

Why the Political World is Flat: An Endogenous “Left” and “Right” in Multidimensional Political Conflict

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Abstract

This paper analyzes a multidimensional model of politics, assuming that political differences reflect matters of opinion, rather than inherent conflicts of interest. In equilibrium, candidates adopt polarized positions and citizens vote for whichever appears to be superior. In general, there are infinitely many such equilibria, but correlations across issues eliminate all but two. One of these aggregates information more efficiently, and is therefore likely to be focal, but political issues that are bundled sub-optimally can also persist in equilibrium. Even in the best equilibrium, substantial information loss is inevitable. The equilibrium characterization closely matches common political rhetoric, suggesting that a model such as this might be implicit in popular political discourse.

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1 Introduction

The world of public policy is complex and multifaceted. Elected officials must decide tax policy, foreign policy, health policy, education policy, immigration policy, social policy, and many others, each of which encompasses numerous more narrow issues that are themselves complex and multifaceted. Voters must consider all of the same issues in order to properly evaluate candidates, in addition to personal characteristics such as integrity and management skills. The number of dimensions required to model such an environment is enormous; indeed, every line of legislation could be viewed as a separate dimension, along which policy could be adjusted. In contrast, existing political economic models are almost exclusively one-dimensional.

Multidimensional analysis has been stymied for a variety of reasons. In Plott's (1967) straightforward extension of the one-dimensional Downsian model, for example, equilibrium simply doesn't exist.¹ This is sometimes interpreted as a prediction of political chaos, in that challengers should always be able to unseat incumbents, and that successive majority votes could cycle or lead to *any* eventual outcome (McKelvey, 1979), but as Tullock (1981) points out, such instability is not evident empirically. The probabilistic voting models of Hinich (1978) and Coughlin and Nitzan (1982) do exhibit equilibria, but candidates converge to the political center, a result that arises in one dimension as well, but is at odds with evidence that, empirically, candidates tend to be quite polarized.² Policy motivated candidates can diverge (Wittman, 1983; Calvert, 1985), but only slightly (McMurray 2014a), unless uncertainty is substantial. Equilibrium polarization can occur in the citizen-candidate models of Osborne and Slivinski (1996) and Besley and Coate (1997), but only if candidates tie in expectation, whereas empirical margins of victory are often quite large.³ The latter model also requires that strategic voters coordinate to ignore more moderate entrants, which generates a problem of indeterminacy: in the authors' words (p. 94), "basically *any* pair of candidates who split the voters evenly can be an equilibrium".⁴

The second reason for the almost-universal restriction to one dimension is that, empirically, the political world looks surprisingly one-dimensional. That is, voters' positions on political issues appear to be correlated, and so can be succinctly summarized by voter *ideology*, as indexed on a one-dimensional scale ranging from liberal to moderate to conservative. In the words of Converse (1964, p. 207), "...if a person is opposed to the expansion of social security, he is probably a conservative and is probably opposed as well to any nationalization of private industries, federal aid to education, sharply progressive income taxation, and so forth." Poole and Rosenthal (1997, 2001) formalize this statistically: ideology estimates for a one-dimensional spatial model correctly predict almost 90% of the individual roll call votes cast by members of the U.S. House and Senate between 1789 and 1998. They cite similar findings for the European Parliament and the U.N. General Assembly, as well as the British, French, Czech, and Polish parliaments. Shor and McCarty (2011) find similar results for U.S. state legislatures, and also find similar results using responses to a political survey, instead of roll call votes.⁵ Using the same survey, Shor (2011) then finds voter responses only slightly less consistently one-dimensional.

¹See also Duggan and Fey (2005). Duggan and Jackson (2005) prove the existence of mixed-strategy equilibria, but as Austen-Smith and Banks (2005) discuss, the empirical relevance of mixed strategies is unclear in this context. The proof is also not constructive, and so cannot characterize equilibrium behavior.

²See Lee, Moretti, and Butler (2004), the references in Wittman (1983), and especially Bafumi and Herron (2010) and Shor (2011). Another empirical weakness of these models is that if candidates converge then equilibrium voting behavior is purely random, and unrelated to voters' policy preferences.

³Mueller (2003, ch. 11) reports, for example, that U.S. governors have historically won reelection by an average margin of 23%.

⁴See also Eguia (2007).

⁵Poole and Rosenthal (1985) obtain similar results using interest group ratings, instead of roll call votes, to measure legislator ideology.

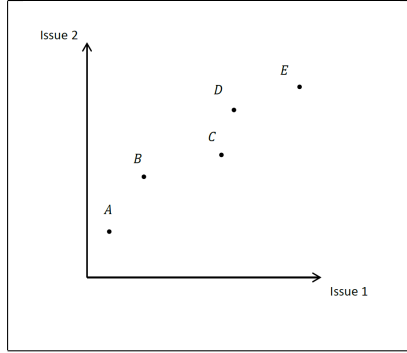


Figure 1: Correlated voter ideal points

Why preferences across issues are so consistently correlated is a puzzle largely unaddressed by existing literature (see Section 2). An even more challenging puzzle is that correlated preferences do not actually resolve the theoretical conundrum described above. Figure 1 illustrates this for five voters in two dimensions: preferences are highly correlated, so that voters' ideal points follow the same order in either dimension. In this case, it is tempting to label voter *C* as the median voter, and predict that, in equilibrium, candidates will cater to this voter's preferences on both issues. To the contrary, however, Plott's (1967) analysis implies that no equilibrium exists: if one candidate caters to *C*, the other can win the election with a position slightly to the northwest, attracting voters *A*, *B*, and *D*. For correlation to induce the Median Voter Theorem, voters' positions on the two issues would have to be *perfectly* correlated; in that case, of course, the model would again be one-dimensional.

This paper presents an election model with multiple policy dimensions, in which voter preferences are correlated across issues, equilibrium always exists, candidates are polarized, the problem of multiple equilibria is largely mitigated, and equilibrium behavior appears one dimensional. The formal analysis treats only a two-dimensional model, but Section 5 explains how this can be extended to accommodate arbitrary dimensions, and also to reproduce auxiliary insights from one-dimensional analyses.

The literature above models voter ideology as a preference parameter, deriving for example from a voter's wealth, which could determine his demand both for redistribution (Romer 1975; Meltzer, Richard 1981) and for public goods (Bergstrom, Goodman 1973). The model below instead adopts the common-values framework of Condorcet (1785). That is, there is some vector of policies that are ultimately best for society, in some objective sense, and all citizens prefer policy vectors as close as possible to this optimum. Policy disagreements stem not from fundamental conflicts of interest, but rather from differences of opinion regarding which policies are optimal. These opinions are modeled as private signals, each correlated with the truth.

To many, Condorcet's (1785) framework has seemed appropriate for juries and other committee settings, but inappropriate for broader political contexts. After all, policies inevitably create winners and losers—or, even if help everyone, benefit some more than others. On the surface, this seems to invalidate the key assumption of common values. On the other hand, however, the most important political objectives are ultimately quite universal: voters across the political spectrum desire world peace and economic stability, for example, and wish to reduce crime, corruption, pollution, and poverty. These issues are exceedingly complex, however, so wide differences of opinion are inevitable: do financial instability and rising health care costs result because regulation is insufficient, or excessive? Does military aggression deter terrorist activity, or provoke it? Are the poor helped or harmed by minimum wage laws and workers' unions? Preferences aside,

experts and non-experts alike predict different outcomes for each of these policies, and so naturally prescribe different courses of action. Over time, opinions evolve, and “preferences” drift accordingly.⁶ Indeed, many expend great effort trying to convert or persuade others—often successfully—via debate, endorsements, policy research, and so on.⁷

Another consideration in favor of a common-value approach is that, in deciding how to vote, citizens seem already to take one another’s preferences into account, supporting policies that do not benefit themselves, but seem to benefit the group. Policies such as minimum wages, food assistance, unemployment insurance, social security, and public education remain quite popular, for example, even among citizens who help pay for, but do not receive these services. Even wealth redistribution, which is inherently zero-sum, is typically discussed in the language of public goods: to liberals, including many who are wealthy, redistribution makes society better for all, by promoting fairness, providing a social “safety net” against uninsurable risks, and preserving democracy from the undue influence of wealthy elites; to conservatives, including many who are poor, redistribution makes society worse, by taking unfair advantage of a wealthy minority, squelching incentives for effort, investment, caution, and innovation, and subjecting democracy to abuse by political elites.⁸ As I explain in McMurray (2013), even a slight concern for others can be amplified dramatically in political arenas, because actions impact large numbers of peers simultaneously. Thus, even citizens who are basically selfish may actually view politics as if through the eyes of a social planner.⁹ Preference aggregation then occurs internally, before votes are ever cast, and the only differences that remain on election day are disagreements regarding the true future impact of policy proposals, or the true best interests of society.¹⁰

In a one-dimensional version (McMurray 2014a) of the model below, citizens who believe the optimal policy is left of center vote for whichever candidate is on the left, while those who believe the optimum is right of center vote for the candidate on the right. The analysis below focuses on truth motivated candidates who, like ordinary citizens, prefer policies as close as possible to the social optimum. Bolstered in their beliefs by the popular support that each will have received if they win, such candidates adopt polarized platforms in equilibrium, and citizens then vote for whoever seems closer to the optimum. An analogous equilibrium

⁶Magleby (1984, ch. 9) documents the volatility of policy preferences, and the field experiments of Gilens (2001), Luskin, Fishkin, and Jowell (2002), and Banerjee et al. (2010) reveal a causal impact of information. Broader ideological positions are more stable, but do drift, especially for young voters (Jennings and Markus, 1984), and sometimes shift abruptly. In the wake of the 2008 financial crisis, for example, former U.S. Federal Reserve chairman Alan Greenspan famously testified before Congress of being suddenly convinced that the ideology motivating his earlier efforts at financial deregulation had been “wrong” (Andrews, Edmund L. 10/23/2008. “Greenspan Concedes Error on Regulation”, *New York Times*. <http://www.nytimes.com/2008/10/24/business/economy/24panel.html>).

⁷In their review of empirical literature, DellaVigna and Gentzkow (2010) conclude that patterns of political persuasion are broadly consistent with Bayesian updating.

⁸According to Fong (2001), this is more than just rhetoric: preferences for redistribution correlate empirically with voters’ beliefs regarding the prevalence of poverty, and about the relative importance of luck and effort for economic success, more strongly than with demographic variables such as their own incomes. See also the references cited in McMurray (2014a).

⁹One way to formalize this logic is with a standard model of altruism, in which voter i ’s utility function $U_i = u_i + \sum_{j \neq i} \alpha_i u_j$ places positive weight α_i on the well-being u_j of each of his peers, and so can be rewritten as a weighted average

$$U_i = (1 - \alpha_i) u_i + \alpha_i n \bar{u} \tag{1}$$

of his own well-being u_i and the average well-being $\bar{u} = \frac{1}{n} \sum_{j=1}^n u_j$ of the population. When the number n of peers is large, the second term in (1) dominates, even if α_i is close to zero. In other words, even a citizen who is almost purely selfish should base political decisions almost entirely on his perception of the common good, rather than his private interest.

¹⁰In principle, conflicts of interest could remain even between social planners, who prioritize common objectives differently, or aggregate preferences according to different social welfare functions. Reasons for such conflict are far less obvious, however, than reasons for policy conflicts (e.g. differences in income). If two planners agreed, for example, that one policy would be best under a utilitarian criterion but another would be best under a Rawlsian criterion, they could proceed to discuss reasons why one welfare function or the other more accurately reflects the true interests of society, and eventually become unified.

exists in two or more dimensions, with candidates adopting polarized policy positions, and citizens voting for whoever seems superior. Now, however, there are infinitely many ways to divide the electorate into “left” and “right” halves, and thus infinitely many equilibria.

As noted above, it appears empirically that voters’ opinions are correlated across issues. In an information setting, this actually seems quite natural, because of the inherent logical connections between issues. For the purposes of ending an economic recession, for example, it might turn out to be the case that fiscal stimulus is effective while monetary stimulus is not, or vice versa, but *ex ante*, it is more likely either that both forms of stimulus are beneficial (i.e. because the macroeconomy functions more or less as Keynesian models predict) or that both are wasteful (i.e. as in more classical models). Similarly, a citizen who believes markets to be basically efficient may oppose a variety of regulations that are all supported by those who view market failures as rampant.

To capture these logical links across issues, the analysis below considers the case in which the policy that is truly optimal in one dimension is correlated with the policy that is truly optimal in another. Even if this correlation is weak, the equilibrium effect of this is dramatic: instead of an infinite number of directions in which equilibrium positions can be polarized, there are exactly two. These correspond to the most basic ways issues can be bundled: in the context of the macroeconomic example above, there is a *minor equilibrium* in which one candidate proposes only fiscal stimulus, while the other proposes only monetary stimulus, and a *major equilibrium* in which one candidate proposes both forms of stimulus, while the other proposes neither.

The major equilibrium is more polarized than the minor equilibrium, but aggregates information more effectively, and so is Pareto superior. Because of this, the major equilibrium is likely to be focal, in the sense of Schelling (1960). In that sense, the model offers a unique behavioral prediction. On the other hand, it also warns that an inefficient bundling of political issues could be perpetuated in equilibrium.

The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 introduces the model, and Section 4 analyzes equilibrium incentives, first for uncorrelated issues, and then for correlated issues. Section 5 discusses the robustness of the model’s results to a variety of extensions, arguing that insights from a one-dimensional analysis—such as voter abstention, electoral mandates, and political pandering—apply here too, because political behavior reduces to a single dimension in equilibrium. Section 6 elaborates on the nature of ideological correlations, and applies the insights of the model to explain political rhetoric that is prevalent in public discourse, including that of minor parties. Section 7 concludes, and proofs of analytical results are provided in the appendix.

2 Literature

None of the papers above discuss the puzzle of unidimensionality in politics, but many spatial models attribute ideological heterogeneity to variations in income, which determines the demand for both redistribution (Romer, 1975; Meltzer and Richard, 1981) and public goods (Bergstrom and Goodman, 1973); if income jointly determines the demand for multiple public goods or multiple forms of redistribution, this might implicitly provide a justification for unidimensionality, as well. On the other hand, however, income is not likely to induce a *perfect* correlation across issues, which, as Section 1 discusses, would be necessary for supporting equilibrium in a standard multidimensional spatial model. Furthermore, as I discuss at length in McMurray (2014a), the relationship between income and ideology is problematic empirically: most notably, many wealthy voters favor wealth redistribution, while many of the poor do not.¹¹

¹¹Moreover, income correlations are sometimes contradictory: environmental protection and military investment are both textbook examples of public goods, but the former is traditionally favored by liberals, and the latter by conservatives.

The topic of unidimensionality does arise in the communication models of Spector (2000) and DeMarzo, Vayanos, and Zwiebel (2003).¹² Spector (2000) considers two groups of agents who start with different prior beliefs about a commonly-valued, multidimensional state variable. As these groups learn and communicate over time, their beliefs converge in every dimension except the direction of prior disagreement, because communication in this direction lacks credibility. The logic of Battaglini (2002) suggests, however, that adding a third set of beliefs would restore credibility (even in higher dimensions) by allowing each group to infer information from the cross-section of messages from the other two. By contrast, learning in the model below is impaired unless there are as many political parties as political issues, as Section 5 emphasizes.

DeMarzo et al. (2003) show that repeatedly circulating the same information through a social network leads eventually to consensus, but first reduces all agents' opinions to a single dimension, with orientation depending on the network structure. This requires that agents are only boundedly rational, however, and mistake repeated information for new information; otherwise, beliefs would not change after the first round of communication. In the model below, agents are fully rational. Informational impediments stem from the inherent discreteness of political action, relative to the space of possible signals, and political orientation is determined by the logical structure of issues.

3 The Model

A society consists of N citizens where, following Myerson (1998, 2000), N is drawn from a Poisson distribution with mean n . Together, these citizens must choose a pair $x = (x_1, x_2)$ of policies from the set \mathcal{X} , which is taken to be the unit disk.¹³ If the origin $(0, 0)$ represents a pair of status quo policies, for example, then the electorate can depart from the status quo in any direction, up to some maximal distance, normalized to one. It is often convenient to represent policies using polar coordinates (r_x, θ_x) , where $r_x = \|x\| = \sqrt{x_1^2 + x_2^2}$ is the Euclidean norm, or distance from the origin, and θ_x is the angle formed between x and the horizontal axis. In terms of its polar coordinates, the Cartesian coordinates of x are given by $x_1 = r_x \cos(\theta_x)$ and $x_2 = r_x \sin(\theta_x)$. A policy pair can also be represented as a column vector $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. Multiplying x by the matrix

$$T_\delta = \begin{bmatrix} \cos(\delta) & -\sin(\delta) \\ \sin(\delta) & \cos(\delta) \end{bmatrix} \quad (2)$$

then produces a *rotation* of x , which is a vector $T_\delta x$ with the same magnitude as x , but polar angle $\theta_x + \delta$, as Figure 2 illustrates.

There is a particular pair $z = (z_1, z_2) \in \mathcal{X}$ of policies that are optimal for society; if information were perfect, every citizen would prefer these. Specifically, citizens share a common utility function $u(x|z) = -\|x - z\|^2$ which decreases quadratically in the distance $\|x - z\| = \sqrt{(x_1 - z_1)^2 + (x_2 - z_2)^2}$ between the policy vector implemented and the optimal policy vector. Conditional on available information Ω (and dropping terms that do not depend on the policy outcome), then, expected utility is given by

$$E_z [u(x|z) | \Omega] = -\|x - E(z|\Omega)\|^2, \quad (3)$$

¹²Duggan and Martinelli (2011) and Egorov (2014) take unidimensionality as an exogenous constraint on communication, but show how the orientation of political conflict can be influenced by a monolithic media or by candidate messaging, respectively.

¹³Section 5.1 discusses the generalization to $k > 2$ dimensions, as well as the possibility of a policy space $\mathcal{X} = [-1, 1]^k$ that is a Cartesian product of intervals.

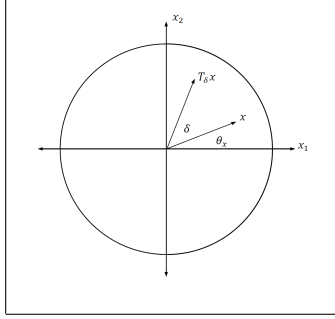


Figure 2: A rotation $T_\delta x$ of a policy vector x

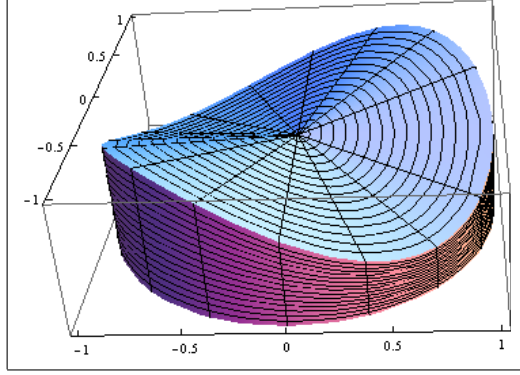


Figure 3: The joint density $f(z)$, for the case of $\rho > 0$.

which decreases quadratically in the distance between the policy vector implemented and the citizen's updated expectation $E(z|\Omega)$ of the optimal policy vector.¹⁴

The prior density of $z = (z_1, z_2)$ is common knowledge, and is given by

$$f(z; \rho) = \frac{1}{\pi} \left(1 + \rho \frac{z_1 z_2}{\|z\|} \right) = \frac{1}{\pi} [1 + \rho r_z \cos(\theta_z) \sin(\theta_z)]. \quad (4)$$

Intuitively, the most important feature of $f(z)$ is that it accommodates the possibility that the policies z_1 and z_2 that are optimal in each dimension are correlated, as Section 1 suggests they should be. With this particular specification, the correlation coefficient $Corr(z_1, z_2) = \frac{1}{4}\rho$ is proportional to the parameter $\rho \in [-1, 1]$. For positive ρ , (4) is illustrated in Figure 3. If $\rho = 0$ then $f(z)$ reduces to a uniform density, meaning that every feasible policy pair is equally likely to be optimal.

The properties of (4) that are sufficient for the results below are given by Conditions 1 through 3. In addition to implying the correlation between z_1 and z_2 , these conditions provide monotonicity and symmetry that make the analysis tractable. Condition 1 states that, for positive ρ , $f(z)$ increases in the direction of the *main diagonal* (i.e. the line defined by $z_1 = z_2$) and decreases in the direction of the *minor diagonal* (i.e. defined by $z_1 = -z_2$). Condition 2 states that, when $\rho = 0$, the likelihood of z being optimal does not depend on its direction from the origin. In either case, Condition 3 states that $f(z)$ does not depend on the order or orientation of z_1 and z_2 , except that reversing the orientation of one of the two dimensions is equivalent to reversing the sign of ρ .

¹⁴Quadratic utility loss is standard both in political economy and in statistics, but is largely for ease of exposition.

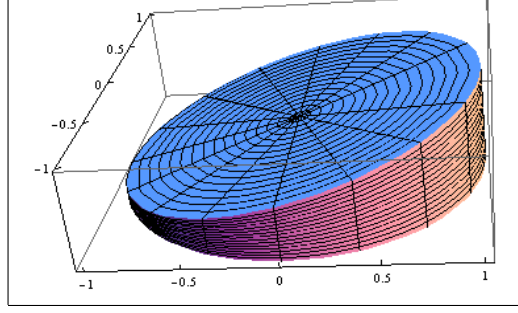


Figure 4: The perimeter of conditional density $g(s|z)$, for z on the horizontal axis.

Condition 1 (Correlative monotonicity) If $z \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is positive (negative) then $\nabla f(z) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ has the same (opposite) sign as ρ and is increasing (decreasing) in ρ . If $z \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is positive (negative) then $\nabla f(z) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ has the opposite (same) sign as ρ and is decreasing (increasing) in ρ .

Condition 2 (Radial symmetry) If $\rho = 0$ then $f(T_\delta z) = f(z)$ for any δ and for any $z \in \mathcal{X}$.

Condition 3 (Dimensional symmetry) $f(z_2, z_1) = f(-z_1, -z_2) = f(z_1, z_2)$ and $f(z_1, z_2; -\rho) = f(-z_1, z_2; \rho) = f(z_1, -z_2; \rho)$. Equivalently, $f(r_z, \frac{\pi}{2} - \theta_z) = f(r_z, \theta_z + \pi) = f(r_z, \theta_z)$ and $f(r_z, \theta_z; -\rho) = f(r_z, -\theta_z; \rho) = f(r_z, \pi - \theta_z; \rho)$.

Citizens' private opinions regarding the location of the optimal policy vector are determined by pairs $s_i = (s_{i1}, s_{i2}) \in \mathcal{X}$ of informative private signals. Suppressing the i subscript, these signals are drawn independently from the conditional density

$$\begin{aligned} g(s|z) &= \frac{1}{\pi} (1 + s_1 z_1 + s_2 z_2) \\ &= \frac{1}{\pi} [1 + r_s r_z \cos(\theta_s - \theta_z)]. \end{aligned} \quad (5)$$

This density is illustrated in Figure 4, for z on the horizontal axis. The unconditional distribution of s is uniform. Conceptually, the most important feature of (5) is that signals are informative of the truth.

The properties of (5) that are sufficient for the results below are given by Conditions 4 through 7. In addition to implying that s is correlated with z , these conditions provide monotonicity and symmetry that make the analysis tractable. Condition 4 states that signals become more likely in the direction of z . Condition 5 states that signals of the same magnitude are equally likely whether they lie clockwise from the truth or counter-clockwise from the truth. Condition 6 states that rotating z merely rotates the entire distribution of signals by the same amount. Condition 7 is actually implied by Conditions 4 and 5, but states that reversing the orientation of z_1 or z_2 merely reverses the orientation of s_1 or s_2 , respectively.

Condition 4 (Directional monotonicity) $\nabla_s g(s|z) \cdot z > 0$ for any $s, z \in \mathcal{X}$.

Condition 5 (Error symmetry) If $r_s = r_{s'}$ and $s \cdot z = s' \cdot z$, where $s, s', z \in \mathcal{X}$, then $g(s|z) = g(s'|z)$.

Condition 6 (Rotational consistency) $g(T_\delta s | T_\delta z) = g(s|z)$ for any δ and for any $s, z \in \mathcal{X}$.

Condition 7 (Axis symmetry) $g(-s_1, s_2 | -z_1, z_2) = g(s_1, -s_2 | z_1, -z_2) = g(s_2, s_1 | z_2, z_1) = g(s_1, s_2 | z_1, z_2)$. Equivalently, $g(r_s, \pi - \theta_s | r_z, \pi - \theta_z) = g(r_s, \theta_s | r_z, \theta_z)$.

By Bayes' rule, the posterior density of z is given by

$$f(z|s) = \frac{1}{\pi} [1 + r_s r_z \cos(\theta_s - \theta_z)] [1 + \rho r_z \cos(\theta_z) \sin(\theta_s)]$$

and a citizen's private expectation of z_1 reduces to a simple linear function of his signals:

$$\begin{aligned} E(z_1|s) &= \int_0^{2\pi} \int_0^1 [r_z \cos(\theta_z)] f(z|s) r_z dr_z d\theta_z \\ &= \frac{1}{4} r_s \cos(\theta_s) + \frac{1}{20} \rho r_s \sin(\theta_s) \\ &= \frac{1}{4} s_1 + \frac{1}{20} \rho s_2, \end{aligned} \tag{6}$$

where integration requires tedious repetition of standard trigonometric identities, but is otherwise straightforward. Similarly,

$$E(z_2|s) = \frac{1}{20} \rho s_1 + \frac{1}{4} s_2. \tag{7}$$

If z_1 and z_2 are uncorrelated then $E(z_1|s)$ depends only on s_1 and $E(z_2|s)$ depends only on s_2 , but if $\rho > 0$ then each signal conveys information about the optimal policy in *both* dimensions. The distribution of signals is continuous, so despite their common objective, citizens develop a myriad of different opinions regarding what combination of policies is optimal.

Citizens do not vote directly for policies. Instead, there are two candidates, A and B , who choose platform policy pairs $x_A = (x_{A1}, x_{A2}) \in \mathcal{X}$ and $x_B = (x_{B1}, x_{B2}) \in \mathcal{X}$. Each citizen votes (at no cost) for one of the two candidates. The candidate $w \in \{A, B\}$ who receives the most votes (breaking ties, if necessary, by a fair coin toss) wins the election, takes office, and implements her platform policies. Informally, candidates can be viewed as citizens themselves, in the spirit of Osborne and Slivinski (1996) and Besley and Coate (1997), championing policies that they personally believe to be optimal. Formally, candidates do share voters' objective function (3), and so prefer policies as close as possible to z , but for simplicity do not vote, and do not receive private signals; instead, they must take cues from voters. The possibility that candidates might hold valuable private information of their own is an important direction for future extension, but given the current model's other assumptions, candidate signals would have essentially no consequence for voter or candidate behavior, because the informational content of one or two additional signals would be overwhelmed by what is inferred from the N citizens.¹⁵

Let \mathcal{V} denote the set of voting strategies $v : \mathcal{X} \rightarrow \{A, B\}$, which specify a vote choice $v(s)$ for every possible pair $s \in \mathcal{X}$ of signals. For tractability, voters and candidates move simultaneously, so that equilibrium voting is a best response to candidate platforms x_A and x_B , but v is not an explicit function of these variables. The equilibrium concept used below is Bayesian Nash equilibrium. With the assumption of Poisson population uncertainty, such equilibria are necessarily symmetric, in that citizens each play the same voting strategy.¹⁶ The ultimate policy outcome depends on the realizations of N and z , and of the private information s of each citizen, in combination with voter and candidate strategies.

¹⁵For a spatial model in which voters instead learn from candidates, see Kartik, Squintani, and Tinn (2012).

¹⁶In games of Poisson population uncertainty, the finite set of citizens who actually play the game is a random draw from an infinite set of potential citizens, for whom strategies could be defined (see Myerson 1998). The distribution of opponent behavior is therefore the same for any two individuals within the game (unlike a game between a finite set of players), implying that a best response for one citizen is a best response for all.

4 Equilibrium Analysis

4.1 Candidate platforms

Depending on the voting strategy v that citizens follow, a candidate is more likely to win the election in certain states of the world than in other states. If she knew that she had won the election, therefore, a candidate could form posterior belief

$$f(z|w=j) = \frac{\Pr(w=j|z)f(z)}{\Pr(w=j)} \quad (8)$$

and update her expectation $E(z|w=j)$ of the optimal policy as follows.¹⁷

$$E(z_k|w=j) = \int_{\mathcal{X}} z_k f(z|w=j) dz. \quad (9)$$

When she chooses her platform policy, of course, a candidate does not know whether she will win or lose. If she loses, however, her platform choice will not matter. Therefore, as Proposition 1 now states, a candidate commits in advance to whatever policy she would expect to be optimal in the event that she wins.

Proposition 1 *For $j \in \{A, B\}$, $v \in \mathcal{V}$, and $x_{-j} \in \mathcal{X}$, if x_j^{br} is a best response to (v, x_{-j}) then $x_j^{br} = E(z|w=j)$.*

For most voting strategies, the two candidates are favored in different states of the world, and so form different expectations upon winning. Anticipating this, candidates adopt distinct policy positions even though they are ex ante identical. A candidate's expectation $E(z|w=j)$ of the optimal policy vector depends implicitly on the strategy $v \in \mathcal{V}$ followed by voters, but does not depend on the platform choice of the candidate's opponent. The search for an equilibrium can therefore focus on the strategy followed by voters: if citizens follow a voting strategy v^* , and have no incentive to deviate from this strategy when candidates respond with $x_A^* = E(z|w=A)$ and $x_B^* = E(z|w=B)$ as prescribed in Proposition 1, then (v^*, x_A^*, x_B^*) is an equilibrium. Examining voter incentives is thus the task of the following section.

4.2 Voting

Based on his private information alone, a citizen would prefer whichever platform is closest to $E(z|s)$. That is, there would be a line of policies equidistant from the two platforms, such that citizens with expectations on one side of the line prefer to vote A , while those on the other side prefer to vote B . A linear threshold on $E(z|s)$ would simply translate into a linear threshold on s , because (6) and (7) are linear in s_1 and s_2 .¹⁸ Voting decisions are more complicated than this, however, because a citizen's influence depends on the behavior of other voters. In addition to his own information, therefore, a citizen must consider the voting strategy of his peers.

¹⁷Lebesgue integration throughout this paper is with respect to the standard measure. Written in Cartesian or polar coordinates,

$$\int_{\mathcal{X}} dz = \int_{-1}^1 \int_{-\sqrt{1-z_2^2}}^{\sqrt{1-z_2^2}} dz_1 dz_2 = \int_0^{2\pi} \int_0^1 r_z dr_z d\theta_z.$$

¹⁸Linearity is a product of the specific functional forms adopted in Section 3, of course, but a threshold of some shape in the space of signals would emerge as long as $E(z|s)$ is monotonic in s . A sufficient condition for monotonicity is that s and z are *affiliated*, in the sense of Milgrom and Weber (1982).

If the optimal policy vector is $z \in \mathcal{X}$ and citizens follow the voting strategy v then each votes for candidate j with probability

$$\phi(j|z) = \int_{\mathcal{X}} I_{v(s)=j} g(s|z) ds, \quad (10)$$

where $I_{v(s)=j}$ is an indicator function that equals one if $v(s) = j$ and zero otherwise, and $d(s)$ denotes incremental changes in both dimensions of s . By the decomposition property of Poisson random variables, the numbers N_A and N_B of A and B votes are independent Poisson random variables with means $n\phi(A|z)$ and $n\phi(B|z)$, respectively, and the joint probability of vote totals $N_A = a$ and $N_B = b$ is therefore

$$\psi(a, b|z) = \frac{e^{-n}}{a!b!} [n\phi(A|z)]^a [n\phi(B|z)]^b. \quad (11)$$

Candidate A thus wins the election by a margin of exactly $m \geq 0$ votes (alternatively, B “wins” by $-m$ votes) with probability

$$\pi_A(m|z) = \pi_B(-m|z) = \sum_{k=0}^{\infty} \psi(k+m, k|z), \quad (12)$$

while B wins by $m \geq 0$ votes (or A “wins” by $-m$ votes) with probability

$$\pi_B(m|z) = \pi_A(-m|z) = \sum_{k=0}^{\infty} \psi(k, k+m|z), \quad (13)$$

and the total probability with which candidate j wins the election is

$$\Pr(w = j|z) = \sum_{m=1}^{\infty} \pi_j(m|z) + \frac{1}{2} \pi_j(0|z). \quad (14)$$

By the environmental equivalence property of Poisson games, an individual from within the game reinterprets N_A and N_B as the numbers of A and B votes cast by his peers (Myerson, 1998); by voting herself, he can add one to either total. A vote for candidate j only influences the election outcome, however, if it is *pivotal* (event piv_j), meaning that the candidates otherwise tie but j loses the tie-breaking coin toss, or j wins the coin toss but loses the election by one vote. In state z , these occur with probability

$$\Pr(piv_j|z) = \frac{1}{2} \pi_j(0|z) + \frac{1}{2} \pi_j(-1|z), \quad (15)$$

so the total probability of influencing the election is

$$\Pr(piv|z) = \Pr(piv_A|z) + \Pr(piv_B|z). \quad (16)$$

Depending on the voting strategy of a citizen’s peers, a vote is more likely to be pivotal in some states of the world than in others. It is unlikely in any case, of course, but since a vote only matters when it is pivotal, a citizen optimally updates his beliefs as if this will occur. That is, as Proposition 2 now states, he votes for the candidate whose platform is closest to $E(z|piv, s)$, because in situations where his own vote makes a difference, this policy vector is optimal in expectation.¹⁹

Proposition 2 *The voting strategy v^{br} is a best response to $v \in \mathcal{V}$ and $x_A, x_B \in \mathcal{X}$ if and only if $v^{br}(s) = \arg \min_{j \in \{A, B\}} \|x_j - E(z|piv, s)\|$ for all $s \in \mathcal{X}$.*

¹⁹A common objection to strategic voting models is that voters in the real world do not seem familiar with or even capable of the calculus of pivotal voting. As Section 4.6 notes, however, strategic voting in this model is also socially optimal, which means that a citizen could behave *as if* she were strategic without thinking about pivot probabilities, by determining the socially optimal voting strategy and then following it.

According to Proposition 2, there is indeed a line of policies equidistant from the two candidates' platforms, such that citizens with expectations on one side of the line prefer to vote A , while those on the other side prefer to vote B . It is $E(z|piv, s)$ that matters, however, not $E(z|s)$. Unlike $E(z|s)$, a linear threshold on $E(z|piv, s)$ does not reduce to a linear threshold on s , because $\Pr(piv|z)$ is non-linear. It seems reasonable to conjecture that $E(z|piv, s)$ is at least monotonic in s , so that a threshold on $E(z|piv, s)$ translates into a threshold of some shape in the space of signals, but the non-linearity of $\Pr(piv|z)$ makes even this difficult to prove. To make the analysis tractable, therefore, the following section restricts attention to a particular class of threshold strategies, which exhibit additional symmetry and monotonicity.²⁰

4.3 Half-space strategies

Definition 1 defines a *half-space strategy* which, as its name suggests, merely divides the electorate in half, in the direction of some unit vector h . That is, voters whose signals lie in the direction of h vote for candidate B , while those with signals in the opposite direction vote A . Definition 1 imposes the restriction that $\theta_h \in (-\frac{\pi}{2}, \frac{\pi}{2}]$, so that A voters are on the left and B voters are on the right, but this is of course without loss of generality.

Definition 1 $v_h \in \mathcal{V}$ is a half-space strategy if h is a unit vector with $\theta_h \in (-\frac{\pi}{2}, \frac{\pi}{2}]$ and $v(s) = B$ when the dot product $h \cdot s$ is positive and $v(s) = A$ otherwise. A Bayesian Nash equilibrium (x_A^*, x_B^*, v_h^*) is a half-space equilibrium if v_h^* is a half-space strategy.

One convenient aspect of half-space strategies is that the threshold that divides A and B voters is linear.²¹ As Lemma 1 now states, half-space strategies also produce voting outcomes that exhibit convenient symmetry and monotonicity.

Lemma 1 If $v_h \in \mathcal{V}$ is a half-space strategy then the following hold for all $j \in \{A, B\}$, $z \in \mathcal{X}$, $a, b \in \mathbb{Z}_+$, and $m \in \mathbb{Z}$.

1. *Monotonic voting:* $\frac{\partial}{\partial z_k} \phi(B|z)$ and $\frac{\partial}{\partial z_k} \Pr(w = B|z)$ have the same sign as h_k , for $k = 1, 2$.
2. *Symmetric voting in orthogonal state:* if $h \cdot z = 0$ then $\phi(A|z) = \phi(B|z) = \frac{1}{2}$, implying that $\psi(a, b|z) = \psi(b, a|z)$, $\pi_A(m|z) = \pi_B(m|z)$, and $\Pr(w = A|z) = \Pr(w = B|z) = \frac{1}{2}$.
3. *Rotationally consistent voting:* for any δ , $\phi(j|T_\delta z; v_{T_\delta h}) = \phi(j|z; v_h)$, implying that $\psi(a, b|T_\delta z; v_{T_\delta h}) = \psi(a, b|z; v_h)$, $\pi_j(m|T_\delta z; v_{T_\delta h}) = \pi_j(m|z; v_h)$, $\Pr(w = j|T_\delta z; v_{T_\delta h}) = \Pr(w = j|z; v_h)$, $\Pr(piv_j|T_\delta z; v_{T_\delta h}) = \Pr(piv_j|z; v_h)$, and $\Pr(piv|T_\delta z; v_{T_\delta h}) = \Pr(piv|z; v_h)$.
4. *Directional symmetry:* $\phi(A|z) = \phi(B|-z)$, implying that $\psi(a, b|z) = \psi(b, a|-z)$, $\pi_j(m|z) = \pi_j(-m|-z)$, $\Pr(w = A|z) = \Pr(w = B|-z)$, and $\Pr(piv_A|z) = \Pr(piv_B|-z)$, and therefore that $E(z|w = A) = -E(z|w = B)$ and $\Pr(piv|z) = \Pr(piv|-z)$.
5. *Expected tie:* $\Pr(w = A) = \Pr(w = B) = \frac{1}{2}$.

Part 1 of Lemma 1 simply notes that B votes are more frequent when the voting strategy is oriented in the same direction as the optimal policy (or, equivalently, when the optimal policy lies in the direction of h). Part 2 states that when the optimal policy pair lies exactly on the voting threshold (i.e. h and z are perpendicular) the two candidates receive equal vote shares. Part 3 states that rotating h merely rotates the

²⁰This restriction also eliminates “babbling” equilibria, in which citizens ignore their private signals and in turn are ignored by candidates.

²¹In higher dimensions, as Section 5.1 discusses, A and B voters are divided by a hyperplane, with normal vector h . In three dimensions, for example, a plane divides the policy space into hemispheres.

states of the world in which particular outcomes occur. In particular, Part 4 states that voting outcomes in state $-z$ are mirror images of those in state z . These two states are equally likely, so as Part 5 states, candidates tie in expectation.

By Part 4 of Lemma 1, a half-space strategy tends to favor the two candidates in opposite states of the world. A consequence of this, as Proposition 1 now states, is that candidates form opposite expectations upon winning the election and, anticipating this, adopt policy platforms that are distinct, but mirror images of each other.

Proposition 3 *If $v_h \in \mathcal{V}$ is a half-space strategy then $x_A^{br}(v_h) = -x_B^{br}(v_h) \neq 0$.*

Lemma 1 and Proposition 3 characterize voting outcomes and candidate responses when voters follow a half-space strategy. The following two sections proceed to analyze equilibrium. Section 4.4 begins with the case of $\rho = 0$, and Section 4.5 then considers the case of positive correlation $\rho > 0$.

4.4 Uncorrelated States

If $\rho = 0$ then z_1 and z_2 are uncorrelated. In that case, as Section 3 notes, $f(z)$ reduces to a uniform density, which in particular exhibits radial symmetry (Condition 2). That is, the optimal policy pair is equally likely to lie in any direction from the origin. In that case, as Theorem 1 now states, any unit vector h defines a half-space strategy v_h that, together with candidates' best responses, constitutes an equilibrium. In such an equilibrium, candidates take policy positions in the direction of h , symmetric around the origin.

Theorem 1 *Let $\rho = 0$. If v_h^* is any half-space strategy and $x_j^* = E(z|w = j; v_h^*)$ for $j = A, B$ then (x_A^*, x_B^*, v_h^*) is a half-space equilibrium, with $x_A^* = -x_B^* \neq 0$.*

In characterizing an equilibrium, Theorem 1 contrasts with the non-existence results of Plott (1967) and others. Now, however, there is an entire continuum of equilibria. As in Besley and Coate (1997), the problem of multiple equilibria makes it difficult to make clear behavioral or welfare predictions. On the other hand, as the following section shows, most of these equilibria are not robust.

4.5 Correlated States

Assuming that z_1 and z_2 are uncorrelated, Theorem 1 states that any half-space strategy can be sustained in equilibrium. As Section 1 discusses, however, logical connections between issues suggest that z_1 and z_2 should be correlated. With that motivation, this section considers the case of $\rho > 0$. The main result of this is that only two of the equilibria identified above are robust. Specifically, Theorem 2 states the existence of a *major equilibrium* oriented along the main diagonal of the policy space, with candidate platforms in quadrants 1 and 3, and a *minor equilibrium* oriented on the opposite diagonal, with candidate platforms in quadrants 2 and 4. These are illustrated in Figure 5. Theorem 3 then states that no other half-space strategy can support an equilibrium.

Theorem 2 *If $\rho > 0$ then there exists a major equilibrium (x_A^+, x_B^+, v_{h^+}) with $x_j^+ = E(z|w = j; v_{h^+})$ for $j = A, B$ and $\theta_{h^+} = \frac{\pi}{4}$, as well as a minor equilibrium (x_A^-, x_B^-, v_{h^-}) with $x_j^- = E(z|w = j; v_{h^-})$ for $j = A, B$ and $\theta_{h^-} = -\frac{\pi}{4}$.*

Theorem 3 *If $\rho > 0$ and v_h is a half-space strategy with $\theta_h \notin \{-\frac{\pi}{4}, \frac{\pi}{4}\}$ then, whenever n is sufficiently large, (x_A, x_B, v_h) is not an equilibrium for any $x_A, x_B \in \mathcal{X}$.*

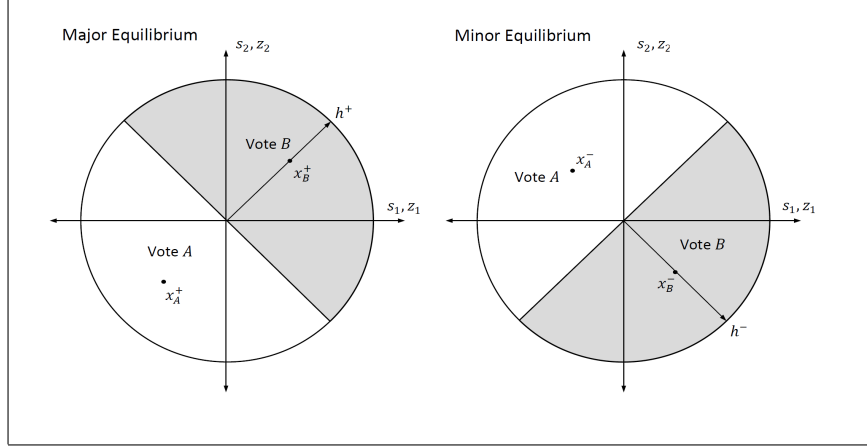


Figure 5: Major and minor equilibria.

Figure 6 gives some intuition for Theorems 2 and 3 by illustrating a half-space strategy defined by the first standard basis vector $h = e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, which divides the electorate along the vertical axis, as indicated by the shaded and unshaded regions. Citizens who follow such a strategy vote on the basis of s_1 alone, ignoring s_2 completely. If z_1 and z_2 were uncorrelated, therefore, as in Section 4.4, then candidates would learn nothing about z_2 , and would adopt policy positions along the horizontal axis. When $\rho > 0$, however, candidate B recognizes that if z_1 is positive then z_2 is likely positive as well, and so adopts a platform strictly above the horizontal axis. By symmetric reasoning, A takes a position below the horizontal axis.

When candidates adopt platforms off the horizontal axis, voters have the incentive to base their voting behavior on s_2 , in addition to s_1 . In fact, if his peers follow the original voting strategy, a voter should put extra weight on s_2 , because his vote is most likely to be pivotal when $z_1 = 0$, in which case z_2 is all that matters for his vote choice. Thus, the best-response threshold vector h^{br} is further from the horizontal axis than x_B , and $h = e_1$ is not an equilibrium. If voters base their behavior partly on s_2 , of course, then candidates should respond with platforms that are still farther from the horizontal axis, and closer to the main diagonal. This logic applies unless voters play equal weight on s_1 and s_2 , so that candidates infer the same information about both issues, and in response, adopt platforms on the major diagonal. This occurs in the major equilibrium.

In a minor equilibrium, candidate B infers from voters that z_1 is positive; by itself, this should imply that z_2 is positive, or at least less negative than she would suppose if z_1 and z_2 were uncorrelated. At the same time, however, she infers from voters that z_2 is negative, suggesting that z_1 is less positive than she would otherwise suppose. These conflicting messages have offsetting influence on candidate B 's beliefs, so in the end she is just as confident in her conclusion that z_2 is negative as she is that z_1 is positive, so x_B^{br} lies on the minor diagonal. He is less confident about either issue, however, than she would be in a major equilibrium, where voters send messages that are mutually reinforcing—namely, that z_1 is likely positive and that z_2 is likely positive. With x_A^{br} and x_B^{br} on the minor diagonal, so is $h^{br}(x_A^{br}, x_B^{br})$.

Since candidates in a minor equilibrium infer conflicting information about the signs of z_1 and z_2 , they are less confident in their posterior opinions than candidates in a major equilibrium, whose information is mutually reinforcing. As a consequence, candidates in a minor equilibrium tend to remain more moderate, as Proposition 4 now states.

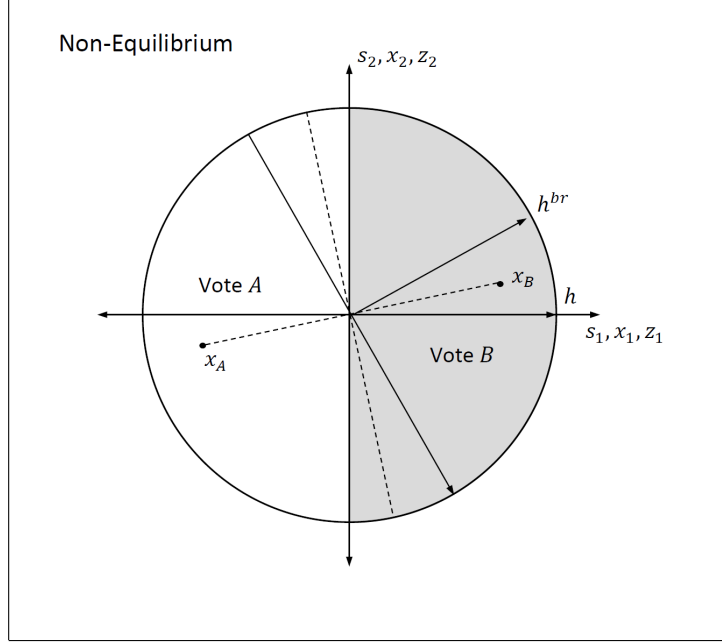


Figure 6: Non-equilibrium threshold τ .

Proposition 4 *If $\rho > 0$ then $\|x_j^+\| > \|x_j^-\|$.*

4.6 Welfare

The fact that policy outcomes are more extreme in a major equilibrium and more moderate in a minor equilibrium begs the question of which equilibrium is better for society. Defining social welfare $W(x_A, x_B, v)$ is uncontroversial in this model, unlike many settings, because voters and candidates share the same objective function, rewritten here from (19).

$$W(x_A, x_B, v) = E_{w,z} [u(x_w|z)] = \int_{\mathcal{X}} \left[\sum_{j=A,B} \Pr(w = j|z) u(x_j|z) \right] f(z) dz. \quad (17)$$

Common utilitarian arguments in favor of moderation and compromise do not apply here, because a voter benefits not from a policy that is close to his current opinion, but from a policy that is close to whatever is actually optimal. In fact, Proposition 5 states that the major equilibrium is actually superior, even though policies are more extreme.

Proposition 5 *If $\rho > 0$ then $W(x_A^+, x_B^+, v_{h^+}) > W(x_A^-, x_B^-, v_{h^-})$.*

To see the intuition for Proposition 5, note that v_{h^+} and v_{h^-} specify the same voter behavior, but in different states of the world. When z happens to be in quadrant 1 or quadrant 3, v_{h^+} does an excellent job of identifying the right quadrant, but v_{h^-} does not; similarly, v_{h^-} is effective at distinguishing between states of the world in quadrants 2 and 4, but v_{h^+} is not. Since quadrants 1 and 3 occur more frequently, v_{h^+} is the more informative voting strategy. The proof of Proposition 5 therefore proceeds in two steps: first, the strategy combination (x_A^-, x_B^-, v_{h^-}) is rotated by $\frac{\pi}{2}$, so that candidate positions are as extreme as before, but voter and candidate behavior is oriented along the major, instead of the minor diagonal, thus

providing a welfare improvement. Welfare then improves again when candidates move to the positions x_A^+ and x_B^+ that are optimal responses to v_{h^+} .

Theorem 2 establishes that there are multiple equilibria in this model. Typically, this makes it difficult to predict behavior specifically. The result in Proposition 5 that the major equilibrium is Pareto superior, however, makes this equilibrium an obvious target for coordination, and therefore *focal* in the sense of Schelling (1960). On the other hand, the presence of a minor equilibrium reflects the possibility that political issues could be inefficiently bundled, and that such inefficiency could be self-perpetuating, in the sense that it is consistent with equilibrium, so given that others in the electorate are bundling issues inefficiently, an individual wishes to use the same inefficient bundling. The welfare difference between equilibria is small if ρ is small, but large if ρ is large.

5 Extensions

This section lists a variety of modifications of the model of Section 3 to which the equilibrium results of Section 4 seem to extend readily. Section 5.1 considers changes to the policy environment, and Section 5.2 considers changes in the objectives or opportunities of voters and candidates. The latter largely discusses whether, and to what extent, insights from one-dimensional models extend to multiple dimensions.

5.1 Policy Environment

Cartesian Products

Defining \mathcal{X} as the unit disk has the perhaps unrealistic feature that the choice of x_1 renders certain choices of x_2 infeasible. An alternative is to specify \mathcal{X} as a Cartesian product of intervals, which in two dimensions is a square. In that case, the densities (4) and (5) could be replaced by $f(z) = \frac{1}{4}(1 + \rho z_1 z_2)$ and $g(s|z) = \frac{1}{4}(1 + s_1 z_1)(1 + s_2 z_2)$, which exhibit properties similar to the functions used in the model of Section 3. A square model lacks the rotational symmetry of the unit disk, so it is difficult to characterize voter and candidate incentives in arbitrary directions from the origin. It is straightforward to show, however, that threshold strategies that ignore one or the other dimension (i.e. $h = e_1$ or $h = e_2$) are not consistent with equilibrium, except in the case of $\rho = 0$, while at the same time, threshold strategies h^+ and h^- oriented toward the major or minor diagonals support equilibria for any ρ . As above, the major equilibrium Pareto dominates the minor equilibrium, and so is likely to be focal. Thus, specifying \mathcal{X} as a Cartesian product of intervals produces qualitatively similar results.

Discrete Issues

In some cases, the nature of uncertainty is inherently discrete. In the macroeconomic example of Section 1, for instance, stimulus should be large if the economy behaves according to a Keynesian model, and small if the economy behaves according to a Classical model. Moderate-sized stimulus is feasible, but is known not to be optimal in either case. If the policy space is a square $\mathcal{X} = [-1, 1]^2$, as in the previous subsection, then this possibility can be accommodated simply by imposing the restriction that $z_k \in \{-1, 1\}$ for $k = 1, 2$, so that the optimal policy in either dimension lies at one of the two extremes of the policy space (say, with prior distribution $P(z) = \frac{1}{4}(1 + \rho z_1 z_2)$ and signal density $g(s|z) = \frac{1}{4}(1 + s_1 z_1)(1 + s_2 z_2)$). The analysis then proceeds as before, and yields the same results.

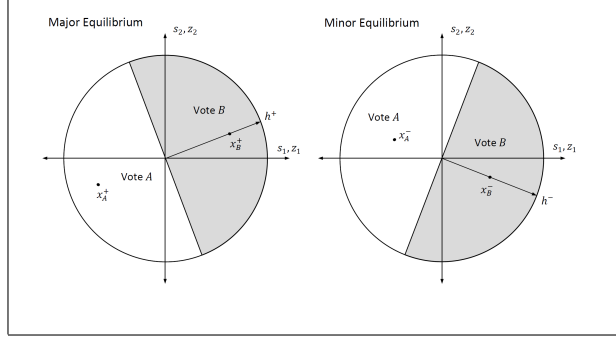


Figure 7: Major and minor equilibria when issue 1 is more important than issue 2

Asymmetric Issue Importance

The analysis of Section 4 relies heavily on symmetry. Some forms of asymmetry seem entirely plausible, however, and at any rate should be explored, since perfect symmetry is a knife-edge condition. For example, the utility function (3) treats both issues identically, but more generally it may be that issue 1 is more important than issue 2 (or vice versa). This section therefore extends the model by generalizing (3) as follows,

$$E_z [u(x|z) | \Omega] = - \left\{ (1 + \alpha) [x_1 - E(z_1 | \Omega)]^2 + (1 - \alpha) [x_2 - E(z_2 | \Omega)]^2 \right\},$$

where $\alpha \in [0, 1)$. The model of Section 3 corresponds to the case of $\alpha = 0$.

In the symmetric version of the model, equilibria exist along the major and minor diagonals, whether issues are correlated with each other or not. When $\alpha > 0$, however, these orientations cannot be sustained in equilibrium in either case. If h is on the main diagonal then candidates infer the same information as before, and therefore still adopt platforms that lie on the diagonal, in response. Previously, however, this would have rendered voters along the minor diagonal (i.e. those for whom $s_1 = -s_2$) indifferent between the two candidates, thereby sustaining an equilibrium; now, those in quadrant 2 (i.e. for whom $s_1 = -s_2 < 0$) have a strict preference for candidate A, while those in quadrant 4 (i.e. for whom $s_1 = -s_2 > 0$) prefer B. Thus, the best response vector h^{br} now lies below the main diagonal.

If $\rho > 0$ and h lies on the horizontal axis then, just as in the analysis of Section 4, h^{br} lies above the horizontal axis. If h^{br} lies below h when h is on the main diagonal and above h when h is on the horizontal axis, there exists an equilibrium threshold vector h^+ somewhere between the horizontal axis and the main diagonal, which characterizes its own best response. This is analogous to the major equilibrium of Section 4. By symmetric reasoning, there is an equilibrium vector h^- between the minor diagonal and the horizontal axis. Thus, the major and minor equilibria are no longer perpendicular, as Figure 7 illustrates, but both still exist. As before, the major equilibrium Pareto dominates. Figure 7 also makes clear that, as α approaches one, the two equilibria converge to a single equilibrium, oriented along the horizontal axis. This is perfectly intuitive, since in that case voters no longer care at all about the second dimension, so equilibrium is identical to that of a one-dimensional model.

Multiple Dimensions

With $k > 2$ separate issues to be decided, the policy space \mathcal{X} above can be extended from a disk or square to a k -dimensional hyperball or hypercube, with optimal policies on each issue denoted by z_1, z_2, \dots, z_k . If these policies are uncorrelated, as in Section 4.4, then extending the logic of Theorem 1 would imply

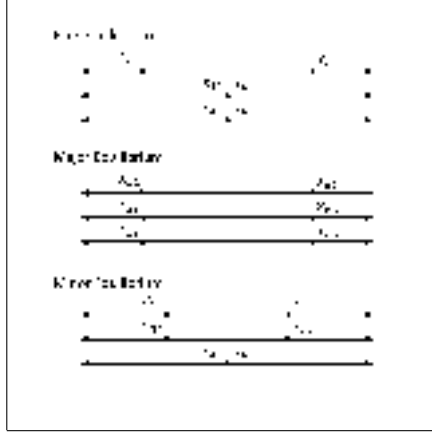


Figure 8: Equilibrium (and non-equilibrium) candidate positions in three dimensions with symmetric prior distribution.

that any hyperplane that divides the policy space into equal halves would define a half-space strategy that can be supported in equilibrium. By the logic of Theorem 3, however, correlation across issues is likely to eliminate many of these equilibria.

To see this, consider the case of three dimensions, with a correlation matrix

$$\begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix}, \quad (18)$$

and policy vectors x_A and x_B that differ only in the first dimension, as illustrated in the top portion of Figure 8. In response to such candidate platforms, voters should base their behavior only on their estimates of z_1 . If Candidate B wins the election, then, she will deduce that z_{B1} is likely positive, which is consistent with her platform position, but she can also infer that z_{B2} and z_{B3} are likely positive, in contrast with her platform.

If the prior distribution of the optimal policy vector z is symmetric such that the pair-wise correlation $\rho > 0$ between any pair of policies is the same (as in the example above) then the logic of Theorem 2 can extend to guarantee the existence of a major equilibrium analogous to that described in Section 4.5, with one candidate taking a left position on every issue, and the other candidate taking a right position on every issue. This is illustrated in the center portion of Figure 8.

With a symmetric prior as above, the logic of Theorem 2 can extend to guarantee not one, but multiple equilibria analogous to the minor equilibrium of Section 4.5. For three dimensions, one such equilibrium is illustrated in the bottom portion of Figure 8: each candidate takes a leftist position on one of the first two issues and a rightist position on the other, and both candidates propose the same platform position on the third issue. In this example, candidate B can infer in equilibrium that z_1 is positive, which suggests that z_3 is positive as well. He also infers that z_2 is negative, however, which suggests that z_3 is negative. These conflicting messages about z_3 exactly cancel out, so that convergent platforms in this dimension are consistent with equilibrium.

Of course, shuffling which of the three dimensions corresponds to which of a candidate's three policy positions produces six minor equilibria of the type illustrated in Figure 8. With four dimensions, there would be twelve equilibria in which candidates adopt platforms that are leftist on one issue, rightist on

another, and centrist on the other two, and twelve additional equilibria in which platforms are each leftist on two issues and rightist on two issues. Clearly, the number of equilibria proliferates rapidly as the cardinality of the state space expands. However, there is only ever one major equilibrium, and extending the logic of Proposition 5, this equilibrium Pareto dominates any of the others, and is thus the most likely target for coordination. Moreover, the growing number of minor equilibria would seem to make it increasingly difficult to coordinate on any one of those. Nevertheless, as before, the caution remains that an inefficient bundling of issues could persist in equilibrium.

5.2 Voters and Candidates

Office Motivated Candidates

In the model above, candidates care only about policy; winning office is merely a means to that end. A common alternative is to assume that candidates value office for its own sake, as in Downs (1957), and are willing to compromise on policy in an effort to win. The one-dimensional model of McMurray (2014a) analyzes this possibility, and shows that policy moderation gives candidates a competitive advantage, so that office motivated candidates are less polarized than those with pure truth motivation. In the extreme case in which candidates are purely office motivated, candidates converge to the political center, just as in traditional median voter theorems. Nothing about that logic is special to the one-dimensional model, however, so office motivation should mitigate polarization in the multidimensional setting of this paper, as well.²²

Abstention

With common values and private information, Feddersen and Pesendorfer (1996) show that uninformed voters have an incentive to abstain from voting altogether, even if voting is costless, to avoid a *swing voter's curse* of inadvertently overturning an informed decision. McMurray (2014a) shows that the same curse actually applies to voters with moderate policy opinions as well, even when they are highly informed. This is because such a citizen has a precise estimate of the location of the optimal policy, but views the two candidates as roughly equidistant from the optimum, making it difficult to determine whose position is superior. In one dimension, the equilibrium consequence of this is that citizens with strong convictions that the optimal policy is left or right of center vote A or B , respectively, but ideological moderates remain neutral, and abstain from voting.

Since voting behavior in the multidimensional model ultimately collapses to a single dimension, the same logic applies here as in the one-dimensional setting, to predict strategic abstention. What matters now is the projection of a voter's policy opinions onto the diagonal, as illustrated in Figure 9 for a major equilibrium. As before, citizens may be centrist either because they receive private signals that the optimal policy lies in the center, or because they simply lack expertise, and thus put little weight on their own opinions, and remain politically neutral. Now, however, there is a third type of citizen who will abstain: those whose opinions are extreme, but orthogonal to the equilibrium—that is, citizens who are strongly liberal on one issue and strongly conservative on another. Such a citizen is confident in his beliefs about the location of the optimal policy, but views both candidates as far from the truth, and abstains out of ambivalence over whose platform is slightly better, and whose is slightly worse.

²²Note that this contrasts with the multidimensional probabilistic voting literature described in Section 2, where policy motivated candidates converge and office motivation leads to equilibrium non-existence.

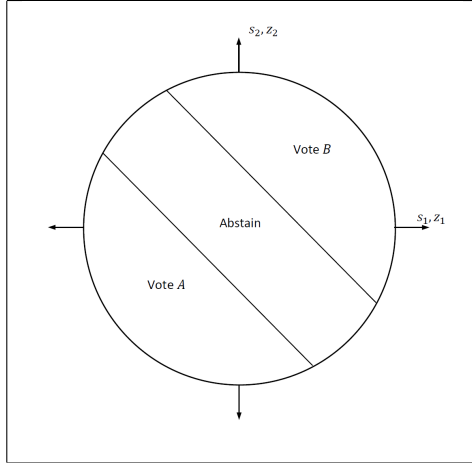


Figure 9: Equilibrium with Abstention

Electoral Mandates

In the equilibrium above, candidates base their policy platforms on their *ex-ante* expectations of what they will learn from voters. In McMurray (2014b) I show for a one-dimensional model that, *ex post*, candidates can infer additional information from the margin of victory, and have incentive to adjust their policy positions accordingly. That is, a candidate who wins by a landslide is more confident that the truth is on her side than a candidate who wins only narrowly. Anticipating such a response, voters shift attention from the rare event of a pivotal vote to the ever-present opportunity to (slightly) impact the winning candidate’s beliefs. In that case, as I discuss in that paper, voter behavior can be meaningfully interpreted as a *mandate* to candidates, in the common sense that citizens vote in an effort to push the eventual policy outcome either to the left or to the right. The welfare consequence of this is a strengthened version of Condorcet’s (1785) jury theorem: large electorates only choose the better of two alternatives, but can potentially steer the winning candidate toward the optimal policy from an entire continuum of possibilities.

If candidates are allowed to adjust their policy positions *ex post*, after learning the electoral outcome, the logic of electoral mandates extends to a multidimensional setting. In the major equilibrium, for example, a large margin will push candidate *A*’s policy choice to the southwest, or push candidate *B*’s policy choice to the northeast, along the main diagonal. With two candidates, however, the margin of victory remains one-dimensional, even as the policy space is expanded, and generically, the optimum lies off the diagonal, even when ρ is large. Thus, the ability to further extend the jury theorem is limited: even a large electorate cannot identify the social optimum from within a multidimensional set of alternatives. In other words, even if individual votes are efficiently aggregated into a collective decision, substantial information loss is inevitable, as voters reduce their multidimensional signals into binary voting decisions. Thus, democracy suffers from a “curse of dimensionality”, in that policy questions are extremely complex and multi-faceted, but political communication is one-dimensional.²³

²³Besley and Coate (2000) make a similar observation regarding the inefficiency of bundling multiple policies into a single piece of legislation. For this reason, they advocate citizens’ initiatives, which allow separate dimensions to be decided separately.

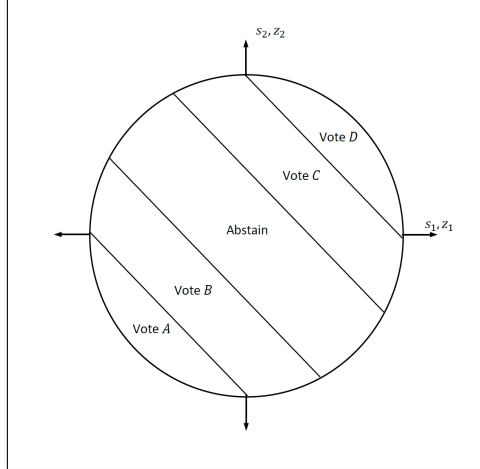


Figure 10: Equilibrium with Abstention and Multiple Candidates

Multiple Candidates

When winning politicians respond to electoral mandates, votes for extreme parties communicate more extreme private information, and therefore exert more extreme influence on the beliefs of the winning candidate, and the ultimate policy outcome. Because of this, I show in McMurray (2014b) for the one-dimensional setting that voters may have an incentive to support minor parties who are unlikely to win the election, in contrast with pivotal voting models where they focus attention on the two front-runners. Applying this logic to the major equilibrium of this multidimensional model would produce an equilibrium in which multiple candidates take positions on the main diagonal, and citizens vote on the basis of their private signals, projected onto the same diagonal, as illustrated in Figure 10.

With two policy dimensions, other equilibrium configurations would also be possible. For example, four candidates could adopt platforms in each of the four quadrants, enabling voters to communicate which of the four quadrants their signals are in. The resulting mandate would be multidimensional, and a large electorate could once again steer the winning candidate precisely to the optimal policy vector. Of course, eliminating the political curse of dimensionality requires at least as many candidates as policy dimensions, which unfortunately is infeasible in higher dimensions. Thus, information loss remains inevitable.

6 Applications

6.1 Ideological Consistency

To many observers, the consistent empirical correlation of voter attitudes across issues is somewhat of a puzzle. As Shor (2011) expresses, “it is not clear why environmentalism necessarily hangs together with a desire for more union prerogatives, but it does”. As discussed in Section 2, this is a puzzle not sufficiently addressed in existing literature. The model above provides an explanation, however, which is that ideological consistency derives from logical connections across issues. To reiterate the example of Section 1, Keynesian macroeconomic theory prescribes both fiscal and monetary stimulus, but classical theory views both forms of stimulus as wasteful. Thus, a voter’s positions on the two forms of stimulus are correlated. Mathematically, $E(z_1|s)$ is correlated with $E(z_2|s)$ because s_1 and s_2 are correlated with z_1 and z_2 , which are correlated with each other.

It is actually possible to point to potential sources of logical correlation, even between seemingly unrelated issues such as environmentalism and union support. Both could stem, for example, from a general belief that business leaders are categorically unethical, willing to abuse employees or environmental resources in pursuit of profits. In fact, this belief could engender support for minimum wage laws as well, and a host of other pro-labor policies. Positions on these issues might also depend on voters' beliefs about the extent to which ethical behavior is rewarded or penalized in the marketplace, about the extent of firms' market power, and about the reliability of government regulators. By a similar token, as Section 1 notes, citizens who believe in market efficiency may oppose a variety of regulations that are all supported by those who view market failures as rampant. As Alesina and Angeletos (2005) conjecture, the demand for economic redistribution in its variety of forms could depend on whether voters attribute wealth largely to luck or to effort.

Of course, the quote above is likely not intended to mean that there is *no* connection between environmentalism and union support, but rather that the logical link is much weaker than the correlation in political behavior. This interpretation is entirely consistent with the perspective of this paper, as emphasized in Section 4.5, in that a little bit of correlation goes a long way. That is, *any* non-zero correlation, no matter how small, is sufficient to orient the equilibrium, producing behavior that is identical to the case of $\rho = 1$. Actually, the model predicts candidate positions exactly on the diagonal of the policy space. Empirical estimates do not suggest quite this level of consistency even for candidates, but Shor (2011) does find U.S. legislators to be more ideologically consistent than voters. He also finds legislators' policy positions to be more extreme than voters' on average; in McMurray (2014a), I relate this finding to the result that candidates are more confident than the average voter, having their beliefs bolstered in equilibrium by public support.

6.2 Political Party Alignment

In addition to explaining ideological consistency, the model above provides a natural framework for formally characterizing various well-known political philosophies. The U.S. Libertarian party, for example, takes liberal positions on social issues (e.g. immigration, abortion, marriage) and conservative positions on fiscal issues (e.g. taxes, regulation). On its website, the party emphasizes the logical consistency of these positions, arguing that while Democrats favor personal liberty and Republicans favor economic liberty, the Libertarian party favors both.²⁴ To illustrate this, the party founder, David Nolan, created a diagram, reproduced here as Figure 11.

Figure 11 bears a striking resemblance to Figure 5 (adapted to a square-shaped policy space, as described in Section 5.1), with left-wing Democrats and right-wing Republicans aligned along the minor diagonal. In the language of the model, this reflects the Libertarian view that the U.S. electorate is stuck in an inferior equilibrium: welfare would improve if politics could be reoriented toward the Libertarian party (and a competitor, that favors regulating both personal and economic spheres).

Rotating the Nolan chart 90 degrees would reverse the orientation, of course, placing Democrats and Republicans on the major diagonal. It may be, therefore, that the current political orientation is actually optimal. In other words, the Libertarian claim can be viewed as an assertion about the correlation structure of the state variables: personal freedom is likely desirable if economic freedom is, and vice versa. Alternative correlation structures would endorse other perspectives. For example, conservative positions could be unified by the logical connection between preserving both the social and economic traditions of the nation's

²⁴See www.lp.org, accessed 2/7/13.

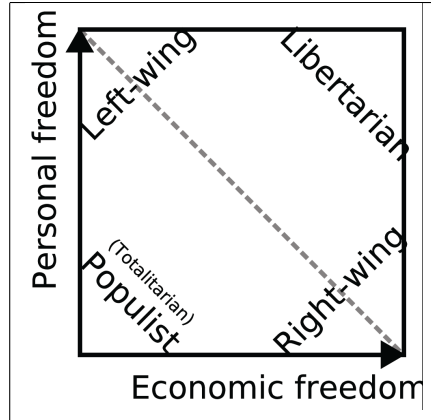


Figure 11: Nolan chart (11/26/2006 version) created by U.S. Libertarian party founder, reproduced from upload.wikimedia.org/wikipedia/commons/3/3e/Nolan-chart.svg (accessed 10/5/2012).

founders (e.g. limited government, traditional gender roles, etc.) while liberal or progressive positions seek to modernize on both fronts.

There may be many ways to logically organize a set of political issues, and it is difficult empirically to establish any as superior to the rest, since welfare cannot readily be measured. Furthermore, the most correct orientation likely involves many more dimensions than two. In this light, the genesis of Green parties in the U.S., Europe, and elsewhere could be viewed as an effort to broaden political discourse to neglected issues, such as the environment.

The bundling of political issues may also vary across countries, and over time. For example, prominent Republicans have recently advocated that the party shift its stances on immigration and same-sex marriage.²⁵ Political realignments have occurred periodically throughout history, which could be interpreted as a natural response to evolving public opinion. With a large number of issues, of course, a voter's private opinions are unlikely to coincide exactly with the positions of any political party.

7 Conclusion

Without an empirical measure of voter welfare, the model above may be unable to establish the optimal bundling of political issues. It does, however, offer a framework that resembles a variety of well-known perspectives, suggesting that an information model such as this might be implicit in much of modern political discourse. Such similarities corroborate the model's basic assumptions, including the premise that political conflicts stem largely from differences of opinion, rather than fundamental conflicts of interest. This is important because, as I emphasize in McMurray (2014a), information models and preference models in general may have very different normative implications.

For tractability, this paper has basically followed the information structure of Condorcet (1785): private signals are independent, and in the aggregate, fully reveal the true state of the world; voters start with common priors, and do not suffer from psychological biases or cognitive limitations. One consequence of these assumptions is that disagreements are quite fragile: if a voter learned that he held a minority opinion,

²⁵See www.nytimes.com/2013/03/20/us/politics/gop-opposition-to-immigration-law-is-falling-away.html and www.nytimes.com/2013/02/26/us/politics/prominent-republicans-sign-brief-in-support-of-gay-marriage.html (accessed 3/27/03).

this should be a sufficiently strong signal to make him reverse his position, and join the majority. Thus, the present model offers a plausible characterization of a single election, but offers no explanation of why political disagreements persist, even after election results are made public.

Ladha (1992) shows that correlated signals make the majority more prone to mistakes, which should make a minority voter more willing to stick to his private opinion. Biased beliefs about either the reliability of one's own expertise or the expertise or motives of others could have a similar effect. Identifying and analyzing informational frictions is an important direction for future research, but the model of perfect rationality is a natural starting point, as well as a useful benchmark to which more elaborate models can be compared. Intuitively, extensions of the information structure seem unlikely to alter the main results of the analysis above. In particular, equilibrium is likely to exist quite generally, as in the probabilistic voting literature, because beliefs, unlike preferences, move continuously in response to small changes in candidate positions or voter behavior. Symmetric informational impediments are likely to preserve the infinite number of possible equilibrium orientations, while correlation across issues reduces this to a single orientation in the direction of correlation, and an opposite orientation, which aggregates information less effectively.

To the extent that optimal policies in various dimensions are highly correlated, most private information can be preserved within a single dimension. As Section 5 discusses, however, some information loss is inevitable, especially if issues are only weakly correlated, in reducing complex political issues to a single dimension. This underscores the importance of informal channels of political communication, such as public opinion surveys, political demonstrations, and letters to public officials, which may shape political outcomes in ways that binary votes simply cannot.

8 Appendix

Proposition 1 For $j \in \{A, B\}$, $v \in \mathcal{V}$, and $x_{-j} \in \mathcal{X}$, if x_j^{br} is a best response to (v, x_{-j}) then $x_j^{br} = E(z|w = j)$.

Proof. If candidates adopt platforms x_A and x_B and citizens follow the voting strategy v then a candidate's expected utility is given by

$$E_{w,z} [u(x_w); v, x_A, x_B] = E_z \left[\sum_{j=A,B} \Pr(w = j|z; v) u(x_j|z) \right]. \quad (19)$$

The only term in (19) that depends on candidate j 's own platform is $u(x_j|z)$, so differentiating with respect to x_{jk} yields

$$\begin{aligned} \frac{\partial}{\partial x_{jk}} E_{w,z} [u(x_w); v, x_A, x_B] &= 2E_z [\Pr(w = j|z; v) (z_k - x_{jk})] \\ &= 2 \Pr(w = j) [E(z_k|w = j; v) - x_{jk}], \end{aligned}$$

which is zero if and only if $x_{jk} = E(z_k|w = j)$. The second derivative $-2 \Pr(w = j)$ is negative, establishing this as a maximum. ■

Proposition 2 The voting strategy v^{br} is a best response to $v \in \mathcal{V}$ and $x_A, x_B \in \mathcal{X}$ if and only if $v^{br}(s) = \arg \min_{j \in \{A, B\}} \|x_j - E(z|piv, s)\|$ for all $s \in \mathcal{X}$.

Proof. In terms of (15) and (16), the expected benefit of voting B , relative to voting A , is

$$\Delta E_{w,z} [u(x_w) | s] = E_z \{ [u(x_B|z) - u(x_A|z)] \Pr(piv_B|z) | s \}$$

$$\begin{aligned}
& -E_z \{ [u(x_A|z) - u(x_B|z)] \Pr(\text{piv}_A|z) | s \} \\
= & E_z \{ [u(x_B|z) - u(x_A|z)] \Pr(\text{piv}|z) | s \} \\
= & E_z \left(\left\{ \sum_{k=1,2} \left[-(x_{Bk} - z_k)^2 + (x_{Ak} - z_k)^2 \right] \right\} \Pr(\text{piv}|z) | s \right) \\
= & E_z \left\{ \left[\sum_{k=1,2} 2(x_{Bk} - x_{Ak})(z_k - \bar{x}_k) \right] \Pr(\text{piv}|z) | s \right\} \\
= & \Pr(\text{piv}) \sum_{k=1,2} 2(x_{Bk} - x_{Ak}) [E(z_k | \text{piv}, s) - \bar{x}_k] \\
= & \Pr(\text{piv}) \{ \|x_A - E(z | \text{piv}, s)\| - \|x_B - E(z | \text{piv}, s)\| \},
\end{aligned}$$

where $\bar{x}_k = \frac{x_{Ak} + x_{Bk}}{2}$ is the average policy position of the two candidates in dimension $k \in \{1, 2\}$. This benefit is positive if and only if x_B is closer than x_A to $E(z | \text{piv}, s)$. ■

Lemma 1 *If $v_h \in \mathcal{V}$ is a half-space strategy then the following hold for all $j \in \{A, B\}$, $z \in \mathcal{X}$, $a, b \in \mathbb{Z}_+$, and $m \in \mathbb{Z}$.*

1. *Monotonic voting:* $\frac{\partial}{\partial z_k} \phi(B|z)$ and $\frac{\partial}{\partial z_k} \Pr(w = B|z)$ have the same sign as h_k , for $k = 1, 2$.
2. *Symmetric voting in orthogonal state:* if $h \cdot z = 0$ then $\phi(A|z) = \phi(B|z) = \frac{1}{2}$, implying that $\psi(a, b|z) = \psi(b, a|z)$, $\pi_A(m|z) = \pi_B(m|z)$, and $\Pr(w = A|z) = \Pr(w = B|z) = \frac{1}{2}$.
3. *Rotationally consistent voting:* for any δ , $\phi(j|T_\delta z; v_{T_\delta h}) = \phi(j|z; v_h)$, implying that $\psi(a, b|T_\delta z; v_{T_\delta h}) = \psi(a, b|z; v_h)$, $\pi_j(m|T_\delta z; v_{T_\delta h}) = \pi_j(m|z; v_h)$, $\Pr(w = j|T_\delta z; v_{T_\delta h}) = \Pr(w = j|z; v_h)$, $\Pr(\text{piv}_j|T_\delta z; v_{T_\delta h}) = \Pr(\text{piv}_j|z; v_h)$, and $\Pr(\text{piv}|T_\delta z; v_{T_\delta h}) = \Pr(\text{piv}|z; v_h)$.
4. *Directional symmetry:* $\phi(A|z) = \phi(B| -z)$, implying that $\psi(a, b|z) = \psi(b, a| -z)$, $\pi_j(m|z) = \pi_j(-m| -z)$, $\Pr(w = A|z) = \Pr(w = B| -z)$, and $\Pr(\text{piv}_A|z) = \Pr(\text{piv}_B| -z)$, and therefore that $E(z|w = A) = -E(z|w = B)$ and $\Pr(\text{piv}|z) = \Pr(\text{piv}| -z)$.
5. *Expected tie:* $\Pr(w = A) = \Pr(w = B) = \frac{1}{2}$.

Proof. 1. Rotating the basis vectors by $\frac{\pi}{2} - \theta_h$, so that the normal vector h lies at $(0, 1)$, the expected vote share (10) of candidate j simplifies as follows:

$$\begin{aligned}
\phi(B|z) &= \int_{\{s \in \mathcal{X} : s \cdot h > 0\}} g(s|z) ds \\
&= \int_{\{s \in \mathcal{X} : s \cdot \frac{\pi}{2} > 0\}} g(s | T_{\frac{\pi}{2} - \theta_h} z) ds \\
&= \int_{-1}^1 \int_0^{\sqrt{1-s_1^2}} \frac{1}{\pi} \left\{ \begin{aligned} & 1 + s_1 [z_1 \cos(\frac{\pi}{2} - \theta_h) - z_2 \sin(\frac{\pi}{2} - \theta_h)] \\ & + s_2 [z_1 \sin(\frac{\pi}{2} - \theta_h) - z_2 \cos(\frac{\pi}{2} - \theta_h)] \end{aligned} \right\} ds_2 ds_1 \\
&= \int_{-1}^1 \int_0^{\sqrt{1-s_1^2}} \frac{1}{\pi} \left\{ \begin{aligned} & 1 + s_1 [z_1 \sin(\theta_h) - z_2 \cos(\theta_h)] \\ & + s_2 [z_1 \cos(\theta_h) + z_2 \sin(\theta_h)] \end{aligned} \right\} ds_2 ds_1.
\end{aligned}$$

Then differentiating, converting to polar coordinates, and applying a standard trigonometric identity,

$$\begin{aligned}
\frac{\partial}{\partial z_1} \phi(B|z) &= \int_{-1}^1 \int_0^{\sqrt{1-s_1^2}} \frac{1}{\pi} [s_1 \sin(\theta_h) + s_2 \cos(\theta_h)] ds_2 ds_1 \\
&= \int_0^\pi \int_0^1 \frac{1}{\pi} [r_s \cos(\theta_s) \sin(\theta_h) + r_s \sin(\theta_s) \cos(\theta_h)] r_s dr_s d\theta_s
\end{aligned}$$

$$\begin{aligned}
&= \int_0^\pi \int_0^1 \frac{1}{\pi} r_s^2 \cos(\theta_s + \theta_h) dr_s d\theta_s \\
&= \frac{2}{3\pi} \cos(\theta_h) \\
&= \frac{2}{3\pi} h_1.
\end{aligned}$$

A similar derivation shows that $\frac{\partial}{\partial z_1} \phi(B|z) = \frac{2}{3\pi} h_2$. In both cases, $\frac{\partial}{\partial z_k} \phi(B|z)$ has the same sign as h_k , for $k = 1, 2$.

Conditional on the total number k of voters (and on the state variable z), the number of B votes follows a binomial distribution, with probability parameter $\phi(B|z)$. The probability that B votes exceed A votes therefore increases with this parameter. Summing over all k , this implies that the total probability $\Pr(w = B|z)$ of a B victory is increasing in $\phi(B|z)$, so $\frac{\partial}{\partial z_k} \Pr(w = B|z)$ has the same sign as $\frac{\partial}{\partial z_k} \phi(B|z)$.

2. If h is orthogonal to z then either $h = T_{\pi/2}z$ or $h = T_{-\pi/2}z$. In the first case, the substitution $\delta = \theta_z - \theta_s$ yields

$$\begin{aligned}
\phi(A|z) &= \int_{\theta_z - \pi}^{\theta_z} \int_0^1 \frac{1}{\pi} [1 + r_s r_z \cos(\theta_s - \theta_z)] r_s dr_s d\theta_s \\
&= \int_0^\pi \int_0^1 \frac{1}{\pi} [1 + r_s r_z \cos(-\delta)] r_s dr_s d\delta,
\end{aligned}$$

while the substitution $\delta = \theta_s - \theta_z$ yields

$$\begin{aligned}
\phi(B|z) &= \int_{\theta_z}^{\theta_z + \pi} \int_0^1 \frac{1}{\pi} [1 + r_s r_z \cos(\theta_s - \theta_z)] r_s dr_s d\theta_s \\
&= \int_0^\pi \int_0^1 \frac{1}{\pi} [1 + r_s r_z \cos(\delta)] r_s dr_s d\delta.
\end{aligned}$$

These expressions are equal, since $\cos(-\delta) = \cos(\delta)$; a similar derivation applies if $h = T_{-\pi/2}z$. The other equalities stated in the lemma then follow immediately from the definitions (11) through (14), together with the fact that $\phi(A|z) + \phi(B|z) = \Pr(w = A|z) + \Pr(w = B|z) = 1$ for any z .

3. The rotational consistency of $\phi(j|z)$ follows from the rotational consistency of $g(s|z)$ (Condition 6):

$$\begin{aligned}
\phi(j|T_\delta z; v_{T_\delta h}) &= \int_{\{s \in \mathcal{X} : s \cdot T_\delta h > 0\}} g(s|T_\delta z) ds \\
&= \int_{\{s \in \mathcal{X} : T_\delta s \cdot T_\delta h > 0\}} g(T_\delta s|T_\delta z) ds \\
&= \int_{\{s \in \mathcal{X} : s \cdot h > 0\}} g(s|z) ds \\
&= \phi(B|z; h),
\end{aligned}$$

where the second equality merely rotates the basis vectors. The rotational consistency of ψ , π_j , $\Pr(w = j|z)$, $\Pr(\text{piv}_j|z)$, and $\Pr(\text{piv}|z)$ then follow immediately from (11) through (14).

4. The first five equalities are simply special cases of Part 3 of this lemma, since $-z = T_\pi z = T_{-\pi}z$. The final equalities then follow from (9) and (16).

5. The symmetry derived in Part 4, together with the dimensional symmetry (Condition 3) of the prior density, imply that

$$\Pr(w = A) = \int_{\mathcal{X}} \Pr(w = A|z) f(z) dz$$

$$\begin{aligned}
&= \int_{\mathcal{X}} \Pr(w = B | -z) f(-z) dz \\
&= \int_{\mathcal{X}} \Pr(w = B | z) f(z) dz \\
&= \Pr(w = B),
\end{aligned}$$

where the third equality is simply a change of variables. Since these probabilities sum to one, they must both equal one-half. ■

Proposition 3 *If $v_h \in \mathcal{V}$ is a half-space strategy then $x_A^{br}(v_h) = -x_B^{br}(v_h) \neq 0$.*

Proof. Proposition 1 states that $x_j^{br}(v_h) = E(z|w = j)$ for $j = A, B$. Symmetry follows immediately from Part 4 of Lemma 1. To see that platforms diverge, suppose that h lies in Quadrant 1, so that $h_1, h_2 > 0$ (other cases can be treated analogously). In that case, by Part 1 of Lemma 1, $\Pr(w = B|z)$ is increasing in z_k and $\Pr(w = A|z)$ is decreasing in z_k , for $k = 1, 2$. Therefore,

$$\begin{aligned}
E(z_k|w = B) - E(z_k|w = A) &= \int_{\mathcal{X}} z_k \left[\frac{\Pr(w = B|z) f(z)}{\Pr(w = B)} - \frac{\Pr(w = A|z) f(z)}{\Pr(w = A)} \right] dz \\
&= 2 \int_{\mathcal{X}} z_k [\Pr(w = B|z) - \Pr(w = A|z)] f(z) dz \\
&> 2 \int_{\mathcal{X}} z_k f(z) dz \\
&= 0,
\end{aligned}$$

where the first equality follows from Part 5 of Lemma 1. ■

Theorem 1 *Let $\rho = 0$. If v_h^* is any half-space strategy and $x_j^* = E(z|w = j; v_h^*)$ for $j = A, B$ then (x_A^*, x_B^*, v_h^*) is a half-space equilibrium, with $x_A^* = -x_B^* \neq 0$.*

Proof. This proof begins with the special case of h equal to the Euclidean basis vector $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, which defines a half-space strategy v_{e_2} oriented along the vertical axis. In that case, $s \cdot h > 0$ if and only if $s_2 > 0$, so vote shares are symmetric around the vertical axis:

$$\begin{aligned}
\phi(B | -z_1, z_2) &= \int_{-1}^1 \int_0^{\sqrt{1-s_1^2}} g(s_1, s_2 | -z_1, z_2) ds_2 ds_1 \\
&= \int_{-1}^1 \int_0^{\sqrt{1-s_1^2}} g(-s_1, s_2 | z_1, z_2) ds_2 ds_1 \\
&= \int_{-1}^1 \int_0^{\sqrt{1-s_1^2}} g(s_1, s_2 | z_1, z_2) ds_2 ds_1 \\
&= \phi(B | z_1, z_2).
\end{aligned}$$

In this derivation, the second equality reflects axis symmetry and the third equality is simply a change of variables. By a similar derivation, $\phi(A | -z_1, z_2) = \phi(A | z_1, z_2)$. From the definitions (11) through (16) it is then clear that $\psi(a, b|z)$, $\pi_j(m|z)$, $\Pr(w = j|z)$, $\Pr(piv_j|z)$, and $\Pr(piv|z)$ do not depend on the sign of z_1 either.

Since $\Pr(w = j | -z_1, z_2) = \Pr(w = j | z_1, z_2)$, and by the radial symmetry of f (Condition 2), (9) reduces so that $E(z_1|w = j) = 0$:

$$E(z_1|w = j) \Pr(w = j)$$

$$\begin{aligned}
&= \int_{-1}^0 \int_{-\sqrt{1-s_1^2}}^{\sqrt{1-s_1^2}} z_1 \Pr(w = j|z_1, z_2) f(z_1, z_2) dz_2 dz_1 + \int_0^1 \int_{-\sqrt{1-s_1^2}}^{\sqrt{1-s_1^2}} z_1 \Pr(w = j|z_1, z_2) f(z_1, z_2) dz_2 dz_1 \\
&= \int_0^1 \int_{-\sqrt{1-s_1^2}}^{\sqrt{1-s_1^2}} (-z_1) \Pr(w = j| -z_1, z_2) f(-z_1, z_2) dz_2 dz_1 + \int_0^1 \int_{-\sqrt{1-s_1^2}}^{\sqrt{1-s_1^2}} z_1 \Pr(w = j|z_1, z_2) f(z_1, z_2) dz_2 dz_1 \\
&= \int_0^1 \int_{-\sqrt{1-s_1^2}}^{\sqrt{1-s_1^2}} (z_1 - z_1) \Pr(w = j|z_1, z_2) f(z_1, z_2) dz_2 dz_1 \\
&= 0.
\end{aligned}$$

By Propositions 1 and 3, candidates therefore respond with distinct platforms along the vertical axis: $x_{A1}^{br}(v_h) = x_{B1}^{br}(v_h) = 0$ and $x_{A2}^{br}(v_h) < 0 < x_{B2}^{br}(v_h)$. By Proposition 2, then, a citizen prefers to vote B if and only if $E(z_2|piv, s)$ is positive. This expectation has the same sign as $E(z_2|s)$, because by the derivation above, combined with Part 4 of Lemma 1, a pivotal vote is equally likely for positive and negative z_2 :

$$\begin{aligned}
\Pr(piv|z_1, -z_2) &= \Pr(piv| -z_1, -z_2) \\
&= \Pr(piv|z_1, z_2).
\end{aligned}$$

From (7) it is clear that, when $\rho = 0$, $E(z_2|s)$ has the same sign as s_2 . Thus, if citizens vote according to the half-space strategy v_{e_2} and candidates adopt best-response platforms then a voter's best response is to follow the same half-space strategy. In other words, $[x_A^{br}(v_{e_2}), x_B^{br}(v_{e_2}), v_{e_2}]$ constitutes a half-space equilibrium.

The next step of this proof is to show that equilibrium is preserved under any rotation of h . This is intuitive: rotating h merely rotates the dividing line between types who vote A and types who vote B , leading to the same outcomes, but in different states of the world. Since the distribution $f(z)$ of states of the world exhibits radial symmetry when $\rho = 0$, this leads to identical outcomes, except with a new orientation. Formally, $T_\delta s \cdot T_\delta h > 0$ if and only if $s \cdot h > 0$, implying that

$$\begin{aligned}
\phi(j|T_\delta z; v_{T_\delta h}) &= \int_{\{s: s \cdot T_\delta h > 0\}} g(s|T_\delta z) ds \\
&= \int_{\{s: T_\delta s \cdot T_\delta h > 0\}} g(T_\delta s|T_\delta z) ds \\
&= \int_{\{s: s \cdot h > 0\}} g(s|z) ds \\
&= \phi(j|z; v_h),
\end{aligned}$$

where the second equality is merely a rotation of basis vectors and the third equality follows from the rotational consistency of $g(s|z)$ (Condition 6). In turn, this implies that $\psi(a, b|T_\delta z; v_{T_\delta h}) = \psi(a, b|z; v_h)$, $\pi_j(m|T_\delta z; v_{T_\delta h}) = \pi_j(m|z; v_h)$, $\Pr(w = j|T_\delta z; v_{T_\delta h}) = \Pr(w = j|z; v_h)$, and $x_j^{br}(v_{T_\delta h}) = E(z|w = j; v_{T_\delta h}) = T_\delta E(z|w = j; v_h) = T_\delta x_j^{br}(v_h)$, where the latter also relies on the radial symmetry of f (Condition 2). In other words, rotating h merely rotates candidates' best-response platforms by the same amount. Also, $\Pr(piv|T_\delta z; v_{T_\delta h}) = \Pr(piv|z; v_h)$, implying that $E(z|piv, T_\delta s; v_{T_\delta h}) = T_\delta E(z|piv, s; v_h)$ (again using the radial symmetry of f), and therefore that $x_j^{br}(v_{T_\delta h}) \cdot E(z|piv, T_\delta s; v_{T_\delta h}) = T_\delta x_j^{br}(v_h) \cdot T_\delta E(z|piv, s; v_h) > 0$ if and only if $x_j^{br}(v_h) \cdot E(z|piv, s; v_h) > 0$. Thus, rotating a half-space strategy from h to $T_\delta h$ simply rotates the best response threshold by the same amount, and the existence of an equilibrium with $h = e_2$ implies the existence of an equilibrium with $h = T_\delta e_2$ for any δ . ■

Theorem 2 *If $\rho > 0$ then there exists a major equilibrium (x_A^+, x_B^+, v_{h^+}) with $x_j^+ = E(z|w = j; v_{h^+})$ for $j = A, B$ and $\theta_{h^+} = \frac{\pi}{4}$, as well as a minor equilibrium (x_A^-, x_B^-, v_{h^-}) with $x_j^- = E(z|w = j; v_{h^-})$ for $j = A, B$ and $\theta_{h^-} = -\frac{\pi}{4}$.*

Proof. The optimality of candidate platforms $x_j^* = E(z|w = j; v_h^*)$ is stated in Proposition 1. As discussed above, it therefore suffices to show that v_{h^+} and v_{h^-} are the best responses to (x_A^+, x_B^+, v_{h^+}) and (x_A^-, x_B^-, v_{h^-}) , respectively.

1. (Major Equilibrium) Let $\theta_{h^+} = \frac{\pi}{4}$, but rotate the basis vectors by $\frac{\pi}{4}$ so that, under the rotated coordinate system, $\theta_{h^+} = 0$. In other words, in this rotated coordinate system, $h^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, so $s \cdot h^+ > 0$ if and only if $s_1 > 0$. Voting behavior is therefore symmetric with respect to s_2 , implying that vote shares are symmetric with respect to z_2 ,

$$\begin{aligned} \phi(j|z_1, -z_2) &= \int_{-1}^1 \int_0^1 g(s_1, s_2|z_1, -z_2) ds_1 ds_2 \\ &= \int_{-1}^1 \int_0^1 g(s_1, -s_2|z_1, -z_2) ds_1 ds_2 \\ &= \int_{-1}^1 \int_0^1 g(s_1, s_2|z_1, z_2) ds_1 ds_2 \\ &= \phi(j|z_1, z_2), \end{aligned}$$

where the second equality simply relabels s_2 and the third equality follows from Condition 7, which in the original coordinate system implied that $g(s_2, s_1|z_2, z_1) = g(s_1, s_2|z_1, z_2)$, but in this rotated system implies that $g(s_1, -s_2|z_1, -z_2) = g(s_1, s_2|z_1, z_2)$. By definitions (8) and (11) through (14), the symmetry of $\phi(j|z)$ further implies that $\psi(a, b|z_1, -z_2) = \psi(a, b|z_1, z_2)$, $\pi_j(m|z_1, -z_2) = \pi_j(m|z_1, z_2)$, $\Pr(w = j|z_1, -z_2) = \Pr(w = j|z_1, z_2)$, and $f(z_1, -z_2|w = j) = f(z_1, z_2|w = j)$, implying that the winning candidate infers no information about the second dimension of z :

$$\begin{aligned} E(z_2|w = j) &= \int_{-1}^1 \int_{-\sqrt{1-z_1^2}}^0 z_2 f(z_1, z_2|w = j) dz_2 dz_1 + \int_{-1}^1 \int_0^{\sqrt{1-z_1^2}} z_2 f(z_1, z_2|w = j) dz_2 dz_1 \\ &= \int_{-1}^1 \int_0^{\sqrt{1-z_1^2}} (-z_2) f(z_1, -z_2|w = j) dz_2 dz_1 + \int_{-1}^1 \int_0^{\sqrt{1-z_1^2}} z_2 f(z_1, z_2|w = j) dz_2 dz_1 \\ &= 0, \end{aligned}$$

where the second equality merely relabels z_2 .

Anticipating these symmetric expectations, candidates adopt policy platforms on the horizontal axis (which, in the original coordinate system, was the main diagonal): $x_{A1}^{br}(v_{h^+}) < 0 < x_{B1}^{br}(v_{h^+})$ and $x_{j2}^{br}(v_{h^+}) = 0$ for $j = A, B$ (where the inequalities follow from Proposition 3). That is, $x_B^{br}(v_{h^+})$ and h^+ lie in the same direction, while $x_A^{br}(v_{h^+})$ lies in the opposite direction. In response to such platforms, a citizen should vote B if and only if $E(z_1|piv, s) > 0$. For the voting strategy v_{h^+} (which is v_0 in the rotated coordinate system) this inequality holds if and only if $E(z_1|s) > 0$.

The expectation $E(z|s)$ from the rotated coordinate system can be rewritten in the original coordinate system as $T_{-\frac{\pi}{4}} E(z|s) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} E(z_1|s) \\ E(z_2|s) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} E(z_1|s) + E(z_2|s) \\ -E(z_1|s) + E(z_2|s) \end{pmatrix}$, so the condition that $E(z_1|s) > 0$ in the rotated coordinate system is equivalent to the condition that $E(z_1|s) + E(z_2|s) > 0$ in the original coordinate system. From (6) and (7) it is clear that this requires $(\frac{1}{4} + \frac{1}{20}\rho)(s_1 + s_2) > 0$, or

$s_1 + s_2 > 0$, or $s \cdot h^+ > 0$. In other words, v_{h^+} is its own best response, implying that (x_A^+, x_B^+, v_{h^+}) is a half-space equilibrium.

2. (Minor Equilibrium) Let $\theta_{h^-} = -\frac{\pi}{4}$, but rotate the basis vectors by $-\frac{\pi}{4}$ so that, under the rotated coordinate system, $\theta_{h^-} = 0$. In other words, in this rotated coordinate system, $h^- = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, so $s \cdot h^- > 0$ if and only if $s_1 > 0$. The analysis then proceeds just as in the case of a major equilibrium, except that returning to the original coordinate system, $E(z|s)$ now becomes $T_{\frac{\pi}{4}} E(z|s) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} E(z_1|s) \\ E(z_2|s) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} E(z_1|s) - E(z_2|s) \\ E(z_1|s) + E(z_2|s) \end{pmatrix}$, so the condition that previously implied $E(z_1|s) > 0$ now implies $E(z_1|s) > E(z_2|s)$ which, according to (6) and (7), is equivalent to $(\frac{1}{4} - \frac{1}{20}\rho) s_1 > (\frac{1}{4} - \frac{1}{20}\rho) s_2$, or $s_1 > s_2$, or $s \cdot h^- > 0$. In other words, v_{h^-} is its own best response, implying that (x_A^-, x_B^-, v_{h^-}) is a half-space equilibrium. ■

Theorem 3 *If $\rho > 0$ and v_h is a half-space strategy with $\theta_h \notin \{-\frac{\pi}{4}, \frac{\pi}{4}\}$ then, whenever n is sufficiently large, (x_A, x_B, v_h) is not an equilibrium for any $x_A, x_B \in \mathcal{X}$.*

Proof. Consider a half-space strategy defined by h , with $|h_1| > |h_2|$, and suppose $h_1 > 0$ (symmetric arguments apply if $h_1 < 0$). In that case, the equalities in steps 1 and 2 of the proof of Theorem 2 must be replaced with inequalities: $E(z_1|w=B) > E(z_2|w=B)$ and $E(z_1|w=B) > -E(z_2|w=B)$. In other words, x_{B2} may be positive or negative, but $x_{B1} > |x_{B2}|$. Similarly, $x_{A1} < |x_{A2}|$. As before, platforms are divergent and equidistant from the origin. It is no longer the case, however, that h and x_B lie in the same direction. Instead, $\theta_{x_B} > \theta_h$. To see this, first note that h and x_B would lie in the same direction if it were the case $\rho = 0$, as explained in the proof of Theorem 1. With positive ρ , however, the joint density $f(z; \rho)$ places more weight on z values close to the main diagonal, so $E(z|w=j)$ is closer to the main diagonal than before, implying that $\theta_{x_B} > \theta_h$.

As before, the optimal response for a voter is to vote B if and only if $x_B \cdot E(z|piv, s) > 0$. Previously, however, the fact that candidate platforms aligned with h implied that voting was symmetric with respect to candidate positions, and a voter could infer nothing from presuming that his vote would be pivotal. Now, the event of a pivotal vote makes realizations of z more likely that are close to the voting threshold. If $\theta_{E(z|s)} > \theta_h$, therefore, then it must be that $\theta_{E(z|s)} > \theta_{E(z|piv, s)} > \theta_h$. The best response threshold vector h^{br} must align with $E(z|s)$, so this implies that $\theta_{h^{br}} > \theta_{x_B} > \theta_h$. In other words, h no longer characterizes its own best response. If $|h_1| < |h_2|$ is inconsistent with equilibrium then, by symmetry, $|h_2| < |h_1|$ is inconsistent with equilibrium, as well. Thus, every equilibrium of the model must be one of the two types listed above. ■

Proposition 4 *If $\rho > 0$ then $\|x_j^+\| > \|x_j^-\|$.*

Proof. The threshold vector $h^- = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$ is a rotation of $h^+ = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$ by the amount $-\frac{\pi}{2}$. The two voting thresholds produce identical behavior, but in different states of the world:

$$\begin{aligned} \phi(B|T_{-\pi/2}z; h^-) &= \int_{\{s \in \mathcal{X}: s \cdot h^- > 0\}} g(s|T_{-\pi/2}z) ds \\ &= \int_{\{s \in \mathcal{X}: T_{\pi/2}s \cdot h^+ > 0\}} g(s|T_{-\pi/2}z) ds \\ &= \int_{\{s \in \mathcal{X}: s \cdot h^+ > 0\}} g(T_{-\pi/2}s|T_{-\pi/2}z) ds \end{aligned}$$

$$\begin{aligned}
&= \int_{\{s \in \mathcal{X}: s \cdot h^+ > 0\}} g(s|z) ds \\
&= \phi(B|z; h^+),
\end{aligned}$$

where the third equality reflects a rotation of the basis vectors and the fourth equality follows from the rotational consistency of g (Condition 6). This matters when $\rho > 0$ because some states are more likely than others; by the dimensional symmetry of f (Condition 3), rotating the state variable by $\frac{\pi}{2}$ has the same effect as reversing the sign of the correlation coefficient. Thus, a candidate learns the same amount about z_1 from voters who follow v_{h^-} as she would learn about z_2 , if ρ were negative, from voters who follow v_{h^+} .

$$\begin{aligned}
E(z_1|w = j; \rho, v_{h^-}) &= \frac{\int_{\mathcal{X}} z_1 \Pr(w = j|z; v_{h^-}) f(z; \rho) dz}{\Pr(w = j; v_{h^-})} \\
&= 2 \int_{\mathcal{X}} \left[z_1 \cos\left(-\frac{\pi}{2}\right) - z_2 \sin\left(-\frac{\pi}{2}\right) \right] \Pr(w = j|T_{-\pi/2}z; v_{h^-}) f(T_{-\pi/2}z; \rho) dz \\
&= 2 \int_{\mathcal{X}} z_2 \Pr(w = j|T_{-\pi/2}z; v_{h^-}) f(T_{-\pi/2}z; \rho) dz \\
&= 2 \int_{\mathcal{X}} z_2 \Pr(w = j|z; v_{h^+}) f(z; -\rho) dz \\
&= E(z_2|w = j; -\rho, v_{h^+}).
\end{aligned}$$

In this derivation, the second equality merely reflects a change of variables, by rotating the basis vectors of the support of z .

Assessing whether x_j^+ or x_j^- has greater magnitude for v_{h^+} is therefore equivalent to determining whether x_B^+ is larger (and x_A^+ is more negative) with positive or negative ρ . The answer is that it is larger when ρ is positive:

$$\begin{aligned}
&E(z_2|w = B; \rho) - E(z_2|w = B; -\rho) \\
&= 2 \int_{\mathcal{X}} r_z \sin(\theta_z) \Pr(w = B|z) f(z; \rho) dz - 2 \int_{\mathcal{X}} r_z \sin(\theta_z) \Pr(w = B|z) f(z; -\rho) dz \\
&= 2 \int_{\mathcal{X}} r_z \sin(\theta_z) \Pr(w = B|z) [f(z; \rho) - f(z; -\rho)] dz \\
&= 2 \int_{-\pi/4}^{3\pi/4} \int_0^1 r_z^2 \sin(\theta_z) \Pr(w = B|z) [f(z; \rho) - f(z; -\rho)] dr_z d\theta_z \\
&\quad + 2 \int_{-\pi/4}^{3\pi/4} \int_0^1 r_z^2 \sin(\theta_z + \pi) \Pr(w = B|T_\pi z) [f(T_\pi z; \rho) - f(T_\pi z; -\rho)] dr_z d\theta_z \\
&= 2 \int_{-\pi/4}^{3\pi/4} \int_0^1 r_z^2 \sin(\theta_z) [\Pr(w = B|z) - \Pr(w = B|T_\pi z)] [f(z; \rho) - f(z; -\rho)] dr_z d\theta_z \\
&= 2 \int_{\pi/4}^{3\pi/4} \int_0^1 r_z^2 \sin(\theta_z) [\Pr(w = B|z) - \Pr(w = B|T_\pi z)] [f(z; \rho) - f(z; -\rho)] dr_z d\theta_z \\
&\quad + 2 \int_{\pi/4}^{3\pi/4} \int_0^1 r_z^2 \sin\left(\theta_z - \frac{\pi}{2}\right) [\Pr(w = B|T_{-\pi/2}z) - \Pr(w = B|T_{\pi/2}z)] [f(T_{-\pi/2}z; \rho) - f(T_{-\pi/2}z; -\rho)] dr_z d\theta_z \\
&= 2 \int_{\pi/4}^{3\pi/4} \int_0^1 r_z^2 \sin(\theta_z) [\Pr(w = B|z) - \Pr(w = B|T_\pi z)] [f(z; \rho) - f(z; -\rho)] dr_z d\theta_z \\
&\quad + 2 \int_{\pi/4}^{3\pi/4} \int_0^1 r_z^2 \cos(\theta_z) [\Pr(w = B|T_{-\pi/2}z) - \Pr(w = B|T_{\pi/2}z)] [f(z; -\rho) - f(z; \rho)] dr_z d\theta_z
\end{aligned}$$

$$\begin{aligned}
&= 2 \int_{\pi/4}^{3\pi/4} \int_0^1 r_z^2 \left\{ \begin{array}{l} \sin(\theta_z) [\Pr(w = B|z) - \Pr(w = B|T_\pi z)] \\ - \cos(\theta_z) [\Pr(w = B|T_{-\pi/2} z) - \Pr(w = B|T_{\pi/2} z)] \end{array} \right\} [f(z; \rho) - f(z; -\rho)] dr_z d\theta_z \\
&= 2 \int_{\pi/4}^{\pi/2} \int_0^1 r_z^2 \left\{ \begin{array}{l} \sin(\theta_z) [\Pr(w = B|z) - \Pr(w = B|T_\pi z)] \\ - \cos(\theta_z) [\Pr(w = B|T_{-\pi/2} z) - \Pr(w = B|T_{\pi/2} z)] \end{array} \right\} [f(z; \rho) - f(z; -\rho)] dr_z d\theta_z \\
&\quad + 2 \int_{\pi/4}^{\pi/2} \int_0^1 r_z^2 \left\{ \begin{array}{l} \sin(\pi - \theta_z) [\Pr(w = B|r_z, \pi - \theta_z) - \Pr(w = B|r_z, -\theta_z)] \\ - \cos(\pi - \theta_z) [\Pr(w = B|r_z, \frac{3\pi}{2} - \theta_z) - \Pr(w = B|r_z, \frac{\pi}{2} - \theta_z)] \end{array} \right\} \\
&\quad \times [f(r_z, \pi - \theta_z; \rho) - f(r_z, \pi - \theta_z; -\rho)] dr_z d\theta_z \\
&= 2 \int_{\pi/4}^{\pi/2} \int_0^1 r_z^2 \left\{ \begin{array}{l} \sin(\theta_z) [\Pr(w = B|z) - \Pr(w = B|T_\pi z)] \\ - \cos(\theta_z) [\Pr(w = B|T_{-\pi/2} z) - \Pr(w = B|T_{\pi/2} z)] \end{array} \right\} [f(z; \rho) - f(z; -\rho)] dr_z d\theta_z \\
&\quad + 2 \int_{\pi/4}^{\pi/2} \int_0^1 r_z^2 \left\{ \begin{array}{l} -\sin(\theta_z) [\Pr(w = B|r_z, \pi - \theta_z) - \Pr(w = B|r_z, -\theta_z)] \\ + \cos(\theta_z) [\Pr(w = B|r_z, \frac{\pi}{2} - \theta_z) - \Pr(w = B|r_z, \frac{3\pi}{2} - \theta_z)] \end{array} \right\} \\
&\quad \times [f(r_z, \theta_z; -\rho) - f(r_z, \theta_z; \rho)] dr_z d\theta_z \\
&= 2 \int_{\pi/4}^{\pi/2} \int_0^1 r_z^2 \left\{ \begin{array}{l} \sin(\theta_z) \left[\begin{array}{l} \Pr(w = B|z) - \Pr(w = B|T_\pi z) \\ + \Pr(w = B|T_{-\pi/2} z) - \Pr(w = B|r_z, -\theta_z) \end{array} \right] \\ - \cos(\theta_z) \left[\begin{array}{l} \Pr(w = B|T_{-\pi/2} z) - \Pr(w = B|r_z, -\theta_z) \\ \Pr(w = B|z) - \Pr(w = B|T_\pi z) \end{array} \right] \end{array} \right\} [f(z; \rho) - f(z; -\rho)] dr_z d\theta_z \\
&= 2 \int_{\pi/4}^{\pi/2} \int_0^1 r_z^2 \left[\begin{array}{l} \Pr(w = B|z) - \Pr(w = B|r_z, -\theta_z) \\ + \Pr(w = B|T_{-\pi/2} z) - \Pr(w = B|T_\pi z) \end{array} \right] [\sin(\theta_z) - \cos(\theta_z)] [f(z; \rho) - f(z; -\rho)] dr_z d\theta_z \\
&> 0.
\end{aligned}$$

In this derivation, the third, fifth, and eighth equalities use change of variables. The second-to-last equality and the inequality follow because $\frac{\pi}{4} < \theta_z < \frac{\pi}{2}$ implies that

$$\begin{aligned}
&\Pr(w = B|z) = \Pr\left(w = B|r_z, \frac{\pi}{2} - \theta_z\right) \\
&> \Pr(w = B|r_z, \pi - \theta_z) = \Pr(w = B|T_{-\pi/2} z) \\
&> \Pr(w = B|T_{\pi/2} z) = \Pr(w = B|r_z, -\theta_z) \\
&> \Pr(w = B|T_\pi z) = \Pr(w = B|r_z, \theta_z + \pi) = \Pr\left(w = B|r_z, \frac{3\pi}{2} - \theta_z\right)
\end{aligned}$$

and that $\sin(\theta_z) > \cos(\theta_z)$. ■

Proposition 5 *If $\rho > 0$ then $W(x_A^+, x_B^+, v_{h+}) > W(x_A^-, x_B^-, v_{h-})$.*

Proof. As a preliminary step, note that the symmetry of f (specifically, Condition 3), $u(x|z)$, and v_{h+} are such that welfare (17) can be rewritten as

$$\begin{aligned}
W(x_A, x_B, v_{h+}) &= \int_{\{z \in \mathcal{X}: z_1 + z_2 > 0\}} \left[\begin{array}{l} \sum_{j=A,B} u(x_j|z_1, z_2) \Pr(w = j|z_1, z_2) f(z_1, z_2) \\ + \sum_{j=A,B} u(x_j| -z_1, -z_2) \Pr(w = j| -z_1, -z_2) f(-z_1, -z_2) \end{array} \right] dz \\
&= 2 \int_{\{z \in \mathcal{X}: z_1 + z_2 > 0\}} \sum_{j=A,B} u(x_j|z_1, z_2) \Pr(w = j|z_1, z_2) f(z_1, z_2) dz \\
&= 2 \int_{\{z \in \mathcal{X}: z_2 > |z_1|\}} \sum_{j=A,B} \left[\begin{array}{l} u(x_j|z_1, z_2) \Pr(w = j|z_1, z_2) f(z_1, z_2) \\ + u(x_j|z_2, z_1) \Pr(w = j|z_2, z_1) f(z_2, z_1) \end{array} \right] dz \\
&= 4 \int_{\{z \in \mathcal{X}: z_2 > |z_1|\}} \sum_{j=A,B} u(x_j|z_1, z_2) \Pr(w = j|z_1, z_2) f(z_1, z_2) dz
\end{aligned}$$

$$= 4 \int_{\{z \in \mathcal{X}: z_2 > z_1 > 0\}} \sum_{j=A,B} \left[\begin{array}{c} u(x_j|z_1, z_2) \Pr(w = j|z_1, z_2) f(z_1, z_2) \\ + u(x_j| - z_1, z_2) \Pr(w = j| - z_1, z_2) f(-z_1, z_2) \end{array} \right] dz. \quad (20)$$

Given candidate platforms x_A and x_B and equilibrium voting strategy v_{h+} , therefore, the welfare benefit of having a positive correlation instead of a negative correlation can be written as

$$\begin{aligned} & W(x_A, x_B, v_{h+}; \rho) - W(x_A, x_B, v_{h+}; -\rho) \\ &= 4 \int_{\{z \in \mathcal{X}: z_2 > z_1 > 0\}} \sum_{j=A,B} \left\{ \begin{array}{c} u(x_j|z_1, z_2) \Pr(w = j|z_1, z_2) [f(z_1, z_2; \rho) - f(z_1, z_2; -\rho)] \\ + u(x_j| - z_1, z_2) \Pr(w = j| - z_1, z_2) [f(-z_1, z_2; \rho) - f(-z_1, z_2; -\rho)] \end{array} \right\} dz \\ &= 4 \int_{\{z \in \mathcal{X}: z_2 > z_1 > 0\}} [f(z_1, z_2; \rho) - f(-z_1, z_2; \rho)] \sum_{j=A,B} \left[\begin{array}{c} u(x_j|z_1, z_2) \Pr(w = j|z_1, z_2) \\ - u(x_j| - z_1, z_2) \Pr(w = j| - z_1, z_2) \end{array} \right] dz \\ &= 4 \int_{\{z \in \mathcal{X}: z_2 > z_1 > 0\}} \left[\begin{array}{c} f(z_1, z_2; \rho) \\ - f(-z_1, z_2; \rho) \end{array} \right] \\ & \quad \times \sum_{j=A,B} \left[\begin{array}{c} [u(x_j|z_1, z_2) - u(x_j| - z_1, z_2)] \Pr(w = j|z_1, z_2) \\ + u(x_j| - z_1, z_2) [\Pr(w = j|z_1, z_2) - \Pr(w = j| - z_1, z_2)] \end{array} \right] dz. \end{aligned} \quad (21)$$

The second portion of this summation reduces to

$$\begin{aligned} & u(x_A| - z_1, z_2) \left[\begin{array}{c} \Pr(w = A|z_1, z_2) \\ - \Pr(w = A| - z_1, z_2) \end{array} \right] + u(x_B| - z_1, z_2) \left[\begin{array}{c} \Pr(w = B|z_1, z_2) \\ - \Pr(w = B| - z_1, z_2) \end{array} \right] \\ &= u(x_A| - z_1, z_2) \left[\begin{array}{c} \Pr(w = B| - z_1, z_2) \\ - \Pr(w = B|z_1, z_2) \end{array} \right] + u(x_B| - z_1, z_2) \left[\begin{array}{c} \Pr(w = B|z_1, z_2) \\ - \Pr(w = B| - z_1, z_2) \end{array} \right] \\ &= [u(x_B| - z_1, z_2) - u(x_A| - z_1, z_2)] [\Pr(w = B|z_1, z_2) - \Pr(w = B| - z_1, z_2)], \end{aligned}$$

which is positive when $z_2 > z_1 > 0$ because $\Pr(w = B|z_1, z_2) > \Pr(w = B| - z_1, z_2)$ and $u(x_B| - z_1, z_2) > u(x_A| - z_1, z_2)$. Moreover, since equilibrium platforms lie on the main diagonal (i.e. $x_{j1} = x_{j2}$ for $j = A, B$), the utility difference

$$u(x_j|z_1, z_2) - u(x_j| - z_1, z_2) = -(z_1 - x_{j2})^2 - (z_2 - x_{j2})^2 + (-z_1 - x_{j2})^2 + (z_2 - x_{j2})^2$$

reduces to $4x_{j2}z_1$. Therefore, the welfare difference (21) exceeds

$$\begin{aligned} & 4 \int_{\{z \in \mathcal{X}: z_2 > z_1 > 0\}} [f(z_1, z_2; \rho) - f(-z_1, z_2; \rho)] \sum_{j=A,B} 4x_{j2}z_1 \Pr(w = j|z_1, z_2) dz \\ &= 4 \int_{\{z \in \mathcal{X}: z_2 > z_1 > 0\}} z_1 [f(z_1, z_2; \rho) - f(-z_1, z_2; \rho)] \left\{ \begin{array}{c} x_{A2} [1 - \Pr(w = B|z_1, z_2)] \\ + x_{B2} \Pr(w = B|z_1, z_2) \end{array} \right\} dz \\ &= 32x_{B2} \int_{\{z \in \mathcal{X}: z_2 > z_1 > 0\}} z_1 [f(z_1, z_2; \rho) - f(-z_1, z_2; \rho)] \left[\Pr(w = B|z_1, z_2) - \frac{1}{2} \right] dz \end{aligned} \quad (22)$$

where the second equality follows because platforms are symmetric, implying that $x_{A2} = -x_{B2}$.

By symmetry, $z_1 = z_2 = 0$ would imply that $\phi(B|z) = \frac{1}{2}$, and therefore that $\Pr(w = B|z_1, z_2) = \frac{1}{2}$. Since $\Pr(w = B|z_1, z_2; v_{h+})$ is increasing in $\phi(B|z)$, which (by Part 1 of Lemma 1) is increasing in both z_1 and z_2 , $z_2 > z_1 > 0$ implies that $\Pr(w = B|z_1, z_2) > \frac{1}{2}$. Also, $z_2 > |z_1|$ implies that $z \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} > 0$ and $z \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} < 0$, so by Condition 1, $\nabla f(z) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \nabla f(z) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \nabla f(z) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is positive, and increasing in ρ . Thus, for $z_2 > z_1 > 0$, f increases in its first argument between $(-z_1, z_2)$ and (z_1, z_2) . Together, these

observations imply that (22) is positive and increasing in ρ , meaning that welfare is higher when ρ is positive than when it is negative.

Given this lengthy preliminary step, the proof of the proposition is relatively simple. First, note that the voting strategy v_{h^+} is merely a rotation of v_{h^-} by the amount $\frac{\pi}{2}$ (i.e. $h^+ = T_{\pi/2}h^-$), and consider the (non-equilibrium) strategy combination $(T_{\pi/2}x_A^-, T_{\pi/2}x_B^-, v_{h^+})$ in which candidate behavior is rotated by the same amount. This strategy combination specifies the same behavior for voters and candidates as (x_A^-, x_B^-, v_{h^-}) , but in different states of the world. The latter could therefore be viewed as welfare-equivalent to the former, except with a negative correlation coefficient:

$$\begin{aligned} W(x_A^-, x_B^-, v_{h^-}; \rho) &= \int_{\mathcal{X}} \sum_{j=A,B} u(x_j^-|z) \Pr(w = j|z; h^-) f(z; \rho) dz \\ &= \int_{\mathcal{X}} \sum_{j=A,B} u(T_{\pi/2}x_j^-|T_{\pi/2}z) \Pr(w = j|T_{\pi/2}z; T_{\pi/2}h^-) f(T_{\pi/2}z; -\rho) dz \\ &= \int_{\mathcal{X}} \sum_{j=A,B} u(T_{\pi/2}x_j^-|z) \Pr(w = j|z; h^+) f(z; -\rho) dz \\ &= W(T_{\pi/2}x_A^-, T_{\pi/2}x_B^-, v_{h^+}; -\rho), \end{aligned}$$

where the fact that $f(z; \rho) = f(T_{\pi/2}z; -\rho)$ follows from the directional symmetry of f (Condition 3). Since the platforms $T_{\pi/2}x_A^-$ and $T_{\pi/2}x_B^-$ lie on the main diagonal, the preliminary argument above applies here, implying that $(T_{\pi/2}x_A^-, T_{\pi/2}x_B^-, v_{h^+})$ provides higher welfare. Moving from $(T_{\pi/2}x_A^-, T_{\pi/2}x_B^-, v_{h^+})$ to $(x_A^+, T_{\pi/2}x_B^-, v_{h^+})$ thus improves welfare, and moving to (x_A^+, x_B^+, v_{h^+}) improves welfare a second and third time, as the two candidates optimize in turn. ■

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