deviate from basic strategies assuming (a) and (b) are true and $\delta > \underline{\underline{\delta}}$.

First consider $\mathbf{q^{t-1}} \in \mathbf{Z}$. If i is the proposer, consider the votes of other legislators. With a proposal $y^t = q^{t-1}$, legislators are indifferent and vote for q^{t-1} under the indifference rule. Consider legislator i's incentives to make a deviation proposal. The best potential deviation inside Z gives legislator i a dynamic payoff of $\frac{u(z_{\max})}{1-\delta}$, and with the status quo legislator i's dynamic payoff is $\frac{u(q_i^{t-1})}{1-\delta}$. If $u(q_i^{t-1}) < u(z_{\max})$, legislator i has an incentive to deviate to an allocation in Z unless a minimal-winning coalition will reject the proposal. If $W(z) = \emptyset$ for all $z \in Z$, there is no deviation in Z that can defeat q^{t-1}. Legislator i therefore has no incentive to propose this deviation.

To deviate to a policy $z' \notin Z$, legislator i must find an m-member coalition to accept z'. A deviation proposal must include the legislator with the m^{th} largest allocation for any $z \in Z$ and the smallest of these is $z_m^{\min}$. This deviation is rejected by the m-member coalition if

$$\begin{aligned} u(z_i') + \delta v^* &\leq \frac{u(z_m^{\min})}{1-\delta} \\ \Leftrightarrow (1-\delta)u(z_i') + \delta\overline{u} &\leq u(z_m^{\min}) \\ \Leftrightarrow \delta &\geq \underline{\underline{\delta}}(z_i') \equiv \frac{u(z_i')-u(z_m^{\min})}{u(z_i')-\overline{u}}. \end{aligned}$$

Since $\underline{\underline{\delta}}(z_i')$ is increasing in z_i', $\delta \geq \underline{\underline{\delta}}(1) = \frac{u(1)-u(z_m^{\min})}{u(1)-\overline{u}}$ is sufficient to ensure no such deviation is possible.

Second, consider $\mathbf{q^{t-1}} \notin \mathbf{Z}$. Repeating the argument above, some majority prefers $z \in Z$ to any $z' \notin Z$ if $\delta > \underline{\underline{\delta}}(1)$, so the equilibrium proposal is accepted. The proposer also prefers any $z \in Z_i$ to any $z' \notin Z$. Legislator i does not have an incentive to propose any policy in $Z \setminus Z_i$, since the dynamic payoff is strictly less than the dynamic payoff $\frac{u(z_{\max})}{1-\delta}$ from the equilibrium proposal.

Necessity: We show necessity by way of contradiction. Suppose Z can be supported by a coalition equilibrium, and suppose $W(z') \neq \emptyset$ for some $z' \in Z$. Then there exists a $z'' \in Z$ such that m legislators strictly prefer z'' to z'. Consequently, a majority prefers to vote for a deviation to z'', so z' cannot be supported as a coalition equilibrium. Condition (a) thus is necessary. Next, suppose that Z is supported by a coalition equilibrium and Condition (b) is not satisfied. Suppose that there is a $z' \in Z$ such that $v^* \geq v_i(z')$ for some i in each $M \in \mathcal{M}$. Then a minimal-winning coalition will not accept the proposal $z' \in Z$, since

$u(q_i^{t-1}) + \delta v^* < v_i(z')$ does not hold for a minimal-winning coalition. Hence, $v_i(z') > v^*$ for i in M is necessary, and $v_i(z') > v^* \Leftrightarrow u(z_m^{\min}) > \overline{u}$. Finally we show that a lower bound on the discount factor, $\underline{\underline{\delta}} \leq \frac{u(1)-u(z_m^{\min})}{u(1)-\overline{u}}$ is necessary. Consider a status quo policy $z \in Z$ such that every legislator has strictly positive utility. Order the elements of z such that $z_1 \geq \ldots \geq z_n > 0$ and suppose $z_m = z_m^{\min}$. Suppose legislator n proposes a deviation policy $z' \notin Z$. The most profitable deviation gives legislators $i = m, \ldots, n-1$ some allocation z_i' where $u(z_i') > u(z_i)$, and legislator n keeps the allocation $z_n' = 1 - \sum_{i=m}^{n-1} z_i'$ for itself. Suppose $u(z_n') > u(z_n)$, then for δ close to zero, such a proposal is voted for, but if $\delta \geq \underline{\underline{\delta}}(z_i') = \frac{u(z_m')-u(z_m^{\min})}{u(z_m')-\overline{u}}$ legislator m will reject the proposal. By a similar argument if the status quo is $z' \notin Z$ legislator m must have a strict incentive to accept the proposal $z \in Z$, hence it is necessary for $\delta > \underline{\underline{\delta}}(z_i')$, where $z' \notin Z$ gives the m-member coalition strictly higher utility than $z \in Z$. The most binding constraint $\underline{\underline{\delta}}$ is thus determined by the elements of Z. Since $\underline{\underline{\delta}}(z_i')$ is increasing in z_i' we have $\underline{\underline{\delta}} \leq \underline{\underline{\delta}}(1) = \frac{u(1)-u(z_m^{\min})}{u(1)-\overline{u}}$. ■

Dynamic Coalitions in the Extended Model

Proposition 2e. *In the extended model, for each i, there exists a $\underline{\underline{\delta_i}} \leq \frac{u(1)-u(z_m^{\min})}{u(1)-p_i u(z_{\max})-(1-p_i)\overline{\overline{u}}}$ such that the set Z is supported by a coalition equilibrium if and only if $\delta_i > \underline{\underline{\delta_i}}$ for all i and:*

- (a) *No policy in Z can be defeated by another policy in Z; i.e., $W(z) = \emptyset$ for all $z \in Z$.*
- (b) *At least a majority M of legislators is punished if the coalition dissolves; i.e., $v_i(z) > v_i^*$ for all $i \in M$, for all $z \in Z$, or equivalently $u(z_m^{\min}) > p_i u(z_{\max}) + (1-p_i)\overline{\overline{u}}$ for all i.*

The proof is analogous to the proof of Proposition 2.

Proof of Lemma 2

By way of contradiction suppose Z is supportable as a coalition equilibrium, and there are more than m distinct payoffs in z for some $z \in Z$. Consider the status quo $q^{t-1} = z$ giving dynamic payoff $\frac{u(z_i)}{1-\delta}$ to legislator i. Since there are more than m distinct allocations in z and $u(\cdot)$ is strictly increasing, there exists a proposal $y^t \in Z$ that is a permutation of z such that $u(y_i^t) > u(z_i)$ for at least m legislators. That is, consider that z is ordered such that $z_1 \geq \ldots \geq z_n$. Next consider an alternate policy z' such that $z_i' = z_{i+1}$ for $i \neq 1$, and $z_1 = z_n$. Since z' is a permutation of z and Z is symmetric, $z' \in Z$ and gives dynamic payoff $\frac{u(z_i')}{1-\delta}$ to legislator i. Since there are at least $m+1$ distinct payoffs, there are at least m legislators that receive a strictly higher dynamic payoff under z', hence a minimal-winning coalition will vote for the deviation z', contradicting that Z is supportable as a coalition equilibrium.

Proof of Proposition 3

Proposition 1 shows part (a) and (b) are sufficient for an equilibrium to exist. We show below that they are necessary.

By Lemma 2 a necessary condition for Proposition 2 part (a) to be satisfied is that there are $m = 2$ or fewer distinct allocations in each $z \in Z$, implying $z_i = z_j$ for some i, j, for each $z \in Z$.

Next, for any two policies $z, z' \in Z$, if $z_i = z_j$, and $z'_j = z'_k$ for any i, j, k, it must be that $z_i = z_j = z'_j = z'_k$. Suppose not, then assume $z_i = z_j > z'_j = z'_k$. Since both policies are in Z, both are absorbing states, and if $q^{t-1} = z'$ legislators j and k would vote to change the policy to a permutation of z in which they receive the allocations z_i and z_j. This violates Condition (a) of Proposition 2.

The arguments above show that it is necessary for each policy in S to have two identical allocations which we will denote by c. So $S = \{(c, c, w), (y, c, c)\}$, for example. This also implies $z_m = z_2 = c$ for all $z \in Z$. We show (a) in Proposition 1 is necessary, that is, $z_{\max} = c$ for all $z \in Z$. Suppose $|S| = 1$ and $z_{\max} > c$, then $\overline{u} > u(c) = u(z_m)$ which is a contradiction of Condition (b). Clearly, $z_{\max} \not< c$, since that violates the definition of $z_{\max}$. Next suppose $|S| > 1$. If the maximum allocation for any $z \in Z$ is strictly larger than c, then Proposition 2 Condition (b) is again violated by the previous argument. Suppose $z_i = c$ for some $z \in Z$ and $z'_j > c$ for some $z' \in Z$. Then $z_{\max} \geq z'_j > c$. Furthermore, any element of Z with its maximum allocation $z''_{\max} = c$ is not in Z_j for any j (since Z_j gives j the highest possible allocation in Z). Then $\overline{u} \geq u(c) = u(z_m)$ which violates Proposition 2 Condition (b). So for each $z \in Z$, we must have $z_2 = c = z_{\max}$. If Proposition 1 Condition (b) fails, then $z = (c, c, c) \forall z \in Z$, and this violates Proposition 2 Condition (b) that $\overline{u} < u(c)$.

Implementation uncertainty

The maintained assumption on implementation uncertainty ensures that when uncertainty is realized the policy remains in the feasible set X. Assumption 1 gives details of the specification of implementation uncertainty.

Assumption 1. *If a proposal $y^t = q^{t-1}$ is adopted, with probability $1 - \eta$ the policy implemented equals the proposal, and with probability $1 > \eta \geq 0$ the policy is distorted by a uniformly distributed shock $\tilde{\theta}^t$ with mean zero and support $[-\underline{\theta}, \underline{\theta}]$. (i) For $y^t \in Z(c)$ if the realization θ^t is such that $c - \theta^t \geq 0$, the legislators in the coalition receive $1 - c + \theta^t$ and $c - \theta^t$. If $c - \theta^t < 0$ for legislator ℓ, ℓ receives 0 and the other coalition member ℓ' receives 1. (ii) For a proposal $y^t = q^{t-1} \notin Z(c)$, if a legislator ℓ receives 1 in y^t and $1 + \theta^t \geq 1$, ℓ receives 1. If $1 + \theta^t < 1$, ℓ receives $1 + \theta^t$ and one other legislator selected at random receives $-\theta^t$. If only two legislators receive positive allocations in $y^t = (1 - x_\ell, x_\ell, 0)$, where $0 < x_\ell \leq \frac{1}{2}$, they receive $1 - x_\ell + \theta^t$ and $x_\ell - \theta^t$, respectively, if $x_\ell - \theta^t \geq 0$. If $x_\ell - \theta^t \leq 0$, ℓ receives 1 and ℓ' receives 0. If all three legislators receive positive allocations in y^t, the allocations with the shock are $x_\ell + \alpha_\ell \theta^t, \ell = i, j, k$, where $|\alpha_\ell| \leq 1, \ell = i, j, k$, and $\alpha_i + \alpha_j + \alpha_k = 0$. If $x_{\ell'} + \alpha_{\ell'} \theta^t \leq 0$ for some ℓ', ℓ' receives 0 and $-\alpha_{\ell'} \theta^t$ is allocated randomly among the other legislators.*

If a proposal $y^t \neq q^{t-1}$ is adopted, with probability $1 - \gamma$ the policy implemented equals the proposal, and *with probability* $1 > \gamma \geq 0$ *the policy is distorted from the proposal by a uniformly distributed shock* $\tilde{\varepsilon}^t$ *with support* $[-\underline{\varepsilon}, \underline{\varepsilon}]$. *(iii) For* $y^t \in Z(c)$ *the allocations are as in (i) with the realization* ε^t *replacing* θ^t. *(iv) For* $y^t \notin Z(c)$ *the allocations are as in (ii) with the realization* ε^t *replacing* θ^t.

Specific-policy equilibrium policies

For notational convenience define $z_{ij}(c) \in Z(c)$ as the policy that allocates $1-c$ to legislator i, c to legislator j, and 0 to legislator k, so legislator i receives the highest allocation and legislator j is the coalition partner.

Proof of Lemma 3

Let $v_i(y^t), i = 1, 2, 3$, denote the continuation value when the proposal is y^t, conditional on no shock being realized. Let $v_i(y^{\varepsilon^t})$ denote the continuation value when y^t is implemented conditional on the shock ε^t being realized, and $v_i(y^{\theta^t})$ denote the continuation value when y^t is implemented conditional on the shock θ^t being realized.

The continuation value $\hat{v}_1$ when $q^{t-1} \notin Z(c)$ is given by

$$\begin{aligned}
\hat{v}_1 \quad = \quad & (1-\gamma)\left[\frac{1}{3}\left[\frac{1}{2}(1-c+\delta v_1(z_{12}(c)))\right] + \left[\frac{1}{2}(1-c+\delta v_1(z_{13}(c)))\right]\right] \\
& + \frac{1}{3}\left[\frac{1}{2}\left(c+\delta v_1(z_{21}(c))\right) + \frac{1}{2}\delta v_1(z_{23}(c))\right] \\
& + \frac{1}{3}\left[\frac{1}{2}\left(c+\delta v_1(z_{31}(c))\right) + \frac{1}{2}\delta v_1(z_{32}(c))\right] \\
& + \gamma\left[\frac{1}{3}\left[\frac{1}{2}(1-c+E^t\tilde{\varepsilon}^t+\delta v_1(z_{12}^{\varepsilon^t})) + \frac{1}{2}(1-c+E^t\tilde{\varepsilon}^t+\delta v_1(z_{13}^{\varepsilon^t}))\right]\right] \\
& + \frac{1}{3}\left[\frac{1}{2}(c-E^t\tilde{\varepsilon}^t+\delta v_1(z_{21}^{\varepsilon^t})) + \frac{1}{2}\delta v_1(z_{23}^{\varepsilon^t})\right] \\
& + \frac{1}{3}\left[\frac{1}{2}(c-E^t\tilde{\varepsilon}^t+\delta v_1(z_{31}^{\varepsilon^t})) + \frac{1}{2}\delta v_1(z_{32}^{\varepsilon^t})\right], \qquad (11)
\end{aligned}$$

and the continuation values $\hat{v}_2$ and $\hat{v}_3$ for the other legislators are analogous. By symmetry $\hat{v}_i = \hat{v}, i = 1, 2, 3$.

Given $q^{t-1} \notin Z(c)$, the policy $z_{ij}^{\varepsilon^t}$ resulting from a proposal $y^t \neq q^{t-1}$ is not in $Z(c)$ with probability one, so the continuation values $v_i(z_{ij}^{\varepsilon^t}) = \hat{v}, i, = 1, 2, 3,\ i \neq j$. For $q^{t-1} = z_{ij}(c)$ the allocation $z_{ij}^{\theta^t} \in Z(c)$ with probability zero, so the continuation value $v_i(z_{ij}^{\theta^t}) = \hat{v}, i = 1, 2, 3,\ i \neq j$. For $q^{t-1} \in Z(c)$ the dynamic payoffs $v_i(z_{12}(c)), i = 1, 2, 3$, are given by

$$v_1(z_{12}(c)) \quad = \quad (1-\eta)[1-c+\delta v_1(z_{12}(c))] + \eta[1-c+E^t\tilde{\theta}^t+\delta\hat{v}] \qquad (12)$$

$$v_2(z_{12}(c)) \quad = \quad (1-\eta)[c+\delta v_2(z_{12}(c))] + \eta[c+E^t\tilde{\theta}^t+\delta\hat{v}] \qquad (13)$$

$$v_3(z_{12}(c)) \quad = \quad (1-\eta)\delta v_3(z_{12}(c)) + \eta\delta\hat{v}. \qquad (14)$$

Continuation values for the other policies in $Z(c)$ are defined analogously. Solving (11)-(14) and the analogous conditions simultaneously yields the continuation values in Lemma 3.

Proof of Lemma 4

The proof of part (i) proceeds by checking incentives to deviate from basic strategies given the continuation values in Lemma 3.

Suppose $\mathbf{q^{t-1}} \in \mathbf{Z(c)}$, so $q^{t-1} = z_{ij}(c)$ for some i and j. Consider the incentives of legislators to vote for the equilibrium proposal. With basic strategies the proposal $z_{ij}(c) \in Z$ is the same as the status quo, so legislators vote for the status quo.

Consider i's incentive to propose a deviation. Proposing $z_{ji}(c)$ or $z_{ki}(c)$ with j and k voting for the proposal changes the status quo if approved, and since $c \leq \frac{1}{2}$ and $\eta \leq \gamma$, $z_{ij}(c)$ is preferred by i. Formally, the incentive constraint is

$$(1-\eta)[1-c+\delta v_i(z_{ij}(c))] + \eta[1-c+E^t\tilde{\theta}^t + \delta\hat{v}] \geq (1-\gamma)[c+\delta v_i(z_{ji}(c))] + \gamma[c - E^t\tilde{\varepsilon}^t + \delta\hat{v}],$$

which can be verified using $\eta \leq \gamma$.

Proposing $z_{ik}(c)$ results in a change in the status quo if approved, so i prefers to propose $z_{ij}(c)$, since by stochastic dominance

$$(1-\eta)[1-c+\delta v_i(z_{ij}(c))] + \eta[1-c+E^t\tilde{\theta}^t + \delta\hat{v}] \geq (1-\gamma)[1-c+\delta v_i(z_{ik}(c))] + \gamma[1-c+E^t\tilde{\varepsilon}^t + \delta\hat{v}].$$

Proposing $z_{jk}(c)$ or $z_{kj}(c)$ changes the status quo if approved, and i prefers to propose $z_{ij}(c)$, since

$$(1-\eta)[1-c+\delta v_i(z_{ij}(c))] + \eta[1-c+E^t\tilde{\theta}^t + \delta\hat{v}] \geq (1-\gamma)\delta v_i(z_{jk}(c)) + \gamma\delta\hat{v}.$$

The best proposal deviation for i outside the set $Z(c)$ gives 1 to i with i and k voting for the proposal. Proposer i prefers not to deviate if and only if

$$(1-\eta)[1-c+\delta v_i(z_{ij}(c))] + \eta[1-c+E^t\tilde{\theta}^t + \delta\hat{v}] \geq 1 - \gamma\tfrac{\underline{\varepsilon}}{4} + \delta\tfrac{1}{3(1-\delta)},$$

where $1-\gamma\frac{\underline{\varepsilon}}{4} = 1-\gamma+\gamma\left[\int_{-\underline{\varepsilon}}^{0}(1+\varepsilon^t)\frac{d\varepsilon^t}{2\underline{\varepsilon}} + \int_0^{\underline{\varepsilon}}\frac{d\varepsilon^t}{2\underline{\varepsilon}}\right]$ is i's expected truncated payoff in period t. Then legislator i does not deviate if

$$c \leq c^u \equiv \tfrac{\underline{\varepsilon}\gamma(1-\delta(1-\eta))}{4} + \tfrac{2\delta(1-\eta)}{3}.$$

Consider j's incentives to propose a deviation. Proposals $z_{ki}(c)$ or $z_{ik}(c)$ give j an allocation of 0, and j

has no incentive to propose these over $z_{ij}(c)$, since

$$(1-\eta)[c+\delta v_j(z_{ij}(c))]+\eta[c+E^t\tilde{\theta}^t+\delta\hat{v}] \geq (1-\gamma)[\delta v_j(z_{ki}(c))]+\gamma[E^t\tilde{\varepsilon}^t+\delta\hat{v}].$$

If j proposes $z_{kj}(c)$, j and i vote against it as above.

If j proposes $z_{ji}(c)$ or $z_{jk}(c)$, j receives $1-c$ in expectation in the current period, and with probability $1-\gamma$ the continuation value is $v_j(z_{ji}(c)) = v_i(z_{ij}(c))$ and with probability γ the continuation value is $\hat{v}$. Legislator j has no incentive to deviate if and only if

$$\begin{aligned}(1-\eta)[c+\delta v_j(z_{ij}(c))]+\eta[c-E^t\tilde{\theta}^t+\delta\hat{v}] &\geq (1-\gamma)[1-c+\delta v_j(z_{jk}(c))]+\gamma[1-c+E^t\tilde{\varepsilon}^t+\delta\hat{v}] \\ \Leftrightarrow c &\geq \tfrac{3-2\delta(\gamma-\eta)}{3(2-\delta(\gamma-\eta))} = c^*. \end{aligned} \tag{15}$$

If j proposes $y^t \notin Z(c)$, the best proposal gives 1 to j (j and k vote for it), and the expected truncated payoff is $1-\gamma\frac{\underline{\varepsilon}}{4}+\delta\frac{1}{3(1-\delta)}$. Legislator j prefers the equilibrium proposal if and only if

$$\begin{aligned}(1-\eta)[c+\delta v_j(z_{ij}(c))]+\eta[c-E^t\tilde{\theta}^t+\delta\hat{v}] &\geq 1-\gamma\tfrac{\underline{\varepsilon}}{4}+\delta\tfrac{1}{3(1-\delta)} \\ \Leftrightarrow c &\geq c^\ell \equiv \tfrac{3-2\delta(1-\eta)}{3}-\tfrac{\underline{\varepsilon}\gamma(1-\delta(1-\eta))}{4}. \end{aligned} \tag{16}$$

Note that $c^\ell + c^u = 1$.

Consider k's incentive to propose a deviation. As shown above if $c \in [\max\{c^*, c^\ell\}, c^u]$, i and j prefer $z_{ij}(c)$ to any other allocation. Hence, any proposal by k different from $z_{ij}(c)$ will be rejected. Legislator k's payoff is the same if he proposes $z_{ij}(c)$ and it is accepted, or proposes another allocation that is rejected, hence legislator k has no incentive to deviate from the equilibrium strategies.

Suppose $\mathbf{q^{t-1}} \notin \mathbf{Z(c)}$. Consider j's incentive to vote for the equilibrium proposal $z_{ij}(c)$. The best status quo for j, gives 1 to j. Legislator j along with i vote for $z_{ij}(c)$ rather than the status quo if and only if

$$1-\eta\frac{\underline{\theta}}{4}+\delta\frac{1}{3(1-\delta)} < (1-\gamma)[c+\delta v_j(z_{ij}(c))]+\gamma[c-E^t\tilde{\varepsilon}^t+\delta\hat{v}],$$

where $1-\eta\frac{\underline{\theta}}{4} = 1-\eta+\eta\left[\int_{-\underline{\theta}}^{0}(1+\theta^t)\frac{d\theta^t}{2\underline{\theta}}+\int_0^{\underline{\theta}}\frac{d\theta^t}{2\underline{\theta}}\right]$ is the expected truncated payoff in period t. Legislator j accepts the proposal if

$$c > \frac{3-\delta(2+\gamma-3\eta)-\frac{3}{4}\eta\underline{\theta}(1-\delta(1-\eta))}{3(1-\delta(\gamma-\eta))} = c^o.$$

Since $\eta\underline{\theta} \leq \gamma\underline{\varepsilon}$, $c^o \geq c^\ell$, so c^ℓ is not binding.

Consider i's incentive to make a proposal other than $z_{ij}(c)$. Using the continuation values in (1), $z_{ij}(c)$ gives i the highest payoff among proposals in $Z(c)$, so there is no incentive to make any other proposal in

$Z(c)$.

If q^{t-1} gives 1 to i, i strictly prefers $z_{ij}(c)$ to the status quo if and only if

$$\begin{aligned} 1-\eta\tfrac{\underline{\theta}}{4}+\delta\tfrac{1}{3(1-\delta)} &< (1-\gamma)[1-c+\delta v_i(z_{ij}(c))]+\gamma[1-c+E^t\tilde{\varepsilon}^t+\delta\hat{v}] \\ \Leftrightarrow c &< \hat{c}_1 \equiv \tfrac{2\delta(1-\gamma)+\frac{3}{4}\eta\underline{\theta}(1-\delta(1-\eta))}{3(1-\delta(\gamma-\eta))}. \end{aligned}$$

Since $\eta\underline{\theta} \leq \gamma\underline{\varepsilon}$, $c^u \geq \hat{c}_1$, so c^u is not binding. Note that $\hat{c}_1 = 1-c^o$.

If i does not receive 1 in q^{t-1}, i prefers a proposal $z_{ij}(c)$ to a proposal that gives 1 to i if and only if

$$\begin{aligned} 1-\gamma\tfrac{\underline{\varepsilon}}{4}+\delta\tfrac{1}{3(1-\delta)} &\leq (1-\gamma)[1-c+\delta v_i(z_{ij}(c))]+\gamma[1-c+E^t\tilde{\varepsilon}^t+\delta\hat{v}], \\ \Leftrightarrow c &\leq \hat{c}_2 \equiv \tfrac{2\delta(1-\gamma)+\frac{3}{4}\gamma\underline{\varepsilon}(1-\delta(1-\eta))}{3(1-\delta(\gamma-\eta))}. \end{aligned}$$

Note that $\hat{c}_1 \leq \hat{c}_2$, since $\eta\underline{\theta} \leq \gamma\underline{\varepsilon}$, so $\hat{c}_2$ is not binding.

To prove part (ii) it is straightforward to show that c^* is decreasing in γ and increasing in η. Evaluating c^* at $\gamma = 1$ and $\eta = 0$ yields $c^* = \frac{3-2\delta}{3(2-\delta)}$, which implies that $c^* > \frac{1}{3}$, for all $\gamma \in [0,1)$ and $\eta \in [0,1)$. Consequently, if $(1-c, c, 0)$ is a coalition equilibrium proposal, $c - \underline{\varepsilon} > 0$, and hence $c - \underline{\theta} > 0$. Also, $1-c+\underline{\varepsilon} < 1$, so $1-c+\underline{\theta} < 1$. Then $\underline{\varepsilon} \leq \frac{1}{3}$ is sufficient for the coalition allocations c and $1-c$ to be in $[0,1]$.■

Proof of Lemma 5

The lower bound $c^* \leq \frac{1}{2}$ for all $\gamma \geq \eta$, $\delta > 0$. The lower (upper) bound c^o $(1-c^o)$ is strictly less (greater) than $\frac{1}{2}$ for $\delta > \delta^o = \frac{3-\frac{3}{2}\eta\underline{\theta}}{4-\gamma-3\eta-\frac{3}{2}(1-\eta)\eta\underline{\theta}}$. ■

Proof of Lemma 6

It is straightforward to show that $\delta^o < 1 \Leftrightarrow (\gamma, \eta) \in R(\underline{\theta})$. ■

Proof of Corollary 7

The difference $c^* - c^o$ is increasing in δ for $\gamma \leq \frac{2}{3}$. To show this, differentiation yields

$$\begin{aligned} \tfrac{\partial(c^*-c^o)}{\partial\delta} &= -\tfrac{\gamma-\eta}{3[2-\delta(\gamma-\eta)]^2}+\tfrac{(2-\frac{3}{4}\eta\underline{\theta})(1-\delta(1-\eta))}{3[1-\delta(\gamma-\eta)]^2} \\ &> -\tfrac{\gamma-\eta}{3[2-\delta(\gamma-\eta)]^2}+\tfrac{(2-\frac{3}{4}\eta\underline{\theta})(1-\delta(1-\eta))}{3[2-\delta(\gamma-\eta)]^2}. \end{aligned} \tag{17}$$

If $\gamma = \eta$, the first line of (17) is positive. If $\gamma = \eta$, the second line is positive if $2-3\gamma+\eta[1-\frac{3}{4}\underline{\theta}(1-\gamma)] > 0$, which is the case for $\gamma \leq \frac{2}{3}$. The greater lower bound is then c^* if and only if $\delta \geq \delta^+$, where δ^+ in (3) is obtained by equating c^o and c^* in (2). ■

Characterization of c^+ in terms of γ

The following lemma characterizes c^+ in terms of the probability γ of implementation uncertainty with $c^+ = c^*$ for low γ and $c^+ = c^o$ for higher γ.

Lemma 10. *$c^+ = c^*$ for $\gamma \leq \gamma^e$ and $c^+ = c^o$ for $\gamma > \gamma^e$, where*

$$\gamma^e \equiv 1 + \tfrac{1}{8\delta}\left[3\eta\underline{\theta}(1-\delta(1-\eta)) - [(8-3\eta\underline{\theta})(1-\delta(1-\eta))(16+(8-3\eta\underline{\theta})[1-\delta(1-\eta)])]^{\frac{1}{2}}\right]. \tag{18}$$

Proof. The bound c^* is decreasing in γ, and c^o is increasing in γ, so the difference $c^* - c^o$ is decreasing in γ. The greater lower bound is then c^* if and only if $\gamma \geq \gamma^e$, where γ^e in (18) is obtained by equating c^o and c^* in (2). ■

Proof of Corollary 8

Substituting $\gamma = \eta$ into c^* given in (2) yields $c^* = \frac{1}{2}$. The condition $\eta < \frac{1}{3\underline{\theta}}[4 - 2(4 - \frac{3}{2}\underline{\theta})^{\frac{1}{2}}]$ implies $(\eta, \eta) \in R(\underline{\theta})$, so $\delta^o < 1$ by Lemma 6 and hence $c^o < \frac{1}{2}$. ■

Tolerant coalition equilibrium

Proof of Lemma 9

For $c \in \zeta$ the dynamic payoffs are given by

$$\begin{aligned}
\bar{v}_i(z_{ij}(c)) &= (1-\eta)\left[1 - c + \delta\bar{v}_i(z_{ij}(c))\right] + \eta[1 - c + E^t\tilde{\theta}^t + \delta\bar{v}_i(z_{ij}^{\theta^t})] && (19)\\
\bar{v}_j(z_{ij}(c)) &= (1-\eta)\left[c + \delta\bar{v}_j(z_{ij}(c))\right] + \eta[c - E^t\tilde{\theta}^t + \delta\bar{v}_j(z_{ij}^{\theta^t})] && (20)\\
\bar{v}_k(z_{ij}(c)) &= (1-\eta)\delta\bar{v}_k(z_{ij}(c)) + \eta\delta\bar{v}_k(z_{ij}^{\theta^t}), && (21)
\end{aligned}$$

where

$$\begin{aligned}
\bar{v}_\ell(z_{ij}^{\theta^t}) = {} & \int_{-\underline{\theta}}^{c+\underline{c}-1} \bar{v}_\ell(z_{ij}(c-\theta^t))\tfrac{1}{2\underline{\theta}}d\theta^t + \int_{c+\underline{c}-1}^{c-\underline{c}} \bar{v}_\ell(z_{ij}(c-\theta^t))\tfrac{1}{2\underline{\theta}}d\theta^t \\
& + \int_{c-\underline{c}}^{\underline{\theta}} \bar{v}_\ell(z_{ij}(c-\theta^t))\tfrac{1}{2\underline{\theta}}d\theta^t. && (22)
\end{aligned}$$

In the first and third integrals in (22), $c - \theta^t \notin \zeta$, so $\bar{v}_\ell(z_{ij}(c-\theta^t)) = \bar{v}_\ell(z_{ij}(c'))$ for some $c' \notin \zeta$, where $\bar{v}_\ell(z_{ij}(c'))$ is not a function of c or θ according to the equilibrium strategies.

In the second integral in (22) $c - \theta^t \in \zeta$. Conjecture that for $c \in \zeta$, $\bar{v}_i(z_{ij}(c))$ is linear in $1-c$, $\bar{v}_j(z_{ij}(c))$ is linear in c and that these are given by $\bar{v}_i(z_{ij}(c)) = a_i + b_i(1-c)$ and $\bar{v}_j(z_{ij}(c)) = a_j + b_jc$. Then $\bar{v}_i(z_{ij}(c-\theta^t)) = a_i + b_i(1-c+\theta^t)$, and $\bar{v}_j(z_{ij}(c-\theta^t)) = a_j + b_j(c-\theta^t)$. Conjecture that $\bar{v}_k(z_{ij}(c))$ is constant in c. Then for $\ell = i, j$

$$\bar{v}_\ell(z_{ij}^{\theta^t}) = \frac{\bar{v}_\ell(z_{ij}(c'))(2\underline{c}-1+2\underline{\theta})}{2\underline{\theta}} + \frac{(1-2\underline{c})(2a_\ell+b_\ell)}{4\underline{\theta}} \tag{23}$$

$$\bar{v}_k(z_{ij}^{\theta^t}) = \frac{\bar{v}_\ell(z_{ij}(c'))(2\underline{c}-1+2\underline{\theta})}{2\underline{\theta}} + \frac{(1-2\underline{c})a_k}{2\underline{\theta}}. \tag{24}$$

Substituting into (19)-(21) and matching coefficients gives

$$\begin{aligned} a_i = a_j &= \frac{\delta\eta(1-2\underline{c})}{2[1-\delta(1-\eta)][2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})]} + a_k \\ a_k &= \frac{\delta\eta v_k(z_{ij}(c'))(2\underline{c}-1+2\underline{\theta})}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})} \\ b_i = b_j &= \frac{1}{1-\delta(1-\eta)}. \end{aligned}$$

Substituting the coefficients and simplifying (23)-(24) gives

$$\bar{v}_\ell(z_{ij}^{\theta^t}) = \frac{(1-2\underline{c})[1-2\bar{v}_\ell(z_{ij}(c'))(1-\delta)]}{2[2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})]} + \bar{v}_\ell(z_{ij}(c')) \tag{25}$$

$$\bar{v}_k(z_{ij}^{\theta^t}) = \frac{(1-2\underline{c})\bar{v}_k(z_{ij}(c'))(1-\delta)}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})} + \bar{v}_k(z_{ij}(c')). \tag{26}$$

Simplifying (19)-(21) gives

$$\bar{v}_i(z_{ij}(c)) = \frac{1-c}{1-\delta(1-\eta)} + \beta_i \tag{27}$$

$$\bar{v}_j(z_{ij}(c)) = \frac{c}{1-\delta(1-\eta)} + \beta_j \tag{28}$$

$$\bar{v}_k(z_{ij}(c)) = \beta_k, \tag{29}$$

where for $\ell = i, j$,

$$\begin{aligned} \beta_\ell &= \frac{\eta\delta\bar{v}_\ell(z_{ij}(c'))}{1-\delta(1-\eta)} + \frac{\eta\delta(1-2\underline{c})[1-2(1-\delta)\bar{v}_\ell(z_{ij}(c'))]}{2[1-\delta(1-\eta)][2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})]} \text{ and} \\ \beta_k &= \frac{\eta\delta\bar{v}_k(z_{ij}(c'))}{1-\delta(1-\eta)} + \frac{\eta\delta(1-2\underline{c})(1-\delta)\bar{v}_j(z_{ij}(c'))}{[1-\delta(1-\eta)][2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})]}. \end{aligned}$$

For $q^{t-1} \neq z_{ij}(c)$ for all $c \in \zeta$, the continuation value $\bar{v}_\ell(q^{t-1})$ is, using the equilibrium strategies,

$$\begin{aligned} \bar{v}_\ell(q^{t-1}) = {} & (1-\gamma)\left[\tfrac{1}{3}\left[\tfrac{1}{2}(1-c+\delta\bar{v}_\ell(z_{ij}(c))) + \tfrac{1}{2}(1-c+\delta\bar{v}_\ell(z_{ik}(c)))\right]\right. \\ & +\tfrac{1}{3}\left[\tfrac{1}{2}\left(c+\delta\bar{v}_\ell(z_{ji}(c))\right) + \tfrac{1}{2}\delta\bar{v}_\ell(z_{jk}(c))\right] \\ & \left.+\tfrac{1}{3}\left[\tfrac{1}{2}\left(c+\delta\bar{v}_\ell(z_{ki}(c))\right) + \tfrac{1}{2}\delta\bar{v}_\ell(z_{kj}(c))\right]\right] \\ & +\gamma\left[\tfrac{1}{3}\left[\tfrac{1}{2}\left(1-c+E^t\tilde{\varepsilon}^t+\delta\bar{v}_l(z_{ij}^{\varepsilon^t})\right) + \tfrac{1}{2}\left(1-c+E^t\tilde{\varepsilon}^t+\delta\bar{v}_\ell(c_{ik}^{\varepsilon^t})\right)\right]\right. \\ & +\tfrac{1}{3}\left[\tfrac{1}{2}\left(c-E^t\tilde{\varepsilon}^t+\delta\bar{v}_\ell(c_{ji}^{\varepsilon^t})\right) + \tfrac{1}{2}\delta\bar{v}_\ell(c_{jk}^{\varepsilon^t})\right] \\ & \left.+\tfrac{1}{3}\left[\tfrac{1}{2}(c-E^t\tilde{\varepsilon}^t+\delta\bar{v}_\ell(z_{ki}^{\varepsilon^t})) + \tfrac{1}{2}\delta\bar{v}_\ell(c_{kj}^{\varepsilon^t})\right]\right], \end{aligned} \tag{30}$$

where $\bar{v}_\ell(z_{ij}(c)), \ell = i, j, k$, are given by (27)-(29) and

$$\begin{aligned}\bar{v}_\ell(z_{ij}^{\varepsilon^t}) &= \int_{-\underline{\varepsilon}}^{c+\underline{c}-1} \bar{v}_\ell(z_{ij}(c-\varepsilon^t))\tfrac{1}{2\underline{\varepsilon}}d\varepsilon^t + \int_{c+\underline{c}-1}^{c-\underline{c}} \bar{v}_\ell(z_{ij}(c-\varepsilon^t))\tfrac{1}{2\underline{\varepsilon}}d\varepsilon^t \\ &+ \int_{c-\underline{c}}^{\underline{\varepsilon}} \bar{v}_\ell(z_{ij}(c-\varepsilon^t))\tfrac{1}{2\underline{\varepsilon}}d\varepsilon^t. \end{aligned} \tag{31}$$

In the first and third integrals in (31), $c - \varepsilon^t \notin \zeta$, so $\bar{v}_\ell(z_{ij}(c-\varepsilon^t)) = \bar{v}_\ell(q^{t-1})$. In the second integral in (31) $c - \varepsilon^t \in \zeta$, so $\bar{v}_\ell(z_{ij}(c-\varepsilon^t))$ is given by (27)-(29). Then substituting from (27)-(29) and simplifying gives

$$\bar{v}_i(z_{ij}^{\varepsilon^t}) = \left(\tfrac{1-2\underline{c}}{2\underline{\varepsilon}}\right)\tfrac{\underline{\theta}[1-2(1-\delta)\bar{v}_i(q^{t-1})]}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})} + \bar{v}_i(q^{t-1}) \tag{32}$$

$$\bar{v}_j(z_{ij}^{\varepsilon^t}) = \left(\tfrac{1-2\underline{c}}{2\underline{\varepsilon}}\right)\tfrac{\underline{\theta}[1-2(1-\delta)\bar{v}_j(q^{t-1})]}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})} + \bar{v}_j(q^{t-1}) \tag{33}$$

$$\bar{v}_k(z_{ij}^{\varepsilon^t}) = -\left(\tfrac{1-2\underline{c}}{3\underline{\varepsilon}}\right)\tfrac{\underline{\theta}(1-\delta)\bar{v}_k(q^{t-1})}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2\underline{c})} + \bar{v}_k(q^{t-1}). \tag{34}$$

By symmetry $\bar{v}_i(q^{t-1}) = \bar{v}_j(q^{t-1}) = \bar{v}_k(q^{t-1}) = \bar{v}_\ell(q^{t-1})$. Substituting (32)-(34) into (30) and solving gives

$$\bar{v}_\ell(q^{t-1}) = \hat{v} = \tfrac{1}{3(1-\delta)}, \ell = i, j, k. \tag{35}$$

This proves part (ii) of the lemma.

To prove part (i), by part (ii) $\hat{v} = \frac{1}{3(1-\delta)}$ is the continuation payoff for any allocation such that $c \notin \zeta$, hence $\bar{v}_\ell(z_{ij}(c')) = \hat{v} = \frac{1}{3(1-\delta)}$, for $c' \notin \zeta$. Substituting $\bar{v}_\ell(z_{ij}(c')) = \frac{1}{3(1-\delta)}$ into (27)-(29) yields (8).

To prove part (iii), first note that the numerator of (6) is nonnegative, since $\underline{c} \leq \frac{1}{2}$. Using $\underline{\theta} \geq 1 - 2\underline{c}$ from Assumption 4 the denominator of (6) yields

$$2\underline{\theta}(1-\delta(1-\eta)) - \delta\eta(1-2\underline{c}) \geq \underline{\theta}[2-\delta(2-\eta)] > 0,$$

so $\nu(\underline{c}) \geq 0$. If $\underline{c} < \frac{1}{2}$, the numerator is strictly positive. ■

Proof of Lemma 7

The proof proceeds by checking incentives to deviate from basic strategies supporting $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$. The following lemma identifies the continuation values when a coalition dissolves, which is required to check incentives to deviate.

Lemma 11. *If $q^{t-1} = z_{ij}(c)$ for $c \in \zeta$, the continuation value $\bar{v}_\ell(z_{ij}^{\varepsilon^t})$ for $c - \underline{\varepsilon} < \underline{c}$ when $y^t \neq q^{t-1}$ is proposed is given by*

$$\bar{v}_i(z_{ij}^{\varepsilon^t}) = \bar{v}_j(z_{ij}^{\varepsilon^t}) = \tfrac{1}{3(1-\delta)} + \tfrac{\underline{\theta}}{\underline{\varepsilon}}\nu(\underline{c}) \tag{36}$$

$$\bar{v}_k(z_{ij}^{\varepsilon^t}) = \tfrac{1}{3(1-\delta)} - 2\tfrac{\underline{\theta}}{\underline{\varepsilon}}\nu(\underline{c}). \tag{37}$$

If $q^{t-1} = z_{ij}(c)$ *for* $c \in \zeta$, *the continuation value* $\bar{v}_\ell(z_{ij}^{\theta^t})$ *for* $c - \underline{\theta} < \underline{c}$ *when* $y^t = z_{ij}(c)$ *is proposed is given by*

$$\bar{v}_i(z_{ij}^{\theta^t}) = \bar{v}_j(z_{ij}^{\theta^t}) = \tfrac{1}{3(1-\delta)} + \nu(\underline{c}) \tag{38}$$

$$\bar{v}_k(z_{ij}^{\theta^t}) = \tfrac{1}{3(1-\delta)} - 2\nu(\underline{c}). \tag{39}$$

Proof. The first part follows from substituting $\bar{v}_\ell(q^{t-1}) = \frac{1}{3(1-\delta)}$ into (32)–(34). The second part follows from substituting $\bar{v}_\ell(z_{ij}(c')) = \frac{1}{3(1-\delta)}$ into (25) and (26). ■

The continuation values in (36)-(39) are constant in c because of Assumption 4 and the uniformly distributed shocks. The continuation values $\bar{v}_\ell(z_{ij}^{\varepsilon^t})$ and $\bar{v}_\ell(z_{ij}^{\theta^t})$, $\ell = i, j$, are greater than $\frac{1}{3(1-\delta)}$ because a proposal $z_{ij}(c), c \notin \zeta$ could result in a tolerant coalition allocation, whereas it equals the specific-policy allocation with probability 0. Note that $\bar{v}_\ell(z_{ij}^{\theta^t}) \geq \bar{v}_\ell(z_{ij}^{\varepsilon^t}), \ell = i, j$, and $\bar{v}_k(z_{ij}^{\theta^t}) \leq \bar{v}_k(z_{ij}^{\varepsilon^t})$, since $\underline{\varepsilon} \geq \underline{\theta}$. If $\underline{\theta} = \underline{\varepsilon}$, $\bar{v}_\ell(z_{ij}^{\theta^t}) = \bar{v}_\ell(z_{ij}^{\varepsilon^t}), \ell = i, j, k$.

We show in the next Lemma that the optimal proposal for the originator of a coalition gives the proposer $1 - \underline{c}$.

Lemma 12. *For* $y^t \neq q^{t-1} \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, *the optimal proposal by the originator* i *of a tolerant coalition is* $z_{i\ell}(\underline{c}), \ell = j, k$.

Proof. Legislator i proposes $z_{ij}(c) \in \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, which yields an expected dynamic payoff $EU_i(c)$ given by

$$EU_i(c) = (1-\gamma)[1 - c + \delta \bar{v}_i(z_{ij}(c))] + \gamma[1 - c + E^t \tilde{\varepsilon}^t + \delta \bar{v}_i(z_{ij}^{\varepsilon^t})], \tag{40}$$

where $\bar{v}_i(z_{ij}(c))$ is given in (8) and $\bar{v}_i(z_{ij}^{\varepsilon^t})$ is given in (36). From Lemma 11 $\bar{v}_i(z_{ij}^{\varepsilon^t})$ does not depend on c, so differentiating (40) yields

$$\tfrac{dEU_i(c)}{dc} = -1 - \tfrac{\delta(1-\gamma)}{1-\delta(1-\eta)} < 0.$$

Consequently, i prefers the lowest $c \in \zeta$, so $c = \underline{c}$ is optimal. ■

We now check incentives to deviate from basic strategies.

Suppose $\mathbf{q^{t-1}} \in \bigcup_{\mathbf{c}=\underline{\mathbf{c}}}^{\frac{1}{2}} \mathbf{Z(c)}$. Then $q^{t-1} = z_{ij}(c)$ for some i and j and some $c \in \zeta$. Consider the incentives of legislators to accept the equilibrium proposal. With basic strategies the proposal is the same as the status quo, so legislators vote for the status quo.

Consider i's and j's incentives to propose a deviation. The lowest allocation for i or j when the status quo is in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ is $\underline{c}$, so consider $q^{t-1} = z_{ij}(\underline{c})$ and j's incentives. Since i's payoff is strictly higher, i has no incentive to deviate if j does not have an incentive to deviate.

From Lemma 12 the best deviation proposal for j in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ gives j the allocation $1-\underline{c}$. Legislator j will not propose this if

$$\begin{aligned}(1-\eta)[\underline{c}+\delta\bar{v}_j(z_{ij}(\underline{c}))]+\eta[\underline{c}-E^t\tilde{\theta}^t+\delta\bar{v}_j(z_{ij}^{\theta^t})] &\geq (1-\gamma)[1-\underline{c}+\delta\bar{v}_j(z_{ji}(\underline{c}))]+\gamma[1-\underline{c}+E^t\tilde{\varepsilon}^t+\delta\bar{v}_j(z_{ji}^{\varepsilon^t})] \\ \Leftrightarrow \underline{c} &\geq c^{**}. \end{aligned} \tag{41}$$

If j proposes $y^t \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, the best proposal such that the realized policy is not in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ gives 1 to j. Legislator j has no incentive to deviate if

$$\begin{aligned}(1-\eta)[\underline{c}+\delta\bar{v}_j(z_{ij}(\underline{c}))]+\eta[\underline{c}-E^t\tilde{\theta}^t+\delta\bar{v}_j(z_{ij}^{\theta^t})] &\geq 1-\gamma\tfrac{\underline{\varepsilon}}{4}+\delta\tfrac{1}{3(1-\delta)} \\ \Leftrightarrow \underline{c} &\geq c^{\ell\ell}.\end{aligned}$$

Legislator j may propose $y^t \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ such that the realized policy has some probability of being in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$. Consider a policy that awards $1-a^1$ to legislator j, where $0 \leq a^1 \leq \underline{c}$. Legislator j has no incentive to deviate if

$$(1-\eta)[\underline{c}+\delta\bar{v}_j(z_{ij}(\underline{c}))]+\eta[\underline{c}-E^t\tilde{\theta}^t+\delta\bar{v}_j(z_{ij}^{\theta^t})] \geq (1-\gamma)[1-a^1+\delta\hat{v}]+\gamma E^t[1-a^1+\tilde{\varepsilon}^t+\delta\bar{v}_j(y^{\varepsilon^t})], \tag{42}$$

where $\bar{v}_j(y^{\varepsilon^t})$ is the continuation payoff from the realized policy. This realized policy may be in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ or not. If the realized allocation can be less than $\underline{c}$, then by Assumption 1

$$\begin{aligned}& (1-\gamma)[1-a^1+\delta\hat{v}]+\gamma E^t[1-a^1+\tilde{\varepsilon}^t+\delta\bar{v}_j(y^{\varepsilon^t})] \\ = {} & (1-\gamma)(1-a^1)+\gamma E^t(1-a^1+\varepsilon^t) \\ & +\delta\left[(1-\gamma)\hat{v}+\gamma\int_{-\underline{\varepsilon}}^{a^1+\underline{c}-1}\hat{v}\tfrac{d\varepsilon^t}{2\underline{\varepsilon}}+\gamma\int_{a^1+\underline{c}-1}^{a^1-\underline{c}}\bar{v}_j(z_{ji}(a^1-\varepsilon^t))\tfrac{d\varepsilon^t}{2\underline{\varepsilon}}+\gamma\int_{a^1-\underline{c}}^{\underline{\varepsilon}}\hat{v}\tfrac{d\varepsilon^t}{2\underline{\varepsilon}}\right] \\ = {} & 1-a^1-\gamma\tfrac{(\underline{\varepsilon}-a^1)^2}{4\underline{\varepsilon}}+\delta\left[\hat{v}(1-\gamma+\tfrac{\gamma}{2\underline{\varepsilon}}(2\underline{c}+2\underline{\varepsilon}-1))+\gamma\tfrac{3(1-\delta)\frac{1}{2}+\delta\eta+3(1-\delta)\eta\nu(\underline{c})}{3(1-\delta)(1-\delta(1-\eta))}\left(\tfrac{1-2\underline{c}}{2\underline{\varepsilon}}\right)\right].\end{aligned}$$

The most attractive deviation maximizes this expected payoff with respect to a^1. This expected dynamic payoff is decreasing in a^1, so the optimal $a^1=0$. This is the case described above, hence there is no incentive to deviate if $c \geq c^{\ell\ell}$.

If the realized allocation cannot be less than $\underline{c}$, i.e.,$1-\underline{c}-\underline{\varepsilon} > a^1 > \underline{c}-\underline{\varepsilon}$

$$E^t[1-a^1+\tilde{\varepsilon}^t+\delta\bar{v}_j(y^{\varepsilon^t})] = 1-a^1+\int_{-\underline{\varepsilon}}^{a^1-\underline{c}}\left(\varepsilon^t+\delta\bar{v}_j(z_{ji}(a^1-\varepsilon^t))\right)\tfrac{d\varepsilon^t}{2\underline{\varepsilon}}+\int_{a^1-\underline{c}}^{a^1}\left(\varepsilon^t+\delta\hat{v}\right)\tfrac{d\varepsilon^t}{2\underline{\varepsilon}}+\int_{a^1}^{\underline{\varepsilon}}\delta\hat{v}\tfrac{d\varepsilon^t}{2\underline{\varepsilon}}. \tag{43}$$

The right side of (42) is quadratic and strictly concave in a^1 with a maximum at $\hat{a}^1$ given by

$$\hat{a}^1 = \tfrac{-(2\underline{\varepsilon}-\gamma\underline{\varepsilon})(1-\delta(1-\eta))+\delta\gamma(\frac{2}{3}-\underline{\varepsilon}+\eta\delta\nu(\underline{c}))}{\gamma(1+\delta\eta)}, \tag{44}$$

which must satisfy the constraints $1-\underline{c}-\underline{\varepsilon} > \hat{a}^1 > \underline{c}-\underline{\varepsilon}$. Let the corresponding lower bound be $\hat{c}^{\ell\ell} \equiv c^{\ell\ell}(\hat{a}^1)$, so j (and also i) vote for $z_{ij}(\underline{c})$ if $\underline{c} \geq \hat{c}^{\ell\ell}$.

Consider k's incentive to propose a deviation. If $\underline{c} \in [\max\{c^{**}, c^{\ell\ell}, \hat{c}^{\ell\ell}\}, \frac{1}{2}]$, i and j prefer $z_{ij}(c)$ for all $c \in [\underline{c}, 1-\underline{c}]$ to any new allocation. Hence, any proposal by k other than the status quo will be defeated. Hence legislator k has no incentive to deviate from the equilibrium strategies.

Suppose $\mathbf{q^{t-1}} \notin \bigcup_{\mathbf{c}=\underline{\mathbf{c}}}^{\frac{1}{2}} \mathbf{Z}(\mathbf{c})$. Consider j's incentive to vote for the equilibrium proposal $z_{ij}(\underline{c})$. The best status quo for j such that the realized policy from the status quo is not in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ gives 1 to j. Legislator j votes for $z_{ij}(\underline{c})$ rather than the status quo, and i also votes for $z_{ij}(c)$, if and only if

$$\begin{aligned}(1-\gamma)[\underline{c}+\delta\bar{v}_j(z_{ij}(\underline{c}))] + \gamma[\underline{c} - E^t\tilde{\varepsilon}^t + \delta\bar{v}_j(z_{ij}^{\varepsilon^t})] &> 1-\eta\tfrac{\theta}{4}+\delta\tfrac{1}{3(1-\delta)} \\ \Leftrightarrow \underline{c} &> c^{oo}.\end{aligned}$$

The status quo $q^{t-1} \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ may be such that the policy when implementation uncertainty is realized is in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ with some probability. Consider that the status quo gives $1-a^2$ to legislator j. Legislator j votes for $z_{ij}(\underline{c})$ if and only if

$$(1-\gamma)[\underline{c}+\delta\bar{v}_j(z_{ij}(\underline{c}))] + \gamma[\underline{c} - E^t\tilde{\varepsilon}^t + \delta\bar{v}_j(z_{ij}^{\varepsilon^t})] > (1-\eta)[1-a^2+\delta\hat{v}] + \eta E^t[1-a^2+\tilde{\theta}^t+\delta\bar{v}_j(y^{\theta^t})].$$

The right side of the incentive constraint is quadratic and strictly concave in a^2 with a maximum at $\hat{a}^2$ given by

$$\hat{a}^2 = \tfrac{-(2\underline{\theta}-\eta\underline{\theta})(1-\delta(1-\eta))+\delta\eta(\frac{2}{3}-\underline{\theta}+\eta\delta\nu(\underline{c}))}{\eta(1+\delta\eta)}.$$

The optimal $\hat{a}^2$ must satisfy the constraints analogous to those for a^1. Let the corresponding lower bound be $\hat{c}^{oo} \equiv c^{oo}(\hat{a}^2)$, so legislator j (and also i) votes for $z_{ij}(\underline{c})$ if $\underline{c} \geq \hat{c}^{oo}$.

Consider i's incentive to propose $y^t \neq z_{ij}(\underline{c})$. By Lemma 9 $z_{ij}(\underline{c})$ gives i the highest dynamic payoff among proposals in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$, so there is no incentive to make any other proposal in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$. Since legislator j has no incentive to deviate when receiving 1 or $1-\hat{a}^2$ and legislator i's allocation is at least as great as j's, legislator i has no incentive to deviate to receive 1 or $1-\hat{a}_2$.

Consider the case in which i does not receive 1 in q^{t-1}. Legislator i prefers a proposal $z_{ij}(\underline{c})$ to a proposal that gives 1 to i if and only if

$$(1-\gamma)[1-\underline{c}+\delta\bar{v}_i(z_{ij}(\underline{c}))] + \gamma[1-\underline{c} + E^t\tilde{\varepsilon}^t + \delta\bar{v}_i(z_{ij}^{\varepsilon^t})] \quad \geq \quad 1-\gamma\tfrac{\varepsilon}{4}+\delta\tfrac{1}{3(1-\delta)}.$$

Note that the right side of the inequality is less than $1-\eta\frac{\theta}{4}+\delta\frac{1}{3(1-\delta)}$ since $\eta\underline{\theta} \leq \gamma\underline{\varepsilon}$, so $\underline{c} < 1-c^{oo}$ is sufficient for this to be satisfied.

Legislator i may also propose $y^t \notin \bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ such that the realized proposal is in $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ with positive probability. From the previous analysis the optimal proposal has $\hat{a}^1$. Legislator i prefers $z_{ij}(\underline{c})$ if and only if[23]

$$(1-\gamma)[1-\underline{c}+\delta\bar{v}_i(z_{ij}(c))]+\gamma[1-\underline{c}+E^t\tilde{\varepsilon}^t+\delta\bar{v}_i(z_{ij}^{\varepsilon^t})] \quad \geq \quad (1-\gamma)[1-\hat{a}^1+\delta\hat{v}]+\gamma E^t[1-\hat{a}^1+\tilde{\varepsilon}^t+\delta\bar{v}_j(y^{\varepsilon^t})] \tag{45}$$

$$\Leftrightarrow \hat{\hat{c}}_2 \quad \geq \quad \underline{c}.$$

For this to be a feasible upper bound, we require $\hat{\hat{c}}_2 \geq \frac{1}{2}$, which is shown in Lemma 14.

Note that $\hat{\hat{c}}_2 \geq \underline{c}$ implies that $c^{**} \geq \hat{c}^{\ell\ell}$ so $\hat{c}^{\ell\ell}$ is not binding. The right side of (45) is the same as the right side of (42), and the left side of (45) is the same as the (41). In addition the left side of (41) and (42) are the same, so (45) implies the constraint giving c^{**} is tighter than the constraint giving $\hat{c}^{\ell\ell}$, so $\hat{c}^{\ell\ell}$ is not binding. ■

Proof of Lemma 8

To show that $c^{**} \leq \frac{1}{2}$, first totally differentiate (4) to show that c^{**} is increasing in $\underline{\theta}$. Since c^{**} is increasing in $\underline{\theta}$, the maximum is at $\underline{\theta} = \underline{\varepsilon}$, in which case

$$\begin{aligned} c^{**} &= c^* + \tfrac{\delta(\gamma-\eta)(1-\delta)\nu(c^{**})}{2-\delta(\gamma-\eta)} \\ &= \tfrac{3-\delta(\gamma-\eta)(2-3(1-\delta)\nu(c^{**}))}{3(2-\delta(\gamma-\eta))}. \end{aligned}$$

Then,

$$c^{**} < \tfrac{1}{2} \iff 0 < \delta(\gamma-\eta)(1-6(1-\delta)\nu(c^{**})).$$

Substituting for $\nu(c^{**})$ and rearranging yields

$$1 \quad > \quad (1-\delta)\tfrac{1-2c^{**}}{(2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2c^{**}))}$$

$$\begin{aligned} \Leftrightarrow 2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2c^{**}) &> (1-\delta)(1-2c^{**}) \\ \Leftrightarrow 2\underline{\theta}(1-\delta(1-\eta))-(1-2c^{**})(1-\delta(1-\eta)) &> 0 \\ \Leftrightarrow 2\underline{\theta} &> 1-2c^{**}. \end{aligned}$$

[23]Legislator j also votes for $z_{ij}(\bar{c})$ if $\underline{c} \leq 1-\hat{c}^{oo}$.

The last line holds since by Assumption 4 we have $1 - 2c^{**} \leq \underline{\theta} < 2\underline{\theta}$.

For $\delta > \delta^o$, $c^o < \frac{1}{2}$, and since $c^{oo} \leq c^o$, $c^{oo} < \frac{1}{2}$. Note that $c^{\ell\ell} \leq c^{\ell} \leq c^o$ so $c^{\ell\ell} < \frac{1}{2}$ for $\delta > \delta^o$.

■

Proof of Proposition 5

The difference between the continuation values in (8) and (1) for the coalition originator i receiving $1 - c$ under the status quo q^{t-1} is

$$\bar{v}_i(q^{t-1}) - v_i(q^{t-1}) = \tfrac{\delta\eta\nu(\underline{c})}{1-\delta(1-\eta)},$$

which is positive for $\eta > 0$ and $\underline{c} < \frac{1}{2}$. If $\eta = 0$, the continuation values are the same. The same argument establishes the result for the coalition partner receiving c. ■

Proof of Proposition 6

By Lemma 7 basic strategies constitute a coalition equilibrium if $\underline{c}^+ \equiv \max\{c^{**}, c^{oo}, \hat{c}^{oo}, c^{\ell\ell}\}$ and $\hat{\hat{c}}_2 \geq \frac{1}{2}$. The proof of Proposition 6 proceeds first by establishing that $\hat{\hat{c}}_2 \geq \frac{1}{2}$ for δ large enough. We then show for δ large enough the greatest lower bound on the coalition member's allocation is c^{**}. By Lemma 8, $c^{**} \leq \frac{1}{2}$. Hence $\bigcup_{c=\underline{c}}^{\frac{1}{2}} Z(c)$ is supported by a coalition equilibrium for all $\underline{c} \in [c^{**}, \frac{1}{2}]$, which is non-empty since $c^{**} \leq \frac{1}{2} \leq \hat{\hat{c}}_2$.

The following lemma gives the value for $\hat{a}^1$ when $\eta = 0$ and δ is large enough. The value of $\hat{a}^1$ is required to calculate $\hat{\hat{c}}_2$.

Lemma 13. *For $\eta = 0$ and $\delta \geq \hat{\delta} \equiv \frac{(1-\underline{c})\gamma + 2\underline{\varepsilon}(1-\gamma)}{2\underline{\varepsilon}(1-\gamma)+\frac{2}{3}\gamma}$, we have $\hat{a}^1 = 1 - \underline{c} - \underline{\varepsilon}$.*

Proof. For $\eta = 0$ the optimal $\hat{a}^1$ in (13) is

$$\hat{a}^1 = \frac{1}{\gamma}\left(-\underline{\varepsilon}(2-\gamma)(1-\delta) + \delta\gamma\left(\frac{2}{3} - \underline{\varepsilon}\right)\right),$$

which is valid if and only if $1 - \underline{c} - \underline{\varepsilon} > \hat{a}^1 > \underline{c} - \underline{\varepsilon}$. Then,

$$\begin{aligned} 1 - \underline{c} - \underline{\varepsilon} > \hat{a}^1 \quad &\Longleftrightarrow \quad 1 - \underline{c} > -\frac{1}{\gamma}(2\underline{\varepsilon}(1-\delta)(1-\gamma)) + \frac{2}{3}\delta \\ \hat{a}_1 > \underline{c} - \underline{\varepsilon} \quad &\Longleftrightarrow \quad \underline{c} < -\frac{1}{\gamma}(2\underline{\varepsilon}(1-\delta)) + \frac{2}{3}\delta. \end{aligned} \tag{46}$$

The condition in (46) is violated for $\delta \geq \hat{\delta}$ given by

$$\hat{\delta} \equiv \frac{(1-\underline{c})\gamma + 2\underline{\varepsilon}(1-\gamma)}{2\underline{\varepsilon}(1-\gamma) + \frac{2}{3}\gamma},$$

which is strictly less than 1 for all $\underline{c} > \frac{1}{3}$, which is required by the lower bound c^{**}. Consequently, for $\delta \geq \hat{\delta}$, $\hat{a}^1 = 1 - \underline{c} - \underline{\varepsilon}$. That is, the best deviation proposal maximizes the probability that the allocation is in a tolerated interval $[\underline{c}, 1-\underline{c}]$. ■

The next lemma establishes for δ large enough $\hat{\hat{c}}_2 \geq \frac{1}{2}$.

Lemma 14. *For $\eta = 0$ we have $\hat{\hat{c}}_2 \geq \frac{1}{2}$ for all $\delta > \max\{\hat{\delta}, \bar{\delta}\}$ where* $\bar{\delta} \equiv \frac{\underline{\varepsilon} - \frac{\gamma}{4\underline{\varepsilon}}(2\underline{\varepsilon}-\frac{1}{2})^2}{\frac{1}{6}(1-\gamma)+\underline{\varepsilon}-\frac{\gamma}{4\underline{\varepsilon}}(2\underline{\varepsilon}-\frac{1}{2})^2}$.

Proof. The incentive constraint in (45) that gives $\hat{\hat{c}}_2$ evaluates to

$$\frac{1-\underline{c}}{1-\delta} - \frac{\delta\gamma}{1-\delta}\left(\frac{2}{3}-\underline{c}\right) + \frac{\delta\gamma(1-2\underline{c})}{(1-\delta)12\underline{\varepsilon}} \geq 1 - a^1 + \delta\hat{v} - \gamma\frac{(\underline{\varepsilon}-a^1)^2}{4\underline{\varepsilon}} + \frac{\delta\gamma}{(1-\delta)2\underline{\varepsilon}}\left((a^1-\underline{c}+\underline{\varepsilon})\left(\frac{2}{3}-\frac{1}{2}(a^1+\underline{c}+\underline{\varepsilon})\right)\right), \quad (47)$$

where the right side is maximized at $\hat{a}^1$. For $\delta \geq \hat{\delta}$, $\hat{a}^1 = 1-\underline{c}-\underline{\varepsilon}$ from Lemma 13, so the right side of (47) simplifies to

$$\frac{1-\underline{c}}{1-\delta} - \frac{\delta\gamma}{1-\delta}\left(\frac{2}{3}-\underline{c}\right) + \frac{\delta\gamma(1-2\underline{c})}{(1-\delta)12\underline{\varepsilon}} \geq \underline{c} + \underline{\varepsilon} + \delta\hat{v} - \gamma\frac{(2\underline{\varepsilon}+\underline{c}-1)^2}{4\underline{\varepsilon}} + \frac{\delta\gamma(1-2\underline{c})}{(1-\delta)12\underline{\varepsilon}}. \quad (48)$$

Then (48) is satisfied if the difference Δ between the two sides is positive, where

$$\Delta \equiv (1-2\underline{c}-\underline{\varepsilon}) + \frac{\delta(1-\gamma)}{1-\delta}\left(\frac{2}{3}-\underline{c}\right) + \frac{\gamma}{4\underline{\varepsilon}}(\underline{c}+2\underline{\varepsilon}-1)^2. \quad (49)$$

The expression for Δ is quadratic in $\underline{c}$. The second derivative of Δ with respect to $\underline{c}$ is $\frac{\partial^2\Delta}{\partial\underline{c}^2} = \frac{\gamma}{2\underline{\varepsilon}} > 0$ so Δ is convex in $\underline{c}$. At $\underline{c} = 1$ the derivative of Δ with respect to $\underline{c}$ evaluates to $\frac{\partial\Delta}{\partial\underline{c}} = -2 - \frac{\delta(1-\gamma)}{(1-\delta)} + \gamma < 0$, so the derivative is negative for all feasible values of $\underline{c}$. Then $\hat{\hat{c}}_2$ is the value of $\underline{c}$ such that $\Delta = 0$. By the implicit function theorem we can show that $\underline{c}$ is increasing in δ, so if $\hat{\hat{c}}_2 = \frac{1}{2}$ for some $\bar{\delta}$, then $\hat{\hat{c}}_2 \geq \frac{1}{2}$ for all $\delta \geq \bar{\delta}$. By the implicit function theorem

$$\frac{\partial\underline{c}}{\partial\delta} = -\frac{\frac{\partial\Delta}{\partial\delta}}{\frac{\partial\Delta}{\partial\underline{c}}}.$$

Differentiating Δ with respect to δ yields

$$\frac{d\Delta}{d\delta} = \frac{(1-\gamma)\left(\frac{2}{3}-\underline{c}\right)}{(1-\delta)^2},$$

which is positive for $\underline{c} < \frac{2}{3}$, and otherwise $\underline{c} \geq \frac{2}{3} > \frac{1}{2}$. Consequently, if $\Delta_{|\underline{c}=\frac{1}{2}} \geq 0$ for $\delta = \bar{\delta}$ then $\hat{\hat{c}}_2 \geq \frac{1}{2}$ for all $\delta \geq \bar{\delta}$. The following characterizes $\bar{\delta}$. Evaluating $\Delta_{|\underline{c}=\frac{1}{2}}$ yields

$$\Delta_{|\underline{c}=\frac{1}{2}} = -\underline{\varepsilon}(1-\gamma) - \frac{\gamma}{2} + \frac{\delta(1-\gamma)}{6(1-\delta)} + \frac{\gamma}{16\underline{\varepsilon}}.$$

The term $-\underline{\varepsilon}(1-\gamma) - \frac{\gamma}{2} + \frac{\gamma}{16\underline{\varepsilon}}$ is nonnegative for $\underline{\varepsilon} \leq \underline{\varepsilon}^o(\gamma) \equiv \frac{\sqrt{\gamma}-\gamma}{4(1-\gamma)}$, $\gamma \in (0,1)$, in which case $\Delta_{|\underline{c}=\frac{1}{2}} \geq 0$ for all $\delta \in [0,1)$. For $\underline{\varepsilon} > \underline{\varepsilon}^o(\gamma)$, $\Delta_{|\underline{c}\geq\frac{1}{2}} \geq 0$ if and only if

$$\delta \geq \bar{\delta}(\gamma, \underline{\varepsilon}) \equiv \frac{1}{1 + \frac{1-\gamma}{6(\underline{\varepsilon}(1-\gamma)+\frac{\gamma}{2}-\frac{\gamma}{4\underline{\varepsilon}})}} < 1. \quad (50)$$

Consequently, for $\delta \geq \bar{\delta}(\gamma, \underline{\varepsilon})$, $\Delta_{|\underline{c}=\frac{1}{2}} \geq 0$, and $\hat{c}_2 \geq \frac{1}{2}$. The set R^T of $(\gamma, \underline{\varepsilon})$ such that $\bar{\delta}(\gamma, \underline{\varepsilon}) < 1$ is $R^T = \{(\gamma, \underline{\varepsilon}) | \underline{\varepsilon} > \underline{\varepsilon}^o(\gamma), \gamma \in (0,1)\}$, but since $\Delta_{|\underline{c}=\frac{1}{2}} \geq 0$ for all δ when $\underline{\varepsilon} \leq \underline{\varepsilon}^o(\gamma)$, the bound $\bar{\delta}(\gamma, \underline{\varepsilon})$ in (50) holds for all $(\gamma, \underline{\varepsilon}) \in (0,1) \times [1-2\bar{c}, \bar{c}]$. ■

The next lemma shows for δ large enough $c^{**} \geq \max\{c^{oo}, c^{\ell}\}$.

Lemma 15. *For $\eta = 0$ there exists a δ^* and a δ^{ℓ} such that for all $\delta \geq \delta^*$ we have $c^{**} \geq c^{oo}$, and for all $\delta \geq \delta^{\ell}$ we have $c^{**} \geq c^{\ell\ell}$.*

Proof. The difference between c^{**} and c^{oo} is

$$c^{**} - c^{oo} = \frac{3 - 2\delta\gamma + \frac{\delta\gamma}{4\underline{\varepsilon}}}{3(2 - \delta\gamma(1 - \frac{1}{6\underline{\varepsilon}}))} - \frac{3 - \delta(2+\gamma) - \frac{\delta\gamma}{4\underline{\varepsilon}}}{3(1 - \delta\gamma(1 + \frac{\delta\gamma}{6\underline{\varepsilon}}))}. \tag{51}$$

Evaluating (51) at $\delta = 0$ yields $(c^{**} - c^{oo})_{|\delta=0} = -\frac{1}{2}$. Taking the limit as $\delta \to 1$ yields

$$\sup_{\delta\to 1}(c^{**} - c^{oo}) = \frac{(1-\gamma)^2 + \frac{\gamma}{12\underline{\varepsilon}}}{9(2 - \gamma(1 - \frac{1}{6\underline{\varepsilon}}))(1 - \gamma(1 + \frac{1}{6\underline{\varepsilon}}))} > 0.$$

By the mean value theorem there exists one or more solutions to $c^{**} - c^{oo} = 0$ in $(0,1)$. Let the largest of these be denoted by δ^*.

For $\eta = 0$, $c^{\ell\ell} = c^{\ell}$, so similarly, $c^{**} - c^{\ell}$ is positive as $\delta \to 1$ and negative for $\delta = 0$. Let $\delta^{\ell} \in (0,1)$ denote the largest δ such that $c^{**} - c^{\ell} = 0$.

The difference $c^{**} - c^*$ is positive, since

$$\begin{aligned} c^{**} - c^* &= \frac{3 - 2\delta\gamma + \frac{\delta\gamma}{4\underline{\varepsilon}}}{3(2-\delta\gamma) + \frac{\delta\gamma}{2\underline{\varepsilon}}} - \frac{3 - 2\delta\gamma}{3(2-\delta\gamma)} \\ &= \frac{\delta^2\gamma^2}{36\underline{\varepsilon}(2-\delta\gamma)(3(2-\delta\gamma) + \frac{\delta\gamma}{2\underline{\varepsilon}})} > 0. \end{aligned} \tag{52}$$

To determine the relation between δ^o and δ^*, evaluate the difference $c^{**} - c^{oo}$ at $\delta = \delta^o = \frac{3}{4-\gamma}$, which yields

$$(c^{**} - c^{oo})_{|\delta=\delta^o} = -\frac{\gamma}{2(8 - 5\gamma + \frac{\gamma}{2\underline{\varepsilon}})} < 0.$$

This implies that $\delta^* > \delta^o$. Since $\delta^* > \delta^o$, $c^* > c^{\ell}$ from the proof of Lemma 2, so $c^{**} > c^{\ell}$. ■

It is straightforward to show for $\eta = 0$, $\hat{a}^2 = 0$, hence $\hat{c}^{oo} = c^{oo}$. Then, for $\delta > \delta^{\zeta} \equiv \max\{\delta^*, \delta^{\ell}, \hat{\delta}, \bar{\delta}\} > \delta^o$, c^{**} is the greatest lower bound. This completes the proof of Proposition 6. ■

Proof of Proposition 7

To prove (iii) and (iv), differentiate c^{**} to obtain

$$\frac{dc^{**}}{d\delta} = \frac{\delta}{\gamma}\frac{dc^{**}}{d\gamma} = -\frac{\gamma}{3(2\delta\gamma(1 - \frac{1}{6\underline{\varepsilon}}))^2} < 0.$$

Properties (i) and (ii) are also straightforward to show. ■

Proof of Proposition 8

The difference $c^{**} - c^{*}$ is positive from (52), and the result follows from Corollary 7 and Proposition 6. ■

Battaglini-Palfrey experiment results

Table 1: Duration of Coalitions

		Frequency									
Match	**# Rounds**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	10	3	2	0	1	0	1	0	0	0	0
2	4	1	1	1	0						
3	10	1	0	0	0	1	0	0	0	1	0
4	5	1	0	0	2	1					
5	2	1	2								
6	1	2									
7	8	0	0	0	0	0	0	1	2		
8	1	2									
9	3	0	1	2							
10	5	0	0	0	2	1					
11	7	0	2	0	2	0	1	1			
12	2	2	1								
13	1	0									
14	5	2	0	0	2	1					
15	3	2	3	0							
16	5	1	1	0	2	1					
17	1	3									
18	2	3	1								
19	7	3	3	2	0	0	0	0			
20	3	2	1	2							
Total		29	18	7	11	5	2	2	3	1	0

NB: 70 committees

Table 2: Transition Probabilities – Continuous Experiment (All Rounds)

	Outcome q^t				
Status quo q^{t-1}	M12	M13	M23	U	D
M12 (46)	0.522	0.130	0.217	0.087	0.043
M13 (55)	0.145	0.582	0.073	0.091	0.109
M23 (32)	0.063	0.250	0.313	0.281	0.094
U (92)	0.087	0.054	0.011	0.848	0.000
D (23)	0.217	0.130	0.087	0.043	0.522

Table 3: Transition Probabilities – Continuous Experiment (Rounds after First)

	Outcome q^t				
Status quo q^{t-1}	M12	M13	M23	U	D
M12 (42)	0.524	0.119	0.214	0.095	0.048
M13 (47)	0.128	0.596	0.085	0.064	0.128
M23 (24)	0.083	0.333	0.292	0.208	0.083
U (88)	0.080	0.034	0.011	0.875	0.000
D (23)	0.217	0.130	0.087	0.043	0.522

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