

# Competitive Policy Entrepreneurship

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## **Abstract**

In political organizations, the process of developing new policies often involves competing policy entrepreneurs who make productive investments to make their proposals more appealing to decisionmakers. We analyze how entrepreneurs' extremism and costs of crafting high-quality proposals affect patterns of competition and policy outcomes. A centrist decisionmaker can benefit from extremism of proposers and proposals, once we account for proposals' endogenously-determined quality. Lower costs spur investment, but entrepreneurs extract some ideological rents. When the contest is highly asymmetric, one entrepreneur almost always wins, but the decisionmaker benefits from the threat of competition.

# 1 Introduction

In political organizations, the process of developing new policies typically involves competing actors. For example, a legislature may consider bills drafted by different committees or interest groups. In bureaucratic politics, each subunit within a government agency may develop its own proposal for consideration by the agency head. Moreover, this pattern is not restricted to the public sector; on the contrary, many NGOs, universities, and firms have different factions that exert effort to craft competing proposals that they hope will be implemented.

We use the term “policy entrepreneur” to refer to an individual, faction, or interest group that takes the initiative to develop a policy, without any guarantee that it will be adopted. Of course, policy entrepreneurs often disagree—both with each other and with decisionmakers—about a variety of things. These disagreements may be ideological, or they may be about the organization’s mission and the relative importance of different objectives. Yet despite their disagreements, people in a political organization usually have some interests in common. To the extent that there are overarching organizational goals, they (*ceteris paribus*) prefer policies that more effectively achieve them. When possible they prefer to save money, or to make money in the case of a for-profit firm. And, other things being equal, they prefer to enhance the organization’s status and prestige.

To understand competitive policy entrepreneurship, we build on previous research on all-pay contests (e.g., Che and Gale 2003, Siegel 2009). The foundation of our analysis is a spatial model of policy, which is standard in political economy but atypical of contest models. A policy in our model has two dimensions: an *ideological* dimension over which players have different preferences, and a *quality* dimension that is common value. The participants in the contest, i.e., the entrepreneurs, are fundamentally *policy motivated*—they care about the ideology and quality of the policy that is ultimately implemented, and their motive for winning is purely instrumental. Because our model features *spillovers*, it is related to the symmetric all-pay contest that Baye, Kovenock, and de Vries (2012) use to analyze auctions, R&D contests, litigation, and price competition. The spillovers in our model are more complex, however, because the entrepreneurs care about, and make choices on, both the ideological dimension and the quality dimension. Thus, our model is more appropriate for analyzing policy development in political organizations.

The sequence of our game is as follows. Two competing policy entrepreneurs simultaneously choose specific ideological locations at which to develop policies and also how much

to invest in producing quality. Each entrepreneur has an ideal ideological outcome  $x_i$  and marginal cost of developing quality  $\alpha_i$ . Their investments are costly and cannot be combined or transferred to other policies. A decisionmaker with an ideal ideological outcome  $x_D = 0$  located between them chooses one of the entrepreneurs' proposals, a reservation policy, or any other ideological location for which no quality has been developed. The decisionmaker can neither commit to a decision rule nor pay the entrepreneurs to reward them for developing particular policies. Rather, he must simply choose among the available options. This assumption reflects the fact that leaders in many political organizations have access to a very-limited set of rewards and punishments (Moe 1984) and, more importantly, a fundamental feature of political economy is that commitment is often difficult or impossible, due to lack of external enforcement.

To gain the support of the decisionmaker, entrepreneurs in our model use a combination of ideological concessions and productive quality investments. The primary reason an entrepreneur invests in quality is to reduce her need to make ideological concessions. An important intermediate result of the analysis is that ideologically-extreme policies are not bad for a centrist decisionmaker—in equilibrium, when extreme policies are developed they are not only higher quality, but also strictly better for the decisionmaker.

We first show that equilibria are in two-dimensional mixed strategies. We provide necessary and sufficient conditions for equilibrium, and show that strategies can be characterized by a univariate probability distribution over the decisionmaker's utility and simple functions that associate each utility with a specific combination of ideology and quality. The equilibrium probability distributions are characterized by a straightforward system of differential equations and boundary conditions. Next, we show that equilibria exist and are unique, and we provide an analytical characterization of equilibrium strategies and players' payoffs.

For generic asymmetric parameters, participation in the contest is asymmetric; one entrepreneur is more engaged, i.e., she enters the contest with probability 1, whereas the other one sometimes sits out. The probability that the less-engaged entrepreneur sits out is a function of the two entrepreneurs' preferences and costs, and for extremely asymmetric values of these parameters it converges to 1. However, this does not imply that the model functions as if the less-engaged entrepreneur did not exist (in which case the more-engaged entrepreneur could extract all quality benefits for herself, in the form of ideological rents). Rather, the seldomly-realized threat of potential entry can induce the more-engaged entrepreneur to develop policies that benefit the decisionmaker.

We also show that the more-engaged entrepreneur may not dominate the contest. Rather,

if she is more ideologically-motivated yet faces a sufficiently large cost disadvantage, her opponent is more likely to win the contest, despite being less likely to enter. On the other hand, if the more-engaged entrepreneur is both more ideologically-extreme and more cost-effective at developing quality, then compared to her opponent she will develop policies that are (in a first-order stochastic sense) more extreme and also better for the decisionmaker.

The model provides intuitive comparative statics. Each entrepreneur is worse off when her opponent's costs decrease. Lower costs make it cheaper to develop any given level of quality, and thus easier to realize ideological gains. As an entrepreneur's costs decrease, she develops more-extreme policies, and her opponent develops moderate ones. The effect of increasing one entrepreneur's ideological extremism is, for the most part, similar to decreasing her costs: her policies become more extreme, her opponent's policies become moderate, and her opponent is worse off.

We also analyze a symmetric variant in which the two entrepreneurs have the same marginal cost of developing quality ( $\alpha_L = \alpha_R = \alpha$ ) and ideological ideal points that are symmetrically located on either side of the decisionmaker ( $|x_L| = |x_R| = x$ ). This is the purest form of competitive entrepreneurship because neither side is advantaged. To analyze the effects of *polarization of interests* we vary the ideological distance  $x$  between the entrepreneurs and the decisionmaker. We show that more ideologically-extreme entrepreneurs produce policies that are first-order stochastically more ideologically-extreme and better for the decisionmaker. Their extremism gives them a greater incentive to make productive investments to capture ideological rents, and competition prevents them from fully extracting the benefits of their additional investments.

We also consider the effect of decreasing the common quality-development cost  $\alpha$ , e.g., due to subsidies for policy development or a technology shock. We show that policies become first-order stochastically more extreme, but the decisionmaker's utility nevertheless increases. Interestingly, the effect on entrepreneurs' utility is nonmonotonic, because lower costs make it cheaper to compensate the decisionmaker for ideological losses but also increase the intensity of competition. If cost are high to begin with, a competition effect dominates and cost decreases make the entrepreneurs worse off. However, if costs are low to begin with, a cost effect dominates and decreases make the entrepreneurs better off.

**Literature** The canonical approach to studying endogenous development of high-quality policies is Crawford and Sobel's (1982) model, in which policies and outcomes are ordered in a unidimensional space and linked via a common additive shift. In that framework, information

is *invertible* (Callander 2008), in the sense that knowing how to achieve a liberal outcome is also sufficient to know how achieve a conservative one. The canonical approach is well-suited to understanding the strategic use of expertise when the appropriate policy to enact depends on a single unknown underlying factor, such as the severity of global warming or the size of an enemy’s army. The key strategic tension is that privately-informed experts worry that their information will be expropriated to implement outcomes that do not reflect their preferences. Such models have been widely applied to study the development of expertise and, within political science, the institutional determinants of high-quality policymaking (Bendor and Meirowitz 2004).

Our model, in contrast, assumes that quality is *policy-specific* (Ting 2011, Hirsch and Shotts 2012). It is thus better suited to empirical domains where information and expertise are not readily transferable across different approaches to the same organizational problem.<sup>1</sup> For example, information about how to design an effective and equitable school voucher program cannot be used to improve the quality of public schools. Similarly, when one division within a firm develops a new product, this doesn’t help another division that is developing a completely different product that it wants the firm to focus on. Or, if we consider adoption of a “policy” to be the election of a particular party to control the government, then a party that makes productive investments in its own capacity to govern—e.g., by developing a well thought-out platform or by improving recruitment and training of its candidates and bureaucrats—knows that the benefits of its investments are only realized in the event that it actually wins office.<sup>2</sup>

Because quality is policy-specific in our model, an entrepreneur does not need to worry about being expropriated, but rather attempts to *exploit* her investments to encourage the decisionmaker to select her policy. This effect is akin to Aghion and Tirole’s (1997) “real authority,” in that a decisionmaker who wishes to benefit from an entrepreneur’s efforts must select her policy. However, the investments are wasted if that policy is not selected.

Our model is closely related to a growing political economy literature on strategic development of *valence*, i.e., common value dimensions of policy, in both single-actor and competitive sequential models. Lax and Cameron (2007) consider a sequential model of costly

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<sup>1</sup>See Callander (2011a, 2011b) for models in which learning about one policy option provides information that is useful for small policy changes, but not necessarily for major ones.

<sup>2</sup>This example, like a few others later in the paper, stretches the definition of “organization” from its typical usage, to include an entire polity. We note, however, that a political system is a way of organizing collective decision making.

development of high-quality Supreme Court opinions; Ting (2011) and McCarty (2012) analyze development of bureaucratic expertise; and Hirsch and Shotts (2012) and Hitt, Volden, and Wiseman (2011) analyze the development of high-quality Congressional legislation. To our knowledge, our model is the first simultaneous competitive model of endogenous valence development.

Our model is also analytically related to models of competing political candidates who choose ideological platforms and make costly up-front investments to increase their chances of electoral victory. Wiseman (2006) studies candidates who sequentially select platforms and a level of costly electoral support, and Ashworth and Bueno de Mesquita (2009) model candidates who simultaneously choose policy platforms and then levels of campaign spending. Our model is distinct from this literature in that investments in policy quality are productive and common value. However, our technique for characterizing equilibria can be adapted and used to analyze costly campaign expenditures.

At a broader level, our model provides a new approach to studying competition for intra- and inter-organizational influence. One strand of the literature analyzes competitive informational lobbying with either general policy-relevant information (Gilligan and Krehbiel 1989, Battaglini 2003) or information specific to a binary set of alternatives (Dewatripont and Tirole 1999). Another strand considers influence via transfers to a decisionmaker or decisionmakers that are either contractible (Grossman and Helpman 1994) or non-contractible (Grosseclose and Snyder 1996). Finally, intra-firm influence of various forms is analyzed in several models, including Milgrom and Roberts’s (1988) model of self-promotion by subordinates, and Rotemberg and Saloner’s (1994) model of competitive project investments by divisions within a firm. Our model is distinct from each of these literatures—it is non-informational, the set of available alternatives is a continuum, and influence-generating investments are policy-specific, productive, and non-contractible.

The paper proceeds as follows. Section 2 introduces the model. Section 3 develops concepts and notation, and then presents some general results. Section 4 provides an analytical characterization of equilibria and general comparative statics. Section 5 considers the symmetric model, and Section 6 considers specific asymmetric variants. Section 7 concludes.

## 2 The Model

We analyze a two-stage game of policy development and choice played by two competing entrepreneurs and a decisionmaker. Policies in the model have two components: *ideology*  $y \in \mathbb{R}$  and *quality*  $q \in [0, \infty) = \mathbb{R}^+$ . Thus, a policy is a point in a subset of two-dimensional

real space,  $b = (y, q) \in \mathbb{R} \times \mathbb{R}^+ = \mathbb{B}$ . Players' utility functions  $U_i(b)$  over the two dimensions are additive, and quality is valued equally by all players:

$$U_i(b) = q - (x_i - y)^2$$

where  $x_i$  denotes player  $i$ 's ideological ideal point. We assume without loss of generality that the decisionmaker's ideal ideology is  $x_D = 0$ , and furthermore assume that the entrepreneurs are on opposite sides of the decisionmaker, i.e.,  $\text{sign}(x_i) \neq \text{sign}(x_j)$ .

In the *policy development* stage, each entrepreneur  $i \in N = 2$  simultaneously develops a policy  $b_i = (y_i, q_i) \in \mathbb{B}$  with ideology  $y_i$  and quality  $q_i \geq 0$ . We assume for simplicity that the cost of developing quality  $q_i$  is  $c_i(q_i) = \alpha_i q_i$  where  $\alpha_i > 1$ . Thus, the cost is linear and independent of ideology  $y_i$ , and policies with 0 quality are costless. The *net benefit* of producing quality is  $(1 - \alpha_i) q_i < 0$ , so an entrepreneur will only develop quality to increase the probability that her policy will be selected.

In the *policy choice* stage, the decisionmaker chooses from the set of newly-developed policies  $\mathbf{b} \in \mathbb{B}^N$  or a reservation policy  $b_0 = (0, 0)$ , i.e., the decisionmaker's ideal ideology with 0 quality. Implicitly, we assume that the decisionmaker can choose freely from the 0-quality policies, and that quality is *policy-specific* (Hirsch and Shotts 2012).

With only one entrepreneur, our model would be technically similar to Snyder's (1991) model of vote-buying without price discrimination—the entrepreneur produces just enough quality to induce the decisionmaker to choose her policy over the reservation policy, and balances the costs of developing quality against the ideological benefits of moving policy in her direction. (See, e.g., the single-proposer model that Hitt, Volden, and Wiseman (2011) use to analyze variation in legislators' effectiveness or ability to craft bills).

In contrast, we focus on competitive policy entrepreneurship when different entrepreneurs or factions can develop new proposals. In our model, entrepreneurs compete for the decisionmaker's support by simultaneously making costly, quality-increasing investments that are specific to a particular ideology. Because the cost of investing in quality is paid up-front, the game is an all-pay contest (Che and Gale 2003; Siegel 2009, 2010).

Our model has two main differences from previous work on all-pay contests. First, entrepreneurs are *policy motivated* rather than rent seeking (as in Tullock 1980 and Baye, Kovenock, and de Vries 1993). They care about which policies are implemented even if they lose, so the contest features spillovers (as in Baye, Kovenock, and de Vries 2012). Second, in our model the investments made to gain influence are *productive*, and not simply transfers to the decisionmaker. These differences stem from the fact that our model is designed to



apply to political organizations, where people have both divergent ideological interests and common organizational interests.

### 3 Preliminary Analysis

In this section we introduce notation, and provide necessary and sufficient conditions for equilibrium as well as a general characterization. All proofs are in the Appendix.

A strategy for the decisionmaker,  $w(\mathbf{b}) : \mathbb{B}^N \rightarrow \Delta(N \cup 0)$ , is a mapping from each profile of policies  $\mathbf{b}$  to a probability distribution over the winner, where  $w(\mathbf{b}) = 0$  denotes choosing the reservation policy  $b_0$ . We introduce additional notation to characterize decisionmaker strategies that are subgame perfect.

**Definition 1** *Let the **score**  $s(y, q)$  of a policy be the decisionmaker's utility, i.e.,*

$$s(y, q) = U_D(y, q) = q - y^2.$$

*A decisionmaker strategy  $w(\mathbf{b})$  is subgame perfect i.f.f. only policies with the highest score win, i.e.,*

$$\forall \mathbf{b} \text{ and } i, w_i(\mathbf{b}) > 0 \text{ i.f.f. } (y_i, q_i) \in \arg \max_{(y, q) \in \mathbf{b} \cup b_0} \{s(y_i, q_i)\}.$$

An entrepreneur  $i$  thus wins the contest if her policy gives the decisionmaker higher utility than both her opponent's policy and the reservation policy. In the event of ties, the decisionmaker may randomize arbitrarily. We use the term *score* to refer to the decisionmaker's utility, which plays a similar role as in Siegel (2009). Developing a policy with a higher score is strictly worse for an entrepreneur, conditional on winning. Also, the entrepreneur who develops the higher-score policy wins, provided that the score beats the decisionmaker's utility from the reservation policy, i.e.,  $s(b_i) \geq s(0, 0) = 0$ .

Unlike Siegel (2009), however, a policy is more than just a score—there are a *continuum* of policies with different ideologies that lie on the same indifference curve for the decisionmaker. These policies have different costs to develop; a policy with ideology  $y$  and score  $s$  must have quality  $s + y^2$ , so entrepreneur  $i$ 's cost to develop it is  $\alpha_i(s + y^2)$ . In addition, the policies are valued differently by different players; entrepreneur  $i$ 's utility from policy  $(s, y)$  is  $U_i(y, s + y^2) = -x_i^2 + s + 2x_i y$ . It is thus helpful to introduce notation for these quantities, which allow us to think of an entrepreneur's problem as the choice of a *score curve*  $s$  and an ideology  $y$  to develop along that score curve.

**Definition 2** *Player  $i$ 's utility for a policy  $(s, y)$  with score  $s$  and ideology  $y$  is*

$$V_i(s, y) = U_i(y, s + y^2) = -x_i^2 + s + 2x_i y.$$

*The up-front cost to an entrepreneur of developing the policy herself is  $-\alpha_i(s + y^2)$ .*

Figure 1 depicts the game in ideology-quality space for entrepreneurs who are equidistant from the decisionmaker. The decisionmaker's indifference curves, i.e., the policies with equal score, are depicted by green lines.

**Necessary and Sufficient Conditions** An entrepreneur's pure strategy  $b_i$  is a two-dimensional element  $(y_i, q_i)$  of  $\mathbb{B}$  consisting of an ideology and a quality. A mixed strategy  $\sigma_i$  is a probability measure over the Borel subsets of  $\mathbb{B}$ . A strategy profile is  $(\sigma, w(\mathbf{b}))$ , a strategy for each entrepreneur and a decisionmaker decision rule  $w(\mathbf{b})$ .

We first establish that in any equilibrium, there is zero probability that there are two distinct available policies over which the decisionmaker is indifferent. The absence of “score ties” is an intuitive consequence of the all-pay nature of investing in quality—if an entrepreneur knew that her policy might tie with her opponent's policy or the reservation policy, she could invest up front in a bit more quality to break the tie.<sup>3</sup>

**Lemma 1** *In equilibrium, the probability the entrepreneurs develop new policies  $b_i \neq b_0$  with the same score as the reservation policy ( $s(b_i) = s(b_0)$ ) or each other ( $s(b_i) = s(b_j)$ ) is 0.*

Lemma 1 allows us to solve for two-dimensional equilibrium strategies by applying a substitution method to an entrepreneur's choice  $(s_i, y_i)$  of score and ideology. The reason to work with  $(s_i, y_i)$  rather than ideology and quality  $(y_i, q_i)$  is that, given the opponent's strategy  $\sigma_{-i}$ , two policies  $(s_i, y'_i)$  and  $(s_i, y''_i)$  with the same score win the policy contest with the same probability. This is 0 if  $s_i < 0$  is worse than the reservation policy, and if  $s_i > 0$  it is the probability  $\Pr(s(b_{-i}) \leq s_i)$  that her opponent  $-i$  produces a lower-score policy. This generates the following essential Lemma.

**Lemma 2** *Let  $F_i(s)$  denote the CDF of  $\max\{0, s(b_i)\}$ . At any score  $s_i > 0$  where the score CDF  $F_{-i}(\cdot)$  of  $i$ 's opponent has no atom, developing the policy  $(s_i, y_i^*(s_i))$ , where  $y_i^*(s_i) = F_{-i}(s_i) \cdot \frac{x_i}{\alpha_i}$ , is strictly better for  $i$  than developing any other policy  $(s_i, y_i)$ .*

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<sup>3</sup>Proving this property is more complex than in all-pay contests without spillovers, because the utility from tying can be a complicated function of the opponent's policies and the decisionmaker's decision rule.

Lemma 2 states that for almost every score  $s_i > 0$ , entrepreneur  $i$ 's best combination of ideology  $y_i$  and quality  $q_i$  to generate that score is unique. Crucially, the optimal ideology-quality combination does not depend on the specific policies that her opponent develops. Instead, it is simply  $F_{-i}(s_i) \cdot \frac{x_i}{\alpha_i}$ , a weighted average of the entrepreneur and decisionmaker's ideal ideologies, multiplied by the probability  $F_{-i}(s_i)$  that her opponent develops a lower-score policy.<sup>4</sup>

Lemmas 1 and 2 jointly imply that in equilibrium, player  $i$  can compute her expected utility as if her opponent always develops policies of the form  $(s_{-i}, y_{-i}^*(s_{-i}))$ . Thus, entrepreneur  $i$ 's utility from developing any  $(s_i, y_i)$  with  $s_i > 0$  where her opponent's score CDF  $F_{-i}$  has no atom (or if a tie would be broken in her favor) can be written as

$$\Pi_i^*(s_i, y_i; F) = \underbrace{-\alpha_i (s_i + y_i^2)}_{\text{quality cost}} + \underbrace{F_{-i}(s_i) \cdot V_i(s_i, y_i)}_{\text{Pr win} \cdot \text{utility if win}} + \underbrace{\int_{s_i}^{\infty} V_i(s_{-i}, y_{-i}^*(s_{-i})) dF_{-i}}_{\text{utility when lose}}. \quad (1)$$

Her utility from developing the *best* policy with score  $s_i > 0$  is  $\Pi_i^*(s_i, y_i^*(s_i); F)$ . We denote this as  $\Pi_i^*(s_i; F)$  and use it to characterize conditions for equilibrium.

**Lemma 3** *A profile  $(\sigma, w(\mathbf{b}))$  is a SPNE i.f.f. it satisfies three conditions.*

1. **(No Ties)** *The probability the entrepreneurs develop new policies  $b_i \neq b_0$  with the same score as the reservation policy ( $s(b_i) = s(b_0)$ ) or each other ( $s(b_i) = s(b_j)$ ) is 0.*
2. **(Ideological Optimality)** *With probability 1, each entrepreneur develops policies that either*

- (a) *generate score  $s(y_i, q_i) < 0$  and have quality  $q_i = 0$ , or*
- (b) *generate score  $s(y_i, q_i) \geq 0$  and satisfy  $y_i = y_i^*(s(y_i, q_i))$ .*

3. **(Score Optimality)** *For all  $i$  and  $s_i$  in the support of  $F_i$ ,  $s_i \in \arg \max_{s_i} \{\Pi_i^*(s_i; F)\}$ .*

The Lemma provides necessary and sufficient conditions for equilibrium that do not depend on decisionmaker's decision rule  $w(\mathbf{b})$ . Entrepreneurs cannot be exerting effort

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<sup>4</sup>Lemma 2 is reminiscent of Che's (1993) simplification of two-dimensional procurement auctions to choice of a score. However, in that model the optimal price-quality combination at a score is independent of the opponent's strategy, whereas in our all-pay model the optimal ideology-quality combination depends on  $F_{-i}(s_i)$ . This dependence on the other player's CDF is similar to Lemma 6 of Che and Gale (2003).

on policies that are no better than the reservation policy or that might tie each other. The policies they generate must be ideologically-optimal. Finally, a score  $s_i$  can be in the support of  $i$ 's score CDF  $F_i$  if and only if developing the ideologically-optimal policy for that score would maximize  $i$ 's utility when a tie would be broken in her favor.

**Equilibrium Characterization** Lemma 3 provides necessary and sufficient conditions on score CDFs for equilibrium. We now characterize the equilibrium in more detail. We say that an entrepreneur is *active* when she develops a policy with strictly positive score and hence positive quality.<sup>5</sup> Obviously, both entrepreneurs must be active with strictly positive probability. If one were inactive (say  $-i$ ), her score CDF would be  $F_{-i}(s) = 1, \forall s \geq 0$ . Her opponent would thus develop  $\left(0, \frac{x_i}{\alpha_i}\right)$ , i.e., a new policy with the same score as the reservation policy, which violates no ties.

In addition, all equilibria must be in mixed strategies. No ties implies that in any pure strategy profile one entrepreneur's policy has a strictly lower score and hence loses, so she would be strictly better off developing no policy. Thus, in equilibrium both entrepreneurs mix over both the ideological locations and qualities of policies they develop, according to a strategy profile  $\sigma$  that generates no ties, is ideologically-optimal, and induces CDFs  $(F_i(\cdot), F_{-i}(\cdot))$  satisfying score optimality. While characterizing score-optimal CDFs seems potentially complex, the next result states that all such profiles satisfy simple conditions.

**Proposition 1** *A profile of CDFs  $F$  satisfies score optimality i.f.f. it satisfies the following boundary conditions and differential equations.*

1. **Boundary Conditions:**  $F_k(0) > 0$  for at most one  $k \in \{L, R\}$ , and  $\min \{F_i^{-1}(1)\} = \bar{s} \forall i$ .
2. **Differential Equations:** For all  $i \in N$  and  $s \in [0, \bar{s}]$ ,  

$$\alpha_i - F_{-i}(s) = f_{-i}(s) \cdot 2x_i \left( \left( \frac{x_i}{\alpha_i} \right) F_{-i}(s) - \left( \frac{x_{-i}}{\alpha_{-i}} \right) F_i(s) \right).$$

Proposition 1 implies that equilibria have a straightforward form. First, at least one entrepreneur is *always* active—thus, competition is always strictly beneficial for the decision-maker. The other entrepreneur may also always be active ( $F_i(0) = 0$ ) or be inactive with strictly positive probability ( $F_i(0) > 0$ ). Second, when entrepreneur  $i$  is active, she mixes

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<sup>5</sup>No ties rules out zero-score positive-quality policies. An inactive entrepreneur can develop the reservation policy or another 0-quality policy.

smoothly over the ideologically-optimal policies  $(s, y_i^*(s)) = \left(s, \frac{x_i}{\alpha_i} F_{-i}(s)\right)$  with scores in the interval  $[0, \bar{s}]$  according to the CDF  $F_i(s)$ .

The differential equations that generate equilibrium score CDFs arise intuitively from the requirement that both entrepreneurs are indifferent over developing all ideologically-optimal policies with scores in  $[0, \bar{s}]$ . The left hand side of each differential equation is  $i$ 's net marginal cost of producing a higher-score policy, given a fixed probability  $F_{-i}(s)$  of winning the contest; the entrepreneur pays marginal cost  $\alpha_i > 1$  for sure, but with probability  $F_{-i}(s)$  her policy is chosen and she enjoys a marginal benefit of 1 (because she values quality). The right hand side represents  $i$ 's marginal ideological benefit of producing a higher score. Doing so increases by  $f_{-i}(s)$  the probability that her policy wins, which changes the ideological outcome from her opponent's optimal ideology  $y_{-i}^*(s) = \left(\frac{x_{-i}}{\alpha_{-i}}\right) F_i(s)$  at score  $s$  to her own optimal ideology  $y_i^*(s) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s)$ .

Figure 2 summarizes a mixed strategy equilibrium of the game, with symmetrically located entrepreneurs ( $-x_L = x_R$ ) and a cost advantage for the right entrepreneur ( $\alpha_L > \alpha_R$ ). The top panel depicts entrepreneurs' score CDFs. The right entrepreneur is always active due to her cost advantage ( $F_R(0) = 0$ ), whereas the left entrepreneur is sometimes inactive ( $F_L(0) > 0$ ). The right entrepreneur's policies are better for the decisionmaker in a first-order stochastic sense; we later show that this property is a general feature of the game with symmetric ideologies and asymmetric costs.

The bottom panel depicts the ideological locations and quality of the policies over which each entrepreneur mixes—a parametric plot of  $(y_i^*(s), s + (y_i^*(s))^2)$  for  $s \in [0, \bar{s}]$ . The ideological locations of entrepreneur  $i$ 's policies extend out to  $\frac{x_i}{\alpha_i}$ , which is the policy she would develop absent competition. The right entrepreneur exploits her cost advantage to develop more ideologically-extreme policies at every score, and overall her policies are first-order stochastically more extreme. This is a general feature of symmetric ideologies paired with asymmetric costs.

A notable feature of the equilibrium is that more-extreme policies are not merely higher-quality than less-extreme ones. They are also higher-score, so the additional quality *overcompensates* the decisionmaker for his ideological losses. Thus, the decisionmaker prefers the ideologically-extreme policies in the support of each entrepreneur's strategy over the ideologically-moderate ones. This is a general property, which follows immediately from ideological optimality. If two policies  $(y'_i, s'_i)$  and  $(y''_i, s''_i)$  have scores  $s'_i < s''_i$ , the higher-score one wins the contest with strictly higher probability  $F_{-i}(s''_i) > F_{-i}(s'_i)$ . Thus, the ideology

$y_i'' = y_i^*(s_i'') = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s_i'')$  of the higher-score policy is more extreme than the ideology  $y_i' = y_i^*(s_i') = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s_i')$  of the lower-score one. Intuitively, a policy that gives greater utility to the decisionmaker will be paired with a more-extreme ideology because it has a higher chance of being selected, so the entrepreneur is more willing to pay the sure costs of developing quality for the uncertain benefits of ideological change. As noted in the following result, a surprising implication is that when a competing faction chooses to develop more-extreme policies, such policies are better for the decisionmaker.

**Corollary 1** *For two policies  $(y_i', q_i'), (y_i'', q_i'')$  in the support of  $i$ 's strategy  $\sigma_i$ , the more ideologically-extreme policy  $(y_i'' > y_i')$  is both higher-quality  $(q_i'' > q_i')$  and preferred by the decisionmaker  $(s(y_i'', q_i'') > s(y_i', q_i'))$ .*

## 4 Analytical Characterization

Proposition 1 can be used to numerically compute equilibrium score CDFs for particular parameter values, but our results thus far ensure neither existence nor uniqueness. We now provide a characterization that ensures both, and identify some straightforward properties.

**Proposition 2** *Define the following notation.*

- Let  $\epsilon_i(p) = \left(\frac{\alpha_i - p}{\alpha_i - 1}\right)^{|x_i|} = e^{\left(\int_p^1 \frac{|x_i|}{\alpha_i - q} dq\right)}$  be entrepreneur  $i$ 's engagement at probability  $p$ , a function that decreases from  $\epsilon_i(0) = \left(\frac{\alpha_i}{\alpha_i - 1}\right)^{|x_i|}$  to  $\epsilon_i(1) = 1$ .
- Let  $i$ 's engagement at probability 0 be  $\epsilon_i$ , and let  $k$  denote the less-engaged entrepreneur at probability 0.
- Let  $p_i(\epsilon) = \epsilon_i^{-1}(\epsilon) = \alpha_i - (\alpha_i - 1) \epsilon^{\frac{1}{|x_i|}}$  be the unique probability such that  $i$ 's engagement is equal to  $\epsilon$ .

The unique score CDFs satisfying Proposition 1 are  $F_i^*(s) = p_{-i}(\epsilon^*(s))$ , where  $\epsilon^*(s)$  is the inverse of  $s^*(\epsilon) = 2 \sum_j |x_j| \cdot \left( \ln\left(\frac{\epsilon_k}{\epsilon}\right) - \left| \frac{x_j}{\alpha_j} \right| \cdot (p_j(\epsilon) - p_j(\epsilon_k)) \right)$ .

The unique equilibrium score CDFs  $(F_i^*, F_{-i}^*)$  can be understood through the function  $\epsilon_i(p) = \left(\frac{\alpha_i - p}{\alpha_i - 1}\right)^{|x_i|}$ , which we call entrepreneur  $i$ 's engagement at probability  $p$ . This quantity captures an entrepreneur's willingness to develop policies whose probability of winning the contest is  $\geq p$ . Whether an entrepreneur always enters the contest or sometimes sits out

depends on her *engagement*  $\epsilon_i \equiv \epsilon_i(0)$ . The entrepreneur with the lower engagement, whom we denote as  $k$  throughout the rest of the paper, mixes between entering and not entering the contest, i.e.,  $F_k(0) > 0$  if  $\epsilon_k < \epsilon_{-k}$ . The more-engaged entrepreneur  $-k$  always enters.

When entrepreneur  $i$  develops a policy at score  $s$ , the probability that she wins the contest is  $F_{-i}(s)$ . Thus, her willingness to develop policies with score  $\geq s$  is equal to  $\epsilon_i(F_{-i}(s))$ , which is decreasing in  $s$ . The key property of equilibrium is that *the entrepreneurs must be equally engaged at each score*  $s \in [0, \bar{s}]$ , i.e.,  $\epsilon_i(F_{-i}^*(s)) = \epsilon_{-i}(F_i^*(s)) \iff$

$$\left( \frac{\alpha_{-i} - F_i^*(s)}{\alpha_{-i} - 1} \right)^{|x_{-i}|} = \left( \frac{\alpha_i - F_{-i}^*(s)}{\alpha_i - 1} \right)^{|x_i|} = \epsilon^*(s) \quad (2)$$

Thus, in equilibrium every score  $s \in [0, \bar{s}]$  is associated with a unique level of engagement  $\epsilon^*(s)$  that is common to both entrepreneurs. Proposition 2 analytically characterizes the inverse of this function, which is uniquely pinned down by the boundary conditions on  $(F_i^*, F_{-i}^*)$ . It is necessarily decreasing in  $s$ , because higher scores must be associated with a greater probability of winning, and hence lower engagement.

The main equilibrium quantities are then easily derived from the following:  $\epsilon^*(s)$ , the equilibrium engagement associated with each score  $s$ ;  $\epsilon_i(p)$ , entrepreneur  $i$ 's engagement when she wins the contest with probability  $p$ ; and  $p_i(\epsilon)$ ,  $i$ 's probability of winning that yields engagement  $\epsilon$ . The probability  $F_i^*(s)$  that entrepreneur  $i$  develops a policy with score  $\leq s$  is the unique probability of winning the contest  $p_{-i}(\epsilon^*(s))$  such that her competitor  $-i$ 's engagement at score  $s$  is equal to  $\epsilon^*(s)$ . Because  $i$ 's optimal ideology is a linear function of her opponent's score CDF  $F_{-i}(s)$ , i.e.,  $y_i^*(s) = \left( \frac{x_i}{\alpha_i} \right) F_{-i}(s)$ , her unique optimal ideology at each score  $s$  is  $y_i^*(s) = \left( \frac{x_i}{\alpha_i} \right) p_i(\epsilon^*(s))$ .

**Activity** Proposition 2 yields a closed form characterization of the likelihood that each entrepreneur is active. Thus, we can analyze how the ideological extremism and costs of two competing factions determine the probability that each faction will develop a policy proposal. In particular, we consider how one faction's costs and extremism affect the other's activity.

It is easy to verify from the inverse function  $s^*(\epsilon)$  that the engagement associated with score  $s = 0$  is  $\epsilon_k$ , the engagement of the less-engaged entrepreneur  $k$ . Thus, the probability that entrepreneur  $i$  is inactive is  $F_i^*(0) = p_{-i}(\epsilon_k)$ , which gives the following corollary.

**Corollary 2** *In equilibrium*

1. *the more-engaged entrepreneur  $-k$  is always active ( $F_{-k}^*(0) = p_k(\epsilon_k(0)) = 0$ ).*
2. *the less-engaged entrepreneur  $k$  is active with probability  $1 - p_{-k}(\epsilon_k) = (\alpha_{-k} - 1) \left( \epsilon_k^{\frac{1}{|x_{-k}|}} - 1 \right)$ , which is  $< 1$  when  $\epsilon_k < \epsilon_{-k}$ .*
3. *the probability that  $k$  is active is strictly increasing in her engagement  $\epsilon_k$ , strictly increasing in her opponent's costs  $\alpha_{-k}$ , and strictly decreasing in her opponent's extremism  $|x_{-k}|$ .*

Figure 3 is a contour plot of the probability that the less-engaged entrepreneur is active as a function of the ideology  $x_R$  and costs  $\alpha_R$  of the right entrepreneur, holding fixed the left entrepreneur's parameters  $(x_L, \alpha_L)$ . The white curve depicts where the two entrepreneurs are equally engaged, and hence always active. In the purple region, the right entrepreneur is less engaged. Here, decreases in her costs  $\alpha_R$  or increases in her ideological extremism  $x_R$  increase her engagement and thus the probability that she develops a policy. In the blue region, in contrast, the right entrepreneur is more engaged and thus is always active. However, her parameters  $(\alpha_R, x_R)$  influence the probability that the left entrepreneur will be active. Decreases in the right entrepreneur's costs or increases in her extremism accentuate the imbalance in engagement, and decrease the probability that the left entrepreneur will develop a proposal. This comparative static is somewhat surprising given that (as we later show) more-extreme entrepreneurs develop more-extreme policies, which seemingly could give the less-engaged entrepreneur a greater incentive to develop a competing proposal.

Overall, the probability of observing direct competition depends on how evenly the two entrepreneurs are engaged in the contest. As their engagement becomes increasingly asymmetric, the less-engaged one increasingly drops out.

**Relative Strength** Proposition 2 and Equation 2 allow us to characterize the probability that an entrepreneur wins. In terms of score functions, it is  $\int_0^{\bar{s}} \frac{\partial F_i^*(s)}{\partial s} F_{-i}^*(s) ds$ . Applying the engagement equality gives  $\int_0^{\bar{s}} \frac{\partial F_i^*(s)}{\partial s} p_i(\epsilon_{-i}(F_i^*(s))) ds$ , while performing a change of variables (and recalling that  $F_i(0) = p_{-i}(\epsilon_k)$ ) yields the following result.

**Corollary 3** *The probability that entrepreneur  $i$  wins the contest is  $\int_{p_{-i}(\epsilon_k)}^1 p_i(\epsilon_{-i}(p)) dp$ , which is strictly increasing in her ideological extremism  $|x_i|$  and her opponent's costs  $\alpha_{-i}$ , and strictly decreasing in her costs  $\alpha_i$  and her opponent's ideological extremism  $|x_{-i}|$ .*



Thus, each entrepreneur's probability of victory responds naturally to changes in the model's parameters. As either entrepreneur becomes more ideologically motivated or better able to develop quality, her probability of winning increases and her opponent's correspondingly decreases. Note that it is straightforward to evaluate the integral and write the expression in reduced form, but the comparative statics are less transparent.

We can also apply Equation 2's engagement equality to characterize when the more-engaged entrepreneur *score-dominates* the policy contest, developing policies that are first-order stochastically better for the decisionmaker.

**Proposition 3** *The more-engaged entrepreneur  $-k$  score-dominates the contest, i.e.,  $F_{-k}(s) < F_k(s) \forall s \in [0, \bar{s}]$ , i.f.f. she is more engaged at every probability  $p$ , i.e.,  $\epsilon_{-k}(p) > \epsilon_k(p) \forall p \in [0, 1]$ .*

Being more-engaged at probability 0, and thus more likely to enter the contest, is necessary but not sufficient for entrepreneur  $-k$  to score-dominate the contest. Intuitively, the reason is that the entrepreneurs place some intrinsic value on quality. Relative cost advantages become magnified when an entrepreneur develops higher-score policies, which are more likely to be chosen and thus give the entrepreneur the direct benefits of her quality investment. Mathematically, if entrepreneur  $-k$  has higher costs ( $\alpha_{-k} > \alpha_k$ ), then greater engagement at probability 0, i.e.,  $\left(\frac{\alpha_{-k}-1}{\alpha_{-k}}\right)^{|x_{-k}|} > \left(\frac{\alpha_k-1}{\alpha_k}\right)^{|x_k|}$ , is an easier hurdle to satisfy than greater engagement at higher probabilities, i.e.,  $\left(\frac{\alpha_{-k}-1}{\alpha_{-k}-p}\right)^{|x_{-k}|} > \left(\frac{\alpha_k-1}{\alpha_k-p}\right)^{|x_k|}$ .

**Ideology** An important question is how *ideologically extreme* are the policies that the entrepreneurs develop. Proposition 2 can be used to generate analytical characterizations of  $i$ 's average ideological location  $E[y_i]$  and her probability distribution over ideologies. The model can therefore predict how factions in an organization alter the ideology of their proposals, in response to changes in the underlying parameters of competition.

The average ideological location can be derived using our previous results. In terms of score CDFs, it is  $\int_0^{\bar{s}} \frac{\partial F_i^*(s)}{\partial s} y_i(s) ds = \frac{x_i}{\alpha_i} \cdot \int_0^{\bar{s}} \frac{\partial F_i^*(s)}{\partial s} F_{-i}^*(s) ds$  (since  $y_i(s) = \frac{x_i}{\alpha_i} F_{-i}(s)$ ), which is equal to  $\frac{x_i}{\alpha_i} \Pr(i \text{ wins})$ . To derive the full CDF over  $i$ 's policies, observe that  $y_i(s) = F_{-i}(s) \frac{x_i}{\alpha_i} \iff F_{-i}(y_i^{-1}(y_i)) = \frac{y_i}{x_i/\alpha_i}$ , i.e., ideology  $y_i$  is associated with a score  $s$  such that  $F_{-i}(s) = \frac{y_i}{x_i/\alpha_i}$ . The probability that  $i$  develops a policy less extreme than  $y_i$  is the probability  $F_i(y_i^{-1}(y_i))$  that she develops a score  $s$  less than  $y_i^{-1}(y_i)$ , which can be derived by applying the Equation 2's engagement equality.

**Corollary 4** *The average ideology of entrepreneur  $i$ 's policies is  $E[y_i] = \frac{x_i}{\alpha_i} \int_{p_{-i}(\epsilon_k)}^1 p_i(\epsilon_{-i}(p)) dp$ . The ideological extremism  $|y_i|$  of entrepreneur  $i$ 's policies is distributed according to  $G_i(|y_i|) = p_{-i}\left(\epsilon_i\left(\frac{y_i}{x_i/\alpha_i}\right)\right) = \alpha_{-i} - (\alpha_{-i} - 1) \left(\frac{x_i - y_i}{x_i - x_i/\alpha_i}\right)^{\left|\frac{x_i}{x_{-i}}\right|}$ , which is first-order stochastically increasing in  $i$ 's ideological extremism  $|x_i|$ , decreasing in her costs  $\alpha_i$ , decreasing in her opponent's ideological extremism  $|x_{-i}|$ , and increasing in her opponent's costs  $\alpha_i$ .*

Unsurprisingly, when an entrepreneur's extremism  $|x_i|$  increases or her costs  $\alpha_i$  decrease, she reacts by increasing the ideological extremism of her policies. In the former case she is more motivated to exploit quality to realize ideological gains, and in the latter case she is better able to do so. More interestingly, each entrepreneur reacts to increases in her *opponent's* ideological extremism  $|x_i|$  and decreases in her opponent's costs  $\alpha_i$  by *moderating* the ideological location of her own policies. Thus, increased ideological extremism by one faction is necessarily accompanied by greater moderation from the competing faction. These comparative statics resemble those from Lax and Cameron's (2007) sequential model of endogenous quality development for U.S. Supreme Court opinions.

Our result that the cost-advantaged entrepreneur develops more-extreme policies contrasts sharply with Groseclose's (2001) model of electoral competition, which predicts that the higher-quality candidate chooses a moderate ideological platform. However, it is similar to Lax and Cameron's result that more-skillful opinion writers on the Supreme Court will write more-ideological opinions. The reason for the difference is that Groseclose assumes quality to be exogenous, so a candidate can only mitigate the effect of a pre-existing disadvantage or make use of a pre-existing advantage. In contrast, in our model (as well as Lax and Cameron's) quality is endogenous, and the benefit to entrepreneur  $i$  of having a lower cost  $\alpha_i$  is that she finds it easier to craft high-quality policy proposals that are noncentrist yet still appealing to the decisionmaker.

**Payoffs** We wrap up our analysis of the general model by characterizing players' payoffs. Proposition 2 yields a closed form characterization of the maximum score  $\bar{s}$ ; since  $F_i^*(\bar{s}) = 1 = p_{-i}(1)$ , the maximum score is simply the score  $s^*(1)$  associated with an engagement of 1. An entrepreneur's equilibrium utility is equal to her utility  $\Pi_i^*(\bar{s}; F^*)$  from developing the maximum score  $\bar{s}$ , since it is in the support of her strategy.<sup>6</sup> Also, since the CDF of the winning score  $\max\{s_i, s_{-i}\}$  is the product of the score CDFs  $F_i^*(s) F_{-i}^*(s)$ , the

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<sup>6</sup>The equality  $\Pi_{-i}^*(0; F^*) = \Pi_{-i}^*(\bar{s}; F^*) \iff E[s_i] = \frac{x_{-i}^2}{\alpha_{-i}} \left(1 - (F_i(0))^2\right) + 2|x_{-i}E[y_i]| - (\alpha_{-i} - 1)\bar{s}$  also generates the average score  $E[s_i]$ , since  $F_i(0)$  and  $E[y_i]$  are characterized in Corollaries 2 and 4.

decisionmaker's equilibrium utility is  $\int_0^{\bar{s}} s \cdot \frac{\partial}{\partial s} (F_i^*(s) F_{-i}^*(s)) ds$ . This is straightforward to compute by applying the engagement equality and a change of variables. Applying these insights yields the players' equilibrium utilities and comparative statics.

**Proposition 4** *In equilibrium,*

1. *the maximum score is  $\bar{s} = s^*(1) = 2 \sum_j |x_j| \cdot \left( \ln(\epsilon_k) - \left| \frac{x_j}{\alpha_j} \right| (1 - p_j(\epsilon_k)) \right)$ , which is increasing in ideological extremism  $|x_i|$  and decreasing in costs  $\alpha_i$  for all  $i$*
2. *entrepreneur  $i$ 's utility is  $\Pi_i^*(\bar{s}; F^*) = -\left(1 - \frac{1}{\alpha_i}\right) x_i^2 - (\alpha_i - 1) \bar{s}$ , which is decreasing in her opponent's extremism  $|x_{-i}|$  and increasing in her opponent's costs  $\alpha_{-i}$*
3. *the decisionmaker's utility is*

$$\int_{\epsilon_k}^1 s(\varepsilon) \cdot \frac{\partial}{\partial \varepsilon} (p_k(\varepsilon) \cdot p_{-k}(\varepsilon)) d\varepsilon = 2 \int_1^{\epsilon_k} \left( 1 - \prod_j p_j(\varepsilon) \right) \cdot \left( \sum_j \frac{(|x_j|/\alpha_j) p_j(\varepsilon)}{\varepsilon} \right) d\varepsilon.$$

An entrepreneur's utility is written in terms of two components. The first component  $-\left(1 - \frac{1}{\alpha_i}\right) x_i^2$  depends solely on her own parameters, and represents what her utility would be if she could engage in entrepreneurship absent competition. The second component  $-(\alpha_i - 1) \bar{s}$  is the cost generated by competition, which forces her to develop policies that are strictly better for the decisionmaker than the reservation policy, in order to maintain her influence. This cost is increasing in  $i$ 's marginal cost  $\alpha_i$  of developing quality, and increasing in the intensity of competition, as captured by the maximum score  $\bar{s}$ .

The intensity of competition  $\bar{s}$  is affected by the entire profile of parameters in a natural way. It increases if either entrepreneur becomes more extreme, or if either entrepreneur's costs of developing quality decrease.

An interesting implication is that an entrepreneur is worse off if her opponent becomes more willing or able to compete. In particular, an entrepreneur is harmed if her opponent becomes more efficient at developing quality, even though quality is a fully common value dimension. The reason is that the downside of her opponent's ability to exploit quality to achieve noncentrist outcomes outweighs the spillover benefit of the additional quality. It is worth noting that our results on how  $-i$ 's parameters affect  $i$ 's utility are consistent with the casual observation, e.g., from the politics of academic departments, that a faction is often displeased when a competing faction becomes either more-motivated to exert effort on proposals that will shape the future direction of the organization, or more-effective at generating such proposals.

In the next two sections, we examine special cases of our model, including what happens when the entrepreneurs are evenly matched, what happens when one of them is dominant, and what happens when they have different primary motivations.

## 5 Symmetric Competition

We first focus on the special case of symmetric competition. Let  $x \equiv |x_i|$  so the entrepreneurs are equidistant from the decisionmaker and let  $\alpha \equiv \alpha_i$  so they face the same marginal cost of developing quality. Symmetric competition is an important subcase of our general model for two reasons. First, varying the entrepreneurs' extremity is a natural way to analyze the effects of *polarization of preferences*. Second, we can characterize how the cost of developing quality affects decisionmaking. Although everyone benefits from quality, the welfare effects of lower costs are nonobvious, because the entrepreneurs exploit quality to realize ideological gains. We first take advantage of symmetry to characterize the equilibrium in a form simpler than Proposition 2.

**Proposition 5** *If  $x \equiv |x_i|$  and  $\alpha \equiv \alpha_i$ , the unique equilibrium is in symmetric mixed strategies. The entrepreneurs develop policies of the form  $(y_i, s(|y_i|) + y_i^2)$ , where*

1. *the ideological extremity  $|y_i|$  of each entrepreneur's policies is uniform on  $[0, \frac{x}{\alpha}]$*
2. *the score of a policy with ideology  $y_i$  is  $s^*(|y_i|) = 4x \left( x \ln \left( \frac{x}{x - |y_i|} \right) - |y_i| \right)$*
3. *the maximum score is  $\bar{s} = 4x^2 \left( \ln \left( \frac{\alpha}{\alpha - 1} \right) - \frac{1}{\alpha} \right)$ , and each entrepreneur's utility is  $-\left(1 - \frac{1}{\alpha}\right)x^2 - (\alpha - 1)\bar{s} = -4x^2(\alpha - 1) \left( \ln \left( \frac{\alpha}{\alpha - 1} \right) - \frac{3}{4\alpha} \right)$*
4. *the decisionmaker's utility is  $4x^2 \left( \left( \alpha + \frac{1}{2} - \frac{2}{3\alpha} \right) - (\alpha^2 - 1) \ln \left( \frac{\alpha}{\alpha - 1} \right) \right)$ .*

In the symmetric game, the entrepreneurs are both always active and play the identical atomless score CDF. Figure 4 depicts equilibrium policies for different values of  $\alpha$ , holding fixed  $x$ . The ideological distance of each entrepreneur's policies from the decisionmaker is uniformly distributed on  $[0, \frac{x}{\alpha}]$ .

The key simplification produced by symmetry is that the ideological extremity  $|y_i^*(s)|$  of the entrepreneurs' optimal ideologies, and hence their score CDFs, must be identical at every score. This implies that the ideologies of each entrepreneur's policies are uniformly distributed (because  $\Pr(|y_i| \leq |y|) = \frac{y_{-i}^*(y_{-i}^{*-1}(y))}{x_{-i}/\alpha_{-i}} = \frac{|y|}{x/\alpha}$ ), which allows us to easily characterize the equilibrium in terms of the score as a function of ideological extremity.

**Policy Outcomes and Decisionmaker Utility** Because the ideological extremity of policies is uniformly distributed on  $[0, \frac{x}{\alpha}]$ , it is obvious that either an increase in polarization (as measured by  $x$ ) or a decrease in costs (as measured by  $\alpha$ ) leads to more-extreme policies being both developed and adopted, in a first-order stochastic sense. Both of these factors therefore contribute to observable *polarization of outcomes* in the model. However, although the decisionmaker is worse off in the sense of ideology, he is better off overall, both in an expected utility sense and a first-order stochastic sense.

**Proposition 6** *The extremity of the policy outcome and the decisionmaker’s utility are first-order stochastically increasing in polarization  $x$  and decreasing in the cost of quality  $\alpha$ .*

Corollary 1 already showed that for fixed parameters, ideologically-extreme policies in the support of an entrepreneur’s strategy are better for the decisionmaker. Proposition 6 is different—it states that in symmetric environments, factors that induce the entrepreneurs to develop ideologically-extreme policies also induce them to develop policies that are better for the decisionmaker. Intuitively, lower costs make entrepreneurs more *able* to invest in quality to realize ideological gains, resulting in more extreme but better policies. Greater polarization  $x$  makes them more *willing* to do so, to similar effect. Mathematically, the result is easiest to see by considering the effect of decreasing  $\alpha$ , which does not enter the score function  $s(|y|)$  in Proposition 5 and only stretches the range of uniformly distributed ideologies  $[0, \frac{x}{\alpha}]$ . As shown in Figure 4, decreasing  $\alpha$  shifts probability weight towards policies that are more extreme, but also better for the decisionmaker.

It’s worth noting that the presence of a competing faction is essential for the result. If only one entrepreneur could generate quality, she would fully extract its benefits in the form of ideological concessions. This is a general feature of our model, which does not depend on symmetry. It contrasts sharply with Rotemberg and Saloner’s (1994) argument that a firm that must choose among projects can benefit from adopting a narrow focus and eliminating competition. Although we share Rotemberg and Saloner’s focus on productive effort and innovation, the effect of competition is different, because an entrepreneur in our model cares directly about the policy the organization adopts, even if it was developed by someone else. Competition by entrepreneurs with different preferences thus incentivizes each entrepreneur to craft proposals that are more appealing to the decisionmaker.

**Entrepreneur Utility** In Proposition 4 we showed that each entrepreneur is harmed by a decrease in her opponent’s cost of developing quality. Here, we consider how *symmetric* cost

shifts affect the entrepreneurs' utility, e.g., if, their efforts are subsidized by the organization, or if a technological change increases the efficiency of their investments.

**Proposition 7** *In the symmetric model, the marginal cost of developing quality  $\alpha$  has the following effects on entrepreneurs' equilibrium utility.*

1. *As  $\alpha \rightarrow 1$ , an entrepreneur's utility converges to  $U_i(x_i, 0) = 0$  (her utility from her ideal ideology with no quality). As  $\alpha \rightarrow \infty$ , an entrepreneur's utility converges to  $U_i(b_0) = -x_i^2$  (her utility from the reservation policy).*
2. *There exists an  $\hat{\alpha}$  such that the entrepreneurs' utility is decreasing in  $\alpha$  when  $\alpha < \hat{\alpha}$ , and increasing otherwise.*
3. *There exists an  $\bar{\alpha} < \hat{\alpha}$  such that the entrepreneurs benefit from the ability to engage in entrepreneurship when  $\alpha < \bar{\alpha}$ , and are harmed otherwise.*

As the marginal cost of developing quality approaches its marginal benefit, it is as if each entrepreneur can get her ideal ideological outcome at no cost. In contrast, as the cost becomes high, entrepreneurship collapses. Between the limits, the effect of  $\alpha$  is non-monotonic. At low cost levels, competition is most intense but also least costly; here, higher costs harm the entrepreneurs by increasing the price they pay for their efforts. Once quality becomes sufficiently costly ( $\alpha > \hat{\alpha}$ ), however, further cost increases benefit the entrepreneurs by decreasing the intensity of competition.

The proposition also shows when the ability to engage in competitive entrepreneurship benefits the entrepreneurs, relative to simply accepting the reservation policy. In common agency models, e.g., Dixit, Grossman, and Helpman (1997), equal and opposing interest groups are harmed by the ability to lobby. Their counteractive influence doesn't affect policy outcomes, but each group must pay to prevent the decisionmaker from colluding with its competitor. In our model, in contrast, the factions benefit from the ability to develop policies if costs are sufficiently low ( $\alpha < \bar{\alpha}$ ). The reason is that each entrepreneur places some intrinsic value on the quality developed by her influence-seeking opponent.

**Application: Political Polarization** The symmetric model provides a novel lens for analyzing the effects of political polarization. The large literature on this topic (e.g., Brady and Volden 1998, Krehbiel 1998) features two arguments: polarization causes non-centrist policy outcomes, and it is bad for centrists. In our model, polarization leads to non-centrist

outcomes, but is actually good for centrists. The key difference is that most existing work on polarization takes as given the set of available policies, whereas we consider incentives for entrepreneurs to make productive investments in their proposals. An additional difference is that previous work focuses on polarization of the preferences of actors (pivots or veto players) whose approval is necessary for policy enactment, whereas in our model decisionmaking authority remains in the hands of a single centrist.<sup>7</sup>

The literature on signaling games includes some single-decisionmaker models in which actors have shared interests, in the sense that they benefit from variance reduction. The model most directly comparable to ours is Gilligan and Krehbiel (1989), in which two privately-informed experts located symmetrically around a decisionmaker make policy recommendations. In that model, polarization harms the decisionmaker because the experts do not engage in confirmatory signalling in extreme states.<sup>8</sup> Our model is fundamentally different—extreme entrepreneurs place a greater marginal value on shifting ideological outcomes toward their ideal points, magnifying their incentives to invest in quality. Our model thus demonstrates that polarization can be beneficial in political organizations, when it induces competing factions to make productive investments to gain influence.

The symmetric model also generates surprising predictions about observed ideological preferences and the polarization of policy outcomes. Traditional spatial models (since Hotelling 1929, Downs 1957, and Black 1958) feature policy convergence because decisionmakers prefer ideologies close to their ideal points. Our model demonstrates that with a second, endogenous, dimension of policy, this assumption about *primitive* preferences can have very different implications for *observed* preferences. In our model, policy entrepreneurs craft ideologically-extreme policies with sufficient quality to make them more desirable to the decisionmaker. In the symmetric variant (where the entrepreneurs’ strategies are mirror images), this implies that the decisionmaker always prefers and chooses the more ideologically-distant policy. While this implication is no doubt extreme, it highlights the importance of considering, in empirical applications, how strategic actors affect endogenous dimensions of policy (Triossi, Valdivieso, and Villena-Roldan 2013).

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<sup>7</sup>In a companion paper we examine dispersed decisionmaking authority.

<sup>8</sup>Krishna and Morgan (2001) show that Gilligan and Krehbiel’s model with two committees also has a fully-revealing equilibrium, which is criticized by Krehbiel (2001) for being implausible and by Battaglini (2003) for being non-robust.

## 6 Asymmetric Competition

Asymmetric competition is a common feature of politics. Often, one faction has more-extreme ideological preferences or greater expertise and resources to develop high-quality proposals. In this section we consider three special cases of interest: (i) an entrepreneur who is dominant, in the sense of having both more-extreme preferences and lower costs, (ii) entrepreneurs who are equally engaged but with different primary motives, where one has a cost advantage and the other is more ideologically-extreme, and (iii) major asymmetries resulting from very high or low costs or very moderate or extreme preferences. We first review the main comparative statics of the general model from Corollaries 2–4 and Propositions 3 and 4.

**Observation 1** *As an entrepreneur  $i$ 's costs  $\alpha_i$  decrease or her extremism  $|x_i|$  increases,*

1. *her probability of winning the contest increases*
2. *her policies become more extreme and her opponent's become more moderate*
3. *her opponent's utility decreases.*

*Moreover, the probability that the less-engaged entrepreneur  $k$  is active decreases in her own costs  $\alpha_k$  and her opponent's ideological extremism  $|x_{-k}|$ , and increases in her opponent's costs  $\alpha_{-k}$  and her own ideological extremism  $|x_k|$ .*

The main message of the general model is that for many outcomes of interest, increasing an entrepreneur's extremism or reducing her costs has similar effects. Making an entrepreneur either more *willing* or better *able* to exploit quality investments to realize ideological gains increases her strength, induces her to develop extreme policies, forces her opponent to develop moderate policies, and harms her opponent. If she is the less-engaged entrepreneur, she becomes more likely to be active, whereas if she is already more engaged then she further drives her opponent out of the contest.

**A Dominant Entrepreneur** We now consider the special case in which there is a dominant entrepreneur in terms of parameters ( $|x_k| \leq |x_{-k}|$  and  $\alpha_{-k} \leq \alpha_k$  with at least one strict inequality). Recall that  $i$  score-dominates the policy contest if she develops policies that are first-order stochastically better for the decisionmaker, i.e.,  $F_i(s) \leq F_{-i}(s)$ ,  $\forall s \in [0, \bar{s}]$  with a



strict inequality for some scores. We also say that entrepreneur  $i$  is more *ideologically aggressive* if her policies are first-order stochastically more extreme, i.e.,  $G_i(|y|) \leq G_{-i}(|y|)$ ,  $\forall y$  with a strict inequality for some ideologies. These features are characteristic of competition when one entrepreneur is dominant.

**Corollary 5** *If  $\alpha_{-k} \leq \alpha_k$  and  $|x_{-k}| \geq |x_k|$ , with at least one inequality strict, then entrepreneur  $-k$  is more engaged, score dominant, and more ideologically aggressive.*

Greater engagement and score dominance follow from Proposition 3, which states that greater engagement at every probability  $p$ , i.e.,  $\left(\frac{\alpha_{-k}-p}{\alpha_{-k}-1}\right)^{|x_{-k}|} > \left(\frac{\alpha_k-p}{\alpha_k-1}\right)^{|x_k|} \forall p$ , is a necessary and sufficient condition for score dominance. This holds when  $-k$  is both more extreme and has lower costs. First order stochastic dominance of ideologies is then an implication: applying score dominance, entrepreneur  $-k$  develops more-extreme policies at every score, i.e.,  $|y_{-k}^*(s)| = \left|\frac{x_{-k}}{\alpha_{-k}}\right| F_k(s) > \left|\frac{x_k}{\alpha_k}\right| F_{-k}(s) = |y_k^*(s)| \forall s$ , which, combined with score dominance, implies that she is more ideologically aggressive than her opponent.

The subcase of an entrepreneur who is dominant due to lower costs ( $\alpha_{-k} < \alpha_k$ ) despite equally-extreme ideological preferences ( $x_k = x_{-k}$ ) has a natural interpretation. The entrepreneurs may represent two competing factions within a firm or agency. Each leans in favor of one particular approach to a problem, yet one has more staff and money to develop new policy proposals. In these circumstances, the cost-advantaged entrepreneur exploits her advantage to develop policies that reflect her ideological preferences. Interestingly, despite the extremism of her policies, she invests sufficiently in quality to make the decisionmaker probabilistically favor them; she does not overexploit her advantage.

In the subcase of an entrepreneur who is dominant due to a more-extreme ideology ( $|x_{-k}| > |x_k|$ ) despite no greater ability to develop quality ( $\alpha_k = \alpha_{-k}$ ), the model demonstrates that extremism need not be a vice. Greater extremism induces an entrepreneur to value *marginal* ideological gains more, which incentivizes her to produce higher-quality policies. Her extremism does not induce her to be excessively aggressive; her policies are first-order stochastically better for the decisionmaker despite their greater extremism, and her extreme preferences (surprisingly) make her more likely to win the contest.

**Equally-Engaged Entrepreneurs with Different Motives** Next, we consider entrepreneurs who are equally engaged ( $\epsilon_k = \epsilon_{-k}$ ) but with different primary motives for engagement. Let  $j$  have lower costs ( $\alpha_j < \alpha_{-j}$ ) and  $-j$  have more-extreme ideological preferences

( $|x_{-j}| > |x_j|$ ). For example,  $j$  may be a corporate interest group whereas  $-j$  is an environmental NGO with limited resources. The groups' equal engagement implies that both are always active. However, their patterns of policy development differ.

**Proposition 8** *With entrepreneurs who are equally engaged ( $\epsilon_j = \epsilon_{-j}$ ) but have different primary motives for engagement ( $\alpha_j < \alpha_{-j}$  and  $|x_{-j}| > |x_j|$ )*

1. *the cost-advantaged entrepreneur  $j$  is score-dominant and more likely to win*
2. *the more-extreme entrepreneur  $-j$  develops a more-extreme policy  $|y_{-j}^*(s)| > |y_j^*(s)|$  at every score.*

Score dominance of the cost-advantaged entrepreneur follows because they are equally engaged and her cost advantage becomes magnified at higher scores due to the higher likelihood of enjoying the intrinsic benefits of quality. A straightforward consequence is that she is more likely to win the contest.

Demonstrating that the more ideologically-extreme entrepreneur develops more-extreme policies at every score  $s$  (i.e.,  $|y_{-j}(s)| = \frac{|x_{-j}|}{\alpha_{-j}} F_j(s) > \frac{|x_j|}{\alpha_j} F_{-j}(s) = |y_j(s)|$ ) is more involved. It is simple to show that she would develop a more ideologically-extreme policy for any fixed probability of winning despite her greater costs (i.e.,  $\frac{|x_{-j}|}{\alpha_{-j}} > \frac{|x_j|}{\alpha_j}$ ). However, this is counterbalanced by her lower probability of victory at every score. Nevertheless, it can be shown that her tendency toward extremism dominates.

These observations have interesting implications for the decisionmaker's observable choices among the policies that are developed.

**Corollary 6** *If entrepreneurs are equally engaged but have different primary motives, the decisionmaker appears ideologically biased toward the cost-advantaged entrepreneur:*

1. *the cost-advantaged entrepreneur wins the contest whenever her policy is equally or more ideologically-extreme than her opponent's*
2. *the ideologically-motivated entrepreneur sometimes develops a more ideologically-extreme policy that loses the contest*
3. *the cost-advantage entrepreneur wins with probability  $> \frac{1}{2}$ .*

Thus, when the ideology of policies is considered in isolation, the decisionmaker's policy choices appear to be biased towards the entrepreneur with greater resources for policy development. The cost-advantaged entrepreneur tends to win, and is rewarded with victory whenever she develops a policy that is equally or even more ideologically extreme. Conversely, the ideologically-motivated entrepreneur appears to overreach by sometimes developing a more ideologically-extreme policy and losing the contest.

These patterns are consistent with stylized facts about competition between resource-rich interest groups (e.g., firms) and ideologically-motivated ones (e.g., environmental NGOs). However our results are not driven by factors such as backdoor dealings or quid pro quo lobbying expenditures and campaign contributions that could enable corporate interest groups to dominate policy making. Nor do our results stem from irrational behavior by idealistic activists who insist on maintaining ideological purity. Of course, such factors may well contribute to observed patterns of behavior. But our model shows that these patterns can also arise simply due to preference and cost asymmetries between rational actors who make productive investments that improve the quality of their policy proposals.

**Major Asymmetries** Finally, we consider major asymmetries in parameters, which arise when an entrepreneur has very high or low costs or has very moderate or extreme preferences. We first establish that given any of these sources of asymmetry, the less-engaged entrepreneur  $k$  is unlikely to enter the contest. From Corollary 2, we know that her probability of being active is  $(\alpha_{-k} - 1) \left( \epsilon_k^{\frac{1}{|x_{-k}|}} - 1 \right)$ , which converges to zero as: she becomes moderate ( $|x_k| \rightarrow 0$ ), her cost of producing quality becomes high ( $\alpha_k \rightarrow \infty$ ), her opponent's preferences become extreme ( $|x_{-k}| \rightarrow \infty$ ), or her opponent's net cost of producing quality becomes low ( $\alpha_{-k} \rightarrow 1$ ). Thus, equilibrium patterns of activity resemble those of a 1-entrepreneur game because the more-engaged entrepreneur  $-k$  rarely encounters direct competition.

A natural question is whether policy outcomes likewise resemble those of a 1-entrepreneur game. Absent competition, the sole entrepreneur would extract all benefits of quality in the form of ideological gains, leaving the decisionmaker no better off than under the reservation policy. We first show that this is indeed the case when the asymmetry is due to the less-engaged entrepreneur having very moderate preferences or high costs.

**Proposition 9** *When  $|x_k| \rightarrow 0$  or  $\alpha_k \rightarrow \infty$ , the decisionmaker's utility converges to zero.*

The scenario of an entrepreneur with very high costs matches Londregan's (2000) characterization of policymaking in Chile, where both the legislature and the president have formal

proposal power, but the legislature had few resources for policy development. The predictions of our model in this empirical domain are therefore similar to Londregan’s model, in which only the president can develop high-quality policies.

The scenario of a very moderate entrepreneur demonstrates that there is no benefit to the decisionmaker from an entrepreneur who perfectly shares her preferences. Such an entrepreneur stays out of the policy contest because the decisionmaker already represents her interests. The decisionmaker would actually prefer *any* entrepreneur who will generate competition, however extreme, rather than a replica of herself.

Although the benefits of competitive entrepreneurship vanish when a major asymmetry arises from one entrepreneur’s disengagement, our next result demonstrates that this is not the case when it arises from very high engagement of her opponent.

**Proposition 10** *When  $|x_{-k}| \rightarrow \infty$  or  $\alpha_{-k} \rightarrow 1$ , the decisionmaker’s utility is bounded away from zero.*

When the absence of activity by one entrepreneur results from the high engagement of her competitor, the decisionmaker is strictly better off with the possibility of competition. The reason is simple—the *threat* of entry by the less-engaged entrepreneur prevents the more-engaged one from developing policies that are no better than the reservation policy. If she did so, the less-engaged entrepreneur would develop strictly better policies and win. Potential competition thus prevents even a highly-dominant entrepreneur from extracting all the benefits of quality in the form of ideological gains. This observation is crucial for empirical analyses of competitive policy development—in situations where only one faction routinely develops proposals, it cannot be concluded that its actions are unaffected by potential activity from other interested groups.

## 7 Conclusion

This paper develops a model of political organizations in which factions have different ideologies or preferences regarding organizational priorities, yet also agree on certain common objectives. Competing entrepreneurial policy developers can appeal to decision makers by making productive, policy-specific investments to improve the quality of their proposals. Rather than being tailored narrowly to any one specific institution, our model is designed to capture key features of many different political organizations, including legislatures, NGOs, firms, militaries, democratic polities, political parties, and executive branch agencies.

We characterize the equilibrium of the all-pay contest played by two competing entrepreneurs as they generate proposals comprised of two dimensions: ideology and quality. In the analysis, we also develop techniques that can be applied to other environments in which actors compete to have their preferred policies enacted by exerting costly up-front effort, e.g., lobbying (Meirowitz and Jordan 2012) and valence competition in elections (Wiseman 2006, Meirowitz 2008, Ashworth and Bueno de Mesquita 2009). In many models it would be natural to analyze simultaneous choice of ideology and policy, but to the best of our knowledge no model has analyzed the resulting all-pay contest.

Our analysis suggests several avenues for future work. One possibility is to expand the number of possible participants in the contest. This is a natural assumption for organizations in which entrepreneurship occurs at an individual level rather than in teams, or when entrepreneurs come from outside of the organization (e.g., interest groups developing proposals for government policy). It is straightforward to show that when costs are common there always exists an equilibrium in which the two most ideologically-extreme entrepreneurs play their equilibrium strategies in the 2-entrepreneur game, while the others are inactive.

A second possible extension would be to allow policy entrepreneurs to buy support using targeted benefits (pork, as in vote buying models), collective benefits (policy quality, as in our model), or both, and then analyze when they use productive investments in high-quality proposals rather than wasteful targeted vote buying.

A third possible avenue is to consider aspects of institutional design, including subsidies for policy development, endogenous selection of the entrepreneurs, delegation of the decision to a person with different ideological preferences, or design of the decisionmaking mechanism. In a companion paper, we consider how the addition of veto players affects incentives for policy development.

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## 9 Appendix

The supplement has proofs of Lemmas 1 and 3. We refer to the decisionmaker as DM.

**Lemma 2** Player  $i$ 's utility from developing  $(s_i, y_i)$  where  $s_i > 0$  and  $-i$  has no atom is

$$-\alpha_i (s_i + y_i^2) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s(y_{-i}, q_{-i}) > s_i} U_i(y_{-i}, q_{-i}) d\sigma_{-i}. \quad (3)$$

The first order condition with respect to  $y_i$  yields the result.

**Prop. 1 Part 1.** We show that in any equilibrium, the support of both players' score CDFs must be a common interval  $[0, \bar{s}]$ . We first argue that each player's support must be bounded. Boundedness from below is assumed w.l.o.g. To see that  $i$ 's support is bounded from above, first observe that from Lemma 2,  $|y_{-i}^*(s_{-i}; F_i)| \leq \left| \frac{x_{-i}}{\alpha_{-i}} \right|$ . Thus,  $i$ 's utility from developing the reservation policy is bounded from below:  $\Pi_i^*(0, 0; F) \geq V_i\left(0, \frac{x_{-i}}{\alpha_{-i}}\right)$ . It is easy to verify that  $\lim_{s_i \rightarrow \infty} \Pi_i^*(s_i; F) \rightarrow -\infty$  for any  $F$ ; thus unbounded support would require scores that cannot satisfy score optimality.

We next argue that the players' strategies must have common support. Suppose not. Then  $\exists \hat{s}$  in the support of  $i$  and an interval  $[\hat{s} - \varepsilon, \hat{s}]$  over which  $F_{-i}(s)$  is constant. This contradicts score optimality since  $\Pi_i^*(\hat{s}; F) - \Pi_i^*(\hat{s} - \varepsilon; F) = -\alpha_i \varepsilon$ .

Finally, the common support must be the full interval  $[0, \bar{s}]$ . Suppose not. Then there are two scores  $s', s'' \in [0, \bar{s}]$  with  $F_i(s)$  constant over  $[s', s''] \forall i$ . By the argument in the previous paragraph, score optimality is violated if  $F_i(s'') = F_i(s')$  for any  $i$ , so  $F_i(s'') > F_i(s') \forall i$ . But this means both players have atoms at  $s''$  which violates no ties.

*Part 2:*  $F_i$  must be continuous  $\forall i$  over the  $[0, \bar{s}]$ . Otherwise  $\Pi_{-i}^*(s; F)$  would have a discontinuity and score optimality would fail. Continuity and score optimality imply

$$\begin{aligned} \frac{\partial}{\partial s} \Pi_i^*(s; F) &= \left. \frac{\partial \Pi_i^*(s, y_i; F)}{\partial s} \right|_{(s, y_i^*(s))} \quad (\text{by envelope theorem}) \\ &= -\alpha_i + F_{-i}(s) + f_{-i}(s) (V_i(s, y_i^*(s)) - V_i(s, y_{-i}^*(s))) = 0 \quad \forall i, s \in [0, \bar{s}] \end{aligned}$$

which is equivalent to Prop. 1's differential equation. That  $F_i(0) > 0$  for at most one  $i$  follows from no ties. Otherwise  $y_i(0) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(0) \neq 0$ , and  $i$  would be developing  $(0, y_i(0))$  with strictly positive probability (by ideological optimality), violating no ties. ■

**Prop. 2** *Part 1:* We seek a solution to the differential equation in Prop 1. satisfying the boundary conditions. We rewrite the equation as

$$\frac{\alpha_i - F_{-i}(s)}{x_i f_{-i}(s)} = 2x_i \left( \left( \frac{x_i}{\alpha_i} \right) F_{-i}(s) - \left( \frac{x_{-i}}{\alpha_{-i}} \right) F_i(s) \right)$$

which implies  $\frac{\alpha_i - F_{-i}(s)}{x_i f_{-i}(s)} = -\frac{\alpha_{-i} - F_i(s)}{x_{-i} f_i(s)}$ . Letting  $s_i(F_i)$  denote the inverse of  $F_i(s)$ , observing that  $s'_i(F_i) = \frac{1}{f_i(s_i(F_i))}$ , substituting in  $s_i(F_i)$  for  $s$ , and rearranging yields

$$\frac{\alpha_i - F_{-i}(s_i(F_i))}{x_i} = - \left( \frac{\alpha_{-i} - F_i}{x_{-i}} \right) \cdot \frac{\partial}{\partial F_i} (F_{-i}(s_i(F_i))).$$

This is a differential equation on the composite function  $F_{-i}(s_i(F_i))$  giving entrepreneur  $-i$ 's probability of developing a policy with score less than the score  $s_i(F_i)$  associated with  $F_i$ . The following function with an arbitrary constant  $c$  solves the differential equation:

$$F_{-i}(s_i(F_i)) = \alpha_i + c(\alpha_{-i} - F_i)^{-\frac{x_{-i}}{x_i}}.$$

From Prop. 1, the boundary condition  $F_{-i}(s_i(F_i)) = 1$  must be satisfied, as  $F_i(\bar{s}) = F_{-i}(\bar{s}) = 1$ . This implies  $c = -(\alpha_i - 1)(\alpha_{-i} - 1)^{\frac{x_{-i}}{x_i}}$ . Substituting and rearranging yields

$$\left( \frac{\alpha_i - F_{-i}(s_i(F_i))}{\alpha_i - 1} \right)^{x_i} = \left( \frac{\alpha_{-i} - F_i}{\alpha_{-i} - 1} \right)^{-x_{-i}}.$$

Finally, substituting  $F_i(s)$  back in for  $F_i$  yields

$$\left( \frac{\alpha_i - F_{-i}(s)}{\alpha_i - 1} \right)^{|x_i|} = \left( \frac{\alpha_{-i} - F_i(s)}{\alpha_{-i} - 1} \right)^{|x_{-i}|} \iff \epsilon_i(F_{-i}(s)) = \epsilon_{-i}(F_i(s)),$$

i.e., at every score the probabilities of victory must make the entrepreneurs equally engaged.

*Part 2.* Part 1 proves that there is a unique equilibrium engagement  $\epsilon(s)$  associated with every score, where  $\epsilon(s) = \epsilon_i(F_{-i}(s)) = \epsilon_{-i}(F_i(s))$ . It is simple to verify by taking logs and differentiating that  $-\frac{\epsilon(s)}{\epsilon'(s)} = \frac{\alpha_i - F_{-i}(s)}{x_i f_{-i}(s)}$ , and hence

$$-\frac{\epsilon(s)}{\epsilon'(s)} = 2 \left( \left( \frac{x_i}{\alpha_i} \right) F_{-i}(s) - \left( \frac{x_{-i}}{\alpha_{-i}} \right) F_i(s) \right).$$

Letting  $p_i(\epsilon) = \epsilon_i^{-1}(p) = \alpha_i - (\alpha_i - 1)\epsilon^{\frac{1}{|x_i|}}$ , we rewrite in terms of the inverse function  $s(\epsilon)$

$$s'(\epsilon) = -2 \sum_i \frac{(|x_i|/\alpha_i) \cdot p_i(\epsilon)}{\epsilon}.$$

It is then easily verified that  $\int \frac{(|x_i|/\alpha_i) \cdot p_i(\epsilon)}{\epsilon} = |x_i| (\ln(\epsilon) + (|x_i|/\alpha_i) \cdot p_i(\epsilon))$ ; thus,

$$s(\epsilon) = 2 \sum_i |x_i| \left( -\ln(\epsilon) - \left( \frac{|x_i|}{\alpha_i} \right) \cdot p_i(\epsilon) \right) + C.$$

Finally, we set the constant. The score ranges from  $[0, \bar{s}]$ , and score is a decreasing function of engagement, so the maximum engagement  $\epsilon^*(0) = \bar{\epsilon}$  is associated with the minimum score  $s = 0$ . We argue that  $\bar{\epsilon} = \min_i \{\epsilon_i(0)\} = \epsilon_k(0)$ . If the maximum engagement were lower, then  $F_i(0) = p_{-i}(\bar{\epsilon}) > p_{-i}(\epsilon_k(0)) \geq 0 \forall i$  and the boundary condition at score 0 fails. If the maximum engagement were higher, then for entrepreneur  $-k$ ,  $F_{-k}(0) = p_k(\bar{\epsilon}) < p_k(\epsilon_k(0)) = 0$ , a contradiction. Hence,  $C$  must yield  $s(\epsilon_k) = 0$ . The unique solution can be divided up among four additive subterms as

$$s(\epsilon) = 2 \sum_i |x_i| \left( \ln\left(\frac{\epsilon_k}{\epsilon}\right) - \left( \frac{|x_i|}{\alpha_i} \right) \cdot (p_i(\epsilon) - p_i(\epsilon_k)) \right).$$

Entrepreneur  $i$ 's score CDF at  $s$  is the unique probability such that  $-i$ 's engagement equals  $\epsilon(s)$  (the inverse of  $s(\epsilon)$ ), i.e.,  $F_i(s) = p_{-i}(\epsilon(s))$ .

**Prop. 3** For sufficiency:  $\epsilon_{-k}(F_k(s)) = \epsilon_k(F_{-k}(s))$  and  $\epsilon_{-k}(p) > \epsilon_k(p) \forall p \rightarrow F_k(s) > F_{-k}(s)$  as  $\epsilon_i(p)$  is decreasing in  $p$ . For necessity:  $\epsilon_{-k}(F_k(s)) = \epsilon_k(F_{-k}(s))$  and  $F_k(s) > F_{-k}(s) \rightarrow \epsilon_{-k}(F_{-k}(s)) > \epsilon_k(F_{-k}(s))$ . As  $F_{-k}(s)$  maps one to one to  $[0, 1]$  (since  $-k$  is always active) we have  $\epsilon_{-k}(p) > \epsilon_k(p) \forall p$ .

**Prop. 4** *Part 1.* We first show the equilibrium score function  $s^*(\epsilon)$  is increasing in  $x_i$  and decreasing in  $\alpha_i \forall \epsilon$ . Expressing dependence of equilibrium quantities on parameter  $q \in \{x_L, x_R, \alpha_L, \alpha_R\}$ ,  $s(\epsilon; q) = \int_1^\epsilon s'(\epsilon; q) d\epsilon + C(q)$ . Thus

$$\frac{\partial s(\epsilon; q)}{\partial q} = \int_1^\epsilon \frac{\partial s'(\epsilon; q)}{\partial q} d\epsilon + C'(q). \quad (4)$$

Since the constant is chosen so that  $s(\epsilon_k; q) = \int_1^{\epsilon_k(q)} s'(\epsilon; q) d\epsilon + C(q) = 0$  (where  $\epsilon_k(q)$  is shorthand for  $\epsilon_k(0; q)$ )

$$C'(q) = - \int_1^{\epsilon_k} \frac{\partial s'(\epsilon; q)}{\partial q} d\epsilon - \frac{\partial \epsilon_k(q)}{\partial q} s'(\epsilon_k; q).$$

Combining with (4) yields,

$$\begin{aligned} \frac{\partial s(\epsilon; q)}{\partial q} &= - \int_\epsilon^{\epsilon_k} \frac{\partial s'(\epsilon; q)}{\partial q} d\epsilon - \frac{\partial \epsilon_k(q)}{\partial q} s'(\epsilon_k; q) \iff \\ \left(\frac{1}{2}\right) \frac{\partial s(\epsilon; q)}{\partial q} &= \int_\epsilon^{\epsilon_k} \frac{\partial}{\partial q} \left( \sum_i \frac{(|x_i|/\alpha_i) \cdot p_i(\epsilon)}{\epsilon} \right) d\epsilon + \frac{\partial \epsilon_k(q)}{\partial q} \cdot \left( \frac{|x_{-k}|}{\alpha_{-k}} \frac{p_{-k}(\epsilon_k)}{\epsilon_k} \right). \end{aligned}$$

It is straightforward to see that  $s(\epsilon; q)$  is strictly increasing in  $x_i$  and strictly decreasing in  $\alpha_i$ ; the functions  $\epsilon_i(p)$  satisfy the comparative statics (and hence  $\epsilon_k(0)$  does), the inverse functions  $p_i(\epsilon)$  inherit the same comparative statics in  $q$ , so  $(|x_i|/\alpha_i) \cdot p_i(\epsilon)$  in the integral also inherits comparative statics, as does the overall expression.

*Part 2.* A player's utility equals her utility from offering  $\bar{s}$  and the comparative statics follow from Part 1.

*Part 3.* From the main text, the decisionmaker's equilibrium expected utility is  $\int_{\epsilon_k}^1 s(\epsilon) \cdot \frac{\partial}{\partial \epsilon} (p_k(\epsilon) \cdot p_{-k}(\epsilon)) d\epsilon$ . Using integration by parts and observing that  $s'(\epsilon) = -2 \sum_i \frac{(|x_i|/\alpha_i) \cdot p_i(\epsilon)}{\epsilon}$  from the proof of Prop. 2, this equals  $\underbrace{s(\epsilon) p_k(\epsilon) p_{-k}(\epsilon)}_{=\bar{s}} \Big|_{\epsilon_k}^1 - \int_{\epsilon_k}^1 p_k(\epsilon) p_{-k}(\epsilon) s'(\epsilon) d\epsilon = - \int_1^{\epsilon_k} (1 - p_k(\epsilon) p_{-k}(\epsilon)) s'(\epsilon) d\epsilon$ , which reduces to the expression in the Proposition.

**Prop. 5** For  $|x_i| = x$  and  $\alpha_i = \alpha$ , Corollary 4 implies that policy extremism is uniform on  $[0, \frac{x}{\alpha}]$ , score CDFs are identical ( $F_i(s) = F_{-i}(s)$ ), and policies are symmetric ( $y_i(s) = -y_{-i}(s)$ ). Note that  $p_i(\epsilon_k) = 0 \forall i$ , so the equilibrium score function is

$$s^*(\epsilon) = 4x \left( \ln \left( \frac{\epsilon_k}{\epsilon} \right) - \frac{x}{\alpha} p_k(\epsilon) \right). \quad (5)$$

Every score  $s$  is associated with a unique level of engagement and a unique ideological extremism  $y$ . As  $\epsilon(s) = \epsilon_k(F(s)) = \epsilon_k\left(\frac{y(s)}{x/\alpha}\right)$ , the engagement associated with each  $y$  must be  $\epsilon(y) = \epsilon_k\left(\frac{y}{x/\alpha}\right)$ . So score as a function of ideological extremism  $s^*(y)$  is

$$s^*\left(\epsilon_k\left(\frac{y}{x/\alpha}\right)\right) = 4x \left( \ln \left( \frac{\epsilon_k}{\epsilon_k\left(\frac{y}{x/\alpha}\right)} \right) - \frac{x}{\alpha} p_k\left(\epsilon_k\left(\frac{y}{x/\alpha}\right)\right) \right) = 4x \left( x \ln \left( \frac{x}{x-y} \right) - y \right).$$

Note the score as a function of  $y$  does not depend on  $\alpha$ . The maximum score is

$$\bar{s} = s^*\left(\frac{x}{\alpha}\right) = 4x \left( x \ln \left( \frac{x}{x-x/\alpha} \right) - \frac{x}{\alpha} \right) = 4x^2 \left( \ln \left( \frac{\alpha}{\alpha-1} \right) - \frac{1}{\alpha} \right)$$

and expected utilities of the entrepreneurs are straightforward to derive.

The expected utility of the DM is

$$\int_0^{\bar{s}} \frac{\partial (F^2(s))}{\partial s} s \cdot ds \quad (6)$$

since  $F^2(s)$  is the CDF of the maximum score. We can derive the inverse function  $F^{-1}(p)$  by observing that  $F(s) = p_k(\epsilon^*(s)) \rightarrow F^{-1}(p) = s^*(\epsilon_k(p))$ . Substituting into (5) yields

$$F^{-1}(p) = 4x^2 \left( \ln \left( \frac{\alpha}{\alpha-p} \right) - \frac{p}{\alpha} \right).$$

Using this we perform a change of variables on (6) so the DM's expected utility is

$$\int_0^1 \frac{\partial}{\partial p} (p^2) F^{-1}(p) dp = 4x^2 \int_0^1 2p \left( \ln \left( \frac{\alpha}{\alpha - p} \right) - \frac{p}{\alpha} \right).$$

Integration by parts and algebra verifies that the definite integral equals the term inside the parentheses in the Proposition.

**Prop. 6** First order stochastic changes in ideology are obvious as the CDF of  $|y_i|$  is  $\frac{|y_i|}{x/\alpha}$ .

Since  $F(s) = \frac{y(s)}{x/\alpha}$  and  $y(s)$  is unaffected by  $\alpha$  (since  $s(y)$  is unaffected), first-order stochastic decreasing in  $\alpha$  is straightforward. To show  $F(s)$  is first-order stochastically increasing in  $x$ , note that  $F^{-1}(F(s; x); x) = s \rightarrow \frac{\partial F}{\partial x} = -\frac{\partial F^{-1}/\partial x}{\partial F^{-1}/\partial p}$ . Clearly  $F^{-1}(p)$  is increasing in  $p$  and from the proof of Prop. 5,  $F^{-1}(p)$  is increasing in  $x$ , hence  $\partial F/\partial x < 0$  and  $F$  is first-order stochastically increasing in  $x$ .

**Prop. 7** Writing utility as  $-x^2 f(\alpha)$ , where  $f(\alpha) = (\alpha - 1) \left( 4 \ln \left( \frac{\alpha}{\alpha - 1} \right) - \frac{3}{\alpha} \right)$ , we see that  $\lim_{\alpha \rightarrow 1} f(\alpha) = 0$  and  $\lim_{\alpha \rightarrow \infty} f(\alpha) = 1$ . Next, we show  $\exists \alpha^*$  s.t. a)  $f(\alpha)$  is strictly concave below  $\alpha^*$ , b)  $f(\alpha^*) > 1$ , and c)  $f'(\alpha) < 0$  for  $\alpha \geq \alpha^*$ . These properties imply that  $f(\alpha)$  has a unique maximum  $\hat{\alpha} \in (1, \alpha^*)$  and  $f(\hat{\alpha}) > 1$ . Finally, the preceding observations imply that  $f(\alpha) = 1$  at some  $\bar{\alpha} < \hat{\alpha}$ , and that  $f(\alpha) < 1$  for  $\alpha < \bar{\alpha}$  and  $> 1$  for  $\alpha > \bar{\alpha}$ .

Property a) can be shown by taking the second derivative  $f''(\alpha)$  and setting equal to 0; the solution is  $\alpha^* = 3$ . For property b) just evaluate. Property c) can be shown by rearranging the first derivative to be

$$\begin{aligned} & \frac{1}{\alpha^2} \left( 4\alpha^2 \log \left( \frac{\alpha}{\alpha - 1} \right) - (3 + 4\alpha) \right) = \frac{1}{\alpha^2} \left( \int_0^1 \frac{4\alpha^2}{\alpha - q} dq - 2 \int_0^1 (3 + 4\alpha) \right) \\ & = \frac{1}{\alpha^2} \left( \int_0^1 \frac{-4\alpha^2 - 6\alpha + q(6 + 8\alpha)}{\alpha - q} dq \right). \end{aligned}$$

The numerator is clearly  $< 0 \forall q \in [0, 1]$  when  $\alpha \geq \alpha^* = 3$ .

**Prop. 8** *Part 1.* From Prop. 2,  $j$  is more engaged at  $p$  i.f.f.  $\log(\epsilon_j(p)) - \log(\epsilon_{-j}(p)) = \int_p^1 \left( \frac{x_j}{\alpha_j - q} - \frac{x_{-j}}{\alpha_{-j} - q} \right) dq > 0$ , and by assumption we have equal engagement at  $p = 0$ . It is straightforward to show that  $\alpha_j < \alpha_{-j} \rightarrow \frac{\partial}{\partial q} \left( \frac{x_{-j}}{\alpha_{-j} - q} / \frac{x_j}{\alpha_j - q} \right) < 0$ . This implies

1.  $\frac{x_j}{\alpha_j - 1} > \frac{x_{-j}}{\alpha_{-j} - 1}$ . If not then  $\frac{x_{-j}}{\alpha_{-j} - p} > \frac{x_j}{\alpha_j - p} \forall p < 1$  and  $\epsilon_{-j}(0) > \epsilon_j(0)$ , a contradiction.
2.  $\frac{x_{-j}}{\alpha_{-j}} > \frac{x_j}{\alpha_j}$ . If not then  $\frac{x_j}{\alpha_j - p} > \frac{x_{-j}}{\alpha_{-j} - p} \forall p > 0 \rightarrow \epsilon_j(0) > \epsilon_{-j}(0)$ , a contradiction.

3. The log difference is single peaked, since the derivative is  $\frac{x_{-j}}{\alpha_{-j}-p} - \frac{x_j}{\alpha_j-p} > 0$  i.f.f.  $\frac{x_{-j}}{\alpha_{-j}-p} \Big/ \frac{x_j}{\alpha_j-p} = 1$  which happens at most once.

So the entrepreneurs can be equally engaged at most two probabilities, which are  $p = 0$  and  $p = 1$ . Entrepreneur  $j$  is more engaged in a neighborhood around 0 since the entrepreneurs are equally engaged at 0 and  $\frac{x_{-j}}{\alpha_{-j}} > \frac{x_j}{\alpha_j}$  (the derivative of the log difference is positive at 0); hence she is more engaged at all  $p \in (0, 1)$ .

*Part 2.* We wish to show  $\frac{x_{-j}}{\alpha_{-j}} F_j(s) \geq \frac{x_j}{\alpha_j} F_{-j}(s) \forall s \in [0, \bar{s}] \iff \frac{x_{-j}}{\alpha_{-j}} p_{-j}(\epsilon) \geq \frac{x_j}{\alpha_j} p_j(\epsilon) \forall \epsilon \in [1, \epsilon_j]$ . The l.h.s. is strictly greater at  $\epsilon = 1$  (as  $p_j(1) = p_{-j}(1) = 1$  and  $\frac{x_{-j}}{\alpha_{-j}} > \frac{x_j}{\alpha_j}$ ) and equal to the r.h.s. at  $\epsilon = 0$  (as  $\epsilon_j = \epsilon_{-j}$ ). Now we show that the derivative of the l.h.s. is  $<$  the derivative of the r.h.s.  $\forall \epsilon < \epsilon_j$ ; since both sides are decreasing in  $\epsilon$  and equal at  $\epsilon_j = \epsilon_{-j}$ , this proves the desired property. Because  $\epsilon \cdot \frac{\partial}{\partial \epsilon} \left( \frac{x_i}{\alpha_i} p_i(\epsilon) \right) = - \left( 1 - \frac{1}{\alpha_i} \right) \epsilon^{\frac{1}{|\alpha_i|}}$ , the ratio  $\frac{\frac{\partial}{\partial \epsilon} \left( \frac{x_{-j}}{\alpha_{-j}} p_{-j}(\epsilon) \right)}{\frac{\partial}{\partial \epsilon} \left( \frac{x_j}{\alpha_j} p_j(\epsilon) \right)} = \left( \left( 1 - \frac{1}{\alpha_{-j}} \right) / \left( 1 - \frac{1}{\alpha_j} \right) \right) \epsilon^{\frac{1}{|\alpha_{-j}|} - \frac{1}{|\alpha_j|}}$  is strictly decreasing in  $\epsilon$  (as  $|x_{-j}| > |x_j|$ ). The ratio is equal to 1 at  $\epsilon_j = \epsilon_{-j}$ ; hence it is  $> 1 \forall \epsilon < \epsilon_j$ , implying  $\frac{\partial}{\partial \epsilon} \left( \frac{x_{-j}}{\alpha_{-j}} p_{-j}(\epsilon) \right) < \frac{\partial}{\partial \epsilon} \left( \frac{x_j}{\alpha_j} p_j(\epsilon) \right) \forall \epsilon < \epsilon_j$ .

**Prop. 9** To show DM utility converges to zero, rewrite the max score from Prop. 4 as  $2 \left( (|x_k| + |x_{-k}|) \cdot \ln(\epsilon_k) - \frac{x_k^2}{\alpha_k} - x_{-k}^2 \left( 1 - \frac{1}{\alpha_{-k}} \right) \left( \epsilon_k^{\frac{1}{|\alpha_{-k}|}} - 1 \right) \right)$ , which converges to 0 as  $\alpha_k \rightarrow \infty$  or  $x_k \rightarrow 0$ , since  $\epsilon_k \rightarrow 1$ .

**Prop. 10** Suppose DM utility converges to zero. Then for any  $s > 0$ ,  $F_{-k}(s) \rightarrow 1$ . Consider  $\hat{s} = \frac{x_k^2}{2\alpha_k^2}$ . Since  $-k$  only develops policies on her side of 0,  $k$ 's utility from any policy  $-k$  develops at a score  $\leq \hat{s}$  is less than  $-x_k^2 + \hat{s}$ . Thus, rather than staying out (which  $k$  does with strictly positive probability) she could profitably deviate, developing  $(\frac{x_k}{\alpha_k}, \frac{x_k^2}{\alpha_k^2} + \hat{s})$ , improving her utility by at least

$$\begin{aligned} & F_{-k}(\hat{s}) \left[ \left[ - \left( x_k - \frac{x_k}{\alpha_k} \right)^2 + \left( \frac{x_k}{\alpha_k} \right)^2 + \hat{s} \right] - [-x_k^2 + \hat{s}] \right] - \alpha_k \left[ \left( \frac{x_k}{\alpha_k} \right)^2 + \hat{s} \right] \\ &= \frac{x_k^2}{\alpha_k} \left[ 2F_{-k}(\hat{s}) - \frac{3}{2} \right] > 0 \text{ as } F_{-k}(\hat{s}) \rightarrow 1. \end{aligned}$$

## 10 Supplement for Reviewers (For Online Publication)

**Lemma 1** Player  $i$ 's utility for developing policy  $(s, y_i)$  at any score  $s$  where her opponent does not have an atom (and also if a tie would be broken in her favor) is

$$\Pi_i(s, y_i; \sigma_{-i}) = -\alpha_i(s + y_i^2) + F_{-i}(s) \cdot V_i(s, y_i) + \int_{s(y_{-i}, q_{-i}) > s} U_i(y_{-i}, q_{-i}) d\sigma_{-i}.$$

Let  $G_i(y_i; s)$  denote  $i$ 's probability distribution over ideologies conditional on producing a score- $s$  policy, let  $w_i(y_i, y_{-i}; s)$  be the probability that  $i$ 's policy is selected when the players develop policies  $(s, y_i)$  and  $(s, y_{-i})$ , and let  $\bar{y}_t(s)$  be the expected ideological outcome conditional on a tie at score  $s$  (i.e.,  $\int \int (w_i(y_i, y_{-i}; s) y_i + (1 - w_i(y_i, y_{-i}; s)) y_{-i}) dG_i|s \cdot dG_{-i}|s$ ).

*Part 1.* Consider an equilibrium  $(\sigma, w(\mathbf{b}))$  where the first part of the statement fails, so that with strictly positive probability player  $i$  develops policies other than the reservation policy with score  $s(b_0) = 0$ . All such policies must have strictly positive quality. If  $\sigma_{-i}$  doesn't generate an atom at  $s = 0$ ,  $i$ 's utility for developing policy  $(0, y_i)$  is  $\Pi_i(b_0; \sigma_{-i}) - \alpha_i y_i^2$ , and she is strictly better off developing the reservation policy  $b_0$ . So suppose  $\sigma_{-i}$  also generates an atom at  $s = 0$ , of size  $p_{-i}$ . Then  $i$ 's utility for playing according her strategy conditional on generating score 0 is

$$-\alpha_i E[y_i^2] + p_{-i} V_i(0, \bar{y}_t(0)) + \int_{s(y_{-i}, q_{-i}) > s_i} U_i(y_{-i}, q_{-i}) d\sigma_{-i} = U_i^*.$$

But since she can also achieve utility arbitrarily close to  $V_i(0, 0) + \int_{s(y_{-i}, q_{-i}) > s_i} U_i(y_{-i}, q_{-i}) d\sigma_{-i}$  simply by developing the reservation policy  $b_0$  with  $\varepsilon$ -quality, it must be that

$$p_{-i} (V_i(0, \bar{y}_t) - V_i(0, 0)) = 2x_i \cdot \bar{y}_t(0) \geq \alpha_i E[y_i^2] > 0.$$

This cannot be true for both players since  $\text{sign}(x_i) \neq \text{sign}(x_{-i})$ , so we have a contradiction. Intuitively, playing the tie is costly for both players, both could achieve the reservation policy effectively for free instead, and the policy that results from a tie cannot be on average better for both players than the reservation policy due to linearity and opposing ideologies.

*Part 2.* Consider an equilibrium where the second part fails, so that each player  $i$ 's strategy generates an atom at some common  $s > s(b_0) = 0$  of size  $p_i$ . It is straightforward to verify (exploiting the linearity of  $V_i(s, y_i)$ ) that player  $i$ 's utility for playing according to her strategy conditional on generating score  $s$  can be written as both

$$\begin{aligned} & -\alpha_{-i} \text{Var}[y_i|s] + \lim_{s_i \rightarrow s^-} \{\Pi_i(s_i, E[y_i|s]; \sigma_{-i})\} + 2x_i \cdot p_{-i} (\bar{y}_t(s) - E[y_{-i}|s]) \quad \text{and} \\ & -\alpha_{-i} \text{Var}[y_i|s] + \lim_{s_i \rightarrow s^+} \{\Pi_i(s_i, E[y_i|s]; \sigma_{-i})\} + 2x_i \cdot p_{-i} (\bar{y}_t(s) - E[y_i|s]). \end{aligned}$$

Now  $\lim_{s_i \rightarrow s+} \{\Pi_i(s_i, E[y_i|s]; \sigma_{-i})\} \leq \lim_{s_i \rightarrow s+} \left\{ \max_{y_i} \{\Pi_i(s_i, y_i; \sigma_{-i})\} \right\} \leq U_i^*$ , and the same holds true for  $\lim_{s_i \rightarrow s-} \{\Pi_i(s_i, E[y_i]; \sigma_{-i})\}$ . Also,  $\lim_{s_i \rightarrow s+} \{\Pi_i(s_i, E[y_i]; \sigma_{-i})\} \neq \lim_{s_i \rightarrow s-} \{\Pi_i(s_i, E[y_i]; \sigma_{-i})\}$  because  $-i$  has an atom at  $s$ . So one of these terms must be strictly less than  $U_i^*$ . Since  $-\alpha_i \text{Var}[y_i|s] \leq 0$ , both of the third terms must then be weakly positive and at least one must be strictly positive - hence their sum must be strictly positive. Consequently,  $\forall i$

$$x_i p_{-i} \left( \bar{y}_t(s) - \left( \frac{E[y_i|s] + E[y_{-i}|s]}{2} \right) \right) > 0,$$

i.e., the expected ideological outcome conditional on a tie must be better for  $i$  than the midpoint between the expected ideologies of each player's strategy at  $s$ . But this cannot be true for both players since  $\text{sign}(x_i) \neq \text{sign}(x_{-i})$ , so we have a contradiction. ■

**Lemma 3** *Sufficiency.* We first show that ideological optimality and no ties imply that every policy delivers utility  $\leq \Pi_i^*(s, y_i^*(s); F)$  for some  $s$ , which implies that  $i$ 's utility for developing any policy is  $\leq \max_s \{\Pi_i^*(s, y_i^*(s); F)\}$ . We then show that the three conditions imply  $i$ 's utility for playing her strategy is equal to  $\max_s \{\Pi_i^*(s, y_i^*(s); F)\}$ , which means she has no profitable deviation.

*Subpart 1.* First, note that  $i$  can achieve utility equal to  $\Pi_i^*(s, y_i^*(s); F)$  with policy  $(s, y_i^*(s))$  for any  $s > 0$  where her opponent  $-i$  has no atom, and utility arbitrarily close to  $\Pi_i^*(s, y_i^*(s); F)$  for  $s \geq 0$  where her opponent does have an atom, using policy  $(s + \varepsilon, y_i^*(s + \varepsilon))$  for arbitrarily small  $\varepsilon$ .

Second,  $i$ 's exact utility for developing any policy  $(s, y_i)$  with  $s \geq 0$  is

$$\Pi_i^*(s, y_i; F) - p_{-i}(s) \cdot (1 - w_i(y_i, y_{-i}^*(s))) 2x_i(y_i - y_{-i}^*(s)), \quad (7)$$

where  $p_{-i}(s)$  denotes the size of  $-i$ 's atom at  $s$  and  $w_i(y_i, y_{-i}; s)$  is as previously defined. Note that we are applying the no-ties property in the case of  $s = 0$ ; no ties implies that  $p_{-i}(0) > 0 \rightarrow F_i(0) = 0 \rightarrow y_{-i}^*(0) = 0$ , which implies that whenever  $s = 0$  and  $p_{-i}(0) > 0$  and  $i$ 's policy is not selected, the reservation policy—which is equal to  $(0, y_{-i}^*(0))$ —is the outcome.

If  $p_{-i}(s) = 0$  (i.e.,  $-i$  has no atom at  $s$ ) or  $p_{-i}(s) > 0$  but  $w_i(y_i^*(s), y_{-i}^*(s)) = 1$  ( $i$  wins for sure in a tie between ideologically-optimal policies at  $s$ ), then  $i$  achieves utility  $\Pi_i^*(s, y_i^*(s); F) \geq \Pi_i^*(s, y_i; F)$  by developing  $(s, y_i^*(s))$  and the property holds. If instead  $p_{-i}(s) > 0$  ( $-i$  has an atom at  $s$ ) and  $w_i(y_i^*(s), y_{-i}^*(s)) < 1$  ( $i$  will not win in a tie for sure) then  $x_i y_i^*(s) > 0 \geq x_i y_{-i}^*(s)$  (winning at  $s$  is strictly beneficial)  $\rightarrow \Pi_i^*(s, y_i^*(s); F) >$



Eqn (7). And since  $i$  can achieve utility arbitrarily close to  $\Pi_i^*(s, y_i^*(s); F)$  by developing some  $(s + \varepsilon, y_i^*(s + \varepsilon))$ , the property again holds. Finally,  $i$ 's utility for developing a policy  $(s, y_i)$  with  $s < 0$  is  $-\alpha_i(s + y_i^2) + \Pi_i^*(0, 0; F)$  (again applying the no ties property), which is weakly worse than  $\Pi_i^*(0, 0; F)$ . Since  $\Pi_i^*(0, 0; F)$  is  $i$ 's exact utility from developing the reservation policy, the preceding arguments apply.

*Subpart 2.* Suppose a strategy profile satisfies no ties, ideological optimality, and score optimality. Then every  $s \in \text{supp}\{F_i\}$  satisfies  $\Pi_i^*(s, y_i^*(s); F) = \max_s \{\Pi_i^*(s, y_i^*(s); F)\}$  by score optimality, at all such  $s$  where  $-i$  has no atom  $i$ 's utility for developing policy  $(s, y_i^*(s))$  is in fact  $\Pi_i^*(s, y_i^*(s); F)$ , and by no ties the set of  $s \in \text{supp}\{F_i\}$  where  $-i$  has an atom is probability 0; thus  $i$ 's utility from playing her strategy is equal to  $\max_s \{\Pi_i^*(s, y_i^*(s); F)\}$ .

*Necessity.* Necessity of no ties is Lemma 1. We now argue that no ties and equilibrium imply ideological optimality. Suppose not, i.e., we have an equilibrium where no ties holds and ideological optimality fails. Because negative-score policies with positive quality are strictly dominated by developing the reservation policy, some player  $i$  must be placing strictly positive probability on policies  $(y_i, q_i)$  with scores  $s(y_i, q_i) \geq 0$  that satisfy  $y_i \neq y_i^*(s(y_i, q_i); F_{-i})$ . By no ties, at least one such policy  $(\hat{y}_i, \hat{s}_i)$  must deliver  $i$ 's equilibrium utility and not generate a tie with  $-i$ . But then Lemma 2 implies that developing  $(s(y_i, q_i), y_i^*(s(y_i, q_i); F_{-i}))$  would deliver strictly higher utility, a contradiction.

We now argue that no ties, ideological optimality, and equilibrium jointly imply the necessity of score optimality. First, when  $-i$ 's strategy satisfies ideological optimality then  $i$  can achieve utility arbitrarily close to  $\Pi_i^*(s, y_i^*(s); F)$  for any  $s$ , so equilibrium utility must be  $\geq \{\Pi_i^*(s, y_i^*(s); F)\}$ . Second, if  $-i$  has no atom at  $\hat{s}_i$ , then  $i$ 's utility for developing policies  $(s, y_i^*(s))$  in an  $\varepsilon$ -ball around  $\hat{s}_i$  approaches  $\Pi_i^*(\hat{s}_i, y_i^*(\hat{s}_i); F) < \max_{s_i} \{\Pi_i^*(s_i, y_i^*(s_i); F)\}$ , and since the probability is strictly positive for any  $\varepsilon$  we have a contradiction. Third, if  $-i$  has an atom at  $\hat{s}_i$ , then  $i$  cannot be developing policies with scores below  $\hat{s}_i$  within a sufficiently small neighborhood, her probability of developing policies in an  $\varepsilon$ -half ball  $[\hat{s}_i, \hat{s}_i + \varepsilon]$  must be strictly positive for any  $\varepsilon$ , her utility for doing so again approaches  $\Pi_i^*(\hat{s}_i, y_i^*(\hat{s}_i); F) < \max_{s_i} \{\Pi_i^*(s_i, y_i^*(s_i); F)\}$  by right-continuity of  $F_{-i}$ , and we again have a contradiction. All cases are covered, which completes the proof. ■

Figure 1: Setup of the Policy Contest

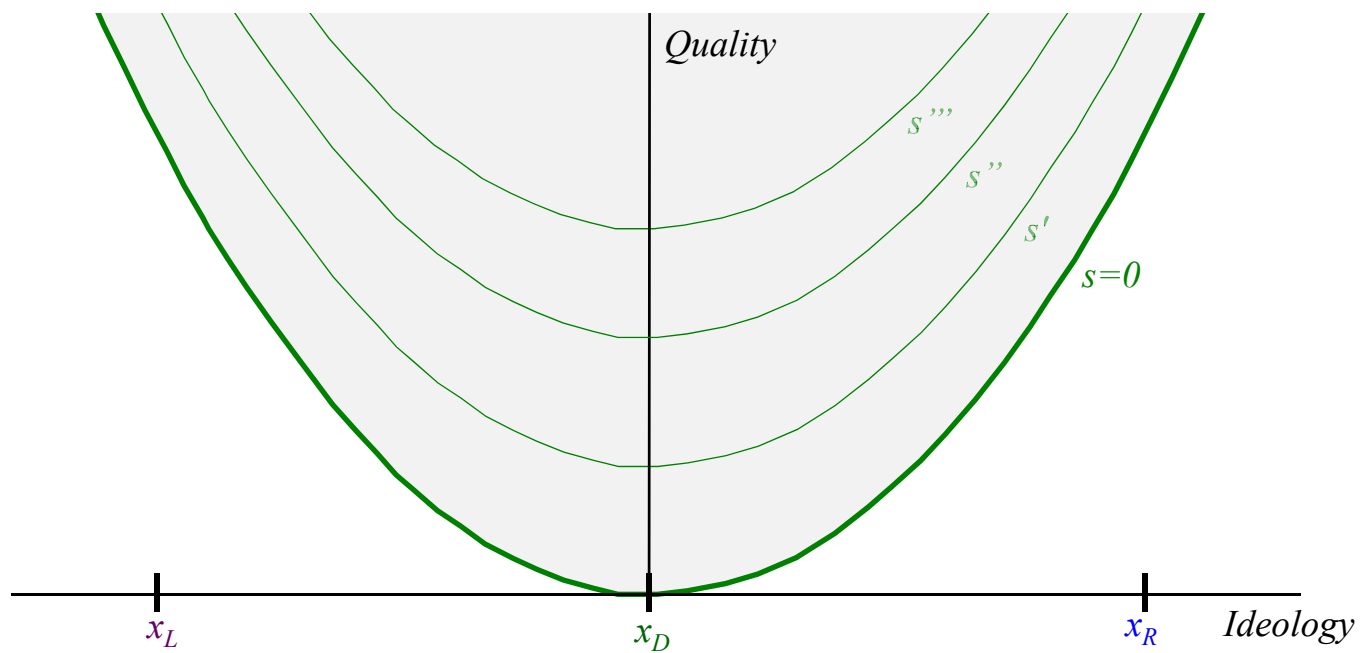


Figure 2: Equilibrium Score CDFs and Policies ( $x_L = -x_R$ ,  $\alpha_L > \alpha_R$ )

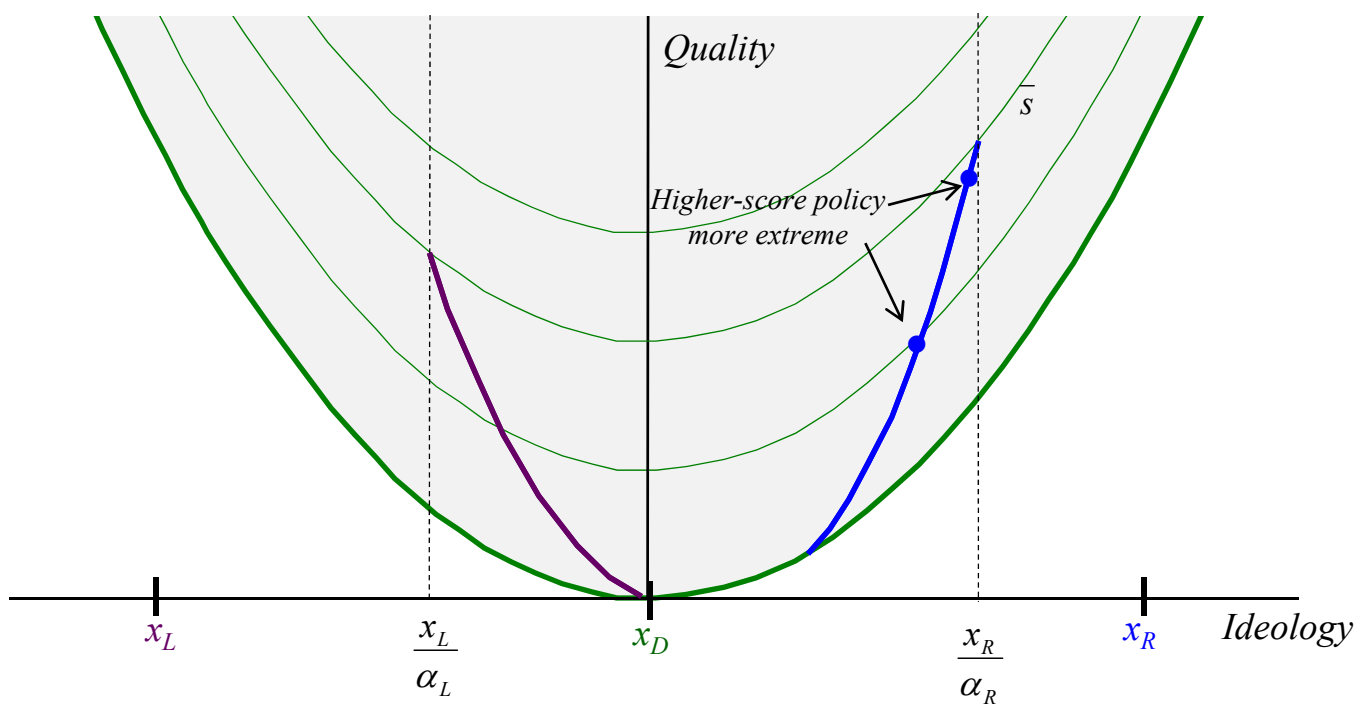
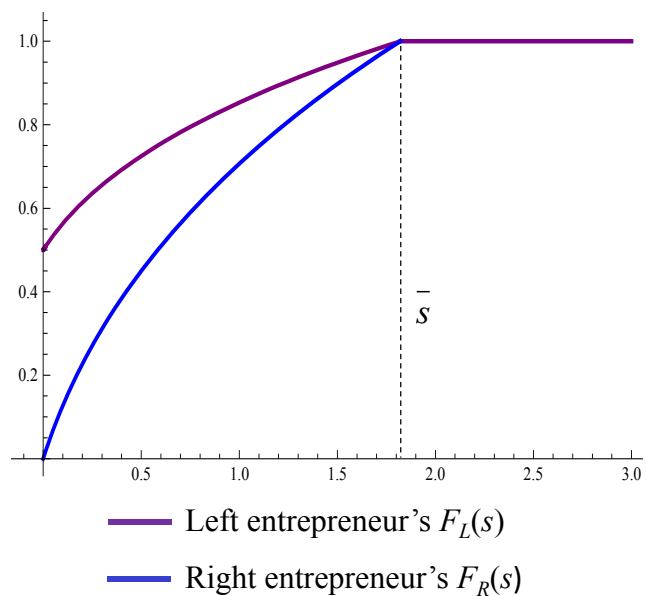


Figure 3: Effect of Right Entrepreneur's Parameters on Probability of Direct Competition ( $x_L = -1$ ,  $\alpha_L = 2$ )

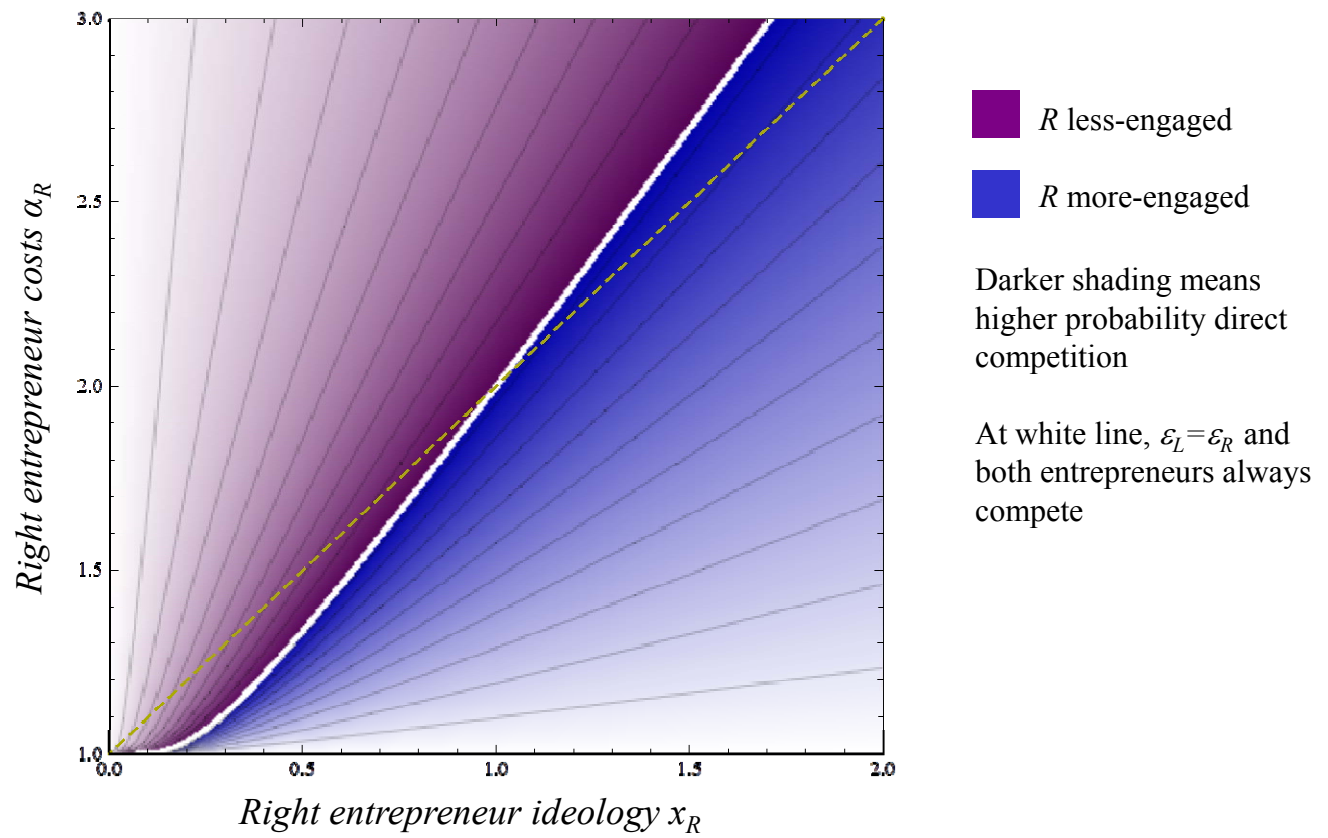


Figure 4: Symmetric Model Equilibrium

