

A Theory of Small Campaign Contributions*

Laurent Bouton[†] Micael Castanheira[‡] Allan Drazen[§]

September 22, 2017

Abstract

We present a model of electorally-motivated, small campaign contributions, an approach not present in the literature, and show it can explain observations that are hard to reconcile with a simple consumption motive. The analysis uncovers interesting effects across small donors and has novel implications for the effect of income inequality on total contributions and election outcomes. We also study the impact of different forms of campaign finance laws, on contribution behavior, probabilities of electoral outcomes, and welfare. We show that these results are consistent with more behaviorally motivated donors when contributions are driven by parties' solicitation of funds. We also show how the model and its results may have important implications for empirical work on campaign contributions.

*Acknowledgements: we hugely benefited from the insights, comments and suggestions by Scott Ashworth, Ethan Bueno de Mesquita, Anthony Fowler, Debraj Ray, Howard Rosenthal, Konstantin Sonin, Richard Van Weelden, by seminar participants at Harris School (U. Chicago), Namur, Pompeu Fabra, and LSE, as well as from comments on a previous step in this research project, in particular the audiences at Georgetown, Warwick, Mannheim, the Priorat Workshop in Theoretical Political Science, and EPSA conference in Edimburgh. This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 637662). Drazen's research is supported by the National Science Foundation, grant SES 1534132.

[†]Department of Economics, Georgetown University and ECARES, Université Libre de Bruxelles.

[‡]ECARES, Université Libre de Bruxelles.

[§]Department of Economics, University of Maryland.

1 Introduction

The role of campaign contributions in elections is a central issue in democracies. Both the popular and academic discussion have largely concentrated on large donors, but in fact campaigns are financed by a mix of contributions from large (and very large) donors, the government (in many countries), and small donors. In the United States, the Federal Election Commission reports that the 2012 presidential campaign cost about \$1.3 billion to the main candidates, out of which individual contributions reached just short of 1 billion.¹ Small contributions (less than \$200 each), added up to \$621 million. Those between \$200 and \$1000 added up to another \$243 million. The numbers tilted slightly more towards small contributions in the 2016 presidential race: Bernie Sanders, for example, raised 202 million dollars from small contributions, for a total budget of 223 million. Hillary Clinton and Donald Trump also each had more than 2 million small donors in the recent election cycle. Interestingly, towards the end of the campaign cycle, contributions come almost exclusively from small donors, as can be seen in Figure 1, which plots histograms of the distribution of the number of contributions for Clinton by dollar value. (In the Appendix, we also show the share of contributions by size.)²

Small donors are important in other countries as well. In Canada, small donors represent about a third of total funds raised for recent campaigns). The figure is similar in the United Kingdom, where a significant share of party funding comes from membership dues and small donations (for instance, the Labour party reported £19.2 million in donations and £9.5 million in membership dues in 2015).³ In Germany, small donors represent over about 53% of campaign resources in the 2012 cycle, with about half of that amount reflecting party membership dues).⁴ Small contributions account for such a significant fraction of total funding because the number of small donors is enormous.

In short, small contributions are extremely important in overall campaign financing. However, much of the theoretical literature has focussed on large donors and a policy

¹<http://www.fec.gov/disclosure/pnational.do?jsessionid=5E34A548A5EEB1D08BBECEA07049DF53.worker1> and <http://www.fec.gov/disclosure/pnational.do>

²PACs and super-PACs do provide large contributions, but one should note that they are also heavily financed by small contributions.

³<http://search.electoralcommission.org.uk/Api/Accounts/Documents/17488>

⁴Most of the rest is public funding; medium and large contributions made up only about 9% of total resources.



Figure 1: Quarter-by-quarter distribution of the 3’471’316 individual contributions to Hillary Clinton’s campaign, from Q2 2015 until Q4 2016 (Source: FEC data). The data displayed here lump together the contributions above \$2700, to ease readability.

influence motive for contributing (“quid pro quo”).⁵ To the best of our knowledge, there is no formal modeling of small campaign contributions, that is, a model which puts the choices of small donors on whether and how much to contribute into an explicit game-theoretic framework. In large part this appears to reflect the view that small campaign contributions are a pure consumption good for those who contribute, analogous to charitable contributions. The basic reasoning is that because such contributions are so small relative to total campaign funding, donors cannot be motivated either by an attempt to buy influence nor by any effect their contributions may have on election outcomes. A consumption motive wins almost by default because of the atomistic nature of individual small donations.⁶

The aim of this paper is to study small campaign contributions in a more formal game-theoretic model where small donors are motivated by the desire to affect election outcomes.

⁵The leading theoretical model is that of Grossman and Helpman (1994, 1996). The empirical literature finds mixed support (Stratmann, 1992; Ansolabehere, de Figueiredo and Snyder, 2003; Gordon, Hafer, and Landa 2007; Chamon and Kaplan 2013, DellaVigna et al. 2015). Hence, it is not clear to what extent large contributions “buy” policy favors or even access to elected politicians. Given our focus on small contributors in this paper, we take no stand on that empirical debate.

⁶Ansolabehere, de Figueiredo, and Snyder (2003) have stressed this view, arguing that the “tiny size of the average contribution made by private citizens suggests that little private benefit could be bought with such donations” (p117) They support their argument with the finding “income is by far the strongest predictor of giving to political campaigns and organizations, and it is also the main predictor of contributing to nonreligious charities” like other normal consumption goods.

This is in addition to or in combination with any consumption motive, which we will argue may actually stimulate electorally motivated contributions in equilibrium. We discuss in detail in section 2 theoretical and empirical arguments supporting an (broadly defined) electoral motive for small donors. We develop a simple model of individual contributor behavior and show how electoral motives can coexist with consumption motives for contributions, as well as how our approach is also consistent with behavioral approaches to small donor behavior.

Because of the strategic interactions that must characterize any model giving a role to electorally-motivated contributions, individual and total contributions may be quite different than those implied by a model of individual choice that ignores such interactions (e.g. a basic model of contributions driven solely by the consumption motive). As a consequence, the equilibrium effects of campaign finance laws may be quite different than conventional wisdom or existing literature suggests.

As will become clear, by “small” donor we mean two things. First, the donor takes the policy of candidates as given, which is simply to say that there is no motive of trading contributions for policy favor. Moreover, and central to our notion of a small donor, each donor takes the behavior of other donors as given. That is, though a donor crucially assumes that his donation has a non-zero (albeit small) positive effect on the probability that his preferred candidate is elected, this is through the numerical effect of his contribution on the aggregate amount the candidate collects rather than through any direct effect on other donors.⁷ There are strategic complementarities between donors, but they are “price takers.” Hence, ‘small’ can refer to donors who make substantial contributions in dollar terms, but who expect neither to receive policy favors in return nor to influence other donors directly.

The plan of the paper is as follows. In the next section we discuss in detail conceptual arguments about how and why the perceived influence of money on the election outcome shapes individual contributions. In section 3 we develop a simple model of individual contributor behavior in which optimizing donors decide how much to contribute to their preferred candidate, taking into account how the probability of a candidate winning the elections depends on total contributions to each of two candidates. We then derive in

⁷In a subsequent paper we are considering the effect that a very large donor may have on other donors in the electoral context we present here.

section 4 the equilibrium level of individual and total contributions, as well as the equilibrium probabilities of election. This allows us to perform comparative statics on, for example, the size of contributions and the preference intensity of donor groups, as well as the income distribution between and within groups. The analysis shows total contributions may rise or fall with increases in income inequality depending on its source, so that increases in income inequality may benefit the leading or the lagging party. In section 5 we show how the same basic results would obtain with “naïve” donors being solicited by electorally-motivated parties, that is, thinking of donors as being more “behavioral” than in our main model of fully rational donors and of parties as the optimizing actor.

We then analyze in section 6 the effects of various campaign finance laws. We find that a cap on individual contributions generally favors the party with the largest number of donors and works against the party with the richest contributors, but local effects are not necessarily monotonic. Caps on total campaign spending necessarily hurt the party with the largest budget, but may also incentivize donors from the lagging party to contribute more, so that the party initially ahead loses more than the direct effect of the legal constraint would suggest. Finally, we study the effect of public subsidies to the campaign budget and find that equal block subsidies to both parties help the party that is behind, while matching subsidies for or taxes on contributions leave election probabilities unchanged only if they affect all donations proportionately. In section 6.3 we consider the welfare implications of how money affects election outcomes and of policies to limit the effect of contributions, focussing on issues of how campaign finance laws may limit the influence of income and may help control the “arms race” of ever-higher aggregate contributions. Section 7 presents conclusions, and further material and proofs are in Appendices.

Our findings should also be relevant for empirical research on campaign contributions, as the different motives for contributions produce qualitatively different donor behavior responses. To give two examples, closeness of an election should have no first-order effect on donors if contributions are simply a consumption good, but will affect contributions that are electorally motivated. Second, an implication of our model is that donors will be induced to contribute more to a candidate who is lagging behind (an “underdog” effect) whereas when contributions are made in exchange for policy favors, the incentive is to give

to the candidate who is ahead. Our approach also has interesting empirical implications for the effect of income inequality on contributions, where effects will depend on whether it is within the group of a candidate’s supporters or between supporters of different candidates. Estimates of the overall income elasticity of contributions may also be biased depending on whether a candidate is ahead or behind.

2 On the Electoral Motive

Logical as it may sound that small donors are too small to be motivated by anything other than a pure consumption motive, there are both theoretical and empirical reasons why electoral motives for small donors, either directly instrumental or behavioral, should not be rejected out of hand.

From a theoretical perspective, “very small” is not zero. That is, a non-zero effect of an individual’s contribution on the election outcome means that an optimizing donor should take this effect, however small, into account. This simple observation proves particularly important when we embrace the presence of a consumption motive for campaign contributions. The presence of the consumption motive guarantees that the opportunity cost of the first dollar of contribution driven by the electoral motive is essential nil. Then, it must be that even an infinitesimal effect on the election outcome will drive additional contributions. Moreover, in that case, it would be erroneous to equate small effect with small electorally motivated contributions. Indeed, the magnitude of electorally motivated contributions prove to depend on the specific form of the utility function, and it is easy to find utility functions (e.g. CARA utility functions) that lead to electorally motivated contribution that are relatively large.

To make this clearer, consider the following simple example. An individual divides his income y between consumption C and campaign contributions q . She also values government expenditure G . Let us first consider the case where the individual believes her contribution q has no effect either on policy directly (the influence motive) or on the election outcome (the electoral motive). She then treat G as a parameter in her decision-making. With only the simple consumption motive – she gets direct utility from contributions q (represented by a utility function $U(C, q; G)$), an individual contributes until $\partial U / \partial C = \partial U / \partial q$. Call this level q^0 . The marginal utility cost of increasing q above q^0 by a

dollar is therefore 0. Now add an electoral effect. Suppose there are two candidates, A and B , who promise expenditures G_A and G_B respectively. The individual prefers candidate A to candidate B , that is, she prefers expenditure level G_A to G_B . Denote the probability that A wins by π_A , so that the electoral motive is represented by $\partial\pi_A/\partial q > 0$. Since the marginal cost of increasing q above q^0 is zero, the individual will be motivated to increase her electoral contribution for any non-zero $\partial\pi_A/\partial q$. The gist of the argument is straightforward: electorally-motivated contributions can become economically significant because their marginal cost is extremely small.

The previous argument supports the idea that, while combined with a consumption motive, a purely instrumental electoral motive may have bite. But, our objective here is not to defend such a purely instrumental electoral motive. Our approach is consistent with a more behavioral perspective. For instance, individual contributors could overestimate the influence of their contribution on the outcome of the election. Another possibility would be a more elaborate consumption motive, in which marginal utility of contributions would directly depend on the marginal effect contributions have on the election outcome (maybe because money is more important in close races, which the media cover more intensely). In section 5, we formally show that a behaviorally-based consumption model – where it is parties rather than individuals who are taking into account the effect of small contributions on electoral outcomes – yields conceptually the same results as those produced by the baseline model. In that alternative model, donors are “naïve” in that they respond positively to their party’s fund-raising efforts according to a simple behavioral rule. Fund-raising is costly and parties strategically decide how to allocate their fund-raising efforts, where the key assumption is that parties believe that money helps them win the election. Under simple and intuitive assumptions about the behavioral rule of donors, equilibrium contributions are the same as if we assumed purely instrumental donors.⁸

Second, empirical regularities also support the importance of a (broadly defined) electoral motive for small donors. First of all, in surveys (Brown *et al.* 1995; Francia *et al.* 2003; Barber 2016), donors overwhelmingly list “to affect an election outcome” as an important motive for giving. Of course, they may be hiding their true motives⁹, or they

⁸Those readers who prefer such an approach may want to skip to section 5 to see the basic set-up and then move to the material directly following (7) and (8) where we begin discussion of the implications of these conditions for electoral equilibrium.

⁹As Barber, Canes-Wrone, and Thrower (2017) put it, information from surveys “lack external validity

may be basing their answers on grossly inflated estimates of their probability of affecting outcomes. But the pervasiveness of this response suggests that an electoral motive, in one form or another, must be present. Moreover, as already mentioned, what is important in considering the electoral motive for individual donors is not whether the contribution actually affected the outcome, but whether donors or parties perceive that contributions have such an influence.

There is also hard evidence supporting the existence of a two-way relationship between campaign contributions and the electoral outcome. First, there is evidence that total campaign spending – and thus total contributions – matter for outcomes. For example, campaign budgets and television access have been found to influence the probability of winning the election (Erikson and Palfrey 1998, 2000; Gerber 2004; Da Silveira and De Mello 2011; Schuster 2016). Second, there is hard evidence suggesting that electoral motivations do influence contributions: first, numerous studies find that ideological proximity is a strong determinant of contributor behavior in different types of contests (see e.g. McCarty, Poole, and Rosenthal 2006; Claasen 2007; Bonica 2014; Barber 2016; Barber, Canes-Wrone, and Thrower 2017). The closeness of the ideological positions of donors and candidates matters because donors care about election outcomes.¹⁰ This is exactly our approach. Second, donations are significantly and positively affected by the (perceived) closeness of the election (Barber *et al.* 2016). While one cannot reject that this is consistent with a consumption motive under a sufficiently rich behavioral modeling of individual choice, at the very least it says that a model of small donors should have the probability of outcomes affecting individual decisions. Again, this is exactly our approach.

Finally, there is another crucial reason for considering alternatives to a pure consumption motive. A simple consumption motive for contributing seems inconsistent with the empirical regularity that election closeness matters and with the donors’ self-reported motivations. One may further note that the positive relation between income and contribution size has been taken as support of the consumption motive (in analogy to a similar finding for charitable contributions; see footnote 6), but the normality of contributions

on whether donors’ self-reported motivations are consistent with revealed behavior.”

¹⁰A related observation from Barber *et al.* (2016) is that contributions are made to legislators who “will represent their professional interests, rather than due to expectations of legislative access or an unsophisticated response to networking.” This too is consistent with an electoral motive rather than simply a consumption motive for giving.

should not in itself be taken as empirical evidence for the consumption motive against other motivations. We show below that it is fully consistent with an electoral motive for small donations.

3 Model

We model a contribution game in which a pre-determined set of donors simultaneously decide how much to contribute to their preferred candidate’s campaign, to increase his chances of election (we identify donors with the pronoun “she” and candidates with “he”). This captures a situation in which donors are “small” in the sense that they take both platforms and the actions of the other donors as given.¹¹ Throughout, we focus on the case of perfectly informed donors.

Candidates. We consider an election with two candidates, A and B , who need funding to run their electoral campaign. The total amount of contributions received by a candidate P is Q_P . We summarize through a *contest success function* (Tullock 1980, Hirshleifer 1989, Baron 1994, Skaperdas and Grofman 1995, Esteban and Ray 2001, Epstein and Nitzan 2006, Konrad 2007, Jia *et al.* 2013, *a.o.*) the fact that P ’s probability of winning the election increases in her funding. A standard argument is that campaign spending finances activities that increase a candidate’s vote totals, such as get out the vote (GOTV) efforts or, especially, advertising (as for example in Baron, 1994, Prat, 2002, Coate 2004a, 2004b, and Morton and Myerson, 2012). We simply assume that these efforts have a positive effect on votes and hence the probability of winning, as in (1) below. In section 6, where we address campaign finance reform, we consider in greater detail some positive and normative implications of how contributions may affect election outcomes.

Given total contributions $\mathbf{Q} = \{Q_A, Q_B\} \in \mathbb{R}_+^2$, P ’s probability of winning the election is given by:

$$\pi_P(\mathbf{Q}) \equiv \frac{(Q_P)^\gamma}{(Q_A)^\gamma + (Q_B)^\gamma} \quad (1)$$

with $\gamma > 0$, such that the winning probability is strictly increasing in Q_P . Note that π_P is everywhere concave in Q_P for $\gamma \leq 1$. Values of $\gamma > 1$ capture the presence of setup costs: π_P is then convex for $Q_P < \bar{Q}_P \equiv \sqrt[\gamma]{\frac{\gamma-1}{\gamma+1} Q_{-P}^\gamma}$. In words, P ’s campaign must reach $\bar{Q}_P$ for

¹¹In a separate project, we study the interactions between large and small donors in a multicandidate setup.

additional contributions to have maximal effect. Figure 1 illustrates the shape of π_A for $\gamma = 1$ (in blue), $\gamma = 2$ (in red), and $\gamma = 3$ (in black) when $Q_B = 1$.

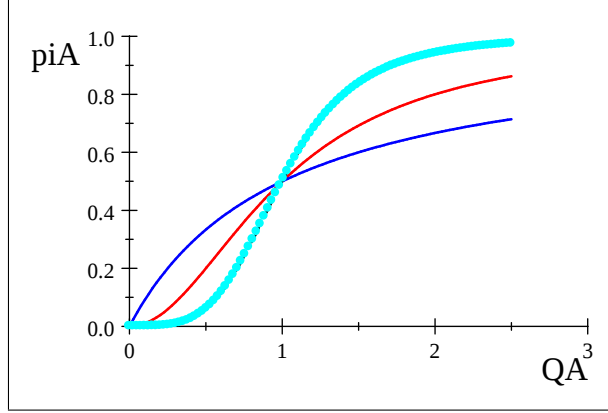


Figure 2: π_A for $Q_B = 1$ and $\gamma = 1$ (blue), $\gamma = 2$ (red), or $\gamma = 3$ (black-dashed)

Candidates are passive in our base model: the players of interest are the donors, who contribute to each candidate's campaign. In Section 5, we show that our results also hold in a model where candidates are the players of interest, and donors naïve.

Donors. A large number of donors must, simultaneously and non-cooperatively, decide how much to contribute to their candidate.¹² Each donor i has a two-dimensional type $(p^i, y^i) \in \{a, b\} \times \mathbb{R}_+$, where $p^i \in \{a, b\}$ identifies who is her favorite candidate/party. Naturally, a -donors support candidate A and b -donors candidate B : small and capital letters are used to avoid confusion between donors and candidates. y^i represents i 's income, which will influence her willingness to contribute.

Income distribution. The n^p donors of type p are distributed in income classes $y^1 < \dots < y^G$ according to some (discrete) distribution function $F^p(y^i)$ with $F^p(0) = 0$, and $F^p(y^G) = 1$. The fraction of type- p donors with income y^i is denoted $f^p(y^i) = F^p(y^i) - F^p(y^{i-1}) \geq 0$, and $\bar{y}^p$ is average income across all p -donors.

Objective function. In line with the motivation discussed in the Introduction, we focus on the electoral motive for contributing to the candidates' electoral campaign. That is, each donor contributes some amount $q_P^i \in [0, \bar{q}]$ to influence the outcome of the election – $\bar{q}$ is the legal contribution limit. In light of the discussion in Section 2, the marginal

¹²A similar setup has been pioneered by Katz *et al.* (1990) for rent-seeking, and by Esteban and Ray (1999, 2001) to analyze conflict situations, in which individuals invest resources to collectively fight over an issue.

cost of contributing must be zero at $q_P^i = 0$ and strictly increasing above that. Assuming iso-elastic cost functions, this amounts to setting $\rho > 1$ in the objective functions (2) and (3):

$$U^a(q_A^i; \mathbf{Q}^{-i}) = \pi_A(q_A^i; \mathbf{Q}^{-i}) v^a - \frac{(q_A^i)^\rho / \rho}{(y^i)^\theta}, \quad (2)$$

$$U^b(q_B^i; \mathbf{Q}^{-i}) = \pi_B(q_B^i; \mathbf{Q}^{-i}) v^b - \frac{(q_B^i)^\rho / \rho}{(y^i)^\theta}, \quad (3)$$

where v^p is the intensity of the donors' preference for their candidate and $\mathbf{Q}^{-i}$ is the vector of contributions by all donors other than i .¹³ The parameter θ will help parametrize the elasticity of contributions with respect to income: for $\theta = 0$, the cost of contributing is independent of income. For $\theta > 0$ instead, this marginal cost is strictly decreasing in y^i . In that case, equilibrium contributions will be increasing in income.

Given individual contributions, the total level of contributions received by party P is:

$$Q_A = \sum_{i=1}^{n^a} q_A^i + \varepsilon_A; \quad Q_B = \sum_{i=1}^{n^b} q_B^i + \varepsilon_B,$$

where ε_A and ε_B represent the prior contributions, personal war chest, and/or the voters' initial support of the two candidates.^{14,15} In the core of the paper, we set them to $\varepsilon_A = \varepsilon_B \rightarrow 0$. In Appendix 2, we show how they influence voluntary contributions when we relax that assumption: a larger ε_A reduces $\sum_{i=1}^{n^a} q_A^i$ but increases Q_A , for instance.

3.1 Donors' Incentives

Before turning to the equilibrium analysis, we highlight two important forces that shape a donor's contribution. Let Q_A^{-i} denote total contributions to candidate A by donors other than i , and rewrite (1) as:

$$\pi_A = \frac{(Q_A^{-i} + q_A^i)^\gamma}{(Q_A^{-i} + q_A^i)^\gamma + (Q_B^{-i})^\gamma}. \quad (4)$$

¹³It is straightforward that types $p^i = a$ want to contribute 0 to B , and conversely for types b .

¹⁴With a focus on why money polarizes politics, Feddersen and Gul (2015) let the probability of winning be a combination of voter support V and monetary contributions M : $\pi_A = \frac{V_A^{1-\alpha} Q_A^\alpha}{V_A^{1-\alpha} Q_A^\alpha + V_B^{1-\alpha} Q_B^\alpha}$. This formulation amounts to setting $\gamma < 1$ and considering asymmetric marginal effects of contributions (see below), which could be integrated into our model. However, since platforms are fixed in our model, we can focus on the simpler case of our model.

¹⁵Technically, winning probabilities are indeterminate for $Q_A = Q_B = \varepsilon_A = \varepsilon_B = 0$. Setting $\varepsilon_A, \varepsilon_B$ positive but small solves that problem.

We can then immediately derive the *marginal effect of a type a 's contribution* on winning probabilities:

$$\pi'_A \equiv \frac{\partial \pi_A}{\partial q_A^i} = \frac{\gamma}{Q_A} \pi_A (1 - \pi_A) = \frac{\gamma}{Q_A} \pi_A \pi_B, \text{ and, similarly:} \quad (5)$$

$$\pi'_B \equiv \frac{\gamma}{Q_B} \pi_A \pi_B. \quad (6)$$

Decomposing these effects identifies two central components of the donors' incentives:

Observation 1 *Given $\{Q_A^{-i}, Q_B^{-i}\}$, a donor's contribution π'_P increases in election closeness $\pi_A \pi_B$ (maximized in $\pi_A = 0.5$) and decreases in Q_P^{-i} (a free-riding effect).*

Taking account of contribution costs, first order conditions produce the following best-responses (for non-binding contribution limits):

$$\text{For types } a : \quad q_A^i = \left((y^i)^\theta \pi'_A v^a \right)^{\frac{1}{\rho-1}} \quad (7)$$

$$\text{For types } b : \quad q_B^i = \left((y^i)^\theta \pi'_B v^b \right)^{\frac{1}{\rho-1}}. \quad (8)$$

Taking the above equations together highlights a two-way interaction between individual contributions q_P^i and the marginal effect of a contribution π'_P . A higher π'_P increases an individual's contribution in (7) and (8), which increases aggregate contributions Q_P . This in turn feeds back into π'_A and π'_B via (5) and (6): both election closeness and free-riding are affected. Importantly, exactly the same strategic interactions would obtain in a model of behaviorally motivated donors who respond to candidate solicitations, as set out in Section 5.

Equations (7) and (8) also make clear the role of income in determining the level of individual contributions: the elasticity of a contribution with respect to income turns out to be $\theta/(\rho - 1)$. This shows that, unless one believes that a donor's willingness to contribute is independent of income ($\theta = 0$), a well-specified electoral motive will predict that contributions are strictly increasing in income – hence, the fact that contributions rise with income is not in itself evidence of a consumption motive. Moreover, it also implies that one cannot estimate the income elasticity of contributions without taking account of the indirect effects of income variations and inequality on total contributions. Indeed, these will also influence π'_P , and therefore equilibrium contributions.

4 Equilibrium Analysis

We focus on *pure strategy Nash equilibria* of this contribution game: each donor's contribution must be a best response to the vector of contributions by all other donors. In this section, we study the properties of *unconstrained equilibria*: we assume that the cap $\bar{q}$ on individual contributions is not binding, and that candidates do not face constraints on spending. The effects of campaign finance laws affecting these constraints are the focus of Section 6.

The first step is aggregate the best responses (7) and (8) to obtain a total contribution for each candidate that is consistent with individual incentives:

$$Q_A = n^a \sum_{i=1}^G q_A^i f^a(y^i) = W_A \times (\pi'_A)^{\frac{1}{\rho-1}} \quad (9)$$

$$Q_B = W_B \times (\pi'_B)^{\frac{1}{\rho-1}}, \quad (10)$$

$$\text{with: } W_P \equiv (v^P)^{\frac{1}{\rho-1}} n^P \sum_{i=1}^G f^P(y^i) \times (y^i)^{\frac{\theta}{\rho-1}}. \quad (11)$$

Note crucially that (9) and (10) are composed of two factors of a different nature. The first, W_P , only contains various parameters that define the primitives of the game. From now on, we thus treat W_A and W_B as *parameters of the model*, and call them the *group's willingness to contribute*. The second factor is π'_P , the marginal effect of contributions, which as discussed already, is endogenous to the donors' actions.

Without loss of generality, we label A the candidate who is *Ahead* and B the candidate who is *Behind*, in the sense that $W_A \geq W_B$. Let:

$$\omega \equiv \frac{(W_B/W_A)^{\gamma(1-\frac{1}{\rho})}}{\left(1 + (W_B/W_A)^{\gamma(1-\frac{1}{\rho})}\right)^2},$$

summarize the asymmetry in willingness to contribute between the two parties. Note that ω is strictly increasing in W_B/W_A for $W_B/W_A \leq 1$ (and decreasing for $W_B/W_A > 1$).

The unconstrained Nash equilibrium of the contribution game is found for the case in which there are no limits on individual contributions, *i.e.* when $\bar{q} = \infty$. Our first proposition identifies sufficient conditions for the existence of an equilibrium, characterizes the equilibrium, and shows that it is unique (most proofs are relegated to Appendix 3):

Proposition 1 *Whenever a pure strategy equilibrium exists, it is unique and characterized*

by the aggregate contributions:

$$(Q_A^*, Q_B^*) = \left(\sqrt[\rho]{\gamma \omega W_A^{\rho-1}}, \sqrt[\rho]{\gamma \omega W_B^{\rho-1}} \right),$$

which result in the following winning probabilities:

$$\pi_P^* = \frac{(W_P)^{\gamma(1-\frac{1}{\rho})}}{(W_A)^{\gamma(1-\frac{1}{\rho})} + (W_B)^{\gamma(1-\frac{1}{\rho})}}. \quad (12)$$

Two sufficient conditions for Pure Strategy Equilibrium existence are:

(1) $\gamma \leq \rho$ and, if $\rho < \gamma$, (2) W_A/W_B not too large.

As we already got a hint of (see Observation 1), equilibrium contributions are affected by free-riding. The fact that A is ahead implies that free-riding is stronger among a -donors:

Observation 2 *In any equilibrium, the ratio of contributions for A and B displays an underdog effect:*

$$\frac{Q_A}{Q_B} = \left(\frac{W_A}{W_B} \right)^{\frac{\rho-1}{\rho}}. \quad (13)$$

That is, equilibrium relative contributions for A are always smaller than A 's intrinsic advantage, W_A/W_B .

Such an underdog effect has already been identified for voters' participation, first by Simon (1954) and Palfrey and Rosenthal (1985), and more recently by Herrera *et al.* (2014) in a model with a contest success function.¹⁶ We are not aware of a similar finding regarding political contributions; to the contrary, the policy influence motive typically used to analyze contributions would predict that contributors are *larger* to the advantaged candidate, that is, the candidate more likely to win. This would lead to a *Bandwagon effect*. Stratmann (1992) shows that PAC contributions do display a bandwagon effect around a threshold strictly below 50%, followed by an underdog effect above that threshold. Bonica (2016, Figure 2) however finds that small donors behave substantially differently: their contributions disproportionately flow to underdogs (about 55% of their funds, instead of 85% for Corporate PACs). Such a fact is difficult to reconcile either with either the

¹⁶In voting models, the underdog effect results from the fact that pivot probabilities are higher for the underdog (see among others Castanheira (2003), Myatt (2012), Agranov, *et al.* (2014)). Here instead, this result is uniquely driven by free riding.

influence or the consumption motives. Very large and small donors thus appear to have different intrinsic motivations.

As is the case for turnout in Herrera et al. (2014), it is crucial to note that free-riding issues cannot reverse A 's initial advantage.¹⁷ As a result, A 's probability of winning does increase in her intrinsic advantage W_A/W_B , but this increase is attenuated by free-riding. In the absence of free-riding, her probability of winning would be $W_A^\gamma/(W_A^\gamma + W_B^\gamma) > \pi_A^*$.

In Appendix 2, we detail additional comparative statics on the importance of money in elections (as parameterized by γ) and on the effect of closeness on total equilibrium contributions ($Q_A^* + Q_B^*$). Our main focus in this section, however, is on the effects of income and income inequality on equilibrium contributions.

4.1 Equilibrium Effects of Income Inequality

The effects of rising income inequality on elections has become a central issue both in public debate and in academic research (see *e.g.* Feddersen and Gul, 2015). The typical perception is that it increases polarization and unduly favors the party with the richest supporters. The following lemma shows how willingness to contribute affects contributions. We then use it to study the effects of income inequality:

Lemma 1 *In equilibrium, Q_A^* is increasing in W_A and in W_B . Q_B^* is decreasing in W_A and increasing in W_B .*

Lemma 1 tells us, first, that increasing willingness to contribute for one candidate always translates into higher equilibrium contributions: Q_P^* is strictly increasing in W_P . The second, and perhaps more interesting, result concerns the effect of changes in support for one candidate on contributions to the *other* candidate. Remember that A is the candidate who is ahead, so that an increase in A 's support reinforces her advantage. By the same token, an increase in B 's support narrows A 's advantage. From Observation 1, this means that the former *reduces election closeness*, $\pi_A\pi_B$, which reduces π_B' and therefore contributions for B . A higher W_B instead *makes the election closer*: it increases $\pi_A\pi_B$, which stimulates contributions both for A and for B .

¹⁷In a different context, Esteban and Ray (2001) show that this is partly due to the shape of the cost function, and partly to the fact that winning the election acts as a public good. We use the qualifier “partly” because they focus on the case in which $\gamma = 1$. For that value of γ , Esteban and Ray (2001, Proposition 3) identify that free-riding effects cannot dominate collective action when payoffs are similar to that of a purely public good, as we have here.

These forces imply that the effect of income inequality actually depends on which candidate benefits from it:

Proposition 2 *Let $\theta > 0$ and $\bar{y}^a > \bar{y}^b$. An increase in between-group income inequality then has different effects if it results from an increase in the income of a -donors or from a drop in the income of b -donors: the former increases Q_A^* and decreases Q_B^* , whereas the latter decreases both Q_A^* and Q_B^* .*

Proposition 2 has a clear empirical implication for the income elasticity of contributions (*i.e.* $\theta/(\rho - 1)$ in our model). Consider a shock to donors' incomes. Estimation based on such a shock could be biased if it fails to control for the effects of this shock on the income distributions across groups and across income classes. For instance, if the shock is such that the income of the richest a contributors increases, the estimated income elasticity for those rich contributors would be biased downwards (contributions increase because of the direct effect, but this increase is reduced by the ensuing rise in free-riding and by the reduced election closeness). No less crucial is to control for the contributors' expectations of whether the candidate they support is ahead or behind. These expectations influence the sign of the indirect effect of other donors' income.

The next proposition considers the effects of *within-group* income inequality:

Proposition 3 *If and only if the income elasticity of contributions is larger than 1, a mean-preserving spread:*

- (1) *of the a -donors' income distribution increases Q_A^* and decreases Q_B^* .*
- (2) *of the b -donors' income distribution increases both Q_A^* and Q_B^* .*

The intuition is that, if the elasticity of contributions to income, $\frac{\theta}{\rho-1}$, is strictly larger than 1, contributions become a convex function of income. Increasing within-group inequality then increases the aggregate willingness to contribute W_P . However, a given increase in inequality does not have the same aggregate effects on the advantaged or in the trailing party (see (1) versus (2) in Proposition 3).

It is also interesting to understand how these aggregate effects play out in terms of individual contributions. To simplify the argument, consider an increase in the income of one of the income classes in a group – say of the richest A -donors. These donors thus

increase their contributions. This increases free riding by other income classes in group a . However, Q_A^* must be increasing in W_A , meaning that the drop in contributions by the other donors is only partial. There is a further cross-party effect: b -donors now face a *lower* incentive to contribute, which further reinforces the gap between A and B . A similar change within group b has the same within group effects, but the opposite cross-group effect: closeness *increases* contributions in the a group, which dampens the boost received by B . The equilibrium effects of income inequality thus always work in favor of the advantaged party.

To summarize these propositions, empirical work on the effects of income inequality on contributions should distinguish between within-group versus between group inequality, what is the source of inequality, and also take account of whether a candidate is leading or trailing.

5 A Model of Naïve Donors and Party Fund-Raising

One may argue that modeling donors as fully rational and strategic in their instrumental behavior lacks realism. That is, in their electorally-driven giving, small donors may display more “behavioral” motivations. For example: (1) donors may mechanically react to media attention and/or party fund-raising efforts, and the media or parties focus more on tighter races¹⁸ – we investigate this possibility below; (2) free-riding effects could be rationalized by individual donors enjoying “feeling important” – they would therefore contribute less if other donors contribute more (note that “herding” effects in consumption would produce the opposite result); (3) candidates may intensify their fund-raising effort on small donors when large donors have contributed less: this would also be consistent with a free-riding result.¹⁹

The purpose of this section is to show that our key results are fully consistent with such behavioral motivations. Comparative statics go in the same direction or can even be identical. We show that a reasonable functional representation of behavioral responses lead to the same first-order conditions, and hence identical results. Hence, whether individual behavior is driven by a purely instrumental electoral motive as above, or by another type of

¹⁸In other words, one could consider the case in which $\pi'_P(\mathbf{Q})$ enters directly the utility function of the consumer of political races. We owe this observation to the seminar participants at the Harris School.

¹⁹We thank Debraj Ray for suggesting some of these alternative scenarios consistent with our results.

behavioral-instrumental motive, the strategic interactions identified in the previous section are key to understanding how aggregate contributions are determined in equilibrium.

To formalize this point, we assume in this section that small donors are “behavioral” in the sense that they mechanically respond to party nudges. Parties, on their side, need to exert a costly effort in order to induce their supporters actually to contribute to their campaign. This change in perspective transforms our model into a “demand-side” model in which parties are the strategic actors, rather than a “supply-side” model in which donors were the strategic actors.

Such an alternative model could be as follows. As in our base model, there are n^p donors of type p , distributed in income classes $y^1 < \dots < y^G$ according to some (discrete) distribution function $F^p(y^i)$, that satisfies the same assumptions as in section 3. We assume that donor i reacts mechanically to her party’s (costly) fund-raising effort, denoted e_P^i . Her contribution q_P^i is increasing and concave in both e_P^i and y^i . We represent this functionally by:

$$\text{For types } a : \quad q_A^i = \left((y^i)^\theta e_A^i \right)^{\frac{1}{2}} \quad (14)$$

$$\text{For types } b : \quad q_B^i = \left((y^i)^\theta e_B^i \right)^{\frac{1}{2}}, \quad (15)$$

where θ parameterizes the donors’ elasticity of contributions exactly like in the instrumental model. The Cobb-Douglas specification is chosen both for simplicity and to relate with the main model.

Parties choose e_P^i to maximize their probability of winning net of the cost of fund-raising (where, for simplicity, we let the cost of soliciting a donor be e_P^i):

$$\begin{aligned} P \text{ maximizes } : \quad & \frac{Q_P^\gamma}{Q_A^\gamma + Q_B^\gamma} - \sum_i e_P^i, \\ & s.t. \quad Q_P = \sum_i q_P^i. \end{aligned}$$

It follows that:

$$e_P^{i*} = \left(\frac{\pi'_P}{2} \right)^2 (y^i)^\theta.$$

Substituting these equilibrium levels of party effort into the donors’ contribution functions (14) and (15) yield:

$$\begin{aligned} q_A^{i*} &= \frac{\pi'_A}{2} (y^i)^\theta \\ q_B^{i*} &= \frac{\pi'_B}{2} (y^i)^\theta, \end{aligned}$$

which is identical (but for the factor $\frac{1}{2}$) to (7) and (8) when $\rho = 2$.

In other words, there exists some form of response by behavioral donors and strategic parties such that the equilibrium level of individual and aggregate contributions are the same as with strategic donors and passive parties. Hence, although it is a perfectly valid empirical question to ask, “How rational are small donors?”, allowing them to be “behaviorally motivated” rather than fully rationally instrumental does not qualitatively change our findings on how electoral motives (here on the part of parties) determine individual contributions, nor on how economic variables and legal constraints (considered in the next section) would influence total contributions and the feedback loops between aggregate and individual contributions.

Analogous to the ethical voter models, for example of Coate and Conlin (2004) or Feddersen and Sandroni (2006), one may think of how an “ethical contributor” would choose individual contributions. That is, if a planner in group A and a planner in group B were to decide of the allocation of contribution effort within their donor group, they would select exactly the same level of contributions if they respectively maximized:

$$U^A = v^a \pi_A - \sum_i \frac{(q_A^i)^{\rho-1}}{(y^i)^\theta \rho} \quad \text{and} \quad U^B = v^b \pi_B - \sum_i \frac{(q_B^i)^{\rho-1}}{(y^i)^\theta \rho}.$$

If instead, they maximize in-group aggregate social value (consistent with Feddersen-Sandroni) of electing their candidate, they would maximize:

$$U^{A'} = n^a v^a \pi_A - \sum_i \frac{(q_A^i)^{\rho-1}}{(y^i)^\theta \rho} \quad \text{and} \quad U^{B'} = n^b v^b \pi_B - \sum_i \frac{(q_B^i)^{\rho-1}}{(y^i)^\theta \rho}.$$

This would require higher individual contributions when there are more donors in a group, to take account of the positive externalities of one contribution on the other donors of the same group. In Section 6.3, we study the social optimum and find that the latter rule-utilitarian solution would actually be detrimental to aggregate welfare. The optimum typically requires some form of cap on contributions. But, to show this, we must first study the potential effects of various campaign finance regulations.

6 Campaign Finance Laws

We study four types of campaign finance laws that are widespread around the world: (1) Caps on individual contributions (used, for example, in the U.S., Canada, Chile, France, Israel, and Japan, among others); (2) Caps on total donations/spending (*e.g.* in many countries in Europe, as well as Chile, Israel, New Zealand, and South Korea) ; (3) Public subsidies to parties (*e.g.* in many countries in Europe, as well as Israel, Japan, and Mexico) either as block subsidies or as proportional subsidies to individual contributions (including tax deductibility of contributions).

6.1 Rationale for campaign finance laws

Campaign finance laws are, very generally speaking, meant to limit the “influence of money in politics”. One rationale is that large contributions buy policy influence outside of any direct effect on voting, that is, trading contributions for policy favors in a “quid pro quo”, as discussed in footnote 5. Such a rationale, as important as it might be in practice, plays no role here as we abstract from the influence motive.²⁰

A second rationale is that campaign spending has exploded because it is like an “arms race” – what is crucial is the level of total contributions *relative* to those of one’s opponent. Hence, the level of money ratchets up without giving either candidate a relative advantage but simply draining resources.

A third argument is that a donor’s influence on elections is determined by the size of her contribution, so that large contributors have undue electoral influence. In that context, contribution caps are meant to ensure that the “voices of small donors” are also heard (this is sometimes referred to as the “equalization” argument). This is central to our paper, where richer donors contribute more simply because they are richer and, all else equal, have a greater effect on election outcomes.

²⁰Coate (2004) considers the negative welfare effects of contributions because they buy policymaker influence. In his setup, contribution limits may increase social welfare not only because they reduce such influence, but also – and because of this – such limits increase the information value of activities that contributions finance.

6.2 Campaign finance laws: the positive effects of caps and subsidies

What we show here is that, due to the strategic complementarities highlighted in the previous sections, even the positive effect of a regulation such as a contribution cap is not straightforward. Among other things, small donors will be affected even if they are not directly capped, an effect almost entirely ignored in the literature. The complementarities central to small donor behavior further suggest that the effects of caps on election outcomes may also be far from simple. In the next subsection, we discuss the positive effects of different campaign finance laws, and their welfare effects in section 6.3.

6.2.1 Caps on individual contributions

The complexity of possible effects is illustrated in the following two propositions: we find that the effects of contribution caps can go in exactly opposite directions, depending on whether the advantage of A results from a wider support (Proposition 4) or from a richer set of donors (Proposition 5). Moreover, the effects need not be monotonic:

Proposition 4 *Consider the case of identical income distributions and preference intensity (v^p) for a - and b -donors, but $n^a > n^b$. In that case:*

- (1) π_A will be **lowest** when the cap is not binding;
- (2) π_A will be **highest** when the cap constrains all donors;
- (3) Depending on the shape of the income distribution, the effects of varying the cap can be non-monotonic.

The main driver of the difference between (1) and (2) is the underdog effect (see Observation 2). With $n^a > n^b$, free riding implies that an a -donor with income y^i contributes less than a b -donor with the same income. When the distributions of income among a - and b -donors are sufficiently similar, any binding cap must therefore constrain b -donors more than a -donors. Candidate A is thus better off with no cap than with a cap, and worse off when the cap is binding for all donors.

Importantly however, this does not imply that the effects of a cap are monotonic, as illustrated in Figure 3.²¹ The reason is that capping high-income donors stimulates

²¹The simulation behind Figure 3 builds on a two-group income distribution with $y_l = 3$ and $y_h = 10$; while we set $\gamma = \rho = 2$, and $v^p = \theta = 1$. The number of low- and high-income donors are: $n_l^a = 60 > n_l^b = 30$ and $n_h^a = 20 > n_h^b = 10$. That is, both income classes are willing to contribute about the same

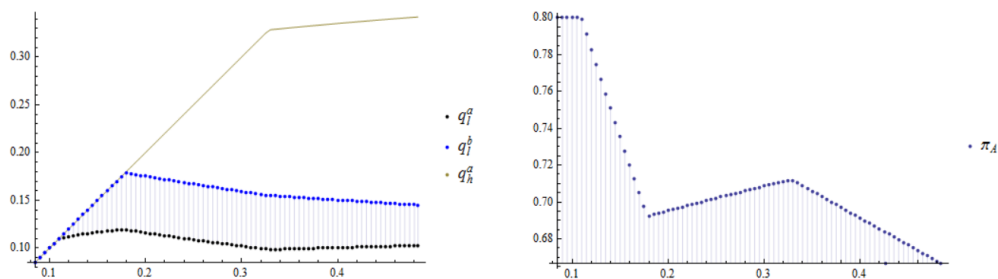


Figure 3: Simulated effect of an individual contribution cap when $n^a > n^b$ but the income distribution is identical across donor groups.

contributions by low-income donors *and* impacts closeness – remember that closer elections stimulate contributions by all income classes in both groups, see Section 1 and Appendix 2. Thus, while the direct effect of the cap favors A (b -donors being more constrained), indirect effects tend to work in the opposite direction, and may dominate.

In our numerical example, the fact that contributions by low-income donors represent about half of the total received by each candidate is sufficient to produce such reversals: the left pane depicts the equilibrium individual contributions by each donor type (except for high-income b -donors who are capped throughout), given the cap on the horizontal axis. The right pane depicts the probability that A wins as a result of these contributions. As one can tell, indirect equilibrium effects dominate for intermediate caps.

Now, contrast these results with the case in which the advantage of A is due to *higher donor income*, rather than a numerically larger donor base:

Proposition 5 *Consider the case in which A and B have equal popular support ($n^a = n^b$) and preference intensity, but a -donors benefit from higher income, by a factor $\alpha > 1$ ($f^a(\alpha y^i) = f^b(y^i)$, $i = 1, \dots, G$). In that case, the effects of a cap are the opposite of the ones in Proposition 4:*

- (1) π_A will be **highest** when the cap is not binding;
- (2) π_A will be **lowest** when the cap constrains all donors;
- (3) Depending on the income distribution, the effects can be non-monotonic.

The intuition and the mechanism of the proof are similar to those of the previous proposition, with the difference that, if a -donors are richer but no more numerous than amount (this proxies the split in the 2015-16 US presidential elections), but there are twice as many a - as b -donors, implying that $W_A = 380$ and $W_B = 190$.

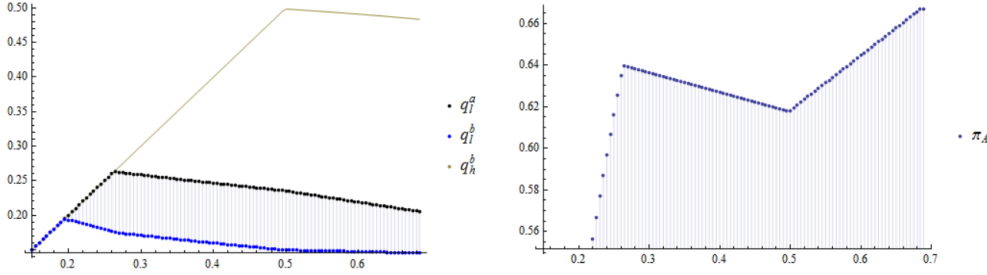


Figure 4: Simulated effect of an individual contribution cap when $y^{i,a} = 2y^{i,b}$ but the number of donors is identical across donor groups.

b -donors, they must be the first constrained. Hence, there are more a than b constrained donors, and any unconstrained a -donor contributes more than the equivalent b -donor. The initial logic is thus the same as above, with the important difference that closeness and free-riding effects now work in the opposite direction, as illustrated in Figure 4.²²

6.2.2 Caps on total spending

Caps on *total* campaign spending, either by parties or by individual candidates are observed in some countries (Ohman, 2012). In our model, campaign spending by a candidate is equal to total contributions by her supporters, so that we could think of limits on the total size of campaign spending as a cap on total contributions. When the cap on total contributions is binding for both candidates, their total contributions are necessarily identical. We thus focus on the interesting case in which the cap only constrains A :

Proposition 6 *Capping total contributions for A increases contributions for B . Therefore, A 's probability of winning decreases by more than the direct effect of the cap would imply. Total contributions $Q_A + Q_B$ may increase or decrease as a result.*

A cap affecting only A increases elections closeness, which stimulates contributions for B , further favoring the latter candidate. This crowding-in effect on Q_B can be so strong as to increase total contributions: $(\bar{Q} + Q_B)$ can actually increase when the cap on total campaign spending ($\bar{Q}$) is tightened. This typically happens when A 's lead of is initially

²²This numerical example also builds on two income classes in each donor group: $y_l^a = 6$ and $y_h^a = 20$, $y_l^b = 3$ and $y_h^b = 10$; $\gamma = \rho = 2$, and $\theta = 1$. Thus a -donors have twice the income of b 's, while their numbers are identical: $n_l^p = 30$ and $n_h^p = 10$, $\forall p$. Hence, as in the previous example, $W_A = 380$ and $W_B = 190$.

large (see Appendix 3).²³

6.2.3 Campaign subsidies

Finally, consider the effects of campaign subsidies. We study two of them: (i) a block subsidy, where the government gives a lump-sum of s dollars to both candidates' campaigns; and (ii) a matching subsidy, where for each donation q_P^i , the government gives an additional m dollars to party P 's campaign. In the presence of subsidies, the total level of contributions received by the two parties become:

$$\tilde{Q}_A = \sum_{i=1}^{n^b} (1+m) q_A^i + s + \varepsilon; \text{ and } \tilde{Q}_B = \sum_{i=1}^{n^b} (1+m) q_B^i + s + \varepsilon. \quad (16)$$

Block subsidies

Consider first a block subsidy s alone, so that $m = 0$ in (16):

Proposition 7 *Block subsidies increase the relative voluntary contributions for A, but decrease the probability that A wins: $\frac{d(Q_A/Q_B)}{ds} > 0$, $\frac{d\pi_A}{ds} < 0$.*

A block subsidy has a direct negative effect on the probability that the most popular party, A , wins. This should not be surprising, since an equal subsidy to both candidates "levels the playing field". However, this direct effect is attenuated by the different reactions of a -donors and b -donors. Somewhat surprisingly, a block subsidy can have a crowding-in effect on individual donations. This happens when the induced effects of closeness are strong enough, as illustrated by the following example: we consider the case of a single level of income: $y^a = 10 = y^b$ but there are 10 times more a -donors than b -donors: $n^a = 100 > n^b = 10$ (like in the other examples, $\gamma = \rho = 2$ and $\theta = 1$). As one can see on Figure 5, Q_A increases in s when s is low, and decreases in s when s is large.²⁴

One direct implication of this proposition is that, neither crowding-in nor crowding-out effects of public subsidies may compensate the direct effect of the subsidy on the probability

²³Note that this effect is different from the one in Che and Gale (1998), who consider an all pay auction. In that auction, expected total contributions are everywhere (weakly) increasing in the cap, except at a point of discontinuity. When the cap is above that level, the high-valuation bidder can make such aggressive bids that the low-valuation bidder shaves his bids significantly. That reduces total contributions.

²⁴We did not find any example in which a block subsidy has a crowding-in effect on individual contributions by b -donors.

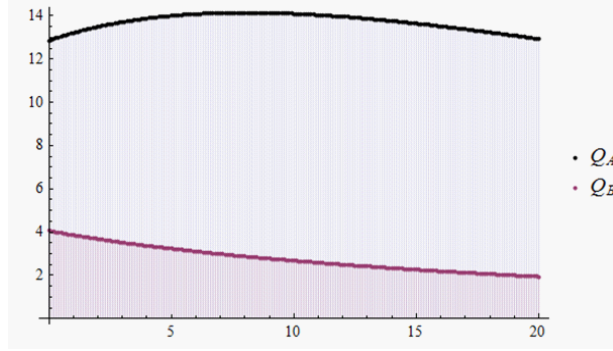


Figure 5: Simulated effect of a block subsidy on total individual contributions when $y^a = y^b$ and $n^a = 100$ and $n^b = 10$.

that A wins. Moreover, for both parties, the sum of total individual contributions plus the block subsidy always increases with the size of the subsidy.

Matching subsidies to and taxes on contributions

A matching subsidy m (which may be negative, that is, a tax on contributions) with no block subsidy ($s = 0$ in (16)) has no effect at all if it applies to all contributions.

Proposition 8 *A campaign contribution matching subsidy of m that applies to all contributions has no effect on the behavior of donors, nor on the outcome of the election. On the other hand, a matching subsidy which applies only to contributions below a certain level can affect election probabilities.*

The first part of the proposition may not be very surprising, given the form of our contest success function. Since the matching subsidy increases each (and hence total) contributions by the same fraction m for both candidates, it has no effect on the *relative* position of the two candidates, and hence no effect on election probabilities. Matching subsidies may affect outcomes for other specifications of the contest success function, but the mechanism behind Proposition 8 makes clear why a general matching subsidy will not have a major effect as it has little or no effect on relative candidate positions. Analogously, there is no reason to anticipate that it should either systematically increase or systematically decrease individual contributions.

A matching subsidy the only applies to contributions below a certain level,²⁵ on the other hand, will have an effect. If the aggregate amount of matched contributions (contribution plus matching funds) rises, contributions of those above the matching threshold will decrease. The overall impact on the election could however go either way.

Turning to *taxes* on contributions, making them dependent on the size of the contribution acts like a *negative* size-dependent match. Since contributions depend positively on income, this would be like a differential tax on contributions, that is a function of income. Such a tax has the possibility of reducing or even eliminating the effect of income on contributions, an issue to which we return in considering the welfare implications of campaign finance laws. Consider the following tax on contributions as a function of income:

Proposition 9 *A tax on contributions equal to $\left[(y^i)^\theta - 1\right] q_P^i$ removes the effect of income inequalities from equilibrium contributions.*

Such a tax means contributions in each income class are independent of the income of active donors.

6.3 Campaign finance laws: welfare considerations

We now consider the welfare implications of campaign finance laws, concentrating on contribution caps. As discussed in section 6.1 above, a key rationale for such restrictions is that unlimited contributions give rich donors disproportionate influence on election outcomes. Another argument was to limit the overall explosion of the size of campaign spending.

The debate for example in the United States, as reflected in U.S. Supreme Court decisions, has been largely framed in terms of issues of ‘freedom of speech’. In the famous *Buckley v. Valeo*, a majority held that limits on campaign spending and individual contributions in the Federal Election Campaign Act of 1971 were unconstitutional because they violated the First Amendment provision on freedom of speech because a restriction on spending “necessarily reduces the quantity of expression”. Similarly, in the 5-4 majority decision in *Citizens United v. FEC*, Justice Kennedy argued that limits on corporate

²⁵In New York City campaigns, for example, donations up to \$175 from New York City residents are matched at a rate of 6:1. In 2013, small donations and matching funds accounted for 71 percent of the individual contributions in the city’s elections. See <https://nyccfb.info/program/impact-of-public-funds>

and union contributions to PACs should be struck down because such limits interfered with free speech, namely the “right of citizens to inquire, to hear, to speak, and to use information to reach consensus.”

Arguments *in favor* of restrictions also relied on such considerations. In *Austin v. Michigan Chamber of Commerce* (1990) the court had upheld previous limit on corporate spending, writing “Corporate wealth can unfairly influence elections.” Analogously, Justice Stevens, in the minority dissent in *Citizens United*, reiterated the “unfair influence” argument, writing that “unregulated expenditures will give corporations ‘unfair influence’ in the electoral process and distort public debate in ways that undermine rather than advance the interests of listeners.”

These arguments can directly be formalized in the framework of our model. Starting from the sum of the donors’ individual utilities, we could consider the following objective function for the social planner (SP):

$$U^{SP} = n^a v^a \pi_A - \sum_{i \in a} \frac{(q_A^i)^\rho}{(y^i)^\theta \rho} + n^b v^b \pi_B - \sum_{i \in b} \frac{(q_B^i)^\rho}{(y^i)^\theta \rho}.$$

In light of the above arguments, however, such a welfare function is inappropriate. Simply because the contribution costs are lower for richer donors would produce the result that richer donors deserve disproportionate influence on the election outcome. Correcting this bias requires setting $\theta = 0$ in the objective function:

$$\begin{aligned} U^{SP} &= n^a v^a \pi_A - \sum_{i \in a} \frac{(q_A^i)^\rho}{\rho} + n^b v^b \pi_B - \sum_{i \in b} \frac{(q_B^i)^\rho}{\rho} \\ &= \left(n^a v^a - n^b v^b \right) \pi_A - \sum_{i \in a} \frac{(q_A^i)^\rho}{\rho} - \sum_{i \in b} \frac{(q_B^i)^\rho}{\rho} + n^b v^b. \end{aligned} \tag{17}$$

The free-speech argument amounts to saying that the group, a or b , with the largest $n^p v^p$ “deserves” winning with the highest possible probability, whether because they are more numerous (larger n^p) or because they have more intense preferences (a larger v^p ; presumably an influence meant to be protected under the First Amendment). However, this requires allowing them to contribute to the campaign of their candidate, which has a cost $\sum_i [(q_A^i)^\rho + (q_B^i)^\rho] / \rho$ in the social welfare function. Limiting the size of the campaign spending may thus conflict with the former objective.

Assume that n^a and n^b represent the total number of *voters* on each side (all voters are potential contributors, but some have zero income available for contributions and hence contribute nothing). Suppose the planner had perfect information about the $n^p v^p$. If (without loss of generality) $n^a v^a > n^b v^b$, his optimum would be to reduce contributions to B to zero, and allow contributions to A to just compensate for the risks introduced by ε_B , as long as the benefit exceeds the cost to group A of these contributions.

Three objections can be raised to this result. First, the outcome the social planner can achieve will depend on the tools at hand, with the issue that the objective function is then itself dependent on the instruments. Second, there is no reason to expect the social planner has sufficient information about the number of voters on each side, nor the intensity of their preferences — otherwise elections would not even be needed. Instead, it is the very contributions by donors that are expected to signal these values, with the issue that the social planner wants to learn about v^p while getting rid of the influence of the donors' income.

The first two objections suggest looking at a constrained information optimum where the tools the social planner has are the campaign finance regulations considered in the previous section. The third objection and the trade-off between revealing information and minimizing the costs of the campaign are actually not simple to address.

In considering the welfare effects of campaign finance laws, we therefore consider how they may bring the ratio of total contributions closer to $\frac{n^a v^a}{n^b v^b}$ (or some monotone function thereof), instead of $\left(\frac{W_A}{W_B}\right)^{\frac{\rho-1}{\rho}}$ like in the unconstrained equilibrium of Proposition 1. The former is the ratio of total contributions that would obtain if there were no income differences between groups, nor free-riding effects. We call *constrained optimum* the joint objective of bringing the contributions ratio closer to $\frac{n^a v^a}{n^b v^b}$ and reducing the costs of the campaign.

6.3.1 Contribution caps

The simplest case to consider is where the groups do not differ in their valuations, that is, $v^a = v^b$, reflecting a situation where individuals in the two groups are believed to care equally on average about election outcomes. Hence, differences in groups reflect size, and perhaps income. We find that, in this case, individual contribution caps are an appropriate

instrument:

Proposition 10 *For $v^a = v^b$, $F^a(0) = F^b(0)$, and $\varepsilon_A = \varepsilon_B \rightarrow 0$, a tight cap on individual contributions can bring welfare arbitrarily close to the constrained optimum.*

A sufficiently tight cap means that all non-zero donors in both groups contribute the same amount, eliminating the effect of income. If the ratio of donors to non-donors (individuals with $y^i = 0$) is the same across groups, *i.e.* $F^a(0) = F^b(0)$, a sufficiently tight cap brings us back to the case of “one man, one vote,” which is implicitly the objective when $v^a = v^b$. This policy even produces a double dividend: on top of bringing the contribution ratio closer to the first best, it also decreases waste. In contrast, a cap on aggregate contributions would decrease costs, but hurt A and therefore move the outcome *away* from the constrained optimum. The same holds for block subsidies in our model.

A more difficult case is when v^a and v^b differ, so that contributions may reflect either differences in income or in preference intensities. If one believes that contributions differ because of differences in the v^P rather than in income between groups, then binding contribution caps would destroy the information that contribution differences convey, which is welfare-reducing. (Note further that capping contributions by the richest induces the less rich to contribute more, to compensate.) If, conversely, contribution differences reflect income disparities rather than disparities in v^P , then caps can move outcomes closer to the optimum by eliminating the “noise” in contribution differences.

6.3.2 Combining caps with taxes on contributions

Although a tax on contributions has not been considered in practice as part of campaign finance legislation, we show it can help address this conundrum (see the next section for a discussion of matching subsidies.) Under the tax to contributions set out in Proposition 9, equilibrium behavior actually leads to contributions that are independent of income. However, there is still a trade-off between the cost of campaign contributions and the revelation of information about preference intensity. The following proposition show that a cap may be used to address that trade-off:

Proposition 11 *Fix $n^a = n^b$ and $\varepsilon_A = \varepsilon_B$, and let contributions be taxed like in Proposition 9. Then, equilibrium contributions are the same as if $\theta = 1$ and social welfare displays*

two local optima: one is with $\bar{q} = 0$ and minimal campaign costs. The other one is with $\bar{q} = \max_i [q_P^{i*}]$ and, effectively, free speech. But any cap in between these two levels must be welfare inferior to one of these two extreme solutions.

The intuition for this result is that, thanks to the tax, a cap constrains contributions of all donors in a same group in the same way. With $v^a > v^b$, the cap first constrains all a donors. If it is tightened further, there is a level, call it χ , for which all donors are capped. It follows immediately that, for any cap $\bar{q} < \chi$, winning probabilities are constant. Any cap tightening is then a Pareto improvement.

For $\bar{q} > \chi$ instead, a cap tightening reduces the probability that A wins. The question is whether this is more than compensated by the decrease in the costs of the campaign. To address this question, we use the envelope theorem: at the equilibrium, the indirect effects of a marginal cap tightening are second order. We thus only need to take account of direct effects, which are $(n^a v^a - n^b v^b) \Delta \pi_A$. Since $\Delta \pi_A$ is negative, the total effect can only be negative.

6.3.3 Matching subsidies

As is made clear in section 6.2.3 above, a general matching subsidy has no effect on election outcomes, while a matching subsidy for donations below a certain level will have an effect. It has been argued that matching of small donor funds can provide a counterweight to “big money”, even more so with the growing importance of internet and social media fundraising.²⁶ The disadvantage of the subsidy in comparison with the tax is that it can only worsen the cost of contributions problem.

6.3.4 Caps on total contributions

Caps on total spending would address the “arms race” nature of contributions, which is well captured by the contest success function, where a proportional increase in Q_A and Q_B would leave election probabilities unchanged. However, contributions are not exogenous – they are motivated by the number of citizens n^p who favor candidate P , as well as the value v^p those citizens give to that candidate’s victory. Hence, limitations on how

²⁶<https://www.brennancenter.org/analysis/money-politics-101-what-you-need-know-about-campaign-finance-after-citizens-united#39>

much an individual may contribute, or on how much in aggregate a candidate may collect may reduce welfare by reducing the probability of victory of the candidate who would deliver higher aggregate welfare. At the same time, as made clear by the role of income in generating contributions, candidate P may receive larger contributions not because $n^P v^P$ is higher, but because her donors are richer.

Finally, note the implication that both candidates and donors may benefit from contribution limits, *even in the absence of any policy distortion*, simply because the contributions “arms race” is de-escalated. The overall amount of resources wasted on the campaign is reduced, *and* the cap increases the probability that the party supported by the largest number of donors wins. The possible optimality of caps does not depend on controlling the ability of groups to buy policy influence. It can arise in a model where contributions are instrumental only to the extent they affect election probabilities.

7 Conclusions

Conventional wisdom is that small donations to political campaigns are a consumption good to the donors. In large part this is a conclusion by default. The basic reasoning is that because each small donation is so small relative to total campaign donations, small donors cannot be motivated either by an attempt to buy influence nor by any effect they may have on election outcomes. A consumption motive is what remains.

As intuitive as this reasoning may sound, in our opinion it misses several basic points. First, “very small” is not zero. Though a zero probability of, for example, affecting the election outcome would imply a zero electorally-motivated contribution, an extremely small but non-zero probability would imply a positive, though small, contribution. Second, strategic complementarity will magnify the effect of small donations even if donors act atomistically (that is, non-cooperatively). Hence, a small increase in the marginal effect of contributions may produce a substantial aggregate impact over and above the effect implied by large number of donors. , but also because of the strategic complementarities across donors. Therefore, individual and total contributions may be quite different than those implied by an individual decision-making model without complementarities, such as a simple consumption motive model. The effect of a change in campaign finance laws, for example, may thus be quite different than conventional wisdom or existing literature

suggests. Both of these suggest the importance of building and studying a model of optimizing small donors who may be driven by instrumental rather than simple consumption motives.

A third observation points in this direction as well. There is significant empirical evidence suggesting that electoral motivations do influence contributions. These include the importance of ideological proximity as a strong determinant of contributor behavior, indicating that donors care about election outcomes; the significant positive effect of perceived closeness of an election on donations; and, simply, the surveys of donors who overwhelmingly list “to affect an election outcome” as an important motive for giving.

The model we present demonstrates that the desire to affect the election outcome can be an important motive for small donors. We show that such a model can reproduce some basic stylized facts. At the same time, we find that a formal decision-theoretic model of electorally-motivated donors yields predictions different than simple intuition may suggest *ex ante*. This is highly relevant to interpreting empirical results on the effect of changes in the characteristics of donors (such as a change in the income distribution) or of campaign finance laws. Our results from a well-specified theoretical model point to pitfalls in empirical estimation of determinants of campaign contributions.

As a final note, our focus on the electoral motive does not rule out alternative decision-theoretic approaches, such as a combination of richer behavioral and electoral motivations for giving. As such the paper should be read not only as an exploration of an electoral motive for small donations, but more generally as an exploration of decision-theoretic approach to small donors in contrast to a simple consumption motive.

References

- [1] Agranov, M., J. Goeree, J. Romero, and L. Yariv (2014). What Makes Voters Turn Out: The Effects of Polls and Beliefs, *mimeo*.
- [2] Ansolabehere, S., J. de Figueiredo, and J. Snyder (2003). Why Is There so Little Money in U.S. Politics?, *Journal of Economic Perspectives*, 17(1): 105-130.
- [3] Barber, M. (2016). Donation Motivations: Testing Theories of Access and Ideology, *Political Research Quarterly*, 69(1): 148-159
- [4] Barber, M., B. Canes-Wrone, and S. Thrower (2017). Ideologically Sophisticated Donors: Which Candidates Do Individual Contributors Finance?, *American Journal of Political Science*, 61(2): 271-88.
- [5] Baron, D. (1994). Electoral Competition with Informed and Uninformed Voters, *American Political Science Review*, 88: 33-47.
- [6] Bonica, A. (2014). Mapping the Ideological Marketplace, *American Journal of Political Science*, 58(2): 367-386.
- [7] Bonica, A. (2016). Avenues of Influence: On the Political Expenditures of Corporations and Their Directors and Executives, *Business and Politics*, forthcoming.
- [8] Bonica, A. and H. Rosenthal (2016). Increasing Inequality in Wealth and the Political Consumption of Billionaires, *mimeo*.
- [9] Brown, C., W. Clifford, L. Powell, and C. Wilcox (1995). *Serious money: Fund-raising and contributing in presidential nomination campaigns*, Cambridge: Cambridge University Press.
- [10] Castanheira, M. (2003). Victory Margins and the Paradox of Voting, *European Journal of Political Economy*, 19: 817-841.
- [11] Chamon, M. and E. Kaplan (2013). The Iceberg Theory of Campaign Contributions: Political Treats and Interest Group Behavior, *American Economic Journal: Economic Policy*, 5(1):1-31.
- [12] Che, Yeon-Koo, and Ian Gale (1998a). Standard auctions with financially constrained bidders. *The Review of Economic Studies* 65(1): 1-21.
- [13] Che, Yeon-Koo, and Ian L. Gale (1998b). Caps on political lobbying. *The American Economic Review* 88(3): 643-651.
- [14] Claassen, R. (2007). Campaign Activism and the Spatial Model: Getting Beyond Extremism to Explain Policy Motivated Participation, *Political Behavior*, 29(3): 369-390.

- [15] Da Silveira, B. and J. De Mello (2011). Campaign Advertising and Election Outcomes: Quasi-natural Experiment Evidence from Gubernatorial Elections in Brazil, *Review of Economic Studies*, 78(2): 590-612
- [16] DellaVigna S., R. Durante, B. Knight, and E. La Ferrara (2016). Market-based Lobbying: Evidence from Advertising Spending in Italy, *American Economic Journal: Applied Economics*, forthcoming
- [17] Epstein, G. and S. Nitzan (2006). The Politics of Randomness, *Social Choice and Welfare*, 27: 423–433.
- [18] Erikson, R. and T. Palfrey (1998). Campaign Spending and Incumbency: An Alternative Simultaneous Equations Approach, *Journal of Politics*, 60(2): 355-373.
- [19] Erikson, R. and T. Palfrey (2000). Equilibria in Campaign Spending Games: Theory and Data, *American Political Science Review*, 94(3): 595-609.
- [20] Esteban, J. and D. Ray (1999). Conflict and Distribution, *Journal of Economic Theory*, 87: 379-415.
- [21] Esteban, J. and D. Ray (2001). Collective Action and the Group Size Paradox, *American Political Science Review*, 95(3): 663-672
- [22] Feddersen, T. and F. Gul (2015). Polarization and Income Inequality: A Dynamic Model of Unequal Democracy, *mimeo*.
- [23] Francia, P., P. Herrnson, J. Green, L. Powell and C. Wilcox (2003). *The Financiers of Congressional Elections: Investors, Ideologues, and Intimates*, New York: Columbia University Press.
- [24] Gerber, A. (2004). Does Campaign Spending Work?: Field Experiments Provide Evidence and Suggest New Theory, *American Behavioral Scientist*, 47: 541-574.
- [25] Gordon, S., C. Hafer, and D. Landa (2007). Consumption or Investment: On Motivations for Political Giving, *Journal of Politics* 69(4): 1057-1072.
- [26] Grossman, G., and E. Helpman (1994). Protection for Sale, *American Economic Review*, 84(4): 833-850.
- [27] Grossman, G., and E. Helpman (1996). Electoral Competition and Special Interest Politics, *Review of Economic Studies*, 63: 265-286.
- [28] Herrera, Helios, Massimo Morelli, and Thomas Palfrey (2014). Turnout and power sharing. *The Economic Journal*, 124(574).
- [29] Hirshleifer, J. (1989). Conflict and Rent-Seeking Success Functions; Ratio vs. Difference Models of Relative Success, *Public Choice* 63: 101-112.

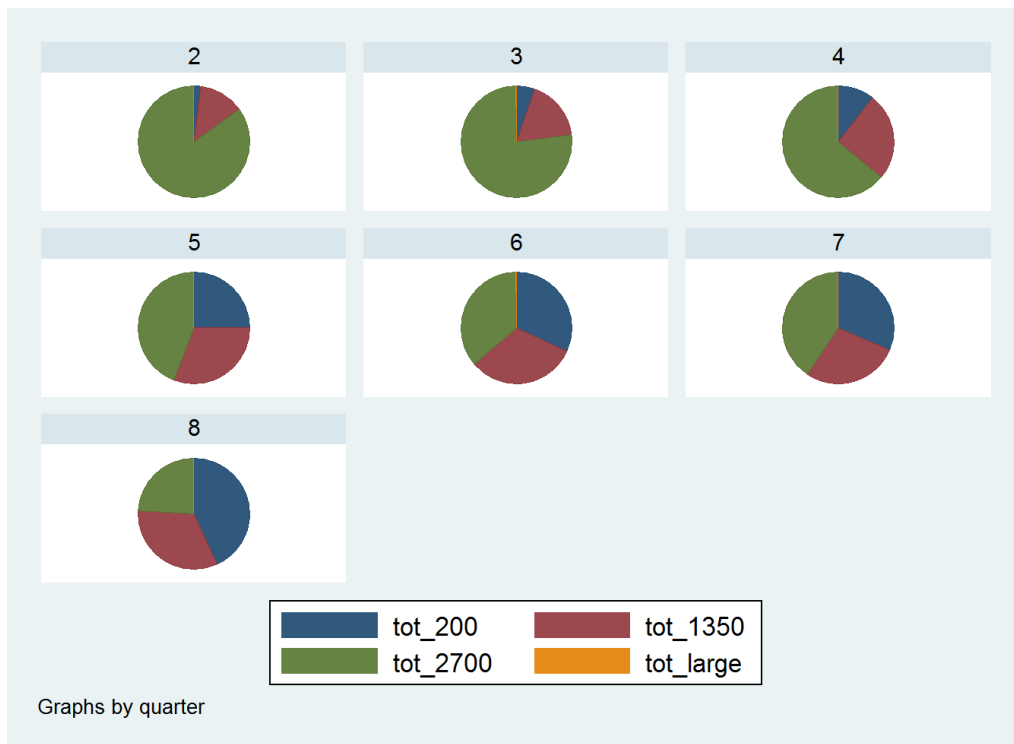
- [30] Jia, H., S. Skaperdas, and S. Vaidya (2013). Contest functions: Theoretical foundations and issues in estimation, *International Journal of Industrial Organization*, 31: 211-222.
- [31] Konrad, K. (2007), Strategy in Contests – an Introduction, *WZB-Markets and Politics Working Paper*, SP II 2007-01.
- [32] McCarty, Poole, and Rosenthal 2006;
- [33] Myatt, D. (2015). A Theory of Voter Turnout, *mimeo*.
- [34] M. Ohman (2012), *Political Finance Regulations around the World*, International Institute for Democracy and Electoral Institutions.
- [35] Palfrey, T. and H. Rosenthal (1985). Voter Participation and Strategic Uncertainty, *American Political Science Review*, 79: 62-78.
- [36] Prat, A. (2002). Campaign Advertising and Voter Welfare, *Review of Economic Studies*, 69(4): 999-1017.
- [37] Schuster, S. (2016). Does Campaign Spending Affect Election Outcomes? New Evidence from Transaction-Level Disbursement Data, *mimeo*.
- [38] Simon, H. (1954). Bandwagon and Underdog Effects and the Possibility of Elections Predictions, *Public Opinion Quarterly*, 18(3): 245-253.
- [39] Skaperdas, S. and B. Grofman (1995). Modeling Negative Campaigning, *American Political Science Review*, 89: 49–61.
- [40] Stratmann, T. (1992). Are Contributors Rational? Untangling Strategies of Political Action Committees, *Journal of Political Economy*, 100(3): 647-664
- [41] Tullock, G. (1980). Efficient Rent Seeking, In: Buchanan, J., R. Tollison, and G. Tullock (Eds.), *Toward a Theory of the Rent-Seeking Society*, Texas A&M University Press, College Station: 97–112.

8 Appendix

Appendix 1. Additional evidence about the distribution of contributions to Hillary Clinton

In the following graph, we subdivided contributions in 4 groups: $[0,200]$, $[201,1350]$, $[1351,2700]$, and $[2701,\infty]$. The first group are for contributions that do not require reporting. The legal limit to individual contributions are \$2700 per election (once for the primary and once for presidential election), which can thus reach a total of \$5400. Out of 3'471'316 contributions, we count three that are above that level, in addition to a number of regular transfers from other financial arms of the Hillary campaign, which we discarded from this analysis.

The quarter-per-quarter results of this decomposition are in the following graph:



One can see that, in 2015, the bulk of campaign finance was coming from contributions between 1350 and 2700. Contributions below \$201 represented respectively 2% and 5% of the total in Q2 and Q3. In 2016, instead, the sum of all contributions above 1350 fell to less than 50% of the total, and represented less than 25% of the flow of contributions by the end of the campaign. Instead, the total value of contributions smaller than \$201 end up representing 43% of the total in Q4 2016.

But the evolution of contributions over the campaign cycle clearly shows that, even if the contribution game may be one of face-to-face meetings and relatively large individual contributions early in the campaign, there is a shift to a more anonymized game among

an extremely large number of very small individual contributions (the 75th percentile of contributions drops from \$2700 in Q2 2015 to \$75 by Q1 2016) . It is hard to believe that, in this later phase, an individual donor's main motivation could be to influence the candidate's platform.

Appendix 2. Additional comparative statics

HETEROGENEOUS POPULAR SUPPORT

This appendix uses the results of Section 3.1. In most of the paper, we assume that the election probability of each party only depends on its campaign spending. This proxies a symmetric situation in which the two parties' popular support is symmetric, meaning that their probability of winning would be 1/2 if campaign spending were to drop to zero. Here, we show that the incentives and some properties we identified extend to the case in which popular support is asymmetric.

We capture the influence of each party's popular support on its probability of winning as a shifter $\varepsilon_P (> 0)$ in the contest success function:

$$\pi_P(\mathbf{Q}) \equiv \frac{(Q_P)^\gamma + \varepsilon_P}{(Q_A)^\gamma + (Q_B)^\gamma + \varepsilon_A + \varepsilon_B}.$$

In that case, the MEC becomes:

$$\pi'_P = \frac{\gamma}{Q_P} \left(\pi_A \pi_B - \frac{\varepsilon_P}{D} \right),$$

where $D = (Q_A)^\gamma + (Q_B)^\gamma + \varepsilon_A + \varepsilon_B$. It is thus essentially the same as in the base model, except for the fact that party- P donors' incentive to contribution is proportionately reduced by the popularity shock experienced by the party.

The underdog effect therefore applies in a complementary way:

$$\frac{\pi'_A}{\pi'_B} = \frac{Q_B}{Q_A} \cdot \frac{1 + \frac{\varepsilon_B}{Q_B^\gamma}}{1 + \frac{\varepsilon_A}{Q_A^\gamma}},$$

where the first factor is the same underdog effect as in the base model, and the second shows that a popularity boost for party A further reinforces the initial underdog effect. Conversely, a popularity boost for party B reduces it – and may reverse it if ε_B increases so much that it compensates B 's financial disadvantage (normalizing ε_A to zero, the condition becomes: $Q_A \geq Q_B (1 + \varepsilon_B/Q_B^\gamma)$).

THE INFLUENCE OF MONEY IN ELECTIONS

Proposition 1 also informs us of how the sensitivity of election outcomes with respect to campaign funding (as captured by the parameter γ) influences the total size of the campaign and the election probabilities:

Observation 3 *A higher γ may translate into costlier or cheaper campaigns ($Q_A^* + Q_B^*$ can be increasing or decreasing in γ). In all cases, a higher γ reinforces the advantage of A (π_A^* converges monotonically to 1 as $\gamma \rightarrow \infty$).*

One effect of election probabilities being more sensitive to contributions (a higher γ) is that it traps donors into a larger “arms race”. Both donor groups are compelled to contribute more, because the marginal effect of contributions increases. That effect always favors A because collecting funds is comparatively less costly for a . This, in turn, produces a second effect that works against the former: the increasing gap between A and B reduces the marginal effect of contributions: $\pi_A \pi_B$ in π'_P falls. This *reduces* the incentive to contribute. Which effect eventually dominates determines whether total contributions increase or decrease in γ .²⁷

THE INFLUENCE OF CLOSENESS ON CAMPAIGN SIZE

Observation 4 *Ceteris paribus, total campaign spending $Q_A^* + Q_B^*$ strictly decreases in W_A/W_B , and hence strictly increases in the closeness of the election, $\pi_A^* \pi_B^*$.*

Proof. Fix $W_A + W_B = \bar{W}$. For that case, we prove that:

$$\frac{d(Q_A^* + Q_B^*)}{dW_A} < 0, \quad \forall W_A > W_B.$$

Indeed:

$$Q_A^* + Q_B^* = \sqrt[\rho]{\gamma \omega W_A^{\rho-1}} + \sqrt[\rho]{\gamma \omega W_B^{\rho-1}} = \gamma^{1/\rho} \omega^{1/\rho} \left(W_A^{\frac{\rho-1}{\rho}} + W_B^{\frac{\rho-1}{\rho}} \right)$$

Hence:

$$\frac{\partial (Q_A^* + Q_B^*)}{\partial W_A} = \frac{1}{\rho} \gamma^{1/\rho} \omega^{\frac{1-\rho}{\rho}} \left(W_A^{\frac{\rho-1}{\rho}} + W_B^{\frac{\rho-1}{\rho}} \right) \frac{d\omega}{dW_A} + (\gamma \omega)^{1/\rho} \left(W_A^{\frac{-1}{\rho}} - W_B^{\frac{-1}{\rho}} \right),$$

where we used the fact that $dW_B/dW_A = -1$. From the definition of ω , the first term is zero for $W_A = W_B$ and strictly negative for $W_A > W_B$. The same holds for the second term. ■

²⁷A similar phenomenon occurs, for example, in “lobby competition” where expected outcomes depend on expenditures relative to competing lobbies. In particular, see Che and Gale (1998a and b). It also relates to the contest effect in Herrera *et al.* (2016).

Appendix 3. Proofs of the Propositions.

Proof of Proposition 1. We are focusing on pure strategies. Even when the pure strategy equilibrium does not exist, there must be a mixed strategy equilibrium, since payoff functions are continuous and bounded above. We are not interested in such MSE, because they are not realistic in our context.

Plugging (5) and (6) into (9) and (10), then taking the ratio between Q_A and Q_B shows that $\frac{Q_A}{Q_B} = \left(\frac{W_A}{W_B}\right)^{\frac{\rho-1}{\rho}}$ in a pure strategy equilibrium. We can therefore substitute for Q_B in (9), and solve for the equilibrium value of Q_A as a function of the exogenous parameters of the game, W_A , W_B , and γ :

$$\begin{aligned} Q_A &= W_A \times (\pi'_A)^{1/(\rho-1)} = W_A \times \left(\frac{\gamma}{Q_A} \times \frac{Q_A^\gamma}{Q_A^\gamma + Q_B^\gamma} \times \frac{Q_B^\gamma}{Q_A^\gamma + Q_B^\gamma} \right)^{1/(\rho-1)} \\ &= W_A \times \left(\frac{\gamma}{Q_A} \times \frac{Q_A^\gamma}{Q_A^\gamma + \left(Q_A (W_B/W_A)^{\frac{\rho-1}{\rho}}\right)^\gamma} \times \frac{\left(Q_A (W_B/W_A)^{\frac{\rho-1}{\rho}}\right)^\gamma}{Q_A^\gamma + \left(Q_A (W_B/W_A)^{\frac{\rho-1}{\rho}}\right)^\gamma} \right)^{\frac{1}{\rho-1}} \\ &= W_A \times \left(\frac{\gamma}{Q_A} \times \frac{\left((W_B/W_A)^{\frac{\rho-1}{\rho}}\right)^\gamma}{\left(1 + \left((W_B/W_A)^{\frac{\rho-1}{\rho}}\right)^\gamma\right)^2} \right)^{\frac{1}{\rho-1}} = W_A \times \left(\frac{\gamma}{Q_A} \times \omega \right)^{\frac{1}{\rho-1}} = \left(\gamma \omega W_A^{\rho-1} \right)^{\frac{1}{\rho}}. \end{aligned}$$

Q_B^* is derived following the same steps, and from the fact that $\frac{x^y}{(1+x^y)^2} = \frac{x^{-y}}{(1+x^{-y})^2}$. The latter implies that ω is identical for A and for B .

Second, equilibrium existence of a pure strategy equilibrium depends on the second order conditions being satisfied for this vector of total contributions. After some simplifications, the SOC for type- a donors can be expressed as:

$$-\gamma \frac{\pi_A^* \pi_B^*}{Q_A^2} (1 + \gamma (\pi_A^* - \pi_B^*)) < (\rho - 1) \frac{(q_A^i)^{\rho-2}}{(y^i)^\theta},$$

which is always satisfied since $\pi_A^* \geq \pi_B^*$. A similar condition must hold for b donors:²⁸

$$-\gamma \frac{\pi_A^* \pi_B^*}{Q_B^2} (1 + \gamma (\pi_B^* - \pi_A^*)) < (\rho - 1) \frac{(q_B^i)^{\rho-2}}{(y^i)^\theta}. \quad (18)$$

²⁸Second order condition amounts to looking at different points of the contest function for a and for b donors. Since a donors contemplate a higher winning probability than b , their SOC is automatically satisfied: they are in the concave part of the CSF. Instead, b donors may be in a spot in which the CSF is convex. That is, a slight decrease in their contribution base would also decrease their individual incentives to contribute. For sufficiently high values of γ , this would reinforce the drop in individual incentives so markedly that total contributions may be driven to 0. In that case, there is no pure strategy equilibrium. The proposition shows that this can never happen if γ is no larger than 2, or –for γ larger– if the contribution bases are not too asymmetric.

Noting that $\pi_A^* \pi_B^* = \omega$, we can rewrite this condition as follows:

$$\begin{aligned} \gamma \omega (\gamma (\pi_A^* - \pi_B^*) - 1) &< (\rho - 1) \frac{\left((y^i)^\theta \pi_B' \right)^{1 - \frac{1}{\rho-1}}}{(y^i)^\theta} Q_B^2 = (\rho - 1) \frac{(\pi_B')^{1 - \frac{1}{\rho-1}}}{(y^i)^{\frac{\theta}{\rho-1}}} (\gamma \omega)^{\frac{2}{\rho}} W_B^{\frac{2(\rho-1)}{\rho}} \\ \gamma \omega (\gamma (\pi_A^* - \pi_B^*) - 1) &< (\rho - 1) \frac{\left(\frac{\gamma \omega}{Q_B} \right)^{1 - \frac{1}{\rho-1}}}{(y^i)^{\frac{\theta}{\rho-1}}} (\gamma \omega)^{\frac{2}{\rho}} W_B^{\frac{2(\rho-1)}{\rho}} \\ \gamma \omega (\gamma (\pi_A^* - \pi_B^*) - 1) &< (\rho - 1) \frac{\left((\gamma \omega / W_B)^{\frac{\rho-1}{\rho}} \right)^{1 - \frac{1}{\rho-1}}}{(y^i)^{\frac{\theta}{\rho-1}}} (\gamma \omega)^{\frac{2}{\rho}} W_B^{\frac{2(\rho-1)}{\rho}} = (\rho - 1) \frac{\gamma \omega W_B}{(y^i)^{\frac{\theta}{\rho-1}}} \\ (\gamma - 1 \geq) \quad \gamma (\pi_A^* - \pi_B^*) - 1 &< (\rho - 1) \frac{\sum n^p f^p (y^j) (y^i)^{\frac{\theta}{\rho-1}}}{(y^i)^{\frac{\theta}{\rho-1}}} (> \rho - 1). \end{aligned}$$

This is automatically satisfied for $\rho \geq \gamma$ (since $\pi_A^* - \pi_B^* \leq 1$), and when $\pi_A^* - \pi_B^* \leq 1/\gamma$ for any other value of ρ and γ . ■

Proof of Lemma 1. From Proposition 1 and the definition of ω , we have:

$$Q_A^* = \frac{\sqrt{\gamma W_A^{\gamma/2+1} W_B^{\gamma/2}}}{W_A^{\gamma/2} + W_B^{\gamma/2}} \text{ and } Q_B^* = \frac{\sqrt{\gamma W_A^{\gamma/2} W_B^{\gamma/2+1}}}{W_A^{\gamma/2} + W_B^{\gamma/2}}$$

Taking derivatives and simplifying yields:

$$\frac{\partial Q_A^*}{\partial W_A} \propto W_A^{\gamma/2} (2 - \gamma) + W_B^{\gamma/2} (2 + \gamma) \text{ and } \frac{\partial Q_A^*}{\partial W_B} \propto W_A^{\gamma/2} - W_B^{\gamma/2}.$$

The latter always positive, and the former is necessarily positive for $\gamma \leq 2$. For $\gamma > 2$, we need to invoke the second order condition for equilibrium existence: we saw that for ν_h^b sufficiently large (one can actually check that the same holds for n_l^b and n_h^b large enough), it can be approximated by: $\pi_A^* - \pi_B^* < 1/\gamma$ (see (18), p38).

Now, the sign of $\frac{\partial Q_A^*}{\partial W_A}$ must be the same as that of:

$$\pi_A^* (2 - \gamma) + \pi_B^* (2 + \gamma) = 2(\pi_A^* + \pi_B^*) + \gamma(\pi_B^* - \pi_A^*),$$

since $W_P^{\gamma/2}$ is the numerator of π_P^* . Substituting the SOC in the second term shows that the latter cannot be smaller than -1 , whereas the former is equal to 2. Hence, $\frac{\partial Q_A^*}{\partial W_A}$ is always be positive when an equilibrium exists. Next,

$$\frac{\partial Q_B^*}{\partial W_B} \propto W_B^{\gamma/2} (2 - \gamma) + W_A^{\gamma/2} (2 + \gamma) \text{ and } \frac{\partial Q_B^*}{\partial W_A} \propto W_B^{\gamma/2} - W_A^{\gamma/2},$$

where the former is always positive and the latter always negative.

$$\frac{W_A^{\gamma/2}}{W_B^{\gamma/2}} (2 - \gamma) + (2 + \gamma) = \frac{\pi_A^*}{\pi_B^*} (2 - \gamma) + (2 + \gamma) > 0$$

■

Proof of Proposition 2. Using the effects of income on W_P in (11), follow the logic of the proof of Lemma 1. ■

Proof of Proposition 3. Remember that $W_P \equiv (v^p)^{\frac{1}{\rho-1}} n^p \sum_{i=1}^G f^p(y^i) \times (y^i)^{\frac{\theta}{\rho-1}}$. A mean-preserving spread of the income distribution is such that $\sum_{i < \bar{y}^p} \Delta f^p(y^i) \times y^i = - \sum_{i > \bar{y}^p} \Delta f^p(y^i) \times y^i$, where $\bar{y}^p$ is the subgroup with mean income in group p , and $\Delta f^p(y^i)$ is the change in density of each income class. If and only if $\frac{\theta}{\rho-1} > 1$, this implies that $\left| \sum_{i < \bar{y}^p} \Delta f^p(y^i) \times (y^i)^{\frac{\theta}{\rho-1}} \right| < \left| \sum_{i > \bar{y}^p} \Delta f^p(y^i) \times (y^i)^{\frac{\theta}{\rho-1}} \right|$ and hence that W_P increases. Applying the proof of Proposition 1 then demonstrates the result. ■

Proof of Proposition 4. Remember that $y^i \in [\underline{y}, \bar{y}]$ with $\underline{y} > 0$ and $\bar{y}$ positive and finite. In that case, there exist two cutoffs q_0 and q_1 for the cap on individual contributions $\bar{q}$, such that: $\forall \bar{q} > q_1$, no donor is constrained and $\forall \bar{q} < q_0$ all donors are constrained. By Proposition 1, for $\bar{q} > q_1$, the ratio of total contributions must be:

$$\frac{Q_A^*}{Q_B^*} = \sqrt{\frac{W_A}{W_B}} = \sqrt{\frac{n^a w^a}{n^b w^b}} = \sqrt{\frac{n^a}{n^b}},$$

and winning probabilities are the ones in Proposition 1. For $\bar{q} < q_0$, all donors contribute $\bar{q}$. Therefore, $Q_A = n^a \bar{q}$ and $Q_B = n^b \bar{q}$. The contribution ratio is thus $\frac{n^a}{n^b}$, and it is immediate to derive that A 's winning probability is then $\pi_A^0 = (n^a)^\gamma / ((n^a)^\gamma + (n^b)^\gamma)$.

For $\bar{q} \in (q_0, q_1)$, Q_A must always be strictly larger than Q_B , otherwise $q_A(y^i) \geq q_B(y^i)$, $\forall y^i$, with a set of income levels such that $q_A^i > q_B^i$, a contradiction. It follows that:

- (1) there is a (possibly empty) set of income levels y^i such that neither a nor b -donors are capped: $q_A^i < q_B^i$
- (2) there is a non-empty set of income levels y^i such that a -donors are uncapped and b -donors are capped: $q_A^i < q_B^i = \bar{q}$
- (3) there is a (possibly empty) set of income levels y^i such that both a and b -donors are capped, $q_A^i = q_B^i = \bar{q}$.

Parts (1) and (2) imply that $\pi_A(\bar{q})$ must be strictly less than π_A^0 . The fact that proportionately more b -donors than a -donors are capped when $\bar{q} > q_0$ implies that their joint contribution capacity is reduced more than a 's. This amounts to letting W_B drop because of a reduction in top b incomes. Following Proposition 1, this increases $\pi_A(\bar{q})$ above π_A^* . ■

Proof of Proposition 5. Define $y^{i,a} = \alpha y^{i,b}$, $\forall i = 1, \dots, G$. Remember that, for any two donors i and j who support the same candidate and are unconstrained by the cap, we must have:

$q^p(y^{i,p})/q^p(y^{j,p}) = (y^{i,p}/y^{j,p})^\theta$. The equilibrium is thus fully characterized by two income cutoff levels $\bar{y}^a(\bar{q})$ and $\bar{y}^b(\bar{q})$ and two “lowest contribution levels” $q^a(y^{1,a})$ and $q^b(y^{1,b})$ such that:

$$\begin{aligned} \text{for } y^{i,p} &< \bar{y}^p(\bar{q}), \quad q^p(y^{i,p}) = q^p(y^{1,p}) (y^{i,p}/y^{1,p})^\theta, \\ \text{for } y^{i,p} &> \bar{y}^p(\bar{q}), \quad q^p(y^{i,p}) = \bar{q}. \end{aligned}$$

First, we show that $q^a(y^{i,a}) > q^b(y^{i,b})$ for all unconstrained donors of some income group i , and hence that more a - than b -donors will be constrained. We prove this by contradiction: the only case in which the fraction of constrained a -donors could be smaller than that of b -donors would be if $\bar{y}^a(\bar{q}) > \alpha \bar{y}^b(\bar{q})$. This would require that $q^b(\bar{y}^b(\bar{q})) > q^a(\alpha \bar{y}^b(\bar{q})) = \alpha^\theta q^a(\bar{y}^b(\bar{q}))$, and thence $q^b(y^i) > \alpha^\theta q^a(y^i)$ for any $y^i < \bar{y}^b(\bar{q})$. But this leads to a contradiction: such contributions would aggregate into $Q_A(\bar{q}) < Q_B(\bar{q})$, which would produce best-response contributions $q^b(\bar{y}^b(\bar{q})) > q^a(\alpha \bar{y}^b(\bar{q}))$, because of free riding.

This establishes that $q^a(y^{i,a}) \geq q^b(y^{i,b})$ for all $i = 1, \dots, G$, and the inequality must be strict for some i . Then, following the same steps as for the proof of Proposition 4 leads to Proposition 5. ■

Proof of Proposition 6 . Applying the same logic as for the proof of Proposition 5, a reduction in Q_A , whether it is the result of a drop in v_h^a or of a legal constraint, must increase contributions q_h^b and q_l^b . The impact on winning probabilities follows immediately.

We use numerical simulations to prove the fact that total contributions may increase or decrease: consider the following example, again with $\gamma = \rho = 2$ and $\theta = 1$, two income groups and the same number of a - and b -donors at each level of income: $n_l^a = 30 = n_l^b$, and $n_h^a = 10 = n_h^b$. The difference with the previous examples is that the high-income a are *much* richer than the high-income b : $y_l^a = 10$, $y_h^a = 100$, $y_l^b = 1$, and $y_h^b = 10$. Figure ?? displays total contributions: one can readily see that relaxing a tight cap produces the expected effect of increasing total contributions ($Q_A + Q_B$). However, the effect is reversed for $\bar{Q} > 13.75$: it is then a *tightening* of the cap that increases total contributions. ■

Proof of Proposition 7. The *Marginal Effect of i 's Contribution to P* can now be written as (for $\varepsilon \rightarrow 0$):

$$\pi'_P = \frac{\gamma}{Q_P + s} \pi_A(\mathbf{Q}, s) \pi_B(\mathbf{Q}, s). \quad (19)$$

Thus, for any s , the two FOCs give:

$$\frac{Q_A}{Q_B} \frac{Q_A + s}{Q_B + s} = \frac{W_A}{W_B} (> 1) \quad (20)$$

This requires that $Q_A > Q_B$. Note also that $\left(\frac{Q_A + s}{Q_B + s}\right)^\gamma = \frac{\pi_A}{\pi_B} = \frac{\pi_A}{1 - \pi_A} (> 1)$, and hence that the former and the latter must move in the same direction as π_A . Note also that $\text{sign}\left(\frac{d\pi_A}{ds}\right) \neq$

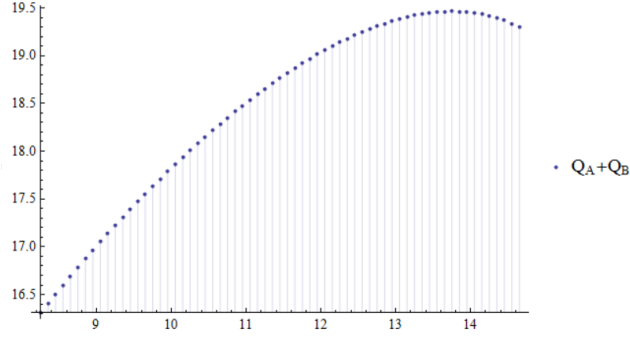


Figure 6: Simulated effect of a cap on total individual contributions when $y_l^a = 10$, $y_h^a = 100$, $y_l^b = 1$, and $y_h^b = 10$, and $n_l^a = 30 = n_l^b$, and $n_h^a = 10 = n_h^b$.

$\text{sign}\left(\frac{d\pi_A\pi_B}{ds}\right)$ since $\pi_A > 1/2$.

Now, we show that $\frac{dQ_A}{ds} < 0$ by contradiction. From (20), we have: $\frac{dQ_A}{ds} < 0 \Leftrightarrow \frac{dQ_{B+s}}{ds} > 0$ with:

$$\begin{aligned} \frac{dQ_{B+s}}{ds} &= \frac{(Q'_A + 1)(Q_B + s) - (Q_A + s)(Q'_B + s)}{(Q_B + s)^2}, \text{ and} \\ \frac{dQ_B}{ds} &= \frac{Q'_A Q_B - Q'_B Q_A}{(Q_B)^2} \end{aligned} \quad (21)$$

$\frac{dQ_B}{ds} < 0$ would impose:

$$Q'_A Q_B < Q'_B Q_A, \quad (22)$$

and we have two cases: (1) $Q'_B < 0$, which would then require that $Q'_A < 0$ as well (since $Q'_A < Q'_B \frac{Q_A}{Q_B} < 0$), and (2) $Q'_B > 0$, which would then require that $(0 \leq) Q'_A < Q'_B \frac{Q_A}{Q_B}$.

Case (1): by (21), $\frac{dQ_{B+s}}{ds} > 0$ iff

$$\underbrace{0}_{\text{by (22)}} > Q'_A Q_B - Q'_B Q_A > Q_A - Q_B + s(Q'_B - Q'_A)$$

To show the contradiction, we prove that the RHS is positive. Since $Q_A - Q_B > 0$, a SC is: $Q'_B > Q'_A$. By (22):

$$Q'_A < Q'_B \frac{Q_A}{Q_B},$$

which is thus more negative than Q'_B . Hence: $Q'_A < Q'_B \frac{Q_A}{Q_B} < Q'_B$.

Case (2): Remember that, by (19),

$$Q_B = \frac{\gamma W_B}{Q_B + s} \pi_A \pi_B.$$

Hence,

$$\frac{dQ_B}{ds} = -Q_B \frac{\frac{dQ_B}{ds} + 1}{Q_B + s} + \frac{\gamma W_B}{Q_B + s} \frac{d(\pi_A \pi_B)}{ds},$$

where the first term is necessarily negative when $\frac{dQ_B}{ds} > 0$, and so is the second term if $\frac{d(\pi_A \pi_B)}{ds} < 0$, i.e. if $\frac{d\pi_A}{ds} > 0$.

This contradicts that $\frac{d\pi_A}{ds}$ can be positive (or zero), for any value of $\frac{dQ_B}{ds}$. ■

Proof of Proposition 8. For $\varepsilon \rightarrow 0$, we can rewrite these total contributions as functions of the total contributions without the matching subsidies:

$$\tilde{Q}_P = (1 + m) \sum_{i=1}^{n^p} q_P^i = (1 + m) Q_P.$$

Plugging that into party P 's probability of winning the election, we get

$$\pi_P(\tilde{\mathbf{Q}}) \equiv \frac{((1 + m) Q_P)^\gamma}{((1 + m) Q_A)^\gamma + ((1 + m) Q_B)^\gamma} = \frac{Q_P^\gamma}{Q_A^\gamma + Q_B^\gamma} = \pi_P(\mathbf{Q}).$$

As a consequence, incentives, and therefore the equilibrium, are the same for any $m \leq 0$. ■

Proof of Proposition 9. With this tax, the cost of contributing q_P^i for a donor with income y^i becomes:

$$\left(q_P^i + \left[(y^i)^\theta - 1 \right] q_P^i \right)^\rho / \left[(y^i)^\theta \rho \right] = (q_P^i)^\rho / \rho.$$

■