A delegation-based theory of expertise

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Abstract

We investigate competition in a delegation framework. An uninformed principal is unable to perform a task herself and must choose between one of two experts to do the job. The experts, who are biased and imperfectly informed, propose action choices simultaneously. In equilibrium experts may exaggerate their biases, taking into account the expected information content of the rival's proposal and the fact that the principal's optimal choice serves to offset this exaggeration - similar to bidders reacting to the winner's curse phenomenon in common value auctions. We show that having a second expert can benefit the principal, even if the two experts have the same biases or if the first expert is known to be unbiased. In contrast with other models of expertise, in our setting the principal prefers experts with equal rather than opposite biases. The principal may also benefit from commitment to an "element of surprise," making an ex post suboptimal choice with positive probability.

1 Introduction

There are many situations in which a principal lacks the knowledge and expertise to perform a certain task, and therefore has to delegate the job to a qualified expert. Examples include a candidate running for office who has to hire an expert to work out her economic agenda, or the CEO of a pharmaceutical company who must delegate building a research and development division to a scientist. Further complicating the principal's situation is that experts tend to have systemic biases, preferring suboptimal actions from the principal's perspective.

In this paper we investigate a model in which a principal has to delegate a task to one of two experts. The need to delegate differentiates our model from models of expertise in which experts send cheap talk recommendations to the principal, such as Krishna and Morgan (2001b). In particular, we consider the following game. First, experts receive noisy and conditionally independent signals of a single dimensional state variable. The principal's ideal action is equal to the state, but each expert has a constant bias (either positive or negative) and a resulting ideal point different from the sender's. Next, the experts simultaneously propose actions. A proposal is assumed to bind the expert to perform the given action whenever the principal delegates the task to him.[1] The principal then chooses one of the two offers, and the corresponding action is taken by the given expert. We are motivated by situations in which the principal originally has much less knowledge about the state

*We thank Peter Landry, Xiao Yu Wang, Navin Kartik, Ricardo Alonso, David McAdams, Curtis Taylor and Scott Kominers for useful feedback. All errors are our own.

[1]Even if the principal might not have the knowledge to verify whether the expert indeed chose the action that he proposed, outside experts might be able to verify if that was the case and hence penalties can be imposed on experts deviating from their proposals.

than the experts, and correspondingly we assume that the principal's prior is improper uniform (diffuse) over a state space represented by the real line.

Our model best applies to situations in which the principal lacks the knowledge to implement or initiate changes in the proposed actions, so all she can do is solicit different proposals and choose one of them. The assumption that proposals commit the experts corresponds to common law, according to which an offer is a statement of terms on which the offeror is willing to be bound, and it shall become binding as soon as it is accepted by the person to whom it is addressed.[2] A different application for our model is political competition: starting from Downs (1957), most papers on political competition in a Hotelling (1929) framework assume that candidates are committed to the policies they announce in the campaign, and the electorate can only choose between the policies announced by the candidates.[3] In this context the bonus corresponds to the rents from being in office. Yet another application for our model is a setting where a legislative body (floor) seeks legislative proposals for the same bill from multiple committees, using a modified rule (see Gilligan and Krehbiel (1989), Krishna and Morgan (2001a)), meaning that the floor cannot amend the proposals and can only accept one of the proposed bills without modification, conforming to the basic assumptions of our model.[4]

We also extend the above baseline model, in which experts only care about the policy outcome (the implemented action relative to the state), to situations in which experts also benefit from being selected. In particular, we allow for a bonus to the chosen expert, either as a monetary payment or as a non-monetary benefit, such as increased prestige in his profession. We investigate two cases, with the bonus amount given exogenously in one case and optimally chosen by the principal in the other.

The above game is very complex in general, due to the size of the strategy space. In this paper we restrict attention to equilibria in which the experts' strategies are *relatively stationary* with respect to signals, meaning that each expert's proposal is equal to his signal plus a constant. We consider focusing on such strategies, that treat states symmetrically, natural in a game with diffuse prior and preferences that are relatively stationary in the state, in which all states are perfectly symmetric. A further motivation comes a companion paper (Ambrus and Kolb, 2016), in which we examine the possibility of extending the concept of ex ante expected payoffs to a larger class of games with diffuse prior (and hence bringing them into the realm of traditional game theory, in which payoffs have to be well-defined for any strategy profile). The companion paper shows that in our game expert strategies need to be restricted to be constant markup in order for well-defined ex ante expected payoffs to exist. In particular, given some weak conditions on the principal's set of strategies, essentially constant markup strategies are the only strategies for which well-defined limit expected payoffs exist for any strategy profile when taking a sequence of proper priors diffusing (converging in a formal sense to the diffuse prior), with the limit not depending on the particular choice of sequence. This result shows that in order to obtain well-defined ex ante expected payoffs

[2] See Treitel (1999), p8.

[3] For theoretical motivations for this assumption, and empirical relevance in the political competition context, see Pétry and Collette (2009), Kartik et al. (2015), and papers cited therein.

[4] Gilligan and Krehbiel (1989) analyze this situation with an additional option to the floor, in the form of not accepting either of the proposals and opting for a status quo outcome. As opposed to our model, Gilligan and Krehbiel (1989) assume perfectly informed experts (committees), which fundamentally changes the strategic interaction.

corresponding to all strategy profiles, one would need to restrict experts' strategies to constant markup ones. In the current paper we do not restrict experts' strategies, simply focus on equilibria in which they play constant markup strategies, and similarly to existing game theoretical models of improper prior (Friedman (1991), Klemperer (1999), Morris and Shin (2002, 2003), Myatt and Wallace (2014)), we only evaluate payoffs in the interim stage (after signal realizations).

Our first result shows that if experts play constant markup strategies then we can restrict attention to the following simple strategies for the principal: always choosing expert 1's offer (effectively delegating the action choice to expert 1), always choosing expert 2's offer, always choosing the minimum of the two offers, and always choosing the maximum of the two offers. In particular, whenever the sum of markups by the experts is positive, the unique best response of the principal is always choosing the smaller of the two offers, while if the sum of the two markups is negative, the unique best response of the principal is always choosing the larger of the two offers.[5]

It is easy to show that a (Bayesian Nash) equilibrium with constant markup strategies always exists in our model, in the form of delegating the task to one of the experts. Formally, one expert always proposing his ideal action conditional on the signal he observes (equal to the signal plus his bias), the other expert proposing his signal minus the first expert's bias, and the principal always delegating the task to the first expert constitutes an equilibrium.[6] The question is whether there exist other equilibria of the game, in which the principal either always chooses the minimum or always chooses the maximum of the two proposals - hence her choice depends nontrivially on the proposals. We assume without loss of generality that the sum of the biases of the two experts is nonnegative.

In our baseline model (only policy preferences), there always exists an equilibrium in which the principal chooses the minimum of the two offers and the experts apply markups above their biases; if both biases are positive, this means that both experts exaggerate their biases. This result is contrary to a naive intuition that experts in competition should move toward the center. We call this equilibrium "upward," as the experts on average exaggerate their signals upwards in their proposals. This result extends to the case where an exogenous bonus is given to the selected expert, as long as the bonus is not too large. To illustrate, suppose that experts have the same biases and the bonus is small. Then in this upward equilibrium both experts propose actions strictly above their ideal actions based purely on their private signals. This is because, similarly to the winner's curse phenomenon in common value auctions, being selected by the principal contains information on the other expert's signal (namely that his signal is higher), changing the optimal action of the expert. In equilibrium, proposals have to be optimal conditional on the event that the other expert's action proposal is higher.

We also show that if the experts' signals are noisy enough then, for a subset of the range of bonuses for which an upward equilibrium exists, there also exists a "downward" equilibrium in which experts

[5]These strategies are also feasible for a principal who can only process information in a coarse way, being only able to make binary comparisons between two offers and lacking the ability to measure the difference between them, as consumers in Kamenica (2008). Therefore such information processing constraints would not hurt the principal in the equilibria we investigate.

[6]There are other Bayesian Nash equilibria on mixed strategies with the same outcome, in which one expert always proposes his ideal point, the other expert "babbles" (randomizes over possible messages he can send), and the principal always delegates the task to the first expert.

propose actions on average *below* their signals, and the maximum of the two proposals is selected by the principal. The strategic forces are similar to those in upward equilibrium: the fact that the maximum of the two offers is selected pushes proposals downwards, and for noisy signals on average experts modify their proposals downward relative to their signals.[7] This type of equilibrium does not exist when the signals are very precise, because then the information conveyed from being selected does not shift the optimal proposals of the experts enough to make markups negative on average. We show that even when the downward equilibrium exists, the principal prefers the upward equilibrium to it, and thus for further welfare comparisons we need only consider upward equilibrium and simple delegation to the less-biased expert.

The feature of the above equilibria that similarly biased experts exaggerate in a particular direction (and hence the principal should choose the proposal least in the direction of the exaggeration) is in line with empirical evidence. For example, Zitzewitz (2001), Bernhardt et al. (2006) and Chen and Jiang (2006) find that financial analysts systematically exaggerate their forecasts relative to unbiased forecasts based on the analysts' information sets, while Iezzoni et al. (2012) report that 55% of doctors in a survey said that in the previous year they had been more positive about patients' prognoses than their medical histories warranted.

We compare the principal's welfare between upward equilibrium and simple delegation in order to find the principal-optimal equilibrium. In general, the comparison is complicated and can go either way, but for several focal cases of interest, the optimum is the upward equilibrium. The principal is always better off in the upward equilibrium when the experts have the same biases (as in settings where all available experts have similar agendas), or when the experts have exactly opposite biases.[8] This result holds even when the bonus is zero and hence there is no competition among experts for being selected. The principal also prefers upward equilibrium to simple delegation when one expert's bias is positive and the other's is zero; this is despite the fact that under simple delegation the unbiased expert's incentives are perfectly aligned with the principal's. The intuition for the result is that in the upward equilibrium the principal extracts some additional information the second expert, which reduces the variance of the chosen action, and this benefit always outweighs any cost associated with higher markups. Applied to a political setting, the latter comparison between upward equilibrium and simple delegation helps explain what voters may otherwise perceive as corruption – a politician may want to seek advice from a biased expert, even if it is common knowledge that she already has access to an unbiased expert.

We also compare the principal's payoffs when experts have equal versus opposite biases, and our results here are in contrast with some of the existing literature. In our model, assuming the upward equilibrium is played, having two experts with identical biases yields a higher payoff than having two antagonist experts with opposite biases. In general, the expected bias of the implemented action is smaller with antagonist experts than with experts having the same bias, but this benefit is outweighed by a higher variance of the implemented action that arises because the expert with the

[7]There can also be an equilibrium in which the principal mixes with a particular probability between accepting the lower or the higher proposal. We provide a partial characterization of such mixed equilibria in the Supplementary Appendix.

[8]The principal's payoffs are continuous in the parameters of the model for a particular type of equilibrium, hence the above comparisons are the same when the absolute values of the biases are close to each other but not exactly equal.

lower bias is selected most of the time, and so the information from the other expert's signal is only utilized to a limited extent. This result contrasts models of competition in persuasion (Milgrom and Roberts (1986), Gentzkow and Kamenica (2015)), in which antagonist experts benefit the principal by pressing each other to reveal more information,[9] and with the multi-sender cheap talk model of Krishna and Morgan (2001b), in which having a second sender with the same bias does not benefit the receiver.[10] See also Shin (1998) and Dewatripont and Tirole (1999) for different types of models making the case for adversarial procedures.

The principal's expected payoff depends in a complicated way on the noise in the experts' signals, and on the amount of the bonus. Hence, for these comparative statics we focus on the case of equally biased experts. Even in this case, the effect of the variance of the experts' signals is ambiguous. An increased precision of experts' signals reduces the variance of the implemented action conditional on the state. For small bonuses, this unambiguously increases the principal's expected payoff. However, for larger bonuses, it might benefit the principal in the upward equilibrium if the experts increase their markups,[11] which can result from increasing the variance of the signals. We provide an exact characterization (for equally biased experts) for when a decrease in the variance of experts' signals benefits the principal.

Increasing the bonus reduces the absolute values of the experts' markups, hence bringing their proposals closer to truthful reporting, both in the upward and downward equilibria. Intuitively, a higher bonus increases competition among experts, leading them to decrease their proposals in the upward equilibrium and increase their proposals in the downward equilibrium. In the upward equilibrium, this initially improves the principal's expected payoff by decreasing the expected bias of the implemented action. There is a threshold level of bonus though at which the expected bias of the implemented action becomes zero, and increasing the bonus above this threshold decreases the principal's payoff. When the bonus comes from exogenous sources, the optimal bonus from the principal's perspective is always strictly positive, and is on the interior of the interval of bonuses for which the upward equilibrium exists. When the bonus is paid by the principal, the optimal bonus amount is always strictly smaller than in the previous case, and depending on the parameters it can be either strictly positive or zero.

In the political competition application of the model the result implies that a small amount of office-seeking motivation can be beneficial for voters, but at higher levels a further increase in office-seek motivation can adversely affect voters' welfare.

We consider two extensions of our model, for equally biased experts. In the first one we allow the principal to commit ex ante to any mixture of simple strategies, and show that for bonuses that

[9] Experts with identical agendas can be better for the principal than experts with opposing agendas in the persuasion model of Bhattacharya and Mukherjee (2013). The mechanism is rather different than in our paper, though: with similar experts an undesirable default action can provide strong incentives for both experts to reveal information.

[10] As opposed to cheap talk models with multiple senders and one receiver, where it tends to be better for information revelation if the senders are oppositely biased from the point of view of the receiver, in committee settings, where committee members can reveal information to each other, it helps information revelation if members have more similar preferences - see for example Li and Suen (2009).

[11] This is related to the chunkiness of the principal's possible choices in our model: for certain parameter values sticking with choosing the minimum of two proposed actions is still optimal for the principal, even though it leads to the implemented action being negatively biased. This can happen if choosing the maximum offer would lead to an even larger positive bias. These are the cases when an increase in the expectation of the minimum offer benefits the principal.

are not too large, such commitment leads to the same outcome as in the upward equilibrium of the original game, hence the ability to commit does not improve the principal's welfare. On the other hand, we show how committing to choosing an inferior offer with some small probability - introducing an element of surprise - can improve the principal's welfare in the case of opposite biases of large magnitude. The intuition is that in upward equilibrium, the expert with the positive bias faces a very large winner's curse, and applies a markup well above his bias. The other expert faces an almost negligible winner's curse and applies a markup just slightly above his bias. Now by threatening to choose the higher offer some of the time, the principal induces the first expert to reduce his markup drastically and the second expert to raise his markup, so both markups move closer to zero. The cost of this deviation to the principal lies in mistakenly choosing the higher offer, but when the magnitude of the biases is sufficiently large, the benefit outweighs the cost. The finding that the principal can benefit from committing to a mixed strategy in certain situations is consistent with an observed pattern of regulatory uncertainty. Ederer et al. (2014) show similarly that commitment to an opaque reward scheme reduces temptation to game the system in a principal-agent environment.

In the second extension we drop the dependence of the unselected expert's payoff on the implemented action, and instead assume that the expert gets a fixed outside option payoff. This variant of the model is more realistic in market transaction situations, such as when experts are car mechanics or doctors. A car mechanic might be biased towards larger repairs than necessary, but typically he does not care about what type of repair is chosen in case a different mechanic is selected to do the job. The analysis of this version of the model is more involved, but we show that under some parameter restrictions similar upward and downward equilibria exist as in the baseline model. The fact that the unselected expert gets a fixed outside payment increases the experts' proposals in upward equilibrium, and decreases them in downward equilibrium.

2 Related Literature

The literature on delegation so far mainly focused on either the question of delegating the action choice versus retaining the right to take the action (Dessein (2002), Li and Suen (2004)) or on optimally constraining the action choices of a particular expert (Holmström (1977), Melumad and Shibano (1991), Alonso and Matouschek (2008)). Krishna and Morgan (2008) investigate how monetary incentives can be used optimally in delegation to a single agent. More related to our investigation are papers introducing policy-relevant private information on the part of candidates into the context of the classic Downs (1957) model of political competition: Heidhues and Lagerlöf (2003), Laslier and Van der Straeten (2004), Loertscher et al. (2012), Gratton (2014), and closest to our model Kartik et al. (2015), as the latter focuses on cases when voters are relatively uninformed. Similarly to our setting, in the above papers politicians receive independent private signals about the state of the world and hence the optimal policy from the electorate's point of view. The main difference relative to our model is that the politicians in the above models do not have policy preferences, and they are purely office-motivated. For this reason neither the own private information nor the rival's private information directly affects their expected payoffs, and the candidates play a zero-sum game. In contrast, in our model the experts' signals are directly payoff-relevant for them, and their interests are partially aligned, as in higher states they would both like to induce higher actions. This leads to different equilibrium dynamics than in Kartik et al. (2015), and to distinct conclusions: in particular, in their model the electorate can never strictly benefit from the presence of a second candidate (relative to just a single one).[12]

There is also a line of literature extending the Downs (1957) model framework to politicians having mixed motivation (having both policy preferences and wanting to win), as in our model, starting with Wittman (1983) and Calvert (1985). Schultz (1996), Martinelli (2001) and Martinelli and Matsui (2002) introduce asymmetric information in this context, but as opposed to our paper, with perfectly informed politicians. This leads to different conclusions, including that full revelation of information is possible in equilibrium when policy preferences are not too extreme. Callander (2011) considers a model of sequential elections with imperfectly informed politicians having mixed motivations, but the issues investigated are different from ours and inherently dynamic: searching for a good policy in a complex environment, by trial and error.

Outside the delegation literature, Prendergast (1993) considers a context in which both a worker and a manager observes the state with noise, and the worker also receives a noisy signal about the manager's observation. In this model the worker has an incentive to cater to the manager and bias her report toward what she thinks the manager's observation is. Gerardi et al. (2009) investigates aggregation of expert opinions through a particular mechanism that approximates the first best outcome if signals are very accurate. Pesendorfer and Wolinsky (2003) investigate the effects of being able to solicit a second opinion from a different expert, in a dynamic model in which experts are not biased but it is costly for them to gather information.

Another line of literature investigates multi-sender extensions of the cheap talk model of Crawford and Sobel (1982), and finds that under certain conditions there can be equilibria in which the receiver can extract full or almost full information from the senders (Gilligan and Krehbiel (1989), Austen-Smith (1993), Wolinsky (2002), Battaglini (2002, 2004), Ambrus and Takahashi (2008), Ambrus and Lu (2014)). As opposed to the above papers, we investigate settings in which the principal cannot solicit information from experts and then take the action choice herself. Lastly, Ottaviani and Sørensen (2006) consider a model with multiple experts with reputational concerns reporting sequentially on privately observed signals. The issues they focus on (potential herding behavior of experts) are very different than in the current paper.

3 Base Model

We consider the following multi-stage game with incomplete information. There are three players: a principal and two experts. The set of states of the world is $\mathbb{R}$, and we assume that the common

[12] Our model approximates the model in Kartik et al. (2015) when the bonus payment is very large and so the agents mainly care about being selected. We find that for very large bonuses the only equilibria in our model involve delegating the action choice to a single agent, which is in line with the result on maximum informativeness of political competition in Kartik et al. (2015). Correspondingly, Kartik et al. (2015) discuss an extension of their model in which they show that allowing a small amount of ideological motivation for the candidates, and assuming that they are close to unbiased from the electorate's point of view implies that in equilibrium one candidate must be winning ex ante with probability close to 1. These results suggest that there is no discontinuity between no policy preference versus a small amount of policy preference for the agents. Our paper mainly focuses on cases in which agents' policy preferences are relatively important.

prior distribution of states is diffuse (improper uniform).

In stage 0 state $\theta \in \mathbb{R}$ realizes. In stage 1 each expert $i = 1, 2$ receives a noisy private signal about the state of the world $s_i = \theta + \epsilon_i$, where $\epsilon_i \sim N(0, \sigma^2)$, and ϵ_1 and ϵ_2 are independent. In stage 2 each expert i proposes an action $a_i \in \mathbb{R}$ to the principal. In stage 3 the principal chooses one of the two experts, who then implements the action he proposed in stage 2.

Let real-valued functions $a_1(s_1)$ and $a_2(s_2)$ denote the strategies of expert 1 and expert 2 respectively, while $C(a_1, a_2) \in \{1, 2\}$ is the principal's choice strategy. If action $a = a_i$ is taken then the principal's payoff is $V(a, \theta) = -(a-\theta)^2$, and the payoff of expert $j = 1, 2$ is $U_j(a, \theta) = -(a-\theta-b_j)^2$. In Section 5, we extend the model to include a bonus payment B to the chosen expert. We call b_i the *bias* of expert i. Without loss of generality, we assume that $b_1 + b_2 \geq 0$ and $b_1 \geq b_2$. We further assume that all parameters of the game are common knowledge.

In the analysis below, we focus on perfect Bayesian equilibria in which experts' strategies are stationary in the following sense: $a_i(s_i) = s_i + k_i$, where $k_1, k_2 \in \mathbb{R}$. In words, each expert applies a constant mark-up to his signal when forming a proposed action. With slight abuse of notation, we use simply $(k_1, k_2, C(a_1, a_2))$ to denote a strategy profile with constant markup strategies.

4 Stationary Equilibria in the Base Model

In this section we characterize all pure strategy stationary equilibria in the delegation game. We start with determining the best response of the principal to all possible pairs of constant markup strategies by the experts, and then investigate the candidate equilibria consistent with this best response behavior.

4.1 Best Responses to Stationary Sender Strategies

Here we analyze the principal's best response to constant markup strategies, and show that it only depends on the sum of markups. As the principal has a quadratic loss function, her expected payoff can be decomposed into losses from the uncertainty about the true state (which is independent of her action) and the losses from the expected difference between the chosen action and the true state. Therefore the principal prefers the offer which is closer to her posterior expectation of the true state. After observing the offers, the principal's expectation about the true state is lower (higher) than the average of the experts' offers if and only if the sum of the markups is positive (negative). Figure 1 illustrates a case where the markups have positive sum.

Let $\arg\min\{a_1, a_2\}$ be defined as $\{1\}$ if $a_1 < a_2$, $\{2\}$ if $a_1 > a_2$, and $\{1, 2\}$ if $a_1 = a_2$. Similarly, let $\arg\max\{a_1, a_2\}$ be defined as $\{1\}$ if $a_1 > a_2$, $\{2\}$ if $a_1 < a_2$, and $\{1, 2\}$ if $a_1 = a_2$.

Theorem 1. *If experts follow constant markup strategies $a_i(s_i) = s_i + k_i$, then*

- *if $k_1 + k_2 > 0$, the principal strictly prefers the lower offer, and $C(a_1, a_2) \in \arg\min\{a_1, a_2\}$;*
- *if $k_1 + k_2 < 0$, the principal strictly prefers the higher offer, and $C(a_1, a_2) \in \arg\max\{a_1, a_2\}$;*
- *if $k_1 + k_2 = 0$, the principal is indifferent between the offers.*

Proof. After observing both offers, the principal updates her belief: $\theta|a_1, a_2 \sim N(\frac{1}{2}(a_1+a_2) - \frac{1}{2}(k_1+k_2), \frac{\sigma^2}{2})$. Therefore the principal's expected utility from choosing offer a_1 or a_2 is:

$$V(a_1) = \mathbb{E}[-(\theta - a_1)^2] = -\mathrm{Var}(\theta) - (\mathbb{E}[\theta] - a_1)^2 = -\frac{\sigma^2}{2} - \left[\frac{1}{2}(a_2 - a_1) - \frac{1}{2}(k_1 + k_2)\right]^2$$
$$V(a_2) = \mathbb{E}[-(\theta - a_2)^2] = -\mathrm{Var}(\theta) - (\mathbb{E}[\theta] - a_2)^2 = -\frac{\sigma^2}{2} - \left[\frac{1}{2}(a_1 - a_2) - \frac{1}{2}(k_1 + k_2)\right]^2.$$

Hence, $V(a_1) - V(a_2) = (a_2 - a_1)(k_1 + k_2)$, which immediately implies the statements in the theorem. ∎

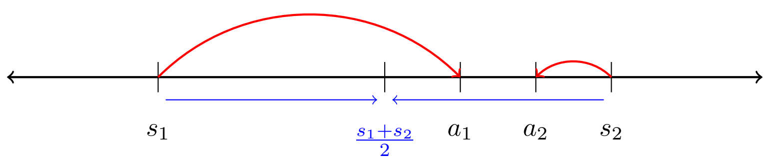

Figure 1: $k_1 > 0, k_2 < 0, k_1 + k_2 > 0$. The principal chooses the lower offer a_1, which lies closer to her expectation $\frac{s_1+s_2}{2}$.

An equilibrium $(k_1, k_2, C(a_1, a_2))$ is said to be an *upward equilibrium*, if on average experts adjust their signals upwards and the lower proposal is accepted: $k_1+k_2 \geq 0$ and $C(a_1, a_2) \in \arg\min(a_1, a_2)$. In upward equilibrium, the principal's updated expectation of the state of the world is lower than the average of the two offers. Her best response is to choose the lower offer, which is closer to her expectation, as demonstrated in Figure 1. Likewise, an equilibrium $(k_1, k_2, C(a_1, a_2))$ is said to be a *downward equilibrium* if $k_1 + k_2 \leq 0$ and $C(a_1, a_2) \in \arg\max(a_1, a_2)$. Note that when $k_1 + k_2 = 0$ then the principal's posterior expectation of θ is exactly the average of the two offers, and she is indifferent between the two. This raises the possibility of equilibria in which the experts play constant markup strategies and the expert mixes between the lower and the higher offer with some fixed probability. We investigate such equilibria in the Supplementary Appendix.

4.2 Simple delegation

In our game there always exist simple pure strategy equilibria in which the principal always chooses the same expert, independently of two offers, in effect delegating the decision to her. In particular, Theorem 1 implies that if expert i chooses constant markup b_i and the other expert chooses constant markup $-b_i$ then the principal is always indifferent between the two offers, and she might as well always choose expert i. Given this strategy of the principal, expert i's best response is choosing exactly markup b_i, which in expectation implements his ideal action. The other expert has no profitable deviation since his proposal is never accepted. While such an equilibrium exists for each of the two experts, it is more natural to consider the one in which the principal always chooses the expert with the smaller absolute bias, who is expert 2 by convention. These observations are summarized in the next proposition.

Proposition 1. *For $i \in \{1,2\}$, an equilibrium exists in which the principal always chooses expert i and markups are $k_i = b_i$ and $k_j = -b_i$ for $j \neq i$. The principal's expected payoff in this equilibrium is $\mathbb{E}[-(s+b_i)^2] = -\sigma^2 - b_i^2$, where $s \sim N(0,\sigma^2)$.*

In the rest of the section we examine equilibria in which the principal's choice between the experts depends in a nontrivial way on the pair of offers proposed.

4.3 Upward equilibrium

Here we investigate strategy profiles $\{(k_1, k_2, a \in \arg\min\{a_1, a_2\}) : k_1 + k_2 \geq 0\}$. We start by computing players' payoffs under such strategy profiles. Let $b(k_1,k_2,L) = \mathbb{E}(a-\theta)$ denote the expected bias of the chosen offer and $Var(k_1,k_2,L) = \mathrm{Var}(a-\theta)$ denote the variance of the chosen offer. Then the principal's utility is: $V(k_1,k_2,L) = -\mathbb{E}(a-\theta)^2 = -b^2(k_1,k_2,L) - Var(k_1,k_2,L)$. Proposition 2 below provides the expanded forms of these expressions, which are useful to our analysis. Here and throughout the rest of the paper, let f and F denote the PDF and the CDF of the distribution $N(0,2\sigma^2)$ and let $z = k_1 - k_2$. Note that the expected bias $b(k_1,k_2,L)$ is strictly less than the expected value of the selected markup, $k_1(1-F(z)) + k_2F(z)$; this is because the lower offer is associated with a noise term which is normally distributed but truncated above and thus has negative expectation.

Proposition 2. *If both experts follow constant markup strategies $a_j(s_j) = s_j + k_j$ and the principal always chooses the lower offer, then*

$$\begin{aligned}
b(k_1,k_2,L) &= -2\sigma^2 f(z) + k_1(1-F(z)) + k_2F(z);\\
Var(k_1,k_2,L) &= \sigma^2 - 4\sigma^4 f^2(z) - 2\sigma^2 z f(z)(2F(z)-1) + z^2F(z)(1-F(z));\\
V(k_1,k_2,L) &= -\sigma^2 + 2(k_1+k_2)\sigma^2 f(z) - k_1^2(1-F(z)) - k_2^2F(z);\\
U_i(k_1,k_2,L) &= -\sigma^2 + 2\sigma^2(k_i+k_j-2b_i)f(z) - (k_j-b_i)^2F(k_i-k_j) - (k_i-b_i)^2F(k_j-k_i).
\end{aligned}$$

Denote the hazard rate $\frac{f(x)}{1-F(x)}$ by $v(x)$ and let $w(x) := \frac{f(x)}{F(x)}$. The hazard rate plays an important role in our analysis. It represents the instantaneous probability that experts' signals differ by x, conditional on differing by at least x.

In upward equilibrium, the principal's choice of the lower offer implies that offers affected by negative realizations of noise are accepted more frequently. Therefore conditional on being chosen, an expert must revise his belief about θ upwards. This induces experts to increase their markups, similarly to bidders shading their bids downwards in a common value auction environment.[13]

Let z^* denote the unique solution[14] to the equation

$$z - \sigma^2\left[v(z) - w(z)\right] = b_1 - b_2. \tag{1}$$

[13] A similar shading behavior emerges in the welfare-maximizing (first best) outcome in Kartik et al. (2015), as interestingly the latter outcome implies always selecting the politician with the larger signal in absolute terms.

[14] That there is a unique solution is shown in the proof of Theorem 2* in the Appendix, which is a generalization of Theorem 2.

Theorem 2. *There exists a unique upward equilibrium, characterized by markups* $k_1^U = b_1 + \sigma^2 v(z^*)$ *and* $k_2^U = b_2 + \sigma^2 w(z^*)$ *with* $k_1^U - k_2^U = z^* \geq b_1 - b_2 \geq 0$.

Notice that the equilibrium markup difference z^* as well as $\mathrm{Var}(a - \theta)$ depend on the biases of the experts only through $b_1 - b_2$. In upward equilibrium expert 1 wins the bonus with probability $1 - F(z^*)$, which is reflected in the expression for b_U. The bonus affects the expected bias in two ways: through the change in experts' probabilities of winning and through the change in markups. In the appendix (Corollary A.1), we give the full expansion of the players' utilities in upward equilibrium.

4.4 Downward equilibrium

Recall that an equilibrium is a downward equilibrium when it belongs to $\{(k_1, k_2, a \in \arg\max(a_1, a_2)) : k_1 + k_2 \leq 0\}$. Proposition 3 is analogous to Proposition 2. Note that $b(k_1, k_2, H)$ now exceeds the expected chosen markup because it is associated with a normally distributed noise term truncated from below.

Proposition 3. *If both experts follow constant markup strategies* $a_j(s_j) = s_j + k_j$ *and the principal always chooses the higher offer, then*

$$
\begin{aligned}
b(k_1, k_2, H) &= k_1 + k_2 - b(k_1, k_2, L);\\
Var(k_1, k_2, H) &= Var(k_1, k_2, L);\\
V(k_1, k_2, H) &= V(-k_1, -k_2, L);\\
U_i(k_1, k_2, H) &= -\sigma^2 - 2\sigma^2(k_i + k_j - 2b_i)f(z) - (k_i - b_i)^2 F(k_i - k_j) - (k_j - b_i)^2 F(k_j - k_i).
\end{aligned}
$$

In downward equilibrium, the principal's strategy of choosing the higher offer implies that, conditional on being chosen, an expert must revise his belief and his markup downward. This downward force must be sufficiently large to ensure that the sum of markups is negative, so that the principal's choice of the higher offer is a best response. Hence, noise must be sufficiently large for downward equilibrium to exist.

Theorem 3. *A downward equilibrium exists if and only if* $b_1 + b_2 \leq \sigma^2(v(z^*) + w(z^*))$. *When it exists, it is unique and characterized by* $k_1^D = b_1 - \sigma^2 w(z^*)$ *and* $k_2^D = b_2 - \sigma^2 v(z^*)$, *with* $k_1^D - k_2^D = z^* \geq b_1 - b_2$.

4.5 Principal-Optimal Equilibrium

In this section we compare the principal's expected utility in upward and downward equilibria, and in the case of simple delegation. We also investigate how the principal's expected payoff in equilibrium depends on the biases of the experts.

Note that the equilibrium markup difference z^* and $\mathrm{Var}(a - \theta)$ stay the same as in upward equilibrium. In downward equilibrium expert 1 wins the bonus with probability $F(z^*)$, which is higher than $1 - F(z^*)$ in upward equilibrium, and this shifts the expected bias higher. Furthermore, in downward equilibrium the bonus motivates experts to increase their markups, as opposed to

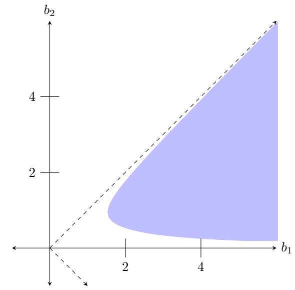

Figure 2: Depending on the biases (b_1, b_2), the principal's strategy under commitment is either simple delegation to expert 2 (shaded region) or choosing the lower offer (unshaded).

upward equilibrium in which the bonus motivates experts to decrease their markups. As a result of the above effects, the expected bias is higher in downward equilibrium. In Corollary A.2 we provide expressions for the players' utilities.

Proposition 4. *The principal prefers upward equilibrium to downward equilibrium whenever both exist.*

Given the above result, we now compare the principal's utility in upward equilibrium and simple delegation to the expert with the smaller absolute bias. In general this comparison is complicated, but in the next proposition we show that for upward equilibrium to be better for the principal, it has to be the case that either both experts have high enough biases, or that they are equally biased.

Proposition 5. *Parameterize biases as $b_1 = b + x$ and $b_2 = b - x$ for some $x, b \geq 0$. For all $x > 0$, there exists a threshold $\bar{b} > 0$ such that the principal prefers simple delegation to the upward equilibrium if and only if $b_1 > \bar{b}$. The principal always prefers upward equilibrium in the following cases: (i) biases are equal, i.e. $x = 0$, and (ii) $b_2 = 0$.*

For intuition behind the above result, first consider the case $x > 0$. The variance of the chosen action in upward equilibrium is always lower than that in simple delegation, and both are independent of b. As b increases, the expected bias $b - (2F(z^*) - 1)x$ in upward equilibrium and $b - x$ in downward equilibrium increase at the same rate. Since the former is higher than the latter, and losses are quadratic, this increase hurts more in upward equilibrium than in simple delegation. Once b is high enough, this disadvantage outweighs the initial advantage of lower variance.

5 Stationary Equilibria with Bonus Payments

Here we generalize the base model, assuming that the chosen expert receives a bonus payment $B \geq 0$ in addition to his quadratic loss. Experts now have two contradictory incentives. To illustrate, recall the upward equilibrium, in which the principal always chooses the lower offer and the resulting winner's curse phenomenon exerts an upward force on experts' offers, leading them to apply markups above their biases. Introducing a positive bonus $B > 0$ induces experts to decrease their markups in order to be selected more frequently. Hence, if the bonus is small, the first force prevails, and the expert sets a markup higher than her bias. If the bonus is large then the second force prevails, and the expert sets a markup lower than his bias. These conclusions are confirmed by formulas in Theorem 2*. If $B < 2\sigma^2$, then the markups are higher than the corresponding biases, and if $B > 2\sigma^2$, the situation reverses. When the bonus becomes sufficiently high, markups become lower than the biases to an extent that the sum of markups is negative. Then the principal prefers to choose the higher offer. Therefore, for very high bonuses upward equilibria do not exist.

Let $\rho := \sigma^2 - \frac{B}{2}$. Equation (1) extends to

$$z - \rho\,[v(z) - w(z)] = b_1 - b_2. \tag{2}$$

Theorem 2*. *There exists a threshold $B_U > 0$ such that an upward equilibrium exists if and only if $B \le B_U$. When it exists, it is unique and characterized by markups $k_1^U = b_1 + \rho v(z^*)$ and $k_2^U = b_2 + \rho w(z^*)$ with $k_1^U - k_2^U = z^* \ge 0$. Moreover, B_U lies in the interval $[2\sigma^2 + 2\sqrt{\pi}\sigma \max(0, b_2), 2\sigma^2 + \sqrt{\pi}\sigma(b_1 + b_2)]$. For $B \le B_U$,*

- $z^* \ge b_1 - b_2 \iff B \le 2\sigma^2 \iff \rho \ge 0$;
- $b(k_1^U, k_2^U, L) = b_U := b_1(1 - F(z^*)) + b_2 F(z^*) - Bf(z^*)$;
- $Var(k_1^U, k_2^U, L) = \sigma^2 - 4\sigma^4 f^2(z^*) - 2\sigma^2 z^* f(z^*)(2F(z^*) - 1) + (z^*)^2 F(z^*)(1 - F(z^*))$.

In the case of equally biased experts, we can obtain a closed form solution for strategies in upward equilibrium.

Proposition 6. *Consider $b_1 = b_2 = b > 0$. Then $B_U = 2\sigma^2 + 2\sqrt{\pi}\sigma b$, and in upward equilibrium, we have the following:*

- $k_1^U = k_2^U = k_U = b + \frac{\rho}{\sigma\sqrt{\pi}}$;
- $b(k_U, k_U, L) = b - \frac{B}{2\sqrt{\pi}\sigma}$;
- $Var(k_U, k_U, L) = \left(1 - \frac{1}{\pi}\right)\sigma^2$;
- $V(k_U, k_U, L) = -\left(b - \frac{B}{2\sqrt{\pi}\sigma}\right)^2 - \sigma^2 + \frac{\sigma^2}{\pi}$;
- $U_i(k_U, k_U, L) = -\rho + \frac{\sigma^2}{\pi} - \frac{B^2}{4\pi\sigma^2}$ *for* $i = 1, 2$.

The next theorem characterizes downward equilibrium. Note that if $B_D < 0$, no downward equilibrium exists. Recall the definition of z^* from (2).

Theorem 3*. *There exists a threshold B_D such that a downward equilibrium exists if and only if $B \le B_D$. When it exists, it is unique and characterized by $k_1^D = b_1 - \rho w(z^*)$ and $k_2^D = b_2 - \rho v(z^*)$, with $k_1^D - k_2^D = z^* \ge b_1 - b_2$ and $B_D \in [2\sigma^2 - \sqrt{\pi}\sigma(b_1 + b_2), 2\sigma^2 - 2\sqrt{\pi}\sigma \max(0, b_2)]$. For $B \le B_D$,*

- $b(k_1^D, k_2^D, H) = b_D = b_1 F(z^*) + b_2(1 - F(z^*)) + Bf(z^*)$;

- $Var(k_1^D, k_2^D, H) = \sigma^2 - 4\sigma^4 f^2(z^*) - 2\sigma^2 z^* f(z^*)(2F(z^*) - 1) + (z^*)^2 F(z^*)(1 - F(z^*))$.

The following corollaries are immediate from Theorems 2* and 3*.

Corollary 1. *A downward equilibrium exists only if an upward equilibrium exists; that is, $B_D \leq B_U$.*

Corollary 2. *For $B \leq B_D$, $k_1^U - b_1 = b_2 - k_2^D$ and $k_2^U - b_2 = b_1 - k_1^D$.*

As before, we can obtain a closed form solution for the case of equally biased experts.

Proposition 7. *Consider $b_1 = b_2 = b > 0$. Then $B_D = 2\sigma^2 - 2\sqrt{\pi}\sigma b$, and in downward equilibrium, we have the following:*

- $k_1^D = k_2^D = k_D = b - \frac{\rho}{\sigma\sqrt{\pi}}$;
- $b(k_D, k_D, H) = b + \frac{B}{2\sqrt{\pi}\sigma}$;
- $Var(k_D, k_D, H) = \left(1 - \frac{1}{\pi}\right)\sigma^2$;
- $V(k_D, k_D, H) = -\left(b + \frac{B}{2\sqrt{\pi}\sigma}\right)^2 - \sigma^2 + \frac{\sigma^2}{\pi}$;
- $U_i(k_D, k_D, H) = -\rho + \frac{\sigma^2}{\pi} - \frac{B^2}{4\pi\sigma^2}$ *for* $i = 1, 2$.

First we establish that whenever both upward and downward equilibria exist, the principal always prefers the former.

Proposition 4*. *For any fixed $B \geq 0$, the principal prefers upward equilibrium to downward equilibrium whenever both exist.*

The intuition for the above result can be summarized as follows. As we pointed out earlier, in any state θ variances of the expected offer $\mathrm{Var}(a-\theta)$ in upward and downward equilibria coincide, but the expected bias $\mathbb{E}(a-\theta)$ is higher in downward equilibrium: $b_D = b_1 F(z^*) + b_2(1 - F(z^*)) + Bf(z^*) \geq b_1(1 - F(z^*)) + b_2 F(z^*) - Bf(z^*) = b_U$. Hence, to conclude that the principal is better off in upward equilibrium it is enough to show that $|b_D| \geq |b_U|$ or, taking into account above, $b_U + b_D \geq 0$. Markup differences in upward and downward equilibria coincide, but the choice rule is opposite, therefore for both experts the probabilities of winning in upward and downward equilibria are complementary. The direct effects of the bonus on b_U and b_D are opposite and equal in absolute value. Therefore $b_U + b_D = b_1 + b_2 \geq 0$, implying that the principal is better off in upward equilibrium.

In the Supplementary Appendix we show that in case the two experts are not equally biased, the expert with the lower bias also prefers upward equilibrium to downward equilibrium, while the expert with the higher bias has the opposite preferences. This is both because the expected action is closer to expert 2's ideal point in upward equilibrium, and closer to expert 1's ideal point in downward equilibrium, and because expert 2 is chosen (and hence receives the bonus) with higher probability in upward equilibrium, and expert 1 is chosen with higher probability in downward equilibrium.

For equal biases, the intuition for why upward equilibrium, when it exists, yields a higher expected payoff to the principal than simple delegation generalizes for $B \geq 0$ as follows. With simple delegation, the expected utility of the principal is $V = -b^2 - \sigma^2$. In contrast, in upward equilibrium the expected bias of the action is $b - \frac{B}{2\sqrt{\pi}\sigma} < b$, and the variance is $\sigma^2 - \frac{\sigma^2}{\pi} < \sigma^2$. The decreased variance of the action implies that the principal prefers upward equilibrium to simple delegation, even when $B = 0$ and therefore the expected action is the same in upward equilibrium as in simple delegation.

6 Comparative Statics

We start this section with comparing the expected payoff the principal can achieve with two equally biased experts, $b_1 = b_2 = b$, to the expected payoff she can achieve with two oppositely biased experts, $b_1 = -b_2 = b$. Theorem 2* implies that the expected bias of the action in the case of equally biased experts is equal to b. In the case of oppositely biased experts the expected bias of the action is $b(1-2F(z^*)) - Bf(z^*)$, simplifying to $b(1-2F(z^*))$ when $B = 0$. Hence, with $B = 0$ the absolute value of the expected bias in the case of oppositely biased experts is lower than in the case of equally biased experts. However, the next Proposition shows that the variance of the action is lower in the symmetric case, and in fact this effect dominates, resulting in the principal preferring to have two equally biased experts. Let $V_{symm}(b)$ be the principal's expected payoff in upward equilibrium when $b_1 = b_2 = b$, let $V_{opp}(b)$ be the principal's expected payoff in upward equilibrium when $b_1 = -b_2 = b$, and let $V_{sim}(b) = -\sigma^2 - b^2$ be the principal's expected payoff in the case of simple delegation to an expert with absolute bias b.

Proposition 8. *For any $b > 0$, $V_{symm}(b) \geq V_{opp}(b) \geq V_{sim}(b)$.*

Proof. Upward equilibrium for opposite biases exists only if $B \leq B_U = 2\sigma^2$. From Theorem 2*

$$V_{symm}(b) = -\sigma^2 - b^2 + 2Bbf(0) + (4\sigma^4 - B^2)f^2(0)$$
$$V_{opp}(b) = -\sigma^2 - b^2 + \left(\sigma^4 - \frac{B^2}{4}\right)\frac{f^2(z^*_{opp})}{F(z^*_{opp})(1-F(z^*_{opp}))} \geq -\sigma^2 - b^2 = V_{sim}(b),$$

where z^*_{opp} is the upward equilibrium markup difference in the case of oppositely biased experts. As $2Bbf(0) \geq 0$ and $\frac{f^2(z)}{F(z)(1-F(z))}$ reaches its maximum at $z = 0$, we get $V_{symm}(b) \geq V_{opp}(b)$ □

6.1 Equally Biased Experts

For the remainder of this section we focus on the case when the experts are equally biased: $b_1 = b_2 = b > 0$. In this setting we can derive closed form solutions for the marginal affects of various parameter values on the principal's expected payoff, greatly simplifying comparative statics exercises.

Recall from Proposition 5 that in the case of equal biases, the principal always prefers upward equilibrium to simple delegation to one of the biased experts. Moreover, using the formula derived in Proposition 8 for the principal's expected utility in upward equilibrium, we can exactly characterize when it is the case that the principal prefers upward equilibrium with two equally biased experts to simple delegation to an unbiased expert.

Corollary 3. *The principal prefers upward equilibrium with equally biased experts to unconstrained delegation to an unbiased expert if and only if $B \in [2\sqrt{\pi}\sigma b - 2\sigma^2, B_u]$. For $B = 0$ this condition is* *equivalent to bias-to-noise ratio being low enough:*

$$\frac{b}{\sigma} \leq \frac{1}{\sqrt{\pi}}.$$

Proof. It is enough to compare $V_{symm} = -(b - \frac{B}{2\sqrt{\pi}\sigma})^2 - \sigma^2 + \frac{\sigma^2}{\pi}$ with utility from single delegation to unbiased expert $-\sigma^2$. $\square$

From here on we investigate comparative statics of the principal's expected payoff in upward equilibrium. First we look at how the principal's expected payoff depends on b, the common bias of the experts. Recall that upward equilibrium exists only for $B \leq B_u = 2\sigma^2 + 2\sqrt{\pi}\sigma b$. The principal's expected payoff in upward equilibrium is $V(k_U, k_U, L) = -(b - \frac{B}{2\sqrt{\pi}\sigma})^2 - (1 - \frac{1}{\pi})\sigma^2$. This expression is maximized at $b^* = \frac{B}{2\sqrt{\pi}\sigma}$, taking the value $-(1 - \frac{1}{\pi})\sigma^2$. This implies that for $B = 0$, it is optimal for the principal to have nonbiased experts, but for $B > 0$, the optimal bias level is strictly positive and increasing in B.

Next we consider how the principal's expected payoff depends on the precision of the experts' signals. The expected bias of the action $b_U = b - \frac{B}{2\sqrt{\pi}\sigma}$ is increasing in σ. Nevertheless, the equilibrium utility of the principal, $V(k_U, k_U, L) = -(b - \frac{B}{2\sqrt{\pi}\sigma})^2 - \sigma^2 + \frac{\sigma^2}{\pi}$, is nonmonotonic in σ, for the following reason. If σ increases, then the expected variance of the action $(1 - \frac{1}{\pi})\sigma^2$ increases, leading to a decrease in the principal's expected payoff. At the same time when the variance of noise σ is small, the value of the expected bias $b - \frac{B}{2\sqrt{\pi}\sigma}$ stays negative and increases, leading to a decrease in absolute value of the expected bias, which increases the principal's expected payoff. After σ reaches the value when B is optimal in exogenous case ($B = 2\sqrt{\pi}\sigma b$) the value of expected bias turns positive and further increase in σ leads to decrease in utility. Hence, optimal value for principal lies on the interval $(0, \frac{B}{2\sqrt{\pi}b})$. The next proposition provides the formal comparative statics of the principal's utility in the precision of the experts' signals.

Let $B > 0$ and σ^* denote the unique positive solution to the equation $(4\pi - 4)\sigma^4 + 2\sqrt{\pi}bB\sigma = B^2$.

Proposition 9. *Consider $b_1 = b_2 = b > 0$.*

If $0 < B < \frac{2(\pi-1)\pi}{(\pi-2)^2}b^2$, there exists a non-empty interval $\sigma \in [\frac{\sqrt{\pi b^2 + 2B} - \sqrt{\pi}b}{2}, \sigma^*)$, where upward equilibrium exists and the principal's expected payoff is increasing in σ; when $\sigma > \sigma^*$, the principal's expected payoff is decreasing in σ.

If $B = 0$ or $B \geq \frac{2(\pi-1)\pi}{(\pi-2)^2}b^2$, the principal's expected payoff is always decreasing in σ.

For a concrete example when the principal's expected payoff increases in σ, consider $b_1 = b_2 = b = 2$ and $B = 9 < \frac{8(\pi-1)\pi}{(\pi-2)^2}$. Then $V(\sigma)$ is increasing on an interval containing $(0.992, 1.085)$.

Lastly, we address the question of how the principal's expected payoff depends on the bonus payment. We start with the case when the bonus payment comes from exogenous sources, and therefore only indirectly affecting the principal's payoff, through influencing experts' strategies in upward equilibrium. While the variance of the action $Var(k_U, k_U, L) = \sigma^2 - \frac{\sigma^2}{\pi}$ does not depend on B, the expected bias $b_U = b - \frac{B}{2\sqrt{\pi}\sigma}$ is decreasing in B. The principal prefers the expected bias to be as close to 0 as possible, so her expected payoff in upward equilibrium is maximized at $B = 2\sqrt{\pi}\sigma b$, where it is equal to $-\sigma^2[1 - \frac{1}{\pi}]$. At $B = 0$, $b_U = b - \frac{B}{2\sqrt{\pi}\sigma} = b \geq 0$ and a small increase in bonus decreases the experts' markup and benefits the principal. However, at $B = B_u$ $b_U = b - \frac{B}{2\sqrt{\pi}\sigma} = -\frac{\sigma}{\sqrt{\pi}} < 0$ and principal prefers to increase markups and correspondingly lower bonus. As a consequence, an intermediate point $B = 2\sqrt{\pi}\sigma b$ is optimal.

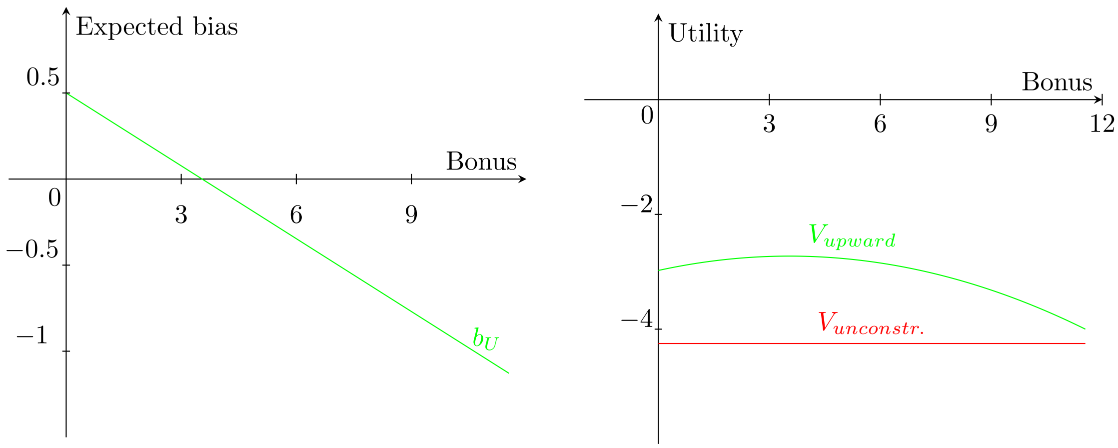

Figure 3: The Expected Bias and the Principal's Payoff (exogenous bonus) for $b = 0.5, \sigma = 2$

Next we consider the situation, when the bonus is paid from the principal's pocket. For this investigation, we append our game (described in Section 2) with a stage 0, preceding stage 1, in which the principal chooses B. We assume that the principal's choice of B becomes public knowledge by stage 1. We also modify the principal's payoff to $V(a, \theta) = -(a - \theta)^2 - B$, corresponding to the assumption that the principal has to pay the bonus.

The next proposition characterizes the optimal bonus choice of the principal in this case.

Proposition 10. *Suppose that bonus B is paid by the principal. The principal's optimal choice of B depends on the bias-to-noise ratio.*
If $\frac{b}{\sigma} \leq \sqrt{\pi}$, then the principal pays no bonus: $B = 0$.
If $\frac{b}{\sigma} > \sqrt{\pi}$, then the principal chooses bonus $B = 2\sqrt{\pi}\sigma^2 \left[\frac{b}{\sigma} - \sqrt{\pi}\right]$. The increase in the principal's expected payoff, relative to when the bonus is restricted to be 0, is equal to $(b - \sqrt{\pi}\sigma)^2$.

Therefore, if the bias-to-noise ratio is high enough, it is optimal for the principal to offer a positive bonus.

At $B = B_u$: $b_U = k_U - \frac{\sigma}{\sqrt{\pi}} = -\frac{\sigma}{\sqrt{\pi}} < 0$ and the principal would like to to decrease the bonus, in order to increase experts' markups and drive the expected bias closer to 0.

At $B = 0$: $b_U = b \geq 0$, and if the principal increases the bonus then the expected bias decreases. Therefore a marginal increase of B from 0 improves the principal's expected payoff whenever the marginal gain from the decrease in expected bias ($\frac{b}{\sqrt{\pi}\sigma}$) exceeds the marginal expense from increasing the bonus (1).

Proposition 10 is illustrated on Figure 3: the principal chooses a strictly positive bonus if the decrease in expected bias exceeds the marginal disutility from increasing bonus: $\frac{b}{\sqrt{\pi}\sigma} > 1$. This for example holds for $b = 2$, $\sigma = 0.5$, but not for $b = 1$, $\sigma = 1$.

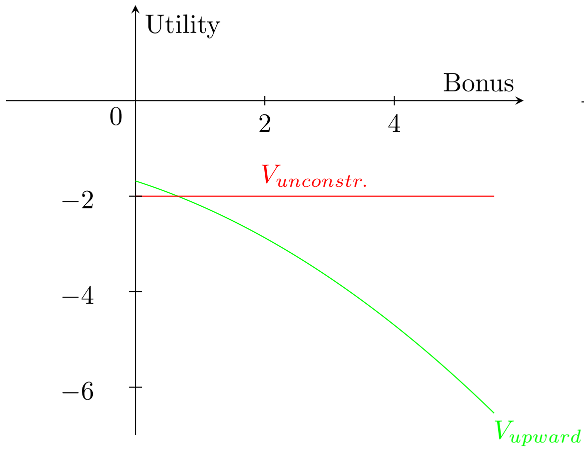

Figure 4: The Principal's Payoff (bonus paid by Principal) for $b = 1$, $\sigma = 1$ on the left; for $b = 2$, $\sigma = 0.5$ on the right

7 Extensions

7.1 Ex Ante Commitment by the Principal

The analysis in the previous sections assumes that the principal plays a best response to the strategies of the experts. Alternatively, we can consider situations in which the principal can ex ante commit to a simple strategy. In particular, assume that the principal can credibly commit to choose the lower offer with any probability $p \in [0, 1]$ and the higher offer with probability $1 - p$. In this section we provide two results. When biases are equal, as long as the bonus is small, commitment power does not help the principal; the principal would choose $p = 1$, which is already an (upward) equilibrium without commitment. On the other hand, when biases are opposite and sufficiently large in magnitude, the principal benefits from commitment to an interior p.

For the moment, restrict attention to the case of equally biased experts: $b_1 = b_2 = b$. In this case we show that for $B \leq 2\sigma^2$ commitment does not improve the principal's payoff, relative to the expected payoff in upward equilibrium in the game without commitment.

If the principal commits to choosing the lower offer with probability p, the experts' FOCs are

$$k_1 = b + \rho \frac{f(z)(2p - 1)}{p(1 - F(z)) + (1 - p)F(z)}$$
$$k_2 = b + \rho \frac{f(z)(2p - 1)}{pF(z) + (1 - p)(1 - F(z))},$$

where $z = k_1 - k_2$.

As $B \leq 2\sigma^2$, by Lemma $S.1$ from Supplementary Appendix, the only equilibrium of the game between the two experts, given the pre-committed strategy of the principal, involves $k_1 = k_2 = k =$

$b+(2p-1)\frac{2\sigma^2-B}{2\sqrt{\pi}\sigma}$. The principal's utility is

$$\begin{aligned} V &= p\left[-\sigma^2-k^2+\frac{2\sigma}{\sqrt{\pi}}k\right]+(1-p)\left[-\sigma^2-k^2-\frac{2\sigma}{\sqrt{\pi}}k\right] \\ &= \frac{\rho(2\sigma^2+B)}{2\pi\sigma^2}(2p-1)^2+\frac{bB}{\sqrt{\pi}\sigma}(2p-1)-\sigma^2-b^2. \end{aligned}$$

The next proposition follows from inspection of the expression above.

Proposition 11. *For $B \le 2\sigma^2$ and $b_1 = b_2$, $p = 1$ is an optimal strategy of the principal under commitment.*

In particular, commitment by the principal results in the same outcome as in the upward equilibrium of the game without commitment.

Next, consider oppositely biased experts and a bonus of $B = 0$. As the common magnitude b of the biases increases, expert 1's winner's curse in the upward equilibrium becomes more severe, and his markup very large. By introducing a small probability of choosing the higher offer, expert 1 is incentivized to reduce his markup, benefiting the principal. The cost to the principal of doing so is in choosing the wrong offer. When b is large, the markup reduction is large and outweighs the cost, and thus the principal can profitably deviate from $p = 1$ to an interior p.

Proposition 12. *Let $b_1 = b > 0$, $b_2 = -b$, and $B = 0$. For sufficiently large b, the optimal p under commitment satisfies $p \in (0, 1)$.*

7.2 Unselected Expert Indifferent over Actions

In the baseline model we assumed that an expert whose offer is not selected is still affected by principal's action. While this is a reasonable assumption in some contexts, in other situations it is more realistic to assume that the expert not selected by the principal receives an outside payoff that is independent of the state and the implemented action. For instance, a car mechanic is unlikely to care what kind of maintenance is done if he is not the one selected for the job. In this extension we assume that if expert i is chosen then expert j's realized payoff is normalized to be 0. We restrict attention the case of equally biased experts: $b_1 = b_2 = b > 0$.

In this version of the model we assume that B is large enough that an expert's expected payoff under simple delegation to that expert is nonnegative; that is, he prefers simple delegation to not being selected at all. Under this condition, the same simple delegation equilibria exist in this version of the model as in the baseline model. Below we show that under some conditions there also exist symmetric pure strategy equilibria that are similar to the ones characterized in the baseline model. For this to be the case, the bonus payment must be neither too low nor too large.

First we examine the conditions for the existence of upward equilibrium. Using the same notation as before, we investigate strategy profiles $\{(k_1, k_2, C(a_1, a_2) \in \arg\min\{a_1, a_2\}) : k_1 + k_2 \ge 0\}$. While for any such profile the principal's payoff does not change, the experts' expected payoffs should be recalculated:

$$U_i(k_i,k_j,L) = \int\limits_{k_i-k_j}^{\infty} \left[B - \left(k_i - b - \frac{t}{2}\right)^2 - \frac{\sigma^2}{2}\right] f(t)\,dt$$
$$= \left[B - \sigma^2 - (k_i-b)^2\right](1 - F(k_i-k_j)) + \left[2\sigma^2(k_i-b) - \frac{1}{2}\sigma^2(k_i-k_j)\right] f(k_i-k_j).$$

Notice that for a fixed constant markup strategy of the other expert, an expert can choose arbitrarily high constant markup and guarantee an expected payoff arbitrarily close to 0.[15] Hence, 0 is a lower bound for experts' equilibrium payoffs.

In what follows, define $\beta := \sqrt{\pi + \frac{B}{\sigma^2} - \frac{5}{2}}$.

Proposition 13. *A symmetric upward equilibrium* $k_1^U = k_2^U = k_U$ *exists if and only if* $B \in \left[\left(\frac{5}{2} - \frac{3\pi(8\pi-11)}{16(\pi-1)^2}\right)\sigma^2, \frac{5}{2}\sigma^2 + 2\sqrt{\pi}b\sigma + b^2\right]$. *When it exists, it is characterized by:*

- $k_1^U = k_2^U = k_U = b + (\sqrt{\pi} - \beta)\,\sigma$;
- $b(k_U,k_U,L) = b + \left(\sqrt{\pi} - \beta - \frac{1}{\sqrt{\pi}}\right)\sigma$;
- $Var(k_U,k_U,L) = \left(1 - \frac{1}{\pi}\right)\sigma^2$;
- $V(k_U,k_U,L) = -\left[b + \left(\sqrt{\pi} - \beta - \frac{1}{\sqrt{\pi}}\right)\sigma\right]^2 - \sigma^2 + \frac{\sigma^2}{\pi}$;
- $U_i(k_U,k_U,L) = \left[\frac{\pi-1}{\sqrt{\pi}}\beta + \frac{7}{4} - \pi\right]\sigma^2$ *for* $i = 1,2$.

In the Appendix we show that $k_U = b + (\sqrt{\pi} - \beta)\sigma > k_U^{bas.} = b + (1 - \frac{B}{2\sigma^2})\frac{\sigma}{\sqrt{\pi}}$, hence in this version of the model experts select higher markups in upward equilibrium than in the baseline model (for parameter values for which upward equilibrium exists in both model versions). The intuition behind this result is that in this alternative version of the model, the relative gain from being selected is reduced by the policy loss (that is not imposed on the expert if not selected). Since we consider B sufficiently large that expected payoffs are nonnegative, the resulting "net bonus" is still nonnegative; being selected is still preferable, conditional on having made the lower offer. It follows that an expert's equilibrium offer in either version is lower than what is ex-post optimal for that expert – that is, optimal after conditioning on both the expert's signal and having the lower offer. The smaller net bonus in the alternative version reduces the expert's incentive to marginally lower his offer in order to more frequently earn the net bonus. This reduction must be met by an offsetting reduction in his incentive to raise his offer, which is enforced through his bidding higher and thus closer to his ex-post optimum; due to quadratic losses, marginal movements toward the ex-post optimum have decreasing marginal benefits.

We note that the qualitative comparison between this extension and the baseline model is dependent upon the modeling of preferences over policy outcomes through losses. Such a model is appropriate for applications where an expert would prefer not to be associated with the project if

[15] For this reason, we do not introduce an explicit participation constraint in this version of the model, even though such a constraint would be natural in many applications.

his action would be sufficiently far from the true state; for example, this would be the case if the expert has a reputation at stake. Alternatively, one could model preferences through gains, using some single-peaked, nonnegative utility function of the distance between the action and the true state. In that model, the comparison above would be reversed, as being selected enhances the bonus and thus experts compete more aggressively by lowering their offers.

Next we turn attention to characterizing the conditions under which a downward equilibrium exists in which the principal always chooses the higher offer. Experts' expected payoffs can be calculated as:

$$\begin{aligned}U_i(k_i, k_j, H) &= \int_{-\infty}^{k_i-k_j} \left[B - \left(k_i - b - \frac{t}{2}\right)^2 - \frac{\sigma^2}{2}\right] f(t)\,dt \\ &= \left[B - \sigma^2 - (k_i - b)^2\right] F(k_i - k_j) - \left[2\sigma^2(k_i - b) - \frac{1}{2}\sigma^2(k_i - k_j)\right] f(k_i - k_j)\end{aligned}$$

As in upward equilibrium, 0 is a lower bound for experts' equilibrium payoffs.

Proposition 14. *Consider $b_1 = b_2 = b > 0$. If $\frac{b}{\sigma} > \frac{3\sqrt{\pi}}{4(\pi-1)}$, then no symmetric downward equilibrium exists. If $\frac{b}{\sigma} \leq \frac{3\sqrt{\pi}}{4(\pi-1)}$, then a symmetric downward equilibrium $k_1^D = k_2^D = k_D$ exists if and only if $B \in \left[\left(\frac{5}{2} - \frac{3\pi(8\pi-11)}{16(\pi-1)^2}\right)\sigma^2, \frac{5}{2}\sigma^2 + 2\sqrt{\pi}b\sigma + b^2\right]$. When it exists, it is characterized by:*

- $k_1^D = k_2^D = k_D = b - (\sqrt{\pi} - \beta)\,\sigma$;
- $b(k_D, k_D, H) = b - \left(\sqrt{\pi} - \beta - \frac{1}{\sqrt{\pi}}\right)\sigma$;
- $Var(k_D, k_D, H) = (1 - \frac{1}{\pi})\sigma^2$;
- $V(k_D, k_D, H) = -\left[b - \left(\sqrt{\pi} - \beta - \frac{1}{\sqrt{\pi}}\right)\sigma\right]^2 - \sigma^2 + \frac{\sigma^2}{\pi}$;
- $U_i(k_D, k_D, H) = \left[\frac{\pi-1}{\sqrt{\pi}}\beta + \frac{7}{4} - \pi\right]\sigma^2$ *for* $i = 1, 2$.

For downward equilibrium, the difference relative to the baseline model is the mirror image of the difference described earlier for upward equilibrium. Again, the bonus is reduced by quadratic losses, but in downward equilibrium this causes markups to decrease, as experts compete less aggressively to make the higher offer.

8 Conclusion

We proposed a model in which a principal can choose between two imperfectly informed experts, introducing the possibility of competition in a delegation framework. We showed that a principal with limited knowledge of the decision environment can benefit from the presence of two experts, relative to a simple unconstrained delegation to one of them, even if the experts have exactly the same bias. The main reason is that in equilibria in which the selection of the expert depends nontrivially on the experts' proposals, information is utilized from both experts' private signals. The option of offering a bonus payment to the selected expert can improve the principal's payoff, by inducing the experts to report more truthfully, but only to a certain point. Lastly, committing with a small probability to choose the (in expectation) inferior proposal can benefit the principal.

As this is the first step in investigating the benefits of multiple choices of experts in a delegation problem, there are many avenues of future research. One is examining multi-dimensional environments, in which different experts differ in their dimensions of specialization. Another direction would be investigating the problem of choosing an expert to delegate a task to with a more general mechanism design approach.

A Appendix

A.1 Proofs for Sections 4 and 5

We first provide an auxiliary lemma, which is proved in Supplementary Appendix.

Lemma A.1. *Let $\epsilon_1, \epsilon_2 \sim N(0, \sigma^2)$, where ϵ_1 and ϵ_2 are independent, and define $\xi(k_1, k_2) := \min(\epsilon_1 + k_1, \epsilon_2 + k_2)$, $\eta(k_1, k_2) := \max(\epsilon_1 + k_1, \epsilon_2 + k_2)$. Then*

$$\begin{aligned}
\mathbb{E}\xi(k_1,k_2) &= -2\sigma^2 f(k_1-k_2) + k_1(1-F(k_1-k_2)) + k_2F(k_1-k_2);\\
\mathbb{E}\eta(k_1,k_2) &= 2\sigma^2 f(k_1-k_2) + k_1F(k_1-k_2) + k_2(1-F(k_1-k_2));\\
\mathbb{E}\xi^2(k_1,k_2) &= \sigma^2 - 2(k_1+k_2)\sigma^2 f(k_1-k_2) + k_1^2(1-F(k_1-k_2)) + k_2^2F(k_1-k_2);\\
\mathbb{E}\eta^2(k_1,k_2) &= \sigma^2 + 2(k_1+k_2)\sigma^2 f(k_1-k_2) + k_1^2F(k_1-k_2) + k_2^2(1-F(k_1-k_2)).
\end{aligned}$$

Proof of Proposition 2. After observing the signal s_i, expert i does a Bayesian update of his beliefs: $\theta|s_i \sim N(s_i, \sigma^2)$ and $s_j|s_i \sim N(s_i, 2\sigma^2)$.

Since the principal chooses the lower offer, she accepts a_i iff $s_j > s_i + k_i - k_j$. Denote by g the PDF of $N(s_i, 2\sigma^2)$. Hence, the expected utility of expert i

$$\begin{aligned}
U_i(k_1,k_2,L) = &\int_{s_i+k_i-k_j}^{\infty} \mathbb{E}\left[B - (a_i-\theta-b_i)^2|s_i,s_j\right] g(s_j)\,ds_j\\
&+ \int_{-\infty}^{s_i+k_i-k_j} \mathbb{E}\left[-(a_j-\theta-b_i)^2|s_i,s_j\right] g(s_j)\,ds_j
\end{aligned}$$

As $(\theta|s_i,s_j) \sim N(\frac{s_j+s_j}{2}, \frac{\sigma^2}{2})$, $a_i = s_i + k_i$, $a_j = s_j + k_j$, we obtain

$$\begin{aligned}
U_i(k_1,k_2,L) = &\int\limits_{s_i+k_i-k_j}^{\infty} \left[B - \left(k_i - b_i - \frac{s_j-s_i}{2}\right)^2 - \frac{\sigma^2}{2}\right] g(s_j)\,ds_j\\
&+ \int\limits_{-\infty}^{s_i+k_i-k_j} \left[-\left(k_j - b_i + \frac{s_j-s_i}{2}\right)^2 - \frac{\sigma^2}{2}\right] g(s_j)\,ds_j.
\end{aligned}$$

Now make a substitution $t = s_j - s_i$ and denote by f and F the PDF and CDF of $N(0, 2\sigma^2)$.

$$U_i(k_1, k_2, L) = \int_{k_i - k_j}^{\infty} \left[B - \left(k_i - b_i - \frac{t}{2} \right)^2 - \frac{\sigma^2}{2} \right] f(t)\, dt + \int_{-\infty}^{k_i - k_j} \left[- \left(k_j - b_i + \frac{t}{2} \right)^2 - \frac{\sigma^2}{2} \right] f(t)\, dt.$$

Note that $U_i(k_1, k_2, L)$ does not depend on signal s_i, which is intuitive for the improper prior. As $\int_a^{\infty} t f(t)\, dt = 2\sigma^2 f(a)$ and $\int_{-\infty}^{\infty} t^2 f(t)\, dt = 2\sigma^2$, we get the expression for $U_i(k_i, k_j, L)$.

Now in state θ, the principal's action a is distributed as $\theta + \xi$, where $\xi = \min(\epsilon_1 + k_1, \epsilon_2 + k_2)$; $\epsilon_1, \epsilon_2 \sim N(0, \sigma^2)$, ϵ_1 and ϵ_2 are independent.

Therefore, from Lemma A.1 the expected bias of the accepted offer is

$$b(k_1, k_2, L) = \mathbb{E}\xi(k_1, k_2) = -2\sigma^2 f(k_1 - k_2) + k_2 F(k_1 - k_2) + k_1(1 - F(k_1 - k_2))$$

and the expected utility of the principal is

$$\begin{aligned} V(k_1, k_2, L) &= -\mathbb{E}(a - \theta)^2 = -\mathbb{E}(\theta + \xi - \theta)^2 = -\mathbb{E}\xi^2(k_1, k_2) \\ &= -\sigma^2 + 2(k_1 + k_2)\sigma^2 f(k_1 - k_2) - k_2^2 - (k_1^2 - k_2^2)(1 - F(k_1 - k_2)). \end{aligned}$$

Finally, the variance of the chosen offer is

$$\begin{aligned} Var(k_1, k_2, L) =& - V(k_1, k_2, L) - b^2(k_1, k_2, L) \\ =& \sigma^2 - 4\sigma^4 f^2(z) - 2\sigma^2 z f(z)(2F(z) - 1) + z^2 F(z)(1 - F(z)). \end{aligned}$$

□

The following lemma provides several useful bounds. The statements are immediate corollaries of Sampford (1953).

Lemma 1. *The following inequalities hold for all $x \in \mathbb{R}$:*

- $0 < v'(x) < \frac{1}{2\sigma^2}$;
- $0 > w'(x) > -\frac{1}{2\sigma^2}$;
- $v''(x) > 0$.

Proof of Theorem 2.* We start by showing that $U_i(k_1, k_2, L)$ is a single-peaked function of k_i. Taking a derivative w.r.t. k_i yields

$$\begin{aligned} U_i'(k_i) &= -2\left[(k_i - b_i)(1 - F(k_i - k_j)) - \rho f(k_i - k_j)\right] \\ &= -2(1 - F(k_i - k_j)) \left[k_i - b_i - \rho \frac{f(k_i - k_j)}{1 - F(k_i - k_j)} \right]. \end{aligned}$$

Let $g(k_i)$ denote the term in square brackets above. Lemma 1 implies that $g'(k_i) = 1 - \rho\lambda(k_i - k_j) \geq 1 - \sigma^2\lambda(k_i - k_j) > 0$. Additionally, we have $\lim_{x\to\pm\infty} g(x) = \pm\infty$. Combining these facts, U_i has a unique critical point, which is a global maximum.

We now look for upward equilibria. The FOCs for the experts are equivalent to:

$$k_1 - b_1 - \rho\frac{f(k_1 - k_2)}{1 - F(k_1 - k_2)} = 0 \tag{3}$$

$$k_2 - b_2 - \rho\frac{f(k_1 - k_2)}{F(k_1 - k_2)} = 0. \tag{4}$$

Subtracting (4) from (3) substituting $z = k_1 - k_2$, we get

$$z - \rho\left[\frac{f(z)}{1 - F(z)} - \frac{f(z)}{F(z)}\right] = b_1 - b_2. \tag{5}$$

Denote $l(z) = -\rho\left[\frac{f(z)}{1-F(z)} - \frac{f(z)}{F(z)}\right] + z$. Using a) from Lemma 1, we obtain

$$l'(z) = 1 - \rho\,[v'(z) - w'(z)] + 1 \geq 1 - \sigma^2[v'(z) - w'(z)] > 0$$

Now $l(z)$ is continuous, strictly increasing on $\mathbb{R}$, and ranges from $-\infty$ to $+\infty$. Therefore (2) has a unique solution, z^*; we use $z(B)$ to denote explicitly the dependence on B.
Using this solution, we get (k_1^U, k_2^U) as the only critical point and check that this point satisfies both initial FOCs. As $U_i(k_i, k_j, L)$ is a single-peaked function of k_i, (k_1^U, k_2^U) is a pair of best responses.

As it was shown in Theorem 1, choosing the lower offer is the BR strategy for the principal iff $k_1 + k_2 \geq 0$, or equivalently

$$b_1 + b_2 - \left[\frac{f(z^*)}{1 - F(z^*)} + \frac{f(z^*)}{F(z^*)}\right] \geq 0.$$

Also the LHS of (2) is equal to 0 at $z = 0$, and therefore $z^* \geq 0$ and $k_1^U - k_2^U \geq 0$.

Define a function $m(B) = b_1 + b_2 + \rho[v(z(B)) + w(z(B)]$; the upward equilibrium exists if and only if $m(B) \geq 0$.
1) For $B \leq 2\sigma^2$: $m(B) \geq 0$, therefore the upward equilibrium exists.
2) Next, we show that $m(B)$ is decreasing in B in the region $B \geq 2\sigma^2$.

$$m'(B) = -\frac{1}{2}[v(z(B)) + w(z(B))] + \rho[\lambda(z(B)) - \lambda(-z(B))]z'(B). \tag{6}$$

Differentiating equation (2) at point B, we get:

$$z'(B) - \rho[\lambda(z(B)) + \lambda(-z(B))]z'(B) + \frac{1}{2}[v(z(B)) - w(z(B))] = 0. \tag{7}$$

By substituting (7), the second term of (6) becomes

$$\begin{aligned}
&+\rho[\lambda(z(B))-\lambda(-z(B))]\frac{-\frac{1}{2}[v(z(B))-w(z(B))]}{1-\rho\left[\lambda(z(B))+\lambda(-z(B))\right]}\\
&\le -\rho[\lambda(z(B))-\lambda(-z(B))]\frac{\frac{1}{2}[v(z(B))-w(z(B))]}{-\rho\left[\lambda(z(B))+\lambda(-z(B))\right]}\\
&=\frac{1}{2}\frac{\lambda(z(B))-\lambda(-z(B))}{\lambda(z(B))+\lambda(-z(B))}[v(z(B))-w(z(B))]\\
\implies m'(B) &\le -\frac{1}{2}[v(z(B))+w(z(B))]+\frac{1}{2}[v(z(B))-w(z(B))]=-w(z(B))<0.
\end{aligned}$$

3) From Lemma 1 the hazard rate v is convex, so for any real x, $v(x)+w(x)=v(x)+v(-x)\geq 2v(0)>0$, and $m(B)$ tends to $-\infty$ as B tends to ∞.

From 1)-3) follows that there exists $B_U : m(B)\geq 0$ iff $B\leq B_U$. Also

$$\left(\frac{B_U}{2}-\sigma^2\right)[v(z(B_U))+w(z(B_U))]=b_1+b_2. \tag{8}$$

As $z(B_U)$ satisfies equation (2), we have:

$$\left(\frac{B_U}{2}-\sigma^2\right)[v(z(B_U))-w(z(B_U))]+z(B_U)=b_1-b_2. \tag{9}$$

From the previous discussion and (8) we have $B_U\geq 2\sigma^2$. Also, (8) and the inequality $v(x)+w(x)=v(x)+v(-x)\geq 2v(0)=\frac{2}{\sqrt{\pi}\sigma}$ give an upper bound on B_U:

$$\left(\frac{B_U}{2}-\sigma^2\right)\frac{2}{\sqrt{\pi}\sigma}\leq b_1+b_2.$$

Subtracting (9) from (8), we get a lower bound on B_U:

$$2b_2=(B_U-2\sigma^2)w(z(B_U))-z(B_U)\leq(B_U-2\sigma^2)w(0)=(B_U-2\sigma^2)\frac{1}{\sqrt{\pi}\sigma}.$$

Finally, we calculate the expected bias of the chosen offer, its variance, and players' utilities:

$$\begin{aligned}
b(k_1^U,k_2^U,L)&=-2\sigma^2 f(z^*)+k_2^U F(z^*)+k_1^U(1-F(z^*))\\
&=-2\sigma^2 f(z^*)+b_2F(z^*)+\rho f(z^*)+b_1(1-F(z^*))+\rho f(z^*)\\
&=b_1(1-F(z^*))+b_2F(z^*)-Bf(z^*);\\
Var(k_1^U,k_2^U,L)&=\sigma^2-4\sigma^4 f^2(z^*)-2\sigma^2 z^* f(z^*)(2F(z^*)-1)+(z^*)^2F(z^*)(1-F(z^*)).
\end{aligned}$$

$\square$

Corollary A.1. *In upward equilibrium,*

- $V(k_1^U,k_2^U,L)=-\sigma^2-b_1^2(1-F(z^*))-b_2^2F(z^*)+B(b_1+b_2)f(z^*)+\left(\sigma^4-\frac{B^2}{4}\right)\frac{f^2(z^*)}{F(z^*)(1-F(z^*))}$;

- $U_1(k_1^U, k_2^U, L) = -\sigma^2 - (b_1 - b_2)^2 F(z^*) + B(1 - F(z^*)) - B(b_1 - b_2) f(z^*) + \left(\sigma^4 - \frac{B^2}{4}\right) \frac{f^2(z^*)}{F(z^*)(1-F(z^*))}$;
- $U_2(k_1^U, k_2^U, L) = -\sigma^2 - (b_1 - b_2)^2 (1 - F(z^*)) + BF(z^*) + B(b_1 - b_2) f(z^*) + \left(\sigma^4 - \frac{B^2}{4}\right) \frac{f^2(z^*)}{F(z^*)(1-F(z^*))}$.

Proof. Immediate from applying Proposition 2 to the markups given by Theorem 2*. □

Proof of Proposition 3. Since the principal chooses the highest offer, she chooses a_i iff $s_j < s_i + k_i - k_j$. Using arguments similar to used in Proposition 2, we find the expected utility of expert i:

$$
\begin{aligned}
U_i(k_1, k_2, H) &= \int_{k_i - k_j}^{\infty} \mathbb{E}[-(a_j - \theta - b_i)^2 | s_j] f(s_j)\, ds_j + \int_{-\infty}^{k_i - k_j} \mathbb{E}[B - (a_i - \theta - b_i)^2 | s_j] f(s_j)\, ds_j \\
&= \int_{k_i - k_j}^{\infty} \left[-\left(k_j - b_i + \frac{s_j}{2}\right)^2 - \frac{\sigma^2}{2} \right] f(s_j)\, ds_j + \int_{-\infty}^{k_i - k_j} \left[B - \left(k_i - b_i - \frac{s_j}{2}\right)^2 - \frac{\sigma^2}{2} \right] f(s_j)\, ds_j \\
&= (B - (k_i - b_i)^2) F(k_i - k_j) - \sigma^2 - (k_j - b_i)^2 [1 - F(k_i - k_j)] - 2\sigma^2 (k_i + k_j - 2b_i) f(k_i - k_j).
\end{aligned}
$$

In state θ the principal's action a is distributed as $\theta + \eta$, where $\eta \sim \max(\epsilon_1 + k_1, \epsilon_2 + k_2)$; $\epsilon_1, \epsilon_2 \sim N(0, \sigma^2)$, ϵ_1 and ϵ_2 are independent.

From Lemma A.1 the expected bias of the accepted offer is

$$
b(k_1, k_2, H) = \mathbb{E}\eta(k_1, k_2) = 2\sigma^2 f(k_1 - k_2) + k_2(1 - F(k_1 - k_2)) + k_1 F(k_1 - k_2).
$$

The expected utility of the principal is

$$
\begin{aligned}
V(k_1, k_2, H) &= -\mathbb{E}(a - \theta)^2 = -\mathbb{E}\eta^2(k_1, k_2) \\
&= -\sigma^2 - 2(k_1 + k_2)\sigma^2 f(k_1 - k_2) - k_2^2 - (k_1^2 - k_2^2) F(k_1 - k_2).
\end{aligned}
$$

The variance of the chosen offer is

$$
\begin{aligned}
Var(k_1, k_2, H) =& - V(k_1, k_2, H) - b^2(k_1, k_2, H) \\
=& \sigma^2 - 4\sigma^4 f^2(z) - 2\sigma^2 z f(z)(2F(z) - 1) + z^2 F(z)(1 - F(z)).
\end{aligned}
$$

□

Proof of Theorem 3.* The proof is analogous to that of Theorem 2*. The FOCs for experts are now:

$$
k_1 - b_1 + \rho \frac{f(k_1 - k_2)}{F(k_1 - k_2)} = 0 \tag{10}
$$

$$
k_2 - b_2 + \rho \frac{f(k_1 - k_2)}{1 - F(k_1 - k_2)} = 0. \tag{11}
$$

Subtracting equation (10) from equation (11) yields (5). Principal optimality holds if and only if $k_1 + k_2 \leq 0$, or equivalently

$$
n(B) := b_1 + b_2 + \left[\frac{f(z^*)}{1 - F(z^*)} + \frac{f(z^*)}{F(z^*)} \right] \leq 0, \tag{12}
$$

where z(B) is given by equation (2). For $B > 2\sigma^2$, we have $n(B) > 0$, and thus a downward equilibrium does not exist. Observe further that $n(2\sigma^2) = b_1+b_2 \geq 0$. Since $m(B)+n(B) = 2(b_1+b_2)$ and $m'(B) < 0$, we have $n'(B) > 0$. It follows that if $n(0) \leq 0$, then there exists $B_D \in [0, 2\sigma^2]$ such that $n(B) \leq 0$ iff $B \leq B_D$. Therefore

$$\left(\frac{B_U}{2} - \sigma^2\right)[v(z(B_D)) + w(z(B_D))] = -(b_1 + b_2). \tag{13}$$

Also $z(B_D)$ satisfies equation (2), and therefore

$$\left(\frac{B_U}{2} - \sigma^2\right)[v(z(B_D)) - w(z(B_D))] + z(B_D) = b_1 - b_2. \tag{14}$$

From the previous discussion and (13) we have $B_D \leq 2\sigma^2$. Also (13) and the inequality $v(x)+w(x) \geq 2v(0) = \frac{2}{\sqrt{\pi}\sigma}$ give the lower bound

$$\left(\frac{B_D}{2} - \sigma^2\right)\frac{2}{\sqrt{\pi}\sigma} \geq -(b_1 + b_2).$$

Summing (14) and (13), we get the upper bound

$$-2b_2 = \left(\frac{B_U}{2} - \sigma^2\right)v(z(B_D)) + z(B_D) \geq (B_D - 2\sigma^2)2v(0).$$

Finally, we compute the following:

$$\begin{aligned}
b(k_1^D, k_2^D, H) &= 2\sigma^2 f(z^*) + k_1^D F(z^*) + k_2^D(1 - F(z^*)) \\
&= 2\sigma^2 f(z^*) + b_1 F(z^*) - \rho f(z^*) + b_2(1 - F(z^*)) - \rho f(z^*) \\
&= b_1 F(z^*) + b_2(1 - F(z^*)) + Bf(z^*); \\
Var(k_1^D, k_2^D, H) &= \sigma^2 - 4\sigma^4 f^2(z^*) - 2\sigma^2 z^* f(z^*)(2F(z^*) - 1) + (z^*)^2 F(z^*)(1 - F(z^*)).
\end{aligned}$$

□

Corollary A.2. *In downward equilibrium,*

$V(k_1^D, k_2^D, H) = -\sigma^2 - b_1^2 F(z^*) - b_2^2(1 - F(z^*)) - B(b_1 + b_2)f(z^*) + \left(\sigma^4 - \frac{B^2}{4}\right)\frac{f^2(z^*)}{F(z^*)(1-F(z^*))}$;

$U_1(k_1^D, k_2^D, H) = -\sigma^2 - (b_1 - b_2)^2(1 - F(z^*)) + BF(z^*) + B(b_1 - b_2)f(z^*) + \left(\sigma^4 - \frac{B^2}{4}\right)\frac{f^2(z^*)}{F(z^*)(1-F(z^*))}$;

$U_2(k_1^D, k_2^D, H) = -\sigma^2 - (b_1 - b_2)^2 F(z^*) + B(1 - F(z^*)) - B(b_1 - b_2)f(z^*) + \left(\sigma^4 - \frac{B^2}{4}\right)\frac{f^2(z^*)}{F(z^*)(1-F(z^*))}$.

Proof. Proposition 3 applied to Theorem 3*. □

Proof of Proposition 4.* From Theorems 2* and 3*

$$V_{upw.} - V_{downw.} = (2F(z^*) - 1)(b_1^2 - b_2^2) + 2B(b_1 + b_2)f(z^*) \geq 0,$$

with equality if and only if either $b_1 + b_2 = 0$ or both $B = 0$ and $b_1 = b_2$. □

Proof of Proposition 5. Delegation to expert 2 alone yields principal utility $V^S(b,x) = -\sigma^2 - (b-x)^2$, while upward equilibrium yields

$$V^U(b,x) = -\sigma^2 - b^2 - x^2 + (2F(z^*) - 1)2bx + (k_1 - b - x)(k_2 - b + x).$$

The difference between these is

$$\begin{aligned} V^U(b,x) - V^S(b,x) &= -2(1 - F(z^*))bx + (k_1 - b - x)(k_2 - b + x) \\ &= -2(1 - F(z^*))bx + \sigma^4 v(z^*)w(z^*). \end{aligned}$$

Recall that z^* is independent of b. If $x = 0$, then the above expression is always positive. For fixed $x > 0$ then the existence of $\bar{b}$ follows from the fact that this expression is decreasing linearly in b and positive for $b = 0$.

Next, consider $b_2 = 0$ and $b_1 = b > 0$. The principal's utility in upward equilibrium is

$$V = -\sigma^2 + 2(k_1 + k_2)\sigma^2 f(z) - k_1^2(1 - F(z)) - k_2^2 F(z),$$

which we aim to show is greater than $-\sigma^2$ as under simple delegation to expert two. Using the expressions $k_1 = b + \sigma^2 \frac{f(z)}{1-F(z)}$ and $k_2 = \sigma^2 \frac{f(z)}{F(z)}$, this is true if and only if

$$2\left(b + \sigma^2 \frac{f}{1-F} + \sigma^2 \frac{f}{F}\right)\sigma^2 f > \left(b^2 + 2b\sigma^2 \frac{f}{1-F} + \sigma^4 \left(\frac{f}{1-F}\right)^2\right)(1-F) + \sigma^4 \left(\frac{f}{F}\right)^2 F.$$

Using $b = z - \sigma^2 \frac{f(2F-1)}{F(1-F)}$ and simplifying, this is equivalent to

$$\sigma^4(4F - 1) > z(zF(1-F) - 2\sigma^2 f(2F - 1)).$$

As $z > 0$, the left hand side is positive; we now show that the right hand side is negative. Let $h(z) := 2\sigma^2 f(2F-1) - zF(1-F)$, which we aim to show is positive. Then $h'(z) = 2f^2 - F(1-F)$. As shown in Sampford (1953), $k(z) := \frac{f^2}{F(1-F)}$ is decreasing for $z \geq 0$. It is easy to verify that $2k(0) > 1$ and that $\lim_{z\to+\infty} k(z) = 0$. It follows that there is a unique positive solution to $h'(z) = 0$. It is also easy to check that $h'(0) > 0$ and that $\lim_{z\to+\infty} h(z) = 0$. Together these facts imply that $h(z) > 0$ for all $z > 0$, as desired. □

Proof of Proposition 6. If $b_1 = b_2 = b > 0$, then upper and lower bounds on B_U coincide, and thus $B_U = 2\sigma^2 + 2\sqrt{\pi}\sigma b$. From Theorem 2*, the experts' markups are $k_1^U = k_2^U = k_U = b + \frac{\rho\sigma}{\sigma\sqrt{\pi}}$ and $z^* = 0$. The other results follow immediately. □

Proof of Proposition 7. If $b_1 = b_2 = b > 0$, then upper and lower bounds on B_D from Theorem 3* coincide, so $B_D = 2\sigma^2 + 2\sqrt{\pi}\sigma b$. The experts' markups are $k_1^D = k_2^D = k_D = b - \frac{\rho}{\sigma\sqrt{\pi}}$ and $z^* = 0$, which implies all other results. □

A.2 Proofs for Section 6

Proof of Proposition 9. From Proposition 6, the symmetric upward equilibrium exists if $B \le 2\sigma^2 + 2\sqrt{\pi}b\sigma$ or, equivalently, $\sigma \ge \frac{\sqrt{\pi b^2+2B}-\sqrt{\pi}b}{2}$. The principal's expected payoff in upward equilibrium is equal to $V = -(b - \frac{B}{2\sqrt{\pi}\sigma})^2 - (1-\frac{1}{\pi})\sigma^2$. Then $V'(\sigma) = \frac{B}{\sqrt{\pi}\sigma^2}(\frac{B}{2\sqrt{\pi}\sigma} - b) - 2(1-\frac{1}{\pi})\sigma = -\frac{1}{2\pi\sigma^3}[(4\pi - 4)\sigma^4 + 2\sqrt{\pi}bB\sigma - B^2] > 0$ if and only if $\sigma < \sigma^*$.

If $B = 0$, $V'(\sigma) < 0$. Otherwise, denote $\sigma_0 = \frac{\sqrt{\pi b^2+2B}-\sqrt{\pi}b}{2} > 0$.

The interval $[\sigma_0, \sigma^*)$ is non-empty if and only if $0 > (4\pi - 4)\sigma_0^4 + 2\sqrt{\pi}bB\sigma_0 - B^2 = (4\pi - 4)\sigma_0^4 - 2B\sigma_0^2 + B[2\sigma_0^2 + 2\sqrt{\pi}b\sigma_0 - B] = (4\pi-4)\sigma_0^4 - 2B\sigma_0^2$ or, equivalently, $0 > (2\pi - 2)\sigma_0^2 - B = (2\pi-2)\sigma_0^2 - (2\sigma_0^2 + 2\sqrt{\pi}b\sigma_0) = (2\pi - 4)\sigma_0^2 - 2\sqrt{\pi}b\sigma_0$. The latter holds if and only if $\sigma_0 < \frac{\sqrt{\pi}b}{\pi-2}$ or, equivalently, $B = 2\sigma_0^2 + 2\sqrt{\pi}b\sigma_0 < \frac{2(\pi-1)\pi}{(\pi-2)^2}b^2$. ◻

Proof of Proposition 10. By Proposition 4*, only upward equilibrium should be considered. Therefore, we seek to maximize the quadratic function $V(k_U, k_U, L) - B = -\left(b - \frac{B}{2\sqrt{\pi}\sigma}\right)^2 - \sigma^2 + \frac{\sigma^2}{\pi} - B$ on the interval $B \in [0, 2\sigma^2 + 2\sqrt{\pi}\sigma b]$.

a) First consider $\frac{b}{\sigma} \le \sqrt{\pi}$. In this case the maximum is achieved at $B = 0$ and is equal to $V(k_U, k_U, L|B = 0) = -b^2 - \left(1 - \frac{1}{\pi}\right)\sigma^2$.

Hence, if $\frac{b}{\sigma} < \sqrt{\pi}$, then the principal pays no bonus.

b) Now consider $\frac{b}{\sigma} \ge \sqrt{\pi}$. In this case the principal achieves maximum in upward equilibrium at $B_R = 2\sqrt{\pi}\sigma\,(b - \sqrt{\pi}\sigma)$ (upward equilibrium exists for this point) and is equal to $V(k_U, k_U, L|B = B_R) = (\pi - 1 + \frac{1}{\pi})\sigma^2 - 2\sqrt{\pi}\sigma b$.

Her gains comparatively to $B = 0$ (if she is legally restricted from paying bonuses) are equal to:

$$V(k_U, k_U, L|B = B_R) - V(k_U, k_U, L|B = 0) = \left(b - \sqrt{\pi}\sigma\right)^2.$$

◻

A.3 Proofs for Section 7

Proof of Proposition 12. Let $V(p)$ denote the principal's utility from commitment to p. That $b_1 + b_2 = 0$ implies $V(0) = V(1)$ is shown in the proof of Proposition 4*. Therefore, it suffices to show that $V'(1) < 0$. Given p, the markups satisfy

$$\begin{aligned}
k_1(p) &= b + \frac{(2p-1)f(z(p))}{W(p)} \\
k_2(p) &= -b + \frac{(2p-1)f(z(p))}{1 - W(p)} \\
\implies z(p) &= 2b + \frac{(2p-1)f(z)(1-2W(p))}{W(p)(1-W(p))},
\end{aligned}$$

where $W(p) := p(1 - F(z(p))) + (1-p)F(z(p))$. Differentiating with respect to p and solving for $z'(1)$ yields

$$z'(1) = \frac{f(4F+1)(2F-1)}{F^2(1-F)^2 - F(1-F)(f'(2F-1) + 2f^2) - f^2(2F-1)^2}.$$

The principal's utility is

$$V(p) = 2(2p-1)f(k_1+k_2) - k_1^2 W - k_2^2(1-W).$$

By differentiating with respect to p, evaluating at $p=1$, substituting in the above expression for $z'(1)$ and simplifying, we obtain

$$\begin{aligned} V'(1) &= \frac{f^2}{F^2(1-F)^2}\left(1+\frac{g_1}{g_2}\right), \text{ where} \\ g_1 &:= (2F-1)(2F(1-F)f' + f^2(2F-1)), \\ g_2 &:= F^2(1-F)^2 - f^2(2F^2-2F+1) - f'(2F-1)F(1-F). \end{aligned}$$

We claim that $g_2 > 0$, and thus it suffices to show that $g_2 < -g_1$ for sufficiently large z. To see this, note that by Lemma 1, $f' < \frac{1-F}{2} - \frac{f^2}{1-F}$, and thus

$$g_2 > F(1-F)^2\left[\frac{1}{2} - \frac{f^2}{F(1-F)}\right].$$

It is easy to verify that $\frac{f^2}{F(1-F)} < \frac{1}{2}$ holds globally, and thus $g_2 > 0$ as desired. To see that $g_2 < -g_1$ for sufficiently large z, note that by algebra this comparison is equivalent to

$$F(1-F) + f'(2F-1) \leq 2f^2. \tag{15}$$

Using Lemma 1 again and simplifying, a sufficient condition for (15) is $2F - \frac{1}{2} < \frac{f^2}{(1-F)^2}$. The left hand side is bounded above by $\frac{3}{2}$, while the right hand side is increasing and unbounded above; thus for sufficiently large z, (15) holds. Finally, since z is increasing and unbounded above as a function of b, the proposition holds. ☐

Proof of Proposition 13. First, we calculate marginal utilities:

$$U_i'(k_i) = -2(k_i-b)[1-F(k_i-k_j)] + \left[\frac{1}{2}\sigma^2 + 2\rho + \frac{1}{4}(k_i+k_j-2b)^2\right] f(k_i-k_j).$$

Here, setting $U_i'(k)=0$ gives two critical points:
$k = b + \sqrt{\pi}\sigma - \sqrt{\left(\pi - \frac{5}{2}\right)\sigma^2 + B}$ and $k = b + \sqrt{\pi}\sigma + \sqrt{\left(\pi - \frac{5}{2}\right)\sigma^2 + B}$.

The second derivative is:

$$\begin{aligned} &U_i''(k_i) = -2[1-F(k_i-k_j)] \\ &+ \left[2(k_i-b) + \frac{1}{2}(k_i+k_j-2b) + \left(\frac{B}{2\sigma^2} - \frac{5}{4}\right)(k_i-k_j) - \frac{1}{8\sigma^2}(k_i-k_j)(k_i+k_j-2b)^2\right] f(k_i-k_j). \end{aligned}$$

We get that only $k^* = b + \sqrt{\pi}\sigma - \sqrt{\left(\pi - \frac{5}{2}\right)\sigma^2 + B}$ is a local maximum of experts' utility functions.

Optimality for the principal holds if and only if $k^* \geq 0$ or, equivalently, $B \leq b^2 + 2\sqrt{\pi}b\sigma + \frac{5}{2}\sigma^2$.

Calculating, we get that $U_i(k^*,k^*,L)=\frac{(\pi-1)\sigma}{\sqrt{\pi}}\sqrt{\left(\pi-\frac{5}{2}\right)\sigma^2+B}-(\pi-\frac{7}{4})\sigma^2$. As we noted earlier, a necessary condition for equilibrium is $U_i(k^*,k^*,L)\geq 0$ or, equivalently, $B\geq(\frac{5}{2}-\frac{3\pi(8\pi-11)}{16(\pi-1)^2})\sigma^2$.

Therefore, upward equilibrium may exist only if $B\in\left[\left(\frac{5}{2}-\frac{3\pi(8\pi-11)}{16(\pi-1)^2}\right)\sigma^2,\frac{5}{2}\sigma^2+2\sqrt{\pi}b\sigma+b^2\right]$.

To finish the proof, we show that if B lies on this interval, then $k=k^*$ is a global maximum of $U_1(k,k^*,L)$.

Denote $g(k)=-2(k-b)+\left[\frac{\sigma^2}{2}+2\rho+\frac{1}{4}(k+k^*-2b)^2\right]v(k-k^*)$.[16]

Then $U_1'(k)=-2(k-b)[1-F(k-k^*)]+\left[\frac{\sigma^2}{2}+2\rho+\frac{1}{4}(k+k^*-2b)^2\right]f(k-k^*)=[1-F(k-k^*)]g(k)$ and $sign(U_1'(k))=sign(g(k))$.

The first and second derivatives of g are

$$g'(k)=-2+\left[\frac{\sigma^2}{2}+2\rho+\frac{1}{4}(k+k^*-2b)^2\right]v'(k-k^*)+\frac{1}{2}(k+k^*-2b)v(k-k^*)$$
$$g''(k)=\left[\frac{\sigma^2}{2}+2\rho-B+\frac{1}{4}(k+k^*-2b)^2\right]v''(k-k^*)+(k+k^*-2b)v'(k-k^*)+\frac{1}{2}v(k-k^*).$$

Consider two cases.

1. $B\in\left[\left(\frac{5}{2}-\frac{3\pi(8\pi-11)}{16(\pi-1)^2}\right)\sigma^2,\frac{5}{2}\sigma^2\right]$. Here $k^*\geq b$.

a) On the interval $k<b$, $U_1'(k)>0$ and hence there is no point of maximum there.

b) On the interval $k\geq b$ we also have that $k+k^*-2b\geq 0$. As all v, v' and v'' are strictly positive functions, $g''(k)>0$. As $g'(k^*)<0$ and $g'(+\infty)>0$, hence there exists $k^{**}>k^*$: for $k<k^{**}$ $g(k)$ is decreasing; for $k>k^{**}$, $g(k)$ is increasing. As $g(b)>0$, $g(k^*)=0$, $g(k^{**})<0$ and $g(+\infty)>0$, then there exists $k_0>k^{**}:g(k_0)=0$. In summary, $g(k)$ is negative only on (k^*,k_0). Consequently, $U_1(k)$ is increasing on $[b,k^*)$, decreasing on (k^*,k_0), increasing for $k>k_0$. Hence, to show that k^* is a maximum on the interval $k\geq b$ it is sufficient to verify that $U_1(k^*)\geq U_1(+\infty)=0$, which was already done.

2. $B\in\left[\frac{5}{2}\sigma^2,\frac{5}{2}\sigma^2+2\sqrt{\pi}b\sigma+b^2\right]$. Here $k^*\leq b$.

a) On the interval $k<k^*$: $U_1'(k)>2(b-k^*)[1-F(k^*-k^*)]+\left[\frac{\sigma^2}{2}+2\rho+\frac{1}{4}(k^*+k^*-2b)^2\right]f(k-k^*)=b-k^*-\frac{b-k^*}{f(0)}f(k-k^*)$ (as k^* is a solution of $\frac{5}{2}\sigma^2-B+(k^*-b)^2=\frac{k^*-b}{f(0)}$). Therefore $U_1'(k)>b-k^*-\frac{b-k^*}{f(0)}f(k-k^*)\geq b-k^*-\frac{b-k^*}{f(0)}f(0)=0$.

b) On the interval $k\in(k^*,b]$: $\frac{U_1'(k)}{f(k-k^*)}=-2(k-b)\frac{1-F(k-k^*)}{f(k-k^*)}+\left[\frac{\sigma^2}{2}+2\rho+\frac{1}{4}(k+k^*-2b)^2\right]<2(b-k^*)\frac{1-F(k^*-k^*)}{f(k^*-k^*)}+\left[\frac{\sigma^2}{2}+2\rho+\frac{1}{4}(k^*+k^*-2b)^2\right]=\frac{b-k^*}{f(0)}-\frac{b-k^*}{f(0)}=0$, hence $U_1'(k)<0$.

c) On the interval $k\in\left(b,2b-k^*+2\sqrt{B-\frac{5}{2}\sigma^2}\right]$ we also have $\frac{\sigma^2}{2}+2\rho+\frac{1}{4}(k+k^*-2b)^2\leq 0$. Then $U_1'(k)=-2(k-b)[1-F(k-k^*)]+\left[\frac{\sigma^2}{2}+2\rho+\frac{1}{4}(k+k^*-2b)^2\right]f(k-k^*)<0$.

d) On the interval $k>2b-k^*+2\sqrt{B-\frac{5}{2}\sigma^2}$ we also have $k+k^*-2b>0$. Hence, on this interval $g''(k)>0$. Also notice that $g(2b-k^*+2\sqrt{B-\frac{5}{2}\sigma^2})<0$. Then two cases are possible: (i) $g'(2b-k^*+2\sqrt{B-\frac{5}{2}\sigma^2})\geq 0$. Then on the whole interval $g'(k)>0$ and $g(k)$ is increasing. As $g(2b-k^*+2\sqrt{B-\frac{5}{2}\sigma^2})<0$ and $g(+\infty)>0$, there exists k_0: $g(k)<0$ for $k<k_0$ and $g(k)>0$ for

[16] Recall that $v(k-k^*)=\frac{f(k-k^*)}{1-F(k-k^*)}$.

$k > k_0$. Now $U_1(k)$ is decreasing for $k < k_0$ and increasing for $k > k_0$. Hence, to show that k^* is a global maximum it is enough to check that $U_1(k^*) \geq U_1(+\infty) = 0$, which was already done.

(ii) $g'(2b - k^* + 2\sqrt{B - \frac{5}{2}\sigma^2}) < 0$. Then there exists $k^{**} > 2b - k^* + 2\sqrt{B - \frac{5}{2}\sigma^2}$: for $k < k^{**}$ $g(k)$ is decreasing; for $k > k^{**}$ $g(k)$ is increasing. As $g(+\infty) > 0$, there exists k_0: $g(k) < 0$ for $k < k_0$ and $g(k) > 0$ for $k > k_0$. Then $U_1(k)$ is decreasing for $k < k_0$ and increasing for $k > k_0$ and k^* is a global maximum as $U_1(k^*) \geq U_1(+\infty) = 0$.

We now verify that $k_U = k^* > k_U^{bas.}$:

$$k_U = b + \left(\sqrt{\pi} - \sqrt{\pi + \frac{B}{\sigma^2} - \frac{5}{2}}\right)\sigma > k_U^{bas.} = b + \left(1 - \frac{B}{2\sigma^2}\right)\frac{\sigma}{\sqrt{\pi}}$$

$$\iff \sqrt{\pi} - \sqrt{\pi + \frac{B}{\sigma^2} - \frac{5}{2}} > \left(1 - \frac{B}{2\sigma^2}\right)\frac{1}{\sqrt{\pi}} \iff \sqrt{\pi} + \frac{1}{\sqrt{\pi}}\left(\frac{B}{2\sigma^2} - 1\right) > \sqrt{\pi + \frac{B}{\sigma^2} - \frac{5}{2}}$$

$$\iff \pi + \frac{B}{\sigma^2} - 2 + \frac{1}{\pi}\left(\frac{B}{2\sigma^2} - 1\right)^2 > \pi + \frac{B}{\sigma^2} - \frac{5}{2} \iff \frac{1}{2} + \left(\frac{B}{2\sigma^2} - 1\right)^2 > 0.$$

□

Proof of Proposition 14. Start with calculation of marginal utilities:

$$U_i'(k_i) = -2(k_i - b_i)F(k_i - k_j) - \left[\frac{\sigma^2}{2} + 2\rho + \frac{1}{4}(k_i + k_j - 2b)^2\right]f(k_i - k_j)$$

Consider the symmetric case: $k_1 = k_2 = k$. The FOCs give two critical points:

$k = b - \sqrt{\pi}\sigma - \sqrt{\left(\pi - \frac{5}{2}\right)\sigma^2 + B}$ and $k = b - \sqrt{\pi}\sigma + \sqrt{\left(\pi - \frac{5}{2}\right)\sigma^2 + B}$.

Next, calculate second derivatives:

$$U_i''(k_i) = -2F(k_i - k_j)$$
$$-\left[2(k_i - b_i) + \frac{1}{2}(k_i + k_j - 2b_i) + \left(\frac{B}{2\sigma^2} - \frac{5}{4}\right)(k_i - k_j) - \frac{1}{8\sigma^2}(k_i - k_j)(k_i + k_j - 2b_i)^2\right]f(k_i - k_j)$$

We get that only $k^* = b - \sqrt{\pi}\sigma + \sqrt{\left(\pi - \frac{5}{2}\right)\sigma^2 + B}$ satisfies SOCs.

In order to satisfy principal optimality we need $k^* \leq 0$ or, equivalently, both $\frac{b}{\sigma} \leq \sqrt{\pi}$ and $B \leq b^2 - \sqrt{\pi}b\sigma + \frac{5}{2}\sigma^2$.

Calculating, we get that $U_i(k^*, k^*, H) = \frac{(\pi-1)\sigma}{\sqrt{\pi}}\sqrt{\left(\pi - \frac{5}{2}\right)\sigma^2 + B} - (\pi - \frac{7}{4})\sigma^2$ (the same as in upward equilibrium). As in upward equilibrium case, a necessary condition is $B \geq \left(\frac{5}{2} - \frac{3\pi(8\pi-11)}{16(\pi-1)^2}\right)\sigma^2$.

From previous arguments downward equilibrium may exist only if $B \in \left[(\frac{5}{2} - \frac{3\pi(8\pi-11)}{16(\pi-1)^2}\right)\sigma^2, b^2 - 2\sqrt{\pi}b\sigma + \frac{5}{2}\sigma^2)$ and $\frac{b}{\sigma} \leq \sqrt{\pi}$. This interval is non-empty if and only if $\frac{b}{\sigma} \leq \frac{3\sqrt{\pi}}{4(\pi-1)}$. Note also that $B \leq b^2 - 2\sqrt{\pi}b\sigma + \frac{5}{2}\sigma^2 \leq \frac{5}{2}\sigma^2$.

To finish the proof we show that if B lies on this interval, then $k = k^*$ is not only a local, but also a global maximum of $U_1(k, k^*, L)$.

Denote $r(k) = -2(k - b) - \left[\frac{\sigma^2}{2} + 2\rho + \frac{1}{4}(k + k^* - 2b)^2\right]w(k - k^*)$ (remind that $w(k - k^*) = \frac{f(k-k^*)}{F(k-k^*)}$).

Then $U_1'(k) = -2(k-b)F(k-k^*) - \left[\frac{\sigma^2}{2} + 2\rho + \frac{1}{4}(k+k^*-2b)^2\right] f(k-k^*) = F(k-k^*)r(k)$ and $sign(U_1'(k)) = sign(r(k))$.

First and second derivatives of $r(k)$ are:

$r'(k) = -2 - \left[\frac{\sigma^2}{2} + 2\rho + \frac{1}{4}(k+k^*-2b)^2\right] w'(k-k^*) - \frac{1}{2}(k+k^*-2b)w(k-k^*)$;

$r''(k) = -\left[\frac{\sigma^2}{2} + 2\rho + \frac{1}{4}(k+k^*-2b)^2\right] w''(k-k^*) - (k+k^*-2b)w'(k-k^*) - \frac{1}{2}w(k-k^*)$.

Notice that as $\frac{b}{\sigma} \leq \sqrt{\pi}$, $B \leq b^2 - \sqrt{\pi}b\sigma + \frac{5}{2}\sigma^2 \leq \frac{5}{2}\sigma^2$ and $k^* = b - \sqrt{\pi}\sigma + \sqrt{\left(\pi - \frac{5}{2}\right)\sigma^2 + B} \leq b$.

a) On interval $k > b$ $U'(k) < 0$, so there is no candidate for maximum there.

b) On interval $k < b$ we also have $k + k^* - 2b \leq 0$. As $w > 0$, $w' < 0$, $w'' > 0$, we have $r''(k) < 0$.

As $r'(-\infty) > 0$ and $r'(k^*) < 0$, there exists $k^{**} < k^* < b$: $r(k)$ is increasing for $k < k^{**}$, $r(k)$ is decreasing for $k > k^{**}$. As also $r(-\infty) < 0$, $r(k^* - 0) > 0$ and $r(k^* + 0) < 0$, there exists k_0: $r(k) > 0$ only on (k_0, k^*). Therefore, $U_1(k)$ is decreasing on $k < k_0$, increasing on (k_0, k^*), decreasing on (k^*, b). Hence, k^* is a global maximum if $U_1(k^*) \geq U_1(-\infty) = 0$, which has already been shown. □

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