Online Appendix

A The Case of Alternating Power ($\pi = 0$)

In this section, we characterize the equilibrium for the case of alternating power ($\pi = 0$). We first present a proof of Proposition 10 which characterizes the GSE when polarization is sufficiently high. We then discuss how the equilibrium changes analytically and numerically when polarization is low.

Proof of Proposition 10: Let $\pi = 0$. When $d = 1$, a GSE is an SE and irreversibility does not bind. We focus on the case of $d < 1$. We first show that a GSE exists with

$$\hat{m}_H = \left(\frac{1}{\delta} - \delta(1-d)^2\right)^{\frac{1-\beta}{2\beta-1}} \alpha_H^{\frac{1-\beta}{1-2\beta}} \left(\frac{1}{1-\beta}\alpha_L^{\frac{\beta}{1-\beta}} + \delta(1-d)^{\frac{\beta}{1-\beta}}\alpha_H^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}} \equiv \phi, \tag{A.1}$$

if the following conditions are satisfied:

$$h(\rho) \equiv \frac{(1-2\beta)\left(1+(1-\beta)\delta(1-d)^{\frac{\beta}{1-\beta}}\rho^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}}}{(1-\delta^2(1-d)^2)^{\frac{\beta}{1-2\beta}}} - \left[1-2\beta+(1-\beta)^2\delta + \frac{\beta\delta^2(1-d)}{\rho}\right] > 0 \tag{A.2}$$

$$\left(\frac{1-d}{\rho}\right)^{\frac{1-2\beta}{1-\beta}} + \frac{\delta(1-d)(1-\beta)}{\rho} + \delta^2(1-d)^2 < \rho^{\frac{2\beta-1}{1-\beta}}. \tag{A.3}$$

Note that $\phi > m_H^*$ and (A.3) is the specialization of (4) in the proof of Proposition 5 for $\pi = 0$. The function $h(\cdot)$ defined above has the following properties:[1] (i) $h(\rho)$ is strictly increasing in ρ; (ii) If $h(\rho) \geq 0$, then $\phi > \frac{m_L^*}{1-d}$; (iii) $h(\frac{1}{1-d}) > 0$.

We prove the proposition using the guess-and-verify approach. Based on the strategy profile, we obtain the first derivative of the value functions as follows

$$V_L'(k) = \begin{cases} \alpha_L^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + (1-d), & k < \frac{m_L^*}{1-d} \\ \alpha_L^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)W_L'((1-d)k), & k \geq \frac{m_L^*}{1-d} \end{cases} \tag{A.4}$$

$$W_L'(k) = \begin{cases} \frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}}, & k < \frac{\phi}{1-d} \\ \frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)V_L'((1-d)k), & k > \frac{\phi}{1-d} \end{cases} \tag{A.5}$$

$$V_H'(k) = \begin{cases} \alpha_H^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + (1-d), & k < \frac{\phi}{1-d} \\ \alpha_H^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)W_H'((1-d)k), & k > \frac{\phi}{1-d} \end{cases} \tag{A.6}$$

[1]Those properties and stated and proved as Lemma C.3.

$$W_H'(k) = \begin{cases} \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}}, & k < \frac{m_L^*}{1-d} \\ \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)V_H'((1-d)k), & k > \frac{m_L^*}{1-d} \end{cases} \quad \text{(A.7)}$$

where by the definition of m_L^*, $\delta W_L'(m_L^*) = 1$, and therefore, $V_L'(k)$ is continuous at $k = \frac{m_L^*}{1-d}$. We now verify that each party's strategy is indeed optimal under the value functions with the first derivatives derived above.

First consider party H. For $\pi = 0$, its constrained optimization problem becomes

$$\max_{y,m} \left\{ v_H(y,k) - y - m + (1-d)k + \delta W_H(m) \right\},$$

subject to $y \geq 0$ and $m \geq (1-d)k$. When $h(\rho) > 0$, from property (ii) of $h(\cdot)$, we have $\phi > \frac{m_L^*}{1-d}$. There are two possible cases: (a) $\phi > \frac{m_L^*}{1-d} \geq m_H^*$; (b) $\phi > m_H^* > \frac{m_L^*}{1-d}$. For (a), from (A.6) and (A.7), we have

$$\begin{aligned} & W_H(\phi) - W_H(m_H^*) - \frac{\phi - m_H^*}{\delta} \\ = & \int_{m_H^*}^{\phi} \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} dk + \int_{\frac{m_L^*}{1-d}}^{\phi} \delta(1-d) \left[\alpha_H^{\frac{1}{1-\beta}} ((1-d)k)^{\frac{2\beta-1}{1-\beta}} + (1-d) \right] dk - \frac{1}{\delta}(\phi - m_H^*) \\ = & \frac{m_H^*}{\beta\delta} h(\rho) > 0, \end{aligned}$$

where the second equality follows from (C.2) in the proof of Lemma C.3. For (b), since $\delta W_H'(\phi) = 1$ by the definition of ϕ and W_H is strictly concave on $(\frac{m_L^*}{1-d}, \phi)$ which contains (m_H^*, ϕ), $\int_{m_H^*}^{\phi} \delta W_H'(k) dk > \int_{m_H^*}^{\phi} dk$, implying $\delta \left[W_H(\phi) - W_H(m_H^*) \right] > \phi - m_H^*$. Thus, for both cases (a) and (b), if $h(\rho) > 0$, we have $\delta \left[W_H(\phi) - W_H(m_H^*) \right] > \phi - m_H^*$. Thus, given party H's optimization problem, m_H^* always yields a strictly lower return than ϕ does. Hence, it is not optimal for party H to choose m_H^* for a low k.

Next, we show that W_H is strictly concave on (ϕ, ∞). For $k \in [\phi, \frac{\phi}{(1-d)^2})$, since $(1-d)k < \frac{\phi}{1-d}$, we have

$$W_H'(k) = \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d) \left[\alpha_H^{\frac{1}{1-\beta}} ((1-d)k)^{\frac{2\beta-1}{1-\beta}} + (1-d) \right].$$

So W_H is strictly concave on $[\phi, \frac{\phi}{(1-d)^2})$. Further, we have

$$\begin{aligned}
& \lim_{k \to \frac{\phi}{(1-d)^2}+} W_H'(k) \\
= & \lim_{k \to \frac{\phi}{(1-d)^2}+} \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\alpha_H^{\frac{1}{1-\beta}}((1-d)k)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)W_H'((1-d)^2 k)\right] \\
= & \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}\left(\frac{\phi}{(1-d)^2}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\alpha_H^{\frac{1}{1-\beta}}\left(\frac{\phi}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\lim_{k\to\phi+} W_H'(k)\right] \\
= & \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}\left(\frac{\phi}{(1-d)^2}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\alpha_H^{\frac{1}{1-\beta}}\left(\frac{\phi}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + (1-d)\right] \\
= & \lim_{k \to \frac{\phi}{(1-d)^2}-} W_H'(k),
\end{aligned}$$

where the second to last equality follows from $\delta W_H'(\phi) = 1$. Thus, W_H' is continuous at $k = \frac{\phi}{(1-d)^2}$. Suppose W_H is strictly concave on $[\frac{\phi}{(1-d)^{2(n'-1)}}, \frac{\phi}{(1-d)^{2n'}})$ for some $n' \in \mathbb{N}$. Then from (A.6) and (A.7), we have

$$W_H'(k) = \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\alpha_H^{\frac{1}{1-\beta}}((1-d)k)^{\frac{2\beta-1}{1-\beta}} + \delta^2(1-d)^2 W_H'((1-d)^2 k).$$

Because W_H is strictly concave on $[\frac{\phi}{(1-d)^{2(n'-1)}}, \frac{\phi}{(1-d)^{2n'}})$, this equation implies that W_H is strictly concave on $[\frac{\phi}{(1-d)^{2n'}}, \frac{\phi}{(1-d)^{2(n'+1)}})$. By induction, W_H is strictly concave on $[\frac{\phi}{(1-d)^{2(n-1)}}, \frac{\phi}{(1-d)^{2n}})$ for any $n \in \mathbb{N}$. Further, since W_H' is continuous at $k = \frac{\phi}{(1-d)^2}$, a similar induction argument based on the equation above shows that W_H' is continuous at $k = \frac{\phi}{(1-d)^{2n}}$ for any $n \in \mathbb{N}$. Hence, W_H is strictly concave on $[\phi, \infty)$. Since $\delta W_H'(\phi) = 1$, $\delta W_H'(k) < 1$ for any $k > \phi$, implying that it is optimal for party H not to invest so that $m_H(k) = (1-d)k$ for $k \geq \frac{\phi}{1-d}$. For $k < \frac{\phi}{1-d}$, ϕ is the global optimum for party H with $m_H(k) = \phi$.

We now turn to party L. For $\pi = 0$, its constrained optimization problem becomes

$$\max_{y,m} \{v_L(y,k) - y - m + (1-d)k + \delta W_L(m)\},$$

subject to $y \geq 0$ and $m \geq (1-d)k$. From (A.5), $W_L(\cdot)$ is not globally concave, we now show that if (A.3) holds, then $W_L(\cdot)$ is well behaved enough such that the first order condition $\delta W_L'(m_L^*) = 1$ remains sufficient for the optimality. We have

$$
\begin{aligned}
&\lim_{k\to\frac{\phi}{1-d}+} W_L'(k) - W_L'(\phi) \\
=\ & \frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}\left(\frac{\phi}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\lim_{k\to\phi+} V_L'(k) - \frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}\phi^{\frac{2\beta-1}{1-\beta}} \\
=\ & \frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}\left(\frac{\phi}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\alpha_L^{\frac{1}{1-\beta}}\phi^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)W_L'((1-d)\phi)\right] - \frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}\phi^{\frac{2\beta-1}{1-\beta}} \\
\le\ & \frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}\left(\frac{\phi}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\alpha_L^{\frac{1}{1-\beta}}\phi^{\frac{2\beta-1}{1-\beta}} + (1-d)\right] - \frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}\phi^{\frac{2\beta-1}{1-\beta}} \\
<\ & \frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}\left(\frac{m_H^*}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\alpha_L^{\frac{1}{1-\beta}}{m_H^*}^{\frac{2\beta-1}{1-\beta}} + (1-d)\right] - \frac{\alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}{m_H^*}^{\frac{2\beta-1}{1-\beta}} \\
=\ & \frac{1}{\delta}\left(\frac{1-d}{\rho}\right)^{\frac{1-2\beta}{1-\beta}} + \frac{(1-\beta)(1-d)}{\rho} + \delta(1-d)^2 - \frac{1}{\delta}\rho^{\frac{2\beta-1}{1-\beta}} < 0,
\end{aligned}
$$

where the second equality follows from $h(\rho) > 0$ (implying $\phi > \frac{m_L^*}{1-d}$ from (ii)), the first inequality follows from the strict concavity of W_L on $[0, \frac{\phi}{1-d})$, $\phi > \frac{m_L^*}{1-d}$, and $\delta W_L'(m_L^*) = 1$, the second inequality follows from $\phi > m_H^*$, and the last inequality follows from (A.3).

Further, since $\phi > \frac{m_L^*}{1-d}$ (from $h(\rho) > 0$), we have

$$
\begin{aligned}
\lim_{k\to\frac{\phi}{1-d}+} V_L'(k) &= \alpha_L^{\frac{1}{1-\beta}}\left(\frac{\phi}{1-d}\right)^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)W_L'(\phi) \\
&< \alpha_L^{\frac{1}{1-\beta}}\phi^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)W_L'((1-d)\phi) = V_L'(\phi),
\end{aligned}
$$

where the inequality follows from the strict concavity of $W_L(\cdot)$ on $[0, \frac{\phi}{1-d})$. We have now shown that $\lim_{k\to\frac{\phi}{1-d}+} W_L'(k) < W_L'(\phi)$ and $\lim_{k\to\frac{\phi}{1-d}+} V_L'(k) < V_L'(\phi)$. Following essentially the same argument as in the proof of Proposition 5 that establishes the optimality of party L's strategy, we can show that for any $n \in \mathbb{N}$,

$$
\begin{aligned}
\lim_{k\to\frac{\phi}{(1-d)^{n-1}}+} W_L'(k) &> \lim_{k\to\frac{\phi}{(1-d)^{n}}+} W_L'(k) \\
\lim_{k\to\frac{\phi}{(1-d)^{n-1}}+} V_L'(k) &> \lim_{k\to\frac{\phi}{(1-d)^{n}}+} V_L'(k).
\end{aligned}
$$

The first inequality implies that for any $n \in \mathbb{N}$, $\lim_{k\to\frac{\phi}{(1-d)^n}+} W_L'(k) < W_L'(\phi) < W_L'(m_L^*)$. Further, we can show that for any $n \in \mathbb{N}$, $W_L'(\cdot)$ is strictly decreasing on $(\frac{\phi}{(1-d)^{n-1}}, \frac{\phi}{(1-d)^n})$, so we have $W_L'(k) < W_L'(m_L^*)$ for any $k > \frac{\phi}{1-d}$. Since $W_L'(k) < W_L'(m_L^*)$ for $k \in (\frac{m_L^*}{1-d}, \frac{\phi}{1-d})$ and $\delta W_L'(m_L^*) = 1$, we have $\delta W_L'(k) < 1$ for $k > \frac{m_L^*}{1-d}$. Given party L's optimization problem, then it is optimal to have $m_L(k) = m_L^*$ for $k < \frac{m_L^*}{1-d}$ and $m_L(k) = (1-d)k$ for $k > \frac{m_L^*}{1-d}$.

From properties (i) and (iii) of $h(\cdot)$, we know that there exists $\bar{\rho} < \frac{1}{1-d}$ such that if $\rho > \bar{\rho}$, (A.2) is satisfied (i.e., $h(\rho) > 0$). For a given $\rho > \bar{\rho}$, (A.3) is satisfied when d is high enough or δ is low enough. Thus, we have obtained the desired conclusion. ∎

The proof above has established the existence of a GSE with $\hat{m}_H = \phi$ under the condition of $h(\rho) > 0$ (i.e., $\rho > \bar{\rho}$). We now discuss the equilibrium outcome when ρ is below the identified the cutoff $\bar{\rho}$, i.e., $h(\rho) < 0$. In the remainder of this section, we assume (A.3) always holds.

Proposition A.1. *There exists a simple equilibrium if* $\phi < \frac{m_L^*}{1-d}$.

Proof. We specialize the sufficient conditions identified in Proposition 5 to the case of $\pi = 0$. Then, a simple equilibrium exists if $\rho < \frac{1}{1-d}$, (A.3), and the following condition

$$(\rho(1-d))^{\frac{1-2\beta}{1-\beta}} + \delta(1-d)(1-\beta)\rho + \delta^2(1-d)^2 < 1 \tag{A.8}$$

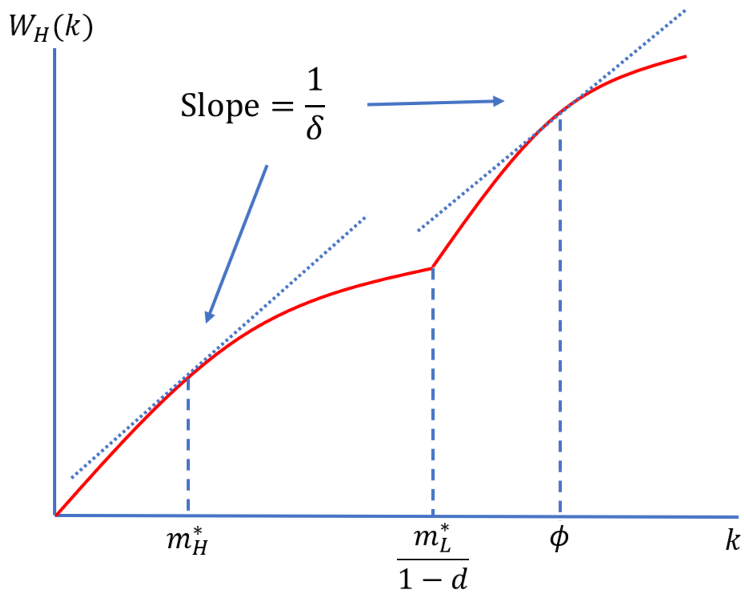
are satisfied. Note that (A.8) is (3) in the proof of Proposition 5 specialized to the case of $\pi = 0$, which can be shown to be equivalent to $\phi < \frac{m_L^*}{1-d}$.[2] Since $m_H^* < \phi$, $\phi < \frac{m_L^*}{1-d}$ implies $m_H^* < \frac{m_L^*}{1-d}$, or equivalently, for $\pi = 0$, $\rho < \frac{1}{1-d}$. Thus, a simple equilibrium exists if (A.3) holds and $\phi < \frac{m_L^*}{1-d}$. We have obtained the desired conclusion. ∎

From (A.8), we can identify a threshold $\tilde{\rho}$ such that $\phi = \frac{m_L^*}{1-d}$ for $\rho = \tilde{\rho}$. If $\rho < \tilde{\rho}$, $\phi < \frac{m_L^*}{1-d}$ and a simple equilibrium exists. Note that $\bar{\rho}$ is identified through $h(\bar{\rho}) = 0$. From (ii) in Lemma C.3, we know $\phi > \frac{m_L^*}{1-d}$ for $\rho \geq \bar{\rho}$. Then, we must have $\tilde{\rho} < \bar{\rho}$. What remains open is the equilibrium characterization for $\rho \in (\tilde{\rho}, \bar{\rho})$ such that $\phi > \frac{m_L^*}{1-d}$ and $h(\rho) < 0$. Our next proposition shows a GSE does not exist when ρ is at this intermediate level.

Proposition A.2. *There does not exist a GSE if* $\phi > \frac{m_L^*}{1-d}$ *and* $h(\rho) < 0$.

Proof. Suppose a GSE exists. Given party L's strategy in a GSE, party H's optimization problem can be illustrated in the figure below.

[2] It should be noted that for $\pi = 0$, (A.8) is also necessary for the existence of an SE because otherwise, for an initial stock $k = \frac{m_L^*}{(1-d)^2}$, party H would have incentives to make a positive investment (for example, to choose ϕ as its target stock).



When the initial stock is low, party H prefers m_H^* to ϕ because $h(\rho) < 0$ (following a similar argument as in the proof of Proposition 10). When the initial stock is high, for example, at $k = \frac{m_L^*}{(1-d)^2}$, due to irreversibility, party H can no longer choose m_H^*. Because the local optimum $\phi > \frac{m_L^*}{1-d}$, party H would choose to make a positive investment to reach ϕ. This contradicts with the definition of a GSE. Thus, a GSE does not exist. ■

Although a GSE does not exist for an intermediate level of ρ, we can still gain some insights about the qualitative feature of this equilibrium through numerical examples. Let $\pi = 0$, $\alpha_L = 0.85$, $\alpha_H = 1.15$, $d = \delta = 0.5$, and $\beta = 0.25$. It can be computed that $\phi > \frac{m_L^*}{1-d}$ and $h(\rho) < 0$ in this example. The figure below shows the numerical characterization of an MPE. In particular, the red line corresponds to the investment strategy $m_H(\cdot)$ for party H and the blue line corresponds to the investment strategy $m_L(\cdot)$ for party L. First and most importantly, the expected steady-state technology stock remains $\frac{m_H^*+m_L^*}{2}$. Second, there is discontinuity in the investment strategy for party H because for a high enough initial stock, party H will find the local optimum ϕ to be globally optimal. Third, discontinuity in party H's investment strategy leads to a discontinuous jump in the investment strategy for party L for some higher level of technology stock. Because this type of discontinuity propagates through the strategic interactions for higher levels of technology stock, an analytical characterization of the equilibrium is challenging. However, the qualitative features of this equilibrium, the expected technology stock coinciding with the case of reversible investment in particular, preserve for all the numerical exercises that we have conducted.

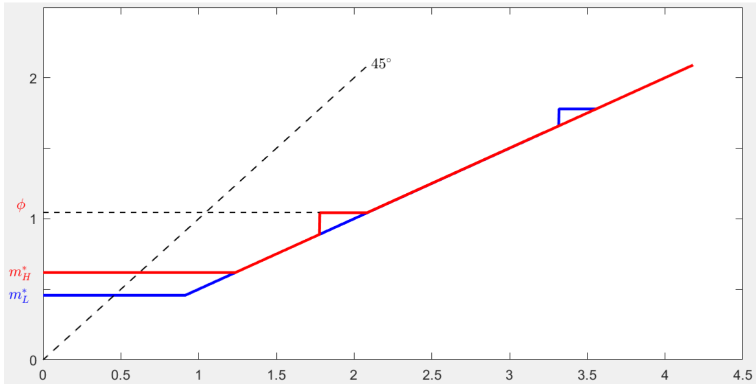


Figure 4 in the main text of our paper is based on the numerical exercises with $\pi = 0$, $\alpha_L + \alpha_H = 2$, $d = \delta = 0.5$, $\beta = 0.25$, and a varying ρ. In this case, $\tilde{\rho} \approx 1.20$ and $\bar{\rho} \approx 1.51$. The expected technology stock jumps upward from $\frac{m_H^* + m_L^*}{2}$ to $\frac{2-d}{2}\phi$ when polarization increases from slightly below $\bar{\rho}$ to slightly above $\bar{\rho}$. This jump in expected technology stock is quite substantial in size. We find at $\bar{\rho} \approx 1.51$, a small increase in polarization raises the expected technology stock from 0.53 to 0.82. It should also be noted that $\tilde{\rho}$ or $\bar{\rho}$ can be less than one for certain parameter values and in those cases, the simple equilibrium or the equilibrium illustrated in the figure above no longer exists.

Moreover, in a GSE with $\hat{m}_H = \phi$, the expected steady state technology stock is given by $\frac{2-d}{2}\phi$. It is clear that ϕ strictly increases in both α_L and α_H. The next proposition establishes the effect of a mean-preserving spread in α_i on ϕ.

Proposition A.3. *Let $\hat{\rho} > 1$ be the unique solution to the equation $1 + \delta(1-d)^{\frac{\beta}{1-\beta}} \rho^{\frac{\beta}{1-\beta}} = \frac{\beta}{1-\beta}\rho$. A mean-preserving spread in α_i increases ϕ if $\rho < \hat{\rho}$ and decreases ϕ if $\rho > \hat{\rho}$.*

Proof. Let $\alpha_L = \alpha - \varepsilon$ and $\alpha_H = \alpha + \varepsilon$. By the construction of ϕ, it suffices to consider how a mean-preserving spread in α_i affects $f(\varepsilon) \equiv \alpha_H \left(\frac{1}{1-\beta} \alpha_L^{\frac{\beta}{1-\beta}} + \delta(1-d)^{\frac{\beta}{1-\beta}} \alpha_H^{\frac{\beta}{1-\beta}} \right)$. We have

$$f'(\varepsilon) = \frac{\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} \left(1 - \frac{\beta}{1-\beta}\rho + \delta(1-d)^{\frac{\beta}{1-\beta}} \rho^{\frac{\beta}{1-\beta}} \right).$$

It is straightforward to show that the equation $1 - \frac{\beta}{1-\beta}\rho + \delta(1-d)^{\frac{\beta}{1-\beta}} \rho^{\frac{\beta}{1-\beta}} = 0$ admits a unique root for $\rho \geq 1$, denoted by $\hat{\rho}$, and we have $\hat{\rho} > 1$. If $\rho > \hat{\rho}$, $f'(\varepsilon) < 0$ and if $\rho < \hat{\rho}$, $f'(\varepsilon) > 0$. Thus, we have obtained the desired conclusion. ∎

According to this proposition, a mean-preserving spread in α_i raises the expected steady state technology stock when polarization is low and lowers it when polarization is high. Because we can have either $\hat{\rho} > \bar{\rho}$ or $\hat{\rho} < \bar{\rho}$, when irreversibility binds, an increase in polarization can first raise and then lower the expected technology stock in the steady state (if $\hat{\rho} > \bar{\rho}$), or can always lower the expected technology stock ($\hat{\rho} < \bar{\rho}$).

B The Case of No Depreciation ($d = 0$)

In this section, we present the proof of Proposition 11 concerning the characterization of a GSE when there is no depreciation ($d = 0$).

Proof of Proposition 11: Let $d = 0$ and investment be irreversible. We focus on the case of $\pi < 1$. (It is straightforward to show that the GSE we specify in what follows extends to the case of $\pi = 1$ with $\hat{m}_H = m_H^*$). We show that there exists a unique GSE with $\hat{m}_L = m_L^*$ and

$$\hat{m}_H = \left(\frac{\delta(\delta + \pi - 2\delta\pi)\alpha_H^{\frac{1}{1-\beta}} + \delta(1-\pi)\frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}}{1 - \delta^2 - 2\delta\pi + 2\delta^2\pi} \right)^{\frac{1-\beta}{1-2\beta}} \equiv \theta. \tag{B.1}$$

For $d = 0$,

$$m_i^* = \left(\frac{1}{\delta} - \pi \right)^{\frac{1-\beta}{2\beta-1}} \left(\pi\alpha_i^{\frac{1}{1-\beta}} + (1-\pi)\frac{\alpha_i \alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta} \right)^{\frac{1-\beta}{1-2\beta}}.$$

For $\pi < 1$, we have

$$\begin{aligned}
& (\delta + \delta\pi^2 - 2\delta\pi)\alpha_H^{\frac{1}{1-\beta}} + (1-\pi)\delta(\delta + \pi - 2\delta\pi)\frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} > 0 \\
\Leftrightarrow \quad & \left((1 - \delta\pi)(\delta + \pi - 2\delta\pi) - \pi(1 - \delta^2 - 2\delta\pi + 2\delta^2\pi) \right) \alpha_H^{\frac{1}{1-\beta}} \\
& + (1-\pi) \left((1 - \delta\pi) - (1 - \delta^2 - 2\delta\pi + 2\delta^2\pi) \right) \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} > 0 \\
\Leftrightarrow \quad & \left(\frac{1}{\delta} - \pi \right) \left(\delta(\delta + \pi - 2\delta\pi)\alpha_H^{\frac{1}{1-\beta}} + \delta(1-\pi)\frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} \right) \\
& > (1 - \delta^2 - 2\delta\pi + 2\delta^2\pi) \left(\pi\alpha_H^{\frac{1}{1-\beta}} + (1-\pi)\frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} \right),
\end{aligned}$$

which implies $\theta > m_H^*$ given the definition of θ and m_H^*.

To establish the existence and uniqueness of such a GSE, we proceed in two steps:

(i) for any GSE, $\hat{m}_L$ and $\hat{m}_H$ can be solved in unique closed forms. (ii) Based on the closed form solutions to $\hat{m}_L$ and $\hat{m}_H$, the strategy profile is indeed an equilibrium.

Step (i): Suppose there exists a GSE with the strategy profile as follows

$$m_i(k) = \begin{cases} \hat{m}_i & \text{if } 0 \leq k \leq \hat{m}_i \\ k & \text{if } k > \hat{m}_i \end{cases},$$

for some $\hat{m}_H \geq \hat{m}_L$, and $y_i(k) = \alpha_i^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}$. Based on this strategy profile and the assumption of no depreciation, the first derivatives of the value functions are given by

$$V_i'(k) = \begin{cases} \alpha_i^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + 1, & k < \hat{m}_i \\ \alpha_i^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta[\pi V_i'(k) + (1-\pi) W_i'(k)], & k > \hat{m}_i \end{cases} \tag{B.2}$$

$$W_i'(k) = \begin{cases} \frac{\alpha_i \alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}}, & k < \hat{m}_j \\ \frac{\alpha_i \alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta[\pi W_i'(k) + (1-\pi) V_i'(k)], & k > \hat{m}_j \end{cases} \tag{B.3}$$

We now show that $\hat{m}_H \neq \hat{m}_L$. Suppose on the contrary $\hat{m}_H = \hat{m}_L$. Then, the optimality of $\hat{m}_i$ guarantees $\lim_{k \to \hat{m}_i -} \delta(\pi V_i'(k) + (1-\pi) W_i'(k)) \geq 1$, because otherwise party i can always increase its payoff by lowering investment in technology marginally (for a low initial stock k). Thus, we have

$$\lim_{k \to \hat{m}_i -} V_i'(k) = \alpha_i^{\frac{1}{1-\beta}} \hat{m}_i^{\frac{2\beta-1}{1-\beta}} + 1 \leq \alpha_i^{\frac{1}{1-\beta}} \hat{m}_i^{\frac{2\beta-1}{1-\beta}} + \lim_{k \to \hat{m}_i -} \delta(\pi V_i'(k) + (1-\pi) W_i'(k)),$$

which implies

$$\begin{aligned} \lim_{k \to \hat{m}_i -} V_i'(k) &\leq \frac{1}{1-\delta\pi} \left[\alpha_i^{\frac{1}{1-\beta}} \hat{m}_i^{\frac{2\beta-1}{1-\beta}} + \delta(1-\pi) \lim_{k \to \hat{m}_i -} W_i'(k) \right] \\ &< \frac{1}{1-\delta\pi} \left[\alpha_i^{\frac{1}{1-\beta}} \hat{m}_i^{\frac{2\beta-1}{1-\beta}} + \delta(1-\pi) \lim_{k \to \hat{m}_i +} W_i'(k) \right] = \lim_{k \to \hat{m}_i +} V_i'(k), \end{aligned}$$

where the second inequality follows from $\lim_{k \to \hat{m}_i -} W_i'(k) < \lim_{k \to \hat{m}_i +} W_i'(k)$ (as in (B.3) with $\hat{m}_i = \hat{m}_j$), and the equality follows from (B.2). Then,

$$\lim_{k \to \hat{m}_i +} \delta(\pi V_i'(k) + (1-\pi) W_i'(k)) > \lim_{k \to \hat{m}_i -} \delta(\pi V_i'(k) + (1-\pi) W_i'(k)) \geq 1,$$

which implies that party i is better off by raising m marginally above $\hat{m}_i$. This leads a contradiction with the optimality of $\hat{m}_i$ for party i for low k. Thus, $\hat{m}_H \neq \hat{m}_L$. Since

$\hat{m}_H \geq \hat{m}_L$ by the definition of a GSE, we have $\hat{m}_H > \hat{m}_L$.

For $d = 0$, we can directly solve for the value functions without exploiting any recursiveness. In particular, from (B.2) and (B.3), for $k > \hat{m}_H$, we have

$$\begin{aligned} V_i'(k) &= \frac{(1-\delta\pi)\alpha_i^{\frac{1}{1-\beta}} + \delta(1-\pi)\frac{\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta}}{(1-\delta\pi)^2 - \delta^2(1-\pi)^2} k^{\frac{2\beta-1}{1-\beta}} \\ W_i'(k) &= \frac{\delta(1-\pi)\alpha_i^{\frac{1}{1-\beta}} + (1-\delta\pi)\frac{\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta}}{(1-\delta\pi)^2 - \delta^2(1-\pi)^2} k^{\frac{2\beta-1}{1-\beta}}, \end{aligned}$$

and for $\hat{m}_H > k > \hat{m}_L$ (we have shown $\hat{m}_L < \hat{m}_H$), we can show

$$\begin{aligned} W_H'(k) &= \frac{\delta(1-\pi)\alpha_H^{\frac{1}{1-\beta}} + \frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}}{1-\delta\pi} k^{\frac{2\beta-1}{1-\beta}} + \frac{\delta(1-\pi)}{1-\delta\pi} \\ V_L'(k) &= \frac{\alpha_L^{\frac{1}{1-\beta}} + \delta(1-\pi)\frac{\alpha_L\alpha_H^{\frac{\beta}{1-\beta}}}{1-\beta}}{1-\delta\pi} k^{\frac{2\beta-1}{1-\beta}}. \end{aligned}$$

Clearly, the function $\pi V_i + (1-\pi)W_i$ is piecewise strictly concave.

We now show $\delta(\pi V_H'(\hat{m}_H) + (1-\pi)W_H'(\hat{m}_H)) = 1$. Specifically,

$$\lim_{k\to\hat{m}_H-} \delta(\pi V_H'(k) + (1-\pi)W_H'(k)) = \lim_{k\to\hat{m}_H+} \delta(\pi V_H'(k) + (1-\pi)W_H'(k)) = 1.$$

By the optimality of $\hat{m}_H$, we have $\lim_{k\to\hat{m}_H-} \delta(\pi V_H'(k) + (1-\pi)W_H'(k)) \geq 1$. Suppose $\lim_{k\to\hat{m}_H-} \delta(\pi V_H'(k) + (1-\pi)W_H'(k)) > 1$. We have

$$\begin{aligned} \lim_{k\to\hat{m}_H-} V_H'(k) &= \alpha_H^{\frac{1}{1-\beta}} \hat{m}_H^{\frac{2\beta-1}{1-\beta}} + 1 \\ &< \alpha_H^{\frac{1}{1-\beta}} \hat{m}_H^{\frac{2\beta-1}{1-\beta}} + \lim_{k\to\hat{m}_H-} \delta(\pi V_H'(k) + (1-\pi)W_H'(k)) \\ &= \alpha_H^{\frac{1}{1-\beta}} \hat{m}_H^{\frac{2\beta-1}{1-\beta}} + \frac{\delta(1-\pi)}{1-\delta\pi}\frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}\hat{m}_H^{\frac{2\beta-1}{1-\beta}} + \lim_{k\to\hat{m}_H-} \delta\left(\pi V_H'(k) + \frac{\delta(1-\pi)^2}{1-\delta\pi}V_H'(k)\right), \quad \text{(B.4)} \\ \lim_{k\to\hat{m}_H+} V_H'(k) &= \alpha_H^{\frac{1}{1-\beta}} \hat{m}_H^{\frac{2\beta-1}{1-\beta}} + \lim_{k\to\hat{m}_H+} \delta(\pi V_H'(k) + (1-\pi)W_H'(k)) \\ &= \alpha_H^{\frac{1}{1-\beta}} \hat{m}_H^{\frac{2\beta-1}{1-\beta}} + \frac{\delta(1-\pi)}{1-\delta\pi}\frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta}\hat{m}_H^{\frac{2\beta-1}{1-\beta}} + \lim_{k\to\hat{m}_H+} \delta\left(\pi V_H'(k) + \frac{\delta(1-\pi)^2}{1-\delta\pi}V_H'(k)\right), \quad \text{(B.5)} \end{aligned}$$

where the second and the last equalities follow from (B.3) and $\hat{m}_H > \hat{m}_L$. (B.4) and (B.5) imply that $\lim_{k\to\hat{m}_H-} V_H'(k) < \lim_{k\to\hat{m}_H+} V_H'(k)$, which, from (B.3) and $\hat{m}_H > \hat{m}_L$

(and $\pi < 1$), further implies $\lim_{k\to\hat{m}_H-} W'_H(k) < \lim_{k\to\hat{m}_H+} W'_H(k)$. Therefore, we have

$$\lim_{k\to\hat{m}_H+} \delta(\pi V'_H(k) + (1-\pi)W'_H(k)) > \lim_{k\to\hat{m}_H-} \delta(\pi V'_H(k) + (1-\pi)W'_H(k)) > 1,$$

contradicting to the optimality of $\hat{m}_H$. Thus, we have $\lim_{k\to\hat{m}_H-} \delta(\pi V'_H(k)+(1-\pi)W'_H(k)) = 1$. Through a similar argument that rules out $\lim_{k\to\hat{m}_H-} \delta(\pi V'_H(k) + (1-\pi)W'_H(k)) > 1$, we can show that $\lim_{k\to\hat{m}_H-} \delta(\pi V'_H(k) + (1-\pi)W'_H(k)) = 1$ implies $\lim_{k\to\hat{m}_H+} \delta(\pi V'_H(k) + (1-\pi)W'_H(k)) = 1$, thus establishing the claim.

From $\delta(\pi V'_H(\hat{m}_H)+(1-\pi)W'_H(\hat{m}_H)) = 1$, it is straightforward to show that $\hat{m}_H = \theta$. Similarly, we can show

$$\lim_{k\to\hat{m}_L-} \delta(\pi V'_L(k) + (1-\pi)W'_L(k)) = \lim_{k\to\hat{m}_L+} \delta(\pi V'_L(k) + (1-\pi)W'_L(k)) = 1,$$

implying $\hat{m}_L = m^*_L$.

Step (ii): To show that the strategy profile with $\hat{m}_H = \theta$ and $\hat{m}_L = m^*_L$ is indeed an equilibrium, we provide further characterization of the value functions. Note that

$$\lim_{k\to\hat{m}_L-} \delta(\pi V'_H(k) + (1-\pi)W'_H(k)) \geq \lim_{k\to\hat{m}_L-} \delta(\pi V'_L(k) + (1-\pi)W'_L(k)) = 1,$$

where equality holds if and only if $\alpha_L = \alpha_H$. Since V'_H is continuous at $k = \hat{m}_L$ and W'_H has a discontinuous upward jump at $k = \hat{m}_L$, we have

$$\lim_{k\to\hat{m}_L+} \delta(\pi V'_H(k) + (1-\pi)W'_H(k)) > \lim_{k\to\hat{m}_L-} \delta(\pi V'_H(k) + (1-\pi)W'_H(k)) \geq 1,$$

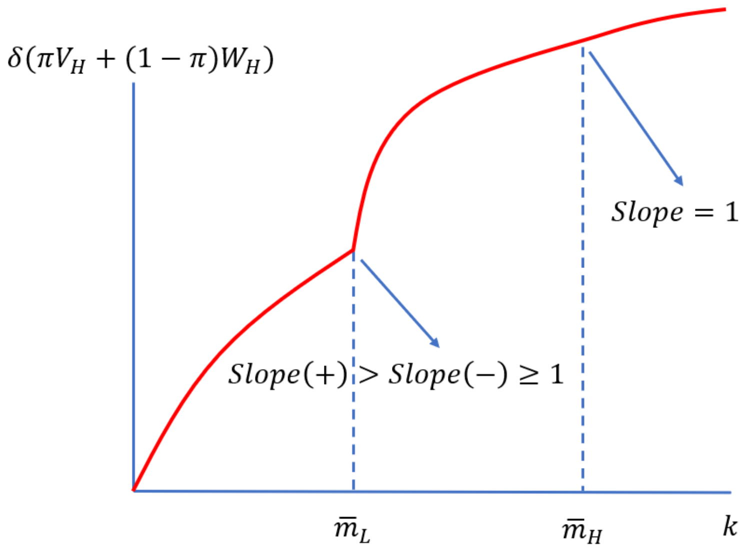
Symmetrically, for party L, we have

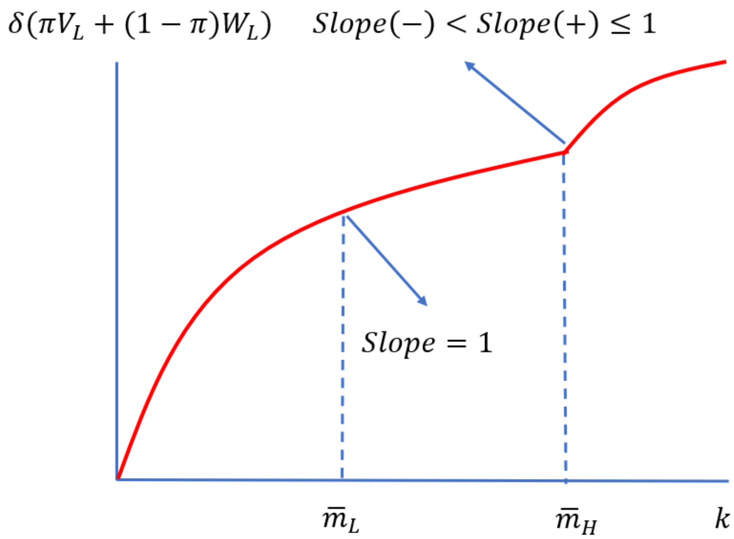
$$\lim_{k\to\hat{m}_H+} \delta(\pi V'_L(k) + (1-\pi)W'_L(k)) \leq \lim_{k\to\hat{m}_H+} \delta(\pi V'_H(k) + (1-\pi)W'_H(k)) = 1.$$

Since both W'_L and V'_L have a discontinuous upward jump at $k = \hat{m}_H$, we have

$$\lim_{k\to\hat{m}_H-} \delta(\pi V'_L(k) + (1-\pi)W'_L(k)) < \lim_{k\to\hat{m}_H+} \delta(\pi V'_L(k) + (1-\pi)W'_L(k)) \leq 1.$$

Based on the derivations so far, we illustrate $\delta(\pi V_i + (1-\pi)W_i)$ as follows





Evidently in the figures above, the strategy profile with $\hat{m}_i$ derived above is optimal. ∎

C Auxiliary Results and Their Proofs

In this section, we collect the auxiliary results and present the proofs for them.

Lemma C.1. $m_L^* \leq m_H^* < \bar{m}_p$.

Proof. To establish $m_L^* \leq m_H^*$, we have

$$\frac{m_L^*}{m_H^*} = \left(\frac{\pi \alpha_L^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta} \alpha_L \alpha_H^{\frac{\beta}{1-\beta}}}{\pi \alpha_H^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta} \alpha_H \alpha_L^{\frac{\beta}{1-\beta}}} \right)^{\frac{1-\beta}{1-2\beta}} = \left(\frac{\pi \rho^{-\frac{\beta}{1-\beta}} + \frac{1-\pi}{1-\beta}}{\pi \rho + \frac{1-\pi}{1-\beta} \rho^{\frac{1-2\beta}{1-\beta}}} \right)^{\frac{1-\beta}{1-2\beta}} \leq 1,$$

where the inequality follows from $\rho \geq 1$ and $\beta \in (0, \frac{1}{2})$. To establish $m_H^* < \bar{m}_p$, we have

$$\begin{aligned}
\frac{\bar{m}_p}{m_H^*} &= \frac{\left(\frac{\delta}{1-\delta+\delta d}\right)^{\frac{1-\beta}{1-2\beta}} (\alpha_L + \alpha_H)^{\frac{1}{1-2\beta}}}{\left(\frac{1}{\delta} - (1-d)\pi\right)^{\frac{1-\beta}{2\beta-1}} \left(\pi \alpha_H^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}}} \\
&= \left(\frac{1-\delta(1-d)\pi}{1-\delta(1-d)}\right)^{\frac{1-\beta}{1-2\beta}} \left(\frac{(\alpha_L+\alpha_H)^{\frac{1}{1-\beta}}}{\pi \alpha_H^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}\right)^{\frac{1-\beta}{1-2\beta}} \\
&\geq \left(\frac{(\alpha_L+\alpha_H)^{\frac{1}{1-\beta}}}{\pi \alpha_H^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}\right)^{\frac{1-\beta}{1-2\beta}},
\end{aligned} \tag{C.1}$$

where the inequality follows from $\pi \leq 1$. We claim $(\alpha_L + \alpha_H)^{\frac{1}{1-\beta}} > \pi \alpha_H^{\frac{1}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}$. Because the right-hand side of this inequality is linear in π, it suffices to show that the inequality holds for $\pi = 0$ and $\pi = 1$. It is straightforward to show that the inequality holds for $\pi = 1$. For $\pi = 0$, the inequality can be simplified as $(1-\beta)(1+\rho)^{\frac{1}{1-\beta}} > \rho$, which can be shown to hold for $\rho \geq 1$ and $\beta \in (0, \frac{1}{2})$. Thus, the claim is established. From (C.1), we must have $\bar{m}_p > m_H^*$. ∎

Lemma C.2. $(\frac{1}{1-\beta})^{\frac{1-\beta}{\beta}}$ *is increasing in* $(1-\beta)$.

Proof. We have $\frac{d \ln\left((\frac{1}{1-\beta})^{\frac{1-\beta}{\beta}}\right)}{d\beta} = \frac{\beta + \ln(1-\beta)}{\beta^2}$. Since $\frac{d \ln(1-\beta)+\beta}{d\beta} = \frac{1}{\beta-1} + 1 < 0$ for $\beta \in (0, \frac{1}{2})$ and $\ln(1-\beta) + \beta = 0$ for $\beta = 0$, we must have $\ln(1-\beta) + \beta < 0$ for $\beta \in (0, \frac{1}{2})$. Thus, $\frac{d \ln\left((\frac{1}{1-\beta})^{\frac{1-\beta}{\beta}}\right)}{d\beta} < 0$, implying that $(\frac{1}{1-\beta})^{\frac{1-\beta}{\beta}}$ is increasing in $(1-\beta)$. ∎

Lemma C.3. *Let* $d < 1$ *and* $\pi = 0$. *The function* $h(\cdot)$ *has the following properties: (i)* $h(\rho)$ *is strictly increasing in* ρ; *(ii) If* $h(\rho) \geq 0$, *then* $\phi > \frac{m_L^*}{1-d}$; *(iii)* $h(\frac{1}{1-d}) > 0$.

Proof. It is straightforward to establish (i). For (ii), we first specialize m_L^* and m_H^* for $\pi = 0$:

$$\begin{aligned}
m_L^* &= \delta^{\frac{1-\beta}{1-2\beta}} (1-\beta)^{\frac{1-\beta}{2\beta-1}} \alpha_L^{\frac{1-\beta}{1-2\beta}} \alpha_H^{\frac{\beta}{1-2\beta}} \\
m_H^* &= \delta^{\frac{1-\beta}{1-2\beta}} (1-\beta)^{\frac{1-\beta}{2\beta-1}} \alpha_H^{\frac{1-\beta}{1-2\beta}} \alpha_L^{\frac{\beta}{1-2\beta}},
\end{aligned}$$

where we note $\frac{m_H^*}{m_L^*}=\rho$ and $\phi > m_H^*$ with ϕ defined as in (A.1). We have

$$\begin{aligned}
& \int_{m_H^*}^{\phi} \frac{\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} dk + \int_{\frac{m_L^*}{1-d}}^{\phi} \delta(1-d)\left[\alpha_H^{\frac{1}{1-\beta}}((1-d)k)^{\frac{2\beta-1}{1-\beta}} + (1-d)\right] dk - \frac{1}{\delta}(\phi - m_H^*) \\
= \; & \frac{1}{\beta}\alpha_H \alpha_L^{\frac{\beta}{1-\beta}}\left(\phi^{\frac{\beta}{1-\beta}} - m_H^{*\,\frac{\beta}{1-\beta}}\right) + \frac{1-\beta}{\beta}\delta(1-d)^{\frac{\beta}{1-\beta}}\alpha_H^{\frac{1}{1-\beta}}\left(\phi^{\frac{\beta}{1-\beta}} - \left(\frac{m_L^*}{1-d}\right)^{\frac{\beta}{1-\beta}}\right) \\
& + \delta(1-d)^2\left(\phi - \frac{m_L^*}{1-d}\right) - \frac{1}{\delta}(\phi - m_H^*) \\
= \; & \left(\frac{1}{\beta}\alpha_H \alpha_L^{\frac{\beta}{1-\beta}} + \frac{1-\beta}{\beta}\delta(1-d)^{\frac{\beta}{1-\beta}}\alpha_H^{\frac{1}{1-\beta}}\right)\phi^{\frac{\beta}{1-\beta}} - \frac{1}{\beta}\alpha_H\alpha_L^{\frac{\beta}{1-\beta}} m_H^{*\,\frac{\beta}{1-\beta}} - \frac{1-\beta}{\beta}\delta\alpha_H^{\frac{1}{1-\beta}} m_L^{*\,\frac{\beta}{1-\beta}} \\
& + \frac{1}{\delta}(\delta^2(1-d)^2 - 1)\phi - \delta(1-d)m_L^* + \frac{1}{\delta}m_H^* \\
= \; & \frac{1-\beta}{\beta}\left(\frac{1}{\delta} - \delta(1-d)^2\right)\phi - \frac{1-\beta}{\beta\delta}m_H^* - \frac{(1-\beta)^2}{\beta}m_H^* \\
& + \frac{1}{\delta}(\delta^2(1-d)^2 - 1)\phi - \delta(1-d)m_L^* + \frac{1}{\delta}m_H^* \\
= \; & \frac{m_H^*}{\beta\delta}\left[(1-2\beta)(1-\delta^2(1-d)^2)\frac{\phi}{m_H^*} - \left((1-2\beta) + (1-\beta)^2\delta + \beta\delta^2(1-d)\frac{m_L^*}{m_H^*}\right)\right] \\
= \; & \frac{m_H^*}{\beta\delta}\left\{\frac{(1-2\beta)\left(1+(1-\beta)\delta(1-d)^{\frac{\beta}{1-\beta}}\rho^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}}}{(1-\delta^2(1-d)^2)^{\frac{\beta}{1-2\beta}}} - \left[1-2\beta+(1-\beta)^2\delta + \frac{\beta\delta^2(1-d)}{\rho}\right]\right\} \\
= \; & \frac{m_H^*}{\beta\delta}h(\rho),
\end{aligned} \tag{C.2}$$

where the third and fifth equations follow from the definition of ϕ, m_H^*, and m_L^*. Because $\ell(x) \equiv \frac{1}{\beta}\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}x^{\frac{\beta}{1-\beta}} - \frac{x}{\delta}$ for $x \geq 0$ attains its maximum at $x = m_H^*$, we have

$$\frac{1}{\beta}\alpha_H\alpha_L^{\frac{\beta}{1-\beta}} m_H^{*\,\frac{\beta}{1-\beta}} - \frac{m_H^*}{\delta} > \frac{1}{\beta}\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}\phi^{\frac{\beta}{1-\beta}} - \frac{\phi}{\delta},$$

or equivalently,

$$\int_{m_H^*}^{\phi} \frac{\alpha_H\alpha_L^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} dk - \frac{1}{\delta}(\phi - m_H^*) < 0. \tag{C.3}$$

Then, if $h(\rho) \geq 0$, it follows from (C.2) and (C.3) that

$$\int_{\frac{m_L^*}{1-d}}^{\phi} \delta(1-d)\left[\alpha_H^{\frac{1}{1-\beta}}((1-d)k)^{\frac{2\beta-1}{1-\beta}} + (1-d)\right] dk > 0,$$

which implies $\phi > \frac{m_L^*}{1-d}$, thus establishing (ii).

For (iii), we have

$$h\left(\frac{1}{1-d}\right) = \frac{(1-2\beta)(1+(1-\beta)\delta)^{\frac{1-\beta}{1-2\beta}}}{(1-\delta^2(1-d)^2)^{\frac{\beta}{1-2\beta}}} - \left(1-2\beta+(1-\beta)^2\delta+\beta\delta^2(1-d)^2\right) \equiv \tilde{h}(\delta).$$

$$\begin{aligned}
\tilde{h}(0) &= (1-2\beta)-(1-2\beta)=0 \\
\tilde{h}'(\delta) &= \frac{(1-\beta)^2(1+(1-\beta)\delta)^{\frac{\beta}{1-2\beta}}}{(1-\delta^2(1-d)^2)^{\frac{\beta}{1-2\beta}}} + \frac{2\beta\delta(1-d)^2(1+(1-\beta)\delta)^{\frac{1-\beta}{1-2\beta}}}{(1-\delta^2(1-d)^2)^{\frac{1-\beta}{1-2\beta}}} - (1-\beta)^2 - 2\beta\delta(1-d)^2 \\
&> (1-\beta)^2 + 2\beta\delta(1-d)^2 - (1-\beta)^2 - 2\beta\delta(1-d)^2 = 0,
\end{aligned}$$

where the inequality holds for $\delta > 0$. Since $\tilde{h}(0) = 0$ and $\tilde{h}'(\delta) > 0$ for $\delta > 0$, we have $\tilde{h}(\delta) > 0$ for any $\delta > 0$, thereby establishing (iii). Thus, we have obtained the desired conclusion. ■

D Extensions

In this section, we provide formal analysis of the four extensions discussed in the concluding section.

D.1 Interaction between Public and Private Sector

In this subsection, we consider interaction between public and private sector. We assume that the incumbent chooses and bears the cost of technology investment k and the private sector chooses and bears the cost of nondurable input y. Let the private sector's utility from the public good be $\alpha v(y_t, k_{t-1})$. Importantly, the private sector's valuation of the public good, α, does not vary with incumbent identity. To simplify notation, we normalize α to be one.

Since the choice of y_t is based on intra-temporal optimization, we again have $y_t = y(k_{t-1}) = k_{t-1}^{\frac{\beta}{1-\beta}}$. Public-good provision depends now on the private sector's response to k. Because of complementarity ($\beta > 0$), y_t increases with k_{t-1}.

Each party's strategy is reduced to a mapping $m_i : \mathbb{R}_+ \to \mathbb{R}_+$. The conditions for an equilibrium are modified as

E1. Given (V_L, W_L, V_H, W_H), party i's strategy is the solution to

$$\max_m \{v_i(y(k), k) + (1-d)k - m + \delta [\pi V_i(m) + (1-\pi) W_i(m)]\},$$

subject to $m \geq (1-d)k$ when investment is irreversible and $m \geq 0$ when investment is reversible.

E2. Given (m_L, m_H), the payoff functions satisfy the following functional equations

$$\begin{aligned} V_i(k) &= v_i(y(k), k) + (1-d)k - m_i(k) + \delta [\pi V_i(m_i(k)) + (1-\pi) W_i(m_i(k))], \\ W_i(k) &= v_i(y(k), k) + \delta [\pi W_i(m_j(k)) + (1-\pi) V_i(m_j(k))]. \end{aligned}$$

Following essentially the same argument as in the benchmark setting of the paper, we can show that there exists a simple equilibrium when polarization is low enough and either depreciation or discounting is high enough. In this simple equilibrium, we have

$$m_i^* = \left(\frac{1}{\delta} - \pi(1-d)\right)^{\frac{1-\beta}{2\beta-1}} \left(\frac{\alpha_i}{1-\beta}\right)^{\frac{1-\beta}{1-2\beta}},$$

implying that m_i^* depends only on α_i but not on α_j, and that m_i^* increases monotonically with π. Moreover, when polarization is high enough such that $(1-d)m_H^* > m_L^*$, again we can show that $\hat{m}_H > m_H^*$ in a generalized simple equilibrium.

D.2 Partial Reversibility

We turn to partial reversibility. Suppose that when a party chooses $z < 0$ (divestment), it reduces the technology stock by $|z|$ and gains a fraction $r \in [0, 1]$ of the value of the divestment. The parameter r represents the degree of reversibility where $r = 1$ corresponds to the (fully) reversible case and $r = 0$ corresponds to the irreversible case. The following proposition characterizes the equilibrium for $r \in (0, 1)$ when depreciation is sufficiently high.

Proposition D.1. *Let $r \in (0, 1)$. If depreciation is sufficiently high, then there exists an equilibrium with $y_i(k) = \alpha_i^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}$ and*

$$m_i(k) = \begin{cases} m_i^* & \text{if } 0 \leq k \leq \frac{m_i^*}{1-d} \\ (1-d)k & \text{if } \frac{m_i^*}{1-d} < k \leq \frac{m_i^{**}}{1-d} \\ m_i^{**} & \text{if } k > \frac{m_i^{**}}{1-d} \end{cases},$$

where m_i^* is defined as in a simple equilibrium and

$$m_i^{**} \equiv \left(\frac{r}{\delta} - (1-d)\pi\right)^{\frac{1-\beta}{2\beta-1}} \alpha_i^{\frac{1-\beta}{1-2\beta}} \left(\pi\alpha_i^{\frac{\beta}{1-\beta}} + \frac{1-\pi}{1-\beta}\alpha_j^{\frac{\beta}{1-\beta}}\right)^{\frac{1-\beta}{1-2\beta}} > m_i^*.$$

Proof. For a sufficiently large d, $\frac{r}{\delta} - (1-d)\pi > 0$ so m_i^{**} is well defined and it is straightforward to show $m_i^{**} > m_i^*$. We let d be sufficiently large such that $(1-d)m_H^{**} < m_L^*$.

We now want to verify that the strategy profile is an equilibrium if d is sufficiently large. Based on the strategy profile, we can show that the value functions satisfy

$$V_i'(k) = \begin{cases} \alpha_i^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + (1-d), & k < \frac{m_i^*}{1-d} \\ \alpha_i^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi V_i'((1-d)k) + (1-\pi)W_i'((1-d)k)\right], & \frac{m_i^*}{1-d} < k < \frac{m_i^{**}}{1-d} \\ \alpha_i^{\frac{1}{1-\beta}} k^{\frac{2\beta-1}{1-\beta}} + (1-d)r, & k > \frac{m_i^{**}}{1-d} \end{cases}$$

$$W_i'(k) = \begin{cases} \frac{\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}}, & k < \frac{m_j^*}{1-d} \\ \frac{\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}} + \delta(1-d)\left[\pi W_i'((1-d)k) + (1-\pi)V_i'((1-d)k)\right], & \frac{m_j^*}{1-d} < k < \frac{m_j^{**}}{1-d} \\ \frac{\alpha_i\alpha_j^{\frac{\beta}{1-\beta}}}{1-\beta} k^{\frac{2\beta-1}{1-\beta}}, & k > \frac{m_j^{**}}{1-d} \end{cases}$$

Because $(1-d)m_H^{**} < m_L^*$, it is straightforward to show that $\pi V_H' + (1-\pi)W_H'$ is piecewise continuous on $(0, \frac{m_L^*}{1-d})$, $(\frac{m_L^*}{1-d}, \frac{m_H^*}{1-d})$, $(\frac{m_H^*}{1-d}, \frac{m_L^{**}}{1-d})$, $(\frac{m_L^{**}}{1-d}, \frac{m_H^{**}}{1-d})$, and $(\frac{m_H^{**}}{1-d}, \infty)$.[3] Moreover, we have

$$\begin{aligned} \lim_{k \to \frac{m_L^*}{1-d}+} \pi V_H'(k) + (1-\pi)W_H'(k) &> \lim_{k \to \frac{m_L^*}{1-d}-} \pi V_H'(k) + (1-\pi)W_H'(k) \\ \lim_{k \to \frac{m_H^*}{1-d}+} \pi V_H'(k) + (1-\pi)W_H'(k) &= \lim_{k \to \frac{m_H^*}{1-d}-} \pi V_H'(k) + (1-\pi)W_H'(k) \\ \lim_{k \to \frac{m_L^{**}}{1-d}+} \pi V_H'(k) + (1-\pi)W_H'(k) &< \lim_{k \to \frac{m_L^{**}}{1-d}-} \pi V_H'(k) + (1-\pi)W_H'(k) \\ \lim_{k \to \frac{m_H^{**}}{1-d}+} \pi V_H'(k) + (1-\pi)W_H'(k) &= \lim_{k \to \frac{m_H^{**}}{1-d}-} \pi V_H'(k) + (1-\pi)W_H'(k). \end{aligned}$$

Since $\frac{m_L^*}{1-d} > m_H^{**} > m_H^*$, party H's strategy is optimal as long as $\lim_{k \to \frac{m_L^*}{1-d}+} \pi V_H'(k) + (1-\pi)W_H'(k) < \frac{r}{\delta}$, which is satisfied for a sufficiently large d. Similarly, we can show the

[3] Here, we assume for simplicity $m_L^{**} > m_H^*$. Our proof easily extends to the case of $m_L^{**} \leq m_H^*$.

piece-wise continuity for $\pi V_L' + (1-\pi)W_L'$ and

$$\begin{aligned}
\lim_{k\to\frac{m_L^*}{1-d}+} \pi V_L'(k) + (1-\pi)W_L'(k) &= \lim_{k\to\frac{m_L^*}{1-d}-} \pi V_L'(k) + (1-\pi)W_L'(k) \\
\lim_{k\to\frac{m_H^*}{1-d}+} \pi V_L'(k) + (1-\pi)W_L'(k) &> \lim_{k\to\frac{m_H^*}{1-d}-} \pi V_L'(k) + (1-\pi)W_L'(k) \\
\lim_{k\to\frac{m_L^{**}}{1-d}+} \pi V_L'(k) + (1-\pi)W_L'(k) &= \lim_{k\to\frac{m_L^{**}}{1-d}-} \pi V_L'(k) + (1-\pi)W_L'(k) \\
\lim_{k\to\frac{m_H^{**}}{1-d}+} \pi V_L'(k) + (1-\pi)W_L'(k) &< \lim_{k\to\frac{m_H^{**}}{1-d}-} \pi V_L'(k) + (1-\pi)W_L'(k).
\end{aligned}$$

Thus, party L's strategy is optimal as long as $\lim_{k\to\frac{m_H^*}{1-d}+} \pi V_L'(k) + (1-\pi)W_L'(k) < \frac{r}{\delta}$, which is also satisfied for a sufficiently large d. Therefore, we have obtained the desired conclusion. ∎

It is straightforward to show that the steady-state distribution of technology stock in the equilibrium that we have established for $r \in (0,1)$ is the same as that under a simple equilibrium, i.e., $k = m_L^*$ with probability $\frac{1}{2}$ and $k = m_H^*$ with probability $\frac{1}{2}$.

Corollary D.1. *For any $r \in [0,1]$, if depreciation is sufficiently high, the steady state distribution of technology stock is m_L^* with probability $\frac{1}{2}$ and m_H^* with probability $\frac{1}{2}$.*

D.3 Cost Sharing

We now consider cost sharing of investment and public good provision. Suppose the share of (opportunity) costs born by the incumbent is η and that by the non-incumbent is $(1-\eta)$. In our baseline model, $\eta = 1$. The following proposition is a direct generalization of Proposition 4 for reversible investment.

Proposition D.2. *If investment is reversible and η is sufficiently high, there exists an equilibrium with $y_i(k) = \left(\frac{\alpha_i}{\eta}\right)^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}$ and $m_i(k) = \tilde{m}_i$ for any k and $i \in \{L, H\}$, where*

$$\tilde{m}_i \equiv \eta^{-\frac{\beta}{1-2\beta}} \left[\frac{\pi \alpha_i^{\frac{1}{1-\beta}} + (1-\pi)\left(\frac{\alpha_i}{1-\beta} - \frac{\beta(1-\eta)\alpha_j}{(1-\beta)\eta}\right)\alpha_j^{\frac{\beta}{1-\beta}}}{\frac{\eta}{\delta} - (\pi\eta + (1-\pi)(1-\eta))(1-d)} \right]^{\frac{1-\beta}{1-2\beta}}.$$

Proof. Since public-good provision is a within-period optimization problem, it is straightforward to show $y_i(k) = \left(\frac{\alpha_i}{\eta}\right)^{\frac{1}{1-\beta}} k^{\frac{\beta}{1-\beta}}$ in any equilibrium. Based on the strategy profile, we can then construct the value functions as follows

$$\begin{aligned} V_i(k) &= \frac{1-\beta}{\beta}\eta^{-\frac{\beta}{1-\beta}}\alpha_i^{\frac{1}{1-\beta}}k^{\frac{\beta}{1-\beta}}+\eta(1-d)k+c_{Vi} \\ W_i(k) &= \left(\frac{\alpha_i}{\beta}\eta^{-\frac{\beta}{1-\beta}}\alpha_j^{\frac{\beta}{1-\beta}}-(1-\eta)\eta^{-\frac{1}{1-\beta}}\alpha_j^{\frac{1}{1-\beta}}\right)k^{\frac{\beta}{1-\beta}}+(1-\eta)(1-d)k+c_{Wi}, \end{aligned}$$

which yield

$$\begin{aligned} V_i'(k) &= \eta^{-\frac{\beta}{1-\beta}}\alpha_i^{\frac{1}{1-\beta}}k^{\frac{2\beta-1}{1-\beta}}+\eta(1-d) \\ W_i'(k) &= \frac{\beta}{1-\beta}\left(\frac{\alpha_i}{\beta}-\frac{(1-\eta)\alpha_j}{\eta}\right)\left(\frac{\alpha_j}{\eta}\right)^{\frac{\beta}{1-\beta}}k^{\frac{2\beta-1}{1-\beta}}+(1-\eta)(1-d), \end{aligned}$$

where c_{Vi} and c_{Wi} are constant with respect to k. Then, we have

$$\begin{aligned} \pi V_i'(k)+(1-\pi)W_i'(k) &= \eta^{-\frac{\beta}{1-\beta}}\left[\pi\alpha_i^{\frac{1}{1-\beta}}+(1-\pi)\left(\frac{\alpha_i}{1-\beta}-\frac{\beta(1-\eta)\alpha_j}{(1-\beta)\eta}\right)\alpha_j^{\frac{\beta}{1-\beta}}\right]k^{\frac{2\beta-1}{1-\beta}} \\ &\quad +\pi\eta(1-d)+(1-\pi)(1-\eta)(1-d). \end{aligned} \tag{D.1}$$

From (D.1), we first note that η cannot be too small because otherwise both parties would choose to raise technology stock to infinity. In particular, we require $\frac{\eta}{\delta}>(\pi\eta+(1-\pi)(1-\eta))(1-d)$, or equivalently,

$$\eta>\frac{\delta(1-d)(1-\pi)}{1+\delta(1-d)(1-2\pi)}. \tag{D.2}$$

Note that (D.2) holds for $\eta\geq\frac{1}{2}$. Moreover, we pick η sufficiently large such that

$$\frac{\beta(1-\eta)}{\eta}<\rho<\frac{\eta}{\beta(1-\eta)}, \tag{D.3}$$

which guarantees $\pi\alpha_i^{\frac{1}{1-\beta}}+(1-\pi)\left(\frac{\alpha_i}{1-\beta}-\frac{\beta(1-\eta)\alpha_j}{(1-\beta)\eta}\right)\alpha_j^{\frac{\beta}{1-\beta}}>0$ for $i\in\{L,H\}$, implying that $(\pi V_i+(1-\pi)W_i)$ is strictly concave. Then, under reversible investment, the first order condition $\delta[\pi V_i'(k)+(1-\pi)W_i'(k)]=\eta$ is sufficient to pin down the optimal target stock. The first order condition can be written explicitly as

$$\eta^{-\frac{\beta}{1-\beta}}\left[\frac{\pi\alpha_i^{\frac{1}{1-\beta}}+(1-\pi)\left(\frac{\alpha_i}{1-\beta}-\frac{\beta(1-\eta)\alpha_j}{(1-\beta)\eta}\right)\alpha_j^{\frac{\beta}{1-\beta}}}{\frac{\eta}{\delta}-(\pi\eta+(1-\pi)(1-\eta))(1-d)}\right]=k^{\frac{1-2\beta}{1-\beta}},$$

where the left hand side is strictly positive because of (D.2) and (D.3). The condition implies the target stock to be $\tilde{m}_i$. Thus, we have obtained desired conclusion. ∎

In a simple equilibrium (with cost sharing), the steady state technology stock will be $\tilde{m}_L$ with probability $\frac{1}{2}$ and $\tilde{m}_H$ with probability $\frac{1}{2}$. When $\eta = 1$, $\tilde{m}_i$ boils down to m_i^*. It is straightforward to show that $\tilde{m}_i$ increases with both α_i and α_j if $\frac{1-\eta}{\eta} < \rho < \frac{\eta}{1-\eta}$ (which holds for a sufficiently large η). The resource effect is captured by the term $(\pi\eta + (1-\pi)(1-\eta))(1-d)$ in the definition of $\tilde{m}_i$. For $\eta > \frac{1}{2}$, the resource effect is positive, similar to the baseline setting. The sign of the technology effect is determined by

$$\begin{aligned}
&\frac{\partial \left[\pi \alpha_i^{\frac{1}{1-\beta}} + (1-\pi)\left(\frac{\alpha_i}{1-\beta} - \frac{\beta(1-\eta)\alpha_j}{(1-\beta)\eta}\right)\alpha_j^{\frac{\beta}{1-\beta}}\right]}{\partial \pi} \\
= \;& \alpha_i^{\frac{1}{1-\beta}} - \left(\frac{\alpha_i}{1-\beta} - \frac{\beta(1-\eta)\alpha_j}{(1-\beta)\eta}\right)\alpha_j^{\frac{\beta}{1-\beta}} \\
= \;& \alpha_j^{\frac{\beta}{1-\beta}}\left[x^{\frac{1}{1-\beta}} - \frac{x}{1-\beta} + \frac{\beta(1-\eta)}{(1-\beta)\eta}\right] \equiv \alpha_j^{\frac{\beta}{1-\beta}} f(x),
\end{aligned}$$

where $x = \frac{\alpha_i}{\alpha_j}$. It is straightforward to show that $f(\cdot)$ strictly decreases on $(0,1)$ and strictly increases on $(1,\infty)$. For $x = 1$, which is the symmetric case with $\alpha_i = \alpha_j$, $f(x)$ attains its minimum, which is given by $f(1) = \frac{\beta(1-2\eta)}{(1-\beta)\eta}$. Thus, the technology effect is negative (higher power persistence leads to lower technology stock) provided that $\eta > \frac{1}{2}$ and polarization is sufficiently low. This finding echoes the comparative static result in the baseline setting.

Further, we consider how an increase in polarization affects the expected steady-state technology stock. For $\pi = 1$, similar to the benchmark setting, an increase in polarization raises the expected steady state technology stock. For $\pi = 0$, the effect becomes ambiguous because of cost sharing. However, if η is sufficiently close to one, we can show that an increase in polarization lowers the expected technology stock.

The discussions above are based on the assumption that the incumbent bears a sufficiently large share of the cost. We now consider the case of equal cost sharing: $\eta = \frac{1}{2}$. It is straightforward to show that $\tilde{m}_L = \tilde{m}_H = \bar{m}_p$ for $\alpha_L = \alpha_H$. If we allow $\alpha_H > \alpha_L$, then the target technology stock and public good provision can exceed the social optimum when party H is the incumbent. Moreover, if polarization is sufficiently high such that (D.3) no longer holds, or more specifically, $\rho > \frac{\eta}{\beta(1-\eta)}$, then party L's target stock can be zero. This can be seen from (D.1) in the proof above. If $\pi = 0$ and $\rho > \frac{\eta}{\beta(1-\eta)} = \frac{1}{\beta}$, then

$$\delta(\pi V_L'(k) + (1-\pi)W_L(k)) < \delta(\pi\eta(1-d) + (1-\pi)(1-\eta)(1-d)) < \delta$$

for any k, where the second inequality follows from (D.2) (for $\eta = \frac{1}{2}$). Thus, party L would divest technology to achieve zero stock for the next period.