Games of School Choice among Boston Students*

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Abstract

Traditionally, children in U.S. are assigned to public schools based on their home address. However due to equity concerns, school choice programs became increasingly popular in the past fifteen years. The best known of these programs, such as those in Boston, Cambridge, Minnesota, and Seattle, rely on a centralized student assignment procedure that we refer as the Boston mechanism. Truthful preference revelation is rarely in the best interest of students under the Boston mechanism and hence they are forced to play a non-trivial preference revelation game. In this paper we characterize the set of Nash equilibrium outcomes of this game which is played at several U.S. school districts every year. Our main result closely connects these real-life practices with the two-sided matching literature and has important policy implications. In particular, we show that transition to Gale-Shapley student-optimal mechanism in these districts would lead to unambiguous efficiency gains.

1   Introduction

Traditionally, children in U.S. are assigned to public schools based on where they live. A difficulty with this practice is that wealthy families can afford to move to an area with good schools while the poor families cannot. Based on such concerns *school choice* programs have become increasingly popular in the past fifteen years (see Abdulkadiroğlu and Sönmez [2003] for a detailed discussion). Under these programs factors other than proximity (such as student preferences) are also considered to determine the assignment of school seats to students.

Among such school choice programs many rely on the centralized student assignment mechanism that is currently used in Boston. Other major school districts that use versions of this mechanism include Cambridge, Minnesota and Seattle. The *Boston mechanism* works as follows:

1. For each school a strict priority ordering of students is determined by the school district. These priorities are based on state and local laws; for instance in Boston it depends on (a) proximity, (b) where the siblings of students study, and (c) a random lottery number to break the ties.

2. Each student submits a preference ranking of the schools. While the submitted preferences may be the true preferences, they may be strategic choices as well.

3. Based on fixed student priorities and submitted preferences, the student assignment is determined in several rounds: In Round 1 only the first choice of each student is considered and students are assigned to their first choice, one at a time, following their priority order until there are no seats left at their first choice. In Round 2, those who cannot be assigned a seat at their first choice (because of low priority) are considered for their second choice (in case seats remain from Round 1), and the process continues similarly until each student is assigned a seat.

While the students can reveal their preferences truthfully, it is rarely in their best interest given that a student may lose his priority at a school unless he ranks it as his first choice.[1] Therefore the Boston mechanism induces a non-trivial simultaneous game among the students. In this paper we characterize Nash equilibrium outcomes of this preference revelation game which is played at several U.S. school districts every year. In order to describe the set of Nash equilibrium outcomes, we shall connect school choice with an important model which has recently played a prominent role in mechanism design literature.

School choice problem (Abdulkadiroğlu and Sönmez [2003]) is closely related to the well-known *college admissions* problem (Gale and Shapley [1962]). The key difference between the two problems is that in the former schools are indivisible objects which shall be assigned to

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[1]In an experimental evaluation of the Boston mechanism, Chen and Sönmez [2003] show that about 80 percent of the subjects mis-represent their preferences under the Boston mechanism. This rate increases among subjects who think there will be stiff competition for their first choice.

students based on student preferences and school priorities whereas in the latter schools themselves are agents who have preferences over students and whose welfare are taken into consideration. While school priorities are determined by the school district based on state/local laws (and/or education policies) and do not necessarily represent school tastes, one can formally treat school priorities as school preferences and hence obtain a college admissions problem for each school choice problem (see Abdulkadiroğlu and Sönmez [2003], Balinski and Sönmez [1999], and Ergin [2002]). Consequently concepts/findings in college admissions have their counterparts in school choice.

The central notion in college admissions is *stability*. Importance of this concept does not diminish in the context of school choice because if a matching is not stable then there is a student-school pair (i, s) such that (1) student i prefers school s to his assignment and (2) either school s has some empty seats or student i has higher priority than another student who is assigned a seat at school s. In either case student i can seek legal action against the school district for not assigning him a seat at school s. It is well-known that there exists a stable matching and furthermore there exists a stable matching which is preferred by any student to any other stable matching (Gale and Shapley [1962]). This matching is known as the *student-optimal stable matching* and it has played a key role in the re-design of U.S. hospital-intern market in 1998 (see Roth [2002], Roth and Peranson [1999]).

A major handicap of the Boston mechanism is that its outcome is not necessarily stable and therefore it is vulnerable to legal action by unsatisfied students and their parents.[2] Another handicap is that it forces students to play a non-trivial preference revelation game on this critical issue. In our main result we characterize the set of Nash equilibrium outcomes of this preference

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[2]For example consider the following statement from the court case Boston's Children First, et al. v. Boston School Committee, et al. Civil Action Number: 99-11330-RGS. United States District Court for the District of Massachusetts. Decided on January 25, 2002.

> In 1996, Nicholas applied to five K-2 kindergarten programs: O'Hearn, Murphy, the Murphy Half-Day Program, Kenny and the East Zone Early Learning Center. Nicholas was admitted to none of his choices. At Nicholas' first choice, the O'Hearn School, all seats were filled by students with permanent seats or by students with sibling preferences. Nicholas' second choice, the Murphy School, admitted six black students with lower priority rankings. Fourteen white students with higher priority rankings, however, were also not admitted. Nicholas' third choice, the Murphy Half-Day Program, was filled by students with permanent seats or by students who had listed Murphy Half-Day as their first choice. All available seats at Nicholas' fourth choice, the Kenny School, were filled by students with permanent seats or by students who had listed Kenny as their first choice. At Nicholas' fifth choice, the East Zone Early Learning Center, all available seats were filled by students who had selected the Center as their first choice. In visiting the East Zone Parent Center, Nicholas' mother, Ellen Dowd, found that there were openings at Nicholas' fourth choice, the Kenny School. Nicholas was given a temporary K-2 seat at Kenny.

In this statement Nicholas' council is emphasizing several instabilities involving Nicholas that are present in the outcome of the Boston mechanism.

revelation game induced by the Boston mechanism: *The set of Nash equilibrium outcomes is equal to the set of stable matchings under the true preferences.*[3] So while the Boston mechanism is not stable, its Nash equilibrium outcomes are. Our result has a number of important policy implications, especially from a mechanism design perspective. Most notably it shows that a possible transition to Gale-Shapley student-optimal stable mechanism may result in significant efficiency gains in Boston, Cambridge, Minneapolis, Seattle and other districts which rely on variants of Boston mechanism. We discuss these policy implications in detail in Section 6.

The organization of the rest of the paper is as follows: In Section 2 we formally define school choice and college admissions. In Section 3 we present the Boston mechanism. In Section 4 we give a detailed example of the preference revelation game induced by the Boston mechanism and present our main characterization result. We discuss the policy implications of our result in Section 5. Finally in Section 6, we provide a generalization to the *controlled choice* model.

2   School Choice and College Admissions

In a **school choice problem** (Abdulkadiroğlu and Sönmez [2003]) there are a number of students each of whom should be assigned a seat at one of a number of schools. Each student has strict preferences over all schools and each school has a strict priority ranking of all students. Each school has a maximum capacity but the total number of seats is no less than the number of schools.

Formally a school choice problem consists of:

1. a set of students $I = \{i_1, \ldots, i_n\}$,

2. a set of schools $S = \{s_1, \ldots, s_m\}$,

3. a capacity vector $q = (q_{s_1}, \ldots, q_{s_m})$,

4. a list of strict student preferences $P_I = (P_{i_1}, \ldots, P_{i_n})$, and

5. a list of strict school priorities $f = (f_{s_1}, \ldots, f_{s_m})$.

Here

- q_s denotes the capacity of school s where $\Sigma_{s \in S} \, q_s \geq |I|$ and

- f_s denotes the priority ordering of students at school s and it is determined by the school district based on state and local laws. For example at many school districts students are given priority at their district schools.

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[3]This result also relates to the *implementation* literature. We can restate our result using implementation theory jargon as follows: The Boston mechanism implements the stable correspondence in Nash equilibria. See Jackson [2001] for a recent and comprehensive survey on implementation theory.

The school choice problem is closely related to the well-known **college admissions problem** (Gale and Shapley [1962]). The college admissions problem has been extensively studied and successfully applied in the American and British entry-level labor markets (see Roth [1984, 1991]). The key difference between the two models is that in school choice schools are "objects" to be consumed by the students whereas in college admissions schools themselves are agents who have preferences over students.

Formally a college admissions problem consists of:

1. a set of students $I = \{i_1, \ldots, i_n\}$,

2. a set of schools $S = \{s_1, \ldots, s_m\}$,

3. a capacity vector $q = (q_{s_1}, \ldots, q_{s_m})$,

4. a list of strict student preferences $P_I = (P_{i_1}, \ldots, P_{i_n})$, and

5. a list of strict school preferences $P_S = (P_{s_1}, \ldots, P_{s_m})$.

Here P_s denotes the strict preference relation of school s over all students.

The theory on college admissions have immediate implications on school choice. That is because, school priorities in the context of school choice can be interpreted as school preferences in the context of college admissions (see Abdulkadiroğlu and Sönmez [2003], Balinski and Sönmez [1999], and Ergin [2002]).

The outcome of both school choice problems and college admissions problems is known as a **matching**. Formally a matching $\mu : I \longrightarrow S$ is a function from the set of students to the set of schools such that no school is assigned to more students than its capacity. Let $\mu(i)$ denote the assignment of student i under matching μ. Note that $\mu^{-1}(s)$ is the set of students each of whom is matched to school s under matching μ.

In the college admissions context, a student-school pair (i, s) is said to **block** a matching μ if

- $sP_i\mu(i)$ and $|\mu^{-1}(s)| < q_s$, or

- $sP_i\mu(i)$ and iP_sj for some student j such that $\mu(j) = s$.

That is, the pair (i, s) blocks a matching μ if either (1) student i prefers school s to its assignment $\mu(i)$ and school s has empty seats under μ, or (2) student i prefers school s to its assignment $\mu(i)$ and school s prefers student i to at least one of the students in $\mu^{-1}(s)$.

A matching is **stable** if there is no student-school pair that blocks it.

Stability has been central to the college admissions literature. It is by now well known that not only the set of stable matchings is non-empty for each college admissions problem, but also there exists a stable matching which is at least as good as any stable matching for any student (Gale and Shapley [1962]). This matching is known as the **student-optimal stable matching**.

Given a school choice problem we refer a matching to be **stable** whenever it is stable for the induced college admissions game that is obtained by interpreting school priorities as school preferences. Stability has an important interpretation in the school choice context: If a matching is not stable then there is a student-school pair (i, s) such that either

- student i prefers school s to his assignment $\mu(i)$ and school s has empty seats, or

- student i prefers school s to his assignment $\mu(i)$ and he has higher priority than another student who is assigned a seat at school s.

In either case student i can seek legal action against the school district for not assigning him a seat at school s.

3 Boston Student Assignment Mechanism

A **student assignment mechanism** is a systematic procedure that selects a matching for each school choice problem. The following mechanism is the most widely used student assignment mechanism in real-life applications of school choice problems.

The Boston Mechanism:

1. For each school a strict priority ordering of students is determined based on state and local laws.[4]

2. Each student submits a preference ranking of the schools.

3. The key phase is the choice of a matching based on fixed priorities and submitted preferences.

   Round 1: In Round 1 only the first choices of the students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their priority order until either there are no seats left or there is no student left who has listed it as his first choice.

   Round 2: Consider the remaining students. In Round 2 only the second choices of these students are considered. For each school with still available seats, consider the students who have listed it as their second choice and assign the remaining seats to these students one at

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[4]For example at Boston the priority ordering is determined based on the following hierarchy:

- First Priority: sibling and walk zone
- Second priority: sibling
- Third priority: walk zone
- Fourth priority: others

Students in the same priority group are ordered based on a previously announced lottery.

a time following their priority order until either there are no seats left or there is no student left who has listed it as his second choice.

In general, at

Round k: Consider the remaining students. In Round k only the kth choices of these students are considered. For each school with still available seats, consider the students who have listed it as their k^th choice and assign the remaining seats to these students one at a time following their priority order until either there are no seats left or there is no student left who has listed it as his k^th choice.

The procedure terminates when each student is assigned a seat at a school.

School districts that currently use Boston mechanism and its variants include Boston, Cambridge, Minneapolis and Seattle. The Boston mechanism is not stable: Under the Boston mechanism a student who is not assigned his first choice is considered for his second choice only after the students who have ranked his second choice as their first choice. Therefore a student loses his priority at a school unless he ranks it as his first choice and hence truthful preference revelation may not be in students' best interest under the Boston mechanism. As field evidence preference manipulation is advocated by the Central Placement and Assessment Center (CPAC) in Minneapolis, where Boston mechanism is in use. Glazerman and Meyer [1994] states:

> The Minneapolis algorithm places a very high weight on the first choice, with second and third choices being strictly backup options. This is reflected in the advice CPAC gives out to parents, which is to make the first choice a true favorite and the other two "realistic," that is, strategic choices.

While it is unclear when the proposed CPAC strategy is optimal, it is apparent that policy makers and parents are aware of the vulnerability of the Boston mechanism to preference manipulation.

4   Nash Equilibria under the Boston Mechanism

In school districts that rely on Boston mechanism, students and their parents play a non-trivial preference revelation game. Under this game, strategies of students are preferences over schools and the outcome is determined by the Boston mechanism. The choice of their stated preferences and especially their stated top choices play a key role in determining the schools they will be assigned. In our main result we characterize the set of Nash equilibrium outcomes of the preference revelation game induced by the Boston mechanism. Before we present our main result, we give a detailed example which illustrates the preference revelation game induced by the Boston mechanism and highlights some of the key points.

Example: There are three students i_1, i_2, i_3 and three schools a, b, c each of which has one seat. Utilities of students as well as their priorities are as follows:

	Utilities		
	a	b	c
i_1	2	1	0
i_2	1	2	0
i_3	2	1	0

Priorities

$a : i_2 - i_3 - i_1$

$b : i_1 - i_2 - i_3$

$c : i_3 - i_1 - i_2$

Each student can submit one of the preferences abc, acb, bac, bca, cab, cba and therefore under the Boston mechanism the following $6 \times 6 \times 6$ simultaneous game is induced: In this game i_1 is the row player, i_2 is the column player and i_3 is the matrix player.

abc

	abc	acb	bac	bca	cab	cba
abc	1,1,0	1,1,0	0,2,2	0,2,2	1,0,2	1,0,2
acb	0,1,1	0,1,1	0,2,2	0,2,2	1,0,2	1,0,2
bac	**1,1,0**	**1,1,0**	1,0,2	1,0,2	1,0,2	1,0,2
bca	**1,1,0**	**1,1,0**	1,0,2	1,0,2	1,0,2	1,0,2
cab	0,1,1	0,1,1	0,2,2	0,2,2	0,2,2	0,2,2
cba	0,1,1	0,1,1	0,2,2	0,2,2	0,2,2	0,2,2

acb

	abc	acb	bac	bca	cab	cba
abc	1,1,0	1,1,0	0,2,2	0,2,2	1,0,2	1,0,2
acb	1,1,0	1,1,0	0,2,2	0,2,2	1,0,2	1,0,2
bac	**1,1,0**	**1,1,0**	1,0,2	1,0,2	1,0,2	1,0,2
bca	**1,1,0**	**1,1,0**	1,0,2	1,0,2	1,0,2	1,0,2
cab	0,1,1	0,1,1	0,2,2	0,2,2	0,2,2	0,2,2
cba	0,1,1	0,1,1	0,2,2	0,2,2	0,2,2	0,2,2

bac

	abc	acb	bac	bca	cab	cba
abc	0,1,1	0,1,1	2,2,0	2,2,0	2,0,1	2,0,1
acb	0,1,1	0,1,1	2,2,0	2,2,0	2,0,1	2,0,1
bac	**1,1,0**	**1,1,0**	1,1,0	1,0,2	1,0,2	1,0,2
bca	**1,1,0**	**1,1,0**	1,1,0	1,0,2	1,0,2	1,0,2
cab	0,1,1	0,1,1	0,2,2	0,2,2	0,1,1	0,1,1
cba	0,1,1	0,1,1	0,2,2	0,2,2	0,1,1	0,1,1

bca

	abc	acb	bac	bca	cab	cba
abc	0,1,1	0,1,1	2,2,0	2,2,0	2,0,1	2,0,1
acb	0,1,1	0,1,1	2,2,0	2,2,0	2,0,1	2,0,1
bac	**1,1,0**	**1,1,0**	1,1,0	1,1,0	1,0,2	1,0,2
bca	**1,1,0**	**1,1,0**	1,1,0	1,1,0	1,0,2	1,0,2
cab	0,1,1	0,1,1	0,2,2	0,2,2	0,1,1	0,1,1
cba	0,1,1	0,1,1	0,2,2	0,2,2	0,1,1	0,1,1

cab

	abc	acb	bac	bca	cab	cba
abc	1,1,0	1,1,0	2,2,0	2,2,0	2,2,0	2,2,0
acb	1,1,0	1,1,0	2,2,0	2,2,0	2,2,0	2,2,0
bac	**1,1,0**	**1,1,0**	1,1,0	1,1,0	1,1,0	1,1,0
bca	**1,1,0**	**1,1,0**	1,1,0	1,1,0	1,1,0	1,1,0
cab	1,1,0	1,1,0	2,2,0	2,2,0	1,1,0	2,2,0
cba	1,1,0	1,1,0	2,2,0	2,2,0	1,1,0	1,1,0

cba

	abc	acb	bac	bca	cab	cba
abc	1,1,0	1,1,0	2,2,0	2,2,0	2,2,0	2,2,0
acb	1,1,0	1,1,0	2,2,0	2,2,0	2,2,0	2,2,0
bac	**1,1,0**	**1,1,0**	1,1,0	1,1,0	1,1,0	1,1,0
bca	**1,1,0**	**1,1,0**	1,1,0	1,1,0	1,1,0	1,1,0
cab	1,1,0	1,1,0	2,2,0	2,2,0	1,1,0	2,2,0
cba	1,1,0	1,1,0	2,2,0	2,2,0	1,1,0	1,1,0

Here

- the payoff vector $(2, 2, 0)$ corresponds to matching $\mu_1 = \begin{pmatrix} i_1 & i_2 & i_3 \\ a & b & c \end{pmatrix}$,

- the payoff vector $(2, 0, 1)$ corresponds to matching $\mu_2 = \begin{pmatrix} i_1 & i_2 & i_3 \\ a & c & b \end{pmatrix}$,

- the payoff vector $(1, 1, 0)$ corresponds to matching $\mu_3 = \begin{pmatrix} i_1 & i_2 & i_3 \\ b & a & c \end{pmatrix}$,

- the payoff vector $(1, 0, 2)$ corresponds to matching $\mu_4 = \begin{pmatrix} i_1 & i_2 & i_3 \\ b & c & a \end{pmatrix}$,

- the payoff vector $(0, 1, 1)$ corresponds to matching $\mu_5 = \begin{pmatrix} i_1 & i_2 & i_3 \\ c & a & b \end{pmatrix}$, and

- the payoff vector $(0, 2, 2)$ corresponds to matching $\mu_6 = \begin{pmatrix} i_1 & i_2 & i_3 \\ c & b & a \end{pmatrix}$.

In the above game the boldface payoff vectors correspond to Nash equilibria. We have two key observations about the Nash equilibria:

1. The strategy profile which corresponds to truthful preference revelation, (abc, bac, abc), is NOT a Nash equilibrium of the above preference revelation game.

2. The vector $(1, 1, 0)$, which is the payoff for matching μ_3, is the payoff for all Nash equilibria. The significance of matching μ_3 is that it is the only stable matching under other true preferences.[5]

We are now ready to present our main result which shows that these observations are not specific to above example.

Theorem 1 *Let P_I be the list of true student preferences and consider the preference revelation game induced by the Boston mechanism. The set of Nash equilibrium outcomes of this game is equal to the set of stable matchings under the true preferences P_I.*

Proof: Let $Q = (Q_1, \ldots, Q_n)$ be an arbitrary strategy profile and let μ be the resulting outcome of the Boston mechanism. Suppose μ is not stable under the true preferences. Then there is a student-school pair (i, s) such that student i prefers school s to his assignment $\mu(i)$ and either

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[5]The matching μ_1 is blocked by the student-school pair (i_3, a), the matching μ_2 is blocked by the student-school pair (i_2, a), the matching μ_4 is blocked by the student-school pair (i_2, a), the matching μ_5 is blocked by the student-school pair (i_1, b), and the matching μ_6 is blocked by the student-school pair (i_1, b).

1. school s has an empty seat under μ, or

2. student i has higher priority at school s than another student who is assigned a seat at school s.

This implies that under the stated preference Q_i student i does not rank school s as his first choice for otherwise he would be assigned a seat at school s. Let Q_i' be any strategy where student i ranks school s as his first choice. Student i is assigned a seat at school s under the profile (Q_{-i}, Q_i') and therefore neither Q is a a Nash equilibrium profile nor μ is a Nash equilibrium outcome. Hence any Nash equilibrium outcome should be stable under the true preferences.

Conversely let μ be a stable matching under the true preferences. Consider a preference profile $Q = (Q_1, \ldots, Q_n)$ where each student i ranks school $\mu(i)$ as his top choice under his stated preferences Q_i. Under the preference profile Q, the Boston mechanism terminates at Round 1 and each student is assigned a seat at his first choice based on the stated preferences. Hence μ is the resulting outcome for the strategy profile Q. Next we show that Q is a Nash equilibrium profile. Consider a student i and a school s such that student i prefers school s to his assignment $\mu(i)$. Since μ is stable, not only all seats of school s are filled under μ but also each student who is assigned a seat at school s under μ has higher priority than student i for school s. Moreover each such student j ranks school s as his first choice under Q_j. Therefore given Q_{-i}, there is no way student i can secure a seat at school s even if he ranks it as his first choice. Therefore Q is a Nash equilibrium strategy profile and μ is a Nash equilibrium outcome. Hence any stable matching under the true preferences is a Nash equilibrium outcome. $\diamondsuit$

Even though the Boston mechanism itself is not stable, by Theorem 1 all Nash equilibrium outcomes of the preference revelation game induced by the Boston mechanism are stable. The outcome of the Boston mechanism is Pareto efficient, provided that students truthfully reveal their preferences. However truthful preference revelation is rarely in the best interest of students, and efficiency loss is expected. Theorem 1 clarifies the nature of this efficiency loss: Stability and Pareto efficiency are not compatible in the context of school choice (see Abdulkadiroğlu and Sönmez [2003], Ergin [2002]).[6] Since all equilibrium outcomes are stable, part of the inefficiency is due to this incompatibility. However out of all equilibrium outcomes there is one, the student-optimal stable matching, which Pareto dominates any other. Therefore in all equilibrium outcomes with the exception of the student-optimal stable matching there is additional efficiency loss.

5 Policy Implications

Gale-Shapley student-optimal stable mechanism is the mechanism that selects the student-optimal stable matching for each school choice problem. Abdulkadiroğlu and Sönmez [2003] show

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[6]For example in the preceding example, matching μ_3 is the only stable matching but it is Pareto dominated by matching μ_1.

that this mechanism has two major advantages over the Boston mechanism:

1. Gale-Shapley student-optimal stable mechanism is stable and therefore it "respects" priorities. The Boston mechanism is not stable and therefore it is vulnerable to legal action by students and their parents.[7]

2. Gale-Shapley student-optimal stable mechanism is **strategy-proof** and therefore truthful-preference revelation is always in students' best interest (Dubins and Freedman [1981], Roth [1982]). On the contrary, truthful preference revelation is rarely in the best interest of students under the Boston mechanism and they are forced to play a non-trivial preference revelation game.

This paper provides a third major advantage of the Gale-Shapley student optimal mechanism over the Boston mechanism: The preference revelation game induced by the Gale-Shapley student-optimal stable mechanism has a dominant strategy equilibrium (which is truthful-revelation) and its outcome is either equal to or Pareto dominates Nash equilibrium outcomes of the Boston mechanism. In that sense the outcome of the Gale-Shapley student-optimal stable mechanism is the best one can hope for under the Boston mechanism and hence Gale-Shapley student-optimal stable mechanism has an efficiency advantage over the Boston mechanism.

Researchers in education tend to evaluate the Boston mechanism based on the stated preferences of students. For example Glenn [1991] argue that in 1991, 74 percent of sixth graders at Boston were assigned to their first choice school. Similarly Glazerman and Meyer [1994] argue that in 1993-94 more than 80 percent of students at Minneapolis were assigned to their first choice school and they conclude

> These numbers imply that student preferences in Minneapolis are quite diverse and that students perceive that there are significant differences in school characteristics. If this were not the case, we might expect most students to apply to a very limited set of schools. As a result, very few students would have been assigned to their preferred school.

In light of the preference revelation game induced by the Boston mechanism, this conclusion is inadequate.[8] It is interesting to note that, under the Nash equilibrium strategies we constructed,

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[7]It is worth emphasizing that, while the Boston mechanism may yield unstable outcomes, this will not happen at Nash equilibria for the true preferences. Any Nash equilibrium outcome is stable under the true preferences immediately by Theorem 1. However students can still seek legal action if the outcome is not stable under the stated preferences whether it is stable or not under the true preferences. Clearly the courts will only care for the stated preferences (such as the court case in footnote 2) which are available in written records.

[8]Glazerman and Meyer [1994] are indeed aware of the problem with their conclusion. They state:

> The second problem with the choice data is that it is not clear that all, or even most, families listed their preferred school as their first choice. Although district staff advised families to do this it is pos-

100 percent of the students are assigned to their first choice school based on their stated preferences. Whether intentional or not, school districts that use Boston mechanism are misleading policy-makers by giving the impression that they are able to accommodate most students' top choices and ironically Boston mechanism is the perfect tool to create this impression.

6 Controlled Choice

One of the major concerns about the implementation of school choice programs is that they may result in racial and ethnic segregation at schools. Because of these concerns, school choice programs in some districts are limited by court-ordered desegregation guidelines. This version of school choice is known as **controlled choice**. In Minneapolis, controlled choice constraints are implemented by imposing type-specific quotas. Under this practice students are partitioned into different groups based on their type (which often depends on their ethnic/racial background) and for each school, type-specific quotas are determined in additional to the capacity of the school. These quotas may be rigid or they may be flexible. For example, in Minneapolis the district is allowed to go above or below the district-wide average enrollment rates by up to 15 percent points in determining the ethnic/racial quotas. Currently in Minneapolis and prior to 1999 in Boston the following variant of the Boston mechanism is used to assign students to public schools.

The Boston Mechanism with Type-Specific Quotas:

1. Students are partitioned based on their types. For each school, in addition to the capacity of the school, type-specific quotas are determined based on the desegregation guidelines.

2. For each school a strict priority ordering of students is determined based on state and local laws.

3. Each student submits a preference ranking of the schools.

4. Based on type-specific quotas, student priorities, and submitted preferences, the student assignment is determined in several rounds.

   Round 1: In Round 1 only the first choices of the students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their priority order unless the quota of a type is full. When that happens, remaining students of that type are rejected and the process continues with the students of other types until either there are no seats left or there is no student left who has listed it as his first choice.

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sible that some families did not if their preferred school was likely to be substantially oversubscribed. Instead, they may have acted strategically and listed an undersubscribed, but highly ranked, school as their first choice.

In general, at

Round k: Consider the remaining students. In Round k only the kth choices of these students are considered. For each school with still available seats, consider the students who have listed it as their kth choice and assign the remaining seats to these students one at a time following their priority order unless the quota of a type is full. When that happens, remaining students of that type are rejected and the process continues with the students of other types until either there are no seats left or there is no student left who has listed it as his kth choice.

The procedure terminates when each student is assigned a seat at a school.

As in the case of the Boston mechanism, this modified version also induces a non-trivial preference revelation game. We need an additional definition in order to characterize the set of Nash equilibrium outcomes of this game.

Given a controlled choice problem, we refer a matching μ to be **weakly stable** if

1. it does not violate the type-specific quotas, and

2. there is no student-school pair (i, s) such that student i prefers school s to his assignment $\mu(i)$ and either

    - school s has not filled its quota for the type of student i and it has an empty seat, or
    - school s has not filled the quota for the type of student i and student i has higher priority than another student (of any type) who is assigned a seat at school s, or
    - school s has filled its quota for the type of student i but student i has higher priority than another student of his own type who is assigned a seat at school s.

Following Kelso and Crawford [1982] and Roth [1991], Abdulkadiroğlu [2002] shows that the set of weakly stable matchings is non-empty. We are ready to characterize the set of Nash equilibrium outcomes of the preference revelation game induced by the Boston mechanism with type-specific quotas.

Theorem 2 *Let P_I be the list of true student preferences and consider the preference revelation game induced by the Boston mechanism with type-specific quotas. The set of Nash equilibrium outcomes of this game is equal to the set of weakly stable matchings under the true preferences P_I.*

Proof: Similar to the proof of Theorem 1.

Many of the key properties on the structure of stable matchings carry over to the set of weakly stable matchings (see Abdulkadiroğlu [2002]). Most notably, given a controlled choice problem, there exists a weakly stable matching which is at least as good as any other weakly stable matching

for any student (Kelso and Crawford [1982], Roth [1991], Abdulkadiroğlu [2002]). The following mechanism selects this **student-optimal weakly stable matching** for each controlled choice problem.

Student-Optimal Weakly Stable Mechanism (Abdulkadiroğlu [2002], Abdulkadiroğlu and Sönmez [2003]):

1. Students are partitioned based on their types. For each school, in addition to the capacity of the school, type-specific quotas are determined based on the desegregation guidelines.

2. For each school a strict priority ordering of students is determined based on state and local laws.

3. Each student submits a preference ranking of the schools.

4. Based on type-specific quotas, student priorities, and submitted preferences, the student assignment is determined in several steps.

   Step 1: Each student proposes to her first choice. Each school tentatively assigns its seats to its proposers one at a time following their priority order. If the quota of a type fills, the remaining proposers of that type are rejected and the tentative assignment proceeds with the students of the other types. Any remaining proposers are rejected.

   In general, at

   Step k: Each student who was rejected in the previous step proposes to her next choice. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time following their priority order. If the quota of a type fills, the remaining proposers of that type are rejected and the tentative assignment proceeds with the students of the other types. Any remaining proposers are rejected.The procedure terminates when no student proposal is rejected and each student is assigned her final tentative assignment.

Theorem 2 shows that policy implications of our main result carry over to the controlled choice model. Most notably, transition to student-optimal weakly stable mechanism is likely to result in Pareto improvements in school districts that currently rely on the Boston mechanism with type-specific quotas.

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