Legislative Bargaining with Changing Political Power

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Abstract

This paper studies legislative negotiations between two parties whose political power changes over time. The model has a unique subgame perfect equilibrium, which becomes very tractable when parties can make offers frequently. This tractability facilitates studying how changes in political power affect implemented policies. An extension of the baseline model analyses how elections influence legislative negotiations when implemented policies affect future political power. Long periods of legislative gridlock may arise when the time until the election is short and parties have similar levels of political power.

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1 Introduction

This paper studies legislative negotiations between two parties whose political power changes over time. Fluctuations in political power are a common feature in democratic countries. For instance, Gallup's polls show that Barack Obama's approval rate was close to 70 percent when he was sworn in as President in January 2009. By July 2009, his approval rate had dropped to around 55 percent, and in January 2010 it was below 50 percent.[1] These fluctuations in the political climate often reflect changes in the public opinion and can have a large impact on the ability of political parties to carry out their legislative agendas. In this paper, I construct a model of legislative bargaining to analyze how time-varying political power affects legislative outcomes. I then use this model to study how the proximity of elections influences legislative policymaking when the policies that parties implement effect their political power.

The model features two political parties that have to bargain over which policy to implement. The parties' relative political power evolves continuously over time as a diffusion process. Parties can make offers at times on the grid $\{0, \Delta, 2\Delta, ...\}$, where $\Delta > 0$ measures the time between bargaining rounds. The parties' level of political power determines their relative bargaining position: the higher a party's political power, the more frequently that party will be making offers in the future. The assumption that the parties' bargaining position fluctuates together with their political power reflects a situation in which the preferences of the voters (i.e., the public opinion) in the different electoral districts is changing over time, and in which individual legislators adjust their choice of which party to support taking into account these changes in the voters' mood.

This legislative bargaining game has a unique subgame perfect equilibrium (SPE). Parties always reach an agreement at the beginning of the negotiations, and the agreement that parties reach depends on their relative political power. The game is difficult to analyze for a fixed time period, but I show that the unique SPE becomes very tractable in the limit as the time period goes to zero. The tractability of the limiting SPE, which is a consequence of the assumption that political power evolves as a diffusion process, allows me to analyze the effect that different features of the environment have on bargaining outcomes. For instance, I identify conditions under which a more volatile political climate benefits the party with less political power and leads to less extreme policies.

I extend this model to study legislative negotiations in the proximity of elections. As before, two political parties bargain over which policy to implement in an environment in which their relative political power is changing over time. The two new features of this

[1] See www.gallup.com.

extension are: (i) there is an upcoming election and the parties' level of political power at the election date determines their chances of winning the vote; and (ii) the policy that parties implement has an effect on the future evolution of their political power, therefore also affecting their chances of winning the election. The model is flexible, allowing for implemented policies to affect the parties' political power in arbitrary ways. This flexibility allows me to study the equilibrium dynamics under different assumptions of how policies affect political power.

The proximity of an election has a substantial effect on the outcomes of legislative negotiations. Unlike the baseline model, when there is an election upcoming the unique equilibrium may involve long periods of gridlock; i.e., delay. These delays occur in spite of the fact that implementing a policy immediately is always the efficient outcome. I show that these periods of legislative inaction can only arise when the time left until the election is short enough. On the contrary, parties are always able to reach a compromise if the election is sufficiently far away. Intuitively, parties cannot to uncouple the direct effect of implementing a policy from its indirect effect on the election's outcome. When the election is close enough, this may reduce the scope of trade to the point that there is no policy that both parties are willing to accept.

The dynamics of legislative negotiations when there is an election upcoming depend on the way in which the policies that parties implement affect their political power. I study different possible ways in which policies may affect political power. The first setting I consider is one in which the party with proposal power sacrifices its future political power when it implements a policy that is close to its ideal point. This trade-off between ideal policies and future political power arises when voters punish parties that implement extreme policies; i.e., policies that are far away from the median voter's ideal point. I show that there will necessarily be gridlock in this setting if parties derive a high enough value to winning the election. Moreover, gridlock is more likely to arise when political power is balanced, with both parties having similar chances of winning the vote.

I also study a setting in which parties bargain over pork-barrel spending, and in which the party that obtains more resources out of the negotiation is able to increase its political power. This setting reflects a situation in which parties are able to broaden their level of support among the electorate by discretionally allocating pork-barrel spending. I show that parties always reach an immediate agreement in this setting. Moreover, I show that an upcoming election leads to a more equal distribution of pork-barrel spending relative to the model without elections.[2]

[2]In the paper I also analyze a setting in which it is always costly in terms of political power for the responder to concede to proposals made by its opponent.

Introducing elections adds a new payoff-relevant state variable to the model: when there is an election upcoming parties care about both their political power and the time left until the election. This additional state variable introduces a new layer of complexity to the analysis, making it harder to obtain a clean characterization of the equilibrium outcome. I sidestep this difficulty by providing bounds to the parties' equilibrium payoffs. These bounds become tight as the election date becomes closer and are easy to compute numerically in the limit as the time period goes to zero. I use these bounds on the parties' payoffs to derive necessary conditions for gridlock to arise, and to analyze how the likelihood of gridlock depends on the time left until the election date and on the parties' level of political power.

1.1 Related literature

Starting with the seminal paper by Baron and Ferejohn (1989), there is a large body of literature that uses non-cooperative game theory to analyze legislative bargaining. Banks and Duggan (2000, 2006) generalize the model in Baron and Ferejohn by allowing legislators to bargain over a multidimensional policy space. A series of papers use these workhorse models to study the effect that different institutional arrangements have on legislative outcomes.[3] The current paper adds to this strand of literature by introducing a model of legislative negotiations in which the parties' political power changes over time, a feature that was previously ignored. I model time-varying political power as a diffusion process. This assumption leads to a tractable characterization of the limiting SPE with frequent offers, allowing for many comparative statics exercises. I use this model to study how the proximity of an election affects legislative outcomes. This extension highlights the importance of electoral considerations in understanding the dynamics of legislative policymaking, and gives new insights as to when gridlock is more likely to arise.[4]

There are other papers that study settings in which policies affect future political power and electoral outcomes. Besley and Coate (1998) study a two period model in which the policy implemented today may change the identity of the policymaker in the future. They show that this link between policies and future power may lead to inefficient policies in the

[3] Winter (1996) and McCarty (2000) analyze models *a la* Baron-Ferejohn with the presence of veto players. Baron (1998) and Diermeier and Feddersen (1998) study legislatures with vote of confidence procedures. Diermeier and Myerson (1999), Ansolabehere *et al.* (2003) and Kalandrakis (2004) study legislative bargaining under bicameralism. Snyder *et al.* (2005) analyze the effects of weighted voting within the Baron-Ferejohn framework. Cardona and Ponsati (2011) analyze the effects of supermajority rules in legislative bargaining within the model of Banks and Duggan.

[4] Diermeier and Vlaicu (2010) construct a legislative bargaining model to study the differences between parliamentarism and presidentialism in terms of their legislative success rate.

present. Bai and Lagunoff (2011) construct an infinite horizon model which also features a link between current policies and future political power. They focus on settings in which the current ruler faces a trade-off between implementing its preferred policy and sacrificing future political power, and characterize the equilibrium dynamics that such a trade-off gives rise to.[5] The current paper adds to this strand of literature by showing that the link between implemented policies and future political power can have a significant effect on legislative outcomes when there is an election upcoming.

This paper shares some features with Dixit, Grossman and Gul (2000), who study a model in which two political parties interact repeatedly and in which the parties' political power evolves over time according to a Markov chain. At each period, the party with more political power can unilaterally decide how to allocate a unit surplus. The authors characterize efficient divisions of the surplus that are self-enforcing over time. The current paper also analyzes a setting with time-varying political power. However, in contrast to Dixit, Grossman and Gul, this paper studies a canonical bargaining model in which parties negotiate over a single policy.

This paper also relates to Simsek and Yildiz (2009), who study a bilateral bargaining game in which the bargaining power of the players evolves stochastically over time. Simsek and Yildiz focus on settings in which players have optimistic beliefs about their future bargaining power. They show that optimism can give rise to costly delays if players expect bargaining power to become more "durable" at a future date. In contrast, there are no differences in beliefs in my model, and bargaining delays can arise when there is an election upcoming and when the policies that parties implement prior to the election influence their chances of winning the vote.

More broadly, this paper relates to the literature on delay and inefficiency in bargaining. Delays in bargaining can arise when players have private information (Kennan and Wilson, 1993), when players bargain over a stochastic surplus (Merlo and Wilson, 1995, 1998), or when players can build a reputation for being irrational (Abreu and Gul, 2000). Inefficiencies may also arise when players hold optimistic beliefs about their bargaining power and update their beliefs as time goes by (Yildiz, 2004), or when outside options are history dependent (Compte and Jehiel, 2004). The current paper provides a new rationale for bargaining inefficiencies by showing that delays may arise when parties cannot uncouple the direct effect of an agreement from its indirect effect on electoral outcomes.

[5]Other papers in this literature are Milesi-Ferretti and Spolaore (1994), Bourguignon and Verdier (2000) and Hassler *et al* (2003).

2 Baseline model

This section introduces the baseline model of legislative bargaining with time-varying political power. Section 2.2 presents the framework. Section 2.3 proves existence and uniqueness of a SPE, characterizes the parties' limiting SPE payoffs as the time period goes to zero, and uses these limiting SPE payoffs to derive some comparative statics results. Section 3 uses this baseline model to study legislative negotiations in the proximity of elections.

2.1 Framework

Let $[0,1]$ be the set of alternatives or policies. Two political parties, $i = 1, 2$, bargain over which policy in $[0,1]$ to implement. The set of times is a continuum $T = [0, \infty)$. However, parties can only make offers at points on the grid $T(\Delta) = \{0, \Delta, 2\Delta, ...\}$, where $\Delta > 0$ measures the time between bargaining rounds. Parties are expected utility maximizers and have a common discount factor $e^{-r\Delta}$ across periods, with $r > 0$. Let $z_i \in [0,1]$ denote party i's ideal policy and assume that $z_1 \neq z_2$. Party i's utility from implementing policy $z \in [0,1]$ is $u_i(z) = 1 - |z - z_i|$. Throughout the paper I maintain the assumption that the parties' ideal points lie at the extremes of the policy space, with $z_1 = 1$ and $z_2 = 0$.[6] Note that this implies that $u_1(z) = z$ and $u_2(z) = 1 - z$ for all $z \in [0,1]$. This model is therefore equivalent to a setting in which parties 1 and 2 are bargaining over how to divide a surplus of size 1.

Unlike models of legislative bargaining *a la* Baron and Ferejohn (1989) and Banks and Duggan (2000, 2006), in this paper I assume that bargaining takes place at the party level. This assumption reflects situations in which the leaders of each party in Congress bargain over an issue on behalf of their respective parties. In this setting, the ideal policies z_1 and z_2 should be interpreted as the ideal policies of each of the parties; for instance, the policies on the parties' platforms.

The model's key variable is an exogenous and publicly observable stochastic process x_t, which measures the parties' relative political power and which determines the bargaining protocol. Let $B = \{B_t, \mathcal{F}_t : 0 \leq t < \infty\}$ be a one-dimensional Brownian motion on the probability space $(\Omega, \mathcal{F}, \mathbf{P})$, where $\{\mathcal{F}_t : 0 \leq t < \infty\}$ is the filtration generated by the Brownian motion. The Brownian motion B drives the process x_t. In particular, I assume that x_t evolves as a Brownian motion with constant drift μ and constant volatility $\sigma > 0$,

[6] The assumption that the parties' ideal policies are at the extremes of the policy space is without loss of generality: if the policy space was $[a,b]$ with $a < z_2 = 0$ and $b > z_1 = 1$, all the alternatives in $[a,0) \cup (1,b]$ would be strictly Pareto dominated by policies in $[0,1]$. Adding these Pareto dominated policies would not change the equilibrium outcome.

with reflecting boundaries at 0 and 1. That is, while $x_t \in (0,1)$ this variable evolves as

$$dx_t = \mu dt + \sigma dB_t. \tag{1}$$

When x_t reaches either 0 or 1, it reflects back. The reflecting boundaries guarantee that $x_t \in [0,1]$ at all times t.[7] Note that the process x_t evolves in continuous time, but parties can only make offers at times $t \in T(\Delta)$. This implies that the speed at which the process x_t evolves remains constant as I vary the time between bargaining rounds Δ. Moreover, this also implies that the process x_t becomes more persistent across bargaining rounds as the time between Δ becomes smaller: for smaller values of Δ the distribution of $x_{t+\Delta}$ conditional $x_t = x$ is more concentrated around x than for larger values of Δ. The assumption that parties can make offers on the grid $T(\Delta)$ makes this a game in discrete time, allowing me to use subgame perfection as a solution concept.

The value of x represents the relative political power of the parties, or their level of support among the electorate. Party 1's political power is increasing in x and party 2's political power is decreasing in x. The parties' relative political power determines the bargaining protocol. In particular, at each bargaining round $t \in T(\Delta)$ the party with more political power has proposal power: party 1 has proposal power if $x_t \geq 1/2$ and party 2 has proposal power if $x_t < 1/2$. The party with proposal power can either make an offer $z \in [0,1]$ to its opponent or pass. If the other party (i.e., the responder) rejects the offer or if the proposer chooses to pass, then play moves to round $t + \Delta$. Otherwise, if at time $t \in T(\Delta)$ the responder accepts its opponent's proposal to implement policy $z \in [0,1]$, party $i = 1,2$ obtains at this date a payoff of $u_i(z)$ and the game ends.[8] The assumption that only the party with more political power has proposal power is for simplicity. In Section 4.3 I show how the model can be extended to allow for more general bargaining protocols.

This bargaining protocol implies that a party's bargaining power is increasing in its political power: party 1's bargaining power increases with x, since a larger x means that party 1 will (on average) be making offers more frequently in the future. Similarly, party 2's political power decreases with x. The assumption that the parties' political power influences their bargaining position reflects a situation in which the preferences of the voters (or the public mood) in the different electoral districts is changing over time, and in which individual legislators adjust their choice of which party to support taking into account these changes in the

[7]See Harrison (1985) for a detailed description of diffusion processes with reflecting boundaries.

[8]This model is a variation of the bilateral bargaining model with time-varying bargaining power in Ortner (2011).

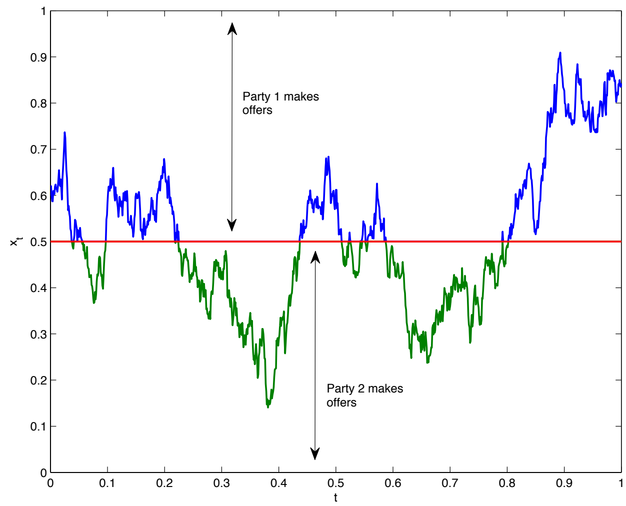

Figure 1: Sample path of x_t.

voters' mood.[9] In such an environment, the party with higher relative political power would have the support of more legislators, and so it would have a stronger bargaining position. With this interpretation in mind, I will sometimes refer to the party with more political power as the *majority party* and to its opponent as the *minority party*.

To illustrate the sequencing of moves in the game, suppose that $x_0 \in [1/2, 1]$. In this case, party 1 has proposal power from $t = 0$ until the first time x_t goes below $1/2$; i.e., until $\tau_1(\Delta) = \inf\{t \in T(\Delta) : x_t < 1/2\}$. At each period $t \in T(\Delta)$ until $\tau_1(\Delta)$ party 1 can either make an offer $z \in [0, 1]$ or pass. If party 2 accepts an offer before $\tau_1(\Delta)$, the bargaining ends and parties collect their payoffs. Otherwise, party 2 becomes proposer between $\tau_1(\Delta)$ and time $\tau_2(\Delta) = \inf\{t \in T(\Delta), t > \tau_1(\Delta) : x_t \geq 1/2\}$. Bargaining continues this way, with parties alternating in their right to make proposals according to the realization of the process x_t, until a party accepts an offer. See Figure 1 for a plot of a sample path of x_t.

Let Γ_Δ denote the legislative bargaining game with time period Δ. I look for the subgame perfect equilibria (SPE) of this game.

Remark 1 Throughout the paper I assume that the parties' political power x_t evolves in continuous time, but that parties can only make offers at times in the grid $T(\Delta) = \{0, \Delta, 2\Delta, ...\}$.

[9] There is empirical evidence showing that legislators respond to the preferences of their constituencies (i.e., Gerber and Lewis, 2004), and that constituents punish legislators that ignore their preferences (i.e., Canes-Wrone *et al*, 2002).

An alternative formulation of this model is to assume that relative political power moves discretely over time according to a transition density $F(x_{t+\Delta}|x_t)$, maintaining the assumption that parties can only make offers at times $t \in T(\Delta)$. For this alternative model to be equivalent to the model above, the transition density $F(x_{t+\Delta}|x_t)$ must be equal to the transition density function of the process that evolves as (1) with reflecting boundaries at 0 and 1. Under this alternative specification the transition density $F(x_{t+\Delta}|x_t)$ would depend on the time period Δ: for small values of Δ the distribution over next period's political power $x_{t+\Delta}$ would be more concentrated around x_t than for larger values of Δ.

Remark 2 Merlo and Wilson (1995) study bargaining games in which the realization of an exogenous stochastic process determines at each period both the size of the surplus over which players are bargaining and the identity of the proposer. The model in this section belongs to the family of games that Merlo and Wilson study: in my model the size of the surplus is constant, and the stochastic process only determines the identity of the proposer at each bargaining round.

2.2 Equilibrium

Let $M_1 = [1/2, 1]$ be the set of states at which party 1 has proposal power and let $M_2 = [0, 1/2)$ be the set of states at which party 2 has proposal power. The following result shows that Γ_Δ has a unique SPE, and characterizes the parties' SPE payoffs. The proof of this and all other results are in the appendix.[10]

Theorem 1 *For any $\Delta > 0$, Γ_Δ has a unique SPE. Parties reach an immediate agreement in the unique SPE. For $i = 1, 2$, let $V_i^\Delta(x)$ denote party i's SPE payoff when relative political power is $x \in [0, 1]$. These payoffs satisfy:*

$$V_i^\Delta(x) = \begin{cases} e^{-r\Delta} E\left[V_i^\Delta(x_{t+\Delta}) \middle| x_t = x \right] & \text{if } x \notin M_i, \\ 1 - e^{-r\Delta} E\left[V_j^\Delta(x_{t+\Delta}) \middle| x_t = x \right] & \text{if } x \in M_i. \end{cases}$$

The content of Theorem 1 can be described as follows. In a SPE, for all $x \in [0, 1]$ the minority party i only accepts offers giving that party a utility equal to its continuation payoff of waiting until the next bargaining round; i.e., a utility equal to $e^{-r\Delta} E\left[V_i^\Delta(x_{t+\Delta}) \middle| x_t = x \right]$. Knowing this, the majority party always makes the lowest offer that its opponent is willing to accept and the game ends with an immediate agreement.

[10] The proof of Theorem 1 adapts arguments in Merlo and Wilson (1995) to the current setting.

By Theorem 1, for all $x \in M_i$

$$V_i^\Delta(x) = 1 - e^{-r\Delta} E\left[V_j^\Delta(x_{t+\Delta})|x_t = x\right] = 1 - e^{-r\Delta} + e^{-r\Delta} E\left[V_i^\Delta(x_{t+\Delta})|x_t = x\right], \quad (2)$$

where the second equality follows since $V_i^\Delta(y) + V_j^\Delta(y) = 1$ for all $y \in [0,1]$. Combining equation (2) with Theorem 1, it follows that

$$V_i^\Delta(x) = (1 - e^{-r\Delta})\mathbf{1}_{\{x \in M_i\}} + e^{-r\Delta} E\left[V_i^\Delta(x_{t+\Delta})|x_t = x\right], \quad (3)$$

where $\mathbf{1}_{\{.\}}$ is the indicator function. Equation (3) shows that party i's payoff when i has proposal power is equal to $1 - e^{-r\Delta}$ plus its expected continuation value. On the other hand, party i's payoff when i is the responder is only equal to its expected continuation value. The term $1 - e^{-r\Delta}$ represents the rent that a party obtains when it makes offers.

Theorem 1 shows existence and uniqueness of a SPE. However, the unique SPE is difficult to analyze for a fixed time period $\Delta > 0$. To obtain a better understanding of the model, the next result characterizes the parties' limiting SPE payoffs as $\Delta \to 0$. These limiting payoffs are very easy to compute, and provide a good approximation of the SPE payoffs for settings in which the time between bargaining rounds is short.

Theorem 2 *There exists functions $V_1^*(\cdot)$ and $V_2^*(\cdot)$ such that $V_i^\Delta(\cdot)$ converges uniformly to $V_i^*(\cdot)$ as $\Delta \to 0$. Moreover, $V_i^*(\cdot)$ solves*

$$rV_i^*(x) = \begin{cases} \mu(V_i^*)'(x) + \frac{1}{2}\sigma^2(V_i^*)''(x) & \text{if } x \notin M_i, \\ r + \mu(V_i^*)'(x) + \frac{1}{2}\sigma^2(V_i^*)''(x) & \text{if } x \in M_i, \end{cases} \quad (4)$$

with boundary conditions $(V_i^*)'(0) = (V_i^*)'(1) = 0$, $V_i^*(1/2^-) = V_i^*(1/2^+)$ and $(V_i^*)'(1/2^-) = (V_i^*)'(1/2^+)$.

Theorem 2 shows that a party's limiting payoffs as $\Delta \to 0$ is the solution to the ordinary differential equation (4) with appropriate boundary conditions. The left-hand side of (4) is party i's limiting payoff measured in flow terms, while the right-hand side of (4) shows the sources of party i's limiting flow payoff. Party i's flow payoff when it has proposal power is equal to the rent it extracts from being proposer, which in the limit as $\Delta \to 0$ is equal to r, plus the expected change in its continuation value coming from changes in x, which is equal to $\mu(V_i^*)'(x) + \frac{1}{2}\sigma^2(V_i^*)''(x)$. On the other hand, party i's flow payoff when it does not have proposal power is given only by the expected change in its continuation value.

The parties' limiting SPE payoffs satisfy four boundary conditions. The boundary conditions $(V_i^*)'(0) = (V_i^*)'(1) = 0$ are a consequence of the nature of the process x_t: since x_t has reflecting boundaries at 0 and 1, party i's payoff becomes "flat" as a function of political power as x approaches either 0 or 1. The boundary condition $V_i^*(1/2^-) = V_i^*(1/2^+)$ guarantees that party i's payoff is continuous on $[0,1]$. Finally, the condition that $(V_i^*)'(1/2^-) = (V_i^*)'(1/2^+)$ guarantees that party i's payoff is differentiable on $[0,1]$.[11]

The solution to the ordinary differential equation in (4) is given by

$$V_i^*(x) = \begin{cases} a_i e^{-\alpha x} + b_i e^{\beta x} & \text{if } x \notin M_i, \\ 1 + c_i e^{-\alpha x} + d_i e^{\beta x} & \text{if } x \in M_i, \end{cases}$$

where $\alpha = (\mu + \sqrt{\mu^2 + 2r\sigma^2})/\sigma^2$, $\beta = (-\mu + \sqrt{\mu^2 + 2r\sigma^2})/\sigma^2$, and where (a_i, b_i, c_i, d_i) are constants determined by the four boundary conditions. Since parties always reach an immediate agreement, the limiting SPE payoffs in Theorem 2 also characterize the policies that parties implement as a function of relative political power. In particular, party 1's payoff is equal to the implemented policy, since $u_1(z) = z$ for all $z \in [0,1]$. Equation (A.4) in Appendix A.2 presents the full expressions for $V_1^*(\cdot)$ and $V_2^*(\cdot)$.

Definition 1 *The political climate is advantageous for party 1 (for party 2) if $\mu \geq 0$ (if $\mu \leq 0$).*

I now present comparative statics results about how the limiting SPE payoffs vary with changes in the volatility and drift of the process x_t. The first result considers how the parties' payoffs (and the implemented policies) depend on the volatility of x_t. Recall that, for $i = 1, 2$, M_i is the set of values of x at which party i is the majority party; i.e., $M_1 = [1/2, 1]$ and $M_2 = [0, 1/2)$.

Proposition 1 *Suppose the political climate is advantageous for party j. Then, the payoff of party $i \neq j$ is increasing in σ for all $x \in M_j$.*

By Proposition 1, a more volatile political power makes the minority party strictly better off when the political climate is advantageous for the majority party. The intuition for this

[11] The intuition as to why this last condition must hold is as follows. Since parties always reach an immediate agreement, $V_1^*(x) + V_2^*(x) = 1$ for all x. If the limiting payoff function of one party was not differentiable at $x = 1/2$, then the limiting payoff function of one of the two parties would have a convex kink at $x = 1/2$. The proof of Theorem 2 shows that this can never be the case: if V_i^* had a convex kink at $1/2$, then for values of Δ small enough party i would have a strict incentive to delay an agreement when x_t is in an interval around $1/2$, contradicting the fact that parties always reach an immediate agreement for any $\Delta > 0$ (Theorem 1).

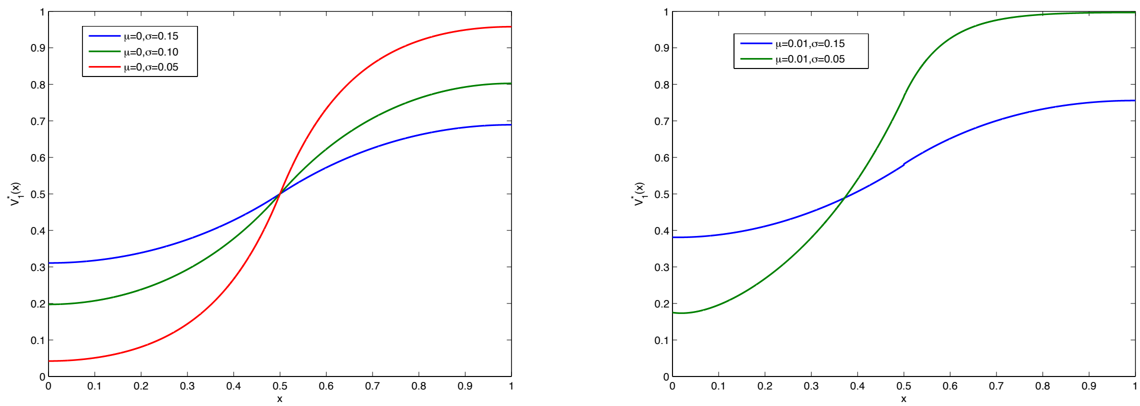

Figure 2: Party 1's payoff. Parameters: $r = 0.05$.

result is as follows. If the political climate is advantageous to party $j \neq i$, an increase in volatility increases the chances that party i will recover political when $x \in M_j$; i.e., when i is the minority party. This improves party i's bargaining position, allowing it to obtain a better deal in the negotiations. When $\mu = 0$ the political climate is advantageous for both parties, so in this case an increase in σ benefits party 1 when $x \in [0, 1/2)$ and benefits party 2 when $x \in (1/2, 1]$.

When $\mu \neq 0$ an increase in volatility may decrease the minority party's payoff if the political climate is advantageous for the minority party. To see the intuition behind this, suppose $\mu > 0$ so that the political climate is advantageous for party 1. If volatility is low, party 1 will expect to regain political power soon when x is slightly below 1/2. In this case, an increase in volatility makes it more likely that party 2 will maintain the right to make offers for longer when x is slightly below 1/2. This improves party 2's bargaining position, and lowers party 1's payoff. Figure 2 illustrates the results in Proposition 1 by plotting party 1's payoff for different values of σ. The left panel considers a case with $\mu = 0$, while the right panel considers a case with $\mu > 0$.

The next result considers how the parties' payoffs change with changes in the drift of x_t.

Proposition 2 *Party 1's payoff is strictly increasing in μ for all $x \in [0, 1]$, and party 2's payoff is strictly decreasing in μ for all $x \in [0, 1]$.*

Proposition 2 shows that party 1's payoff is increasing in μ, and that party 2's payoffs is decreasing in μ. The intuition behind this result is straightforward: a higher μ implies that party 1 will (on average) be making offers more frequently in the future. Thus, party

1's bargaining position improves when μ increases, allowing it to implement a policy that is closer to its most preferred alternative.

3 Elections and legislative gridlock

This section extends the model of Section 2 to study legislative negotiations in the proximity of elections. Section 3.1 presents the extended model. Section 3.2 proves existence and uniqueness of equilibrium. Section 3.3 provides bounds on the parties' equilibrium payoffs and shows how these bounds can be used to analyze how the likelihood of gridlock depends on the parties' level of political power and the time left until the election. Finally, Section 3.4 studies three different applications of the model and derives additional results for those settings.

3.1 A model with elections

I now describe the extension of the model with elections. As in the model of Section 2, parties 1 and 2 bargain over a single issue and must decide which policy in $[0, 1]$ to implement. The set of times is a continuum $T = [0, \infty)$, and parties can only reach an agreement at times $t \in T(\Delta) = \{0, \Delta, 2\Delta, ...\}$. Both parties are expected utility maximizers and share a common discount factor $e^{-r\Delta}$, with $r > 0$. The parties' utility indices over policies $z \in [0, 1]$ are $u_1(z) = z$ and $u_2(z) = 1 - z$.

There are two new features in this extended model. First, there will be an election at a future date $t^* > 0$, with $t^* \in T(\Delta)$. The outcome of this election depends on the parties' level of political power at the election date. In particular, the party with more political power at t^* wins the election: party 1 wins if $x_{t^*} \geq 1/2$, while party 2 wins if $x_{t^*} < 1/2$. The party that wins the election earns at time t^* a payoff equal to $K > 0$. The value of K measures the benefit that parties derive from being in office. For simplicity, I focus on the case in which there is a single election at time t^*. In Section 4.2 I discuss how the results generalize to settings with multiple elections.

The second new feature of this extended model is that the policy that parties implement affects the future evolution of political power. From time $t = 0$ until the time at which parties reach an agreement, relative political power x_t evolves as a Brownian motion with drift μ and volatility $\sigma > 0$ and with reflecting boundaries at 0 and 1. If parties reach an agreement to implement policy $z \in [0, 1]$ at time $t \in T(\Delta)$, this agreement affects the evolution of political power from time t onwards. In particular, I assume that there exists a function

$h : [0,1] \times [0,1] \to \mathbb{R}$, with $x + h(x,z) \in [0,1]$ for all $(x,z) \in [0,1] \times [0,1]$, such that relative political power jumps at time t by $h(x_t, z)$ if at this date parties implement policy z. That is, $x_{t^+} = \lim_{s \downarrow t} x_s = x_t + h(x_t, z)$ if parties implement policy z at time t. Then, from time t^+ onwards relative political power continues to evolve as a Brownian motion with drift μ and volatility σ and with reflecting boundaries at 0 and 1.[12] The function h captures in reduced form the effect that policies have an political power. The assumption that $x + h(x,z) \in [0,1]$ for all x, z guarantees that the parties' relative political power always remains bounded in $[0,1]$. For technical reasons I assume that $h(x, \cdot)$ is continuous for all $x \in [0,1]$.[13]

The bargaining protocol is the same as in the model of Section 2: at each time $t \in T(\Delta)$, party 1 has proposal power if $x_t \geq 1/2$ and party 2 has proposal power if $x_t < 1/2$. The party with proposal power can either make an offer to its opponent or pass. If the responder rejects the offer or if the party with proposal power chooses to pass, then play moves to round $t + \Delta$. Otherwise, if at time t the responder accepts its opponent's proposal to implement policy z, each party $i = 1, 2$ obtains at this date a payoff of $u_i(z)$.

The election is decided at date t^*, with its outcome depending on the value of x_{t^*}. The party that wins the election obtains at time t^* a payoff of K, and the other party obtains a payoff of 0. If parties had reached an agreement before time t^*, then the games ends immediately after the election. Otherwise, if parties have not reached an agreement by time t^*, the party with more political power can either make a proposal immediately after the election (i.e., still at date t^*) or can pass. Bargaining then continues, with parties alternating in their right to make offers according to the realization of the process x_t, until parties reach an agreement.

This specification of the model allows for general ways in which policies can affect the parties' political power: not only do different policies may have a different effect on the level of political power (i.e., for a fixed x, $h(x,z)$ may vary with z); also, the same policy may have a different effect on political power depending on the current level of x (i.e., for a fixed z, $h(x,z)$ may vary with x). Section 3.2 shows existence and uniqueness of equilibrium for this general model and Section 3.3 derives bounds on the parties' equilibrium payoffs. Section 3.4 considers different functional forms of h and derives properties of the equilibrium dynamics under these different settings.

Let $\Gamma_\Delta(t^*)$ denote the game with time period $\Delta > 0$ and election date $t^* > 0$. I look for the

[12] This specification implies that implemented policies only have an instantaneous effect on the parties' relative political power. In Section 4.2 I discuss how the results in this paper would generalize if implemented policies affected the parties' political power in alternative ways.

[13] Continuity of $h(x, \cdot)$ guarantees the existence of an optimal offer for the proposer.

SPE of this game. To guarantee uniqueness, I focus on SPE in which the party responding to offers always accepts proposals that leave that party indifferent between accepting and rejecting, and in which the party with proposal power always makes an acceptable offer to its opponent whenever its indifferent between making the acceptable offer that maximizes its payoff or passing. From now on, I use the word equilibrium to refer to an SPE that satisfies this property.

3.2 Equilibrium

For any measurable function $f : \mathbb{R} \to \mathbb{R}$ and any $s > t \geq 0$, let $E[f(x_s)|x_t = x]$ denote the expectation of $f(x_s)$ conditional on $x_t = x$ assuming that parties don't reach an agreement between times t and s; i.e., assuming that between t and s relative political power evolves as a Brownian motion with drift μ and volatility σ and with reflecting boundaries at 0 and 1.

For all $x \in [0,1]$ and all $t < t^*$, let $Q_i(x,t) := E[\mathbf{1}_{\{x_{t^*} \in M_i\}}|x_t = x]$ be the probability with which at time t party i is expected to win the election conditional on $x_t = x$ if parties don't reach an agreement between t and t^*. Note that these probabilities depend on the value of x: for all $t < t^*$, $Q_1(x,t)$ is increasing in x and $Q_2(x,t) = 1 - Q_1(x,t)$ is decreasing in x. If parties reach an agreement to implement policy z at time $t < t^*$, the probability that party i wins the election is $Q_i^z(x,t) := Q_i(x + h(x,z), t)$.

For $i = 1, 2$ and for any $t < t^*$, let $U_i(z,x,t) := u_i(z) + e^{-r(t^*-t)} K Q_i^z(x,t)$ be the payoff that party i would obtain if parties reached an agreement to implement policy $z \in [0,1]$ at time $t < t^*$ with $x_t = x$: if parties implement policy z at time $t < t^*$, party i earns a payoff $u_i(z)$ and it wins the election at time t^* with probability $Q_i^z(x,t)$. The following result establishes that this game has unique equilibrium payoffs.

Theorem 3 *For any $\Delta > 0$, $\Gamma_\Delta(t^*)$ has unique equilibrium payoffs. For $i = 1, 2$, let $W_i^\Delta(x,t)$ be party i's equilibrium payoff at time $t \in T(\Delta)$ when $x_t = x$. For $t \in T(\Delta)$ and $x \in [0,1]$, these payoffs satisfy:*

(i) if $t > t^*$, $W_i^\Delta(x,t) = V_i^\Delta(x)$,

(ii) if $t = t^*$, $W_i^\Delta(x,t) = K\mathbf{1}_{\{x \in M_i\}} + V_i^\Delta(x)$,

(iii) if $t < t^*$,

$$W_i^\Delta(x,t) = \begin{cases} e^{-r\Delta} E\left[W_i^\Delta(x_{t+\Delta}, t+\Delta) \middle| x_t = x \right] & \text{if } A(x,t) = \emptyset, \\ U_i(z(x,t), x, t) & \text{if } A(x,t) \neq \emptyset, \end{cases}$$

where $A(x,t) = \{z \in [0,1] : U_i(z,x,t) \geq e^{-r\Delta} E[W_i^{\Delta}(x_{t+\Delta}, t+\Delta)|x_t = x]$ *for* $i = 1,2\}$ *and, for all* (x,t) *such that* $A(x,t) \neq \emptyset$,

$$z(x,t) = \begin{cases} \arg\max_{z \in A(x,t)} U_1(z,x,t) & \text{if } x \in M_1, \\ \arg\max_{z \in A(x,t)} U_2(z,x,t) & \text{if } x \in M_2. \end{cases}$$

Parties always reach an agreement at times $t \geq t^*$. *Moreover, parties reach an agreement at times* $t < t^*$ *if and only if* $A(x,t) \neq \emptyset$.

Theorem 3 can be summarized as follows. Part (i) shows that the parties' payoffs at times $t > t^*$ are equal to their payoffs in the game without elections (i.e., the payoffs of Theorem 1). Part (ii), on the other hand, shows that the parties' payoffs at time $t = t^*$ are equal to their payoffs from the election plus their payoffs in the game without elections. Intuitively, for all $t \geq t^*$ the game ends immediately after parties reach an agreement. Therefore, the subgame that starts at any $t \geq t^*$ if parties have failed to reach an agreement before this date is strategically identical to the game in Section 2. This implies that for all such dates the outcome of the bargaining will be identical to the outcome of the game in Section 2: parties will always reach an agreement at time $t \geq t^*$ if they have failed to do so before, and party $i = 1,2$ will obtain a payoff of $V_i^{\Delta}(x_t)$ from this agreement.

Finally, part (iii) of Theorem 3 shows that at each time $t < t^*$ parties will reach an agreement only if there is a policy $z \in [0,1]$ that, if implemented, would leave both parties weakly better-off than waiting until the next period and getting their continuation payoffs (i.e., only if $A(x,t) \neq \emptyset$). In this case, the policy $z(x,t)$ that parties implement is the best policy for the party with proposal power among those policies that both parties are willing to accept. Otherwise, if at time t there is no policy that both parties are willing accept, there is delay at time t.

Theorem 3 establishes uniqueness of equilibrium payoffs and leaves open the possibility of gridlock (i.e., delay). The next result shows that, if there is gridlock in the unique equilibrium, then this gridlock will only occur when the time left until the election is short enough.

Proposition 3 *There exists* $s > 0$ *such that parties always reach an agreement at any time* $t \in T(\Delta)$ *with* $t^* - t > s$. *Moreover, the value of* s *is increasing in* K.

Proposition 3 shows that parties will always reach an agreement when the time left until the election is long enough. Intuitively, the discounted benefit $e^{-r(t^*-t)}K$ of winning the election is small when the election is far away. This limits the effect that implementing a policy has on the parties' payoffs, making it easier for them to reach a compromise. The cutoff $s > 0$ in Proposition 3 is increasing on the benefit K that parties obtain from being in office, and is independent of the way in which implemented policies affect the evolution of the parties' political power (i.e., is independent of the function $h(x,z)$). That is, gridlock may arise when the election is further away if parties attach a higher value to being in office.

3.3 Bounds on payoffs

The election at date $t^* > 0$ introduces an additional state variable to the model: in this setting parties care both about the level of relative political power and about the time left until the election. With this additional state variable, it is no longer possible to obtain a tractable characterization of the parties' payoffs in the limit as $\Delta \to 0$. I sidestep this difficulty by providing bounds on the parties' equilibrium payoffs. These bounds become tight as the election becomes closer, and are easy to compute numerically in the limit as $\Delta \to 0$. Moreover, I show how these bounds can be used to derive necessary conditions for gridlock to arise in equilibrium, and to analyze how the likelihood of gridlock depends on the time left until the election and on the parties' level of political power.

For all $t \in T(\Delta), t < t^*$, for all $x \in [0,1]$ and for $i = 1, 2$, let

$$\underline{W}_i^\Delta(x,t) := E\left[e^{-r(t^*-t)}V_i^\Delta(x_{t^*})|x_t = x\right] + Ke^{-r(t^*-t)}Q_i(x,t),$$

and let $\overline{W}_i^\Delta(x,t) := \underline{W}_i^\Delta(x,t) + 1 - e^{-r(t^*-t)}$. Note that $\underline{W}_i^\Delta(x,t)$ is the expected payoff that party i would obtain if parties delayed an agreement until the election.

Lemma 1 *For all $t \in T(\Delta), t < t^*$, for all $x \in [0,1]$ and for $i = 1, 2$, $W_i^\Delta(x,t) \in [\underline{W}_i^\Delta(x,t), \overline{W}_i^\Delta(x,t)]$.*

Lemma 1 shows that the parties' equilibrium payoffs prior to the election are bounded by $\underline{W}_i^\Delta(x,t)$ and $\overline{W}_i^\Delta(x,t)$. Note that these bounds on payoffs become tight as the election gets closer: $\overline{W}_i^\Delta(x,t) - \underline{W}_i^\Delta(x,t) = 1 - e^{-r(t^*-t)} \to 0$ as $t \to t^*$. Moreover, these bounds don't depend on the way in which policies affect the parties' political power; i.e., they don't depend on $h(x,z)$.

For fixed values of $\Delta > 0$ it is difficult to calculate the bounds $\underline{W}_i^\Delta(x,t)$ and $\overline{W}_i^\Delta(x,t)$. The reason for this is that these bounds depend on the parties' payoffs in the game without elections, and these payoffs are difficult to compute for fixed values of $\Delta > 0$. However, since $V_i^\Delta(\cdot)$ converges uniformly to $V_i^*(\cdot)$, it follows that

$$\underline{W}_i^\Delta(x,t) \to \underline{W}_i^*(x,t) := E\left[e^{-r(t^*-t)}V_i^*(x_{t^*})\middle| x_t = x\right] + Ke^{-r(t^*-t)}Q_i(x,t) \text{ as } \Delta \to 0.$$

Moreover, this convergence is also uniform.[14] Lemma A.5 in Appendix A.5 shows that $\underline{W}_i^*(x,t)$ is the solution to a partial differential equation (PDE). This characterization of $\underline{W}_i^*(x,t)$ as a PDE allows for simple numerical evaluations of the bounds on the parties' payoffs in the limit as $\Delta \to 0$.

The next result uses the bounds on payoffs in Lemma 1 to derive conditions under which there will be delay or agreement at states $(x,t) \in [0,1] \times T(\Delta)$ with $t < t^*$. This result can be used to derive necessary conditions for gridlock to arise in equilibrium, and to analyze how the likelihood of gridlock depends on the state variables of the game: the time left until the election and the parties' level of relative political power.

Proposition 4 *For any time $t \in T(\Delta), t < t^*$ and any $x \in [0,1]$,*

(i) if there exists $i = 1,2$ such that $U_i(z,x,t) < \underline{W}_i^\Delta(x,t)$ for all $z \in [0,1]$, then parties delay an agreement at time t if $x_t = x$;

(ii) if there exists $i = 1,2$ and $z', z'' \in [0,1]$ such that $U_i(z',x,t) \leq \underline{W}_i^\Delta(x,t)$ and $\overline{W}_i^\Delta(x,t) \leq U_i(z'',x,t)$, then parties reach an agreement at time t if $x_t = x$.

Part (i) in Proposition 4 provides necessary conditions for there to be delay at states (x,t) with $t < t^*$. On the other hand, part (ii) in Proposition 4 provides necessary conditions for there to be agreement at states (x,t) with $t < t^*$. These results can be used to analyze the equilibrium dynamics of this model with elections: for each $(x,t) \in [0,1] \times T(\Delta)$, I can use the results in Proposition 4 to check whether parties will be able to reach an agreement or not when the state of the game is (x,t). In the next subsection I illustrate this by analyzing how the proximity of elections affects legislative policymaking under three different applications of this model.

Note that there is a gap between the conditions in the two parts of Proposition 4. That is, there might exist states (x,t) at which the parties' payoffs satisfy neither the conditions in part (i) of Proposition 4 nor those in part (ii). This gap in Proposition 4 arises because I work with bounds on the parties' payoffs. Since the bounds on payoffs become tight as $t \to t^*$, the fraction of states (x,t) that are not covered by either condition in Proposition 4 vanishes as the election becomes closer.

For fixed values of $\Delta > 0$ it is hard to check the conditions in Proposition 4, since it is hard to compute the bounds on payoffs. Recall that $\underline{W}_i^\Delta(x,t)$ converges uniformly to $\underline{W}_i^*(x,t)$

[14]To see that this convergence is uniform, note that $|\underline{W}_i^\Delta(x,t) - \underline{W}_i^*(x,t)| \leq E[e^{-r(t^*-t)}(|V_i^\Delta(x_{t^*}) - V_i^*(x_{t^*})|)|x_t = x]$. Since $V_i^\Delta(x)$ converges uniformly to $V_i^*(x)$, for every $\eta > 0$ there exists $\overline{\Delta} > 0$ such that, for all $(x,t) \in [0,1] \times [0,t^*]$, $E[e^{-r(t^*-t)}(|V_i^\Delta(x_{t^*}) - V_i^*(x_{t^*})|)|x_t = x] < e^{-r(t^*-t)}\eta \leq \eta$ whenever $\Delta < \overline{\Delta}$.

as $\Delta \to 0$. Letting , $\overline{W}_i^*(x,t) := 1 - e^{-r(t^*-t)} + \underline{W}_i^*(x,t)$, it follows that $\overline{W}_i^\Delta(x,t)$ converges uniformly to $\overline{W}_i^*(x,t)$ as $\Delta \to 0$. This observation, together with Proposition 4, leads to the following corollary:

Corollary 1 *For any time $t < t^*$ and any $x \in [0,1]$,*

(i) if there exists $i = 1,2$ such that $U_i(z,x,t) < \underline{W}_i^*(x,t)$ for all $z \in [0,1]$, then there exists $\overline{\Delta} > 0$ such that parties delay an agreement at time t if $x_t = x$ and $\Delta < \overline{\Delta}$.

(ii) if there exists $i = 1,2$ and $z', z'' \in [0,1]$ such that $U_i(z',x,t) < \underline{W}_i^*(x,t)$ and $\overline{W}_i^*(x,t) < U_i(z'',x,t)$, then there exists $\overline{\Delta} > 0$ such that parties reach an agreement at time t if $x_t = x$ and $\Delta < \overline{\Delta}$.

Corollary 1 provides conditions for there to be delay or agreement at states (x,t) when the time between bargaining rounds is small. The conditions in Corollary 1 are easy to check numerically, since $\underline{W}_i^*(x,t)$ solves a PDE.

3.4 Three applications

The equilibrium dynamics in this model with elections will in general depend on the way in which the policies that parties implement affect their political power; i.e., on the function $h(x,z)$. In this subsection, I explore three different ways in which policies affect political power. The goal is to study how the proximity of elections affects the dynamics of legislative policymaking under these three different settings.

3.4.1 Electoral trade-off

I start by considering a setting in which the party with proposal power faces the following trade-off: implementing policies that are close to its ideal point lowers its level of political power, while implementing moderate policies allows it to maintain its political advantage. For instance, this trade-off would arise if voters punish parties that implement policies that are too extreme, i.e., policies that are far away from the median voter's ideal point.

To model this trade-off, I assume that for all $(x,z) \in [0,1] \times [0,1]$,

$$h(x,z) = \begin{cases} -\lambda|z - \frac{1}{2}| & \text{if } x \geq 1/2, \\ \lambda|z - \frac{1}{2}| & \text{if } x < 1/2, \end{cases} \tag{5}$$

where $\lambda \in (0,1]$ measures the effect that implemented policies have on the parties' relative political power. The assumption that $\lambda \in (0,1]$ guarantees that $x + h(x,z) \in [0,1]$ for all

$(x, z) \in [0, 1] \times [0, 1]$. This functional form of $h(x, z)$ captures the trade-off mentioned above, since the majority party sacrifices political power when it implements a policy that its close to its preferred alternative.

Definition 2 *There is* no gridlock *if parties reach an agreement at all states* $(x, t) \in T(\Delta) \times [0, 1]$. *There is* gridlock *if there are states* $(x, t) \in T(\Delta) \times [0, 1]$ *at which parties fail to reach an agreement.*

The following result shows that, in this setting, there will be gridlock whenever parties derive a sufficiently high value from being in office.

Proposition 5 *Suppose $h(x, z)$ is given by equation (5). Then, there exists $\overline{K} > 0$ such that there is gridlock whenever $K > \overline{K}$.*

Figure 3 considers a setting with $K > \overline{K}$ and illustrates the typical patterns of gridlock when $h(x, z)$ satisfies equation (5). The squared areas in the figure are the values of (x, t) at which parties will delay an agreement if Δ is small enough; i.e., states that satisfy the conditions in part (i) of Corollary 1. On the other hand, the shaded areas in the figure are values of (x, t) at which parties will reach an agreement if Δ is small enough, i.e., states that satisfy the conditions in part (ii) of Corollary 1. The white areas are the values of (x, t) that are not covered by either parts of Corollary 1.

Figure 3 shows that parties will delay an agreement when one side has a moderate advantage in terms of political power, and that they will reach an agreement when one party has a very strong bargaining position. To see the intuition for this, consider first states at which the majority party has a small advantage in terms of political power. Note that the majority party has a lot to loose by implementing a policy close to its preferred alternative at such states, since implementing such a policy would have a large negative impact on its electoral prospects. If K is large enough, at such states the majority party will prefer to delay an agreement until the election than to implement a policy close to its preferred alternative and loose its electoral advantage. Moreover, at such states the majority party doesn't want to implement a policy close to $1/2$ either: since it has a moderate political advantage, by delaying an agreement until the election date the majority party would very likely be able to implement a policy that is closer to its ideal point. This implies that at such states any policy $z \in [0, 1]$ would give the majority party a lower payoff than what it could get by delaying an agreement until the election. Thus, by Proposition 4 there must be delay at such a state.

Consider next states at which the majority party has a very strong bargaining position. In this case, the majority party's chances of winning the election would be large even after

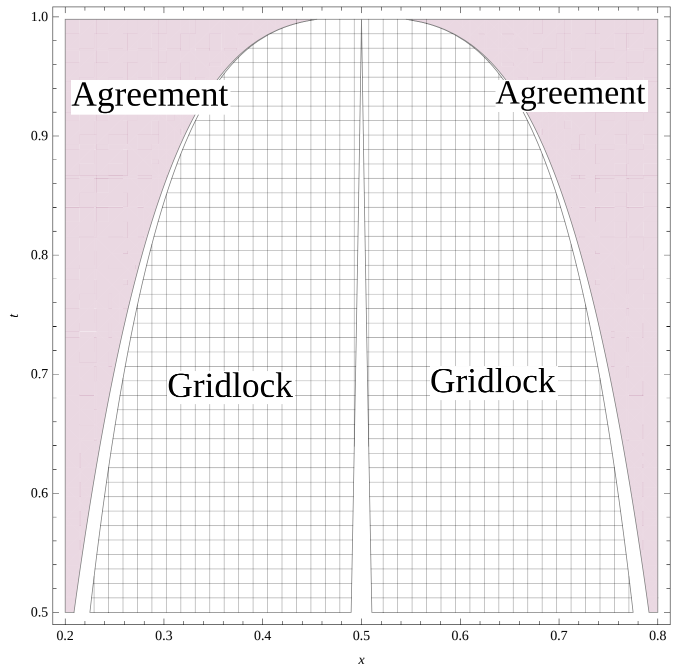

Figure 3: Parameters: $\mu = 0$, $\sigma = 0.1$, $r = 0.05$, $t^* = 1$, $K = 2$ and $\lambda = 0.5$.

implementing a policy that lies relatively close its ideal point. Therefore, the majority party would be willing to implement such a policy at these states. Moreover, the minority party would also be willing to implement such a policy at these states, since this would increase (at least marginally) its chances of winning the election. Thus, at these states parties are able to find a compromise policy that they are both willing to accept.

Finally, for states (x, t) at which parties reach an agreement (i.e., the shaded region in Figure 3), I can obtain bounds on the policies that parties will agree on using the bounds on their payoffs from Lemma 1: since party i's payoff is bounded by $\underline{W}_i^\Delta(x, t)$ and $\overline{W}_i^\Delta(x, t)$, the policy $z^\Delta(x, t)$ that parties agree on at state (x, t) must be such that $U_i(z^\Delta(x, t), x, t) \in [\underline{W}_i^\Delta(x, t), \overline{W}_i^\Delta(x, t)]$. Note that these bounds on policies are easy to compute numerically in the limit as $\Delta \to 0$. Moreover, since $\underline{W}_i^\Delta(x, t) - \overline{W}_i^\Delta(x, t) \to 0$ as $t \to t^*$, these bounds become tight as the election approaches.

3.4.2 Costly concessions

I now consider an environment in which the majority party always benefits when Congress implements a policy. This specification of the model is motivated by empirical evidence showing that voters usually hold the majority party accountable for the job performance of

Congress. That is, voters reward or punish the majority party depending on the performance that Congress has had (i.e., Jones and McDermott, 2004 and Jones, 2010). As journalist Ezra Klein wrote in an article for *The New Yorker*: "...it is typically not in the minority party's interest to compromise with the majority party on big bills – elections are a zero-sum game, where the majority wins if the public thinks it has been doing a good job."[15]

I model this environment by assuming that the majority party's level of political power jumps up discretely if parties reach an agreement to implement any policy. That is, for all $z \in [0, 1]$,

$$h(x, z) = \begin{cases} \min\{g, 1 - x\} & \text{if } x \geq 1/2, \\ -\min\{g, x\} & \text{if } x < 1/2, \end{cases} \tag{6}$$

where g is a strictly positive constant. Note that in this setting it is always costly for the minority party to concede to a policy put forward by its opponent: conceding to a policy lowers its political power by g, leading to a decrease in its electoral chances.

The next result shows there will also be gridlock in this setting if the payoff that parties obtain from winning the election is large enough.

Proposition 6 *Suppose $h(x, z)$ is given by equation (6). Then, there is exists $\overline{K}$ such that there is gridlock if $K > \overline{K}$.*

Proposition 6 shows that there will gridlock in this model when parties attach a high value to being in office. Intuitively, the minority party incurs a cost if it accepts a proposal by its opponent prior to the election, since accepting an offer will negatively affect its electoral chances. When parties attach a high value to winning the election, there are states at which no offer $z \in [0, 1]$ compensates the minority party for this electoral cost. At such states parties will never be able to reach a compromise, since the minority party would strictly prefer to delay an agreement until the election than to implement any policy.

Figure 4 considers a setting with $K > \overline{K}$ and illustrates the typical patterns of gridlock in this model. The squared areas in the figure are values of (x, t) at which parties will delay an agreement if Δ is small; i.e., states that satisfy the conditions in part (i) of Corollary 1. The shaded areas in the figure are values of (x, t) at which parties will reach an agreement if Δ is small; i.e., states that satisfy the conditions in part (ii) of Corollary 1. The white areas are the values of (x, t) that are not covered by either parts of Corollary 1.

Figure 4 shows that parties will delay an agreement when their level of political power is relative balanced, and will reach an agreement when one party has a strong advantage in

[15] "Unpopular Mandate. Why do politicians reverse their positions?," *The New Yorker*, June 25, 2012.

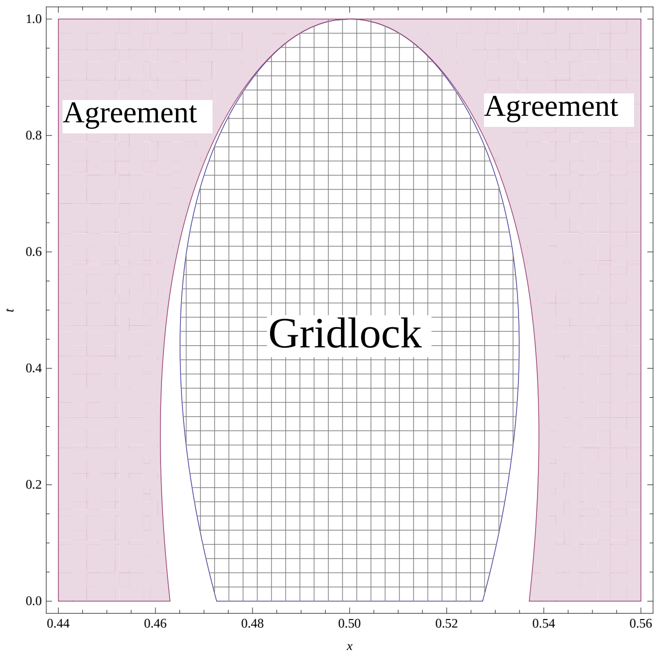

Figure 4: Parameters: $\mu = 0$, $\sigma = 0.1$, $r = 0.05$, $t^* = 1$, $K = 2$ and $g = 0.1$.

terms of political power. Intuitively, the cost that the minority party incurs when it accepts a proposal by its opponent is larger when the level of political power is balanced, since in this case a change in x will have a big impact on the parties' chances of winning the election. If K is large, at these states there will be no policy $z \in [0, 1]$ that would compensate the minority party for its lower electoral chances, and so gridlock will arise. On the other hand, the cost that the minority party incurs by accepting an offer is lower when its opponent has a strong advantage in terms of political power, since in these cases the majority will very likely win the election even if parties don't implement a policy. Therefore, at such states parties are able to reach a compromise.

Finally, for those values of (x, t) at which parties reach an agreement, I can again obtain bounds on the policy that parties will implement using the bounds on payoffs in Lemma 1: for such states (x, t), the policy $z^\Delta(x, t)$ that parties agree on must be such that $U_i(z^\Delta(x, t), x, t) \in [\underline{W}_i^\Delta(x, t), \overline{W}_i^\Delta(x, t)]$.

3.4.3 Pork-barrel spending

I now consider a setting in which parties bargain over how to distribute pork-barrel spending, and in which a party that obtains more resources out of the negotiation can increase its political advantage over its opponent. In this setting, a policy $z \in [0, 1]$ represents the fraction of resources that party 1 obtains from the negotiation, and $1 - z$ is the fraction of resources that party 2 obtains.

To model a situation in which more resources translate into more political power, I assume that for all $x \in [0, 1]$ the function $h(x, z)$ is continuous and increasing in z. Moreover, I assume that $h(x, 0) \leq 0 \leq h(x, 1)$ for all $x \in [0, 1]$. That is, if the outcome of the negotiation is such that a party obtains all the resources, then the political power of that party must be at least weakly larger after the agreement than before.

The following result shows that parties will always reach an immediate agreement in this setting.

Proposition 7 *Suppose that $h(x, z)$ is continuous and increasing in z for all $x \in [0, 1]$. Suppose further that $h(x, 0) \leq 0 \leq h(x, 1)$ for all $x \in [0, 1]$. Then, there is no gridlock. Moreover, for all $t \in T(\Delta), t < t^*$, for all $x \in [0, 1]$ and for $i = 1, 2$,*

$$W_i^\Delta(x, t) = V_i^\Delta(x) + e^{-r(t^*-t)} K Q_i(x, t). \tag{7}$$

Proposition 7 shows that parties always reach an immediate agreement when bargaining over pork-barrel spending. To see the intuition behind this result, recall that policies have two effects on the parties' payoffs: a direct effect, since parties derive utility from the policies they implement, and an indirect effect, since the policies they implement have an effect on their electoral chances. These two effects run in the same direction when parties bargain over pork-barrel spending. Therefore, the party with proposal power is always able to calibrate its offer to leave its opponent indifferent between accepting or rejecting. In equilibrium the responder always accepts such an offer and the game ends with an immediate agreement.

Proposition 7 also characterizes the parties' payoffs in this environment: a party's payoff is equal to its payoff in the game without an election plus its expected payoff from the election. Note that the expected payoff from the election is measured *without* taking into account the effect that the agreement has on the parties' electoral chances. The parties' payoffs in Proposition 7 are easy to compute in limit as the time period goes to zero: since $\lim_{\Delta \to 0} V_i^\Delta(x) = V_i^*(x)$, it follows that $\lim_{\Delta \to 0} W_i^\Delta(x, t) = V_i^*(x) + e^{-r(t^*-t)} K Q_i(x, t)$.

The expression of the parties' payoffs in equation (7) can be used to back out the agreements that parties reach in this setting. Let $z^\Delta(x, t) \in [0, 1]$ be the agreement that parties reach at time $t \in T(\Delta), t < t^*$ when $x_t = x$. Party 1's payoff from this agreement is $U_1(z^\Delta(x, t), x, t) = z^\Delta(x, t) + e^{-r(t^*-t)} K Q_1(x + h(x, z^\Delta(x, t)), t)$. On the other hand, by

Proposition 7 party 1's payoff at (x,t) is $W_1^\Delta(x,t) = V_1^\Delta(x) + e^{-r(t^*-t)} K Q_1(x,t)$. Therefore, $z^\Delta(x,t)$ must be such that

$$z^\Delta(x,t) - V_1^\Delta(x) = e^{-r(t^*-t)} K \left[Q_1(x,t) - Q_1(x + h(x, z^\Delta(x,t)), t) \right]. \tag{8}$$

From equation (8) I can analyze the effect that the election has on the agreement that parties reach. For instance, suppose that x is such that $V_1^\Delta(x) > 1/2$; that is, in the game without elections party 1 gets a larger fraction of the available resources than party 2 when $x_t = x$. Suppose further that the function h is such that $h(x, 1/2) = 0$ for all x; that is, the parties' relative political power remains unchanged if they share the available resources evenly. In this case, the agreement $z^\Delta(x,t)$ that parties reach at time t when $x_t = x$ must be such that $z^\Delta(x,t) \in (1/2, V_1^\Delta(x))$. To see this, note that the left-hand side of (8) would be positive if $z^\Delta(x,t) \geq V_1^\Delta(x)$, while the right-hand side would be negative (since $h(x,\cdot)$ is increasing in z and since $h(x,1/2) = 0$). On the contrary, if $z^\Delta(x,t) \leq 1/2$ then the left-hand side of (8) would be negative and the right-hand side would be positive. By a symmetric argument it must also be that $z^\Delta(x,t) \in [V_1^\Delta(x), 1/2)$ for all x such that $V_1^\Delta(x) < 1/2$. Thus, when parties bargain over pork-barrel spending, an upcoming election leads to a more equal distribution of resources compared to the model without elections. Finally, since $V_1^\Delta(x) \to V_1^*(x)$ as $\Delta \to 0$, it follows from (8) that $z^*(x,t) := \lim_{\Delta \to 0} z^\Delta(x,t)$ solves $z^*(x,t) = V_1^*(x) + e^{-r(t^*-t)} K [Q_1(x,t) - Q_1(x + h(x, z^*(x,t)), t)]$.

4 Discussion

This first part of this section discusses positive implications of the results in this paper. The second part discusses some modeling choices and shows how some of the model's assumptions can be generalized.

4.1 Implications on gridlock and elections

This paper illustrates how electoral considerations can affect the dynamics of legislative policymaking, leading to long periods of gridlock. By Proposition 3, these periods of legislative inaction can only occur when the election is close enough; that is, when the time left until the election is smaller than some value s. The value of s is increasing in the value K that parties attach to winning the election. These results can be used to obtain an estimate on the value that parties derive from winning an election based on observable outcomes of legislative negotiations: if we observe that Congress becomes gridlocked t days before an election, we can use the results in Proposition 3 to obtain a lower bound on the value of K.

In the models of Sections 3.4.1 and 3.4.2, gridlock is less likely to arise in settings in which one party has a very strong bargaining position in Congress; see Figures 3 and 4 above. These results are consistent with the work of Jones (2001), who studies the patterns of gridlock in U.S. Congress. Jones (2001) finds that the level of party polarization in U.S. Congress increases the likelihood of gridlock, but that the magnitude of this increase diminishes as the number of seats under the control of the majority party increases. Put differently, the results in Jones (2001) show that, for any given level of party polarization, the likelihood of gridlock decreases as the number of seats under the control of the majority party increases; i.e., as the bargaining position of the majority party increases.

Finally, the model also predicts that parties will always reach an agreement after the election if they have failed to do so before. Importantly, this result does not depend on there being only one election; see Section 4.2 below. This result suggests that one way to measure whether elections are creating legislative inaction in the U.S. Congress is to count the number of laws that legislators approve during lame duck sessions after elections. If the number of laws approved during lame duck sessions is larger than normal, then the results in this paper suggest that this would be a good indication of a gridlocked Congress prior to the election.

4.2 Modeling assumptions and extensions

Time-varying political power. In this paper I assume that the parties' political power evolves as a diffusion process. In the model of Section 2, this assumption leads to closed form expressions for the parties' payoffs in the limit as $\Delta \to 0$, allowing me to perform comparative statics exercises. In the model with elections, the assumption that political power evolves as a diffusion process allows me to obtain bounds on the parties' payoffs that are easy to compute numerically in the limit as $\Delta \to 0$. I use these bounds on payoffs to derive necessary conditions for gridlock to arise, and to study how the likelihood of gridlock depends on the time left until the election and on the parties' relative political power.

There are other ways to model time-varying political power. For instance, I could instead assume that the parties' political power evolves over time as a Markov chain. However, under this class of processes it would in general not be possible to obtain closed form expressions for the parties' limiting payoffs in the baseline model without elections. Moreover, such processes would also make the analysis of the model with elections less amenable to numerical computations.

Multiple elections. The model in Section 3 assumes that there is only one election at time t^*. This assumption, together with the results in Section 2, implies that parties will always reach an agreement immediately after the election. The analysis in Section 3 generalizes to settings with multiple elections, provided the elections are sufficiently apart in time. For instance, suppose that there is a second election scheduled for time $t^{**} > t^*$. If the time between elections $t^{**} - t^*$ is large enough, then by the results in Proposition 3 parties will reach an agreement immediately after the first election if they haven't done so before. Therefore, a model with multiple elections would deliver a similar equilibrium dynamics than the model with a single election, with gridlock only arising when the next election is close enough.[16]

Implemented policies and political power. This paper assumes that implemented policies have an "instantaneous" effect on the parties' political power: if parties implement policy z at time t, then the parties' relative political power reacts instantaneously after the agreement; i.e., x_t jumps by $h(x_t, z)$ immediately after parties reach an agreement. An alternative (and more general) specification would be to assume that implemented policies affect the law of motion of the process x_t. For instance, implementing policy z at time t could affect the drift and/or volatility of the process that drives the parties' political power going forward. The methods I use in this paper can also be applied to study the equilibrium dynamics in these settings. Indeed, by the same arguments as in Lemma 1, in these settings the parties' equilibrium payoffs would still be bounded by $\underline{W}_i^{\Delta}(x,t)$ and $\overline{W}_i^{\Delta}(x,t)$. Therefore, in this environment I could also use these bounds on payoffs to study how the likelihood of gridlock depends on the time left until the election and on the parties' level of political power.

General bargaining protocols. The models Sections 2 and 3 assume that the party with more political power has proposal power. I now show how this assumption can be relaxed to allow for a broader class of bargaining protocols. For simplicity, I consider only the model without elections.

Consider the model without elections and suppose that at each time $t \in T(\Delta)$ party 1 makes offers with probability $p_1(x_t) \in [0,1]$ and party 2 makes offers with probability $p_2(x_t) = 1 - p_1(x_t)$. Assume further that $p_1(x)$ is continuous and increasing in x. Note that $p_1(x)$ increasing in x captures the idea that party 1's bargaining power is increasing in x, while party 2's bargaining power is decreasing in x.[17]

[16] Moreover, it can be shown that if the elections are sufficiently apart in time, the parties' payoffs after the first election will be close to their payoffs in the game without elections. The proof of this result is available upon request.

[17] With this specification, the bargaining protocol of Sections 2 and 3 can be approximated arbitrarily well by sequence of continuous functions $p_1^n(x)$ converging to the step function $p_1(x)$ with $p_1(x) = 0$ for $x < 1/2$

By arguments similar to those in the proof of Theorem 1, for any time period $\Delta > 0$ this game also has a unique SPE. This unique SPE is again difficult to analyze for any fixed time period $\Delta > 0$. However, this game also becomes very tractable in the limit as $\Delta \to 0$: in Appendix A.7 I show that there exists functions $\hat{V}_1^*$ and $\hat{V}_2^*$ such that party i's SPE payoffs of this game converge uniformly to $\hat{V}_i^*$ in the limit as $\Delta \to 0$. Moreover, for $i = 1, 2$, $\hat{V}_i^*$ solves the following ordinary differential equation,

$$r\hat{V}_i^*(x) \quad = \quad rp_i(x) + \mu(\hat{V}_i^*)'(x) + \frac{1}{2}\sigma^2(\hat{V}_i^*)''(x) \text{ for } x \in [0, 1], \tag{9}$$

with boundary conditions $(\hat{V}_i^*)'(0) = (\hat{V}_i^*)'(1) = 0$. The left-hand side of equation (9) represents party i's limiting payoff measured in flow terms, while the right-hand side shows the sources of this flow payoff. Party i's flow payoff when $x_t = x$ is equal to the expected rent $rp_i(x)$ party i extracts when making offers plus the expected change in its continuation payoff due to changes in political power $\mu(\hat{V}_i^*)'(x) + \frac{1}{2}\sigma^2(\hat{V}_i^*)''(x)$.[18]

5 Conclusion

The first part of this paper constructs a model of legislative bargaining to study how changes in political power affect the outcomes of legislative negotiations. At an abstract level, this model generalizes standard bilateral bargaining games *a la* Rubinstein (1982) to settings in which the player's bargaining power varies over time. The model has a unique SPE, in which parties always reach an immediate agreement. The unique SPE becomes very tractable in the limit as $\Delta \to 0$. This tractability allows me to obtain predictions about how different features of the environment affect the agreements that parties reach.

The second part of the paper uses this bargaining model to study the effect that elections have on legislative outcomes. I show that elections might give rise to long periods of legislative inaction. These delays occur in spite of the fact that implementing a policy immediately is always the efficient outcome. I provide bounds on the parties' equilibrium payoffs. These bounds on payoffs become tight as the election approaches, and are easy to compute numerically in the limit as $\Delta \to 0$. I use these bounds on payoffs to analyze the equilibrium dynamics of this model with elections.

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and $p_1(x) = 1$ for all $x \geq 1/2$.

[18]Note that in this setting party i's payoffs satisfy the same ordinary differential equation for all $x \in [0, 1]$. Therefore, in this case there is no need to impose the boundary conditions $\hat{V}_i^*(1/2^-) = \hat{V}_i^*(1/2^+)$ and $(\hat{V}_i^*)'(1/2^-) = (\hat{V}_i^*)'(1/2^+)$, since any solution to equation (9) will be continuous and differentiable on $[0, 1]$.

A Appendix

A.1 Proof of Theorem 1

Let F^2 be the set of bounded and measurable functions on $[0,1]$ taking values on $\mathbb{R}^2$. Let $\|\cdot\|_2$ denote the sup norm on $\mathbb{R}^2$. For any $f \in F^2$, let $\|f\| = \sup_{z\in[0,1]} \|f(z)\|_2$. Fix $\Delta > 0$, $r > 0$ and let $\delta = e^{-r\Delta}$. Recall that, for $i = 1,2$, M_i is the set of states at which party i is proposer. Define $\psi : F^2 \to F^2$ as follows: for any $f \in F^2$ and for $i = 1,2$, $i \neq j$,

$$\psi_i(f)(x) = \begin{cases} \delta E\left[f_i(x_{t+\Delta})\,|x_t = x\right] & \text{if } x \in M_j, \\ 1 - \delta E\left[f_j(x_{t+\Delta})\,|x_t = x\right] & \text{if } x \in M_i, \end{cases}$$

Note that ψ is a contraction of modulus δ: for any $f, g \in F^2$, $\|\psi(f) - \psi(g)\| \leq \delta \|f - g\|$.

Proof of Theorem 1. To prove Theorem 1 I first assume that the set of SPE is non-empty. I then prove that the game has a SPE. Fix a SPE and let $f_i(x)$ be party i's payoff from this SPE when $x_0 = x$. Let $\overline{M} = \left(\overline{M}_1, \overline{M}_2\right) \in F^2$ and $\overline{m} = (\overline{m}_1, \overline{m}_2) \in F^2$ be the supremum and infimum SPE payoffs. Thus, for all $x \in [0,1]$ and for $i = 1,2$, $f_i(x) \in [\overline{m}_i(x), \overline{M}_i(x)]$.

Note that for all $x \in M_i$, party i's SPE payoff is bounded below by $1 - \delta E[\overline{M}_j(x_{t+\Delta})|x_t = x]$, since in any SPE party j must accept an offer that gives that party a payoff equal to $\delta E[\overline{M}_j(x_{t+\Delta})|x_t = x]$. On the other hand, for all $x \in M_j$ party i's payoffs is bounded below by $\delta E[\overline{m}_i(x_\Delta)|x_t = x]$, since party i can always guarantee this payoff by rejecting party j's offer. Thus, for all $x \in [0,1]$ it must be that $f_i(x) \geq \psi_i\left(\overline{m}_i, \overline{M}_j\right)$. Define $G : F^2 \times F^2 \to F^2$ as $G(M,m) := (\psi_1(m_1, M_2), \psi_2(m_2, M_1))$, so $f_i(x) \geq G_i(\overline{M}, \overline{m})(x)$

At states $x \notin M_i$, party i's payoff is bounded above by $\delta E[\overline{M}_i(x_\Delta)|x_t = x]$, since party j will never make an offer that gives party i a payoff larger than this. Consider next $x \in M_i$, and note that $f_i(x) + f_j(x) \leq 1$. Moreover, by the arguments in the previous paragraph, $f_j(x) \geq \delta E[\overline{m}_j(x_\Delta)|x_t = x]$ for all $x \in M_i$. Combining these inequalities, it follows that $f_i(x) \leq 1 - \delta E[\overline{m}_j(x_\Delta)|x_t = x]$ for all $x \in M_i$. Thus, for all $x \in [0,1]$, $f_i(x) \leq \psi_i\left(\overline{M}_i, \overline{m}_j\right)$. Define $H : F^2 \times F^2 \to F^2$ as $H(M,m) := (\psi_1(M_1, m_2), \psi_2(M_2, m_1))$, so $f_i(x) \leq H_i(\overline{M}, \overline{m})(x)$

Note next that for any M', M'' and m', m'' (all functions in F^2) such that $M_i'(x) \geq M_i''(x)$ for all $x \in [0,1], i = 1,2$ and $m_i'(x) \leq m_i''(x)$ for all $x \in [0,1], i = 1,2$, it must be that $H_i(M', m')(x) \geq H_i(M'', m'')(x)$ for all $x, i = 1,2$ and $G_i(M', m')(x) \leq G_i(M'', m'')(x)$ for all $x, i = 1,2$. It what follows, for any pair $f, g \in F^2$ I will write $f \geq g$ if $f_i(x) \geq g_i(x)$ for all $x \in [0,1], i = 1,2$.

Define the sequences $\{M^n\}$ and $\{m^n\}$ as follows. Let $(M^1, m^1) = \left(\overline{M}, \overline{m}\right)$, and for all

$n \geq 2$ let $(M^n, m^n) = (H(M^{n-1}, m^{n-1}), G(M^{n-1}, m^{n-1}))$. Note that $M^2 = H(M^1, m^1) \geq M^1$ and $m^2 = G(M^1, m^1) \leq m^1$. It follows then by induction and using the observation in the previous paragraph that $\{M^n\}$ is an increasing sequence and $\{m^n\}$ is a decreasing sequence. Moreover, it must be that $m^n \geq 0$ for all n and $M^n \leq 1$ for all n. Thus, both $\{M^n\}$ and $\{m^n\}$ are bounded and monotonic sequences, so there exists M^* and m^* such that $\{M^n\} \to M^*$ and $\{m^n\} \to m^*$. Since the operators H and G are continuous, it must be that $H(M^*, m^*) = M^*$ and $G(M^*, m^*) = m^*$. Therefore, $m^* \leq m^1 = \overline{m} \leq \overline{M} = M^1 \leq M^*$.

Since $G(M^*, m^*) = m^*$, it follows from the definition of G that $\psi_1(m_1^*, M_2^*) = m_1^*$ and $\psi_2(M_1^*, m_2^*) = m_2^*$. Moreover, $H(M^*, m^*) = M^*$ and the definition of H together imply that $\psi_1(M_1^*, m_2^*) = M_1^*$ and $\psi_2(m_1^*, M_2^*) = M_2^*$. Therefore, both (M_1^*, m_2^*) and (m_1^*, M_2^*) are fixed points of ψ. Since ψ is a contraction with a unique fixed point, it follows that $(M_1^*, m_2^*) = (m_1^*, M_2^*)$. Finally, since $m^* \leq \overline{m} \leq \overline{M} \leq M^*$, it must be that $\overline{m} = \overline{M}$.

So far, I showed that if the set of SPE is non-empty, then all SPE are payoff equivalent. I now show that the set of SPE is non-empty. Let $V^\Delta(x) = (V_1^\Delta(x), V_2^\Delta(x))$ be such that $V^\Delta(\cdot) = \psi(V^\Delta(\cdot))$. Note that $V^\Delta(\cdot)$ and satisfies the conditions in Theorem 1. To see that the payoffs $V^\Delta(x)$ can be supported by a SPE, consider the following strategy profile. At every $x \in M_i$, party i makes an offer that gives parties a payoff of $V^\Delta(x) = (V_1^\Delta(x), V_2^\Delta(x))$. At such a state, party $j \neq i$ accepts any offer which gives that party a payoff of at least $V_j^\Delta(x)$, and rejects any offer giving a payoff lower than this. The parties' payoffs from this strategy profile are V^Δ. Moreover, it is easy to see that no party can gain by unilaterally deviating from its strategy at any $x \in [0, 1]$. Hence, this strategy profile is a SPE of Γ_Δ. ■

A.2 Proof of Theorem 2

For all $x \in [0, 1]$, party i's SPE payoffs $V_i^\Delta(x)$ solves equation (3) in the main text. Setting $t = 0$ and solving this equation forward yields

$$V_i^\Delta(x) = E\left[\frac{1 - e^{-r\Delta}}{\Delta} \sum_{k=0}^{\infty} \Delta e^{-rk\Delta} \mathbf{1}_{\{x_{k\Delta} \in M_i\}} \middle| x_0 = x\right]. \tag{A.1}$$

Equation (A.1) implies that $V_i^*(x) := \lim_{\Delta \to 0} V_i^\Delta(x) = E[r \int_0^\infty e^{-rt} \mathbf{1}_{\{x_t \in M_i\}} dt | x_0 = x]$. These arguments show that $V_i^\Delta(\cdot) \to V_i^*(\cdot)$ pointwise as $\Delta \to 0$. However, since $V_1^\Delta(x)$ and $V_2^\Delta(x)$ are monotone in x for all Δ, it follows that this convergence is uniform on $[0, 1]$.[19]

For any $s > 0$, let $p(x, y, s) = \text{Prob}(x_s = y | x_0 = x)$. It is well known that $p(x, y, s)$

[19]Monotonicity of V_1^Δ follows since, for all $t > 0$, the probability that $x_t \geq 1/2$ conditional on $x_0 = x$ is increasing x. Hence, by (A.1), V_1^Δ is increasing in x. Since $V_2^\Delta(x) = 1 - V_1^\Delta(x)$ for all x, V_2^Δ is decreasing.

solves Kolmogorov's backward equation (see, for instance, Bhattacharya and Waymire, 2009, chapter V.6),

$$\frac{\partial}{\partial s}p(x,y,s) = \mu\frac{\partial}{\partial x}p(x,y,s) + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial x^2}p(x,y,s), \tag{A.2}$$

with $\lim_{s\to 0} p(x,y,s) = \mathbf{1}_{\{y=x\}}$ and $\frac{\partial}{\partial x}p(x,y,s)|_{x=0} = \frac{\partial}{\partial x}p(x,y,s)|_{x=1} = 0$ for all $s > 0$. For any $s > 0$ and for $i = 1,2$ let $P_i(s,x) = E[\mathbf{1}_{\{x_s\in M_i\}}|x_0 = x] = \int_{M_i} p(x,y,s)dy$. Since $p(x,y,s)$ solves (A.2) with $\frac{\partial}{\partial x}p(x,y,s)|_{x=0} = \frac{\partial}{\partial x}p(x,y,s)|_{x=1} = 0$ and $\lim_{s\to 0} p(x,y,s) = \mathbf{1}_{\{y=x\}}$, it follows that $P_i(s,x)$ also solves (A.2) with $P_i(0,x) = \mathbf{1}_{\{x\in M_i\}}$ and $\frac{\partial}{\partial x}P_i(s,x)|_{x=0} = \frac{\partial}{\partial x}P_i(s,x)|_{x=1} = 0$ for all $s > 0$.[20] Finally, note that $V_i^*(x) = r\int_0^\infty e^{-rt}P_i(t,x)dt$.

Lemma A1 *$V_1^*(x)$ and $V_2^*(x)$ are continuous in x.*

Proof. For $i = 1,2$ and for every $\varepsilon > 0$, let $V_i^\varepsilon(x) := r\int_\varepsilon^\infty e^{-rt}P_i(t,x)dt$. Since $P_i(t,\cdot)$ is continuous for all $t > 0$,[21] $V_i^\varepsilon(x)$ is continuous for all $\varepsilon > 0$. To show that $V_i^*(x)$ is continuous, it suffices to show that $V_i^\varepsilon(x) \to V_i^*(x)$ uniformly as $\varepsilon \to 0$. To see this, note that for any $\varepsilon > 0$ and any $x \in [0,1]$, $|V_i^*(x) - V_i^\varepsilon(x)| = r\int_0^\varepsilon e^{-rt}P_i(t,x)dt \leq r\int_0^\varepsilon e^{-rt}dt = 1 - e^{-r\varepsilon}$. Therefore, for every $\eta > 0$ there exists $\varepsilon(\eta)$ such that $|V_i^*(x) - V_i^\varepsilon(x)| \leq \eta$ for all $x \in [0,1]$ and all $\varepsilon < \varepsilon(\eta)$. Hence, $V_i^\varepsilon(x) \to V_i^*(x)$ uniformly as $\varepsilon \to 0$. ■

Lemma A2 *$V_1^*(\cdot)$ and $V_2^*(\cdot)$ solve (4) in the main text, with boundary conditions $(V_i^*)'(0) = (V_i^*)'(1) = 0$ and $V_i^*(1/2^-) = V_i^*(1/2^+)$ for $i = 1,2$.*

Proof. The rule of integration by parts implies that for all $x \neq 1/2$,

$$V_i^*(x) = r\int_0^\infty e^{-rs}P_i(s,x)\,ds = -e^{-rs}P_i(s,x)\big|_0^\infty + \int_0^\infty e^{-rs}\frac{\partial P_i(s,x)}{\partial s}ds.$$

Since $-e^{-rs}P_i(s,x)\big|_0^\infty = \mathbf{1}_{\{x\in M_i\}}$, then $V_i^*(x) = \mathbf{1}_{\{x\in M_i\}} + \int_0^\infty e^{-rs}\frac{\partial P_i(s,x)}{\partial s}ds$ for all $x \neq 1/2$. Since P_i satisfies (A.2), for all $x \neq 1/2$

$$\begin{aligned} V_i^*(x) &= \mathbf{1}_{\{x\in M_i\}} + \int_0^\infty e^{-rs}\left(\mu\frac{\partial}{\partial x}P_i(s,x) + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial x^2}P_i(s,x)\right)ds \\ &= \mathbf{1}_{\{x\in M_i\}} + \frac{1}{r}\left[\mu(V_i^*)'(x) + \frac{1}{2}\sigma^2(V_i^*)''(x)\right], \end{aligned}$$

where the second equality follows since $(V_i^*)'(x) = r\int_0^\infty e^{-rs}\frac{\partial P_i(s,x)}{\partial x}ds$ and $(V_i^*)''(x) = r\int_0^\infty e^{-rs}\frac{\partial^2 P_i(s,x)}{\partial x^2}ds$ for all $x \neq 1/2$. Multiplying both sides of this equation by r, it fol-

[20] That is, for all $x \in [0,1]$ and all $s > 0$, $\frac{\partial}{\partial s}P(s,x) = \mu\frac{\partial}{\partial x}P(s,x) + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial x^2}P(s,x)$.

[21] For all $t > 0$, $P_i(t,\cdot)$ is twice differentiable (and hence also continuous), since $P_i(t,x)$ solves (A.2).

lows that V_i^* solves (4) for all $x \in [0,1]$. To pin down the boundary conditions, note that $(V_i^*)'(y) = r\int_0^\infty e^{-rs}\frac{\partial P_i(s,x)}{\partial x}|_{x=y}ds$. Since $\frac{\partial P_i(s,x)}{\partial x}|_{x=0} = \frac{\partial P_i(s,x)}{\partial x}|_{x=1} = 0$ for all s, it follows that $(V_i^*)'(0) = (V_i^*)'(1) = 0$. Finally, since by Lemma A1 V_i^* is continuous, it must be that $V_i^*(1/2^-) = V_i^*(1/2^+)$. ■

Lemma A3 *Fix $y \in (0,1)$ and define $\tau_y = \inf\{t : x_t = y\}$. Let g be a bounded function and let $v(x) = E[e^{-r\tau_y} g(x_{\tau_y}) | x_0 = x]$. Then, for all $x \in [0,y)$, v solves*

$$rv(x) = \mu v'(x) + \frac{1}{2}\sigma^2 v''(x), \tag{A.3}$$

with $v'(0) = 0$ and $v(y) = g(y)$. Similarly, for all $x \in (y,1]$, v solves (A.3) with $v'(1) = 0$ and $v(y) = g(y)$.

Proof. I prove the first statement of the Lemma; the proof of the second statement is symmetric and omitted. Let $\widetilde{v}$ solve (A.3) for all $x \in [0,y)$, with $\widetilde{v}(y) = g(y)$ and $\widetilde{v}'(0) = 0$. Let $f(x,t) = e^{-rt}\widetilde{v}(x)$, so that $f(y,t) = e^{-rt}g(y)$. By Ito's formula, for all $x \in [0,y)$,

$$df(x_t,t) = e^{-rt}\left(-r\widetilde{v}(x_t) + \mu\widetilde{v}'(x_t) + \frac{1}{2}\sigma^2\widetilde{v}''(x_t)\right)dt + e^{-rt}\sigma\widetilde{v}'(x_t)\,dB_t = e^{-rt}\sigma\widetilde{v}'(x_t)\,dB_t,$$

where the second equality follows since $\widetilde{v}(x)$ solves (A.3) for all $x \in [0,y)$. Then, for all $x \in [0,y)$,

$$\begin{aligned} E\left[e^{-r\tau_y} g(x_{\tau_y}) \middle| x_0 = x\right] &= E\left[f(x_{\tau_y},\tau_y) \middle| x_0 = x\right] = \widetilde{v}(x) + E\left[\int_0^{\tau_y} df(x_t,t)\middle| x_0 = x\right] \\ &= \widetilde{v}(x) + E\left[\int_0^{\tau_y} e^{-rt}\sigma\widetilde{v}'(x_t)\,dB_t \middle| x_0 = x\right] = \widetilde{v}(x), \end{aligned}$$

where the last equality follows since $\int_0^{\tau_y} e^{-rt}\sigma\widetilde{v}'(x_t)\,dB_t$ is a martingale with zero expectation. Thus, $\widetilde{v}(x) = v(x)$ for all $x \in [0,y)$, so $v(x)$ solves (A.3) for all $x \in [0,y)$ with $v(y) = g(y)$ and $v'(0) = 0$. ■

Remark A1 Let $\tau(1/2) := \inf\{t : x_t = 1/2\}$. Lemmas A2 and A3 and the fact that $V_1^*(1/2)+V_2^*(1/2) = 1$ imply that, for $i = 1,2$ and all $x \notin M_i$, $V_i^*(x) = E[e^{-r\tau(1/2)}V_i^*(1/2)|x_0 = x] = E[e^{-r\tau(1/2)}(1 - V_j^*(1/2))|x_0 = x]$.

Proof of Theorem 2. By Lemma A2, $V_i^\Delta(x) \to V_i^*(x)$ uniformly as $\Delta \to 0$, with V_i^* satisfying (4) with boundary conditions $(V_i^*)'(0) = (V_i^*)'(1) = 0$ and $V_i^*(1/2^-) = V_i^*(1/2^+)$. To complete the proof of Theorem 2, I now show that V_i^* satisfies $(V_i^*)'(1/2^-) = (V_i^*)'(1/2^+)$

for $i = 1, 2$. Suppose by contradiction that this is not true for some $i = 1, 2$. Since $V_1^*(x) + V_2^*(x) = 1$ for all x, then either V_1^* or V_2^* must have a convex kink at 1/2. Assume that V_1^* has a convex kink. I will show that, in this case, for small values of Δ party 1 has a profitable deviation from the SPE strategies derived in Theorem 1.

For all $\kappa \in (0, 1/2)$ let $\tau^\kappa := \inf\{t : x_t \geq 1/2 + \kappa\}$. Let $U_1^\kappa(x) := E[e^{-r\tau^\kappa}(1 - V_2^*(x_{\tau^\kappa}))|x_0 = x]$, so $U_1^\kappa(x) = V_1^*(x)$ for all $x \geq 1/2 + \kappa$. Since V_1^* has a convex kink at 1/2, there exists $\kappa \in (0, 1/2)$ such that $U_1^\kappa(x) > V_1^*(x) = E[e^{-r\tau(1/2)}(1 - V_2^*(1/2))|x_0 = x]$ for all $x < 1/2$. Consider the following deviation for party 1. For all $x \geq 1/2 + \kappa$, offer $V_2^\Delta(x)$ to party 2 (an offer that party 2 accepts). For all $x < 1/2 + \kappa$, reject all offers when responding, and offer $0 < V_2^\Delta(x)$ to party 2 when proposing (so that these offers are rejected by party 2). For any $\Delta > 0$, define the stopping time $\tau_\Delta^\kappa := \inf\{t \in T(\Delta) : x_t \geq 1/2 + \kappa\}$ and note that the payoff that party 1 gets from following this deviation (when party 2 follows its equilibrium strategy) is $U_1^\Delta(x) = E[e^{-r\tau_\Delta^\kappa}(1 - V_2^\Delta(x_{\tau_\Delta^\kappa}))|x_0 = x]$.

Fix a sequence $\{\Delta_n\} \to 0$. Let $\tau^* := \lim_{n\to\infty} \tau_{\Delta_n}^\kappa$ and note that $\tau^* = \tau^\kappa$ almost surely.[22] Let ξ be such that $U_1^\kappa(x) > V_1^*(x) + \xi$ for all $x \in [0, 1/2]$. Since $V_i^{\Delta_n}(x) \to V_i^*(x)$ uniformly, there exists N such that for all $n > N$, $\left|V_i^{\Delta_n}(x) - V_i^*(x)\right| < \xi/2$ for all $x \in [0, 1]$ and for $i = 1, 2$. For all $n > N$, party 1's payoff from this deviation is

$$U_1^{\Delta_n}(x) = E\left[e^{-r\tau_{\Delta_n}^\kappa}\left(1 - V_2^{\Delta_n}\left(x_{\tau_{\Delta_n}^\kappa}\right)\right)\middle| x_0 = x\right] > E\left[e^{-r\tau_{\Delta_n}^\kappa}\left(1 - V_2^*\left(x_{\tau_{\Delta_n}^\kappa}\right)\right)\middle| x_0 = x\right] - \frac{\xi}{2}.$$

Since $\tau_n^\kappa \to \tau^\kappa$ almost surely, it follows that $U_1^{\Delta_n}(x) \to U_1^\kappa(x)$ as $n \to \infty$. Finally, since $U_1^\kappa(x) - \frac{\xi}{2} > V_1^*(x) + \frac{\xi}{2}$ for all $x \in [0, 1/2]$, and since $V_1^{\Delta_n}(x) < V_1^*(x) + \frac{\xi}{2}$ for all $n > N$, it follows that for n large enough, $U_1^{\Delta_n}(x) > V_1^{\Delta_n}(x)$ for all $x \in [0, 1/2]$. Therefore, if V_1^* has a convex kink at 0, then party 1 has a profitable deviation whenever Δ is small enough. But this contradicts the fact that $V_1^\Delta(x)$ is party 1's SPE payoff, so V_1^* cannot have a convex kink at 1/2. A symmetric proof shows that V_2^* cannot have a convex kink at 1/2 either, so $(V_1^*)'(1/2) = -(V_2^*)'(1/2)$. ■

The unique solution to the system of ODE's in Theorem 2 is

$$V_1^*(x) = \begin{cases} \frac{e^{\alpha/2}\left(\beta e^{-\alpha x} + \alpha e^{\beta x}\right)}{\left(1 + e^{(\alpha+\beta)/2}\right)(\alpha+\beta)} & x \in [0, 1/2], \\ 1 - \frac{e^{-\beta/2}\left(\alpha e^{\beta x} + \beta e^{(\alpha+\beta)} e^{-\alpha x}\right)}{\left(1 + e^{(\alpha+\beta)/2}\right)(\alpha+\beta)} & x \in [1/2, 1], \end{cases} \tag{A.4}$$

[22] To see this, let $D := \{\omega \in \Omega : \tau^*(\omega) > \tau^\kappa(\omega)\}$ be the set of sample paths of x_t such that τ^* is strictly larger than τ^κ. These sample paths are such that x_t hits $1/2 + \kappa$, and then immediately stays below $1/2 + \kappa$ for a positive amount of time. By Blumenthal's Zero-One Law (i.e., Karatzas and Shreve, 1998, page 94), the set of all these paths has measure zero. Hence, $\tau^* = \tau^\kappa$ almost surely.

and $V_2^*(x) = 1 - V_1^*(x)$ for all $x \in [0,1]$.

A.3 Proof of Propositions 1 and 2

Lemma A4 *Let U be a solution to (A.3) with volatility σ, and let W be a solution to (A.3) with volatility $\widetilde{\sigma} > \sigma$. (i) If $U(y) \geq W(y)$ and $U'(y) \geq W'(y)$ for some y, then $U'(x) > W'(x)$ for all $x > y$, and hence $U(x) > W(x)$ for all $x > y$. (ii) If $U(y) \geq W(y)$ and $U'(y) \leq W'(y)$, then $U'(x) < W'(x)$ for all $x < y$, and hence $U(x) > W(x)$ for all $x < y$.*

Proof. I prove part (i) of the Lemma. The proof of part (ii) is symmetric and omitted. To prove part (i), I first show that there exists $\eta > 0$ such that $U'(x) > W'(x)$ for all $x \in (y, y+\eta)$. Since $U, W \in C^2$, this is true when $U'(y) > W'(y)$. Suppose that $U'(y) = W'(y)$. Since U and W solve (A.3) with $U(y) \geq W(y)$ and $U'(y) = W'(y)$, it follows that $W''(y) = 2(rW(y) - \mu W'(y))/\widetilde{\sigma}^2 < 2(rU(y) - \mu U'(y))/\sigma^2 = U''(y)$. Hence, there exists $\eta > 0$ such that $U'(x) > W'(x)$ for all $x \in (y, y+\eta)$.

Suppose next that part (i) in the Lemma is not true, and let $y_1 > y$ be the smallest point with $U'(y_1) = W'(y_1)$. By the paragraph above, $U'(x) > W'(x)$ for all $x \in (y, y_1)$, so $U(y_1) > W(y_1)$. Since U and W solve (A.3), then $W''(y_1) = 2(rW(y_1) - \mu W'(y_1))/\widetilde{\sigma}^2 < 2(rU(y_1) - \mu U'(y_1))/\sigma^2 = U''(y_1)$. But this and $U'(y_1) = W'(y_1)$ together imply that $U'(y_1 - \varepsilon) < W'(y_1 - \varepsilon)$ for $\varepsilon > 0$ small, a contradiction. Thus, it must be that $U'(x) > W'(x)$ for all $x > y$. ■

Proof of Proposition 1. I show that, for $\mu \leq 0$, V_1^* is increasing in σ for all $x \in [0, 1/2]$. The proof that V_2^* is increasing in σ for all $x \in [1/2, 1]$ when $\mu \geq 0$ is symmetric and omitted. Suppose then that $\mu \leq 0$. From equation (A.4), it follows that

$$\frac{\partial V_1^*(1/2)}{\partial \sigma} = -\mu \frac{\mu^2\sigma^2(\alpha+\beta)e^{(\alpha+\beta)/2} + r\sigma^2\left(\left(-1+e^{(\alpha+\beta)/2}\right)\sigma^2 + \sigma^2(\alpha+\beta)e^{(\alpha+\beta)/2}\right)}{\left(1+e^{(\alpha+\beta)/2}\right)\sigma^3(\mu^2+2r\sigma^2)^{3/2}} \geq 0,$$

where the inequality follows since $\mu \leq 0$. Fix $\sigma < \widetilde{\sigma}$, and let V_1^* and $\widetilde{V}_1^*$ denote party 1's limiting payoff under σ and $\widetilde{\sigma}$, respectively. The derivative above implies that $\widetilde{V}_1^*(1/2) \geq V_1^*(1/2)$. By Theorem 2, V_1^* and $\widetilde{V}_1^*$ solve (A.3) on $[0, 1/2]$ (but with different values of volatility), with $(\widetilde{V}_1^*)'(0) = (V_1^*)'(0) = 0$. Note first that it must be that $\widetilde{V}_1^*(0) > V_1^*(0)$. Indeed, if $V_1^*(0) \geq \widetilde{V}_1^*(0)$, then Lemma A4 and the fact that $(\widetilde{V}_1^*)'(0) = (V_1^*)'(0) = 0$ together imply that $V_1^*(1/2) > \widetilde{V}_1^*(1/2)$. But this cannot be, since I have just shown that $V_1^*(1/2) \leq \widetilde{V}_1^*(1/2)$. Therefore, it must be that $\widetilde{V}_1^*(0) > V_1^*(0)$. Let $z > 0$ be the smallest point such that $V_1^*(z) = \widetilde{V}_1^*(z)$, and note that z must be such that $(V_1^*(z))' > (\widetilde{V}_1^*)'(z)$. Using

Lemma A4 again, it follows that $V_1^*(x) > \widetilde{V}_1^*(x)$ for all $x > z$. Since $\widetilde{V}_1^*(1/2) \geq V_1^*(1/2)$, it must be that $z \geq 1/2$, and hence $\widetilde{V}_1^*(x) > V_1^*(x)$ for all $x < 1/2$. ■

Proof of Proposition 2. Since $V_1^*(x) + V_2^*(x) = 1$ for all x, to prove the Proposition 2 it suffices to show that V_1^* is increasing in μ for all $x \in [0, 1/2]$ and that V_2^* is decreasing in μ for all $x \in [1/2, 1]$. From equation (A.4), it follows that

$$\frac{\partial V_1^*(1/2)}{\partial \mu} = \frac{r\sigma^2\left(-1 + e^{(\alpha+\beta)/2}\right) + \mu^2((\alpha+\beta)/2)e^{(\alpha+\beta)/2}}{\left(1 + e^{(\alpha+\beta)/2}\right)\left(\mu^2 + 2r\sigma^2\right)^{3/2}} > 0.$$

Fix $\mu < \widetilde{\mu}$ and let $E[\cdot]$ and $\widetilde{E}[\cdot]$ denote the expectation operator when the drift is μ and $\widetilde{\mu}$, respectively. Let V_i^* and $\widetilde{V}_i^*$ denote party i's payoff under μ and $\widetilde{\mu}$, respectively. Let $\tau = \inf\{t : x_t \geq 1/2\}$. Then, by Remark A1, for all $x \in [0, 1/2]$,

$$\begin{aligned}
\widetilde{V}_1^*(x) &= \widetilde{E}\left[e^{-r\tau}\widetilde{V}_1^*(1/2)|x_0 = x\right] = \widetilde{V}_1^*(1/2)\widetilde{E}\left[e^{-r\tau}|x_0 = x\right] \\
&> V_1^*(1/2)\widetilde{E}\left[e^{-r\tau}|x_0 = x\right] > V_1^*(1/2)E\left[e^{-r\tau}|x_0 = x\right] = V_1^*(x),
\end{aligned}$$

where the first inequality follows since $\widetilde{V}_1^*(1/2) > V_1^*(1/2)$ and the second follows since the expected time until x_t reaches 1/2 is shorter under $\widetilde{\mu}$ than under $\mu < \widetilde{\mu}$. Hence, $\widetilde{V}_1^*(x) > V_1^*(x)\ \forall\ x \in [0, 1/2]$. A symmetric argument shows that $\widetilde{V}_2^*(x) < V_2^*(x)\ \forall\ x \in [1/2, 1]$. ■

A.4 Proofs of Section 3.2

Proof of Theorem 3. Let $W_i^\Delta(x, t)$ denote party i's SPE payoffs at time $t \in T(\Delta)$ with $x_t = x$. Note that the subgame that starts at any time $t \geq t^*$ at which parties haven't yet reached an agreement is identical to the game in Section 2.[23] Therefore, in any SPE parties will reach an agreement at time $t \geq t^*$ if they haven't done so already. Moreover, party i's payoff from this agreement will be equal to $V_i^\Delta(x_t)$. It then follows that $W_i^\Delta(x, t^*) = V_i^\Delta(x) + K \times \mathbf{1}_{\{x \in M_i\}}$ and $W_i^\Delta(x, t) = V_i^\Delta(x)$ for all $t > t^*$.

For $i = 1, 2$ and $t \in T(\Delta), t < t^*$, let $U_i(z, x, t) = u_i(z) + e^{-r(t^*-t)}KQ_i^z(x, t)$ be the payoff that party i gets by implementing policy $z \in [0, 1]$ at time t when $x_t = x$. Since $u_i(\cdot)$ and $h(x, \cdot)$ are continuous and since $Q_i(\cdot, t)$ is also continuous, it follows that $U_i(\cdot, x, t)$ is continuous. Suppose that parties have not reached an agreement by time $t^* - \Delta$, and that $x_{t^*-\Delta} = x$. For $i = 1, 2$, party i's payoff if there is no agreement at time $t^* - \Delta$ is $e^{-r\Delta}E[W_i^\Delta(x_{t^*}, t^*)|x_{t^*-\Delta} = x]$. Let $A_i(x, t^* - \Delta) := \{z \in [0, 1] : U_i(z, x, t^* - \Delta) \geq$

[23] If $t = t^*$, the subgame that starts immediately after the election is identical to the game in Section 2.

$e^{-r\Delta}E[W_i^{\Delta}(x_{t^*}, t^*)|x_{t^*-\Delta} = x]\}$ be the set of policies that give party i a payoff weakly higher than the payoff from delaying an agreement, and let $A(x, t^* - \Delta) := A_1(x, t^* - \Delta) \cap A_2(x, t^* - \Delta)$. If $A(x, t^* - \Delta) = \emptyset$, there is no policy that both parties would agree to implement. In this case, there will be delay at time $t^* - \Delta$, so party i's payoff is $W_i(x, t^* - \Delta) = e^{-r\Delta}E[W_i^{\Delta}(x_{t^*}, t^*)|x_{t^*-\Delta} = x]$ for $i = 1, 2$. Otherwise, if $A(x, t^* - \Delta) \neq \emptyset$ the majority party j offers $z(x, t^* - \Delta) \in \arg\max_{z \in A(x,t^*-\Delta)} U_j(z, x, t^* - \Delta)$, and the minority party accepts this offer.[24] In this case, for $i = 1, 2$, party i's payoff is $U_i(z(x, t^* - \Delta), x, t^* - \Delta)$.

The first paragraph above establishes parts (i) and (ii) of Theorem 3, while the second paragraph establishes part (iii) for $t = t^* - \Delta$. Consider next time $t^* - 2\Delta$. Party i's payoff in case of delay is $e^{-r\Delta}E[W_i^{\Delta}(x_{t^*-\Delta}, t^* - \Delta)|x_{t^*-2\Delta} = x]$. Let $A_i(x, t^* - 2\Delta) = \{z \in [0, 1] : U_i(z, x, t^* - 2\Delta) \geq e^{-r\Delta}E[W_i^{\Delta}(x_{t^*-\Delta}, t^*)|x_{t^*-2\Delta} = x]\}$ and let $A(x, t^* - 2\Delta) = A_1(x, t^* - 2\Delta) \cap A_2(x, t^* - 2\Delta)$. If $A(x, t^* - 2\Delta) = \emptyset$, there is no policy that both parties would agree to implement. In this case, there will be delay at $t^* - 2\Delta$, so for $i = 1, 2$ party i's payoff is $W_i(x, t^* - 2\Delta) = e^{-r\Delta}E[W_i^{\Delta}(x_{t^*-\Delta}, t^*)|x_{t^*-2\Delta} = x]$. Otherwise, if $A(x, t^* - 2\Delta) \neq \emptyset$ the majority party j offers $z(x, t^* - 2\Delta) \in \arg\max_{z \in A(x,t^*-2\Delta)} U_j(z, x, t^* - \Delta)$ and the minority party accepts this offer. In this case, for $i = 1, 2$ party i's payoff is $U_i(z(x, t^* - 2\Delta), x, t^* - 2\Delta)$. Repeating these arguments for all $t \in T(\Delta)$ completes the proof of Theorem 3. ■

Proof of Proposition 3. Note that $W_1^{\Delta}(x, t) + W_2^{\Delta}(x, t) \leq 1 + Ke^{-r(t^*-t)}$ for all $t < t^*$ and all $x \in [0, 1]$. Therefore, there exists $s > 0$ such that $E[e^{-r\Delta}(W_1^{\Delta}(x_{t+\Delta}, t + \Delta) + W_2^{\Delta}(x_{t+\Delta}, t + \Delta))|x_t = x] < 1$ for all t with $t^* - t > s$ and all $x \in [0, 1]$; that is, s is such that $Ke^{-rs} \leq 1 - e^{-r\Delta}$. Note that the value of s is increasing in K. For all such t and for all $x \in [0, 1]$, there exists a policy $z \in (0, 1)$ such that $u_i(z) \geq E[e^{-r\Delta}W_i^{\Delta}(x_{t+\Delta}, t + \Delta)|x_t = x]$ for $i = 1, 2$. Since party i's utility from implementing policy z at time t is weakly larger than $u_i(z)$ regardless of the level of political power x, it follows that $z \in A_i(x, t)$ for $i = 1, 2$ and for all $x \in [0, 1]$. Therefore, $A(x, t) = A_1(x, t) \cap A_2(x, t) \neq \emptyset$ for all $x \in [0, 1]$, and so parties always reach an agreement at t. ■

A.5 Proofs of Section 3.3

Proof of Lemma 1. I first show that $W_i^{\Delta}(x, t) \geq \underline{W}_i^{\Delta}(x, t)$ for all $t < t^*$ and all $x \in [0, 1]$. To see this, note that party i can always unilaterally generate delay at each time $t < t^*$, either by rejecting offers when $x_t \notin M_i$ and by choosing to pass on its right to make offers

[24] There are two things to note. First, the set of policies that maximize party j's payoff is non-empty since $A(x, t^* - \Delta)$ is compact and $U_j(z, x, t^* - \Delta)$ is continuous. Second, by our restriction on SPE, the majority party will always make such an offer even if its indifferent between making this offer or delaying.

when $x_t \in M_i$. At times $t < t^*$, the payoff that party i gets by unilaterally delaying an agreement until time t^* is equal to $E[e^{-r(t^*-t)}V_i^\Delta(x_{t^*})|x_t] + e^{-r(t^*-t)}KQ_i(x_t,t) = \underline{W}_i^\Delta(x_t,t)$. It thus follows that $W_i^\Delta(x,t) \geq \underline{W}_i^\Delta(x,t)$ for all (x,t) with $t < t^*$.

Next, I show that $W_i^\Delta(x,t) \leq \overline{W}_i^\Delta(x,t)$ for all $t < t^*$ and all $x \in [0,1]$. To see this, note $W_1^\Delta(x,t) + W_2^\Delta(x,t) \leq 1 + Ke^{-r(t^*-t)}$ for all $t < t^*$ and all $x \in [0,1]$; that is, the sum of the parties' payoffs is bounded above by the total payoff they would get if they implemented a policy today, which is equal to $u_1(z) + u_2(z) = 1$, plus the sum of the parties' discounted payoff coming from the fact that one party will win the election, which is equal to $Ke^{-r(t^*-t)}$. From this inequality it follows that for all $t < t^*$ and all $x \in [0,1]$

$$\begin{aligned} W_i^\Delta(x,t) &\leq 1 + Ke^{-r(t^*-t)} - W_j^\Delta(x,t) \\ &\leq 1 + Ke^{-r(t^*-t)} - E[e^{-r(t^*-t)}V_j^\Delta(x_{t^*})|x_t = x] - e^{-r(t^*-t)}KQ_j(x,t) \\ &= 1 - e^{-r(t^*-t)} + E[e^{-r(t^*-t)}V_i^\Delta(x_{t^*})|x_t = x] + e^{-r(t^*-t)}KQ_i(x,t), \end{aligned}$$

where the second inequality follows since $W_j^\Delta(x,t) \geq \underline{W}_j^\Delta(x,t)$ and the equality follows since $V_1^\Delta(x) + V_2^\Delta(x) = 1$ for all x and since $Q_1(x,t) + Q_2(x,t) = 1$ for all x and all $t < t^*$. Hence, $W_i^\Delta(x,t) \leq \overline{W}_i^\Delta(x,t)$ for all $t < t^*$ and for all $x \in [0,1]$. ■

Lemma A5 *For $i = 1,2$ and for all $t < t^*$, the function $\underline{W}_i^*(x,t)$ solves*

$$r\underline{W}_i^*(x,t) = \frac{\partial}{\partial t}\underline{W}_i^*(x,t) + \mu\frac{\partial}{\partial x}\underline{W}_i^*(x,t) + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial x^2}\underline{W}_i^*(x,t),$$

with $\underline{W}_i^*(x,t^*) = V_i^*(x) + K\mathbf{1}_{\{x\in M_i\}}$ and $\frac{\partial}{\partial x}\underline{W}_i^*(x,t)|_{x=0} = \frac{\partial}{\partial x}\underline{W}_i^*(x,t)|_{x=1} = 0$.

Proof. For $i = 1,2$ and $t < t^*$ let $w_i(x,t) = E[V_i^*(x_{t^*})|x_t = x]$. Hence, $\underline{W}_i^*(x,t) = e^{-r(t^*-t)}(w_i(x,t) + KQ_i(x,t))$. Note that $Q_i(x,t) = P_i(t^* - t, x)$ (recall from the proof of Theorem 2 that $P_i(s,x) = \Pr[x_s \in M_i|x_0 = x] = \int_{M_i} p(x,y,s)dy$). Since $P_i(s,x)$ solves (A.2) with $P_i(0,x) = \mathbf{1}_{\{x\in M_i\}}$ and $\frac{\partial}{\partial x}P_i(s,x)|_{x=0} = \frac{\partial}{\partial x}P_i(s,x)|_{x=1} = 0$, it follows that, for all $(x,t) \in [0,1] \times [0,t^*)$, $Q_i(x,s)$ solves

$$0 = \frac{\partial}{\partial s}Q_i(x,s) + \mu\frac{\partial}{\partial x}Q_i(x,s) + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial x^2}Q_i(x,s),$$

with boundary conditions $Q_i(x,t^*) = \mathbf{1}_{\{x\in M_i\}}$ and $\frac{\partial}{\partial x}Q_i(x,s)|_{x=0} = \frac{\partial}{\partial x}Q_i(x,s)|_{x=1} = 0$. On the other hand, since $p(x,y,t)$ solves (A.2) with $\lim_{s\to 0} p(x,y,s) = \mathbf{1}_{\{y=x\}}$ and $\frac{\partial}{\partial x}p(x,y,s)|_{x=0} = \frac{\partial}{\partial x}p(x,y,s)|_{x=1} = 0$ for all $s < t^*$, it follows that, for all $(x,t) \in [0,1] \times [0,t^*)$, $w_i(x,s) =$

$E[V_i^*(x_{t^*})|x_t = x] = \int_0^1 V_i(y)p(x,y,t^*-t)dy$ solves

$$0 = \frac{\partial}{\partial s}w_i(x,s) + \mu\frac{\partial}{\partial x}w_i(x,s) + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial x^2}w_i(x,s),$$

with $\lim_{s\to t^*} w_i(x,s) = V_i(x)$ and $\frac{\partial}{\partial x}w_i(x,s)|_{x=0} = \frac{\partial}{\partial x}w_i(x,s)|_{x=1} = 0$.

The analysis above implies that $\underline{W}_i^*(x,t) = e^{-r(t^*-t)}(w_i(x,t) + KQ_i(x,t)) \in C^{2,1}$ for all $(x,t) \in [0,1]\times[0,t^*)$. Note that by the law of iterated expectations $Y_t = e^{-rt}\underline{W}_i^*(x,t)$ is a martingale for all $t < t^*$. By Ito's lemma, for all $(x,t) \in [0,1]\times[0,t^*)$,

$$dY_t = e^{-rt}\left[-r\underline{W}_i^*(x,t) + \frac{\partial}{\partial s}\underline{W}_i(x,s) + \mu\frac{\partial}{\partial x}\underline{W}_i(x,s) + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial x^2}\underline{W}_i(x,s)\right]dt + \sigma\frac{\partial}{\partial x}\underline{W}_i(x,s)dB_t.$$

Since Y_t is a martingale, the term inside the square brackets must be zero. This shows that $\underline{W}_i^*(x,t)$ solves the equation in the statement of the lemma. Finally, the boundary conditions follow from the boundary conditions of $w_i(x,t)$ and $Q_i(x,t)$. ■

Lemma A6 *Fix a time $t \in T(\Delta), t < t^*$ and an $x \in [0,1]$. If there exists an offer $z' \in [0,1]$ and a party $j \in \{0,1\}$ such that $U_j(z',x,t) = E[e^{-r\Delta}W_j^\Delta(x_{t+\Delta},t+\Delta)|x_t = x]$, then parties reach an agreement at time t if $x_t = x$.*

Proof. Suppose such an offer z' exists, and note that this implies that $z' \in A_j(x,t)$ (i.e., party j would accept an offer to implement z' at time t with $x_t = x$). Since $W_1^\Delta(x_{t+\Delta},t+\Delta) + W_2^\Delta(x_{t+\Delta},t+\Delta) \le 1 + Ke^{-r(t^*-t-\Delta)}$ for all $x_{t+\Delta}$, it follows that $W_i^\Delta(x_{t+\Delta},t+\Delta) \le 1 + Ke^{-r(t^*-t-\Delta)} - W_j^\Delta(x_{t+\Delta},t+\Delta)$ for all $x_{t+\Delta}$. Therefore,

$$E\left[e^{-r\Delta}W_i^\Delta(x_{t+\Delta},t+\Delta)|x_t = x\right] \le e^{-r\Delta} + Ke^{-r(t^*-t)} - E\left[e^{-r\Delta}W_j^\Delta(x_{t+\Delta},t+\Delta)|x_t = x\right]. \tag{A.5}$$

Party i's payoff from implementing policy z' at time t with $x_t = x$ is

$$\begin{aligned} U_i(z',x,t) &= 1 + Ke^{-r(t^*-t)} - U_j(z',x,t) \\ &= 1 + Ke^{-r(t^*-t)} - E\left[e^{-r\Delta}W_j^\Delta(x_{t+\Delta},t+\Delta)|x_t = x\right], \end{aligned}$$

where the first equality follows since $U_1(z,x,t) + U_2(z,x,t) = 1 + Ke^{-r(t^*-t)}$ for all $z \in [0,1]$. Combining the equation above with equation (A.5) it follows that $U_i(z',x,t) > E[e^{-r\Delta}W_i^\Delta(x_{t+\Delta},t+\Delta)|x_t = x]$, so that $z' \in A_i(x,t)$. Hence, $A(x,t) = A_1(x,t)\cap A_2(x,t) \neq \emptyset$, so parties reach an agreement at time t if $x_t = x$. ■

Proof of Proposition 4. Let (x,t) be a state satisfying the conditions in part (i) of the

Proposition, and suppose by contradiction that parties reach an agreement at time t when $x_t = x$. Since $U_i(z,x,t) < \underline{W}_i(x,t)$ for all $z \in [0,1]$, this implies that party i's SPE payoff at state (x,t) is strictly lower than $\underline{W}_i(x,t)$, a contradiction to the fact that that $\underline{W}_i(x,t)$ is a lower bound to party i's payoff. Thus, there must be delay at state (x,t).

Next, let (x,t) be a state satisfying the conditions in part (ii) of the Proposition. By Lemma 1

$$E[e^{-r\Delta}\underline{W}_i^{\Delta}(x_{t+\Delta},t+\Delta)|x_t] \leq E[e^{-r\Delta}W_i^{\Delta}(x_{t+\Delta},t+\Delta)|x_t] \leq E[e^{-r\Delta}\overline{W}_i^{\Delta}(x_{t+\Delta},t+\Delta)|x_t]. \tag{A.6}$$

Note further that $E[e^{-r\Delta}\underline{W}_i^{\Delta}(x_{t+\Delta},t+\Delta)|x_t = x] = E[e^{-r\Delta}E[e^{-r(t^*-t-\Delta)}W_i^{\Delta}(x_{t^*},t)|x_{t+\Delta}]|x_t = x] = E[e^{-r(t^*-t)}W_i^{\Delta}(x_{t^*},t)|x_t = x] = \underline{W}_i^{\Delta}(x_t,t)$ and that $E[e^{-r\Delta}\overline{W}_i^{\Delta}(x_{t+\Delta},t+\Delta)|x_t = x] = e^{-r\Delta} - e^{-r(t^*-t)} + \underline{W}_i^{\Delta}(x_t,t) < \overline{W}_i^{\Delta}(x_t,t)$. Since $U_i(\cdot,x,t)$ is continuous and since $U_i(z',x,t) \leq \underline{W}_i^{\Delta}(x,t)$ and $U_i(z'',x,t) \geq \overline{W}_i^{\Delta}(x,t)$ for $z',z'' \in [0,1]$, there exists $z \in [0,1]$ such that $U_i(z,x,t) = E[e^{-r\Delta}W_i^{\Delta}(x_{t+\Delta},t+\Delta)|x_t = x]$. Hence, by Lemma A.6 parties reach an agreement at state (x,t). ■

A.6 Proofs of Section 3.4

Proof of Proposition 5. To prove the Proposition, note first that it must be that either $V_1^{\Delta}(1) > 1/2$ and/or $V_2^{\Delta}(0) > 1/2$.[25]. Therefore, since $E[e^{-r(t^*-t)}V_i^{\Delta}(x_{t^*})|x_t = x] \to V_i^{\Delta}(x)$ as $t \to t^*$, there must exist $i = 1,2$ and $(x,t) \in [0,1] \times T(\Delta), t < t^*$ with $x \in M_i$ such that $E[e^{-r(t^*-t)}V_i^{\Delta}(x_{t^*})|x_t = x] > 1/2$. Suppose that $i = 1$. Party i's payoff from implementing policy z at time t when $x_t = x$ is $U_1(z,x,t) = z + Ke^{-r(t^*-t)}Q_1(x + h(x,z),t)$. Since $E[e^{-r(t^*-t)}V_1^{\Delta}(x_{t^*})|x_t = x] > 1/2$ and since $h(x,z)$ satisfies (5), it follows that $U_1(z,x,t) = z + Ke^{-r(t^*-t)}Q_1(x,t) < \underline{W}_1^{\Delta}(x,t)$ for all $z \leq 1/2$. On the other hand, for all $z > 1/2$ we have that $Q_1(x + h(x,z),t) = Q_1(x - \lambda(z - \frac{1}{2}),t) < Q_1(x,t)$. Let z' be such that $z' = E[e^{-r(t^*-t)}V_1^{\Delta}(x_{t^*})|x_t = x] > 1/2$, and note that $U_1(z,x,t) < \underline{W}_1^{\Delta}(x,t)$ for all $z \in [1/2,z']$. Finally, for all $z \in [z',1]$,

$$\underline{W}_1^{\Delta}(x,t) - U_1(z,x,t) = E[e^{-r(t^*-t)}V_1^{\Delta}(x_{t^*})|x_t = x] - z + Ke^{-r(t^*-t)}\left[Q_1(x,t) - Q_1^z(x,t)\right].$$

Since the term in squared brackets is negative for all $z \in [z',1/2]$, for all such z there exists $K(z) > 0$ such that $\underline{W}_1^{\Delta}(x,t) > U_1(z,x,t)$ if $K > K(z)$. Moreover, it is clear that $K(z)$ is bounded for all $z \in [z',1]$. Letting $\overline{K} = \sup_{z\in[z',z]} K(z)$, it follows that $\underline{W}_1^{\Delta}(x,t) > U_1(z,x,t)$

[25]This follows since $V_1^{\Delta}(x)$ is strictly increasing in x and $V_2^{\Delta}(x) = 1 - V_1^{\Delta}(x)$ is strictly decreasing in x

for all $z \in [0,1]$ whenever $K > \overline{K}$, so by Proposition 4 parties delay an agreement at time t if $x_t = x$. Finally, note that a symmetric argument would establish the result in the Proposition if $i = 2$. ■

Proof of Proposition 6. Fix $x \in M_i \cap (0,1)$ and $t < t^*$. If $h(x,z)$ is given by (6), policy z_j is the policy that maximizes $U_j(\cdot, x, t)$ (where z_j is party j's ideal policy). Moreover, note that for all such (x,t),

$$\underline{W}_j^\Delta(x,t) - U_j(z_j, x, t) = E[e^{-r(t^*-t)}V_j^\Delta(x_{t^*})|x_t = x] - 1 + Ke^{-r(t^*-t)}\left[Q_j(x,t) - Q_j^{z_j}(x,t)\right].$$

Since the term in squared brackets is negative, there exists $\overline{K}$ such that $\underline{W}_j^\Delta(x,t) > U_j(z_j, x, t)$ whenever $K > \overline{K}$. Since z_j maximizes $U_j(\cdot, x, t)$, it follows from Proposition 4 that parties will delay an agreement at time t if $x_t = x$. ■

Proof of Proposition 7. I first show that parties reach an agreement at all states (x,t) with $t \in T(\Delta), t < t^*$. To see this, note that for all (x,t), $U_1(0,x,t) = E[e^{-r(t^*-t)}KQ_1(x + h(0,x))|x_t = x] < \underline{W}_1^\Delta(x,t)$ and $U_1(1,x,t) = 1 + E[e^{-r(t^*-t)}KQ_1(x + h(1,x))|x_t = x] > \overline{W}_1^\Delta(x,t)$. Similarly, $U_2(1,x,t) < \underline{W}_2^\Delta(x,t)$ and $U_2(0,x,t) > \overline{W}_2^\Delta(x,t)$. Proposition 4 (ii) then implies that parties reach an agreement at state (x,t).

Next, I show that the parties' SPE payoffs satisfy equation (7). As a first step to establish this result, I show that for all $x \in M_i$ and all $t \in T(\Delta), t < t^*$, there exists an offer $z \in [0,1]$ such that $U_j(z,x,t) = E[e^{-r\Delta}W_j^\Delta(x_{t+\Delta}, t+\Delta)|x_t = x]$. To see this, note that by the paragraph above there exists z', z'' such that $U_j(z',x,t) < \underline{W}_j^\Delta(x,t)$ and $U_j(z'',x,t) > \overline{W}_j^\Delta(x,t)$. Moreover, note also that $\underline{W}_j^\Delta(x,t) = E[e^{-r\Delta}\underline{W}_j^\Delta(x_{t+\Delta}, t+\Delta)|x_t = x]$ and that $\overline{W}_j^\Delta(x,t) > E[e^{-r\Delta}\overline{W}_j^\Delta(x_{t+\Delta}, t+\Delta)|x_t = x]$. Then, by (A.6) and continuity of $U_j(\cdot, x, t)$, there must exist an offer $z \in [0,1]$ such that $U_j(z,x,t) = E[e^{-r(t^*-t)}W_j^\Delta(x_{t+\Delta}, t+\Delta)|x_t = x]$. Note that this is the best offer for party i among the offers that party j finds acceptable at state (x,t) with $x \in M_i$, and hence is the offer that party i will make in equilibrium.

I use this observation to show that the parties' SPE payoffs satisfy equation (7). The proof is by induction. Consider time $t = t^* - \Delta$. By the previous paragraph, for all $x \in M_i$ party i makes an offer z such that $U_j(z,x,t) = E[e^{-r\Delta}W_j^\Delta(x_{t^*}, t^*)|x_{t^*-\Delta} = x] = E[e^{-r\Delta}V_j^\Delta(x_{t^*})|x_{t^*-\Delta} = x] + e^{-r\Delta}KQ_i(x, t^* - \Delta)$, and party j accepts such an offer. Since $E[e^{-r\Delta}V_j^\Delta(x_{t^*})|x_{t^*-\Delta} = x] = V_j^\Delta(x)$ for all $x \in M_i$, it follows that party j's payoff at any state $(x, t^* - \Delta)$ with $x \in M_i$ is equal to $V_j^\Delta(x) + e^{-r\Delta}KQ_j(x, t^* - \Delta)$. Since parties reach an agreement at $t^* - \Delta$, the sum of their payoffs is $1 + Ke^{-r\Delta}$. Hence, party i's payoff at any state $(t^* - \Delta, x)$ with $x \in M_i$ is equal to $1 + Ke^{-r\Delta} - V_j^\Delta(x) - e^{-r\Delta}KQ_j(x, t^* - \Delta) =$

$V_i(x) + e^{-r\Delta}KQ_i(x, t^* - \Delta)$, where the equality follows since $V_i^\Delta(x) + V_j^\Delta(x) = 1$ for all x and since $Q_i(x,t) + Q_j(x,t) = 1$ for all (x,t) with $t < t^*$. Therefore, the parties' SPE payoffs satisfy equation (7) at $t = t^* - \Delta$.

Suppose next that the parties' SPE payoffs satisfy (7) for $t = t^* - \Delta, t^* - 2\Delta, ..., t^* - n\Delta$, and let $s = t^* - n\Delta$. At states $(x, s-\Delta)$ with $x \in M_i$, party i makes an offer z such that $U_j(z, x, s-\Delta) = E[e^{-r\Delta}W_j^\Delta(x_s, s)|x_{s-\Delta} = x] = E[e^{-r\Delta}V_j^\Delta(x_s)|x_{s-\Delta} = x] + e^{-r(t^*-s-\Delta)}KQ_j(x, s-\Delta)$ (where the last equality follows from the induction hypothesis), and party j accepts such an offer. Since $E[e^{-r\Delta}V_j^\Delta(x_s)|x_{s-\Delta} = x] = V_j^\Delta(x)$ for all $x \in M_i$, it follows that party j's payoff at any state $(x, s-\Delta)$ with $x \in M_i$ is equal to $V_j^\Delta(x) + e^{-r(t^*-(s-\Delta))}KQ_j(x, s-\Delta)$. Since parties reach an agreement at $t^* - \Delta$, the sum of their payoffs is $1 + Ke^{-r(t^*-(s-\Delta)}$. Therefore, party i's payoff at any state $(x, s-\Delta)$ with $x \in M_i$ is equal to $1 + Ke^{-r(t^*-(s-\Delta))} - V_j^\Delta(x) - e^{-r(t^*-(s-\Delta))}KQ_j(x, s-\Delta) = V_i^\Delta(x) + e^{-r(t^*-(s-\Delta))}KQ_i(x, s-\Delta)$, so the parties' payoffs also satisfy equation (7) at $t = s - \Delta$. ■

A.7 General bargaining protocols

Consider game without elections as in Section 2, but with the following bargaining protocol: for all $x \in [0,1]$ party 1 makes offers with probability $p_1(x)$ and party 2 makes offers with probability $p_2(x) = 1 - p_1(x)$. Assume further that $p_1(\cdot)$ is continuous and increasing. By arguments similar to those in Theorem 1, this game has a unique SPE. In the unique SPE parties always reach an immediate agreement. Moreover, party i's SPE payoffs satisfy

$$\begin{aligned} V_i^\Delta(x) &= p_i(x)\left(1 - e^{-r\Delta}E\left[V_j^\Delta(x_{t+\Delta})\middle| x_t = x\right]\right) + (1 - p_i(x))e^{-r\Delta}E\left[V_i^\Delta(x_{t+\Delta})\middle| x_t = x\right] \\ &= p_i(x)(1 - e^{-r\Delta}) + e^{-r\Delta}E\left[V_i^\Delta(x_{t+\Delta})\middle| x_t = x\right], \end{aligned} \tag{A.7}$$

where the equality follows since $V_1^\Delta(y) + V_2^\Delta(y) = 1$ for all $y \in [0,1]$. Setting $t = 0$ and solving (A.7) forward yields

$$V_i^\Delta(x) = E\left[\frac{1 - e^{-r\Delta}}{\Delta}\sum_{k=0}^{\infty}\Delta e^{-rk\Delta}p_i(x_{k\Delta})\middle| x_0 = x\right].$$

Moreover, $\lim_{\Delta\to 0} V_i^\Delta(x) = E[r\int_0^\infty e^{-rt}p_i(x_t)dt|x_0 = x] = \hat{V}_i^*(x)$. Finally, by Corollary 2.4 in chapter 5 of Harrison (1985), for all $x \in [0,1]$ the function $\hat{V}_i^*(x) = E[r\int_0^\infty e^{-rt}p_i(x_t)dt|x_0 = x]$ solves $r\hat{V}_i^*(x) = rp_i(x) + \mu(\hat{V}_i^*)'(x) + \frac{1}{2}\sigma^2(\hat{V}_i^*)''(x)$, with $(\hat{V}_i^*)'(0) = (\hat{V}_i^*)'(1) = 0$.

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