Irreversibility, Complementarity, and the Dynamics of Public Good Provision*

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Abstract

We study public good provision and technology investment when two parties, with differing values of the public good, alternate in power stochastically. In each period, the incumbent decides on public good provision and technological investment that lowers future costs of provision. We obtain the following results for Markov Perfect equilibria of the model. (i) When polarization is low, the steady state distribution is unaffected by the degree of investment reversibility, but when polarization is high, the party favoring the public good invests more, with both parties providing more public good under irreversible investment. (ii) Higher turnover results in increased levels of technology stock and greater public good provision if polarization is low. (iii) Under high turnover, as parties become more polarized, the expected technology stock and public good provision decline initially, but jump upward as irreversibility starts to bind.

Keywords: irreversible investment; public good provision; complementarity; technology effect; resource effect; stochastic turnover; polarization

JEL Codes: C73, D78, H41

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1 Introduction

Policymakers working towards long-term goals are frequently confronted with the difficulty of ensuring the longevity of their policies, given the possibility of political turnover: when power changes hands, policymakers with different preferences will likely reverse unwanted policies that are easy to roll back. A recent instance is the Trump Administration's removal of numerous environmental regulations, subsequently reinstated when Biden took office.[1] When an incumbent takes into account the turnover in power, certain policies may become more appealing because they provide incentives for a future incumbent to adopt policies that are more consistent with the current incumbent's long-term goals.

We formalize these issues in the context of dynamic public good provision. Specifically, we examine a model in which two parties value a public good differently and take turns being in power stochastically. (We refer to the party who favors the public good as party H and the other as party L.) In each period, the incumbent party decides the level of public good to provide as well as the amount to invest in technology. The investment can take different forms required by technological innovation or infrastructural development, and is broadly referred to herein as technology investment. Though it does not deliver any immediate benefits, investment lowers the cost of public good provision in the future because of its complementarity with other inputs: it makes them more productive.

Many interesting questions arise: Since investment may be hard to reverse in many relevant applications, what effect does irreversibility have on public good provision? Does a more competitive political environment, reflected by a high turnover, raise or lower investment and public good provision? How does the difference in the parties' preferences regarding the public good (which we take as a measure of polarization) affect their policy choices in this dynmaic setting?

We answer these questions by studying properties of Markov Perfect equilibria in an infinite-horizon game. We find that each party has a target level of optimal technology stock which they aim to attain. When polarization is low, the parties' target levels are close, and each party will invest to replenish the (depreciated) stock to its own target level, irrespective of the reversibility or irreversibility of investment. As a result, the steady state distribution of technology stock and public good provision remains the same. When polarization is high, however, party L prefers to divest from party H's target stock level, even after depreciation. With irreversible investment, party L is unable to divest

[1] See *New York Times* reports by Popovich, Albeck-Ripka and Pierre-Louis (2021) and Friedman (2022).

and as a result, the technology stock depreciates to a level above its target. This in turn raises the marginal benefit of party H's investment, resulting in it investing more than if investment was reversible. As a result, both parties end up providing more public good in the steady state.

One might expect that the incumbent invests more when it is more likely to retain power. Our analysis indicates that this is not always the case. We identify two dynamic effects of investment: the *technology effect* and the *resource effect*. The technology effect arises from the fact that by investing more in technology, the incumbent reduces the cost of public good provision in the future, thereby incentivizing the next incumbent to increase its provision of the public good. This effect results in more investment under higher power turnover, provided that polarization is low. On the other hand, the resource effect arises from investment increasing the availability of resources in the future, which benefits the next incumbent and therefore leads to less investment under higher power turnover. When the depreciation rate is high, the resource effect is small and the technology effect dominates, resulting in more investment under higher power turnover.

We also uncover interesting implications of an increase in polarization. In a competitive environment characterized by frequent turnover, we find that as the divergence between the parties' preferences grows, the expected technology stock and public good provision decline initially, but jump upward as polarization reaches a threshold and irreversibility starts to bind. Surprisingly, an increase in polarization can enhance public good provision. This occurs because it motivates party H to make substantial investment in technology, capitalizing on its irreversibility. In the current highly polarized political environment in the United States (McCarty (2019)), this irreversibility may be a contributing factor to the Biden administration's focus on infrastructure investment and on technology development rather than relying on regulation as a means of addressing environmental challenges.[2]

Our analysis emphasizing irreversibility of technology investment and its complementarity with other policy instruments provides an explanation for the persistence of certain policies. For example, even though many environmental regulations were rolled back under the Trump administration, tax credits for renewable energy were extended. Our analysis suggests that one reason is that investment in renewable energy technologies such as the "sunshot initiative" by the U.S. Department of Energy has made it more attractive to continue government subsidy of the renewable energy sector.[3] This complements

[2]In an NPR interview, the EPA Administrator Regan emphasized "massive investments" and "evolution of technologies" in response to a question about irreversible policy making.

[3]For a description of the sunshot initiative, see this webpage of the U.S. Department of Energy.

explanations of policy persistence that involve responses from the private sector (see, for example, Coate and Morris (1999)). We discuss how we can modify our model to consider the interplay between public investment and private sector's actions in section 6.

We note that our model can be recast as a partnership game in which two partners take turns making a durable investment in as well as contributing costly effort to a joint project. By incorporating complementarity between investment and effort, our results provide new perspectives into the dynamic incentives of the partners when they lack commitment and may have varying values of their joint asset.[4] The irreversible nature of relationship-specific investment could encourage the partner with a greater value for the joint project to raise investment to motivate the other partner to intensify their efforts.

Related literature. Our paper contributes to several strands of literature. Unlike studies in the literature on dynamic public good provision that focus on either a non-durable or a durable public good with a constant technology (see, for example, Admati and Perry (1991), Marx and Matthews (2000), Compte and Jehiel (2003), Battaglini, Nunnari and Palfrey (2012, 2014)), we incorporate technology investment that reduces future costs of public good provision and thereby uncover (sometimes unexpected) new dynamics. In a closely related paper, Harstad (2020) also explores the implications of technology investment, but he considers a single decision maker who has time-inconsistent preferences,[5] and does not explicitly model interactions between strategic parties who have divergent preferences, which is the focus of our paper.

Although the importance of irreversibility has long been recognized and studied extensively for firms' capital investment decisions,[6] it is under-explored in problems of public good provision. One exception is Battaglini, Nunnari and Palfrey (2014), who analyze a setting with symmetric players and a constant technology and find that irreversibility of the players' contributions to a public good helps mitigate the free-rider problem, consistent with our result.[7] However, in their study, irreversibility is non-binding on

Also see *New York Times* report by Plumer (2017) on the Republican tax bill preserving incentives for renewable energy.

[4]In a widely-used framework introduced in Admati and Perry (1991), partners take turns contributing to a joint project, which is completed when the cumulative contributions reach some threshold. In our model, the partners' contributions are two-dimensional and they receive a flow payoff in each period.

[5]Harstad (2020) discusses hyperbolic discounting and political turnover as potential sources of time inconsistency.

[6]See Arrow (1968) for an investigation in the deterministic context and Dixit and Pindyck (1994) and the references therein for a comprehensive treatment of the stochastic case.

[7]Lockwood and Thomas (2002) study the effect of irreversibility in a dynamic prisoners' dilemma game where irreversibility means that the level of cooperation accumulates over time and does not depreciate. They find that irreversibility has a negative effect on welfare by making it harder to punish a deviation

the equilibrium path and does not give rise to new equilibria; instead, it eliminates less efficient equilibria. In contrast, with asymmetric players and an endogenously evolving technology, our analysis shows that irreversibility does bind on the equilibrium path when polarization is high, leading to qualitatively different equilibria in which the party favoring the public good invests more under irreversibility than under reversibility. Harstad (2023) also shows the importance of irreversibility in a dynamic political economy model, but in a different context of public resource extraction: since extraction is irreversible, the paper finds an asymmetry between a lobby paying for extraction and a donor compensating for conservation, which results in inefficient extraction when they compete.

Our paper also contributes to the literature on dynamic political economy with power turnover. One main insight of previous work is that the possibility of power turnover can motivate the incumbent to use certain policy instruments, such as debt, to constrain the future incumbent's policy choices, leading to inefficiencies (for pioneering papers, see Persson and Svensson (1989) and Alesina and Tabellini (1990); for examples of more recent work, see Battaglini and Coate (2008), Azzimonti (2011), Bouton, Lizzeri and Persico (2020), Foarta and Ting (2023), and Harstad (2023)). In contrast, we show that the strategic use of investment can actually reduce inefficiencies by lowering future cost and steering a future policymaker towards more public good provision. Several political economy models on public good provision have shown that lower turnover reduces distortions by enabling the current policymaker to better internalize the future consequences of their policy choices. This can happen when the public good is durable (Glazer (1989)) or when complementarity of inputs in public good provision is considered (Natvik (2013)). Our result presents a more nuanced picture by showing that the sign of the comparative statics depends on the degree of polarization and the corresponding relative magnitude of the technology effect and resource effect. Bowen, Chen and Eraslan (2014) also find that higher turnover raises public good provision when polarization is low, but through a different mechanism from ours: in their model, allocating more public goods through a mandatory program raises a party's future bargaining position if it loses power.

2 The Model

We consider a stylized economy and a political system with two parties labeled by H and L. Time is discrete and indexed by $t \geq 1$. We consider both finite and infinite time

from the efficient outcome. See Lee, Choi, Choi and Guéron (2023) for a recent experimental study on irreversibility in this setting.

horizons. The two parties alternate in power stochastically, following an exogenously given Markov process. In the initial period, one party is selected to be the incumbent with some exogenous probability and for any period t, the incumbent party in period t will continue to be in power with a probability $\pi \in [0,1]$ in period $t+1$. We refer to π as power persistence.[8] When $\pi = 1$, the incumbent party will be in power forever and when $\pi = 0$, the parties alternate in power.

In every period t, the incumbent party decides how much public good to provide and how much to invest in technology that will lower the cost of public good provision in the future. The initial stock of technology $k_0 \geq 0$ is exogenously given and technology depreciates at a rate $d \in [0,1]$. Thus, given the technology stock at the beginning of period t, k_{t-1}, and investment z_t, the technology stock at the end of period t is

$$k_t = (1-d)k_{t-1} + z_t.$$

The public good provided in period t, denoted by g_t, is an increasing function in a (nondurable) input y_t and k_{t-1}. We assume that $y_t \geq 0$ and for most of the paper, we consider the case in which investment is *irreversible*, that is, $z_t \geq 0$. We make a comparison to what happens if investment is *reversible*, in which case it is possible to have $z_t < 0$ and only $k_t \geq 0$ is required.

Denote the incumbent party by i and the non-incumbent party by j. The parties' utility functions in period t are given by

$$u_i(y_t, z_t, k_{t-1}) = v_i(y_t, k_{t-1}) - y_t - z_t = \alpha_i v(y_t, k_{t-1}) - y_t - z_t,$$
$$u_j(y_t, z_t, k_{t-1}) = v_j(y_t, k_{t-1}) = \alpha_j v(y_t, k_{t-1}),$$

where α_i and α_j are weights that the parties place on the public good and the incumbent incurs the cost of the nondurable input y_t and investment z_t.[9] We assume that $\alpha_H \geq \alpha_L >$

[8] Since the focus of our analysis is how political frictions affect the strategic choice of economic policies, we have assumed that power persistence is exogenously given to simplify the analysis. This assumption has been adopted in many related papers (for example, Alesina and Tabellini (1990), Acemoglu, Golosov and Tsyvinski (2011)). An interesting extension is to study the interplay between power turnover and economic policy. See Bai and Lagunoff (2011) for a study focusing on endogenous turnover and Parihar (2023) for an analysis that incorporates the effect of public good provision on reelection probabilities.

[9] For simplicity we did not explicitly introduce a budget in the model. An alternative formulation is to have a fixed budget in every period and the incumbent decides how much to allocate to a national public good, technology investment, and two local public goods (or transfers) each of which only one party derives utility from. In a Markov Perfect equilibrium, the incumbent allocates 0 to the other party's local public good. Hence, allocation to either the national public good or investment induces an opportunity cost only the incumbent incurs. The same results arise in this alternative formulation as in our model

0, that is, party H places at least as high a weight on the public good as party L does. We use $\rho \equiv \frac{\alpha_H}{\alpha_L}$ to parameterize *polarization* between the parties. A special case is when the two parties are symmetric ($\alpha_H = \alpha_L$), in which case we have the lowest polarization $\rho = 1$.[10] Further, the utility from the public good $v(y_t, k_{t-1})$ is specified to take the form of constant relative risk aversion

$$v(y_t, k_{t-1}) = \frac{(g(y_t, k_{t-1}))^\beta - 1}{\beta} = \frac{(y_t k_{t-1})^\beta - 1}{\beta},$$

where the coefficient of relative risk aversion is $(1 - \beta)$ and the amount of public good provided is $g_t = y_t k_{t-1}$. (Since it takes time for technology to develop and infrastructure to be built, investment in period t becomes productive in period $t + 1$). The function v, which is a composition of the production function of the public good and the utility from it, is increasing and strictly concave in y and k and satisfies the Inada condition with respect to y and k. Assume that $\beta > 0$, which implies that $\frac{\partial^2 v(y,k)}{\partial y \partial k} > 0$, reflecting the complementarity between y and k: a higher technology stock raises the marginal benefit of y. Also assume that $\beta < \frac{1}{2}$, a necessary condition for an interior solution to exist, as will become clear later in our analysis. Each party has a discount factor of δ and seeks to maximize its expected discounted sum of utilities.[11]

Even though the main interpretation we have provided for the model is that y_t is a nondurable input into the public good and k_{t-1} is the technology stock, there are other interpretations that may be suitable depending on the application at hand. One possible interpretation is that y_t is the labor input and k_{t-1} is the capital that goes into producing a public good; another is that y_t is a nondurable public good and k_{t-1} is a durable public good. When framed as a partnership game, we can think of k_{t-1} as the capital stock of the joint project and y_t as the effort contribution.

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provided that the budget is large enough. We discuss what happens with a small budget in section 6.

[10] There are various notions of polarization in dynamic political economy models, depending on the context. In Azzimonti (2011), polarization is parameterized by the (symmetric) weight that agents place on their local public good relative to private consumption; in Bouton, Lizzeri and Persico (2020), it is modelled as a decreased valuation for the public good compared to private goods; in Bowen, Chen and Eraslan (2014), it is parameterized by the ratio of the weights two parties place on the public good relative to private consumption, similar to our notion.

[11] In our model, the state variables consist of the technology stock and the identity of the incumbent. An alternative is to consider two public goods, each produced by a durable and a nondurable input and the parties place potentially different weights on the public goods. Though interesting to study, this alternative model is considerably less tractable since the state variables have an increased dimensionality.

3 Planner's Solution

We first consider a social planner who maximizes the sum of the two parties' payoffs over an infinite horizon.[12] Denote the value function for the planner by V_p. The optimization problem for the planner can be written as

$$V_p(k) = \max_{y,m} v_L(y,k) + v_H(y,k) + (1-d)k - m - y + \delta V_p(m),$$
$$\text{subject to } m \geq (1-d)k, y \geq 0,$$

where m is the technology stock for the next period. A standard argument shows that this problem has a solution. Let $y_p(k)$ be the planner's policy function regarding y. Given the Inada condition, $y_p(k) > 0$ for any $k > 0$ and therefore $y_p(k)$ satisfies $\frac{\partial v_L(y_p(k),k)}{\partial y} + \frac{\partial v_H(y_p(k),k)}{\partial y} = 1$, which implies

$$y_p(k) = (\alpha_L + \alpha_H)^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}.$$

Note that $y_p(k)$ is increasing in k, reflecting the complementarity between y and k.

Let $\bar{m}_p$ satisfy $\delta V_p'(\bar{m}_p) = 1$, which implies that

$$\bar{m}_p = (\alpha_L + \alpha_H)^{\frac{1}{1-2\beta}} \left[\frac{\delta}{1-\delta+\delta d}\right]^{\frac{1-\beta}{1-2\beta}}.$$

Note that $\bar{m}_p$ is constant in k, and it is the planner's target technology stock provided that investment in technology is positive. If $k > \frac{\bar{m}_p}{1-d}$, then the planner makes 0 investment. The following proposition summarizes the planner's optimal investment decision. (All proofs are relegated to the Appendix.)

Proposition 1. *If technology stock is below $\bar{m}_p/(1-d)$, the planner invests to reach $\bar{m}_p$; if it is above $\bar{m}_p/(1-d)$, the planner makes 0 investment.*

We illustrate the planner's investment policy function $m_p(k)$ in Figure 1. The solution has a unique steady state at $k = \bar{m}_p$ for any $d > 0$.[13] If $d = 0$, then any $k \geq \bar{m}_p$ is a steady state and $k = \bar{m}_p$ is the steady state reached in equilibrium starting from an initial stock

[12] A similar solution can be derived over a finite horizon, which we omit here to save space.

[13] For simplicity, we have assumed that the cost of investment is linear, which implies that we reach the steady state immediately if the initial stock is below $\frac{\bar{m}_p}{1-d}$, and we gradually reach the steady state through depreciation if the initial stock is above $\frac{\bar{m}_p}{1-d}$ and $0 < d < 1$. If we introduce convexity in the investment cost, then gradualism also occurs with a low initial stock.

lower than $\bar{m}_p$. If investment is reversible, then the planner's solution is to either invest or divest to reach $\bar{m}_p$, and the unique steady state is $k = \bar{m}_p$ for any $d \geq 0$. Note that if there is no power turnover, that is, if $\pi = 1$, then party i's optimal solution is analogous to the planner's solution with α_i replacing $\alpha_L + \alpha_H$.

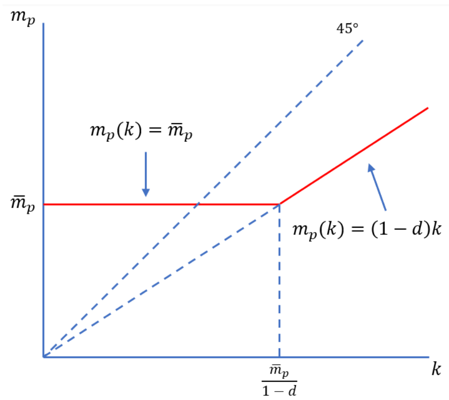

Figure 1: Planner's Solution

4 Three-period Game

We analyze the three-period game to gain some understanding of the forces at work. Our analysis focuses on irreversible investment and we also discuss the case of reversible investment for comparison. We solve for subgame perfect equilibria, and for expositional simplicity, we consider the case with no discounting ($\delta = 1$). Let y_{it} and z_{it} denote the amount of nondurable input and technology investment chosen by incumbent i in period t. Let $m_{it} = (1-d)k_{t-1} + z_{it}$ denote the technology stock chosen for the next period $t+1$.

Note that allocation to y is an intra-period problem, and the first order condition yields $y_{it}(k_{t-1}) = \alpha_i^{\frac{1}{1-\beta}} k_{t-1}^{\frac{\beta}{1-\beta}}$. We next consider the incumbent's investment decision. In the last period, since investment is irreversible, incumbent i always chooses $z_{i3} = 0$ in equilibrium. In the second period, in addition to affecting current payoffs, the incumbent's choices can have a dynamic effect because investment increases technology stock, which raises public good provision in the next period. Note, however, that the incumbent's investment decision in period 2 does not affect investment decisions in period 3. We find that incumbent i's equilibrium investment strategy in period 2 takes a simple form analogous to the planner's solution: there exists m_{i2}^* (the period-2 ideal level for incumbent i) such that if $k_1 \leq \frac{m_{i2}^*}{1-d}$, then $z_{i2}(k_1) = m_{i2}^* - (1-d)k_1$, implying that $m_{i2}(k_1) = m_{i2}^*$, and if $k_1 > \frac{m_{i2}^*}{1-d}$, then $z_{i2}(k_1) = 0$, implying that $m_{i2}(k_1) = (1-d)k_1$.

Turning now to period 1, note that the incumbent's investment decision not only affects the technology stock (and thus public good provision) in the next period but also potentially affects the investment decision of the next incumbent. We say that the irreversibility constraint does not bind in equilibrium when party i is the incumbent if for any k_0 such that $z_{i1}(k_0) > 0$, we have $(1-d)m_{i1}(k_0) \leq m^*_{j2}$ for $j \in \{L, H\}$. That is, irreversibility does not bind if the incumbent invests in period 1 to reach a level of technology stock that neither party wants to reverse in period 2. In this case, we find that there exists m^*_{i1} such that if $k_0 \leq \frac{m^*_{i1}}{1-d}$, then $m_{i1}(k_0) = m^*_{i1}$. Moreover, if investment is reversible, then incumbent i either invests or divests to reach m^*_{i1} in period 1. Conversely, we say that the irreversibility constraint binds in equilibrium when party i is the incumbent if for some k_0 such that $z_{i1}(k_0) > 0$, we have $(1-d)m_{i1}(k_0) > m^*_{j2}$ for some $j \in \{L, H\}$. That is, irreversibility binds if the incumbent makes a positive investment in period 1 to reach a level of technology stock that at least one party wants to reverse in period 2. The next proposition establishes that the irreversibility constraint does not bind in equilibrium when the depreciation rate is high enough but binds for party H when it is low enough. We say that the initial technology stock is low if $k_0 \leq \frac{m^*_{i1}}{1-d}$.[14]

Proposition 2. *If depreciation is sufficiently high, then irreversibility does not bind and the incumbent invests the same amount in the first period in equilibrium whether investment is irreversible or reversible for a low initial technology stock; if depreciation is sufficiently low, then irreversibility binds when party H is the incumbent and it invests a different amount when investment is irreversible than when reversible.*

To understand this result, it is useful to think of m^*_{i1} as incumbent i's period-1 target level when ignoring investment's potential effects on the stock level chosen by the next incumbent. If investment is reversible, the next incumbent will either invest or divest to reach its target level in period 2. In this case, the investment choice made by the incumbent in period 1 does not affect the next incumbent's chosen level of technology stock, making m^*_{i1} incumbent i's equilibrium choice. Under irreversibility, if depreciation is high, then $(1-d)m^*_{i1} < m^*_{j2}$ for any $i, j \in \{L, H\}$, implying that choosing m^*_{i1} will not prevent the next incumbent from reaching its target level of technology stock. Therefore the incumbent's equilibrium choice is still m^*_{i1} in period 1. If depreciation is low enough, however, this is no longer true. Specifically, for a sufficiently low d, we can show $(1-d)m^*_{H1} > m^*_{i2}$ for any $i \in \{L, H\}$, implying that choosing m^*_{H1} will prevent the next

[14] We consider a low initial stock to avoid having a binding irreversibility constraint due to the initial stock being high.

incumbent from reaching its target level. The binding irreversibility constraint in period 2 renders m^*_{H1} no longer optimal for party H in period 1.

In Proposition 2, we investigated how depreciation affects investment decisions in period 1, fixing the other parameters. Next, we turn to the effect of polarization. We find that even with high depreciation, irreversibility still binds when party H is the incumbent in the first period provided that polarization is sufficiently high. Specifically, suppose that neither party binds itself in equilibrium, that is, $(1-d)m^*_{i1} < m^*_{i2}$ for any $i \in \{L, H\}$, which holds when depreciation is high.[15] When polarization is high enough such that $(1-d)m^*_{H1} > m^*_{L2}$,[16] raising investment above m^*_{H1} will motivate party L to provide more public good if it comes in power in the next period while not binding party H itself should it continue to be in power. This results in party H investing more under irreversibility.

Proposition 3. *If polarization is sufficiently high, then party H invests more in the first period in equilibrium when investment is irreversible than when it is reversible.*

We next turn to the infinite-horizon game where we find certain parallels as in the three-period game. The infinite-horizon model enables us to explore long-term behavior in steady state distributions and investigate the implications of the comparative statics.

5 Infinite-horizon Game

We consider stationary *Markov Perfect equilibria* (Maskin and Tirole (2001)) for the infinite-horizon game. A Markov strategy depends only on payoff-relevant states, which are the technology stock k and the identity of the incumbent i in our model. Given the technology stock k, when party i is in power, it decides on the level of public good provision by choosing the amount of the input y and on the investment in the technology, z, which determines the technology stock in the following period, m. Hence, party i's Markov strategy $s_i = (y_i, m_i)$ consists of a pair of $y_i(k) : \mathbb{R}_+ \to \mathbb{R}_+$ and $m_i(k) : \mathbb{R}_+ \to \mathbb{R}_+$ such that $m_i(k) \geq (1-d)k$.

To each strategy profile $s = (s_L, s_H)$ and each party i, we associate functions $V_i(\cdot; s)$ and $W_i(\cdot; s)$ where $V_i(k; s)$ is party i's dynamic payoff when it is in power in the current period and $W_i(k; s)$ is party i's dynamic payoff when party $j \neq i$ is in power in the current period, when the technology stock is k and the strategy profile s will be played from the current period onward. We suppress the dependence of V_i and W_i on s from now on.

[15] This holds if $(1-d)^{\frac{1-2\beta}{1-\beta}} + \delta\pi(1-d) < 1$ in the three-period model. As shown in section 5, neither party binds itself in a Markov Perfect equilibrium in the infinite-horizon game.

[16] For any given $d < 1$, this inequality holds if polarization is sufficiently high.

A *Markov Perfect equilibrium* is described by a strategy profile s and the payoff functions (V_L, W_L, V_H, W_H) such that the following conditions hold:

E1. Given (V_L, W_L, V_H, W_H), strategy s_i is the solution to the following problem

$$\max_{y,m} \left\{ v_i(y,k) + (1-d)k - m - y + \delta \left[\pi V_i(m) + (1-\pi) W_i(m) \right] \right\},$$

subject to $y \geq 0,\ m \geq (1-d)k$.

E2. Given s, the payoff functions satisfy the following functional equations

$$\begin{aligned}
V_L(k) &= v_L(y_L(k),k) + (1-d)k - m_L(k) - y_L(k) + \delta \left[\pi V_L(m_L(k)) + (1-\pi) W_L(m_L(k)) \right], \\
V_H(k) &= v_H(y_H(k),k) + (1-d)k - m_H(k) - y_H(k) + \delta \left[\pi V_H(m_H(k)) + (1-\pi) W_H(m_H(k)) \right], \\
W_L(k) &= v_L(y_H(k),k) + \delta \left[\pi W_L(m_H(k)) + (1-\pi) V_L(m_H(k)) \right], \\
W_H(k) &= v_H(y_L(k),k) + \delta \left[\pi W_H(m_L(k)) + (1-\pi) V_H(m_L(k)) \right],
\end{aligned}$$

If investment is reversible, we define a Markov Perfect equilibrium in the same way except that in **E1**, we replace $m \geq (1-d)k$ with $m \geq 0$. In what follows, we refer to a Markov Perfect equilibrium as an equilibrium for simplicity.

5.1 Reversible Investment

As a benchmark, we first consider the case when investment is reversible. Let

$$m_i^* = \left(\frac{1}{\delta} - (1-d)\pi \right)^{\frac{1-\beta}{2\beta-1}} \alpha_i^{\frac{1-\beta}{1-2\beta}} \left(\pi \alpha_i^{\frac{\beta}{1-\beta}} + \frac{1-\pi}{1-\beta} \alpha_j^{\frac{\beta}{1-\beta}} \right)^{\frac{1-\beta}{1-2\beta}} < \bar{m}_p.$$

Proposition 4. *If investment is reversible, there exists an equilibrium such that $m_i(k) = m_i^*$ for any k and $y_i(k) = \alpha_i^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}$.*

The proposition says that the equilibrium takes a simple form with certain resemblance to the planner's solution: party i has a target level of technology stock m_i^* such that it invests or divests (depending on the current stock) to reach it in equilibrium. Note that m_i^* is lower than the socially efficient level with $m_L^* \leq m_H^* < \bar{m}_p$.[17] Allocation to the input y_i is increasing in the technology stock, stemming from their complementarity. Hence, a higher technology stock leads to a higher level of public good provision. In this equilibrium, the unique steady state distribution of technology stock is m_L^* and m_H^* each with probability $\frac{1}{2}$.

[17] We state the inequalities as a lemma and include its proof in the Online Appendix C.

We prove the proposition by the guess-and-verify method. In what follows, we provide some intuition for why equilibrium takes this simple form. From **E1**, the first order conditions are given by

$$\begin{aligned} \frac{\partial v_i(y_i(k),k)}{\partial y} &= 1, & \text{(FO1)} \\ \delta[\pi V_i'(m_i(k)) + (1-\pi)W_i'(m_i(k))] &= 1. & \text{(FO2)} \end{aligned}$$

Equation (FO1) says that the marginal benefit of y equals 1, implying that $y_i(k) = (\alpha_i k^\beta)^{\frac{1}{1-\beta}}$. Since incumbent i places weight α_i instead of $\alpha_L + \alpha_H$ on the public good as compared to the planner, a lower amount of resources is allocated to y for any k. Still, allocation to y is increasing in the technology stock.

Equation (FO2) says that the discounted marginal benefit of technology investment equals 1. The marginal benefit of investment has two parts: with probability π, the incumbent continues to stay in power next period and in this case, the marginal benefit of investment is $V_i'(m_i(k))$; with probability $1-\pi$, the other party comes in power in the next period and in this case, the marginal benefit of investment is $W_i'(m_i(k))$. Since the discounted marginal product of investment equals 1, it follows that $m_i(k)$ is constant in k when the first order condition holds. Together with the last two equations in **E2**, this implies

$$W_i'(k) = \frac{\partial v_i(y_j(k),k)}{\partial y} y_j'(k) + \frac{\partial v_i(y_j(k),k)}{\partial k} > 0.$$

This captures the strategic motive of investment: should incumbent i lose power, a higher technology stock will both motivate the other party to allocate more to y and make it more productive, thus raising the provision of the public good and player i's payoff.

The direct effect of investment in technology is captured by $V_i'(k)$, and by the Envelope Theorem (Benveniste and Scheinkman (1979)), we have

$$V_i'(k) = \frac{\partial v_i(y_i(k),k)}{\partial k} + (1-d) > 0.$$

An increase in investment works through two channels as well if the incumbent continues to be in power: a higher technology stock leads to a higher level of public good provision; additionally, the investment required to reach its target technology stock level at the next period is lower, which is reflected by the $(1-d)$ term. The values of m_i^* are obtained through the first order condition $\pi V_i'(m_i^*) + (1-\pi)W_i'(m_i^*) = \frac{1}{\delta}$.

5.2 Simple equilibrium: irreversibility does not bind

We now turn to the case of irreversible investment. We show that under certain conditions, there exists an equilibrium which shares many properties as the equilibrium characterized in Proposition 4 with reversible investment. In particular, it has the same steady state distribution, and in this sense, irreversibility does not bind under these conditions.

We call an equilibrium a *simple equilibrium* (SE) if the incumbent i's strategy satisfies

$$m_i(k) = \begin{cases} m_i^* & \text{if } 0 \leq k \leq \frac{m_i^*}{1-d} \\ (1-d)k & \text{if } k > \frac{m_i^*}{1-d} \end{cases},$$

$$y_i(k) = \alpha_i^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}.$$

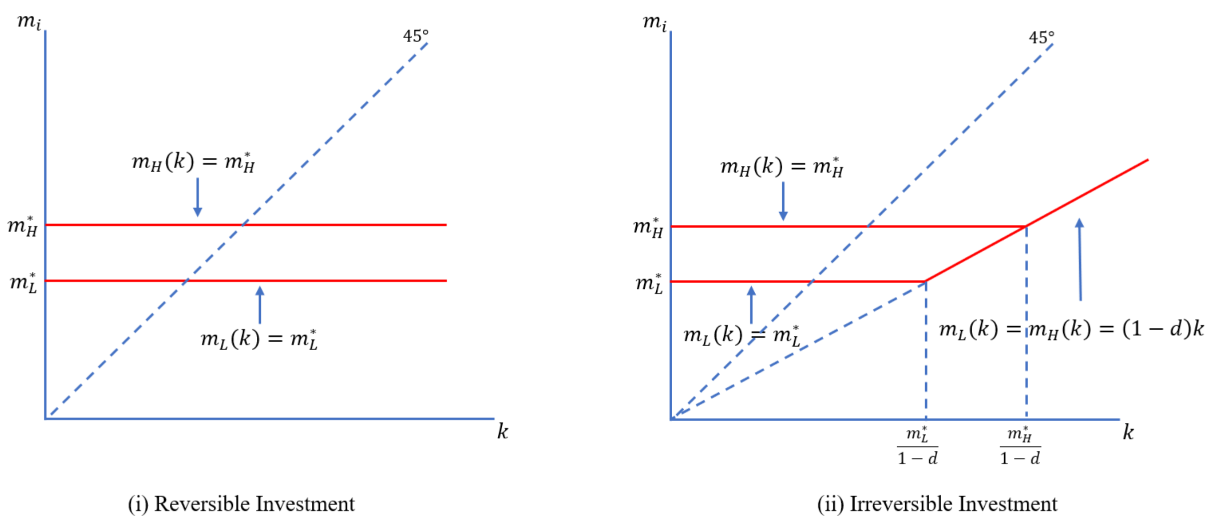

Figure 2: Equilibrium under Low Polarization

In this equilibrium, we still have $y_i(k) = \alpha_i^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}$. The main difference between a simple equilibrium and the equilibrium under reversibility is that now $m_i(k) = m_i^*$ only if the current stock is below $\frac{m_i^*}{1-d}$. This is because under irreversibility, the incumbent can invest but not divest. Hence, if k is high enough such that the next period's stock (without any investment) is already at or above its target level, then the incumbent invests zero and lets the stock depreciate. Still, in a simple equilibrium, the unique steady state distribution of k is m_L^* and m_H^* each with probability $\frac{1}{2}$, the same as under reversibility. The technology stock fluctuates between m_L^* and m_H^*, depending on the incumbent's identity. The parties' equilibrium investment strategies are illustrated in the right panel of Figure 2 (the left panel illustrates the reversible case). The following proposition establishes conditions under which a simple equilibrium exists under irreversibility.

Proposition 5. *With irreversible investment, a simple equilibrium exists if polarization is low enough and either depreciation or discounting is high enough.*

To gain some intuition for the conditions, recall that m_i^* satisfies the first order condition $\pi V_i'(m_i^*) + (1-\pi)W_i'(m_i^*) = \frac{1}{\delta}$. The derivations using the first order conditions assume that $m_i(m_j^*) = m_i^*$. This holds under reversibility since incumbent i either invests or divests to reach m_i^* for any k, including $k = m_j^*$. Under irreversibility, since $m_H^* \geq m_L^*$, this still holds when $i = H$; for $i = L$, this requires that $m_H^* \leq \frac{m_L^*}{1-d}$ because otherwise incumbent L invests 0 when $k = m_H^*$, resulting in $m_L(m_H^*) = (1-d)m_H^* > m_L^*$. Since $\frac{m_H^*}{m_L^*}$ is increasing in polarization ρ, this condition holds when polarization is low or depreciation is high.[18] Moreover, the first order conditions imply that m_i^* is locally optimal, but to ensure that it is globally optimal when $k \leq \frac{m_j^*}{1-d}$ requires more conditions, since the value function W_i is not globally concave given the policy functions. Global optimality is ensured if depreciation or discounting is sufficiently high.[19] Intuitively, under irreversibility, incumbent i potentially wants to raise investment beyond m_i^* to motivate the next incumbent to provide more public good, but this will not be optimal if depreciation or discounting is high enough. Indeed, in the extreme case of $d = 1$, there is no difference between reversible and irreversible investment since the technology depreciates completely after one period and there will be no stock remaining to reverse. Similarly, if discounting is high, then the incumbent is impatient and will not find it optimal to invest beyond m_i^*.

Comparative statics. We next investigate comparative statics of the technology stock levels in the steady state distribution, m_L^* and m_H^*. These results apply to equilibria under reversibility as well.

Proposition 6. *(i) For both $i \in \{L, H\}$, m_i^* is increasing in both α_L and α_H. (ii) If depreciation is sufficiently high, then m_H^* is decreasing in power persistence if polarization is low and m_L^* is decreasing in power persistence irrespective of polarization.*

Part (i) says that an incumbent's investment in technology is increasing in both parties' weights on the public good, implying that raising one party's weight on public good increases both parties' investment in equilibrium. Straightforwardly, increasing party i's weight on the public good strengthens party i's incentive to invest in technology. It also

[18]Specifically, $\frac{m_H^*}{m_L^*} = [(\pi\rho + \frac{1-\pi}{1-\beta}\rho^{\frac{1-2\beta}{1-\beta}})/(\pi\rho^{-\frac{\beta}{1-\beta}} + \frac{1-\pi}{1-\beta})]^{\frac{1-\beta}{1-2\beta}}$, which is increasing in ρ.

[19]High discounting means a low discount factor δ.

strengthens party j's incentive to invest since the technology will be put to better use when the opponent values the public good more.

Part (ii) discusses comparative statics with respect to power persistence, parameterized by π. One might expect that the incumbent invests more when it is more likely to hold on to power. Our result shows that this is not necessarily true. Intuitively, by investing more in technology, the incumbent raises the marginal benefit of y in the future and thereby motivates the next incumbent to provide more public good, which serves the role of consumption smoothing (the *technology effect*). This effect is stronger when the future incumbent is party H since it places a higher weight on the public good. Hence, if the current incumbent is party L, the technology effect results in more investment under higher power turnover; and this also holds if the current incumbent is party H provided that polarization is low.[20] Another effect of investment is to make more resources available in the future so that the next incumbent can make less investment to reach its target stock level (the *resource effect*). Since the resource effect benefits the current incumbent only if it continues to stay in power, this effect results in less investment under higher power turnover. When the depreciation rate is high, the resource effect is small and the technology effect dominates, resulting in higher investment under higher power turnover. In section 5.3, we will return to the question of how power turnover affects investment and show that the comparative statics in Proposition 6 are robust even with no depreciation.

Remark 1. *As β goes to 0, the complementarity between y and k disappears since $v(y,k)$ becomes $\ln(y)+\ln(k)$. In this limiting case, the optimal policy of y_i no longer depends on k and $m_i^* = (\frac{1}{\delta} - (1-d)\pi)^{-1}\alpha_i$ is increasing in π since investment does not motivate the future incumbent to allocate more resources to the public good. This implies that in a setting in which the incumbent provides a nondurable and a durable public good without any complementarities (or provides just one of them), the provision of the nondurable public good is independent of power turnover and the provision of the durable public good is decreasing in power turnover. The finding that higher power turnover can lead to more investment and a higher level of public good provision relies importantly on the complementarity between y and k.*

Another interesting question is how polarization affects investment and public good provision in equilibrium. Recall that in the unique equilibrium steady state distribution,

[20] Specifically, the condition on polarization being low is $\rho < (\frac{1}{1-\beta})^{\frac{1-\beta}{\beta}}$ when d goes to 1. Note that $(\frac{1}{1-\beta})^{\frac{1-\beta}{\beta}}$ is increasing in $(1-\beta)$, the coefficient of relative risk aversion (see the Online Appendix C for a proof). Since investment serves the purpose of consumption smoothing, the condition is easier to satisfy for a more risk-averse party H.

each party is in power with probability $\frac{1}{2}$ and $k = m_H^*$ with probability $\frac{1}{2}$ and $k = m_L^*$ with probability $\frac{1}{2}$. Hence, the expected technology stock is $\frac{m_L^*+m_H^*}{2}$ and expected level of public good is $\frac{1}{2}[(\pi\alpha_H^{\frac{1}{1-\beta}} + (1-\pi)\alpha_L^{\frac{1}{1-\beta}})(m_H^*)^{\frac{1}{1-\beta}} + (\pi\alpha_L^{\frac{1}{1-\beta}} + (1-\pi)\alpha_H^{\frac{1}{1-\beta}})(m_L^*)^{\frac{1}{1-\beta}}]$. To see the effects of polarization in a simple equilibrium, consider a mean-preserving spread of α_i, the weight placed on the public good by the incumbent.

Proposition 7. *As the parties become more polarized (in the sense of a mean-preserving spread of α_i), the expected steady state technology stock and level of public good provision in a simple equilibrium becomes higher under high power persistence and lower under low power persistence.*

The result implies that in a competitive political environment, polarization discourages the accumulation of technology stock and provision of public good on average in a simple equilibrium. We revisit this question in the next subsection and show that some surprising comparative statics results emerge when irreversibility binds.

5.3 Generalized simple equilibrium: irreversibility can bind

If $\frac{m_H^*}{m_L^*} > \frac{1}{1-d}$, which happens if depreciation is low enough or polarization is high enough, a simple equilibrium does not exist with irreversible investment. We next consider a broader class of equilibria. An equilibrium is a *generalized simple equilibrium* (GSE) if incumbent i's strategy satisfies

$$m_i(k) = \begin{cases} \hat{m}_i & \text{if } 0 \leq k \leq \frac{\hat{m}_i}{1-d} \\ (1-d)k & \text{if } k > \frac{\hat{m}_i}{1-d} \end{cases},$$

for some $\hat{m}_i$ with $\hat{m}_H \geq \hat{m}_L$, and $y_i(k)$ is the same as defined in a simple equilibrium. So a GSE has the same qualitative features as an SE but does not impose a specific value for $\hat{m}_i$. Analogous to the three-period game, we say that irreversibility does not bind in a GSE if $(1-d)\hat{m}_H \leq \hat{m}_L$ and binds if $(1-d)\hat{m}_H > \hat{m}_L$. That is, irreversibility does not bind if party H's target technology stock is such that party L does not want to divest from it and irreversibility binds otherwise.[21]

Proposition 8. *In a generalized simple equilibrium with irreversible investment, we have $\hat{m}_L = m_L^*$; moreover, $\hat{m}_H = m_H^*$ if irreversibility does not bind and $\hat{m}_H > m_H^*$ if irreversibility binds.*

[21] We focus on whether party L wants to divest from party H's (depreciated) target stock because neither party wants to divest from its own target stock and party H does not want to divest from party L's target stock in a GSE.

Proposition 8 implies that party L's optimal technology stock is the same whether investment is reversible or irreversible. Intuitively, party L will not bind a future incumbent no matter who comes into power and therefore it makes the same amount of investment as if it were reversible. However, if party H is the incumbent and irreversibility binds, then the marginal return of its investment is higher than when investment is reversible since party L will be motivated to provide more public good should it come into power in the future. This results in a higher target technology stock for party H.

Figure 3 illustrates an equilibrium when irreversibility binds. In this example, polarization is sufficiently high such that $(1-d)m_H^* > m_L^*$, as illustrated in panel (i), and a simple equilibrium does not exist when investment is irreversible. Instead, we have a generalized simple equilibrium illustrated in panel (ii): the target stock for party H exceeds m_H^* because the binding irreversibility constraint now provides extra incentives to invest.

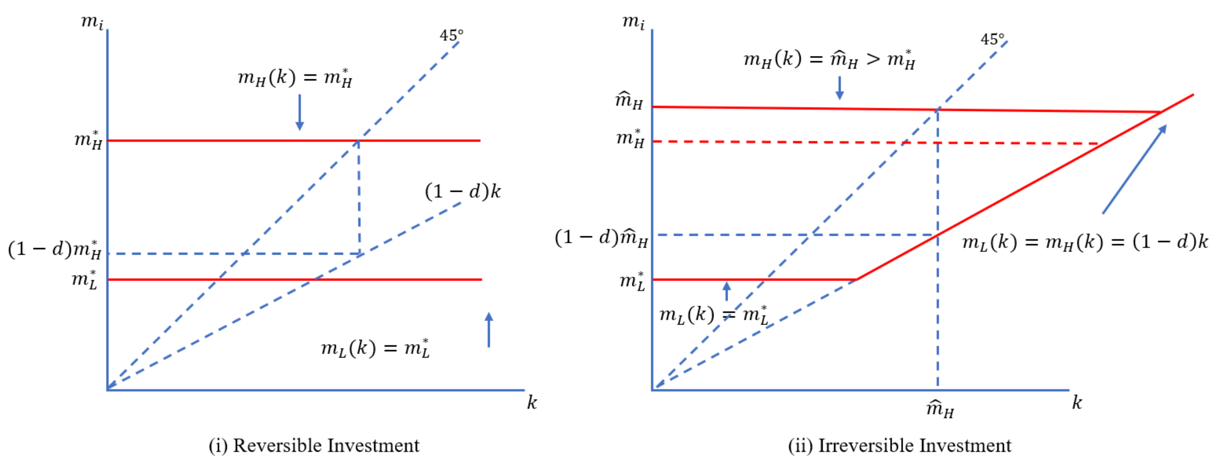

Figure 3: Equilibrium under High Polarization

5.3.1 Comparison of reversible and irreversible investment

We started our analysis of the infinite-horizon game with simple equilibria, in which the steady state distribution is invariant to whether investment is reversible or not. However, as shown in the preceding analysis, irreversibility can bind, resulting in a different steady state distribution than under reversibility. In this case, technology stock still fluctuates between m_L^* and m_H^* when investment is reversible, but when investment is irreversible, more than two levels of technology stock emerge in the steady state distribution and the levels of public good provision are also more variable.[22] This is because when party H is in power, it immediately sets the technology stock to its target $\hat{m}_H$, but party L does not make any investment if next period's stock is above its target m_L^*. Hence, the longer party L is in power, the lower the technology stock becomes, and a higher power turnover implies that the technology stock is less likely to be at party L's target level. We find that the expected steady state technology stock is higher when investment is irreversible, which is formally stated in the next proposition.

Proposition 9. *In a generalized simple equilibrium, the expected levels of technology stock and public good provision are higher in the steady state when investment is irreversible than when it is reversible, and strictly so when irreversibility binds.*

It is instructive to compare our findings to Battaglini, Nunnari and Palfrey (2014) (BNP for short), a paper that also investigates the effect of irreversibility on public good contribution in a dynamic setting. A finding common in both papers is that irreversibility helps mitigate the free-riding problem, but there are important differences. In BNP, irreversibility is not binding on the equilibrium path and it does not give rise to any new equilibrium but eliminates less efficient equilibria. In our model, irreversibility does bind on the equilibrium path for certain parameter ranges (characterized in the subsections that follow), resulting in qualitatively different equilibria then when investment is reversible. Our analysis highlights the importance of incorporating asymmetry in preferences in a fuller understanding of the effect of irreversibility in the dynamics of public good provision, a consideration absent in BNP since the players in their model are symmetric.

Providing a general closed-form characterization of GSE and its comparative statics is challenging. To gain additional insights into equilibrium properties when irreversibility binds, we study the cases of alternating power ($\pi = 0$) and no depreciation ($d = 0$).[23]

5.3.2 Alternating power

Suppose $\pi = 0$, that is, the parties alternate in control. In the three-period model, we have found that even under high depreciation, irreversibility binds if polarization is sufficiently high. We find parallels in the infinite-horizon game.

[22] The steady state distribution of technology stock is $\hat{m}_H$ with probability $\frac{1}{2}$, $(1-d)^n\hat{m}_H$ with probability $\frac{\pi^{n-1}(1-\pi)}{2}$ and m_L^* with probability $\frac{\pi^{n'}}{2}$ where $n' \geq 1$ is the largest integer satisfying $(1-d)^{n'}\hat{m}_H > m_L^*$. For $d = 0$, the steady state technology stock stays constant at $\hat{m}_H$.

[23] The basic idea of the characterization of those two cases is similar to the establishment of a simple equilibrium (Proposition 5), so we present the proofs for Propositions 10 and 11 in the Online Appendix A and B.

As discussed earlier, a necessary condition for the existence of a simple equilibrium is $\frac{m_H^*}{m_L^*} < \frac{1}{1-d}$. With $\pi = 0$, we have $\frac{m_H^*}{m_L^*} = \rho$. Hence, if $\rho > \frac{1}{1-d}$, simple equilibrium does not exist and irreversibility binds. We find that there is a threshold value of $\bar{\rho}$ such that if $\rho > \bar{\rho}$, a GSE exists with $\hat{m}_H > m_H^*$.

Proposition 10. *Let $\pi = 0$. With irreversible investment, a generalized simple equilibrium exists with $\hat{m}_H > m_H^*$ if polarization is high enough and either depreciation or discounting is high enough.*

Under high polarization, $\hat{m}_H > (1-d)m_L^*$, implying that in the steady state, party L makes no investment and just lets the stock depreciate, whereas party H makes investment $d \cdot \hat{m}_H$ to restore it to $\hat{m}_H$ when in power. Hence, the steady state distribution is that each party is in power with probability $\frac{1}{2}$ and $k = \hat{m}_H$ and $k = (1-d)\hat{m}_H$ each with probability $\frac{1}{2}$. Unlike in a simple equilibrium, party L's target level of technology stock no longer enters the steady state and as a result, only party H's target level matters. Note that $\hat{m}_H$ is increasing in both α_L and α_H, so increasing either party's weight on the public good still has a positive effect on the steady state technology stock level.

Effect of an increase in polarization. If polarization is sufficiently low, then we have a simple equilibrium, whose steady state distribution coincides with that under reversibility. In this case, an increase in polarization results in a lower expected technology stock in the steady state, as shown in Proposition 7 and illustrated by the left solid green curve (for $\rho \in [1, \tilde{\rho}]$) in Figure 4.

For sufficiently high polarization, irreversibility binds, resulting in higher expected technology stock than when reversible. The steady state now depends only on party H's target $\hat{m}_H$, illustrated by the right solid green curve (for $\rho > \bar{\rho}$) in Figure 4.[24] If polarization is in some intermediate range ($\tilde{\rho} < \rho < \bar{\rho}$), a GSE does not exist because in this case, it is optimal for party H to bind party L when the stock is high but not do so when stock is low. Numerical results show that the steady state expected technology stock is still $\frac{m_L^*+m_H^*}{2}$, coinciding with that under reversibility.[25]

Strikingly, the expected steady state technology stock jumps upward at the threshold $\bar{\rho}$ because $\hat{m}_H > m_H^*$ and $(1-d)\hat{m}_H > m_L^*$ when irreversibility binds. Hence, an increase in

[24] A further increase in polarization can have a non-monotonic effect on $\hat{m}_H$. Specifically, an increase in polarization raises $\hat{m}_H$ if ρ is below some threshold but lowers $\hat{m}_H$ otherwise. If this threshold is above $\bar{\rho}$, an increase in polarization can first raise and then lower expected technology stock in the steady state. See the Online Appendix A for details of this and a characterization of $\tilde{\rho}$.

[25] For ρ between $\tilde{\rho}$ and $\bar{\rho}$, we provide in the Online Appendix A a proof of the non-existence of a GSE and explain the qualitative features of equilibrium derived from the numerical analysis.

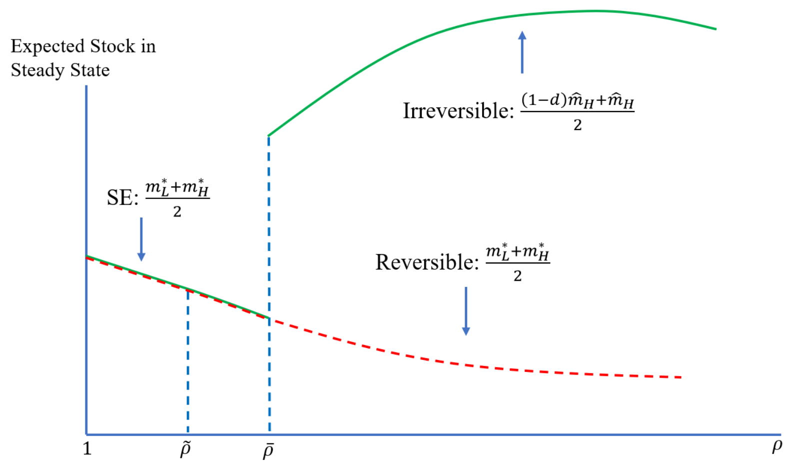

Figure 4: Polarization and Expected Steady State Technology Stock

polarization can raise the expected technology stock and public good provision discontinuously. Intuitively, an increase in polarization at $\bar{\rho}$ activates the irreversibility constraint, motivating party H to increase investment substantially so that a high technology stock ensures that public good provision does not fall too low should power changes hands.

With π fixed for the case of alternating power, the question of how equilibrium outcomes vary with power persistence does not arise. Next, we turn to the case of no depreciation and discuss the role of power persistence when irreversibility binds.

5.3.3 No depreciation

Suppose the technology stock does not depreciate ($d = 0$). As shown in the proposition below, irreversibility always binds irrespective of the degree of polarization, again parallel to what we have established in the three-period model.

Proposition 11. *Let $d = 0$. With irreversible investment, there exists a unique generalized simple equilibrium with $\hat{m}_L = m_L^*$ and $\hat{m}_H > m_H^*$. Moreover, $\hat{m}_H$ is decreasing in power persistence if and only if polarization is low.*

For any initial technology stock below $\hat{m}_H$, the steady state technology level is $\hat{m}_H$ regardless of which party is in power. This implies that only party H's optimal technology stock matters in the steady state.[26] We now revisit the role of power persistence when irreversibility binds. Because neither party makes any investment in the steady state, the

[26]Any $k > \hat{m}_H$ is also a steady state, but reaching it requires that we start from the same initial stock and is therefore not of much interest.

resource effect disappears as more investment today simply raises the technology stock without crowding out future investment. With only the technology effect remaining, we obtain a comparative static result analogous to that in a simple equilibrium: party H invests more under higher power turnover if polarization is low.[27]

6 Discussion

We conclude by discussing some variations, extensions, and directions for future research.[28]

Interaction between public and private sector. In our model, the incumbent chooses both technology investment and the level of public good provision. An interesting variant is to consider a public good being jointly produced by the public and the private sector.[29] Specifically, suppose the incumbent still chooses how much to invest in a technology but now the amount of the nondurable input is determined by a private sector. Similar to the original model, given complementarity, an increase in the technology stock prompts the private sector to allocate more input, thereby leading to a higher provision of the public good.[30] Assuming the private sector's preference for public good provision is not influenced by incumbent identity, this variant is similar to a setting in which the incumbent provides a durable public good. In this case, the incumbent's target level of technology stock no longer depends on the preference of the other party and higher power turnover reduces public good provision. However, the effect of irreversibility still remains: when polarization is high enough, irreversibility results in more investment by party H and more public good is provided in the steady state.

Partial reversibility. We have compared equilibrium outcomes of reversible and irreversible investments. An interesting question is what happens with partial reversibility. Suppose that when a party chooses $z < 0$ (divestment), it reduces the technology stock by $|z|$ and gains a fraction $r \in [0, 1]$ of the value of the divestment. The parameter r

[27]Indeed, the condition for polarization being low is $\rho < (\frac{1}{1-\beta})^{\frac{1-\beta}{\beta}}$, the same as the condition in a simple equilibrium when there is no resource effect.

[28]We provide formal results for the discussions in this section in the Online Appendix D.

[29]Ostrom, Ostrom and Savas (1977) discuss co-production of public goods where the users of public service function as co-producers since their efforts affect the value of the final product (examples include education and public health). Besley and Ghatak (2001) study public-private partnerships in the delivery of public goods. Additionally, several other papers study settings in which public investment enhances the productivity of a private sector in producing a private good (see, for example, Besley and Coate (1998) and Azzimonti (2015)). None of these papers addresses the issue of irreversibility.

[30]For example, infrastructure investment such as building a more extensive network of charging stations encourages switching to electric vehicles, thereby reducing emission.

represents the degree of reversibility: $r = 1$ corresponds to full reversibility and $r = 0$ corresponds to irreversibility.

Although a comprehensive analysis of partial reversibility is beyond the scope of our paper, some of our results can be readily extended to accommodate it. Recall that under full reversibility, each party i invests or divests to reach its optimal stock level m_i^* in equilibrium. In a simple equilibrium under irreversibility, each party i invests to reach m_i^* when the stock is below the threshold $\frac{m_i^*}{1-d}$ and invests 0 otherwise since divestment yields no return. Under partial reversibility, when the rate of depreciation is sufficiently high, an analogous equilibrium arises: each party i still invests to reach m_i^* when the stock is below the threshold $\frac{m_i^*}{1-d}$; now that each unit of divestment is worth $r \in (0,1)$, there is another local optimum, denoted by $m_i^{**} > m_i^*$, where the (discounted) marginal cost of divestment equals r. If the stock is above the threshold $\frac{m_i^{**}}{1-d}$, party i will divest to reach m_i^{**}. When the stock lies between the threshold $\frac{m_i^*}{1-d}$ and $\frac{m_i^{**}}{1-d}$, party i neither invests nor divests since the (discounted) marginal value of investment is below 1 and the (discounted) marginal cost of divestment exceeds r. Note that this results in the same steady state distribution of technology stock as in a simple equilibrium.

Cost sharing. In our paper, the incumbent bears all the opportunity costs of investment and public good provision. The comparative static results for the steady state in a simple equilibrium continue to hold if the incumbent bears a sufficiently large share of the cost. If the cost falls equally on both parties, the equilibrium outcome changes in two substantive ways. First, it is possible that party H's target technology stock and public good provision exceed the social optimum (planner's solution). Second, when polarization is sufficiently high, party L may have no incentives to invest in technology even when the stock is low.

Efficiency under a small budget. As discussed in section 2, We did not explicitly incorporate a budget constraint for expositional simplicity. Similar results arise in an alternative formulation in which the incumbent allocates a fixed budget to a national public good, two local public goods (or transfers) and investment in technology provided that the budget is large enough. What happens if the budget is small? Interestingly, with a sufficiently small budget, numerical results show that efficiency is achieved in equilibrium. This is because with a small budget, the planner does not allocate any resources to local public goods, and the same holds true for the incumbent in the dynamic game. Hence, the allocation problem becomes a tradeoff between provision of public good and investment. Since the parties's interests are aligned on this dimension of tradeoff, they make the same choice in equilibrium, which also coincides with the planner's solution.

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7 Appendix

Proof of Proposition 1: Consider the planner's optimization problem

$$\max_{y,m}(\alpha_L + \alpha_H)\frac{(yk)^\beta - 1}{\beta} + (1-d)k - m - y + \delta V_p(m)$$

subject to $m \geq (1-d)k$ and $y \geq 0$. The first order condition for y yields

$$y_p(k) = (\alpha_L + \alpha_H)^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}.$$

We establish the policy function concernin $m_p(\cdot)$ by the guess-and-verify approach. We postulate that planner makes the technology investment as stated in the proposition and obtain the slope of the value function as follows

$$V_p'(k) = \begin{cases} (\alpha_L + \alpha_H)^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + (1-d), & \text{for } k < \frac{\bar{m}_p}{1-d}, \\ (\alpha_L + \alpha_H)^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)V_p'((1-d)k), & \text{for } k > \frac{\bar{m}_p}{1-d}. \end{cases} \tag{1}$$

For $d > 0$, we have

$$V_p'(\bar{m}_p) = (\alpha_L + \alpha_H)^{\frac{1}{1-\beta}} \bar{m}_p^{\frac{2\beta-1}{1-\beta}} + (1-d) = \delta^{-1},$$

where the second equality follows from the definition of $\bar{m}_p$. Then, it is straightforward to show that V_p' is continuous at $k = \frac{\bar{m}_p}{1-d}$. Moreover, for $d = 0$, we have

$$\lim_{k \to \frac{\bar{m}_p}{1-d}+} V_p'(k) = \frac{(\alpha_L + \alpha_H)^{\frac{1}{1-\beta}} \bar{m}_p^{\frac{2\beta-1}{1-\beta}}}{1-\delta} = (\alpha_L + \alpha_H)^{\frac{1}{1-\beta}} \bar{m}_p^{\frac{2\beta-1}{1-\beta}} + 1 = \lim_{k \to \frac{\bar{m}_p}{1-d}-} V_p'(k),$$

where the second equality follows from the definition of $\bar{m}_p$. From (1), V_p is strictly concave on $(0, \frac{\bar{m}_p}{1-d})$ and we have shown that V_p' is continuous at $k = \frac{\bar{m}_p}{1-d}$. Using the recursive structure of V_p' (as in (1)), we can establish by induction that V_p is strictly concave on $(\frac{\bar{m}_p}{(1-d)^{n-1}}, \frac{\bar{m}_p}{(1-d)^n})$ and V_p' is continuous at $k = \frac{\bar{m}_p}{(1-d)^n}$ for any $n \in \mathbb{N}$, implying that V_p is strictly concave on $(0, \infty)$. Since $\delta V_p'(\bar{m}_p) = 1$, the optimal policy for the planner is to invest to reach $\bar{m}_p$ for $k < \frac{\bar{m}_p}{1-d}$ and makes no investment otherwise. ∎

Proof of Proposition 2: Allocation to y is an intra-period optimization problem, so irrespective of reversibility of investment, we have $y_{it}(k_{t-1}) = \alpha_i^{\frac{1}{1-\beta}} k_{t-1}^{\frac{\beta}{1-\beta}}$. We next consider the investment in technology. Denote the value function in period t for party i by V_{it} when it is the incumbent and by W_{it} when it is the non-incumbent.

First consider the case of reversible investment. For the last period (period 3), the incumbent divests the technology stock to zero by choosing $z_{i3}(k_2) = -(1-d)k_2$. Thus, the value functions are given by

$$V_{i3}(k_2) = \frac{1-\beta}{\beta} \alpha_i^{\frac{1}{1-\beta}} k_2^{\frac{\beta}{1-\beta}} + (1-d)k_2 \text{ and } W_{i3}(k_2) = \frac{1}{\beta} \alpha_i \alpha_j^{\frac{\beta}{1-\beta}} k_2^{\frac{\beta}{1-\beta}}.$$

Solving by backward induction, for period 2, we have

$$m_{i2}(k_1) = \left(\frac{1}{\delta} - (1-d)\pi\right)^{\frac{1-\beta}{2\beta-1}} \left(\pi \alpha_i^{\frac{1}{1-\beta}} + \frac{(1-\pi)\alpha_i}{1-\beta} \alpha_j^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}} \equiv m_i^*,$$

and the slopes of the value functions for period 2 are given by

$$V_{i2}'(k_1) = \alpha_i^{\frac{1}{1-\beta}} k_1^{\frac{2\beta-1}{1-\beta}} + (1-d) \text{ and } W_{i2}'(k_1) = \frac{1}{1-\beta} \alpha_i \alpha_j^{\frac{\beta}{1-\beta}} k_1^{\frac{2\beta-1}{1-\beta}}.$$

Given the (slopes of) the value functions, the optimization problem for the incumbent in period 1 is essentially the same, so we also have $m_{i1}(k_0) = m_i^*$.

We now turn to the case of irreversible investment. In period 3, since investment is irreversible, the incumbent i makes zero investment. Thus, the value functions are

$$V_{i3}(k_2) = \frac{1-\beta}{\beta}\alpha_i^{\frac{1}{1-\beta}}k_2^{\frac{\beta}{1-\beta}} \text{ and } W_{i3}(k_2) = \frac{1}{\beta}\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}k_2^{\frac{\beta}{1-\beta}}.$$

For period 2, we have

$$m_{i2}(k_1) = \delta^{\frac{1-\beta}{1-2\beta}}\left(\pi\alpha_i^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}} \equiv m_{i2}^*,$$

for $k_1 < \frac{m_{i2}^*}{1-d}$ and $m_{i2}(k_1) = (1-d)k_1$ for $k_1 \geq \frac{m_{i2}^*}{1-d}$. We obtain the first derivative of the value functions as follows

$$V_{i2}'(k_1) = \begin{cases} \alpha_i^{\frac{1}{1-\beta}}k_1^{\frac{2\beta-1}{1-\beta}} + (1-d), & \text{if } k_1 < \frac{m_{i2}^*}{1-d} \\ \alpha_i^{\frac{1}{1-\beta}}k_1^{\frac{2\beta-1}{1-\beta}} + \delta\alpha_i(1-d)^{\frac{\beta}{1-\beta}}\left[\pi\alpha_i^{\frac{\beta}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_j^{\frac{\beta}{1-\beta}}\right]k_1^{\frac{2\beta-1}{1-\beta}} & \text{if } k_1 > \frac{m_{i2}^*}{1-d}, \end{cases}$$

$$W_{i2}'(k_1) = \begin{cases} \frac{1}{1-\beta}\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}k_1^{\frac{2\beta-1}{1-\beta}}, & \text{if } k_1 < \frac{m_{j2}^*}{1-d} \\ \frac{1}{1-\beta}\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}k_1^{\frac{2\beta-1}{1-\beta}} + \delta\alpha_i(1-d)^{\frac{\beta}{1-\beta}}\left[\frac{\pi}{1-\beta}\alpha_j^{\frac{\beta}{1-\beta}} + (1-\pi)\alpha_i^{\frac{\beta}{1-\beta}}\right]k_1^{\frac{2\beta-1}{1-\beta}} & \text{if } k_1 > \frac{m_{j2}^*}{1-d}. \end{cases}$$

The optimization problem in period 1 is more complicated because the value functions in period 2 are piece-wise strictly concave but are not necessarily globally concave. For simplicity, let $k_0 = 0$. The first order condition $\delta[\pi V_{i2}'(m_{i1}^*) + (1-\pi)W_{i2}'(m_{i1}^*)] = 1$ yields potentially three local optima: (i) m_i^* if $m_i^* < \frac{m_{L2}^*}{1-d}$; (ii) $m_{i1}^{*'}$ which is given by

$$\begin{aligned} m_{H1}^{*'} &\equiv \left(\frac{1}{\delta} - (1-d)\pi\right)^{\frac{1-\beta}{2\beta-1}}\left[\pi\alpha_H^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_H\alpha_L^{\frac{\beta}{1-\beta}} + \right. \\ &\quad \left. (1-\pi)\delta\alpha_H(1-d)^{\frac{\beta}{1-\beta}}\left(\frac{\pi}{1-\beta}\alpha_L^{\frac{\beta}{1-\beta}} + (1-\pi)\alpha_H^{\frac{\beta}{1-\beta}}\right)\right]^{\frac{1-\beta}{1-2\beta}} \\ m_{L1}^{*'} &\equiv \delta^{\frac{1-\beta}{1-2\beta}}\left[\pi\alpha_L^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_L\alpha_H^{\frac{\beta}{1-\beta}} + \pi\delta\alpha_L(1-d)^{\frac{\beta}{1-\beta}}\left(\pi\alpha_L^{\frac{\beta}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_H^{\frac{\beta}{1-\beta}}\right)\right]^{\frac{1-\beta}{1-2\beta}}, \end{aligned}$$

if $\frac{m^*_{L2}}{1-d} < m^{*'}_{i1} < \frac{m^*_{H2}}{1-d}$; (iii) $m^{*''}_{i1}$ which is given by

$$\begin{aligned} m^{*''}_{i1} \;\equiv\; & \delta^{\frac{1-\beta}{1-2\beta}} \Bigg[\pi\alpha_i^{\frac{1}{1-\beta}} + \frac{(1-\pi)\alpha_i}{1-\beta}\alpha_j^{\frac{\beta}{1-\beta}} + \pi\delta\alpha_i(1-d)^{\frac{\beta}{1-\beta}}\left(\pi\alpha_i^{\frac{\beta}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_j^{\frac{\beta}{1-\beta}}\right) \\ & + (1-\pi)\delta\alpha_i(1-d)^{\frac{\beta}{1-\beta}}\left(\frac{\pi}{1-\beta}\alpha_j^{\frac{\beta}{1-\beta}} + (1-\pi)\alpha_i^{\frac{\beta}{1-\beta}}\right)\Bigg]^{\frac{1-\beta}{1-2\beta}}, \end{aligned}$$

if $m^{*''}_{i1} > \frac{m^*_{H2}}{1-d}$. It is straightforward to show that $\lim_{d\to 1} m^*_i = \lim_{d\to 1} m^{*'}_{i1} = \lim_{d\to 1} m^{*''}_{i1} = m^*_{i2}$ whereas $\lim_{d\to 1} \frac{m^*_{i2}}{1-d} = \infty$. Thus, for d sufficiently large, we have $m^{*'}_{i1} < \frac{m^*_{L2}}{1-d}$ and $m^{*''}_{i1} < \frac{m^*_{L2}}{1-d}$, ruling out the optimality of $m^{*'}_{i1}$ and $m^{*''}_{i1}$ and implying that m^*_i is the global optimum. Note the argument above holds for any $k_0 < \frac{m^*_i}{1-d}$. For a low initial stock, the period-1 optimum $m^*_{i1} = m^*_i$, irrespective of whether investment is reversible or irreversible.

When d is sufficiently small, we claim that $m^*_H \geq \frac{m^*_{H2}}{1-d} \geq \frac{m^*_{L2}}{1-d}$, which renders m^*_H no longer (locally) optimal for party H. To see this, we have $m^*_H \geq \frac{m^*_{H2}}{1-d}$ if and only

$$f(d) \equiv (1-d) - (1-\delta\pi(1-d))^{\frac{1-\beta}{1-2\beta}} \geq 0.$$

It is straightforward to show that $f(0) \geq 0$, $f(1) < 0$, and $f'(d) < 0$ for $d \in [0,1]$, so there exists $\bar{d} \in [0,1)$ (note $\bar{d} = 0$ if and only if $\pi = 0$) such that $f(\bar{d}) = 0$ and $f(d) \geq 0$ if and only if $d \leq \bar{d}$. Thus, if $d \leq \bar{d}$, $m^*_H \geq \frac{m^*_{H2}}{1-d}$. Note that $m^{*'}_{H1} > m^*_H$ and $m^{*''}_{H1} > m^*_H$ for any $d < 1$. Then, when d is sufficiently small, party H's period-1 target stock under reversible investment m^*_H is no longer optimal when investment is irreversible.[31] ■

<u>**Proof of Proposition 3:**</u> As we have shown in the proof of Proposition 2, there exists $\bar{d} \in [0,1)$ such that $(1-d) - (1-\delta\pi(1-d))^{\frac{1-\beta}{1-2\beta}} < 0$, or equivalently, $m^*_H < \frac{m^*_{H2}}{1-d}$, if and only if $d > \bar{d}$. Let $d \in (\bar{d}, 1)$.

Following the derivations in the proof of Proposition 2, we can further rewrite $m^*_H > \frac{m^*_{L2}}{1-d}$ explicitly as

$$\frac{\pi\rho + \frac{1-\pi}{1-\beta}\rho^{\frac{1-2\beta}{1-\beta}}}{\pi\rho^{-\frac{\beta}{1-\beta}} + \frac{1-\pi}{1-\beta}} > \frac{1-\delta\pi(1-d)}{(1-d)^{\frac{1-2\beta}{1-\beta}}},$$

which holds for ρ sufficiently large. Pick ρ large enough such that the inequality above holds. Then, $m^*_H > \frac{m^*_{L2}}{1-d}$. It follows from the previous proof that the period-1 optimal level of technology stock for party H is either $m^{*'}_{H1}$ or $m^{*''}_{H1}$. For $d < 1$, $m^{*'}_{H1} > m^*_H$ and

[31] The same argument applies to party L except for $\pi = 0$, because when $\pi = 0$, $m^{*'}_{L1} = m^*_L$ so $m^*_L > \frac{m^*_{L2}}{1-d}$ is not sufficient to rule out the optimality of m^*_L.

$m_{H1}^{*''} > m_H^*$. Thus, we have obtained the desired conclusion. ■

Proof of Proposition 4: It is straightforward to establish the optimality of $y_i(\cdot)$. Based on the policy function $m_i(k) = m_i^*$, we construct the value functions as follows

$$V_i(k) = \frac{1-\beta}{\beta}\alpha_i^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}} + (1-d)k + c_{Vi} \text{ and } W_i(k) = \frac{1}{\beta}\alpha_i\alpha_j^{\frac{\beta}{1-\beta}} k^{\frac{\beta}{1-\beta}} + c_{Wi}.$$

where c_{Vi} and c_{Wi} are constant in k. All the value functions are strictly concave and

$$V_i'(k) = \alpha_i^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + (1-d) \text{ and } W_i'(k) = \frac{1}{1-\beta}\alpha_i\alpha_j^{\frac{\beta}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}}.$$

Under reversible investment, next period's stock m satisfies $\delta[\pi V_i'(m)+(1-\pi)W_i'(m)] = 1$, which admits a unique solution m_i^*. We have obtained the desired conclusion. ■

Proof of Proposition 5: We first show that a simple equilibrium exists if the following conditions are satisfied:

$$\frac{m_H^*}{m_L^*} = \left(\frac{\pi\rho + \frac{1-\pi}{1-\beta}\rho^{\frac{1-2\beta}{1-\beta}}}{\pi\rho^{-\frac{\beta}{1-\beta}} + \frac{1-\pi}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}} < \frac{1}{1-d}, \tag{2}$$

$$\frac{1}{(1-\beta)\pi\rho^{\frac{\beta}{1-\beta}} + (1-\pi)} - \frac{(\rho(1-d))^{\frac{1-2\beta}{1-\beta}} + \delta(1-d)\pi\rho^{\frac{1-2\beta}{1-\beta}} + \delta(1-d)(1-\pi)(1-\beta)\rho}{(1-\beta)\pi\rho^{-\frac{\beta}{1-\beta}} + (1-\pi)} > \frac{\delta(1-d)^2(1-\pi)}{\frac{1}{\delta} - (1-d)\pi}, \tag{3}$$

$$\frac{\left(1 - \delta(1-d)\pi - (1-d)^{\frac{1-2\beta}{1-\beta}}\right)\rho^{\frac{2\beta-1}{1-\beta}} - \delta(1-d)(1-\pi)(1-\beta)\rho^{-1}}{(1-\beta)\pi\rho^{\frac{\beta}{1-\beta}} + (1-\pi)} > \frac{\delta(1-d)^2(1-\pi)}{\frac{1}{\delta} - (1-d)\pi}, \tag{4}$$

From (2), $(1-d)m_H^* < m_L^*$. Since $m_H^* \geq m_L^*$, $d > 0$. It is straightforward to show that in any equilibrium, $y_i(k) = \alpha_i^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}$. We then take a guess and verify approach. Based on the strategy profile of a simple equilibrium, we have

$$V_i'(k) = \begin{cases} \alpha_i^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + (1-d), & k < \frac{m_i^*}{1-d} \\ \alpha_i^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi V_i'((1-d)k) + (1-\pi)W_i'((1-d)k)\right], & k > \frac{m_i^*}{1-d} \end{cases} \tag{5}$$

$$W_i'(k) = \begin{cases} \frac{\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}}, & k < \frac{m_j^*}{1-d} \\ \frac{\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi W_i'((1-d)k) + (1-\pi)V_i'((1-d)k)\right], & k > \frac{m_j^*}{1-d} \end{cases} \tag{6}$$

We now verify that each party's strategy is indeed optimal under the value functions with the first derivatives derived above.

First consider party H. From (5) and (6), it is clear that V'_H and W'_H are continuous and strictly decreasing on $(0, \frac{m^*_L}{1-d})$. We now show that V'_H and W'_H are continuous and strictly decreasing on $(\frac{m^*_L}{(1-d)^n}, \frac{m^*_L}{(1-d)^{n+1}})$ for any $n \in \mathbb{N}$. Suppose $n = 1$. Since $(1-d)m^*_H < m^*_L$, $\frac{m^*_H}{1-d} < \frac{m^*_L}{(1-d)^2}$. For $k \in (\frac{m^*_L}{1-d}, \frac{m^*_H}{1-d})$, $V'_H(k) = \alpha_H^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + (1-d)$. For $k \in (\frac{m^*_H}{1-d}, \frac{m^*_L}{(1-d)^2})$,

$$\begin{aligned} V'_H(k) &= \alpha_H^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi V'_H((1-d)k) + (1-\pi)W'_H((1-d)k)\right] \\ &= \alpha_H^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi(1-d) + \left(\pi\alpha_H^{\frac{1}{1-\beta}} + (1-\pi)\frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}\right)((1-d)k)^{\frac{2\beta-1}{1-\beta}}\right], \end{aligned}$$

where the second equality follows from $(1-d)k < \frac{m^*_L}{1-d} \leq \frac{m^*_H}{1-d}$. Thus,

$$\begin{aligned} \lim_{k\to\frac{m^*_H}{1-d}+} V'_H(k) &= \alpha_H^{\frac{1}{1-\beta}}\left(\frac{m^*_H}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi(1-d) + \left(\pi\alpha_H^{\frac{1}{1-\beta}} + (1-\pi)\frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}\right)(m^*_H)^{\frac{2\beta-1}{1-\beta}}\right] \\ &= \alpha_H^{\frac{1}{1-\beta}}\left(\frac{m^*_H}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + (1-d) = \lim_{k\to\frac{m^*_H}{1-d}-} V'_H(k), \end{aligned}$$

where the second equality follows from $(m^*_H)^{\frac{2\beta-1}{1-\beta}} = \left(\frac{\delta}{1-\delta\pi(1-d)}\right)^{-1}\left(\pi\alpha_H^{\frac{1}{1-\beta}} + (1-\pi)\frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}\right)^{-1}$. Since V'_H is continuous and strictly decreasing on $(\frac{m^*_L}{1-d}, \frac{m^*_H}{1-d})$ and $(\frac{m^*_H}{1-d}, \frac{m^*_L}{(1-d)^2})$, and V'_H is continuous at $\frac{m^*_H}{1-d}$ as shown above, V'_H is continuous and strictly decreasing on $(\frac{m^*_L}{1-d}, \frac{m^*_L}{(1-d)^2})$. Further, for $k \in (\frac{m^*_L}{1-d}, \frac{m^*_L}{(1-d)^2})$, we have

$$\begin{aligned} W'_H(k) &= \frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi W'_H((1-d)k) + (1-\pi)V'_H((1-d)k)\right] \\ &= \frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\left(\pi\frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} + (1-\pi)\alpha_H^{\frac{1}{1-\beta}}\right)((1-d)k)^{\frac{2\beta-1}{1-\beta}} + (1-\pi)(1-d)\right], \end{aligned}$$

where the second equality follows from $(1-d)k < \frac{m^*_L}{1-d} \leq \frac{m^*_H}{1-d}$. Clearly, W'_H is also continuous and strictly decreasing on $(\frac{m^*_L}{1-d}, \frac{m^*_L}{(1-d)^2})$. Using the recursive structure of V'_H and W'_H as in (5) and (6), we now prove by induction that V'_H and W'_H are continuous and strictly decreasing on $(\frac{m^*_L}{(1-d)^n}, \frac{m^*_L}{(1-d)^{n+1}})$ for any $n \in \mathbb{N}$. Suppose this statement is true for some $n' \in \mathbb{N}$. Consider $k \in (\frac{m^*_L}{(1-d)^{n'+1}}, \frac{m^*_L}{(1-d)^{n'+2}})$. Since $k > \frac{m^*_L}{(1-d)^{n'+1}} \geq \frac{m^*_L}{(1-d)^2} > \frac{m^*_H}{1-d}$, from

(5) and (6), we have

$$\begin{aligned}
V_H'(k) &= \alpha_H^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi V_H'((1-d)k) + (1-\pi)W_H'((1-d)k)\right] \\
W_H'(k) &= \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi W_H'((1-d)k) + (1-\pi)V_H'((1-d)k)\right].
\end{aligned}$$

Since $(1-d)k \in (\frac{m_L^*}{(1-d)^{n'}}, \frac{m_L^*}{(1-d)^{n'+1}})$, and we know that V_H' and W_H' are continuous and strictly decreasing on $(\frac{m_L^*}{(1-d)^{n'}}, \frac{m_L^*}{(1-d)^{n'+1}})$, from the two equations above, we must have V_H' and W_H' are continuous and strictly decreasing on $(\frac{m_L^*}{(1-d)^{n'+1}}, \frac{m_L^*}{(1-d)^{n'+2}})$. We have shown the statement holds for $n = 1$. By induction, it must hold for any $n \in \mathbb{N}$.

Next, we want to show that for any $n \in \mathbb{N}$,

$$\lim_{k \to \frac{m_L^*}{(1-d)^n}-} V_H'(k) \leq \lim_{k \to \frac{m_L^*}{(1-d)^n}+} V_H'(k) \quad \text{and} \quad \lim_{k \to \frac{m_L^*}{(1-d)^n}-} W_H'(k) \leq \lim_{k \to \frac{m_L^*}{(1-d)^n}+} W_H'(k),$$

with at least one inequality being strict. It is straightforward to show that the statement holds for $n = 1$. We again prove by induction that it holds for any $n \in \mathbb{N}$. Suppose the statement holds for some $n' \in \mathbb{N}$. Then, we have

$$\begin{aligned}
\lim_{k \to \frac{m_L^*}{(1-d)^{n'+1}}-} V_H'(k) &= \lim_{k \to \frac{m_L^*}{(1-d)^{n'+1}}-} \alpha_H^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi V_H'((1-d)k) + (1-\pi)W_H'((1-d)k)\right] \\
&\leq \lim_{k \to \frac{m_L^*}{(1-d)^{n'+1}}+} \alpha_H^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi V_H'((1-d)k) + (1-\pi)W_H'((1-d)k)\right] \\
&= \lim_{k \to \frac{m_L^*}{(1-d)^{n'+1}}+} V_H'(k),
\end{aligned}$$

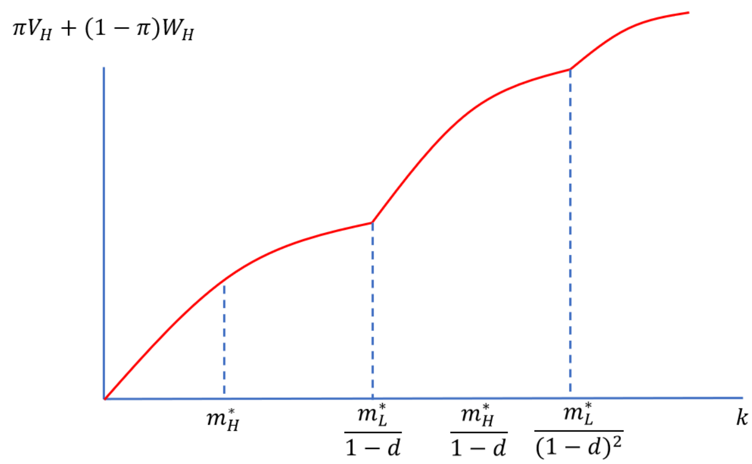
where the two equalities follow from $\frac{m_L^*}{(1-d)^{n'+1}} \geq \frac{m_L^*}{(1-d)^2} > \frac{m_H^*}{1-d}$ and (5), and the inequality follows from the fact that the statement holds for n'. Similarly, we have

$$\begin{aligned}
\lim_{k \to \frac{m_L^*}{(1-d)^{n'+1}}-} W_H'(k) &= \lim_{k \to \frac{m_L^*}{(1-d)^{n'+1}}-} \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi W_H'((1-d)k) + (1-\pi)V_H'((1-d)k)\right] \\
&\leq \lim_{k \to \frac{m_L^*}{(1-d)^{n'+1}}+} \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi W_H'((1-d)k) + (1-\pi)V_H'((1-d)k)\right] \\
&= \lim_{k \to \frac{m_L^*}{(1-d)^{n'+1}}+} W_H'(k).
\end{aligned}$$

For $\pi \in (0,1)$, both weak inequalities are strict, and otherwise, at least one weak inequality is strict. It immediately follows from the two inequalities that for any $n \in \mathbb{N}$,

$$\lim_{k \to \frac{m_L^*}{(1-d)^n}-} \pi V_H'(k) + (1-\pi) W_H'(k) \le \lim_{k \to \frac{m_L^*}{(1-d)^n}+} \pi V_H'(k) + (1-\pi) W_H'(k),$$

with the inequality being strict for $\pi \in (0,1)$ (and for $\pi = 0$ when n is an odd number). Thus, the continuation value in the objective function for party H, $\pi V_H + (1-\pi) W_H$, is piecewise strictly concave, as illustrated in the figure below.



It is straightforward to show that $\delta[\pi V_H'(m_H^*) + (1-\pi) W_H'(m_H^*)] = 1$. Hence, when k is below $\frac{m_H^*}{1-d}$, m_H^* is locally optimal on $[(1-d)k, \frac{m_L^*}{1-d})$. To pin down the conditions for a simple equilibrium, we look for conditions such that (i) m_H^* is globally optimal when k is small and (ii) party H has no incentives to invest when k is high (no interior local optimum above $\frac{m_L^*}{1-d}$). Equivalently, we look for conditions such that for any $n \in \mathbb{N}$,[32]

$$\lim_{k \to \frac{m_L^*}{(1-d)^n}+} \delta[\pi V_H'(k) + (1-\pi) W_H'(k)] \le 1.$$

We claim that it is sufficient to have $W_H'(m_H^*) > \lim_{k \to \frac{m_L^*}{1-d}+} W_H'(k)$, or equivalently, $W_H'(m_H^*) > \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} \left(\frac{m_L^*}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)[\pi W_H'(m_L^*) + (1-\pi) V_H'(m_L^*)]$, which can be

[32]Note that m_H^* can still be globally optimal for a low initial stock when this inequality does not hold, but in that case, when the initial stock is relatively high, party H may have incentives to invest because there will be other interior local optimum than m_H^*.

further simplified as (3). To establish this, we first introduce some notation:

$$\nu_n \equiv \lim_{k \to \frac{m_L^*}{(1-d)^n}+} V_H'(k) \quad \text{and} \quad \omega_n \equiv \lim_{k \to \frac{m_L^*}{(1-d)^n}+} W_H'(k).$$

Since $m_H^* \geq m_L^*$, $\omega_0 = W_H'(m_L^*) \geq W_H'(m_H^*) > \omega_1$, where the last inequality follows from (3). Since V_H is strictly concave on $[0, \frac{m_H^*}{1-d})$, V_H' is continuous at $\frac{m_H^*}{1-d}$, and $m_H^* < \frac{m_L^*}{1-d}$, $\nu_0 = V_H'(m_L^*) \geq V_H'(m_H^*) > \nu_1$. We now prove by induction that $\omega_{n-1} > \omega_n$ and $\nu_{n-1} > \nu_n$ for any $n \in \mathbb{N}$. We have shown this holds for $n = 1$. Suppose it holds for some $n' \in \mathbb{N}$. Then,

$$\begin{aligned}
\omega_{n'+1} &= \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} \left(\frac{m_L^*}{(1-d)^{n'+1}} \right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)[\pi \omega_{n'} + (1-\pi)\nu_{n'}] \\
&< \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} \left(\frac{m_L^*}{(1-d)^{n'}} \right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)[\pi \omega_{n'-1} + (1-\pi)\nu_{n'-1}] = \omega_{n'},
\end{aligned}$$

where the equality follows from $\frac{m_L^*}{(1-d)^{n'+1}} \geq \frac{m_L^*}{1-d}$, $\beta \in (0, \frac{1}{2})$, $\omega_{n'-1} > \omega_{n'}$, and $\nu_{n'-1} > \nu_{n'}$. For $n' > 1$, similarly, we have

$$\begin{aligned}
\nu_{n'+1} &= \alpha_H^{\frac{1}{1-\beta}} \left(\frac{m_L^*}{(1-d)^{n'+1}} \right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)[\pi \nu_{n'} + (1-\pi)\omega_{n'}] \\
&< \alpha_H^{\frac{1}{1-\beta}} \left(\frac{m_L^*}{(1-d)^{n'}} \right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)[\pi \nu_{n'-1} + (1-\pi)\omega_{n'-1}] = \nu_{n'},
\end{aligned}$$

where the last equality follows from $\frac{m_L^*}{(1-d)^{n'}} \geq \frac{m_L^*}{(1-d)^2} > \frac{m_H^*}{1-d}$ for $n' > 1$. For $n' = 1$, the derivation above applies to the symmetric case of $\alpha_L = \alpha_H$ (so that $m_L^* = m_H^*$). For the asymmetric case of $\alpha_L < \alpha_H$, when $n' = 1$, however, we have $\frac{m_L^*}{(1-d)^{n'}} = \frac{m_L^*}{1-d} < \frac{m_H^*}{1-d}$ so the last equality in the derivation above does not hold any more. Instead, we have

$$\begin{aligned}
\nu_2 &= \alpha_H^{\frac{1}{1-\beta}} \left(\frac{m_L^*}{(1-d)^2} \right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)[\pi \nu_1 + (1-\pi)\omega_1] \\
&< \alpha_H^{\frac{1}{1-\beta}} \left(\frac{m_L^*}{(1-d)} \right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi V_H'(m_H^*) + (1-\pi) W_H'(m_H^*)\right] \\
&= \alpha_H^{\frac{1}{1-\beta}} \left(\frac{m_L^*}{(1-d)} \right)^{\frac{2\beta-1}{1-\beta}} + (1-d) = \nu_1,
\end{aligned}$$

where the first equality follows from $\frac{m_L^*}{(1-d)^2} > \frac{m_H^*}{1-d}$, the inequality follows from $\beta \in (0, \frac{1}{2})$,

$V_H'(m_H^*) > \nu_1$, and $W_H'(m_H^*) > \omega_1$, the second equality follows from $\delta[\pi V_H'(m_H^*) + (1 - \pi)W_H'(m_H^*)] = 1$, and the last equality follows from $\frac{m_L^*}{1-d} < \frac{m_H^*}{1-d}$ (for $\alpha_L < \alpha_H$). Thus, $\nu_{n'+1} < \nu_{n'}$ for $n' = 1$.

We have shown for $n' \in \mathbb{N}$, $\nu_{n'+1} < \nu_{n'}$ and $\omega_{n'+1} < \omega_{n'}$. Since $\nu_1 < \nu_0$ and $\omega_1 < \omega_0$, by induction, we establish the claim. Then, for any $n \in \mathbb{N}$, we have

$$\delta[\pi\nu_n + (1-\pi)\omega_n] \leq \delta[\pi\nu_1 + (1-\pi)\omega_1] < \delta\left[\pi V_H'(m_H^*) + (1-\pi)W_H'(m_H^*)\right] = 1,$$

thus establishing the optimality of party H's strategy.

We now turn to party L's optimization problem. Similarly, we can show that (i) V_L' and W_L' are continuous and strictly decreasing on $(0, \frac{m_H^*}{1-d})$ and on $(\frac{m_H^*}{(1-d)^n}, \frac{m_H^*}{(1-d)^{n+1}})$ for any $n \in \mathbb{N}$. and (ii) for any $n \in \mathbb{N}$,

$$\lim_{k \to \frac{m_H^*}{(1-d)^n}-} V_L'(k) \leq \lim_{k \to \frac{m_H^*}{(1-d)^n}+} V_L'(k), \quad \textit{and} \quad \lim_{k \to \frac{m_H^*}{(1-d)^n}-} W_L'(k) \leq \lim_{k \to \frac{m_H^*}{(1-d)^n}+} W_L'(k),$$

where at least one inequality is strict. Thus, the continuation value in the objective function for party L, $\pi V_L + (1-\pi)W_L$, is also piecewise strictly concave. To establish the optimality of party L's strategy, it is sufficient that for any $n \in \mathbb{N}$,

$$\lim_{k \to \frac{m_H^*}{(1-d)^n}+} \delta[\pi V_L'(k) + (1-\pi)W_L'(k)] \leq 1.$$

The above condition is guaranteed if $W_L'(m_H^*) > \lim_{k \to \frac{m_H^*}{1-d}+} W_L'(k)$. To see this, first note that V_L' is strictly decreasing and continuous on $[0, \frac{m_H^*}{1-d}]$ so $V_L'(m_H^*) > V_L'(\frac{m_H^*}{1-d})$. Using a similar induction argument as above, we can show that for any $n \in N$,

$$\lim_{k \to \frac{m_H^*}{(1-d)^{n-1}}+} V_L'(k) > \lim_{k \to \frac{m_H^*}{(1-d)^n}+} V_L'(k), \quad \text{and} \quad \lim_{k \to \frac{m_H^*}{(1-d)^{n-1}}+} W_L'(k) > \lim_{k \to \frac{m_H^*}{(1-d)^n}+} W_L'(k),$$

which imply

$$\begin{aligned} \lim_{k \to \frac{m_H^*}{(1-d)^n}+} \delta[\pi V_L'(k) + (1-\pi)W_L'(k)] &< \delta[\pi V_L'(m_H^*) + (1-\pi)W_L'(m_H^*)] \\ &\leq \delta[\pi V_L'(m_L^*) + (1-\pi)W_L'(m_L^*)] = 1, \end{aligned}$$

where the last equality follows from the definition of m_L^* and (5) and (6). Further, the condition $W_L'(m_H^*) > \lim_{k\to \frac{m_H^*}{1-d}+} W_L'(k)$, or equivalently, $W_L'(m_H^*) > \frac{\alpha_L\alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}\left(\frac{m_H^*}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)[\pi W_L'(m_H^*)+(1-\pi)V_L'(m_H^*)]$, which can be simplified as (4). Thus, we have shown that a simple equilibrium exists if (2)–(4) are satisfied.

Last, it is straightforward to show that (2) is satisfied if ρ is low enough (or d is high enough) and (3) and (4) are satisfied if d is high enough. Let $\delta \to 0$. Then, (3) and (4) are reduced to $W_H'(m_H^*) > \frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}\left(\frac{m_L^*}{1-d}\right)^{\frac{2\beta-1}{1-\beta}}$ (or equivalently, $m_H^* < \frac{m_L^*}{1-d}$) and $W_L'(m_H^*) > \frac{\alpha_L\alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}\left(\frac{m_H^*}{1-d}\right)^{\frac{2\beta-1}{1-\beta}}$, which holds for any $d>0$. Thus, (2)– (4) are also satisfied if ρ is low enough and δ is small enough. We have obtained the desired conclusion. ∎

Proof of Proposition 6: Based on the definition of m_i^*, it is straightforward to show that m_i^* is increasing in both α_L and α_H (for $\pi<1$). To prove (ii), consider

$$\begin{aligned}\frac{\partial m_H^*}{\partial \pi} &= \frac{1-\beta}{1-2\beta}\left(\frac{1}{\delta}-(1-d)\pi\right)^{\frac{1-\beta}{2\beta-1}}\alpha_H^{\frac{1-\beta}{1-2\beta}}\left(\pi\alpha_H^{\frac{\beta}{1-\beta}}+\frac{1-\pi}{1-\beta}\alpha_L^{\frac{\beta}{1-\beta}}\right)^{\frac{\beta}{1-2\beta}}\left(\alpha_H^{\frac{\beta}{1-\beta}}-\frac{\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}\right)\\ &\quad+\frac{1-\beta}{1-2\beta}\left(\frac{1}{\delta}-(1-d)\pi\right)^{\frac{2-3\beta}{2\beta-1}}\alpha_H^{\frac{1-\beta}{1-2\beta}}\left(\pi\alpha_H^{\frac{\beta}{1-\beta}}+\frac{1-\pi}{1-\beta}\alpha_L^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}}(1-d),\end{aligned}$$

which implies $\frac{\partial m_H^*}{\partial \pi}<0$ if $\alpha_H^{\frac{\beta}{1-\beta}} < \frac{\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}$ and d is sufficiently close to one such that the second term in the expression above is sufficiently small. The former holds when polarization is relatively low (i.e. $\rho < (\frac{1}{1-\beta})^{\frac{1-\beta}{\beta}}$). Moreover, we have

$$\begin{aligned}\frac{\partial m_L^*}{\partial \pi} &= \frac{1-\beta}{1-2\beta}\left(\frac{1}{\delta}-(1-d)\pi\right)^{\frac{1-\beta}{2\beta-1}}\alpha_L^{\frac{1-\beta}{1-2\beta}}\left(\pi\alpha_L^{\frac{\beta}{1-\beta}}+\frac{1-\pi}{1-\beta}\alpha_H^{\frac{\beta}{1-\beta}}\right)^{\frac{\beta}{1-2\beta}}\left(\alpha_L^{\frac{\beta}{1-\beta}}-\frac{\alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}\right)\\ &\quad+\frac{1-\beta}{1-2\beta}\left(\frac{1}{\delta}-(1-d)\pi\right)^{\frac{2-3\beta}{2\beta-1}}\alpha_L^{\frac{1-\beta}{1-2\beta}}\left(\pi\alpha_L^{\frac{\beta}{1-\beta}}+\frac{1-\pi}{1-\beta}\alpha_H^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}}(1-d),\end{aligned}$$

where the first term is always negative, so $\frac{\partial m_H^*}{\partial \pi}<0$ if d is sufficiently close to one such that the first term dominates. Thus, we have obtained the desired conclusion. ∎

Proof of Proposition 7: Let $\alpha_L=\alpha-\varepsilon$ and $\alpha_H=\alpha+\varepsilon$. We consider how changes in ε, which induce a mean-preserving spread of α_i, affect the outcome. In a simple equilibrium, the expected technology stock in the steady state distribution, Em, is given by

$$\begin{aligned}Em &\equiv \frac{m_L^* + m_H^*}{2} = \frac{1}{2}\left(\frac{1}{\delta} - (1-d)\pi\right)^{\frac{1-\beta}{2\beta-1}} \\ &\cdot \left[\left(\pi\alpha_L^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_L\alpha_H^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}} + \left(\pi\alpha_H^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}}\right],\end{aligned}$$

and the expected public good provision in the steady state distribution, Ep, is given by

$$\begin{aligned}Ep &\equiv \frac{1}{2}\left(\pi\alpha_L^{\frac{1}{1-\beta}} m_L^{*\frac{\beta}{1-\beta}} + (1-\pi)\alpha_H^{\frac{1}{1-\beta}} m_L^{*\frac{\beta}{1-\beta}} + \pi\alpha_H^{\frac{1}{1-\beta}} m_H^{*\frac{\beta}{1-\beta}} + (1-\pi)\alpha_L^{\frac{1}{1-\beta}} m_H^{*\frac{\beta}{1-\beta}}\right) \\ &= \frac{1}{2}\left(\frac{1}{\delta} - (1-d)\pi\right)^{\frac{\beta}{2\beta-1}} \cdot \left[\left(\pi\alpha_L^{\frac{1}{1-\beta}} + (1-\pi)\alpha_H^{\frac{1}{1-\beta}}\right)\left(\pi\alpha_L^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_L\alpha_H^{\frac{\beta}{1-\beta}}\right)^{\frac{\beta}{1-2\beta}}\right. \\ &\left. + \left(\pi\alpha_H^{\frac{1}{1-\beta}} + (1-\pi)\alpha_L^{\frac{1}{1-\beta}}\right)\left(\pi\alpha_H^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}\right)^{\frac{\beta}{1-2\beta}}\right].\end{aligned}$$

Because $\frac{\partial Em}{\partial \varepsilon}$ and $\frac{\partial Ep}{\partial \varepsilon}$ are continuous in π, it suffices to show that in a simple equilibrium, a mean-preserving spread of α_i has a positive effect on Em and Ep for $\pi = 1$ and a negative effect for $\pi = 0$. When $\pi = 1$, we have

$$\begin{aligned}Em &= \frac{1}{2}\left(\frac{1}{\delta} - (1-d)\right)^{\frac{1-\beta}{2\beta-1}}\left(\alpha_L^{\frac{1}{1-2\beta}} + \alpha_H^{\frac{1}{1-2\beta}}\right), \\ Ep &= \frac{1}{2}\left(\frac{1}{\delta} - (1-d)\right)^{\frac{\beta}{2\beta-1}}\left(\alpha_L^{\frac{1}{1-2\beta}} + \alpha_H^{\frac{1}{1-2\beta}}\right),\end{aligned}$$

where $\alpha_i^{\frac{1}{1-2\beta}}$ is convex in α_i. Therefore, a mean preserving spread raises Em and Ep.

When $\pi = 0$, we have

$$\begin{aligned}Em &= \frac{1}{2}\left(\frac{\delta}{1-\beta}\right)^{\frac{1-\beta}{1-2\beta}}\alpha_L^{\frac{\beta}{1-2\beta}}\alpha_H^{\frac{\beta}{1-2\beta}}(\alpha_H + \alpha_L) \\ Ep &= \frac{1}{2}\left(\frac{\delta}{1-\beta}\right)^{\frac{\beta}{1-2\beta}}\alpha_L^{\frac{\beta}{1-2\beta}}\alpha_H^{\frac{\beta}{1-2\beta}}(\alpha_H + \alpha_L).\end{aligned}$$

Taking log on both sides for the first equation, we have

$$\ln(Em) = \ln(\frac{1}{2}) + \frac{1-\beta}{1-2\beta}\ln(\frac{\delta}{1-\beta}) + \frac{\beta}{1-2\beta}\ln(\alpha_L) + \frac{\beta}{1-2\beta}\ln(\alpha_H) + \ln(\alpha_H + \alpha_L).$$

A mean preserving spread leaves $(\alpha_L + \alpha_H)$ constant. Since $\ln \alpha_i$ is concave in α_i and $\frac{\beta}{1-2\beta} > 0$, it follows that a mean-preserving spread of α_i lowers Em. Similarly, we can show that a mean-preserving spread of α_i also lowers Ep.

Since $\frac{\partial Em}{\partial \varepsilon}$ and $\frac{\partial Ep}{\partial \varepsilon}$ are continuous in π, the findings above extend to the neighborhood of $\pi = 0$ and $\pi = 1$.

Further, we show a stronger result that there exists a threshold for power persistence such that a mean-preserving spread of α_i raises Em for π greater than that threshold and lowers Em for π below that threshold. To see this, consider

$$\frac{\partial Em}{\partial \varepsilon} = \frac{1-\beta}{2(1-2\beta)} \left(\frac{1}{\delta} - \pi(1-d)\right)^{\frac{1-\beta}{2\beta-1}} \left(\pi \alpha_H^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta} \alpha_H \alpha_L^{\frac{\beta}{1-\beta}}\right)^{\frac{\beta}{1-2\beta}} (\pi\psi_1 + (1-\pi)\psi_2),$$

with

$$\begin{aligned}
\psi_1 &\equiv -\frac{\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}\psi_3 + \frac{\alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta} \\
\psi_2 &\equiv \left(-\frac{\alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta} + \frac{\beta}{(1-\beta)^2}\alpha_L \alpha_H^{\frac{2\beta-1}{1-\beta}}\right)\psi_3 + \left(\frac{\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} - \frac{\beta}{(1-\beta)^2}\alpha_H \alpha_L^{\frac{2\beta-1}{1-\beta}}\right) \\
\psi_3 &\equiv \left(\frac{\pi \alpha_L^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{\pi \alpha_H^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}\right)^{\frac{\beta}{1-2\beta}}.
\end{aligned}$$

We have

$$\frac{\partial(\psi_3^{\frac{1-2\beta}{\beta}})}{\partial \pi} = \frac{(1-\beta)\alpha_H \alpha_L^{\frac{1+\beta}{1-\beta}}\left(1-\rho^{\frac{2\beta}{1-\beta}}\right)}{\left((1-\beta)\pi\alpha_H^{\frac{1}{1-\beta}} + (1-\pi)\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}\right)^2} \leq 0,$$

where the weak inequality holds with equality if an only if $\rho = 1$. Since $\rho > 1$ for any $\varepsilon > 0$, ψ_3 strictly decreases with π. Moreover, when $\pi = 0$, $\psi_3 = (\alpha_L/\alpha_H)^{\frac{\beta}{1-\beta}}$ and $\psi_2 = \frac{\beta}{(1-\beta)^2}\left(\alpha_L^{\frac{1}{1-\beta}}\alpha_H^{-1} - \alpha_H \alpha_L^{\frac{2\beta-1}{1-\beta}}\right) < 0$. Since $-\frac{\alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta} + \frac{\beta}{(1-\beta)^2}\alpha_L\alpha_H^{\frac{2\beta-1}{1-\beta}} < 0$, ψ_2 is strictly increasing with π. Then, there exists $\tilde{\pi} \leq 1$ such that $\psi_2 < 0$ if $\pi < \tilde{\pi}$ and $\psi_2 > 0$ if $\pi > \tilde{\pi}$ (when $\tilde{\pi} = 1$, ψ_2 is non-positive for any π). Since $\psi_3 < 1$, $\psi_1 > 0$ for any π. Thus, for $\pi > \tilde{\pi}$, $\pi\psi_1 + (1-\pi)\psi_2 > 0$. For $\pi \leq \tilde{\pi}$, since ψ_2 is non-positive and increasing in π, and ψ_1 strictly increases with π because ψ_3 is strictly decreasing with π, $\pi\psi_1 + (1-\pi)\psi_2$ is strictly increasing in π for $\pi \leq \tilde{\pi}$. For $\pi = 0$, $\pi\psi_1 + (1-\pi)\psi_2 = \psi_2 < 0$. For $\pi = \tilde{\pi}$, if $\tilde{\pi} = 1$, $\pi\psi_1 + (1-\pi)\psi_2 = \psi_1 > 0$ and if $\tilde{\pi} < 1$, $\pi\psi_1 + (1-\pi)\psi_2 = \pi\psi_1 > 0$. Therefore, there exists

$\bar{\pi} \in (0, \tilde{\pi})$ such that when $\pi < \bar{\pi}$, $\pi\psi_1 + (1-\pi)\psi_2 < 0$ and $\pi > \bar{\pi}$, $\pi\psi_1 + (1-\pi)\psi_2 > 0$. Since $\frac{\partial Em}{\partial \varepsilon}$ has the same sign as $\pi\psi_1 + (1-\pi)\psi_2$, a mean-preserving spread in α raises Em when $\pi > \bar{\pi}$ and lowers it when $\pi < \bar{\pi}$. ■

Proof of Proposition 8: The case of $d = 0$ will be covered by Proposition 11. We focus on the case of $d > 0$ here. In any GSE, it is straightforward to show that $y_i(k) = \alpha_i^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}$. Based on the strategy profile of a GSE, we have

$$V_i'(k) = \begin{cases} \alpha_i^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + (1-d), & k < \frac{\hat{m}_i}{1-d} \\ \alpha_i^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi V_i'((1-d)k) + (1-\pi)W_i'((1-d)k)\right], & k > \frac{\hat{m}_i}{1-d} \end{cases} \quad (7)$$

$$W_i'(k) = \begin{cases} \frac{\alpha_i \alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}}, & k < \frac{\hat{m}_j}{1-d} \\ \frac{\alpha_i \alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi W_i'((1-d)k) + (1-\pi)V_i'((1-d)k)\right], & k > \frac{\hat{m}_j}{1-d} \end{cases} \quad (8)$$

For party L, for its target stock $\hat{m}_L$ to be globally optimal for a sufficiently low k, $\hat{m}_L$ has to be locally optimal on the interval $(0, \frac{\hat{m}_L}{1-d})$. Since V_L and W_L are strictly concave on $(0, \frac{\hat{m}_L}{1-d})$, the necessary condition for $\hat{m}_L$ to be locally optimal is given by $\delta[\pi V_L'(\hat{m}_L) + (1-\pi)W_L'(\hat{m}_L)] = 1$. By (7) and (8), this can be explicitly written as

$$\delta\left[\pi\left(\alpha_L^{\frac{1}{1-\beta}}(\hat{m}_L)^{\frac{2\beta-1}{1-\beta}} + (1-d)\right) + (1-\pi)\frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}(\hat{m}_L)^{\frac{2\beta-1}{1-\beta}}\right] = 1,$$

which implies $\hat{m}_L = m_L^*$.

For party H, we consider two cases: $\hat{m}_H \le \frac{\hat{m}_L}{1-d}$ (irreversibility does not bind) and $\hat{m}_H > \frac{\hat{m}_L}{1-d}$ (irreversibility binds). Consider $\hat{m}_H \le \frac{\hat{m}_L}{1-d}$. Since $\lim_{k \to \frac{\hat{m}_L}{1-d}-} W_H'(k) < \lim_{k \to \frac{\hat{m}_L}{1-d}+} W_H'(k)$ whereas we can show that V_H' is continuous at $k = \frac{\hat{m}_L}{1-d}$, $\lim_{k \to \frac{\hat{m}_L}{1-d}-} \pi V_H'(k) + (1-\pi)W_H'(k) < \lim_{k \to \frac{\hat{m}_L}{1-d}+} \pi V_H'(k) + (1-\pi)W_H'(k)$ (for $\pi < 1$). Then, $\frac{\hat{m}_L}{1-d}$ cannot be optimal for party H, because otherwise party H could do better by investing to reach a level slightly higher than $\frac{\hat{m}_L}{1-d}$. Therefore, we have $\hat{m}_H < \frac{\hat{m}_L}{1-d}$. Since V_H and W_H are strictly concave on $(0, \frac{\hat{m}_L}{1-d})$, the first order condition for party H can be written as

$$\delta\left[\pi\left(\alpha_H^{\frac{1}{1-\beta}}(\hat{m}_H)^{\frac{2\beta-1}{1-\beta}} + (1-d)\right) + (1-\pi)\frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}(\hat{m}_H)^{\frac{2\beta-1}{1-\beta}}\right] = 1,$$

which implies $\hat{m}_H = m_H^*$. Thus, if $\hat{m}_H \le \frac{\hat{m}_L}{1-d}$, we have $\hat{m}_H = m_H^*$.

We now turn to the case of $\hat{m}_H > \frac{\hat{m}_L}{1-d}$. Since $\frac{\hat{m}_L}{1-d} < \hat{m}_H < \frac{\hat{m}_H}{1-d}$, the first order condition for party H can be written as

$$\begin{aligned} 1 &= \delta[\pi V_L'(\hat{m}_H) + (1-\pi)W_L'(\hat{m}_H)] \\ &= \delta\left[\pi\left(\alpha_H^{\frac{1}{1-\beta}}(\hat{m}_H)^{\frac{2\beta-1}{1-\beta}} + (1-d)\right)\right. \\ &\quad \left. +(1-\pi)\left(\frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}(\hat{m}_H)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)[\pi W_H'((1-d)\hat{m}_H) + (1-\pi)V_H'((1-d)\hat{m}_H)]\right)\right] \\ &> \delta\left[\pi\left(\alpha_H^{\frac{1}{1-\beta}}(\hat{m}_H)^{\frac{2\beta-1}{1-\beta}} + (1-d)\right) + (1-\pi)\frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}(\hat{m}_H)^{\frac{2\beta-1}{1-\beta}}\right], \end{aligned}$$

where the second equality follows from (7) and (8). By construction, we have

$$\delta\left[\pi\left(\alpha_H^{\frac{1}{1-\beta}}(m_H^*)^{\frac{2\beta-1}{1-\beta}} + (1-d)\right) + (1-\pi)\frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}(m_H^*)^{\frac{2\beta-1}{1-\beta}}\right] = 1,$$

implying $\hat{m}_H > m_H^*$. Thus, if irreversibility binds, that is, $\hat{m}_H > \frac{\hat{m}_L}{1-d}$, then $\hat{m}_H > m_H^*$. ∎

Proof of Proposition 9: We have shown that when irreversibility does not bind, the GSE is an SE, and therefore the steady state distribution of technology stock and public good provision is the same irrespective of whether investment is reversible or irreversible. We now focus on when irreversibility binds: $\hat{m}_H > \frac{\hat{m}_L}{1-d}$. In this case, we have shown that $\hat{m}_H > m_H^*$ and $\hat{m}_L = m_L^*$. When $d = 0$, the technology stock is at $\hat{m}_H$ with probability one in the steady state and it is straightforward to show that the expected technology stock and public good provision are strictly higher under irreversible investment than under reversible investment. In what follows, we consider $d > 0$.

We first consider the steady state distribution (invariant measure) of technology stock. The distribution is over $(\hat{m}_H, (1-d)\hat{m}_H, ..., (1-d)^{n'}\hat{m}_H, \hat{m}_L)$ with the probability vector $(p_0, p_1, ..., p_{n'}, p_{n'+1})$, where n' is the largest natural number such that $(1-d)^{n'}\hat{m}_H > \hat{m}_L$. The probability vector satisfies

$$p_0 = (1-\pi)\sum_{n=1}^{n'+1} p_n + \pi p_0,\ p_1 = (1-\pi)p_0,\ p_{n+1} = \pi p_n,\ n = 1, ..., n'-1,\ \text{ and }\ p_{n'+1} = \pi p_{n'} + \pi p_{n'+1},$$

which yield $p_0 = \frac{1}{2}$, $p_n = \frac{\pi^{n-1}(1-\pi)}{2}$ $(n = 1, ..., n')$, $p_{n'+1} = \frac{\pi^{n'}}{2}$. Thus, the expected technology stock under irreversible investment, Em, is given by

$$Em = \frac{\hat{m}_H}{2} + \sum_{n=1}^{n'} \frac{\pi^{n-1}(1-\pi)}{2}(1-d)^n \hat{m}_H + \frac{\pi^{n'}}{2} m_L^* > \frac{m_H^* + m_L^*}{2},$$

where the inequality follows from $\hat{m}_H > m_H^*$ and $(1-d)^n \hat{m}_H > \hat{m}_L = m_L^*$ for $n = 1, ..., n'$. Hence, when irreversibility binds, the expected steady state technology stock is higher under irreversible investment than under reversible investment.

Similarly, in the steady state, the expected public good provision for $d > 0$ equals

$$\begin{aligned} Ep \;=\; & \frac{1}{2}\left(\pi\alpha_H^{\frac{1}{1-\beta}} + (1-\pi)\alpha_L^{\frac{1}{1-\beta}}\right)\hat{m}_H^{\frac{\beta}{1-\beta}} + \frac{\pi^{n'}}{2}\left(\pi\alpha_L^{\frac{1}{1-\beta}} + (1-\pi)\alpha_H^{\frac{1}{1-\beta}}\right) m_L^{*\,\frac{\beta}{1-\beta}} \\ & + \sum_{n=1}^{n'} \frac{\pi^{n-1}(1-\pi)}{2}\left(\pi\alpha_L^{\frac{1}{1-\beta}} + (1-\pi)\alpha_H^{\frac{1}{1-\beta}}\right)((1-d)^n \hat{m}_H)^{\frac{\beta}{1-\beta}} \\ > \; & \frac{1}{2}\left(\pi\alpha_H^{\frac{1}{1-\beta}} + (1-\pi)\alpha_L^{\frac{1}{1-\beta}}\right) m_H^{*\,\frac{\beta}{1-\beta}} + \frac{1}{2}\left(\pi\alpha_L^{\frac{1}{1-\beta}} + (1-\pi)\alpha_H^{\frac{1}{1-\beta}}\right) m_L^{*\,\frac{\beta}{1-\beta}}, \end{aligned}$$

implying that the expected steady state public good provision is higher under irreversible investment than under reversible investment, when irreversibility binds. ∎