Explicit and implicit status-quo determination in dynamic bargaining: Theory and application to FOMC directive *

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Abstract

The paper analyses recurring group decision making problems under uncertainty regarding future preferences of the actors involved, problems faced by many real world committees. The need for continuity in policy making implies that past decisions play an important role in determining subsequent ones, creating dynamic procedural linkages. In this context we compare two bargaining protocols: i) the *implicit status-quo protocol* under which present period policy serves as the status-quo for the next period and ii) the *explicit status-quo protocol* under which the decision in the current period involves both current policy and a (possibly different) status-quo for the ensuing period. We show that the two bargaining protocols lead to notably different policy outcomes. In Stationary Markov Perfect equilibrium, unique under slight refinement, the difference is most marked in the periods of common interest. These are still characterized by disagreement under implicit status-quo bargaining, while under explicit status-quo bargaining they lead to the policy decisions that fully reflect the congruent preferences of the committee members. Furthermore, the former bargaining protocol leads quickly to gridlock with constant policy unresponsive to the varying preferences in the committee, something the latter bargaining protocol does not deliver. However, the implicit status-quo protocol prevents abuse of proposal power, which is possible under the explicit status-quo. With this insight, we re-interpret explanations for the existence of 'asymmetry' in the Federal Open Market Committee (FOMC) directive.

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Several other members indicated that they would have preferred to tighten at that meeting. ... The asymmetric directive, which held prospect of near-term tightening, once again allowed FOMC to reach a consensus.

Meyer (2004), page 83

And we know from firsthand accounts that Greenspan was holding back an FOMC that was eager to raise rates.

Blinder and Reis (2005), page 58

1 Introduction

We study recurring group decision making problems with the preferences of the actors involved varying over time. As a leading example, consider periodic meetings of a monetary policy committee. In every period, the preferences of each committee member will be affected by host of factors, such as the state of the economy, his view of future economic development, his opinion about the strength of monetary policy transmission channels, or judgment about the suitable inflation monetary policy should aim for. Inevitably, most of those factors will change stochastically over time, opening the possibility for renegotiation of decisions reached at an earlier stage. Of course, there are many other examples of recurring decision making with varying preference, both in the economic and the political spheres.

With changing preferences of the involved parties, bargaining over a decision at any given point in time will proceed under varying degrees of disagreement. In the monetary policy committee case, ambiguity of information the committee holds can provoke disagreement over the most appropriate policy in some periods, but can lead to agreement in other periods when the information becomes more definite. Uncertainty over the future then implies uncertainty about the extent of future disagreement as well.

Recurring decisions naturally create dynamic linkages that need to be considered. First, strategic linkages arise due to the repeated nature of the interaction. The current action of any given decision maker will take into account its possible impact on the future behaviour of the remaining decision makers, with the repeated interaction allowing for cooperative outcomes unattainable in static settings. This is analysed in the folk theorems literature in general and in the political arena context in particular by Dixit, Grossman, and Gul (2000) and Maggi and Morelli (2006). We abstract away from these strategic linkages in recurring decisions by focusing on Stationary Markov Perfect equilibria.

Instead, we concentrate on the second type of linkages, procedural ones. These involve the role past decisions play during the determination of subsequent ones. These linkages stem from the need to ensure continuity in policy making. Protocols in place ensure that some policy is chosen even in the case of a decision not being reached.

The simplest of such protocols is the one under which a policy, once established, becomes the status-quo for the ensuing round of bargaining. Inaction, no change in a given policy or contract, leaves the previous decision in place. For example, in most countries personal income tax rates apply until changed. Labour unions negotiate agreements with firms regarding wage and employment levels which are effective until renegotiated. In effect, current policy *implicitly* determines status-quo.

However, there are several prominent examples of decision processes that enlarge the space of current decisions to include provisions for the future. These yield not only current policy but also *explicitly* determine a (potentially different) status-quo for the next round of negotiations. Legislative *sunset provisions* specify a time horizon for the statute or regulation in question, after which it automatically terminates. These are often found in tax laws or in laws impinging civil liberties, most prominently in the US Economic Growth and Tax Relief Reconciliation Act of 2001 and US Patriot Act of 2001. Regulatory *escape clauses* are another example of the present policy and the status-quo being distinct.[1]

In this paper, we investigate decisions reached in recurring negotiations with stochastically changing preferences of the actors involved. The status-

[1] Regulatory escape clauses can be viewed as temporary modifications of the regulatory rules in light of changed conditions with no change implied for the future. For example, the European Commission in the context of the Stability and Growth Pact initiates an excessive deficit procedure with a given country if its annual budget deficit exceeds 3% of its GDP, but can refrain from doing so if the breach of the limit is associated with, for example, a prolonged period of slow economic growth. This can be interpreted as the European Commission temporarily increasing the 3% limit during recessions but keeping it intact for the future, or in other words having different current and status-quo deficit limits.

quo for a given round of bargaining is determined endogenously during the previous negotiations. We call the distinction between the new status-quo being implicitly or explicitly decided upon *bargaining protocol* and ask how the bargaining protocol influences decisions reached.

We are motivated by normative, positive and theoretical questions. On the normative side we analyse how the different bargaining protocols influence the ability of the committee to respond to the changing preferences of the involved parties. The procedural linkages mentioned above imply that the behaviour of each decision maker will reflect both current and future incentives. This might imply that even in 'agreement' periods, characterized by similar current preferences of the decision makers, their behaviour might be driven by efforts to affect future decisions. We will show below that enlarging the space of current decisions to include provisions for the future via the explicit status-quo bargaining protocol delivers policies better tailored to changing circumstances. A potential downside of allowing for such provisions comes from the resulting increase in proposal power. Consequently, from the utilitarian perspective none of the bargaining protocols clearly dominates the other and we lay out conditions under which one or the other of them should be endorsed.

Our positive motivation builds on one of our motivating examples, monetary policy committees. Monetary policy in most central banks is decided upon by a committee composed of several members convening with regular frequency. The policy usually consists of the bank's operating target, its interest rate. In most central banks the interest rate decided in a given committee meeting serves also as a status-quo for the next meeting. Inaction leads to no change in monetary policy stance, hence the status-quo is implicitly determined by a given decision.

In contrast, the Federal Open Market Committee (FOMC), the decision body of the US Federal Reserve System, issues at the close of each meeting operating instructions for the Federal Reserve Bank of New York known as the *domestic policy directive.* The directive contains not only the decision about current policy but also a statement concerning the FOMC's expectation of future policy stance. Viewing the 'asymmetry', 'bias' or 'tilt' in the policy directive as explicitly specifying a status-quo policy possibly different from the current one allows us to gain deeper understanding of the FOMC decision making process.[2] Our model then suggests a novel rationale for the existence of the asymmetry.

The theoretical motivation is to advance growing dynamic bargaining literature. While acknowledging endogeneity of the status-quo in recurring decision making situations, this literature has invariably assumed that the status-quo is equal to the policy decision of the previous bargaining round. While this is a natural assumption in many environments, some environments might be more appropriately modelled as having an explicit status-quo bargaining protocol. Our analysis of explicit status-quo bargaining is not only, to our knowledge, novel in the literature, but also highlights differences the two bargaining protocols bring by analysing them in an otherwise identical model setup.

In the model, a committee composed of two members, one of whom possesses fixed proposal power, takes repeated decisions on a policy from a one-dimensional policy space over which each of the committee members has single peaked preferences represented by a bliss point. Every period is randomly selected to be either an agreement or disagreement one, with only the present period type being common knowledge. In agreement periods, the two members share a common bliss point whereas in the disagreement periods the bliss points of the two members differ. While certainly a crude simplification of the continuum on which conflict of preferences can take place, the agreement/disagreement dichotomy allows us to clearly illustrate the effect of the bargaining protocol on policy outcomes.

Besides the period type, every committee meeting is characterized by a one-dimensional status-quo. Under the implicit status-quo bargaining protocol, the status-quo is pitched against a proposed policy with the winning alternative being both the current policy outcome and the next period status-quo. Under the explicit status-quo bargaining protocol, the status-quo is pitched against a joint proposal for a policy and a new status-quo. If the committee selects the proposed pair, this proposal determines the current policy outcome and a possibly different future status-quo, otherwise, the status-quo becomes both the policy implemented today and the future status-quo.

We first show existence and uniqueness in a certain well defined sense

[2] See the opening part of section 5 for a discussion of why the asymmetry might constitute a status-quo.

of Stationary Markov Perfect equilibrium (S-MPE) under both bargaining protocols (propositions 1 and 4). The lack of general S-MPE existence results and typically ill behaved induced preferences over the 'state' variable in dynamic bargaining models (Baron, 1996; Baron and Herron, 2003; Kalandrakis, 2004; Duggan and Kalandrakis, 2010) make this a nontrivial exercise and we are forced to work with induced preferences that typically lack monotonicity, concavity and continuity. Adding further stochastic elements would allow us to use existing results on existence of S-MPE in dynamic bargaining context.[3] We refrain from doing so, limiting generality of our results to cases of sufficiently but not excessively strong conflict between the two players. On the other hand, this allows us to characterize equilibria of the model to a greater extent.

For the implicit status-quo bargaining protocol, we show that in equilibrium negotiations display inefficiency in agreement periods; the committee members are unable to agree on a policy corresponding to their common bliss point (proposition 2). The intuition for this result is the dual role of policy under the implicit status-quo bargaining protocol. Policy serves not only as policy but also determines the future status-quo. Moreover, we show that bargaining quickly reaches a point of gridlock, with the policy outcomes unresponsive to changing preferences (proposition 2). Once in gridlock, the two players have antithetic preference over policy even in agreement periods, as it determines the future status-quo and affects their future bargaining positions. Explicit status-quo bargaining reverses both of these results. In equilibrium, it leads to the policy outcomes corresponding to the common committee members' bliss point in the agreement periods (proposition 3) and does not lead to the gridlock as the policy outcomes remain responsive to the changing preferences of the committee members (proposition 5).

One possible side effect of explicit status-quo bargaining comes from the increase of proposal power relative to implicit status-quo bargaining. Allowing for proposals with different policy and status-quo creates room for the proposer to push through policies fully reflecting her preferences. Those are too extreme for the rest of the committee and the committee as a

[3] Duggan and Kalandrakis (2010) overcome the ill behaved induced preferences problem by adding noise elements to players' preferences and to the policy status-quo transition mechanism. This 'smooths out' the induced preferences and allows them to prove existence of S-MPE in a very general dynamic bargaining model.

whole might prefer different bargaining protocols in different environments (proposition 6).

Finally, we show that these results carry over to a committee composed of an odd number of members with preferences similar to those in the benchmark model (proposition 7). This allows us to shift attention to the FOMC decision making process and examine the role of the asymmetry in its directive. We focus mainly on its role as a predictor of future policy changes and as an instrument to achieve more consensual FOMC decisions. Our model delivers these two predictions and also suggests a novel explanation for the existence of the asymmetry as a tool which allows the FOMC chairman to maintain his dominant position in the committee.

The model we build belongs to the dynamic bargaining literature that assumes that the status-quo during a given round of bargaining is endogenously determined during previous bargaining rounds. Differently from most of the existing literature (Baron, 1996; Baron and Herron, 2003; Kalandrakis, 2004; Bernheim, Rangel, and Rayo, 2006; Battaglini and Coate, 2007; Baron, Diermeier, and Fong, 2011; Battaglini and Palfrey, 2011) we focus on an environment with stochastic preferences and abstract from distributional issues analysed in many of the mentioned papers.

Despite its obvious appeal, the dynamic bargaining literature with time-varying preferences is rather scarce. Battaglini and Coate (2008) build a dynamic model of legislative bargaining with general and targeted public spending. In their model, the status-quo is fixed but the intertemporal link is created by accumulated public debt while the time-varying preferences stem from a stochastic value of general public spending. Diermeier and Fong (2009) build a similar model. Riboni and Ruge-Murcia (2008) analyse a model similar to ours with the implicit status-quo bargaining protocol. They analytically solve the two period version of their model and resort to numerical simulation of the infinite period version. Dziuda and Loeper (2010) also analyse a model closely related to ours with the implicit status-quo bargaining protocol. In their model, a two member committee takes repeated decisions over a binary agenda with the preference parameter of each of the committee members being a continuous random variable distributed on the real line. In our model, it is the preference parameter that takes on two values with the policy being a continuous variable. However, none of the papers mentioned above analyses how expanding the space of current decisions to include provisions for the future changes policy outcomes and ability of the committee members to renegotiate in the changing environment, something our explicit status-quo bargaining protocol does. It is the comparison between the two bargaining protocols or institutions we are interested in.

Another strand of literature related to this paper is the literature investigating the effect of linking decisions. In Jackson and Sonnenschein (2007) agents are constrained to represent their preferences across decision problems such that the representation corresponds to the underlying distribution of their preferences. The main result of their paper is that linking large numbers of decisions leads to approximate ex ante Pareto efficiency. In Casella (2005) agents can store their votes and use them in future meetings when their preferences are more intense. This typically leads to ex ante welfare improvement over non-storable votes. Hortala-Vallve (2010) proves similar result in a setting where agents can distribute a given number of votes freely across a predetermined number of issues. The first mentioned paper improves efficiency by putting constraints on the misrepresentation of preferences allowed for, while the two latter papers improve efficiency by relaxing the one-person-one-vote constraint. In the context of this literature, our explicit status-quo bargaining protocol, by relaxing the 'policy equal to status-quo' constraint, can be viewed as relaxing constraint on the committee decision making but also as removing constraint on the proposal power.

We proceed as follows. The next section lays out the theoretical model. Section 3 solves for the equilibrium in a two period version. It is meant to build intuition for the infinite horizon version and to show that the key results are not sensitive to changes in the foresight horizon. Section 4 contains all the theoretical results. These describe equilibria for both of the bargaining protocols, discuss conditions under which one of them should be preferred and show that the model applies equally well to larger committees. Section 5 applies these results to the FOMC decision making, demonstrates that the model can replicate stylized facts about its decision patterns and suggests a novel interpretation of the asymmetric FOMC directive.

2 Model

We analyse the effect of bargaining protocol on dynamic policy making in a simple model. Policies in the model are set by a committee composed of two members. The first member is the chairman, who has policy proposal power and whom we denote by C (she). The second committee member is denoted by P (he) and has policy approval power. The voting rule used by the committee is simple majority with ties decided against C's proposal so that in order for C's proposal to pass, consent of both committee members is required.[4]

The committee sets policy p_t in each period t of an infinite horizon. The utility player $i \in \{C, P\}$ receives from the path of policies $\mathbf{p} = \{p_0, p_1, p_2, \ldots\}$ is given by

$$U_i(\mathbf{p}) = \sum_{t=0}^{\infty} \delta^t u_{i,t}(p_t)$$

where $\delta \in [0, 1)$ is common discount factor. Instantaneous utility $u_{i,t}(p_t)$ of each player is

$$u_{i,t}(p_t) = -(p_t - \pi^* - \varepsilon_{i,t})^2$$

where π^* is a common component in the committee members' preferences and $\varepsilon_{i,t}$ is a stochastic time-varying $i-$specific preference shock.

The timing of actions in period t is as follows. First, nature determines $\varepsilon_{i,t}$ according to the process specified below and the committee convenes with x_t being the default option. Both $\varepsilon_{i,t}$ and x_t are common knowledge. Second, the chairman C proposes a pair $\gamma_t = \{p_t, q_t\}$ against default option $\bar{\gamma}_t = \{x_t, x_t\}$. Third, voting takes place between γ_t and $\bar{\gamma}_t$. If P prefers γ_t it is implemented (C always votes for her proposal), players receive their payoffs from the offered policy p_t and the offered status-quo q_t becomes default option for the next period, i.e. $x_{t+1} = q_t$. If P prefers $\bar{\gamma}_t$, players receive their payoffs from the default policy x_t and the default status-quo x_t becomes default option for the next period, i.e. $x_{t+1} = x_t$. Finally, the committee adjourns and the game moves into period $t + 1$.

In the text we refer to the pair $\gamma_t = \{p_t, q_t\}$ C proposes as to (C's) *proposal* or *offer*, call its first element p_t *proposed (offered) policy* and its

[4] An alternative assumption that would not change any of the results is C making a take it or leave it offer to P. We opt for the voting rules specification as it naturally adapts once we expand the committee below.

second element q_t *proposed (offered) status-quo*. The pair $\bar{\gamma}_t = \{x_t, x_t\}$ is then *default option* or simply *default* and we abuse notation slightly in calling x_t by the same term.

Without loss of generality we assume that C whose utility maximizing offer γ_t coincides with the default option $\bar{\gamma}_t$ proposes $\bar{\gamma}_t$ instead of proposing a policy she knows would be rejected. It is also easy to see that in any equilibrium of the game it has to be the case that if P is indifferent between default $\bar{\gamma}_t$ and C's proposal γ_t he votes for γ_t. As a result C's offer γ_t is always accepted and we do not need to distinguish between proposed and accepted policies.

Up to this point the model generates dynamic policy making in that the proposed (and hence accepted) status-quo q_t from period t becomes the default option x_{t+1} for the $t+1$ period. To study how this feature interacts with the bargaining protocol used by the committee, we contrast two versions of the model. The first model version and bargaining protocol is with *implicit status-quo*. Under this bargaining protocol C's proposals are constrained to those that satisfy $p_t = q_t$ so that the t period status-quo q_t, and hence $t+1$ period default option x_{t+1}, is implicitly defined by the t period policy p_t. The second model version and bargaining protocol is with *explicit status-quo*. Under this bargaining protocol t period status-quo q_t, and hence $t+1$ period default option x_{t+1}, is explicitly determined during the committee bargaining.

To close the model we need to specify the distribution of the preference shocks $\varepsilon_{i,t}$. We assume those are generated according to

$$\varepsilon_{i,t} = \begin{cases} -\phi \text{ for } i = C \text{ and } \phi \text{ for } i = P & \text{with probability } r_d \\ 0 \text{ for } i \in \{C, P\} & \text{with probability } 1 - r_d \end{cases}$$

where $\phi > 0$ and $r_d \in [0, 1]$. In words, there are two types of periods. With probability r_d bliss points in the instantaneous utility functions of C and P are $\pi^* - \phi$ and $\pi^* + \phi$ respectively. We call those *disagreement* periods or D periods for short. The second type of period occurs with probability $1 - r_d$ and are called *agreement* periods or A periods for short. In these, bliss points in the instantaneous utility functions of both players are π^*.

Several comments regarding our modelling choices are in order. First, completely breaking the link between policy and status-quo and giving all the proposal power to C is motivated by our interest in the trade-off the explicit status-quo bargaining protocol creates. On the one hand, it should lead to more efficient policy outcomes, but it also opens the door to an abuse of proposal power. We want to see the full effect on both sides and thus opt for arguably strong assumptions.

Second, having A and D periods in the model reflects our belief that in recurrent decision making this is a natural assumption. We could have chosen either purely ideological or purely common preferences, which our model indeed includes as special cases with $r_d = 1$ or $r_d = 0$. However, it is easy to show that under both specifications the bargaining protocol plays no role. It is the interaction with the time varying preferences that creates an interesting problem to study.

3 Two period model

To build intuition for the results below, we first solve a two period version of the model. All the results are easily derived using backward induction and we state them without formal proofs.

Lemma 1 (Last period). *For the last period default option x_1 and both bargaining protocols, equilibrium policy proposals $p_{A,1}(x_1)$ and $p_{D,1}(x_1)$, in A and D periods respectively, satisfy*

$$p_{A,1}(x_1) = \pi^*$$
$$p_{D,1}(x_1) = f(x_1, \phi)$$

where $f(x, \phi) = \max\{\min\{2(\pi^* + \phi) - x, x\}, \pi^* - \phi\}$.

In the last period there is no procedural link with the future via the status-quo and hence the bargaining protocol plays no role. It is thus easy to see why the two policy makers decide on π^* in A periods as it is a bliss point in their common utility function.

D period policy then reflects conflict in the committee. P's acceptance set consists of a symmetric interval around his bliss point $\pi^* + \phi$ with one boundary given by default option x_1, $[2(\pi^* + \phi) - x_1, x_1]$. C maximizes her utility with bliss point at $\pi^* - \phi$ by proposing minimum of P's acceptance (the min term) but only if she cannot propose her bliss point (the max term). The parameter ϕ captures the interval of disagreement, for $x_1 \in$

$[\pi^* - \phi, \pi^* + \phi]$, there is no other policy except for x_1 the two policy makers are willing to agree on. P would reject any policy below x_1 and C does not want to offer any policy above x_1 and thus $f(x_1, \phi) = x_1$.[5]

C's and P's expected utilities before nature determines the type of last period, as a function of x_1 (and hence as a function of the first period status-quo),

$$\mathbb{E}\left[U_{C,0}(x_1)\right] = -r_d(p_{D,1}(x_1) - \pi^* + \phi)^2 + (1 - r_d) \cdot 0$$
$$\mathbb{E}\left[U_{P,0}(x_1)\right] = -r_d(p_{D,1}(x_1) - \pi^* - \phi)^2 + (1 - r_d) \cdot 0,$$

reflect intertemporal preferences of the two policy makers and are interesting for several reasons. First, both are non-concave and non-monotone. $\mathbb{E}[U_{C,0}(x_1)]$ and $\mathbb{E}[U_{P,0}(x_1)]$ are non-increasing and non-decreasing respectively for $x_1 \leq \pi^* + \phi$ and vice-versa for $x_1 \geq \pi^* + \phi$. This is the reason why we cannot work with equilibria associated with well-behaved (concave, monotone) value functions as in, for example, Battaglini and Coate (2007, 2008), as the ill-behaved intertemporal preferences are an inherent feature of the model.

Second, potential future conflict spills over to the current period through the conflict in the intertemporal preferences. P prefers default option x_1 as close to $\pi^* + \phi$ as possible while C prefers it as far away from $\pi^* + \phi$ as possible. Thus the committee members have an incentive to manipulate x_1 in the first period as it determines their bargaining positions. Under implicit status-quo bargaining this is done via the enacted policy and under explicit status-quo bargaining this is done via the enacted status-quo.

Third, $\mathbb{E}[U_{C,0}(x_1)]$ and $\mathbb{E}[U_{P,0}(x_1)]$ are constant for $x_1 \notin (\pi^* - \phi, \pi^* + 3\phi)$. For the first period under explicit status-quo bargaining this means that whenever $z \notin (\pi^* - \phi, \pi^* + 3\phi)$ is an equilibrium status-quo proposal for some default option, so is $z' \notin (\pi^* - \phi, \pi^* + 3\phi)$. However, this multiplicity has no effect on the last period policy. No matter whether z or z' is proposed, last period P's acceptance set includes, on a policy dimension unique, unconstrained maximizer of C's overall utility.

[5] Notice also that the interval where $f(\cdot, \cdot)$ is not constant, the interval of default options for which C's proposal makes P indifferent between accepting and rejecting and interval of default options for which C cannot implement her bliss point, all coincide. This is a more general feature of the model, and it will hold for the first period irrespective of the type of period or bargaining protocol, and motivates our choice of equilibrium refinement (definition 3) for the infinite horizon model.

Lemma 2 (First period, D)**.** *For the first period default option x_0, implicit status-quo protocol policy proposal $p^I_{D,0}(x_0)$ and explicit status-quo protocol policy and status-quo proposals $p^E_{D,0}(x_0)$ and $q^E_{D,0}(x_0)$ in D periods satisfy*

$$p^I_{D,0}(x_0) = p^E_{D,0}(x_0) = q^E_{D,0}(x_0) = f(x_0, \phi).$$

For the implicit status-quo protocol this reflects conflict in terms of both current and intertemporal preferences. The same holds for the explicit status-quo, but C can, in principle, offer a policy different from the status-quo. To see the nature of her trade-off, C can either concede on the policy dimension in order to gain a better bargaining position on the status-quo dimension, or vice versa. The strength of those two forces, to satisfy instantaneous or intertemporal utility, then determines her equilibrium proposal. As we will see below, the two forces exactly cancelling each other, which leads to $p^E_{D,0}(x_0) = q^E_{D,0}(x_0)$, is a result specific to the two period model.

Lemma 3 (First period, A)**.** *For the first period default option x_0, equilibrium policy proposal under the implicit and explicit status-quo protocol, $p^I_{A,0}(x_0)$ and $p^E_{A,0}(x_0)$ respectively, in A periods satisfy*

$$\begin{aligned} p^I_{A,0}(x_0) &= f(x_0, \phi\kappa') \\ p^E_{A,0}(x_0) &= \pi^* \end{aligned}$$

where $\kappa' = \frac{\delta r_d}{1+\delta r_d} \leq \frac{1}{2}$.

First A periods reveal the key difference between the two bargaining protocols. Under the implicit status-quo bargaining, policy serves two roles. It is a policy in the standard sense but also determines future bargaining positions. Agreeing on π^* in A period would entail, at least for one of the players, giving up bargaining position relative to x_0. Combining current preferences favouring π^* and intertemporal preferences favouring $\pi^* - \phi$ ($\pi^* + \phi$) for C (P) makes A periods 'lesser disagreement' periods with the degree of conflict given by $\phi\kappa'$. The more probable the true D periods are and the more players care about future, the more of the conflict spills over to A periods.

Explicit status-quo bargaining on the other hand implies π^* is implemented in A periods. With the policy makers' preferences aligned on the policy dimension and, crucially, with the policy and status-quo possibly different, C does not compromise her bargaining position by proposing π^* policy. To the contrary, this allows C to propose status-quo that improves her bargaining position. She has a room to do so since proposing π^* on the policy dimension has made P better off compared to the default option x_0.

A key advantage of the two period model just discussed is that it delivers key predictions about the difference in policy outcomes under the two bargaining protocols in a relatively simple framework. On the other hand, with a fixed time horizon we are unable to discuss the evolution of policies in the long-run, and the fixed horizon also raises concerns about robustness of the results presented. For this reason we turn to the infinite horizon version of the model next.

4 Infinite horizon model

This section solves the infinite horizon dynamic bargaining model for the two bargaining protocols. For technical reasons we restrict the proposal space along any dimension to lie in a convex compact subset X of $\mathbb{R}$. Hence for both C's proposals and default options, we have $\gamma_t, \bar{\gamma}_t \in X^2 \subseteq \mathbb{R}^2$. However, it will become apparent from the model equilibria below that with X taken to be 'sufficiently large', this assumption is without loss of generality.

We focus on Stationary Markov Perfect Equilibria (S-MPE) where strategies in a given period depend only on the type of that period and on the default option for that period, i.e. only on payoff relevant variables. Focusing on the S-MPE we can drop all time subscripts. We denote by $x \in X$ the default option for a given period with the understanding that it is composed of a default policy status-quo pair $\bar{\gamma}(x) = \{x, x\} \in X^2$. Any policy is always denoted by (appropriately subscripted) $p \in X$ and any status-quo is always denoted by $q \in X$.

For this model, S-MPE will be a combination of several components. For C, we are looking for four functions, two of them mapping the space of default options X into proposed policies for each type of period, $p_D(x), p_A(x) : X^2 \to X$, and the remaining two mapping X into the proposed status-quo, $q_D(x), q_A(x) : X^2 \to X$. Formally, $\rho_C = \{p_D(x), p_A(x), q_D(x), q_A(x)\} : X^4 \to X^4$ denotes C's strategy and her proposal in period $i \in \{A, D\}$ given default option x is $\gamma_i(x) = \{p_i(x), q_i(x)\}$. For P, his strategy given period

$i \in \{A, D\}$ and default option x maps the combination of $\bar{\gamma}(x)$ and $\gamma_i(x)$ into his vote, hence it is a mapping $\rho_P : X^8 \rightarrow \{yes, no\}^2$.

It has to be acknowledged that our definition of ρ_C and ρ_P does not allow for mixed strategies. For P this is driven by the already mentioned fact that in any equilibrium P's voting strategy has to be to vote for C's proposal $\gamma_i(x)$ whenever indifferent between $\gamma_i(x)$ and $\bar{\gamma}(x)$ for $i \in \{A, D\}$. For C the reason behind focusing on pure strategies is twofold. First, we have assumed above that C whose utility maximizing proposal coincides with the default option $\bar{\gamma}(x)$ indeed proposes $\bar{\gamma}(x)$ instead of coming up with a proposal she knows would be rejected. Second, below we focus on a certain class of equilibria (see definition 3) for which it will be true that C's indifference among K proposals $\{\gamma_i^1(x), \ldots, \gamma_i^K(x)\}$ for some default option $x \in X$ and $i \in \{A, D\}$ will imply indifference by P among the same proposals. As a result, in case of C's indifference between two or more proposals we can pick one $\gamma_i^k(x)$ out of $\{\gamma_i^1(x), \ldots, \gamma_i^K(x)\}$ without changing the equilibrium (via changing the equilibrium value functions defined below) and hence we can think of ρ_C as a function instead of thinking of ρ_C as a distribution on X^4. With this qualification in mind, our definition of S-MPE is as follows.

Definition 1 (Stationary Markov Perfect Equilibrium)**.** *A pair of strategies* $\rho^* = \{\rho_C^*, \rho_P^*\}$ *constitutes S-MPE if it constitutes subgame perfect equilibrium.*

Notice that any given pair of strategies $\rho = \{\rho_C, \rho_P\}$ for given x and given path of A and D periods generates a unique path of implemented policies $\{p_0, p_1, \ldots\}$. Taking expectations over all possible paths gives a continuation value function for each policy maker who knows x but does not know whether the next period will be an A or D one,

$$V_C^\rho(x) = \mathbb{E}\left[\sum_{t=0}^{\infty} -\delta^t (p_t - \pi^* + \phi \mathbb{I}_D(t))^2\right]$$
$$V_P^\rho(x) = \mathbb{E}\left[\sum_{t=0}^{\infty} -\delta^t (p_t - \pi^* - \phi \mathbb{I}_D(t))^2\right]$$

where $\mathbb{I}_D(t)$ is D-period indicator function and the superscript ρ captures dependence on given ρ. Having the continuation value functions, we observe these can be equivalently derived as

$$V_C^\rho(x) = r_d\left[-(p_D(x) - \pi^* + \phi)^2 + \delta V_C^\rho(q_D(x))\right] + (1 - r_d)\left[-(p_A(x) - \pi^*)^2 + \delta V_C^\rho(q_A(x))\right]$$
$$V_P^\rho(x) = r_d\left[-(p_D(x) - \pi^* - \phi)^2 + \delta V_P^\rho(q_D(x))\right] + (1 - r_d)\left[-(p_A(x) - \pi^*)^2 + \delta V_P^\rho(q_A(x))\right].$$

Finally, we denote by $A_i^\rho(x)$ P's acceptance set in period $i \in \{A, D\}$ given default option x and strategies ρ. The acceptance sets are given by

$$A_D^\rho(x) = \{\{p, q\} \in X^2 | -(p - \pi^* - \phi)^2 + \delta V_P^\rho(q) \geq -(x - \pi^* - \phi)^2 + \delta V_P^\rho(x)\}$$
$$A_A^\rho(x) = \{\{p, q\} \in X^2 | -(p - \pi^*)^2 + \delta V_P^\rho(q) \geq -(x - \pi^*)^2 + \delta V_P^\rho(x)\}$$

and both are nonempty as $\bar{\gamma}(x) \in A_i(x)$ for $i \in \{A, D\}$.

With this notation, C's problem can be restated in terms of a pair of the usual Bellman functional equations

$$\begin{aligned} U_D^\rho(x) &= \max_{\{p,q\}\in A_D^\rho(x)} \{-(p - \pi^* + \phi)^2 + \delta r_d U_D^\rho(q) + \delta(1 - r_d)U_A^\rho(q)\} \\ U_A^\rho(x) &= \max_{\{p,q\}\in A_A^\rho(x)} \{-(p - \pi^*)^2 + \delta r_d U_D^\rho(q) + \delta(1 - r_d)U_A^\rho(q)\} \end{aligned} \tag{1}$$

where C's continuation value function V_C^ρ will be the probability-weighted sum of the value functions of the two optimization problems, i.e. $V_C^\rho = r_d U_D^\rho + (1 - r_d)U_A^\rho$. An alternative definition of S-MPE that exploits the recursive structure of the model and that we use is the following.

Definition 2 (Stationary Markov Perfect Equilibrium)**.** *A pair of strategies $\rho^* = \{\rho_C^*, \rho_P^*\}$ constitutes a S-MPE if for all $x \in X$ and any period $i \in \{A, D\}$*

1. *C's proposal strategy ρ_C^* solves (1)*

2. *P votes for C's proposal $\gamma_i(x)$ if and only if $\gamma_i(x) \in A_i^{\rho^*}(x)$.*

An equivalent way to express the requirement of the S-MPE is to say we are looking for ρ giving rise to V_C^ρ and V_P^ρ such that when C and P maximize their utility in the current period, their optimal behaviour is indeed expressed as ρ. If we can find such a ρ then by the one deviation principle we have an equilibrium.

Below, when we discuss S-MPE for the two bargaining protocols, it will become apparent that many of them satisfy an additional restriction in P being, for a given default option, indifferent between accepting and rejecting

C's offer, provided C's proposal differs from the unconstrained maximizer of her overall utility. Another way to view this is that as long as the default option x provides P with any real bargaining power, C's proposal will provide him with the minimum utility sufficient for her proposal to pass. We call S-MPE satisfying this feature Conflict S-MPE (CS-MPE). Denoting by γ^{ρ}_{CD} and γ^{ρ}_{CA} solutions to the two optimization problems in (1) when the restrictions on $\{p, q\}$ to lie in P's acceptance sets are removed, CS-MPE is defined as follows.

Definition 3 (Conflict Stationary Markov Perfect Equilibrium)**.** *A pair of strategies $\rho^* = \{\rho^*_C, \rho^*_P\}$ constitutes a CS-MPE if for all $x \in X$ and any period $i \in \{A, D\}$*

1. *ρ^* constitute a S-MPE*
2. *P is indifferent between $\gamma_i(x)$ and $\bar{\gamma}(x)$ provided $\gamma_{Ci} \notin A^{\rho}_i(x)$.*

Our focus on CS-MPE has another rationale as it can be viewed as a focus on equilibria with the minimum winning coalition property. Whenever C is constrained by the other committee member her proposal will make P indifferent between accepting and rejecting. Assuming P is a median member of some larger committee with C's proposal accepted if and only if P accepts, something we show in the context of larger committee in the proposition 7 below, CS-MPE will imply C establishes minimum winning coalitions supporting her proposals. This is reminiscent of the result by Duggan and Kalandrakis (2010) (see part 4 of their theorem 1) who show that minimum winning proposals are a natural feature of equilibria in dynamic bargaining models.[6]

From here on we focus on the equilibrium strategies and we drop the superscript ρ whenever the chance of confusion is minimal. Finally, for the bargaining protocol with implicit status-quo all results of this section additionally require any policy status-quo pair to have both of its elements equal.

[6] Formally the game just described can be viewed as a mapping from a pair of value functions, one for each player, into a new pair of value functions. The CS-MPE assumption makes sure this mapping is 'well behaved'. Without it, the way in which C reconciles indifference between two proposals has real consequences for P, inducing jumps in his value function.

Equilibrium with implicit status-quo

In this section we prove equilibrium existence and uniqueness result for the bargaining protocol with implicit status-quo. We then discuss predictions of the equilibrium about the evolution of policies under implicit status-quo bargaining.

Proposition 1 (S-MPE with implicit status-quo)**.** *Assume* $\delta^2 r_d(3-2r_d) \leq 1-\delta(1-r_d)$*. Then there exists unique CS-MPE. Equilibrium proposals satisfy*

$$p_D(x) = q_D(x) = max\,\{min\{z \in X | z \in A_D(x)\}, \gamma_{CD}\}$$
$$p_A(x) = q_A(x) = max\,\{min\{z \in X | z \in A_A(x)\}, \gamma_{CA}\}$$

for $\forall x \in X$*, where* $\gamma_{CD} = \pi^* - \phi$ *and* $\gamma_{CA} = \pi^* - \phi\delta r_d$*.*

Proof. See appendix A1.

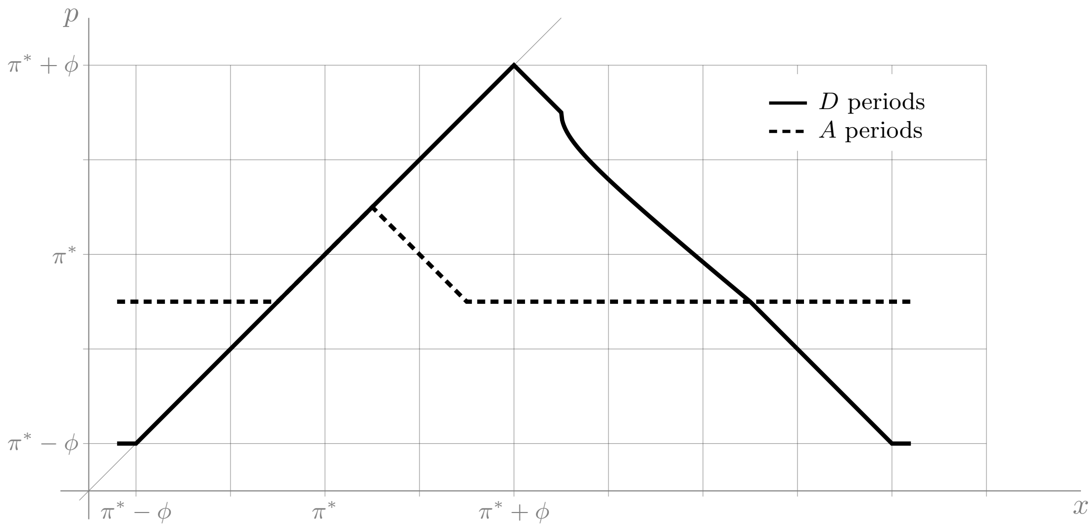
In words, for a given type of period $i \in \{A, D\}$ and default option x, C proposes the lowest policy out of P's acceptance set $A_i(x)$, provided the policy that is an unconstrained maximizer of her overall utility would be rejected, that is provided $\gamma_{Ci} \notin A_i(x)$.

The strategy of the proof follows. Existence follows by construction. We conjecture that the construction will give us a CS-MPE which allows us to derive P's continuation value function V_P and hence his acceptance sets A_A and A_D. Given the acceptance sets we conjecture that C's proposal strategy will be the one given in the proposition allowing us to derive her continuation value function V_C. Having the proposal strategy we note it indeed generates V_P and we confirm strategies generated by V_P and V_C satisfy definition 3 showing that the construction is CS-MPE.

To prove uniqueness of the CS-MPE, we note that it has to generate a unique V_P. What we then need to show is uniqueness of the solution to C's dynamic optimization program (1) given acceptance sets generated by V_P. We show this using an extended version, which we prove, of the theorem guaranteeing existence and uniqueness of solutions to Bellman functional equations from Stokey and Lucas (1989).

The assumption on $\{\delta, r_d\}$ in proposition 1 ensures existence of the CS-MPE equilibrium. The assumption can be alternatively expressed as $\delta \leq \varphi(r_d)$ where $\varphi(0) = \varphi(1) = 1$ and $\min_{r_d \in (0,1)} \varphi(r_d) = 7/9$ so that in effect

Figure 1: Equilibrium policy with implicit status-quo
$\pi^* = 2,\ \phi = 1,\ \delta = 0.5,\ r_d = 0.5$



we are ruling out cases where the 'future looms large' as δ approaches unity. When this happens the requirement on C's proposals under CS-MPE, to bring P to indifference between accepting and rejecting when unable to propose the unconstrained maximum of her overall utility, might fail in D periods. Intuitively, with δ large C focuses primarily on her bargaining position captured by the V_C function when determining which policy to propose. With the V_C function non-monotone, C might propose a policy strictly inside A_D, in effect disregarding her instantaneous utility. When this happen the equilibria become cumbersome to characterize due to non-continuity of V_P so that we rule those cases out by assumption.

To see how the equilibrium from proposition 1 looks in graphical form, figure 1 shows a particular parametrization for $\pi^* = 2, \phi = 1, \delta = 0.5, r_d = 0.5$. While proving proposition 1 we show that depending on the values of δ and r_d, the equilibrium falls into one of four (mutually exclusive) cases. For all four of those cases the A period proposed policy $p_A(x)$ has exactly the same shape as the one given in the figure, with the constant part given by γ_{CA} evaluated at particular values of $\{\delta, r_d\}$.

However, there are case dependent differences regarding the shape of the D period proposed policy $p_D(x)$. What is common to all of them is the constant and then linear increasing part for low values of x. Nevertheless, the default option x for which $p_D(x)$ reaches a maximum in general differs depending on the values of δ and r_d and the 'right' part of $p_D(x)$ (decreasing part in figure 1) is not necessarily monotone or even continuous. One common feature is that it eventually decreases to γ_{CD} where it becomes a constant function again.

Figure 1 (and proposition 1) shows that the equilibrium shares several features with the equilibrium in the two period version of the model. It is a CS-MPE, P is indifferent between accepting and rejecting unless C proposes the unconstrained maximizer of her utility. Furthermore, A periods are in effect lesser disagreement periods with degree of conflict captured by $\phi\delta r_d$ and the committee members are failing to agree on π^*, common bliss point in their instantaneous utility functions. Basic intuition for this result is again the dual role of policy under the implicit status-quo bargaining, it enters policy makers' instantaneous utility while at the same time determining their bargaining position. On the other hand it is the A periods during which C forgoes her bargaining position. By proposing $p_A(x)$ closer to π^* relative to the default option x, she compromises her intertemporal preferences in exchange for the current ones. D periods are then truly disagreement periods and C is fully using her proposal power to steer policy towards her most preferred one.

In order to discuss the long-term policy outcomes generated in equilibrium, we find it helpful to define a set of default options x which, when reached, implies a constant path of default options irrespective of the type of period. Constant default options then imply policies alternating between two (not necessarily) different values, one for A periods and the other for D periods. We call such a set a set of *stable default options* and define it along with two notions of efficiency in the following definition.

Definition 4 (Stable default options and efficiency)**.** *Set $S \subseteq X$ defined by*

$$S = \{x \in X | q_A(x) = q_D(x) = x\}$$

is called set of stable default options (stable set).

We say bargaining displays A-efficiency whenever

$$p_A(x) = \pi^*.$$

We say bargaining displays D-efficiency if

$$p_D(x) = p^*$$

for some $p^* \in [\pi^* - \phi, \pi^* + \phi]$ *across* D *periods.*

The rationale behind the definition of stable set is that once the bargaining reaches $x \in S$, resulting status-quo outcomes are constant forever for any path of A and D periods. If additionally we have $p_A(x) = p_D(x)$ for all $x \in S$ we can say that the bargaining outcomes are unresponsive to the changing preferences of the committee members.

Our notion of efficiency then comes from a static Pareto efficient mechanism implementing an infinite sequence of policies in the current environment. As we show in appendix A2, such a mechanism implements π^* in A periods and $p^* \in [\pi^* - \phi, \pi^* + \phi]$ in D periods. The notion of A-inefficiency whenever $p_A(x) \neq \pi^*$ comes from the fact that the policy makers fail to agree on their current-period most preferred policy π^* due to their concerns about their bargaining position in the future. Given A period and default x such that $p_A(x) \neq \pi^*$, if they could sign a binding contract specifying that the next period default option will be x irrespective of today's policy (which they would set to π^*), both of them would be made better off. The notion of D-inefficiency on the other hand stresses the fact that both policy makers have a preference for policy smoothing. Finally, note that our notion of A-efficiency looks at each A period individually while D-efficiency compares policy decisions reached in different D periods.

Discussing equilibrium policy outcomes is further complicated by the fact that those will in general depend on the default x with which bargaining starts and on the path of A and D periods which is stochastic. Nevertheless, denoting by $x^t(x) \in X$ the default option after t periods of equilibrium play starting with default option x and some path of A and D periods, the following proposition captures the key features.

Proposition 2 (Policy outcomes with implicit status-quo)**.** *CS-MPE from proposition 1 generates policy and status-quo decisions satisfying following.*

1. *If* $x \in S$ *then the policy outcomes display D-efficiency in all subsequent periods*
2. *If* $x \in S$ *then* $p_A(x) = p_D(x) \in [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d]$

For initial default option x_0 being continuous random variable with pdf $f(x_0)$ defined on X, for any $t = 1, 2, \ldots$

3. $\int_{x_0 \in X} \mathbb{P}(x^t(x_0) \notin S) f(x_0) dx_0 \leq r_d^t$

4. $\int_{x_0 \in X} \mathbb{P}(p_A(x^t(x_0)) = \pi^*) f(x_0) dx_0 = 0$ *unless* $r_d = 0$.

Proof. See appendix A1.

Recalling the equilibrium in figure 1 the intuition behind the result is straightforward. For any default option $x \in S$ we have policies constant not only in D periods (part one) but also in A periods (part two). For the third part, for any default option x in A period, policy and hence status-quo reaches S immediately and can stay out of S only for the path of D periods with the probability of t consecutive D periods being r_d^t. The last part then comes from the fact that the set of default options in X that can bring π^* as a policy outcome in the future for some combination of A and D periods has zero measure.

What proposition 2 says is that in CS-MPE from proposition 1 under the bargaining protocol with implicit status-quo, bargaining outcomes eventually become stable for any distribution of initial default option (part three). When this happens the policy outcomes display D-efficiency (part one) on the one hand but become unresponsive to the changing preferences of the two policy makers on the other (part two) with the policy constant henceforth. At the same time, unless $r_d = 0$ for any distribution of initial default option the chance that the bargaining satisfies A-efficiency is zero both on the path to S and once it is reached (part four). In other words, in the CS-MPE under the implicit status-quo bargaining there is no equilibrium force that would bring the bargaining outcome eventually to A-efficiency.

Equilibrium with explicit status-quo

We now show how policy outcomes change when C's proposals are not restricted to those with policy and status-quo equal. The first result we prove is that policy in A periods is equal to π^* for any default option. The logic behind the result is that since in the A periods the preferences of the two policy makers are aligned along the policy dimension, there is no reason they should not be able to reach an agreement on π^*, as doing so needs not compromise their bargaining position embodied in status-quo. The intuition is confirmed by the proposition.

Proposition 3 ($p_A(x)$ with explicit status-quo)**.** *In any S-MPE for any default option $x \in X$*

$$p_A(x) = \pi^*.$$

Proof. See appendix A1.

A key strength of proposition 3 is that it applies to any S-MPE under the explicit status-quo bargaining protocol and shows that this bargaining protocol allows the committee members to reach consensus in A periods. What the proposition does not ensure is existence of such S-MPE, which is what the next proposition does.

Proposition 4 (S-MPE with explicit status-quo)**.** *Assume $\delta \geq \frac{1}{5r_d}$, $\delta \geq 1 - r_d^2$ and $\delta \leq 1 - \frac{(1-r_d)^2}{2}$. Then there exists a unique CS-MPE in terms of associated value functions V_C and V_P. Equilibrium proposals satisfy*

1. *$p_A(x) = \pi^*$ for $\forall x \in X$*
2. *$V_C(x) \leq V_C(q_A(x))$ for $\forall x \in X$*
3. *$V_C(q_D(x)) \leq V_C(q_D(x'))$ for $x, x' \in X$ satisfying $A_D(x) \subseteq A_D(x')$*
4. *C proposes γ_{CD} (γ_{CA}) for $\forall x \in X$ such that $\gamma_{CD} \in A_D(x)$ ($\gamma_{CA} \in A_A(x)$)*

where $\gamma_{CD} = \{\pi^* - \phi, z\}$ and $\gamma_{CA} = \{\pi^*, z'\}$ for some $z, z' \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)$.

Proof. See appendix A1.

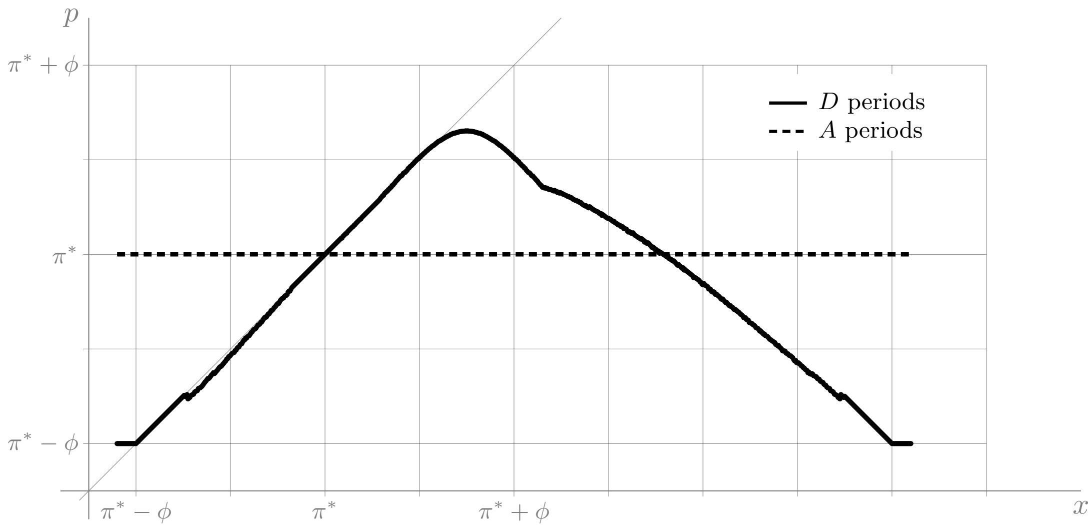
In words, equilibrium under explicit status-quo bargaining involves policy equal to π^* in A periods (part one) with C using A periods to improve her bargaining position (part two). Because C can improve her bargaining position in A periods, she is willing to surrender more of it in D periods in which P has more bargaining power (part three). Finally, C's unconstrained proposals are γ_{CA} and γ_{CD} in A and D periods respectively, implementing C's instantaneous utility bliss point and status-quo that maintains her bargaining position (part four).

The idea of the proof is similar to the proof of proposition 1. For the existence part we partially characterize the equilibrium, conjecturing first that we are characterizing CS-MPE. This gives us the V_P function and associated acceptance sets A_A and A_D. We prove these are well behaved, which allows us to prove existence of C's continuation value function V_C as a solution to C's dynamic optimization program (1). We then confirm that the proposal strategies generated by V_C indeed satisfy definition 3 of CS-MPE. Uniqueness in terms of associated value functions then follows from the uniqueness of V_P in any CS-MPE and resulting uniqueness of V_C.

The key difficulty in the proof of proposition 4 and the source of the assumptions on $\{\delta, r_d\}$ is confirming that proposal strategies associated with V_C indeed satisfy the definition of CS-MPE. What we need to ensure is that intertemporal incentives are strong enough (first two conditions) so that C is willing to use the status-quo dimension of her proposal space in A periods to bring P to indifference between accepting and rejecting as the definition of CS-MPE demands. On the other hand we need to make sure that the intertemporal incentives are not too strong (third condition). When this happens, in D periods P is willing to accept a wide range of policies when offered an even slightly more favourable status-quo compared to the default option. One of those policies is C's D period most preferred policy $\pi^* - \phi$. With proposals involving $\pi^* - \phi$ policy possibly violating requirements of CS-MPE, we need to make sure that C foregoes only little of her bargaining position exactly for those values of $\{\delta, r_d\}$, somewhat paradoxically, when the bargaining position is most valuable.

Figures 2 and 3 show equilibrium proposals from proposition 4 on the policy and status-quo dimension respectively for the same values of parameters used in figure 1. Even though we do not have an explicit expression for V_C we use computer simulation to estimate V_C and associated equilibrium proposal policies (see appendix A3 for details of the numerical simulation). From proposition 4 we know that proposals on the status-quo dimension need not be unique and involve $z, z' \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)$ for defaults such that $\gamma_{CD} \in A_D(x)$ and $\gamma_{CA} \in A_A(x)$. When this happens figures 2 and 3 always use $z = z' = \pi^* - \phi$. Notice also that $\delta = 0.5$ and $r_d = 0.5$ used in the figures do not satisfy the assumption on $\{\delta, r_d\}$ from proposition 4. Nevertheless, given the simulated V_C and associated proposal strategies it is easy to confirm those satisfy the definition of CS-MPE and hence that the

Figure 2: Equilibrium policy with explicit status-quo
$\pi^* = 2,\ \phi = 1,\ \delta = 0.5,\ r_d = 0.5$

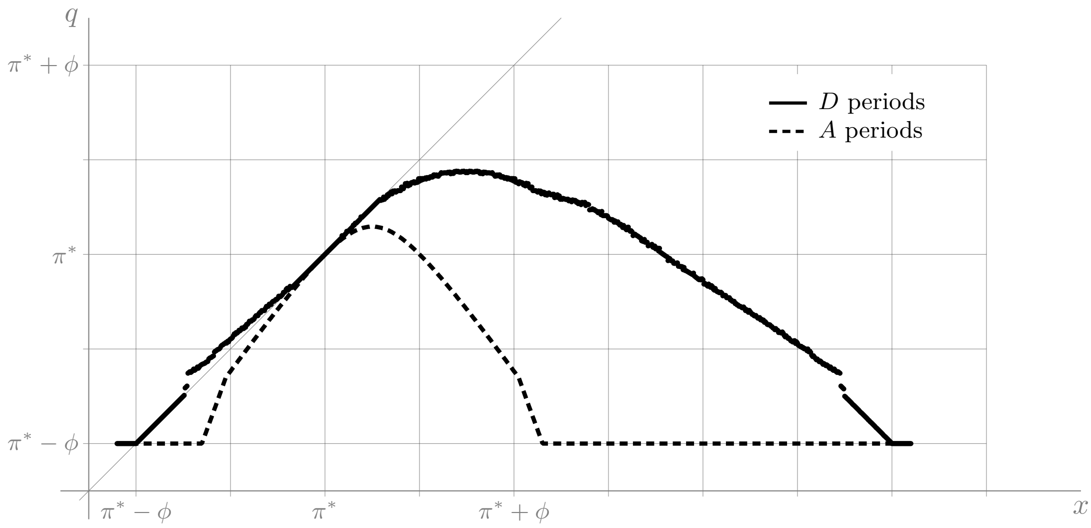


assumptions on $\{\delta, r_d\}$ in proposition 4 are sufficient but not necessary for existence of CS-MPE.

Figures 2 and 3 along with the proposition 4 show that under explicit status-quo bargaining, equilibrium in the infinite horizon again resembles equilibrium in the two period version of the model. It is a CS-MPE equilibrium, A period policy proposals are equal to π^* and C uses A periods to gain a better bargaining position. For any default option x, by offering π^* on the policy dimension P is made better off compared to $\bar{\gamma}(x) = \{x, x\}$, which allows C to gain a better bargaining position on the status-quo dimension in terms of proposing $q_A(x)$ providing her with higher intertemporal utility compared to x.

This in turn makes C willing to forego some of her bargaining position in D periods, a feature not present in the two period model. Intuitively, with C knowing she can gain bargaining position in future A periods without sacrificing on the policy dimension, she is willing to forego some of that bargaining position in D periods in exchange for a more favourable policy outcome. In the two period model any future A period is necessarily the last one with no bargaining position to be gained and hence with C not willing to trade-off policy for status-quo or vice versa in the first period,

Figure 3: Equilibrium status-quo with explicit status-quo
$\pi^* = 2,\ \phi = 1,\ \delta = 0.5,\ r_d = 0.5$



even though it might be a disagreement one.

The absence of an explicit expression for V_C and subsequently for C's proposal strategies further complicates characterization of the policy outcomes under explicit status-quo bargaining. Nevertheless, we are able to prove following.

Proposition 5 (Policy outcomes with explicit status-quo). *CS-MPE from proposition 4 generates policy and status-quo decisions satisfying the following.*

1. *If $x \in S$ then the policy outcomes display D-efficiency in all subsequent periods*

2. *If $x \in S$ then $p_A(x) = \pi^*$ and $p_D(x) = \pi^* - \phi$ for almost all $x \in S$*

For an initial default option x_0 being a continuous random variable with pdf $f(x_0)$ defined on X, for any $t = 1, 2, \dots$

3. $\int_{x_0 \in X} \mathbb{P}(x^t(x_0) \notin S) f(x_0) dx_0 \leq 1 - r_d \int_{x_0 \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)} f(x_0) dx_0$
$$-(1 - r_d) \int_{x_0 \in X \setminus (\pi^* + \phi\delta r_d - \kappa, \pi^* + \phi\delta r_d + \kappa)} f(x_0) dx_0$$

4. $\int_{x_0 \in X} \mathbb{P}(p_A(x^t(x_0)) = \pi^*) f(x_0) dx_0 = 1$

where $\kappa = \phi\sqrt{\delta r_d(3 + \delta r_d)}$.

Proof. See appendix A1.

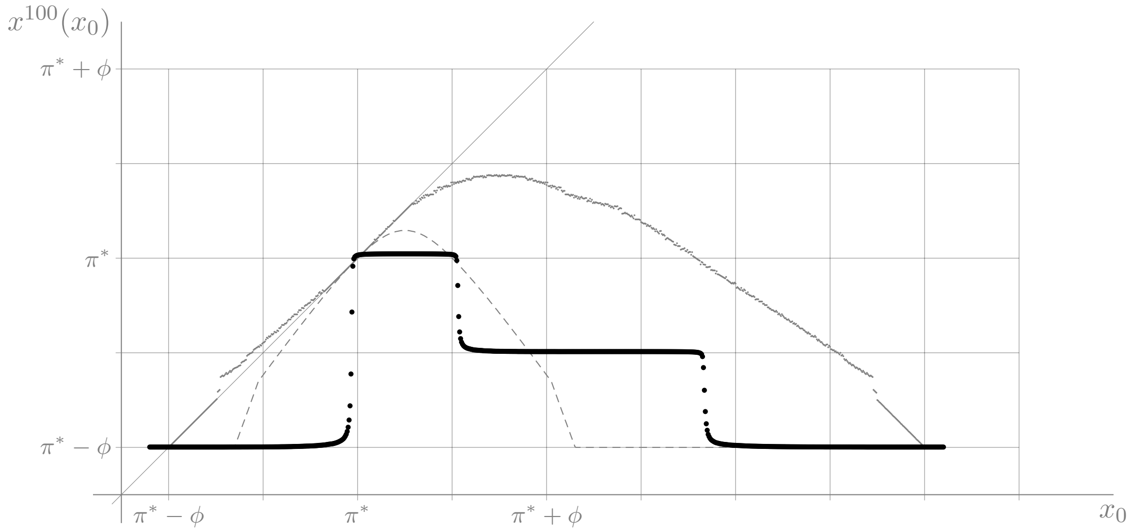
What the proposition 5 says is that in CS-MPE from proposition 4 under the bargaining protocol with explicit status-quo, when the bargaining outcomes become stable they display D-efficiency (part one). When this happens policy outcomes will be π^* in A periods and $\pi^* - \phi$ in D periods except for a finite set of discrete values of default options in S (part two). Indeed, in the proof of proposition 5 we show that the only other candidate default option for inclusion in the S set is $x = \pi^*$. With $\pi^* \in S$ we would have $p_A(\pi^*) = p_D(\pi^*) = \pi^*$. As a result under the explicit bargaining protocol, even when the bargaining outcomes become stable they almost always still reflect the changing preferences of the two policy makers unlike in proposition 2 for implicit status-quo bargaining and are both A-efficient and D-efficient, where the former holds no matter whether the bargaining has reached the stable set or not (part four).

Another difference explicit status-quo bargaining brings is that we cannot put an upper bound on the probability of the bargaining staying outside the stable set that would converge to zero over time (part three). We know from the proof of proposition 4 that for an initial period being an A (D) one and initial default option x_0 satisfying $x_0 \in X \setminus (\pi^* + \phi\delta r_d - \kappa, \pi^* + \phi\delta r_d + \kappa)$ ($X \setminus (\pi^* - \phi, \pi^* + 3\phi)$), we can set C's equilibrium proposal on the status-quo dimension equal to $q_A(x_0) = q_D(x_0) = \pi^* - \phi$ and $q_A(\pi^* - \phi) = q_D(\pi^* - \phi) = \pi^* - \phi$ such that the bargaining becomes stable in the initial period and remains so. When those conditions fail, convergence of the default option to the stable set S remains an open question.

To shed light on the convergence question we generated 10.000 one hundred period long random paths of A and D periods for the parameter values used in figures 2 and 3. For each path, we derived status-quo proposed in the last period $x^{100}(x_0)$ as a function of the initial default option x_0. Averaging over all the 10.000 paths gives figure 4, also depicting (thin lines) equilibrium status-quo offers $q_D(x)$ and $q_A(x)$.

Looking at figure 4, for default options x with $q_D(x) < \pi^*$ and $q_A(x) < \pi^*$, C proposes $q_A(x) < x$ in A periods improving her bargaining position by more than by how much it loses it in D periods by proposing $q_D(x) \geq x$. As a result status-quo in the long term converges to $\pi^* - \phi$, or more precisely

Figure 4: Long-run status-quo with explicit status-quo
average over 10.000 random 100 period long paths
$\pi^* = 2,\ \phi = 1,\ \delta = 0.5,\ r_d = 0.5$



to the $X \setminus (\pi^* - \phi, \pi^* + 3\phi)$ set out of which figure 4 selects $\pi^* - \phi$. In terms of policy outcomes this implies convergence to $\pi^* - \phi$ in D periods and to π^* in A periods with C becoming effectively a dictator in the committee. We call such a committee *authoritarian* or the game being in *authoritarian regime*. However, C has to build up her dominant position gradually over time using A periods to improve her bargaining position and until the status-quo reaches $\pi^* - \phi$, she still has to take into account preferences of the other committee member when crafting her proposal.

For default options x with $q_D(x) > \pi^*$ and $q_A(x) > \pi^*$, status-quo in the long term converges to π^* with C never proposing status-quo that would start the convergent process to $\pi^* - \phi$ discussed above. Such a status-quo is not in P's acceptance set in A periods and would involve considerable loss on the policy dimension in D periods. With the status-quo converging to π^* policy outcomes converge to the same value in both types of periods with the committee becoming consensual and the D period policy outcomes midway in between the preferences of the committee members. We call such a committee *collegial* or the game being in *collegial regime*.

Finally, for default options x with $q_D(x) > \pi^*$ and $q_A(x) < \pi^*$, the long term outcome of the bargaining depends crucially on the nature of the first period. If the bargaining starts with an A period, C is able to start the convergent process towards $\pi^* - \phi$ and the committee eventually becomes authoritarian. Should the bargaining start with D period, C's proposal starts the convergence to π^* and the committee eventually becomes collegial. The line in figure 4 between π^* and $\pi^* - \phi$ then reflects the fact that a proportion r_d of the paths converges to $\pi^* - \phi$ whereas the remaining paths converge to π^*.

Notice the strong path dependency displayed by the model. For some default options the committee eventually becomes authoritarian, for some default options it eventually becomes collegial and for some default options the first period plays a crucial role in determining whether the committee becomes of the former or latter type.

Comparison of the bargaining protocols

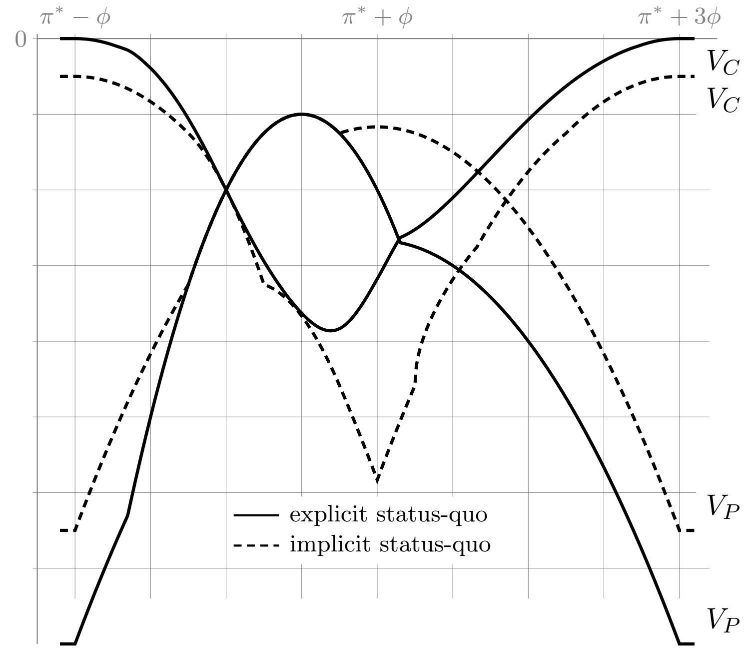
First, we want to provide an answer to the question of comparison between the bargaining protocols from the perspective of the two policy makers. Assume C and P before starting the game just analysed and before the first default option is known, have an option to choose between the bargaining protocols. Would they prefer either of the protocols and does it depend on their beliefs about the initial default option?

Figure 5 illustrates the answer to this question. It depicts the value functions of both policy makers for the two bargaining protocols. All the functions are based on the analytical results except for the V_C function in the model with explicit status-quo, which comes from the simulation exercise.

Note first that the intuition about C preferring the bargaining protocol with explicit status-quo as it relaxes the constraint on her optimization problem is misleading as it does not take into account changes in P's strategic behaviour. Nevertheless, figure 5 suggests C indeed prefers explicit status-quo bargaining protocol for any beliefs about the initial default option.

For P figure 5 suggests he is indifferent between the two bargaining protocols for intermediate values of initial default and strictly prefers bargaining under implicit status-quo otherwise. The intuition behind this result is that for the default options x for which P is indifferent between C's proposal $\gamma_i(x)$ and $\bar{\gamma}(x)$ for $i \in \{A, D\}$ under both bargaining protocols, his continuation value is equal under the two protocols. On the other hand for the

Figure 5: Equilibrium value functions
$\pi^* = 2,\ \phi = 1,\ \delta = 0.5,\ r_d = 0.5$



default options for which C is able to extract all the bargaining power in the long term under the bargaining with explicit status-quo, P prefers the other bargaining protocol as he retains some influence over the enacted policies, which then reflect, at least to some extent, his preferences.

Denoting by $V_i^j(x)$ the value function of player $i \in \{C, P\}$ under bargaining protocol $j \in \{E, I\}$ for default option x, the next proposition then shows that the situation depicted in figure 5 is a general feature of the model.

Proposition 6 (Policy makers' choice over bargaining protocol)**.**

1. *$V_C^E(x) - V_C^I(x) \geq 0$ for $x \in X$*

2. *$V_P^E(x) - V_P^I(x) \leq 0$ for $x \in X$ where the inequality is strict for $x \in X \setminus [\pi^* - \phi\delta r_d, \pi^* + 3\phi\delta r_d]$*

3. $V_C^E(x) - V_C^I(x) + V_P^E(x) - V_P^I(x) = k$, *where*

$$\begin{aligned} &k \geq 0 && \text{for } x \in [\pi^* - \phi\delta r_d, \pi^* + 3\phi\delta r_d] \\ &k = -\frac{2\phi^2\delta r_d(1-r_d)}{1-\delta} && \text{for } x \in X \setminus (\pi^* - \phi, \pi^* + 3\phi) \end{aligned}$$

Proof. See appendix A1.

The third part of the proposition shows choice over the bargaining protocol by players who are also uncertain over the role they will play in the game. Another interpretation is that it shows which bargaining protocol is preferred from the utilitarian perspective. For non-extreme values of the default option it is the explicit status-quo bargaining protocol. It allows for the A-efficient policy outcomes and the initial bargaining position of the P player prevents C from using her proposal power to determine D period policy fully according to her preferences.

On the other hand, for extreme values of the default option the implicit status-quo protocol dominates from the utilitarian perspective. Although it does not deliver A-efficiency it prevents C from fully using her proposal power. The explicit status-quo would allow C to hold on to her bargaining power, becoming a dictator in the committee.

The proposition also shows by how much the implicit status-quo protocol dominates for $x \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)$, i.e. for default options generating the authoritarian regime under explicit status-quo bargaining. The difference increases with δ, is maximized for $r_d = \frac{1}{2}$ and equal to zero for $r_d \in \{0, 1\}$. The intuition for the effect of r_d comes from the benefits and costs of the explicit status-quo protocol. It delivers A-efficiency but creates too much proposal power, implying extreme policies viewed from the perspective of the committee as a whole. With $r_d = 1$ we need not be concerned either with A-efficiency, as there are no A periods, or with the excessive proposal power, as there are no A periods during which C gives up her bargaining position under the implicit status-quo protocol. At the other extreme, with $r_d = 0$ the model is a common preference one with no concerns over efficiency or excessive proposal power present as well.

Multi-member committee

Finally, to prepare for the next section, we want to show that the results presented above apply to any N member committee with fixed chairman, common preferences in A periods and D period preference shocks $-\phi$ and ϕ of the chairman and (not necessarily fixed) median committee member respectively. We call such a committee *essentially two-member* one and define it as follows.

Definition 5 (Essentially two-member committee)**.** *We say a committee composed of N (odd) members is an essentially two-member one if it possesses a fixed chairman with proposal power, has common preference for π^* in A periods, i.e. $\varepsilon_{i,t} = 0$ for $i \in \{1, \ldots, N\}$, and its D period preference parameters satisfy either*

1. *$\varepsilon_{i,t} = \phi_i$ for $i \in \{1, \ldots, N\}$ and $\phi_C = -\phi$, $\phi_m = \phi$ are chairman's and median member's preference parameters respectively,*

or

2. *$\varepsilon_{i,t} = -\phi$ for $i = C$ and $(N-1) \times 1$ vector of remaining preference parameters $\varepsilon_t = \{\varepsilon_{i,t}\}'_{i \in \{1,\ldots,N\} \setminus \{C\}}$ satisfies $\varepsilon_t = \phi + \nu_t$ where ν_t is (possibly each D period specific) vector of random variables with number of negative, zero, positive elements equal to $\frac{N-3}{2}$, 2, $\frac{N-3}{2}$ respectively and $\mathbb{E}[\nu_{i,t}] = 0$ for $i \in \{1, \ldots, N-1\}$ where $\nu_{i,t}$ is i-th element of ν_t.*

In words, any committee is essentially a two-member one if there is a fixed chairman with proposal power and D period preference shock equal to $-\phi$, the whole committee has common preferences in A periods and the D period preference parameters of the remaining committee members satisfy one of the conditions from the definition. The first condition requires the D period preference parameters to be fixed across periods for a given committee member and existence of a median member (among N members) with a preference shock equal to ϕ. The second condition allows for time varying D period preferences but requires those to be equal to ϕ on average and requires existence of two (each D period possibly different) median committee members (among $N-1$ members) with preference shock equal to ϕ. The reason for requiring two median members is that for the second condition we are now choosing among the $N-1$ non-chairman members, which is an even number, and need an equal number of those members with higher and lower preference shock $\phi + \nu_{i,t}$.

Next we need to rule out equilibria that possibly arise due to the committee members voting against their preferences as they realize they are not pivotal. Following Baron and Kalai (1993) we restrict attention to *stage-undominated* voting strategies that for all members $n \in \{1, \ldots, N\}$, all periods $i \in \{A, D\}$, all default options $x \in X$ and all proposals $\gamma(x) \in X^2$ satisfy

$$n \text{ votes for } \gamma(x) \text{ (against } \bar{\gamma}(x)) \Leftrightarrow \gamma(x) \in A_{i,n}(x).$$

where $A_{i,n}(x)$ is acceptance set of player n in period i and default option x.

With the preliminaries established, we are able to prove the following proposition asserting that the results presented above can be equally applied to any larger committee.

Proposition 7 (Committee with more than two members)**.** *Bargaining (policy, status-quo) outcomes under both bargaining protocols for any essentially two-member committee with its members using state-undominated strategies correspond to the bargaining outcomes of a game played between the committee chairman and player with median preference shock and thus to the results presented above.*

Proof. See appendix A1.

5 Re-interpretation of asymmetric FOMC directive

In this section we interpret the asymmetry in FOMC directive in light of our model. We first discuss several reasons that make us believe that the FOMC decision making process is better viewed as proceeding under the explicit status-quo bargaining protocol. Adopting this perspective, we show that the model can replicate existing stylized facts about FOMC decision making. Finally, we discuss a novel interpretation of the asymmetry our model provides.

The structure of the model above is largely inspired by the decision making process in most modern central banks (see Mahadeva and Sterne, 2000, for further details). Typically a committee of several members with a well defined chairman is responsible for repeated decisions on a single monetary policy instrument with a goal to anchor inflation to some predetermined level. While having a single objective, the committee members do not always agree on the most appropriate stance of monetary policy, with differences driven both by personal preferences as well as by the external economic environment. Indeed, Chappell, McGregor, and Vermilyea (2005) show significant statistical differences in both intercepts and effects of economic variables in the 'individual reaction functions' of the FOMC members (see Blinder, 2007, for a discussion of possible causes of the preference heterogeneity in monetary policy committees). This opens up the possibility of time-varying disagreement which our model captures in the agreement/disagreement dichotomy. Finally, in most central banks the monetary policy instrument serves also as the status-quo for the next committee meeting.

FOMC, the decision body of the US Federal Reserve System, makes monetary policy decisions but also decides on the 'asymmetry', 'bias' or 'tilt' in its directive. What is formally known as the domestic policy directive is a set of operating instructions sent to the Open Market Trading Desk at the Federal Reserve Bank of New York. Every directive, in addition to current policy, specifies FOMC's expectations regarding future policy, specifying either asymmetry towards policy tightening or easing (asymmetric) or no change (symmetric). In its original form, the asymmetry has been used between 1983 and 1999 (see Thornton and Wheelock, 2000, for historal account), evolved endogenously, and FOMC has never clarified the meaning it has in its decision making. Additionally, the meaning seems to have evolved over time.[7]

We interpret asymmetry in the FOMC directive as a possible difference between the current policy and a status-quo for several reasons. First, its original intent has been to specify a contingency under which the Open Market Trading Desk would change the FOMC operating target before the next FOMC meeting. The transcript of the discussion during the first FOMC meeting to specify asymmetry in the directive reveals this intention. Chairman Volcker summarized that the whole proposed directive 'says we don't

[7] FOMC transcripts reveal a certain ambiguity regarding the meaning of the asymmetric directive. Chairman Greenspan, when asked this question by one of the new FOMC members, answers that FOMC does not have a 'specific formulation. Asymmetry merely means a general sense of the Committees's disposition or the direction' of its bias (Federal Reserve System, 2011, July 5-6, 1994 transcript, p. 69).

want to tighten right now but we do contemplate easing if the aggregates are noticeably, or quite visibly, soft' (Federal Reserve System, 2011, February 8-9, 1983 transcript, p. 83).

Second, it authorized the FOMC's 'chairman to notch up the Fed funds rate if necessary before the next regular meeting' (Greenspan, 2007, p. 102). Intermeeting change in the FOMC operating target, whether upon contingency or at the chairman's discretion, then implies change in the status-quo in between the committee meetings. At any given meeting the committee might find itself facing a default option different from the policy agreed upon at the previous meeting.

Third, the FOMC used the asymmetry to signal its future intentions over the intermediate horizon.[8] A difference between the policy and the status-quo then comes from credibility concerns.[9] Should the FOMC signal its intention to, say, tighten monetary policy without eventually doing so, its credibility would be compromised. Chairman Greenspan saying 'And I'm concerned about the credibility of the [FOMC] sitting with an asymmetric directive time and time again when the purpose of that is essentially to signal an intermediate trend' (Federal Reserve System, 2011, August 17, 1993 transcript, p. 36) lends itself to this explanation. On another occasion, after six consecutive meetings with no change in policy but asymmetry towards tightening, chairman Greenspan in his opening statement of the 'policy go-around' part of the FOMC meeting says that 'It is quite evident that we have come to a point, as we suggested we might at the last meeting, [...] We have to 'deliver'' (Federal Reserve System, 2011, March 25, 1997 transcript, p. 44). The three reasons taken together make us believe it is more appropriate to think of the FOMC as having the explicit status-quo bargaining protocol.

Notwithstanding the ambiguity regarding the meaning of the asymmetric directive, it generated several papers investigating its role in FOMC decision making. Three hypotheses have been put forward. First, the *authorizing intermeeting policy adjustments* hypothesis holds that the asymmetry gave

[8] Until early 1999 the directive has been published only as a part of FOMC minutes few days after its next meeting. Hence its immediate signalling role was rather limited. As Blinder (2007) notes, the long lag between the meeting and the publication of the minutes means that the 'minutes draw little press or market attention when they are published'.

[9] Recently, several central banks started publishing expected future policy paths along with their current monetary policy decision (see Kahn, 2007, for details). Present argument would apply to those central banks as well.

the FOMC chairman discretion to adjust policy stance in between regularly scheduled FOMC meetings. Chappell, McGregor, and Vermilyea (2007) confirm this hypothesis using data from the 1987 to 1992 period during which the intermeeting policy adjustments were common and refute it for the 1993 to 1999 period during which the intermeeting policy adjustments were rare. Thornton and Wheelock (2000) refute the hypothesis using data from the 1983 to 1999 period. However, with the intermeeting policy adjustments rare in the second half of their sample, they are effectively refuting it for the first half contradicting results of Chappell et al. (2007).

Second, the *predicting future policy changes* hypothesis holds that the asymmetry predicts the direction of future policy changes and increases their likelihood. Regarding the direction part of the hypothesis Thornton and Wheelock (2000), Lapp and Pearce (2000) and Pakko (2005) all confirm it using data from the 1983 to 1999, 1984 to 1998 and 1984 to 2003 periods respectively. Evidence on the likelihood part of the hypothesis is mixed with Thornton and Wheelock (2000) refuting it while Lapp and Pearce (2000) and Pakko (2005) reach an opposite conclusion.

Third, the *consensus building* hypothesis holds that the asymmetry allowed FOMC chairman to craft consensus among the FOMC members. Thornton and Wheelock (2000), Meade (2005) and Chappell et al. (2007) all confirm this hypothesis using data from the 1983 to 1999, 1989 to 1997 and 1987 to 1992 periods respectively, while the last paper refutes it for the 1993 to 1999 sample.

With our model silent on the intermeeting policy adjustment hypothesis, we focus on the latter two hypotheses and ask if our model is consistent with either of them.[10] In order to see how the two hypotheses are reflected in FOMC decision making, we use data about its decisions. For each of 48 meetings between February 4, 1994 and December 12, 1999 (inclusive) we record the change in the federal funds rate target and the adopted asymmetry in FOMC directive.[11] The reason for focusing on the period starting

[10] Intermeeting policy adjustments by FOMC chairman have become increasingly rare during the 1990's. For example, in the 1994 through 1999 period there have been only two intermeeting changes (see Thornton and Wheelock, 2000, for further details).

[11] While the federal funds rate target has not become FOMC's operating target until August 1997 with extent of restraint on commercial bank reserve positions being its operating target prior, there is considerable consensus that FOMC has been shifting its focus from the restraint on reserve positions to the federal funds rate as its operating target well before 1997 (see Thornton and Wheelock, 2000, for detailed discussion).

with the February 4, 1994 meeting is that it marks beginning of FOMC's practice of announcing target changes immediately upon making them and the beginning of the practice of making target changes almost exclusively at the regular FOMC meetings. The December 12, 1999 meeting is then the last meeting before the asymmetry in FOMC directive was replaced by a 'balance of risk assessment'.[12]

Existing hypotheses vs. simulated data

One way to generate theoretical predictions of our model is to simulate random path of equilibrium policy and status-quo proposals. To do so we took C's equilibrium proposal strategies depicted in figures 2 and 3 and generated 100.000 random 100 period long paths of policy and status-quo decisions, $\{p_1, \ldots, p_{100}\}$ and $\{q_1, \ldots, q_{100}\}$, with initial default option x_0 uniformly distributed on the $[\pi^* - \phi, \pi^* + \phi]$ interval.[13] We classify each meeting in period $t \in \{2, \ldots, 100\}$ of a given path as resulting in policy increase, no change or decrease depending on whether $p_t - p_{t-1} \geq \chi$, $|p_t - p_{t-1}| < \chi$ or $p_t - p_{t-1} \leq -\chi$ respectively. Each meeting also generates asymmetry towards increase, no change or decrease depending on whether $q_t - p_t \geq \chi$, $|q_t - p_t| < \chi$ or $q_t - p_t \leq -\chi$ respectively. We set $\chi = 0.075$ in order to match approximately the empirical ratio of the number of meetings resulting in no policy change to the number of meetings resulting in policy change (2.20). Finally, we rescale all data in the simulated sample to mach the number of meetings in the FOMC sample (48).

Table 1 shows data for the predicting future policy changes hypothesis, recording policy change during the given meeting and asymmetry adopted during the previous meeting. FOMC data clearly show support for the direction part of the hypothesis with FOMC never decreasing (increasing) the federal funds rate target with tightening (easing) asymmetry in its directive adopted previously. Similar holds for the simulated data with asymmetry towards policy increase (decrease) never followed by policy decrease (increase)

[12] Alternatively we could have focused on the period up to March 30, 1999 meeting after which FOMC began its practice of publishing statement immediately after each meeting that also included asymmetry contained in its directive (Farka, 2010), but none of the results would be substantially altered. Nor would the results change had we taken our data to start with the February 8-9, 1983 meeting, the very first one to specify the asymmetry in FOMC directive.

[13] We also experimented with either 2 period long paths or x_0 distributed uniformly on $[\pi^* - \phi, \pi^* + 3\phi]$ with little change in the results.

Table 1: Predicting future policy changes hypothesis
$t+1$ period policy change and t period asymmetry

t period asymmetry	$t+1$ period policy change					
	FOMC sample			Simulated sample		
	+	0	−	+	0	−
+	7	14	0	1	1	0*
0	3	18	4	7	25	0
−	0	1	1	0*	7	7

Note: Number of meetings in each cell. FOMC sample from February 4, 1994 to December 12, 1999. Simulated data rescaled and rounded to 48 meetings. * zero before rounding.

during the subsequent meeting.

For the likelihood part of the hypothesis, which holds that the asymmetric directive is associated with higher likelihood of policy change, results are mixed. FOMC meetings data in table 1 show that FOMC changed the federal funds rate target at 15 of its 48 meetings (31.3%) while conditional on asymmetric directive adopted at a previous meeting, FOMC changed the federal funds rate target at 8 of 23 meetings (34.8%). A simple proportions test of the hypothesis that 34.8% equals 31.3% (as opposed to the alternative of the former percentage being higher) yields insignificant test statistics (p-value 0.36).[14] Simulated data then show policy change at 15 out of 48 meetings (31.3%) and conditional on asymmetric directive at 8 out of 16 meetings (50.0%) with test statistics for the test of 50.0% being equal to 31.3% (with the same alternative as above) marginally significant (p-value 0.05).

In order to test the consensus building hypothesis we replicate the argument from Thornton and Wheelock (2000). They argue that the asymmetry in FOMC directive serves a consensus building role, with the asymmetric directives adopted more often during the meetings with no policy change as opposed to meetings with a policy change. Table 2 shows data for the consensus building hypothesis, recording policy change during given meeting and asymmetry adopted during the same meeting.

FOMC data in table 2 show that the asymmetric directive has been

[14] This test is based on normal approximation of binomial with the test statistic equal to $(r' - r)/\sqrt{r(1-r)/n}$ standard normal distributed. In this case $r' = 0.348$, $r = 0.313$ and $n = 23$. We use similar test as Thornton and Wheelock (2000) for comparability.

Table 2: Consensus building hypothesis
t period policy change and t period asymmetry

t period asymmetry	t period policy change			
	FOMC sample		Simulated sample	
	$+/-$	0	$+/-$	0
$+/-$	3	20	8	8
0	12	13	7	25

Note: Number of meetings in each cell. FOMC sample from February 4, 1994 to December 12, 1999. Simulated data rescaled to 48 meetings.

adopted at 23 out of 48 meetings (47.9%) while conditional on no policy change at the same meeting the asymmetric directive has been adopted at 20 out of 33 meetings (60.6%). Using the same test as above to test the hypothesis that 60.6% equals 47.9% (as opposed to the alternative of the former percentage being higher) produces marginally significant test statistics (p-value 0.07). For the simulated data we obtain asymmetric directive adopted at 16 out of 48 meetings (33.3%) and conditional on no policy change asymmetric directive adopted at 8 out of 33 meetings (24.2%) with test statistic for the test of 24.2% being equal to 33.3% (with the same alternative as above) insignificant (p-value 0.87).

Existing hypotheses vs. authoritarian regime

Comparison of the simulated and FOMC decision data faces two possible objections. First, it is dependent on the choice of values for the model parameters. Second, empirical literature on FOMC decision making often notes dominance of chairman Greenspan (see for example Chappell et al., 2005). Hence comparison to the simulated data, which capture convergence to the authoritarian or the collegial regimes explained in the context of discussion of figure 4, might not be appropriate.

Table 3 shows the comparison the model generates assuming the bargaining has already converged to the authoritarian regime. For the policy, AA and DD paths generate no change while AD and DA paths generate policy decrease and increase respectively, as $p_D(x) = \pi^* - \phi$ and $p_A(x) = \pi^*$ in the authoritarian regime. For the asymmetry we have $q_D(x) = q_A(x) = \pi^* - \phi$ in the authoritarian regime and hence A periods produce asymmetry to-

Table 3: Authoritarian regime
predicting future policy/consensus building hypothesis

path	probability	asymmetry		policy change
		$t-1$	t	t
AA	$(1-r_d)^2$	–	–	0
AD	$r_d(1-r_d)$	–	0	–
DD	r_d^2	0	0	0
DA	$r_d(1-r_d)$	0	–	+

wards policy decrease while D periods produce asymmetry towards no policy change.

It is apparent from table 3 that even in the authoritarian regime the asymmetry has an ability to predict the direction of future policy changes. Asymmetry at the $t-1$ period meeting towards lower policy predicts decrease or no change in the policy during the t period meeting while asymmetry towards no policy change predicts subsequent increase or no change in the policy.

For the increased likelihood of the policy change under the asymmetric directive hypothesis, the probability of the policy change is $2r_d(1-r_d)$ and conditional on asymmetric $t-1$ period asymmetry it is r_d, with the latter larger for $r_d \geq \frac{1}{2}$. For the consensus building hypothesis, the authoritarian regime predicts asymmetric directive adopted with probability $1-r_d$ and conditional on no policy change with probability $\frac{(1-r_d)^2}{(1-r_d)^2+r_d^2}$, with the latter larger for $r_d \leq \frac{1}{2}$. Finally, in the authoritarian regime the ratio of the number of meetings resulting in no policy change to the number of meetings resulting in a policy change is equal to $\frac{(1-r_d)^2+r_d^2}{2r_d(1-r_d)}$. This ratio is larger than 2, the approximate ratio in the FOMC data, either for $r_d \leq \frac{3-\sqrt{3}}{6} \doteq 0.21$ or for $r_d \geq \frac{3+\sqrt{3}}{6} \doteq 0.79$.

As a result, for the high degree of conflict in the committee ($r_d \geq \frac{1}{2}$) the authoritarian regime predicts increased likelihood of policy changes given asymmetric directive adopted during the previous meeting but no consensus building role of the asymmetry. On the other hand for the low degree of conflict in the committee ($r_d \leq \frac{1}{2}$) the authoritarian regime predicts a consensus building role of the asymmetry but not increased likelihood of policy changes under the asymmetric directive.

Adopting the view that the FOMC can be approximated by the authoritarian regime with low degree of conflict, our model then predicts a consensus building role of the asymmetry, its ability to predict direction of future policy changes and the majority of meetings resulting in no change in policy, but not that policy is changed more often under the asymmetric directive.

Novel role of asymmetric directive

Besides capturing major stylized facts about the FOMC decision outcomes, the model suggest a novel role of the asymmetry in its directive. One of the predictions for the explicit status-quo protocol, relative to the implicit status-quo one, is that it allows the chairman to gain or retain a dominant position in the committee. When this happens, the chairman is able to press for policy outcomes fully reflecting her preferences. We call this view of the asymmetric directive the *preservation of supremacy* hypothesis.

Dominance of chairman Greenspan in FOMC is not new. Chappell et al. (2005), Blinder (2007) and Meade (2005) all acknowledge it. Blinder (2007) even goes as far as claiming that it is 'quite possible for the Fed to adopt one policy even though the (unweighted) majority favoured another' and ranks the Federal Reserve System very low in terms of democracy in making monetary policy decisions.[15] Interestingly, the original inclusion of the asymmetry in the directive was made upon the suggestion of then chairman Volcker.

While we cannot rigorously test the preservation of supremacy hypothesis because we lack appropriate counterfactuals, the following anecdotal evidence is at least suggestive of its validity. For six consecutive meetings since the July 2-3, 1996 meeting, FOMC has kept the federal funds rate unchanged, adopting asymmetric directive towards tightening in all those meetings. The series was interrupted by the 25 basis point increase at the March 25, 1997 meeting (with symmetric directive) and followed by another 5 meetings with no change in the federal funds rate and asymmetric directive towards tightening, until the November 12, 1997 meeting.

During the whole period FOMC was receiving signals which would, un-

[15] The ranking includes central banks of (from the least to the most democratic) New Zealand, Canada, Australia, USA, Japan, Switzerland, Euro zone, Sweden and UK. In the first three central banks it is the governor responsible for the policy (see Maier, 2010, for details).

der normal circumstances, call for tighter monetary policy. But, as chairman Greenspan argued, the US economy was not operating under normal circumstances. His explanation for declining unemployment and non-increasing inflation was higher productivity growth, at that time not yet apparent from the economic data. But 'his insight played to an unresponsive audience' (Meyer, 2004, p. 80) with 'many committee members [...] leaning [...] toward an increase' (Greenspan, 2007, p. 171).

During the whole period chairman Greenspan tried to persuade the FOMC members out of tightening monetary policy move. The pattern started with the July 2-3, 1996 meeting with chairman Greenspan arguing that his 'judgment is that in all likelihood, if the Committee does not move at [that] meeting or during the intermeeting period, [it] will do so at the August meeting or later' (Federal Reserve System, 2011, July 2-3, 1996 transcript, p. 89). He made similar argument professing to believe that 'the probability of our having to move [...] is still above 50 percent', and that FOMC confronts 'far greater likelihood that the next move will be up rather than down' (Federal Reserve System, 2011, September 24, 1996 and December 17, 1996 transcripts, p. 29 and 36 respectively).

Chairman Greenspan did not use only the probability of near future policy tightening as his argument. When proposing yet another no change in the federal funds rate, he used asymmetry in the FOMC directive proposing an 'asymmetry that is unlike that at the previous couple of meetings. [...] a real asymmetry' (Federal Reserve System, 2011, February 4-5, 1997 meeting stranscript, p. 104).[16] Meyer (2004, p. 83) summarizes chairman Greenspan's behaviour during the periods as 'speaking like a hawk and walking like a dove'.

Combining chairman Greenspan's dominance in FOMC and his disagreement with many of the FOMC members, we can interpret the episode in light of our model as a series of D periods in the authoritative regime. The model then predicts series of $\pi^* - \phi$ policy choices with the status-quo set at the same level, i.e. with symmetric directive. Discrepancy with the asymmetric directives in FOMC decisions is nevertheless only apparent. Future no change or increase in the policy is in the model associated with symmetric di-

[16] FOMC transcripts reveal some of the committee members becoming increasingly uneasy with the continuing discrepancy between the unchanging policy and the asymmetric directive, such as when Ms. Rivlin remarks that she finds 'meaning of these asymmetries a little mysterious' (Federal Reserve System, 2011, December 17, 1996 transcript, p. 36).

rectives but with asymmetric (tightening) directives in the FOMC decisions. Crucially, it is the explicit status-quo protocol that allows the chairman to preserve his dominance in the committee.

Concluding remarks

We have shown that our model, with the explicit status-quo bargaining protocol, can well represent data generated by the FOMC decision making process. It can replicate existing stylized facts and, additionally, gives us an alternative perspective from which the FOMC decision making can be approached and discussed.

While hardly conclusive, we believe the asymmetry in FOMC directive allowed its chairman to influence US monetary policy, at least to some extent. The two opening quotes of the paper then capture the basic trade-off our model creates under the explicit status-quo, increased efficiency at the potential cost of disproportionate proposal power. The former quote, taken at face value, pertains to the efficiency part of the trade-off. But its meaning pertains to the disproportionate proposal power part. The quote can be taken to mean that the asymmetry allowed the FOMC chairman to carry out policy more to his liking than he would be allowed otherwise. The latter quote then shows that this is a real, not only hypothetical, possibility.[17]

Interestingly, the two quotes refer to the same episode, the 1996-1997 event described above. With hindsight, chairman Greenspan turned out to be correct and among the first to identifying a change in the productivity trend. Indeed, the second opening quote immediately goes on to say 'We give him enormous credit for doing so.' The 'we' does not include everybody (see The Economist, 2006, for an alternative view), but that is another story.

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[17] The former quote is interesting from another perspective. It comes from Laurence H. Meyer who is often viewed as Greenspan's fiercest opponent in the productivity debate.

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A1 Proofs

A1.1 Proof of proposition 1

Preliminaries

To prove the existence part of proposition 1 we construct CS-MPE in a model with implicit status-quo. We are forced to split the equilibria of the model into four distinct cases depending on the parameters δ and r_d. However, the logic of the proof is always the same. For each case we first state C's equilibrium proposal strategies. These are generated conjecturing a CS-MPE equilibrium giving the V_P function with acceptance sets A_A and A_D of which proposal strategies, we conjecture, are minima (unless C's unconstrained optima lie in the appropriate acceptance sets). Next, we specify V_C and V_P generated by the proposal strategies and derive the shape of associated acceptance sets A_A and A_D. In the next step we characterize the shape of C's overall utility in A and D periods deriving her unconstrained maxima in the two periods to be $\gamma_{CA} = \pi^* - \phi\delta r_d$ and $\gamma_{CD} = \pi^* - \phi$ respectively. With P's acceptance sets and C's overall utility, we confirm the proposal strategy originally given is indeed optimal for C and can be written in the form given in the proposition.

Having established existence of CS-MPE by constructing it we next turn to the uniqueness part of proposition 1 by showing that the CS-MPE constructed is the unique one. Here we note that in any CS-MPE, V_P has to be the one derived in the existence part and we establish uniqueness of the solution to C's dynamic optimization problem (1).

Throughout the whole proof we maintain the assumption on $\{\delta, r_d\}$ expressed in the proposition, that is we maintain

Assumption 1. *For any pair $\{\delta, r_d\}$ with $\delta \in [0, 1)$ and $r_d \in [0, 1]$ assume $\delta^2 r_d(3 - 2r_d) \leq 1 - \delta(1 - r_d)$.*

Despite the logic of the proof being rather straightforward, the proof itself is rather lengthy and algebra intensive. Striving to keep its length to a minimum, we sometimes omit proofs of purely algebraic results but always indicate how those can be shown.

Throughout the proof, we often refer to C in D periods as to CD and similarly for P (PD) and by analogy in A periods to CA and PA respectively. To save on notation we denote instantaneous utility of the policy makers by

$$f_{CD}(x) = -(x - \pi^* + \phi)^2 \qquad f_{PD}(x) = -(x - \pi^* - \phi)^2$$
$$f_{CA}(x) = -(x - \pi^*)^2 \qquad f_{PA}(x) = -(x - \pi^*)^2$$

and the overall utility by

$$\begin{aligned} U_{CD}(x) &= f_{CD}(x) + \delta V_C(x) \qquad & U_{PD}(x) &= f_{PD}(x) + \delta V_P(x) \\ U_{CA}(x) &= f_{CA}(x) + \delta V_C(x) \qquad & U_{PA}(x) &= f_{PA}(x) + \delta V_P(x). \end{aligned}$$

Throughout the proof we are forced to work with a series of intervals in the default option space X. Those are always denoted by I_i and are always closed (except where explicitly indicated) and convex subsets of X. The upper boundary of I_i is denoted by I_i^U and lower boundary by I_i^L.

Many of the functions in the proof are defined piecewise. If this is the case then we use the notation $f^{I_i}(x)$ for function $f(x)$ constrained to the appropriate interval. Derivatives are often denoted by primes when no confusion as to with respect to which variable the derivative is being taken is imminent.

It will become apparent that many of the functions we work with are differentiable only in the interior of the intervals but not at the point where the two intervals meet. Taking general $f(x)$, $f'(I_i^U)$ will often fail to exist as $f(x)$ has a kink at I_i^U. If this is the case then $f'^{I_i}(I_i^U)$ will always denote left derivative, i.e. derivative as $x \to I_i^U$ from below, and $f'^{I_i}(I_i^L)$ will denote right derivative, i.e. derivative as $x \to I_i^L$ from above.

It is helpful first to establish following lemmas.

Lemma 4.

$$\begin{aligned} U'_{CD}(x) \geq 0 &\Rightarrow U'_{CA}(x) \geq 0 \qquad & U'_{PD}(x) \geq 0 &\Leftarrow U'_{PA}(x) \geq 0 \\ U'_{CD}(x) \leq 0 &\Leftarrow U'_{CA}(x) \leq 0 \qquad & U'_{PD}(x) \leq 0 &\Rightarrow U'_{PA}(x) \leq 0 \end{aligned}$$

Proof. The lemma follows from the readily verifiable facts that $f'_{CA}(x) > f'_{CD}(x)$ and $f'_{PA}(x) < f'_{PD}(x)$ that naturally assumes differentiability of the V_C and V_P functions. A similar result for V_C and V_P non-differentiable at some specific x but possessing left and right derivatives at x follows by analogy. □

Lemma 5. *Let $h(x)$ and $k(x)$ be two real valued continuously differentiable functions defined on $[t-r, t]$ and $[t, t+r]$ respectively, for some $t, r \in \mathbb{R}$ and $r > 0$. Assume $k(t) = h(t)$ and that the first derivative of the functions satisfies $k'(t+x) \leq -h'(t-x)$ for all positive $x \leq r$. Then $k(t+r) \leq h(t-r)$.*

Proof. Integrating the derivative inequality in the lemma with respect to x from 0 to r gives

$$\begin{aligned} \int_0^r k'(t+z)dz &\leq -\int_0^r h'(t-z)dz \\ k(t+r) - k(t) &\leq h(t-r) - h(t) \\ k(t+r) &\leq h(t-r) \end{aligned}$$

□

Lemma 6. *Define*

$$z(x) = \pi^* + \phi(1-\delta(1-r_d)) - \sqrt{\frac{1-\delta}{1-\delta r_d}(x-\pi^*-\phi)^2 + \phi^2\delta(1-r_d)\left(\frac{4\delta^2 r_d^2}{1-\delta r_d} - (1-\delta)\right)}.$$

Then

$$\begin{aligned} \operatorname{sgn}[z(x)'] &= \operatorname{sgn}[\pi^* + \phi - x] \\ \operatorname{sgn}[z(x)''] &= \operatorname{sgn}[-(4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d))]. \end{aligned}$$

Proof. Denote the term in the square root of $z(x)$ by $T(x)$. Then

$$\begin{aligned} z(x)' &= -\frac{1}{\sqrt{T(x)}}\frac{1-\delta}{1-\delta r_d}(x-\pi^*-\phi) \\ z(x)'' &= -\frac{1}{T(x)^{3/2}}\frac{1-\delta}{(1-\delta r_d)^2}\phi^2\delta(1-r_d)(4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d)). \end{aligned}$$

□

Next we give explicit formulas for the continuation value functions of the two policy makers used throughout the proof. As already mentioned, both of the functions are defined piecewise on the different I_i intervals, but we leave the specific definition of the intervals for later when we will show that in the equilibrium the induced continuation value function of C can be pasted together from the following.

$$
\begin{aligned}
V_C^{I_1}(x) =& V_C^{I_{12}}(x) = -\frac{1-r_d}{1-\delta}\phi^2\delta r_d \\
V_C^{I_2}(x) =& V_C^{I_5}(x) = -\frac{r_d}{1-\delta r_d}\left[(x-\pi^*+\phi)^2+\phi^2\frac{\delta(1-r_d)(1-\delta r_d)}{1-\delta}\right] \\
V_C^{I_3}(x) =& -\frac{1}{1-\delta}\left[(x-\pi^*+\phi r_d)^2+\phi^2 r_d(1-r_d)\right] \\
V_C^{I_4}(x) =& V_C^{I_3}(x)+\frac{8(1-r_d)\delta r_d}{(1-\delta)(1-\delta r_d)}[\phi(x-\pi^*)-\phi^2\delta r_d] \\
V_C^{I_6}(x) =& V_C^{I_{11}}(x) = -\frac{r_d}{1-\delta r_d}\left[(\pi^*+3\phi-x)^2+\phi^2\frac{\delta(1-r_d)(1-\delta r_d)}{1-\delta}\right] \\
V_C^{I_7}(x) =& r_d\left[(2(\pi^*+\phi(1-\delta(1-r_d)))-x-\pi^*+\phi)^2+\delta V_C^{I_4}(2(\pi^*+\phi(1-\delta(1-r_d)))-x)\right] \\
& (1-r_d)\left[(2(\pi^*+\phi\delta r_d)-x-\pi^*)^2+\delta V_C^{I_3}(2(\pi^*+\phi\delta r_d)-x)\right] \\
V_C^{I_8}(x) =& r_d\left[(2(\pi^*+\phi(1-\delta(1-r_d)))-x-\pi^*+\phi)^2+\delta V_C^{I_3}(2(\pi^*+\phi(1-\delta(1-r_d)))-x)\right] \\
& (1-r_d)\left[(2(\pi^*+\phi\delta r_d)-x-\pi^*)^2+\delta V_C^{I_3}(2(\pi^*+\phi\delta r_d)-x)\right] \\
V_C^{I_9}(x) =& r_d\left[-(z(x)-\pi^*+\phi)^2+\delta V_C^{I_4}(z(x))\right]+(1-r_d)\left[-(-\phi\delta r_d)^2+\delta V_C^{I_3}(\pi^*-\phi\delta r_d)\right] \\
V_C^{I_{10}}(x) =& r_d\left[-(z(x)-\pi^*+\phi)^2+\delta V_C^{I_3}(z(x))\right]+(1-r_d)\left[-(-\phi\delta r_d)^2+\delta V_C^{I_3}(\pi^*-\phi\delta r_d)\right]
\end{aligned}
$$

Likewise, P's continuation value function in the equilibrium will be pasted together from the following functions.

$$
\begin{aligned}
V_P^{I_3}(x) =& -\frac{1}{1-\delta}\left[(x-\pi^*-\phi r_d)^2+\phi^2 r_d(1-r_d)\right] \\
=& V_P^{I_4}(x) = V_P^{I_7}(x) = V_P^{I_8}(x) \\
V_P^{I_2}(x) =& -\frac{r_d}{1-\delta r_d}\left[(x-\pi^*-\phi)^2+\phi^2\frac{\delta(1-r_d)(1+3\delta r_d)}{1-\delta}\right] \\
=& V_P^{I_5}(x) = V_P^{I_6}(x) = V_P^{I_9}(x) = V_P^{I_{10}}(x) = V_P^{I_{11}}(x) \\
V_P^{I_1}(x) =& V_P^{I_{12}}(x) = -\frac{\phi^2 r_d}{1-\delta}(4-3\delta(1-r_d))
\end{aligned}
$$

At the time being, use of 12 different I_i's might seem redundant, but as will become apparent the fact that the value functions are identical on some intervals is a coincidence. Indeed, they will be induced by parts of the equilibrium that are different in nature.

Having the V_P function we can explain the rationale behind the $z(x)$ function from lemma 6. Looking at V_P it consists of two quadratic terms that apply on different I_i intervals, a property the U_{PD} function will inherit. The $z(x)$ function then allows us to compare U_{PD} across intervals where it is given by different quadratic terms, or formally, $z(x)$ solves $U_{PD}(x) = U_{PD}(z(x))$ for $x \in I_3 \cup I_4$ and $z(x) \in I_9 \cup I_{10}$. More specifically, as the proposition claims that C implements the policy corresponding to the minimal accepted one, $z(x)$ gives us a lower boundary of A_D for default options in the $I_9 \cup I_{10}$ interval. We do not need similar functions for other intervals as those lower boundaries will be linear functions of the default option x.

We sometimes need to use an inverse of $z(x)$ as well. Formally speaking, as $z(x)$ is not monotone, $z^{-1}(x)$ is not well defined. However, it is apparent there are exactly two solutions x to the equation $k = z(x)$ for a given constant k. Taking the larger of the two, we can define the inverse of the function $z(x)$ as $z^{-1}(x) = \{\max\{y : x = z(y)\}\}$.

Existence

Case 1: Equilibrium for $\delta \leq \frac{1}{1+2r_d}$

For $\delta \leq \frac{1}{1+2r_d}$ the equilibrium offers are

$$p_A(x) = \begin{cases} \pi^* - \phi\delta r_d & \text{for } x \in I_1 \cup I_2 \cup I_5 \cup I_6 \cup I_9 \cup I_{10} \cup I_{11} \cup I_{12} \\ x & \text{for } x \in I_3 \\ 2(\pi^* + \phi\delta r_d) - x & \text{for } x \in I_4 \end{cases}$$

$$p_D(x) = \begin{cases} \pi^* - \phi & \text{for } x \in I_1 \cup I_{12} \\ x & \text{for } x \in I_2 \cup I_3 \cup I_4 \cup I_5 \\ 2(\pi^* + \phi) - x & \text{for } x \in I_6 \cup I_{11} \\ z(x) & \text{for } x \in I_9 \cup I_{10} \end{cases}$$

where

$$\begin{aligned} I_1 &= [x^-, \pi^* - \phi] & I_6 &= [\pi^* + \phi, \pi^* + \phi(2 - 3\delta r_d)] \\ I_2 &= [\pi^* - \phi, \pi^* - \phi\delta r_d] & I_9 &= [\pi^* + \phi(2 - 3\delta r_d), \tau^+] \\ I_3 &= [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d] & I_{10} &= [\tau^+, \pi^* + \phi(2 + \delta r_d)] \\ I_4 &= [\pi^* + \phi\delta r_d, \pi^* + 3\phi\delta r_d] & I_{11} &= [\pi^* + \phi(2 + \delta r_d), \pi^* + 3\phi] \\ I_5 &= [\pi^* + 3\phi\delta r_d, \pi^* + \phi] & I_{12} &= [\pi^* + 3\phi, x^+] \end{aligned}$$

where $\tau^+ = \pi^* + \phi + \phi\sqrt{(1 - \delta r_d)^2 - \frac{4\delta^3 r_d^2(1-r_d)}{1-\delta}}$ (τ^- to be used later is defined analogously with the term in the square root subtracted) and x^- and x^+ are respectively lower and upper boundaries of the policy space X.

To see the term in the square root of τ^+ is always positive, substitute in $\delta = 1/(1 + 2r_d)$ which gives a positive expression. Then, differentiating the term in the square root with respect to δ gives an expression that can be regarded as a cubic equation in δ. It has one real root and the derivative is negative below the root. As the root is always higher than unity, it follows that the original expression has to be positive.

It is straightforward to show that the equilibrium offers induce the continuation value functions given above on the appropriate I_i intervals and that both V_C and V_P are continuous everywhere and differentiable everywhere except at the boundaries of the I_i intervals. Next we need to describe the shape of the U_{PA} and U_{PD} functions.

claim 1 (Shape of U_{PA} and U_{PD})**.** *U_{PA} is increasing on $I_1 \cup I_2 \cup I_3$ and decreasing otherwise. U_{PD} is increasing on $I_1 \cup I_2 \cup I_3 \cup I_4 \cup I_5$ and decreasing otherwise. U_{PA} has a global maximum at $\pi^* + \phi\delta r_d$, U_{PD} has a global maximum at $\pi^* + \phi$ and both functions are quasi-concave.*

Proof. It is straightforward to show that U_{PA} is increasing (and hence U_{PD} as well by lemma 4) on $I_1 \cup I_2 \cup I_3$. Similarly U_{PD} is decreasing (and hence U_{PA} by the same lemma) on $I_6 \cup I_9 \cup I_{10} \cup I_{11} \cup I_{12}$. The remaining two intervals, I_4 and I_5, are easy to show as well. It follows U_{PA} has to have a global maximum at $\pi^* + \phi\delta r_d$, which is the boundary of I_3 with I_4 and U_{PD} has to have a global maximum at $\pi^* + \phi$, which is the boundary of I_5 with I_6. Quasi-concavity then follows. □

The next two claims outline the shape of P's acceptance sets.

claim 2 (Shape of $A_A(x)$)**.** *Let x be the default option. Then*

1. *if $x \in I_3$ then $A_A(x) = \{p : x \leq p \wedge p \leq x'\}$ with $x' = 2(\pi^* + \phi\delta r_d) - x \in I_4$*

2. *if $x \in I_4$ then $A_A(x) = \{p : x' \leq p \wedge p \leq x\}$ with $x' = 2(\pi^* + \phi\delta r_d) - x \in I_3$*

3. *if $x \notin I_3 \cup I_4$ then $\pi^* - \phi\delta r_d \in A_A(x)$.*

Proof. Notice U_{PA} is symmetric around $\pi^* + \phi\delta r_d$, which is its global maximum on $I_3 \cup I_4$. Moreover, for any $x \in I_3$, U_{PA} is increasing up to x and for any $x \in I_4$, U_{PA} is decreasing from x on. Hence the first part follows. A similar argument proves the second part.

To see the third part, notice $U_{PA}(I_3^L) = U_{PA}(I_4^U)$ and $I_3^L = \pi^* - \phi\delta r_d$. The third part then follows by the same argument as in the preceding paragraph about the increasing and decreasing parts of U_{PA}. □

claim 3 (Shape of $A_D(x)$)**.** *Let x be the default option. Then*

1. *if $x \in I_1 \cup I_{12}$ then $\pi^* - \phi \in A_D(x)$*

2. *if $x \in I_2$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = 2(\pi^* + \phi) - x \in I_{11}$*

3. *if* $x \in I_3 \cup I_4$ *then* $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ *where* $x' = z^{-1}(x) \in I_9 \cup I_{10}$

4. *if* $x \in I_5$ *then* $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_6$

5. *if* $x \in I_6$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_5$

6. *if* $x \in I_9 \cup I_{10}$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = z(x) \in I_3 \cup I_4$

7. *if* $x \in I_{11}$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_2$.

Proof. All the parts below use the fact that for $x \leq \pi^* + \phi$, U_{PD} is increasing up to x and for $x \geq \pi^* + \phi$, U_{PD} is decreasing from x on. Also convexity of $A_D(x)$ for given x follows from quasi-concavity of U_{PD}.

For part one, notice $U_{PD}(I_1^U) = U_{PD}(I_{12}^L)$ and $I_1^U = \pi^* - \phi$ which along with the argument in the preceding paragraph gives the result.

For part two, notice U_{PD} is symmetric around $\pi^* + \phi$ for $x \in I_2 \cup I_{11}$. This also proves part seven.

For part three, by quasi-concavity of U_{PD} and the fact that U_{PD} has a global maximum at $\pi^* + \phi$ there must exist an upper boundary of the acceptance set that satisfies $x' \geq \pi^* + \phi$. It is easy to confirm $x' \in I_9 \cup I_{10}$ and that x' has to solve $x = z(x')$, i.e. $x' = z^{-1}(x)$.

For part four, notice U_{PD} is symmetric around $\pi^* + \phi$ for $x \in I_5 \cup I_6$. Hence the fourth part follows. This also proves part five.

For part six, we are looking for x' that solves $U_{PD}(x) = U_{PD}(x')$ with $x \in I_9 \cup I_{10}$. It is easy to confirm $x' = z(x) \in I_3 \cup I_4$ is the solution to this equation. □

The following claim gives the shape of the U_{CD} and U_{CA} functions.

claim 4 (Shape of U_{CA} and U_{CD})**.**

1. U_{CA} *is increasing on* $I_1 \cup I_2$ *and decreasing on* $I_3 \cup I_5 \cup I_6 \cup I_{10} \cup I_{11} \cup I_{12}$

2. U_{CD} *is increasing on* I_1 *and decreasing on* $I_2 \cup I_3 \cup I_4 \cup I_5 \cup I_6 \cup I_{10} \cup I_{11} \cup I_{12}$

3. $U_{CA}(x) \geq U_{CA}(x')$ *where* $x \in I_3$ *and* $x' = 2(\pi^* + \phi\delta r_d) - x \in I_4$

4. $U_{CA}(\pi^* - \phi\delta r_d) \geq \max_{x \in I_9} U_{CA}(x)$

5. $U_{CD}(z(x)) \geq U_{CD}(x')\ \forall x' \in [I_9^L, x]$ *given* $x \in I_9$

6. U_{CA} *has a global maximum at* $\pi^* - \phi\delta r_d$ *and* U_{CD} *at* $\pi^* - \phi$.

Proof. The first part is straightforward given the continuation value functions above, except for I_{10}. To establish $U_{CA}^{I_{10}}$ is decreasing, first note

$$V_C''^{I_{10}}(x) = r_d z(x)'' \left[U_{CD}'^{I_3}(z(x))\right] - r_d \frac{2}{1-\delta}[z(x)']^2.$$

The sign of $z(x)''$ by lemma 6 depends on the sign of $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d)$, which is negative for $\delta \leq 1/(1+2r_d)$, and hence $z(x)''$ is positive. The sign of $U_{CD}'^{I_3}(z(x))$ is negative by part two of this claim and the last term is negative so $V_C''^{I_{10}}(x)$ is negative. It follows $U_{CA}''^{I_{10}}$ is concave so if we can establish that $U_{CA}'^{I_{10}}(I_{10}^L)$ is negative the claim follows.

Evaluating $U_{CA}'^{I_{10}}(x)$ at $I_{10}^L = \tau^+$ gives

$$U_{CA}'^{I_{10}}(\tau^+) = -2\phi\left[1 + \left(\frac{\tau^+ - \pi^* - \phi}{\phi}\right)\left(\frac{1-\delta-2\delta r_d(1-\delta(1-r_d))}{(1-\delta)(1-\delta r_d)}\right)\right]$$

where the term in the brackets is positive. To see this, note that the last term in the equation $1-\delta-2\delta r_d(1-\delta(1-r_d)) > 0$. This can be seen regarding the expression as a quadratic equation in δ. It is negative between the roots. One of the roots is higher than unity and the second one is higher than $1/(1+2r_d)$. This establishes the first part.

For the second part, it is again straightforward to establish most of the results. For I_{10} the claim follows from part one of this claim and lemma 4 and for I_4 the claim follows by assumption 1.

The third part follows readily from the derivatives of U_{CA} on I_3 and I_4 using lemma 5 that can be used as I_3 and I_4 have the same width.

To establish the fourth part where we cannot use the derivative argument as U_{CA} may have local maximum on I_9. First note

$$V_C'^{I_9}(x) = r_d z(x)' \left[U_{CD}'^{I_4}(z(x))\right],$$

which by lemma 6 and part two of this claim is positive. Furthermore f_{CA} is decreasing on I_9. Using the inequality $\max_x f(x) + \max_x g(x) \geq \max_x f(x) + g(x)$ we can derive the upper bound on $U_{CA}^{I_9}$ as we know the maxima of the $f_{CA}^{I_9}$ and $V_C^{I_9}$ functions.

The upper bound is given by

$$f_{CA}(I_9^L) + \delta V_C^{I_9}(I_9^U) \geq \max_{x \in I_9} U_{CA}^{I_9}(x)$$

and we need to show it is lower than $U_{CA}(\pi^* - \phi\delta r_d)$. Some algebra gives

$$1 - 3\delta r_d + 3\delta^2 r_d^2 + \frac{\delta^3 r_d^3}{1-\delta} \geq 0,$$

which holds. To see this, we can disregard the last term in the expression that is positive. Regarding the remaining as a quadratic equation in δ gives a pair of roots both of which are complex and it is easy to confirm the expression has to be positive.

The fifth part is complicated by the fact that U_{CD} may have local maxima on I_9. First note that if we prove $U_{CD}(z(x)) \geq U_{CD}(x)\ \forall x \in I_9$ then we are done by the fact that U_{CD} is decreasing on I_4 and $z(x) \in I_4\ \forall x \in I_9$.

To start, we note the relevant parts of the V_C function can be alternatively expressed as

$$\begin{aligned}
V_C^{I_9}(x) =& r_d[f_{CD}(z(x)) + \delta V_C^{I_4}(z(x))] \\
&+ (1 - r_d)[f_{CA}(\pi^* - \phi\delta r_d) + \delta V_C^{I_3}(\pi^* - \phi\delta r_d)] \\
V_C^{I_4}(x) =& r_d[f_{CD}(x) + \delta V_C^{I_4}(x)] \\
&+ (1 - r_d)[f_{CA}(2(\pi^* + \phi\delta r_d) - x) + \delta V_C^{I_3}(2(\pi^* + \phi\delta r_d) - x)],
\end{aligned}$$

which upon substitution into $U_{CD}(z(x)) - U_{CD}(x)$ simplifies the algebra as the first square brackets disappear. Nevertheless, some lengthy and uninstructive algebra remains and gives

$$U_{CD}(z(x)) - U_{CD}(x) =$$
$$4\phi\left[(x - \pi^*) - \frac{1 - \delta - \delta^2 r_d + \delta^2 r_d^2}{1 - \delta}(z(x) - \pi^*) - \frac{3\phi\delta^3 r_d^2(1 - r_d)}{1 - \delta}\right].$$

with the derivation using

$$(z(x) - \pi^*)^2 = \phi^2(1 - \delta(1 - r_d))^2 + T(x) + 2\phi(1 - \delta(1 - r_d))(z(x) - \pi^*).$$

It is easy to confirm this expression is positive for $x = I_9^L$. Taking the derivative with respect to x then gives

$$[U_{CD}(z(x)) - U_{CD}(x)]' = 4\phi\left[1 - \frac{1 - \delta - \delta^2 r_d + \delta^2 r_d^2}{1 - \delta} z(x)'\right],$$

which is positive. To see this notice $1 - \delta - \delta^2 r_d + \delta^2 r_d^2 > 0$ for $\delta \leq 1/(1 + 2r_d)$ and $z(x)'$ is negative by lemma 6. This proves the fifth part. The sixth part is then a direct consequence of the above. □

It is now easy to confirm the specified offers are indeed an equilibrium and can be written in the way used in proposition 1. By claim 4, CA either implements her unconstrained maximum $\pi^* - \phi\delta r_d$ or minimum of $A_A(x)$. This follows from the shape of A_A given in claim 2, which implies that if $\pi^* - \phi\delta r_d \notin A_A(x)$ for some x then $A_A(x) \in I_3 \cup I_4$.

For CD, the best option is when the unconstrained maximum $\pi^* - \phi$ is available. If she cannot implement $\pi^* - \phi$, then the lowest possible policy is implemented. This follows directly from claim 4 where the only problematic interval is I_9. But in claim 3 we have shown that for $x \in I_4$ the acceptance set takes the form $[x, z^{-1}(x)]$ and for $x \in I_9$ the acceptance set takes the form $[z(x), x]$. But then by part five of claim 4, CD implements as low a policy as possible. This concludes proof of case 1.

Case 2: Equilibrium for $\delta \geq \frac{1}{1+2r_d}$ and $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d) \leq 0$

For $\delta \geq \frac{1}{1+2r_d}$ and $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d) \leq 0$ the equilibrium offers are

$$p_A(x) = \begin{cases} \pi^* - \phi\delta r_d & \text{for} \quad x \in I_1 \cup I_2 \cup I_5 \cup I_6 \cup I_{9-} \cup I_{9+} \cup I_{10} \cup I_{11} \cup I_{12} \\ x & \text{for} \quad x \in I_3 \\ 2(\pi^* + \phi\delta r_d) - x & \text{for} \quad x \in I_4 \cup I_7 \end{cases}$$

$$p_D(x) = \begin{cases} \pi^* - \phi & \text{for} \quad x \in I_1 \cup I_{12} \\ x & \text{for} \quad x \in I_2 \cup I_3 \cup I_4 \cup I_5 \\ 2(\pi^* + \phi(1-\delta(1-r_d))) - x & \text{for} \quad x \in I_7 \\ 2(\pi^* + \phi) - x & \text{for} \quad x \in I_6 \cup I_{11} \\ z(x) & \text{for} \quad x \in I_{9-} \cup I_{9+} \cup I_{10} \end{cases}$$

where

$$\begin{aligned} I_1 &= [x^-, \pi^* - \phi] & I_5 &= (\tau_1^-, \pi^* + \phi] \\ I_2 &= [\pi^* - \phi, \pi^* - \phi\delta r_d] & I_6 &= [\pi^* + \phi, \tau_1^+) \\ I_3 &= [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d] & I_{9+} &= [\tau_1^+, \tau^+] \\ I_4 &= [\pi^* + \phi\delta r_d, \pi^* + \phi(1-\delta(1-r_d))] & I_{10} &= [\tau^+, \pi^* + \phi(2+\delta r_d)] \\ I_7 &= [\pi^* + \phi(1-\delta(1-r_d)), \pi^* + 3\phi\delta r_d] & I_{11} &= [\pi^* + \phi(2+\delta r_d), \pi^* + 3\phi] \\ I_{9-} &= [\pi^* + 3\phi\delta r_d, \tau_1^-] & I_{12} &= [\pi^* + 3\phi, x^+] \end{aligned}$$

where as before $\tau^+ = \pi^* + \phi + \phi\sqrt{(1-\delta r_d)^2 - \frac{4\delta^3 r_d^2(1-r_d)}{1-\delta}}$ and $\tau_1^\pm$ are defined as $\tau_1^- = \pi^* + \phi - \phi\sqrt{\frac{\delta(1-r_d)}{1-\delta}((1-\delta)(1-\delta r_d) - 4\delta^2 r_d^2)}$ and τ_1^+ analogously with the term involving the square root being added.

By the condition on this case, the term under the square root in $\tau_1^\pm$ is positive. To see the term in the square root of τ^+ is positive, follow the same procedure as for case 1 but instead of substituting $\delta = 1/(1+2r_d)$ substitute condition $\delta = 1/(1+r_d)$ that is indeed a weaker condition than the condition defining case 2, $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d) \leq 0$.

It is a matter of simple algebra to confirm that the equilibrium offers induce the continuation value functions specified above where I_{9+} and I_{9-} correspond to I_9. For V_P it is easy to show that the function is continuous everywhere and differentiable everywhere except at the boundaries of the I_i intervals. For V_C it can be shown that it is differentiable everywhere except at the boundaries of the I_i intervals. Regarding continuity, V_C is continuous everywhere except at I_5^L and I_6^U where it jumps in a discrete manner. This is a direct consequence of the equilibrium offers not being continuous at the same points with respect to the default x. We first describe the shape of U_{PA} and U_{PD}.

claim 5 (Shape of U_{PA} and U_{PD})**.** *U_{PA} is increasing on $I_1 \cup I_2 \cup I_3$ and decreasing otherwise. U_{PD} is increasing on $I_1 \cup I_2 \cup I_3 \cup I_4 \cup I_5 \cup I_{9-}$ and decreasing otherwise. U_{PA} has a global maximum at $\pi^* + \phi\delta r_d$ and is quasi-concave. U_{PD} has two local maxima at $\pi^* + \phi(1 - \delta(1 - r_d))$ and $\pi^* + \phi$ the latter of which is also a global maximum. U_{PD} has one local minimum at $\pi^* + 3\phi\delta r_d$.*

Proof. It is easy to show U_{PA} is increasing (and hence U_{PD} as well by lemma 4) on $I_1 \cup I_2 \cup I_3$. Similarly U_{PD} is decreasing (and hence U_{PA} by the same lemma) on $I_7 \cup I_6 \cup I_{9+} \cup I_{10} \cup I_{11} \cup I_{12}$. The remaining three intervals, I_4, I_{9-} and I_5, are equally easy. It follows U_{PA} has a global maximum at $\pi^* + \phi\delta r_d$, which is a boundary of I_3 with I_4 and its quasi-concavity follows. Similarly, U_{PD} has two local maxima. One at the boundary of I_4 and I_7 and the second at the boundary of I_5 and I_6. Also, it follows that a local minimum has to be at the boundary of I_7 and I_{9-}. It is easy to show $\pi^* + \phi$ is the global maximum. ☐

Next we wish to characterize the acceptance sets. As the shape of the A_A is exactly the same as in claim 2 we do not repeat it here. For the A_D we have the following.

claim 6 (Shape of $A_D(x)$)**.** *Let x be the default option. Then*

1. *if $x \in I_1 \cup I_{12}$ then $\pi^* - \phi \in A_D(x)$*
2. *if $x \in I_2$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = 2(\pi^* + \phi) - x \in I_{11}$*
3. *if $x \in I_3 \cup [I_4^L, \pi^* + 2\phi(1 - \delta(1 + r_d/2))]$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = z^{-1}(x) \in I_9 \cup I_{10}$*
4. *if $x \in [\pi^* + 2\phi(1 - \delta(1 + r_d/2)), I_4^U]$ then $A_D(x) = A_D^1(x) \cup A_D^2(x)$ where $A_D^1 = \{p : x \leq p \wedge p \leq x'\}$, $A_D^2 = \{p : x'' \leq p \wedge p \leq x'''\}$, $x + x' = 2(\pi^* + \phi(1 - \delta(1 - r_d))$, $x'' + x''' = 2(\pi^* + \phi)$, $x = z(x'') = z(x''')$, $x' \in I_7$, $x'' \in I_{9-}$ and $x''' \in I_{9+}$*
5. *if $x \in I_7$ then $A_D(x) = A_D^1(x) \cup A_D^2(x)$ where $A_D^1 = \{p : x' \leq p \wedge p \leq x\}$, $A_D^2 = \{p : x'' \leq p \wedge p \leq x'''\}$, $x + x' = 2(\pi^* + \phi(1 - \delta(1 - r_d))$, $x'' + x''' = 2(\pi^* + \phi)$, $x' = z(x'') = z(x''')$, $x' \in I_4$, $x'' \in I_{9-}$ and $x''' \in I_{9+}$*

6. *if* $x \in I_{9-}$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x'' \leq p \wedge p \leq x'''\}$, $A_D^2 = \{p : x \leq p \wedge p \leq x'\}$, $x'' + x''' = 2(\pi^* + \phi(1 - \delta(1 - r_d)),$ $x + x' = 2(\pi^* + \phi)$, $x'' = z(x) = z(x')$, $x'' \in I_4$, $x''' \in I_7$ *and* $x' \in I_{9+}$

7. *if* $x \in [I_{9+}^L, \pi^* + \phi(2 - 3\delta r_d)]$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x'' \leq p \wedge p \leq x'''\}$, $A_D^2 = \{p : x' \leq p \wedge p \leq x\}$, $x'' + x''' = 2(\pi^* + \phi(1 - \delta(1 - r_d)),$ $x + x' = 2(\pi^* + \phi)$, $x'' = z(x) = z(x')$, $x'' \in I_4$, $x''' \in I_7$ *and* $x' \in I_{9-}$

8. *if* $x \in I_5$ *then* $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_6$

9. *if* $x \in I_6$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_5$

10. *if* $x \in [\pi^* + \phi(2 - 3\delta r_d), I_{9+}^U] \cup I_{10}$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = z(x) \in I_3 \cup I_4$

11. *if* $x \in I_{11}$ *then* $A_D(x) = \{x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_2$.

Proof. Parts one through three and eight through eleven are very similar to the relevant parts in claim 3. What we cannot use is the quasi-concavity of U_{PD}. However, it is easy to confirm that the acceptance sets are convex.

Parts four through seven present the key difference compared to claim 3. To see these, first notice for the default options specified, U_{PD} has two peaks. One peak is symmetric around $\pi^* + \phi(1 - \delta(1 - r_d))$ and the second one around $\pi^* + \phi$. It then follows $U_{PD}(x) = U_{PD}(x')$ gives four solutions. One pair symmetric around $\pi^* + \phi(1 - \delta(1 - r_d))$ and the second pair symmetric around $\pi^* + \phi$. It is then a matter of straightforward algebra to work out the appropriate intervals. □

Following claim gives the shape of U_{CA} and U_{CD} functions.

claim 7 (Shape of U_{CA} and U_{CD}).

1. U_{CA} *is increasing on* $I_1 \cup I_2$ *and decreasing on* $I_3 \cup I_{9-} \cup I_5 \cup I_6 \cup I_{10} \cup I_{11} \cup I_{12}$

2. U_{CD} *is increasing on* I_1 *and decreasing on* $I_2 \cup I_3 \cup I_4 \cup I_7 \cup I_{9-} \cup I_5 \cup I_6 \cup I_{10} \cup I_{11} \cup I_{12}$

3. $U_{CA}(x) \geq U_{CA}(x')$ *where* $x \in I_3$ *and* $x' = 2(\pi^* + \phi\delta r_d) - x \in I_4 \cup I_7$

4. $U_{CA}(x'') \geq U_{CA}(x')$ *and* $U_{CD}(x'') \geq U_{CD}(x')$ *for every* $x' \in [I_{9+}^L, x]$ *given* $x \in [I_{9+}^L, \pi^* + \phi(2 - 3\delta r_d)]$ *with* $x'' = 2(\pi^* + \phi) - x \in I_{9-}$.

5. $U_{CA}(\pi^* - \phi\delta r_d) \geq \max_{x \in [\pi^* + \phi(2 - 3\delta r_d), I_{9+}^U]} U_{CA}(x)$

6. $U_{CD}(z(x)) \geq U_{CD}(x')\ \forall x' \in [\pi^* + \phi(2 - 3\delta r_d), x]$ *given* $x \in [\pi^* + \phi(2 - 3\delta r_d), I_{9+}^U]$

7. U_{CA} *has a global maximum at* $\pi^* - \phi\delta r_d$ *and* U_{CD} *at* $\pi^* - \phi$.

Proof. The first part is straightforward given the continuation value functions except for I_{10}. As in claim 4 we have V_C concave on this interval so if we can establish that $U_{CA}^{\prime I_{10}}(I_{10}^L)$ is negative the claim follows. In claim 4 this gave us equation

$$U_{CA}^{\prime I_{10}}(\tau^+) = -2\phi\left[1 + \left(\frac{\tau^+ - \pi^* - \phi}{\phi}\right)\left(\frac{1-\delta-2\delta r_d(1-\delta(1-r_d))}{(1-\delta)(1-\delta r_d)}\right)\right]$$

where we could establish negativity by the fact that $1-\delta-2\delta r_d(1-\delta(1-r_d)) > 0$. For the current case we need to do more work as this inequality might not be satisfied.

Note that $\frac{\tau^+-\pi^*-\phi}{\phi} < 1+\delta r_d$, which can be seen consulting the definition of the I_i intervals. Hence if we can prove the derivative is negative when $\frac{\tau^+-\pi^*-\phi}{\phi}$ is replaced by $1+\delta r_d$ the claim follows. Doing that gives

$$U_{CA}^{\prime I_{10}}(\tau^+) = -4\phi\left[\frac{1-\delta-\delta r_d(1-\delta(1-r_d))(1+\delta r_d)}{(1-\delta)(1-\delta r_d)}\right],$$

which is negative as the term in the square brackets is positive. To see that, take the nominator and substitute $\delta = (1+r_d-\sqrt{1-2r_d+17r_d^2})/(2r_d(1-4r_d))$, which is the solution to the condition defining case 2, and confirm the expression is positive. Next, taking the derivative of the nominator with respect to δ gives a quadratic equation in δ with the derivative being negative between the roots. One of the roots is negative and the second one is higher than unity. This shows the $U_{CA}^{\prime I_{10}}(\tau^+)$ is negative and hence proves the first part of the claim.

The second part of the claim is straightforward using the similar argument as part two of claim 4. Likewise, the third part can be established using the same argument as part three of claim 4 noting that the width of I_3 is the same as the width of $I_4 \cup I_7$.

To see the fourth part, notice that if we show $U_{CA}(x') \geq U_{CA}(x)$ and $U_{CD}(x') \geq U_{CD}(x)$ where $x' = 2(\pi^*+\phi) - x \in I_{9-}$ for every default option $x \in [I_{9+}^L, \pi^* + \phi(2-3\delta r_d)]$ then we are done. However, it is easy to confirm $V_C(x') = V_C(x)$ for x, x' just defined. Hence the claim follows.

The fifth part can be established using a similar argument as in part 4 of claim 4 where the derivation of the upper bound on U_{CA}^{9+} is done using exactly the same values.

To prove the sixth part, again the same argument as in part five of claim 4 can be used. However, the conditions on δ defining case 2 alone are not sufficient to ensure $1-\delta-\delta^2 r_d+\delta^2 r_d^2 > 0$. However, the inequality still holds by virtue of assumption 1. Finally, the last part is a direct consequence of the above. □

Again, putting claims 2, 6 and 7 together proves the specified offers are indeed an equilibrium. CA can either implement her unconstrained optimum $\pi^* - \phi\delta r_d$ and when this policy is not available, she offers as low a policy as possible.

The same logic applies for CD. Using claim 7, CD either offers her unconstrained maximizer $\pi^* - \phi$ and if this is not available she offers as low a policy as possible. This can be seen from the fact that U_{CD} is decreasing over the majority of I_i intervals for policies above $\pi^* - \phi$. When we cannot establish decreasing U_{CD}, claims 7 and 6 imply that whenever any policy from such an interval is available, there is also available another policy that gives CD higher utility, with this policy in turn rejected in favour of the lowest policy available. This concludes the proof of case 2.

Case 3: Equilibrium for $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d) \geq 0$ and $\delta \leq \frac{1}{3r_d}$

For $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d) \geq 0$ and $\delta \leq \frac{1}{3r_d}$ the equilibrium offers are

$$p_A(x) = \begin{cases} \pi^* - \phi\delta r_d & \text{for } x \in I_1 \cup I_2 \cup I_{10-} \cup I_{9-} \cup I_{9+} \cup I_{10+} \cup I_{11} \cup I_{12} \\ x & \text{for } x \in I_3 \\ 2(\pi^* + \phi\delta r_d) - x & \text{for } x \in I_4 \cup I_7 \cup I_8 \end{cases}$$

$$p_D(x) = \begin{cases} \pi^* - \phi & \text{for } x \in I_1 \cup I_{12} \\ x & \text{for } x \in I_2 \cup I_3 \cup I_4 \\ 2(\pi^* + \phi(1 - \delta(1-r_d))) - x & \text{for } x \in I_7 \cup I_8 \\ z(x) & \text{for } x \in I_{10-} \cup I_{9-} \cup I_{9+} \cup I_{10} \\ 2(\pi^* + \phi) - x & \text{for } x \in I_{11} \end{cases}$$

where

$$\begin{aligned} I_1 &= [x^-, \pi^* - \phi] & I_{10-} &= [\pi^* + 3\phi\delta r_d, \tau^-] \\ I_2 &= [\pi^* - \phi, \pi^* - \phi\delta r_d] & I_{9-} &= [\tau^-, \pi^* + \phi] \\ I_3 &= [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d] & I_{9+} &= [\pi^* + \phi, \tau^+] \\ I_4 &= [\pi^* + \phi\delta r_d, \pi^* + \phi(1 - \delta(1-r_d))] & I_{10+} &= [\tau^+, \pi^* + \phi(2 + \delta r_d)] \\ I_7 &= [\pi^* + \phi(1 - \delta(1-r_d)), \pi^* + 2\phi(1 - \delta(1 - r_d/2))] & I_{11} &= [\pi^* + \phi(2 + \delta r_d), \pi^* + 3\phi] \\ I_8 &= [\pi^* + 2\phi(1 - \delta(1 - r_d/2)), \pi^* + 3\phi\delta r_d] & I_{12} &= [\pi^* + 3\phi, x^+]. \end{aligned}$$

Case 3 indeed subsumes two important subcases depending on whether $\delta \leq 1/(1 + r_d)$ holds and one of the subcases can even be split further. However, to economize on space and avoid extensive repetition of similar arguments we have decided to treat all the subcases at once.

We stress that some of the I_i intervals above might not be properly defined. For $\delta \geq 1/(1 + r_d)$ the intervals are exactly as those just given with the qualification that I_{9-} and I_{9+} might not exist if τ^- and τ^+ become complex. If this happens, then I_{10-} and I_{10+} naturally extend all the way to $\pi^*+\phi$. If below we need to distinguish those two cases, we refer to case 3.1 if $\delta \geq 1/(1+r_d)$ with $\tau^\pm$ real and to case 3.2 if $\delta \geq 1/(1+r_d)$ with $\tau^\pm$ complex. The remaining possibility, referred to as case 3.3, is when $\delta \leq 1/(1+r_d)$ in which case I_8 ceases to exist and I_7 extends all the way to $\pi^*+3\phi\delta r_d$. If this happens, I_{10-} also ceases to exist and I_{9-} starts immediately at $\pi^*+3\phi\delta r_d$.

As before, the equilibrium offers induce the continuation value functions given above where I_{9-} and I_{9+} map into I_9 and analogously for $I_{10\pm}$. Both V_C and V_P are continuous everywhere and differentiable everywhere except at the boundaries of the I_i intervals. Proceeding similarly, we first describe the shape of U_{PA} and U_{PD}.

claim 8 (Shape of U_{PA} and U_{PD})**.** *U_{PA} is increasing on $I_1 \cup I_2 \cup I_3$ and decreasing otherwise. U_{PD} is increasing on $I_1 \cup I_2 \cup I_3 \cup I_4 \cup I_{10-} \cup I_{9-}$ and decreasing otherwise. U_{PA} has a global maximum at $\pi^*+\phi\delta r_d$ and is quasi-concave. U_{PD} has two local maxima at $\pi^*+\phi(1-\delta(1-r_d))$ and $\pi^*+\phi$ the former of which is also a global maximum. U_{PD} has one local minimum at $\pi^*+3\phi\delta r_d$.*

Proof. The argument is essentially as in claim 5 adjusting for different intervals. The key difference is that the global maximum is at $\pi^*+\phi(1-\delta(1-r_d))$ and not at $\pi^*+\phi$, something that can be readily verified. □

To characterize the shape of the acceptance sets, A_A described in claim 2 applies for the current case as well and we do not repeat it here. Before we describe A_D let us define another pair of constants $\tau_2^\pm$ given by the expression $\tau_2^- = \pi^*+\phi(1-\delta(1-r_d)) - \phi\sqrt{\frac{\delta(1-r_d)}{1-\delta r_d}(4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d))}$ and analogously for τ_2^+. Notice that by one of the conditions defining case 3, the term in the square root is positive. With this definition we have the following.

claim 9 (Shape of $A_D(x)$)**.** *Let x be the default option. Then*

1. *if $x \in I_1 \cup I_{12}$ then $\pi^*-\phi \in A_D(x)$*
2. *if $x \in I_2$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = 2(\pi^*+\phi)-x \in I_{11}$*
3. *if $x \in [I_3^L, \pi^*+2\phi(1-\delta(1+r_d/2))]$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = z^{-1}(x) \in I_{9+} \cup I_{10+}$*
4. *if $x \in [\pi^*+2\phi(1-\delta(1+r_d/2)), \tau_2^-]$ then $A_D(x) = A_D^1(x) \cup A_D^2(x)$ where $A_D^1 = \{p : x \leq p \wedge p \leq x'\}$, $A_D^2 = \{p : x'' \leq p \wedge p \leq x'''\}$, $x+x' = 2(\pi^*+\phi(1-\delta(1-r_d))$, $x''+x''' = 2(\pi^*+\phi)$, $x = z(x'') = z(x''')$, $x' \in I_7 \cup I_8$, $x'' \in I_{10-} \cup I_{9-}$ and $x''' \in I_{9+} \cup I_{10+}$*

5. *if* $x \in [\tau_2^-, \pi^* + \phi(1-\delta(1-r_d))]$ *then* $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ *where* $x' = 2(\pi^* + \phi(1-\delta(1-r_d))) - x \in I_7 \cup I_8$

6. *if* $x \in [\pi^* + \phi(1-\delta(1-r_d)), \tau_2^+]$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi(1-\delta(1-r_d))) - x \in I_3 \cup I_4$

7. *if* $x \in [\tau_2^+, \pi^* + 3\phi\delta r_d]$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x' \leq p \wedge p \leq x\}$, $A_D^2 = \{p : x'' \leq p \wedge p \leq x'''\}$, $x + x' = 2(\pi^* + \phi(1-\delta(1-r_d))$, $x'' + x''' = 2(\pi^* + \phi)$, $x' = z(x'') = z(x''')$, $x' \in I_3 \cup I_4$, $x'' \in I_{10-} \cup I_{9-}$ *and* $x''' \in I_{9+} \cup I_{10+}$

8. *if* $x \in I_{10-} \cup I_{9-}$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x'' \leq p \wedge p \leq x'''\}$, $A_D^2 = \{p : x \leq p \wedge p \leq x'\}$, $x'' + x''' = 2(\pi^* + \phi(1-\delta(1-r_d))$, $x + x' = 2(\pi^* + \phi)$, $x'' = z(x) = z(x')$, $x'' \in I_3 \cup I_4$, $x''' \in I_7 \cup I_8$ *and* $x' \in I_{9+} \cup I_{10+}$

9. *if* $x \in [I_{9+}^L, \pi^* + \phi(2 - 3\delta r_d)]$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x'' \leq p \wedge p \leq x'''\}$, $A_D^2 = \{p : x' \leq p \wedge p \leq x\}$, $x'' + x''' = 2(\pi^* + \phi(1-\delta(1-r_d))$, $x + x' = 2(\pi^* + \phi)$, $x'' = z(x) = z(x')$, $x'' \in I_3 \cup I_4$, $x''' \in I_7 \cup I_8$ *and* $x' \in I_{10-} \cup I_{9-}$

10. *if* $x \in [\pi^* + \phi(2 - 3\delta r_d), I_{10+}^U]$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = z(x) \in I_3 \cup I_4$

11. *if* $x \in I_{11}$ *then* $A_D(x) = \{x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_2$.

Proof. The proof is very similar to the proof of claim 6 where the key difference arises due to the fact that the higher of the peaks is the one symmetric around $\pi^* + \phi(1-\delta(1-r_d))$. □

To finish the proof of case 3, we need to show C indeed wants to implement as low a policy as possible. The next claim proves that.

claim 10 (Shape of U_{CA} and U_{CD})**.**

1. U_{CA} *is increasing on* $I_1 \cup I_2$ *and decreasing on* $I_3 \cup I_{10-} \cup I_{9-} \cup I_{11} \cup I_{12}$

2. U_{CD} *is increasing on* I_1 *and decreasing on* $I_2 \cup I_3 \cup I_4 \cup I_{10-} \cup I_{9-} \cup I_{11} \cup I_{12}$

3. $U_{CA}(x) \geq U_{CA}(x')$ *where* $x \in I_3$ *and* $x' = 2(\pi^* + \phi\delta r_d) - x \in I_4 \cup I_7 \cup I_8$

4. $U_{CD}(x) \geq U_{CD}(x')$ *where* $x \in I_3 \cup I_4$ *and* $x' = 2(\pi^* + \phi(1-\delta(1-r_d))) - x \in I_7 \cup I_8$

5. $U_{CA}(x'') \geq U_{CA}(x')$ *and* $U_{CD}(x'') \geq U_{CD}(x')$ *for every* $x' \in [I_{9+}^L, x]$ *given* $x \in [I_{9+}^L, \pi^* + \phi(2 - 3\delta r_d)]$ *with* $x'' = 2(\pi^* + \phi) - x \in I_{10-} \cup I_{9-}$.

6. U_{CA} *and* U_{CD} *are decreasing on* $[\pi^* + \phi(2 - 3\delta r_d), I_{10+}^U]$

7. U_{CA} has a global maximum at $\pi^* - \phi\delta r_d$ and U_{CD} at $\pi^* - \phi$.

Proof. The first and second parts of the claim can be readily verified using expressions for the continuation value function V_C.

Part three can be established using lemma 6 where we note that we are allowed to use it given that the width of I_3 is the same as width of $I_4 \cup I_7 \cup I_8$. The same argument gives part four as the width of $I_3 \cup I_4$ is larger than the width of $I_7 \cup I_8$.

To see the fifth part, notice that if we show that $U_{CA}(x') \geq U_{CA}(x)$ and $U_{CD}(x') \geq U_{CD}(x)$ with $x' = 2(\pi^* + \phi) - x \in I_{10-} \cup I_{9-}$ for every default policy $x \in [I^L_{9+}, \pi^* + \phi(2 - 3\delta r_d)$ then we are done. However, it is easy to confirm $V_C(x') = V_C(x)$ for x, x' just defined and the claim follows.

Part six is the key difficulty. Note that by lemma 4 it suffices to show U_{CA} decreasing. However, we cannot rely on concavity of V_C as in claims 4 and 7. Instead we will use the following strategy. Writing $U'_{CA}(x) = f'_{CA}(x) + \delta V'_C(x)$ we replace $V'_C(x)$ by the upper bound on its maximum on the appropriate interval and show the resulting expression is negative, which also proves that U_{CA} is decreasing.

Here we are forced to split the proof according to different cases. For cases 3.1 and 3.2 the interval $[\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]$ falls into I_{10+} and we can write

$$V_C'^{I_{10+}}(x) = r_d z(x)' \left[U_{CD}'^{I_3}(z(x))\right]$$

where we want to find an upper bound on the maximum of $V_C'^{I_{10+}}$ on the interval $[\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]$. To do so notice both of the terms are negative and hence if we can find minima of the two terms treated separately this will give us something that has to be higher than the maximum of $V_C'^{I_{10+}}$.

It is easy to establish $z(x)'$ is decreasing on I_{10+} while the term in the square brackets is increasing on I_{10+}. It follows that if we evaluate $z(x)'$ at I^U_{10+} and $U_{CD}'^{I_3}(z(x))$ at $\pi^* + \phi(2 - 3\delta r_d)$ the resulting expression will give us an upper bound on the maximum of $V'_C(x)$ on $[\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]$. Doing so gives

$$\min_{x \in [\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]} z(x)' \geq -1$$

$$\min_{x \in [\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]} U'_{CD}(z(x)) = -6\phi,$$

which gives us a maximum for V'_C. It is then a matter of straightforward algebra to substitute the maximum into $U'_{CA}(x) = f'_{CA}(x) + \delta V'_C(x)$ and confirm the resulting expression is negative on $[\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]$.

For case 3.3, $\pi^* + \phi(2 - 3\delta r_d) \in I_{9+}$ so that we need to use a similar argument but separately on $[\pi^* + \phi(2 - 3\delta r_d), I^U_{9+}]$ and I_{10+}. We can still use

$$V_C'^{I_{9+}}(x) = r_d z(x)' \left[U_{CD}'^{I_4}(z(x))\right] \qquad V_C'^{I_{10+}}(x) = r_d z(x)' \left[U_{CD}'^{I_3}(z(x))\right]$$

and the fact that $z(x)'$ is decreasing on $I_{9+} \cup I_{10+}$ and $U_{CD}'^{I_4}(z(x))$ with $U_{CD}'^{I_3}(z(x))$ are increasing on I_{9+} and I_{10+} respectively. It follows we need to evaluate $z(x)'$ at I_{9+}^U and I_{10+}^U, $U_{CD}'^{I_4}(z(x))$ at $\pi^* + \phi(2-3\delta r_d)$ and $U_{CD}'^{I_3}(z(x))$ at I_{10+}^L.

The evaluation gives

$$\min_{x \in [\pi^* + \phi(2-3\delta r_d), I_{9+}^U] \cup I_{10+}} z(x)' \geq -1$$

$$\min_{x \in [\pi^* + \phi(2-3\delta r_d), I_{9+}^U]} U'_{CD}(z(x)) = -\frac{2\phi}{(1-\delta)(1-\delta r_d)}(3(1-\delta-\delta r_d+\delta^2 r_d^2) - \delta^2 r_d(1-r_d))$$

$$\min_{x \in I_{10+}} U'_{CD}(z(x)) = -\frac{2\phi}{1-\delta}(1-\delta+2\delta r_d).$$

Upon substitution of the maximum of V_C' into $U'_{CA}(x) = f'_{CA}(x) + \delta V'_C(x)$ the condition for U_{CA} decreasing on I_{10+} becomes

$$\frac{\delta r_d}{1-\delta}(1-\delta+2\delta r_d) - 1 - \sqrt{(1-\delta r_d)^2 - \frac{4\delta^3 r_d^2(1-r_d)}{1-\delta}} \leq 0,$$

which holds. To see this notice that for $r_d \leq 1/2$ we are done. Otherwise, substituting $\delta = 1/(1+r_d)$ confirms the condition holds for maximum δ allowed for case 3.3. The derivative of the condition with respect to δ is positive and hence the condition must hold. Therefore U_{CA} (and hence U_{CD} by lemma 4) is decreasing on I_{10+}.

For $[\pi^* + \phi(2-3\delta r_d), I_{9+}^U]$, upon substitution the corresponding condition is $(1-\delta)(4\delta r_d - 1) - 3\delta^2 r_d^2(1-\delta r_d) + \delta^3 r_d^2(1-r_d) \leq 0$, which holds for case 3.3. To see this regard it as a cubic equation in δ. Solving for the roots, noticing that the condition holds for δ below the lowest root and showing that the lowest root is higher than $1/3r_d$ proves the claim. Finally the last part of the claim follows from all the above. □

Combining the information provided by claims 2, 9 and 10 proves the equilibrium for case 3. CA either offers her unconstrained maximizer $\pi^* - \phi\delta r_d$ and when this policy is not available, then she offers as low a policy as possible. This follows from the information about the intervals over which U_{CA} is decreasing provided by claim 10 and where we cannot use this argument the same claim implies that the minimum policy available gives CA the highest utility among the policies available. The same argument applies for CD and concludes the proof for case 3.

Case 4: Equilibrium for $\delta \geq \frac{1}{3r_d}$

For $\delta \geq \frac{1}{3r_d}$ the equilibrium offers are

$$p_A(x) = \begin{cases} \pi^* - \phi\delta r_d & \text{for } \ x \in I_1 \cup I_2 \cup I_9 \cup I_{10} \cup I_{11} \cup I_{12} \\ x & \text{for } \ x \in I_3 \\ 2(\pi^* + \phi\delta r_d) - x & \text{for } \ x \in I_4 \cup I_7 \cup I_8 \end{cases}$$

$$p_D(x) = \begin{cases} \pi^* - \phi & \text{for } \ x \in I_1 \cup I_{12} \\ x & \text{for } \ x \in I_2 \cup I_3 \cup I_4 \\ 2(\pi^* + \phi(1 - \delta(1 - r_d))) - x & \text{for } \ x \in I_7 \cup I_8 \\ z(x) & \text{for } \ x \in I_9 \cup I_{10} \\ 2(\pi^* + \phi) - x & \text{for } \ x \in I_{11} \end{cases}$$

where

$$\begin{aligned} I_1 &= [x^-, \pi^* - \phi] & I_9 &= [\pi^* + 3\phi\delta r_d, \tau^+] \\ I_2 &= [\pi^* - \phi, \pi^* - \phi\delta r_d] & I_{10} &= [\tau^+, \pi^* + \phi(2 + \delta r_d)] \\ I_3 &= [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d] & I_{11} &= [\pi^* + \phi(2 + \delta r_d), \pi^* + 3\phi] \\ I_4 &= [\pi^* + \phi\delta r_d, \pi^* + \phi(1 - \delta(1 - r_d))] & I_{12} &= [\pi^* + 3\phi, x^+]. \\ I_7 &= [\pi^* + \phi(1 - \delta(1 - r_d)), \pi^* + 2\phi(1 - \delta(1 - r_d/2))] \\ I_8 &= [\pi^* + 2\phi(1 - \delta(1 - r_d/2)), \pi^* + 3\phi\delta r_d] \end{aligned}$$

As in the previous case we have subsumed two subcases and prove the equilibrium for those jointly. The first subcase, referred to as case 4.1, is for $\delta \geq 1/(1 + r_d)$. If this condition holds all the intervals are as those given except for I_9 that does not exist and I_{10} starts at $\pi^* + 3\phi\delta r_d$. For $\delta \leq 1/(1 + r_d)$, referred to as case 4.2, the interval I_8 does not exist and I_7 extends all the way to $\pi^* + 3\phi\delta r_d$.

Once again it is easy to confirm that the strategies given induce continuation value functions on the corresponding intervals. For the current case both V_C and V_P are continuous everywhere and differentiable everywhere except for points where the different I_i intervals meet. Proceeding similarly, we first give the properties of U_{PA} and U_{PD}.

claim 11 (Shape of U_{PA} and U_{PD})**.** *U_{PA} is increasing on $I_1 \cup I_2 \cup I_3$ and decreasing otherwise. U_{PD} is increasing on $I_1 \cup I_2 \cup I_3 \cup I_4$ and decreasing otherwise. U_{PA} has a global maximum at $\pi^* + \phi\delta r_d$, U_{PD} has global maximum at $\pi^* + \phi(1 - \delta(1 - r_d))$ and both functions are quasi-concave.*

Proof. The argument is very similar to the one used to prove claim 1 with minor adjustments for the fact that U_{PD} has a global maximum at $\pi^* + \phi(1 - \delta(1 - r_d))$, which is immediately apparent upon realizing that $\pi^* + \phi(1 - \delta(1 - r_d))$ is a boundary of I_4 and I_7. □

Proceeding to outline the shape of the acceptance sets, for A_A the claim 2 applies for the current case as well and we do not repeat it here. For A_D we have following.

claim 12 (Shape of $A_D(x)$)**.** *Let x be the default option. Then*

1. *if $x \in I_1 \cup I_{12}$ then $\pi^* - \phi \in A_D(x)$*
2. *if $x \in I_2$ then $A_D(x) = \{p : x \le p \wedge p \le x'\}$ where $x' = 2(\pi^* + \phi) - x \in I_{11}$*
3. *if $x \in [I_3^L, \pi^* + 2\phi(1 - \delta(1 + r_d/2))]$ then $A_D(x) = \{p : x \le p \wedge p \le x'\}$ where $x' = z^{-1}(x) \in I_9 \cup I_{10}$*
4. *if $x \in [\pi^* + 2\phi(1 - \delta(1 + r_d/2)), \pi^* + \phi(1 - \delta(1 - r_d))]$ then $A_D(x) = \{p : x \le p \wedge p \le x'\}$ where $x' = 2(\pi^* + \phi(1 - \delta(1 - r_d))) - x \in I_7 \cup I_8$*
5. *if $x \in I_7 \cup I_8$ then $A_D(x) = \{p : x' \le p \wedge p \le x\}$ where $x' = 2(\pi^* + \phi(1 - \delta(1 - r_d))) - x \in I_3 \cup I_4$*
6. *if $x \in I_9 \cup I_{10}$ then $A_D(x) = \{p : x' \le p \wedge p \le x\}$ where $x' = z(x) \in I_3 \cup I_4$*
7. *if $x \in I_{11}$ then $A_D(x) = \{p : x' \le p \wedge p \le x\}$ where $x' = 2(\pi^* + \phi) - x \in I_2$.*

Proof. The proof is very similar to the proof of claim 3 where only minor adjustments have to be made for the current case due to the fact that U_{PD} is symmetric around its global maximum at $\pi^* + \phi(1 - \delta(1 - r_d))$ and hence some of the acceptance sets have to be made symmetric around $\pi^* + \phi(1 - \delta(1 - r_d))$. □

Having the acceptance sets the last thing we need to do is to describe the shape of U_{CA} and U_{CD}. The next claim does that.

claim 13 (Shape of U_{CA} and U_{CD})**.**

1. *U_{CA} is increasing on $I_1 \cup I_2$ and decreasing on $I_3 \cup I_9 \cup I_{10} \cup I_{11} \cup I_{12}$*
2. *U_{CD} is increasing on I_1 and decreasing on $I_2 \cup I_3 \cup I_4 \cup I_9 \cup I_{10} \cup I_{11} \cup I_{12}$*
3. *$U_{CA}(x) \ge U_{CA}(x')$ where $x \in I_3$ and $x' = 2(\pi^* + \phi\delta r_d) - x \in I_4 \cup I_7 \cup I_8$*
4. *$U_{CD}(x) \ge U_{CD}(x')$ where $x \in I_3 \cup I_4$ and $x' = 2(\pi^* + \phi(1 - \delta(1 - r_d))) - x \in I_7 \cup I_8$*
5. *U_{CA} has a global maximum at $\pi^* - \phi\delta r_d$ and U_{CD} at $\pi^* - \phi$.*

Proof. The first and second parts of the claim follow readily using the continuation value function, except for intervals I_9 and I_{10}.

For case 4.1 we do not have to worry about I_9 as it is empty. To show U_{CA} is decreasing on I_{10} we use the same argument as in claim 10. The only difference arises from the fact that in case 3.1 the relevant part of the claim 10 $U'^{I_3}_{CD}(z(x))$ has been evaluated at $\pi^* + \phi(2 - 3\delta r_d)$ whereas for case 4.1 we need to evaluate $U'^{I_3}_{CD}(z(x))$ at $\pi^* + 3\phi\delta r_d$. However, it is easy to confirm that $U'^{I_3}_{CD}(z(\pi^* + 3\phi\delta r_d)) = U'^{I_3}_{CD}(z(\pi^* + \phi(2 - 3\delta r_d)))$ and the argument is essentially the same.

For case 4.2 we need to show the claim for both I_9 as well as I_{10}. Nevertheless, the resulting expressions for the maximum of V'_C on the appropriate intervals are the same as in case 3.3 of the relevant part of claim 10. This is due to the fact that the only change is that I_9 starts at $\pi^* + 3\phi\delta r_d$ not at $\pi^* + \phi(2 - 3\delta r_d)$ but $z(x)$ evaluated at those values is the same. Therefore for the I_{10} interval the claim follows by a similar argument as in claim 10. For I_9 the condition for U_{CA} to be decreasing becomes (note this change is due to the fact that the I_9^L now is different than in claim 10) $-\frac{4\delta^3 r_d^2(1-r_d)}{(1-\delta)(1-\delta r_d)} \leq 0$, which holds.

Finally, parts three and four follow by the use of lemma 6 where we note that we can use it as the width of I_3 is the same as $I_4 \cup I_7 \cup I_8$ (part three) and the width of $I_3 \cup I_4$ is larger than the width of $I_7 \cup I_8$ (part four). Part five then follows from the previous parts. □

By a now familiar argument we do not repeat here we have an equilibrium for case 4.

Uniqueness

First notice any distinct CS-MPE has to give rise to P's continuation value function V_P constructed in the previous part of the proof. It then suffices to show that given V_P, C's dynamic optimization program (1) has a unique solution. In order to do so we first need to establish properties of P's acceptance sets.

claim 14. *For any $x \in X$ the acceptance correspondences $A_D(x)$ and $A_A(x)$ are nonempty, compact valued and upper hemicontinuous.*

Proof. The nonempty part follows from the definition and the compact valued part follows from continuity of V_P along with compactness of X. To prove upper hemicontinuity of the acceptance correspondence

$$A_D(x) = \{p \in X | U_{PD}(p) \geq U_{PD}(x)\}$$

pick two sequences $\{x_\alpha\} \to x$ and $\{p_\alpha\} \to p$ such that $p_\alpha \in A_D(x_\alpha)$ $\forall \alpha$. Note that by non-emptiness of A_D this can be done. We need to show $p \in A_D(x)$.

Suppose $p \notin A_D(x)$. Then

$$U_{PD}(x_\alpha) \leq U_{PD}(p_\alpha)\ \forall \alpha$$
$$U_{PD}(x) > U_{PD}(p).$$

Summing the two inequalities gives

$$U_{PD}(x_\alpha) - U_{PD}(x) < U_{PD}(p_\alpha) - U_{PD}(p)\ \forall \alpha.$$

Taking the limit for $\alpha \to \infty$ on both sides gives a contradiction to continuity of $U_{PD}(\cdot)$. For A_A the proof is analogous and hence omitted. □

We note that although we have proven upper hemicontinuity of the acceptance correspondences, for some of the cases above a stronger result, continuity, holds as well. More specifically, for all cases A_A can be proven continuous and for cases 1 and 4, A_D is continuous as well. Failure of lower hemicontinuity of A_D in cases 2 and 3 is then a consequence of the double peakedness of U_{PD} shown in claims 5 and 8. We can always find a sequence of policies approaching the higher peak as A_D is nonempty. On the other hand it is impossible to find a sequence of policies approaching the lower peak 'from above'. Given that we do not need this stronger result, we state it without proving.

Returning to our main argument, to prove the uniqueness of the CS-MPE we need to show uniqueness of the solution to C's optimization problem (1). The optimization problem can be rewritten as a Bellman type functional equation

$$V_C(x) = r_d \max_{p \in A_D(x)} \{f_{CD}(p) + \delta V_C(p)\} + (1 - r_d) \max_{p \in A_A(x)} \{f_{CA}(p) + \delta V_C(p)\}$$

and we already know the acceptance correspondences are upper hemicontinuous. If we could prove their continuity we would be able to use theorem 4.6 in Stokey and Lucas (1989) to prove uniqueness of V_C solving the functional equation above. It turns out a similar result holds for upper hemicontinuous correspondences as well (with associated value functions upper semicontinuous, not continuous as in Stokey and Lucas, 1989). The following theorem states the result formally.

Theorem 1. *Let X be a convex subset of $\mathbb{R}^n$, $\Gamma : X \twoheadrightarrow X$ nonempty, compact valued and upper hemicontinuous correspondence, $F : A \to \mathbb{R}$ on $A = \{(x, y) \in X \times X |\ y \in \Gamma(x)\}$ bounded and upper semicontinuous function, $SC(X)$ space of bounded upper semicontinuous functions $f : X \to R$ with the sup norm $\|f\| = \sup_{x \in X} |f(x)|$ and $\beta < 1$. Then, the T operator, defined by*

$$(Tf)(x) = \max_{y \in \Gamma(x)} [F(x, y) + \beta f(y)] \tag{2}$$

maps $SC(X)$ into itself and has a unique fixed point $v = Tv$.

Proof. The strategy of the proof is in the following. First, we make sure that a maximum in (2) exists, next we show that Tf is upper semicontinuous (u.s.c.) and, hence, T maps $SC(X)$ into itself. Next, we observe that T is a contraction and, hence, has a unique fixed point, provided that $SC(X)$ is complete. As is customary, we view the normed vector space $(X, \|\cdot\|)$ as a metric space on X with the uniform metric $d(f,g) = \|f-g\|$.

Since the notion of upper semicontinuity is not well known in the economic literature, we provide its definition.

Definition 6 (upper semicontinuous function)**.** *A function $f : X \to \bar{\mathbb{R}}$ on a topological space X is upper semicontinuous at $x \in X$ if, for each $\epsilon > 0$, there exists a neighbourhood U of x such that $f(y) \leq f(x) + \epsilon$ for all y in U. It is upper semicontinuous if it is upper semicontinuous $\forall x \in X$.*

An alternative definition, sometimes used, takes a sequence $\{x_n\}$ and defines u.s.c. as a function that satisfies $x_n \to x \Rightarrow \limsup_n f(x_n) \leq f(x)$ which is, indeed, the same requirement (Bourbaki, 2007, Chapter IV.6, Proposition 4). Yet, another definition requires the set $\{x \in X | f(x) < c\}$ to be open for any $c \in \mathbb{R}$, which is equal to the previous definition (Aliprantis and Border, 2006, Lemma 2.42).

Intuitively, u.s.c. functions are allowed to jump but, when they do so, the value of the function at the jump is 'the higher of the two'. The advantage of the u.s.c. functions is that they possess maxima on compact intervals.

Coming back to the proof, first observe that, for any $x \in X$, the function $F(x,\cdot)+\beta f(\cdot)$ is u.s.c. and is maximized on a compact, non-empty set $\Gamma(x)$, hence, the maximum exists (Aliprantis and Border, 2006, Theorem 2.43).

Furthermore, as Γ is upper hemicontinuous, T is u.s.c. (Aliprantis and Border, 2006, Lemma 17.30) and it is clearly bounded. Hence, $T : SC(X) \to SC(X)$.

Next, we need to make sure that T satisfies conditions under which Blackwell's Theorem (Aliprantis and Border, 2006, Theorem 3.53) holds. Denoting by $B(X)$ the space of bounded functions defined on X, we need T to map a closed linear subspace of $B(X)$ that includes constant functions into itself. Furthermore, we need T to satisfy *monotonicity* and *discounting*.

That $SC(X)$ is a linear subspace of $B(X)$ that includes constant functions follows trivially. To establish that $SC(X)$ is closed, we observe that $B(X)$ is complete and that any complete subset of a complete metric space is closed (Berberian, 1999, Chapter III.4, Theorem 1). Hence, if we can establish that $SC(X)$ is complete, then closedness follows.

To establish that $SC(X)$, with the uniform metric, is a complete metric space, we adopt the approach of the proof of theorem 3.1 in Stokey and Lucas (1989), with appropriate modifications. We find a function f to which a Cauchy sequence of functions $\{f_n\}$ converges, we show the sequence converges in the uniform metric and, finally, that $f \in SC(X)$.

First, fix $x \in X$ and take a sequence $\{f_n(x)\}$, which satisfies

$$|f_n(x) - f_m(x)| \leq \sup_{y \in X} |f_n(y) - f_m(y)| = \|f_n - f_m\|$$

and which satisfies the Cauchy criterion and, hence, converges to a limit $f(x)$.

Second, we need to show that $\{f_n\}$ converges in the uniform metric. Pick $\epsilon > 0$ and $N := N(\epsilon)$, such that $n, m \geq M \Rightarrow \|f_n - f_m\| \leq \epsilon/2$ (which can be done). For any $x \in X$ and all $n, m \geq N$

$$\begin{aligned}|f_n(x) - f(x)| &\leq |f_n(x) - f_m(m)| + |f_m(x) - f(x)| \\ &\leq \|f_n - f_m\| + |f_m(x) - f(x)| \\ &\leq \epsilon/2 + |f_m(x) - f(x)|.\end{aligned}$$

As $f_m(x) \to f(x)$, choose $m(x)$ for each $x \in X$ such that $|f_m(x) - f(x)| \leq \epsilon/2$. As x was arbitrary, it follows that $\|f_n - f\| \leq \epsilon$ for $\forall n \geq N$ and, as ϵ was arbitrary, we have convergence in the uniform metric.

Third, we need to show that f is bounded and u.s.c., the first of which follows readily. To show the u.s.c. part, pick $\epsilon > 0$ and k such that $\|f_k - f\| \leq \epsilon/3$. As $f_n \to f$, this can be done. Then, choose δ such that $\|x - y\|_E < \delta \Rightarrow f_k(y) < f_k(x) + \epsilon/3$ where $\|\cdot\|_E$ is a usual Euclidean distance and it can be done by u.s.c. of f_k. Finally,

$$\begin{aligned}f(y) - f(x) &= f(y) - f_k(y) + f_k(y) - f_k(x) + f_k(x) - f(x) \\ &\leq |f(y) - f_k(y)| + f_k(y) - f_k(x) + |f_k(x) - f(x)| \\ &\leq 2\|f - f_k\| + f_k(y) - f_k(x) \\ &\leq \epsilon.\end{aligned}$$

Furthermore, it is easy to confirm that $g \leq f$ implies $Tg \leq Tf$ (monotonicity) and that there exists $\beta \in (0, 1)$, such that $T(f + c) \leq Tf + \beta c$ for any constant function c (discounting). Hence, by Blackwell's Theorem, T is a contraction and it has a unique fixed point, which concludes the proof. □

With C's optimization problem we can define operator T similarly as in theorem 1 by

$$Tv(x) = r_d \max_{p \in A_D(x)} \{f_{CD}(p) + \delta v(p)\} + (1 - r_d) \max_{p \in A_A(x)} \{f_{CA}(p) + \delta v(p)\}.$$

It is easy to see T satisfies monotonicity and discounting, and existence of a fixed point $Tv = v$ can be proven in a similar way as in theorem 1. The fixed point of T is then C's continuation value function V_C derived in the existence part of the proof, which defines a unique equilibrium proposal strategy for C, proving uniqueness of the CS-MPE.

Notice that instead of using T, we could have used the original formulation of C's optimization problem given in (1) and worked with a pair of mappings defined by

$$T_D u_D(x) = \max_{p \in A_D(x)} \{f_{CD}(p) + \delta r_d u_D(p) + \delta(1 - r_d) u_A(p)\}$$
$$T_A u_A(x) = \max_{p \in A_A(x)} \{f_{CA}(p\ ) + \delta r_d u_D(p) + \delta(1 - r_d) u_A(p)\}$$

using theorem 1 to prove existence of a unique fixed point of T_D for each u_a and existence of a unique fixed point of T_A for each u_d. To complete the proof we would then need to show existence of coincidence solution u_D^*, u_A^* such that u_D^* is a fixed point of T_D for u_A^* and u_A^* is a fixed point of T_A for u_D^*. Using an approach similar to Liu, Agarwal, and Kang (2004) this is possible, but would not give us the uniqueness result that is the focus of this part of the proof.

A1.2 Proof of proposition 2

Using definition 4 of S and the results from the proof of proposition 1 it is easy to see $S = [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d]$ and hence parts one and two. For part three notice $p_A(x) \in S$ for any $x \in X$ and hence $x^t(x) \notin S$ for some $x \in X$ implies that all the t periods generating $x^t(x)$ need to be D periods. As a result we have $\mathbb{P}(x^t(x) \notin S) \leq r_d^t$ for any $x \in X$. Part four follows from the fact that $\mathbb{P}(p_A(x^t(x)) = \pi^*) = 0$ for almost all $x \in X$ except for a finite set of discrete values of zero measure.

A1.3 Proof of proposition 3

Assume there exists S-MPE with $p_A(x) = \pi^* + \varepsilon$ for some $x \in X$ and (not necessarily positive) $\varepsilon \neq 0$. Let $\gamma = \{p_A(x) = \pi^* + \varepsilon, q_D(x)\}$ be C's equilibrium proposal and $\gamma' = \{\pi^* + \varepsilon/2, q_D(x)\}$. By the definition of S-MPE it must be that γ solves C's optimization problem, that is it is a solution to

$$\max_{\{p,q\} \in X^2} \left\{-(p - \pi^*)^2 + \delta V_C(q)\right\}$$
$$\text{s.t.} \ -(p - \pi^*)^2 + \delta V_P(q) \geq -(x - \pi^*)^2 + \delta V_P(x).$$

By continuity of the constraint in p proposal $\gamma' \in A_A(x)$. C's utility from γ' is $-\varepsilon^2/4 + \delta V_C(q_D(x))$ and from γ it is $-\varepsilon^2 + \delta V_C(q_D(x))$. By assumption γ is an equilibrium hence

$$-\varepsilon^2 + \delta V_C(q_D(x)) \geq -\varepsilon^2/4 + \delta V_C(q_D(x)),$$

which implies $\varepsilon^2 \leq \varepsilon^2/4$, a contradiction.

A1.4 Proof of proposition 4

To prove existence of CS-MPE in the model with explicit status-quo bargaining protocol, we proceed as follows. First we give formal meaning to the term C's unconstrained proposals by deriving a global maximum of the V_C function. Then we conjecture that any equilibrium offer $\gamma(x)$ that C makes to P for default option $\bar{\gamma}(x)$ has to make P indifferent between the two options unless C can propose the unconstrained maximizer of her overall utility. This allows us to derive explicit expressions for the continuation value function V_P of the P player and hence the shape of his acceptance sets. Given the acceptance sets we show that those are well behaved and hence that the C's dynamic optimization program has a solution. We then go back and make sure that the equilibrium policies indeed satisfy the original conjecture of making P indifferent between $\gamma(x)$ and $\bar{\gamma}(x)$.

As before we refer to C in D period as to CD and analogously for P and A periods. We keep the notation

$$
\begin{aligned}
f_{CD}(x) &= -(x-\pi^*+\phi)^2 & \qquad f_{PD}(x) &= -(x-\pi^*-\phi)^2\\
f_{CA}(x) &= -(x-\pi^*)^2 & \qquad f_{PA}(x) &= -(x-\pi^*)^2
\end{aligned}
$$

and denote the overall utility by

$$
\begin{aligned}
U_{CD}(p,q) &= f_{CD}(p)+\delta V_C(q) & \qquad U_{PD}(p,q) &= f_{PD}(p)+\delta V_P(q)\\
U_{CA}(p,q) &= f_{CA}(p)+\delta V_C(q) & \qquad U_{PA}(p,q) &= f_{PA}(p)+\delta V_P(q).
\end{aligned}
$$

To prove existence of CS-MPE we need to constrain values of δ and r_d. The following assumption lists all the constraints we need.

Assumption 2. *For any pair $\{\delta, r_d\}$ with $\delta \in [0,1)$ and $r_d \in [0,1]$ assume*

1. $\delta \geq \frac{1}{5r_d}$
2. $\delta \geq 1-r_d^2$
3. $\delta \leq 1-\frac{(1-r_d)^2}{2}$.

Notice that the three requirements are mutually compatible and in general allow for values of δ and r_d with 'enough discounting and conflict'. Denoting the space of possible values for $\{\delta, r_d\}$ by $\mathbb{P} = [0,1) \times [0,1]$ (with the convention that its graphical representation has r_d on the horizontal axis) assumption 2 isolates the north-eastern part of $\mathbb{P}$. Notice also that we are not selecting a measure-zero set out of the $\mathbb{P}$ and hence our existence result will be generic in the sense that S-MPE will exist on some neighbourhood of $\{\delta, r_d\}$ strictly satisfying assumption 2.

claim 15. *Let $X^- = X \setminus (\pi^* - \phi, \pi^* + 3\phi)$ and $z, z' \in X^-$. For any $x \in X^-$ the equilibrium is given by*

$$\begin{aligned} q_A(x) &= z \qquad & p_A(x) &= \pi^* \\ q_D(x) &= z' \qquad & p_D(x) &= \pi^* - \phi. \end{aligned}$$

where the policy strategies are unique. Moreover, for any $x \in X^-$, $V_C(x) = 0$ and $V_P(x) = -\frac{4\phi^2 r_d}{1-\delta}$.

Proof. We first show $\rho = \{q_D(x) = q_A(x) = x, p_D(x) = \pi^* - \phi, p_A(x) = \pi^*\}$ is an equilibrium for any $x \in X^-$. Fix $x \in X^-$. Note that $\{p_D(x) = \pi^* - \phi, x\} \in A_D(x)$ and $\{p_A(x) = \pi^*, x\} \in A_A(x)$ and both increase C's utility compared to $\{x, x\}$. It also follows ρ induces $V_C(x) = 0$ hence C clearly cannot do better. Therefore ρ is an equilibrium.

Having the equilibrium for given x, notice it induces the same path of policy decisions for a fixed path of A and D periods as any $x' \in X^-$. It follows $V_C(x)$ and $V_P(x)$ must be constant on X^-. Therefore the first part of the claim follows.

To show uniqueness of the policy offers notice C's utility strictly decreases by offering anything other than policy specified in the claim.

The fact that $V_C(x) = 0 \; \forall x \in X^-$ follows from the two previous remarks. To show $V_P(x) = -\frac{4\phi^2 r_d}{1-\delta}$ using the constancy of $V_P(x)$ we can write

$$V_P(x) = r_d[-4\phi^2 + \delta V_P(x)] + (1 - r_d)[\delta V_P(x)],$$

which after rearranging gives $V_P(x)$ in the claim. □

claim 16. *Let $X^+ = (\pi^* - \phi, \pi^* + 3\phi)$. Then for all $x \in X^+$, $V_C(x) < 0$.*

Proof. Assume there exists an equilibrium such that $V_C(x) = 0$ for some $x \in X^+$. It follows $V_P(x) = -\frac{4\phi^2 r_d}{1-\delta}$. Take D period, if P rejects today and follows the equilibrium strategy from then on his utility is $f_{PD}(x) - \frac{4\phi^2 \delta r_d}{1-\delta}$ whereas if he accepts (as equilibrium demands) his utility is $f_{PD}(\pi^* - \phi) - \frac{4\phi^2 \delta r_d}{1-\delta}$. For this to be an equilibrium it must be that

$$f_{PD}(x) - \frac{4\phi^2 \delta r_d}{1-\delta} \le f_{PD}(\pi^* - \phi) - \frac{4\phi^2 \delta r_d}{1-\delta},$$

which rewrites as $(x - \pi - \phi)^2 \ge 4\phi^2$ and holds for $x \notin (\pi^* - \phi, \pi^* + 3\phi)$, a contradiction to $x \in X^+$. □

Claims 15 and 16 give precise meaning to the term C's unconstrained maximizer as they imply that $\{\pi^* - \phi, z\}$ maximizes $U_{CD}(p, q)$ and $\{\pi^*, z\}$ maximizes $U_{CA}(p, q)$ for any $z \in X^-$. Denote those by $\gamma_{CD} = \{\pi^* - \phi, z\}$ and $\gamma_{CA} = \{\pi^*, z\}$. Notice that if $\gamma_{CD} \in A_D(x)$ for some default x then γ_{CD} has to be part of C's equilibrium strategy. Similar holds for $\gamma_{CA} \in A_A(x)$.

Next we wish to characterize P's continuation value function V_P conjecturing that if for some default option x we have $\gamma_{CD} \notin A_D(x)$, C's offer $\gamma(x)$ will make P indifferent between $\gamma(x)$ and default option $\bar{\gamma}(x) = \{x, x\}$ and similarly for A periods. The next claim helps in translating the conjecture into V_P.

claim 17. *For any $x \in X^+$ if P is brought to indifference in A periods for default option x, then he is brought to indifference in D periods for the same default option.*

Proof. We prove the converse, i.e. if P is not brought to indifference in D periods, then he is not brought to indifference in A periods.

Note that if P is not made indifferent in D periods, then C's proposal has to be $\{\pi^* - \phi, z\}$ for some $z \in X$. This implies

$$f_{PD}(x) + \delta V_P(x) \leq f_{PD}(\pi^* - \phi) + \delta V_P(z),$$

which after rearranging gives

$$f_{PA}(x) + \delta V_P(x) \leq \delta V_P(z) - [3\phi^2 + 2\phi(x - \pi^*)],$$

where the term in the square brackets is positive for any $x \in X^+$. It then follows that $\{\pi^*, z\} \in A_A(x)$. $\square$

With the help of claim 17 we conjecture that for default options x close to P's D period bliss point $\pi^* + \phi$ he will be made indifferent for both A and D periods and for x further away he will be made indifferent only in D periods. This gives rise to V_P of the following form.

$$V_P(x) = \begin{cases} -\dfrac{1}{1-\delta}\left[(x - \pi^* - \phi r_d)^2 + \phi^2 r_d(1 - r_d)\right] \\ \qquad \text{for} \quad x \in [\pi^* + \phi\delta r_d - \kappa, \pi^* + \phi\delta r_d + \kappa] \\ -\dfrac{r_d}{1-\delta r_d}\left[(x - \pi^* - \phi)^2 + \phi^2 \dfrac{4\delta(1-r_d)}{1-\delta}\right] \\ \qquad \text{for} \quad x \in [\pi^* - \phi, \pi^* + \phi\delta r_d - \kappa] \cup [\pi^* + \phi\delta r_d + \kappa, \pi^* + 3\phi] \\ -\dfrac{4\phi^2 r_d}{1-\delta} \\ \qquad \text{otherwise} \end{cases}$$

with $\kappa = \phi\sqrt{\delta r_d(3 + \delta r_d)}$ where the last constant part applies to x for which $\gamma_{CD} \in A_D(x)$ and $\gamma_{CA} \in A_A(x)$. For future reference denote $\kappa^- = \pi^* + \phi\delta r_d - \kappa$ and $\kappa^+ = \pi^* + \phi\delta r_d + \kappa$. It is easy to confirm V_P is continuous and (strictly) piece-wise concave for $x \in X$ ($x \in X^+$). In the next claim we establish upper hemicontinuity of the acceptance correspondences generated by V_P.

claim 18. *For any $x \in X$ the acceptance correspondences $A_D(x)$ and $A_A(x)$ are nonempty, compact valued and upper hemicontinuous.*

Proof. The nonempty part follows by definition and the compact valued part follows from continuity and the fact that X is compact. To prove upper hemicontinuity of the acceptance correspondence

$$A_D(x) = \{(p, q) \in X^2 | f_{PD}(p) + \delta V_P(q) \geq f_{PD}(x) + \delta V_P(x)\}$$

denote $\mathbf{x} = (x, x)$, $\mathbf{p} = (p, q)$ and $f(\mathbf{p}) = f_{PD}(p) + \delta V_P(q)$.

Pick two sequences $\{\mathbf{x}_\alpha\} \to \mathbf{x}$ and $\{\mathbf{p}_\alpha\} \to \mathbf{p}$ such that $\mathbf{p}_\alpha \in A_D(x_\alpha)\ \forall\alpha$. Note that by non-emptiness of A_D this can be done. We need to show $\mathbf{p} \in A_D(x)$.

Suppose $\mathbf{p} \notin A_D(x)$. Then

$$f(\mathbf{x}_\alpha) \leq f(\mathbf{p}_\alpha)\ \forall\alpha$$
$$f(\mathbf{x}) > f(\mathbf{p}).$$

Summing the two inequalities gives

$$f(\mathbf{x}_\alpha) - f(\mathbf{x}) < f(\mathbf{p}_\alpha) - f(\mathbf{p})\ \forall\alpha.$$

Taking the limit for $\alpha \to \infty$ on both sides gives a contradiction to continuity of $f(\cdot)$. For A_A the proof is analogous and hence omitted. □

Next we want to show C's dynamic optimization problem has a solution. Precise statement of the dynamic program is

$$U_D(x) = \max_{\{p,q\} \in A_D(x)} \{f_{CD}(p) + \delta(r_d U_D(q) + (1 - r_d) U_A(q))\}$$
$$U_A(x) = \max_{\{p,q\} \in A_A(x)} \{f_{CA}(p) + \delta(r_d U_D(q) + (1 - r_d) U_A(q))\},$$

which can alternatively be written as

$$V_P(x) = r_d \max_{\{p,q\} \in A_D(x)} \{f_{CD}(p) + \delta V_C(q)\} + (1 - r_d) \max_{\{p,q\} \in A_A(x)} \{f_{CA}(p) + \delta V_C(q)\}.$$

With the acceptance correspondences possessing properties given in claim 18, existence and uniqueness of the solution to the dynamic program above follow using a similar argument as in proposition 2 for the no directive model. C's equilibrium proposal strategy for default x is then given by $\{p_D(x), q_D(x)\}$ in D periods and by $\{p_A(x), q_A(x)\}$ in A periods, where

$$\{p_D(x), q_D(x)\} \in \arg\max_{\{p,q\} \in A_D(x)} \{f_{CD}(p) + \delta V_C(q)\}$$
$$\{p_A(x), q_A(x)\} \in \arg\max_{\{p,q\} \in A_A(x)} \{f_{CA}(p) + \delta V_C(q)\}.$$

Notice that even though $p_D(x)$, $q_D(x)$, $p_A(x)$ and $q_A(x)$ are correspondences we can always take a unique selection out of each of them. For this reason below we treat those as functions. In the next claim we establish properties of the value functions that solve the dynamic optimization program above.

claim 19. *Under assumption 2 (namely its part one)*

1. *$U_A(x)$, $U_D(x)$ and $V_C(x)$ are all u.s.c.*

2. *$U_A(x) = 0$ for $\forall x \in [x^-, \kappa^-] \cup [\kappa^+, x^+]$, $U_A(x)$ is non-increasing for $\forall x \in [\kappa^-, \pi^* + \phi\delta r_d]$ and non-decreasing for $\forall x \in [\pi^* + \phi\delta r_d, \kappa^+]$*

3. *$U_A(x) = U_A(x')$ with $\frac{x+x'}{2} = \pi^* + \phi\delta r_d$ for $\forall x \in [\kappa^-, \kappa^+]$*

4. *$U_A(x) \geq U_A(x')$ for $\forall x' \in [x, 2(\pi^* + \phi\delta r_d) - x]$ with $x \in [\kappa^-, \pi^* + \phi\delta r_d]$*

5. *$U_D(x) = 0$ for $\forall x \in [x^-, \pi^* - \phi] \cup [\pi^* + 3\phi, x^+]$, $U_D(x)$ is non-increasing for $\forall x \in [\pi^* - \phi, \pi^* + \phi(1 - \delta(1 - r_d))]$ and non-decreasing for $\forall x \in [\pi^* + \phi(1 - \delta(1 - r_d)), \pi^* + 3\phi]$*

6. *$U_D(x) = U_D(x')$ with $\frac{x+x'}{2} = \pi^* + \phi$ for $\forall x \in [x^-, \kappa^-] \cup [2(\pi^* + \phi) - \kappa^-, x^+]$, with $x = z(x')$ where $z(x')$ is a uniquely defined decreasing function mapping the range $[\kappa^+, 2(\pi^* + \phi) - \kappa^-]$ into $[\kappa^-, 2(\pi^* + \phi(1 - \delta(1 - r_d))) - \kappa^+]$ and with $\frac{x+x'}{2} = \pi^* + \phi(1 - \delta(1 - r_d))$ for $\forall x \in [2(\pi^* + \phi(1 - \delta(1 - r_d))) - \kappa^+, \kappa^+]$*

7. *$U_D(x) \geq U_D(x')$ for $\forall x' \in [2(\pi^* + \phi(1 - \delta(1 - r_d))) - x, x]$ with $x \in [\pi^* + \phi(1 - \delta(1 - r_d)), \kappa^+]$ and for $\forall x' \in [z(x), x]$ with $x \in [\kappa^+, 2(\pi^* + \phi) - \kappa^-]$*

8. *$V_C(x)$ is non-increasing $\forall x \in [x^-, \pi^* + \phi\delta r_d]$ and non-decreasing $\forall x \in [\pi^* + \phi(1 - \delta(1 - r_d)), x^+]$.*

Proof. The first part follows immediately from the fact that the value functions are solutions to C's dynamic optimization program and theorem 1.

The second part follows from the fact that $\gamma_{CA} \in A_A(x)$ whenever $x \in [x^-, \kappa^-] \cup [\kappa^+, x^+]$. The non-increasing and non-decreasing parts follow from the fact that $U_{PA}(x, x)$, which defines $A_A(x)$, is under part one of assumption 2 increasing on $[\kappa^-, \pi^* + \phi\delta r_d]$ and decreasing on $[\pi^* + \phi\delta r_d, \kappa^+]$. With the default option x entering C's optimization only as a constraint in the form of $A_A(x)$, it follows $U_A(x)$ has to be non-increasing and non-decreasing on the two intervals respectively.

The third part follows from the fact that $U_{PA}(x, x) = U_{PA}(x', x')$ for x and x' satisfying the condition given in the claim, which implies $A_A(x) = A_A(x')$. Part four then follows from parts two and three.

Part five can be shown in a similar manner as part two, investigating properties of the $U_{PD}(x, x)$ function defining acceptance set $A_D(x)$, using again part one of assumption 2. The sixth part is analogous to part three using the fact that $U_{PD}(x, x) = U_{PD}(x', x')$ for the x and x' defined. Part seven is an implication of parts five and six. Part eight is a direct consequence of parts two and five upon observing that $V_C(x) = r_d U_D(x) + (1-r_d) U_A(x)$. $\square$

The next claim establishes a certain monotonicity property of the equilibrium status-quo offers $q_D(x)$ (as evaluated under the V_C function) that will become useful later. We denote by $A^{\partial}_D(x)$ the boundary of $A_D(x)$ and similarly by $A^{\partial}_A(x)$ the boundary of $A_A(x)$.

claim 20. *Let x be the default option with its associated equilibrium status-quo offer $q_D(x)$. Then for any $x' \in \{q : A_D(q) \subseteq A_D(x)\}$ with its associated equilibrium status-quo offer $q_D(x')$, $V_C(q_D(x)) \geq V_C(q_D(x'))$.*

Proof. Fix x and x' with $x' \in \{q : A_D(q) \subseteq A_D(x)\}$ where the interpretation of x' is that it is a default option with strictly smaller associated P's D period acceptance set. It is immediate that the claim holds for $q_D(x)$ and its associated policy offer $p_D(x)$ with $\{p_D(x), q_D(x)\} \in A_D(x) \setminus A^{\partial}_D(x)$ and any x' such that $\{p_D(x), q_D(x)\} \in A_D(x')$ for then $q_D(x) = q_D(x')$. So assume that $\{p_D(x), q_D(x)\} \in A^{\partial}_D(x)$.

Easy argument shows that for p to be C's equilibrium policy offer for some default option it has to be that $p \in [\pi^* - \phi, \pi^* + \phi]$. We need to construct a set of offers C can be expected to choose from in equilibrium, i.e. those where the policy offer falls into the $[\pi^* - \phi, \pi^* + \phi]$ interval. This will be given as a set $A'_D(x) = \{\{\max\{p, \pi^* - \phi\}, q\} | \{p, q\} \in A^{\partial}_D(x) \wedge p \leq \pi^* + \phi\} \subseteq A_D(x)$ or in words as a subset of $A^{\partial}_D(x)$ for which the policy is smaller than $\pi^* + \phi$ and for which, if the policy falls below $\pi^* - \phi$, it is replaced by $\pi^* - \phi$. It is easy to see that for any default option $x'' \in X$, C's D period equilibrium offer satisfies $\{p_D(x''), q_D(x'')\} \in A'_D(x'')$.

Now with $\{p_D(x), q_D(x)\} \in A^{\partial}_D(x)$ and any $\{p, q\} \in A'_D(x)$ for which $p \leq p_D(x)$ it has to be the case that $V_C(q_D(x)) \geq V_C(q)$. To see this note that

$$
\begin{aligned}
f_{CD}(p_D(x)) + \delta V_C(q_D(x)) &\geq f_{CD}(p) + \delta V_C(q) \\
f_{CD}(p_D(x)) - f_{CD}(p) &\leq 0
\end{aligned}
$$

where the first line follows from the fact that $\{p_D(x), q_D(x)\}$ is C's equilibrium offer and $\{p, q\} \in A_D(x)$ and the second line follows from the fact that $\pi^* - \phi \leq p \leq p_D(x)$.

Next we want to show that for any $\{p, q\} \in A'_D(x)$ for which $p > p_D(x)$, the associated q cannot be part of C's equilibrium offer for x'. To see this note that $\{p, q\} \in A'_D(x)$ with $p > p_D(x)$ and $\{p_D(x), q_D(x)\} \in A^{\partial}_D(x)$ has to satisfy

$$
\begin{aligned}
f_{PD}(x) + \delta V_P(x) &= f_{PD}(p) + \delta V_P(q) \\
&= f_{PD}(p_D(x)) + \delta V_P(q_D(x)).
\end{aligned}
$$

Keeping q and $q_D(x)$ the same, changing x to x' such that $A_D(x') \subseteq A_D(x)$, p and $p_D(x)$ will change to p' and $p'_D(x)$ with $\{p', q\} \in A'_D(x')$ and $\{p'_D(x), q_D(x)\} \in A'_D(x')$ that satisfy

$$\begin{aligned} f_{PD}(x') + \delta V_P(x') &= f_{PD}(p') + \delta V_P(q) \\ &= f_{PD}(p'_D(x)) + \delta V_P(q_D(x)) \end{aligned}$$

(if such a p' does not exist we are done as q cannot be part of equilibrium for x', if such p' does exist, $p'_D(x)$ has to exist as well). Taking the difference of the two systems of equations gives $f_{PD}(p) - f_{PD}(p') = f_{PD}(p_D(x)) - f_{PD}(p'_D(x))$ which rewrites as

$$(p' - p) = (p'_D(x) - p_D(x)) \left[\frac{p_D(x) + p'_D(x) - 2(\pi^* + \phi)}{p + p' - 2(\pi^* + \phi)} \right]$$

where the term in the square brackets is positive and strictly larger than unity (all p, p', $p_D(x)$ and $p'_D(x)$ are in the $[\pi^* - \phi, \pi^* + \phi]$ interval and $p_D(x) < p$ and $p_D(x)' < p'$). This in turn implies that $p' - p > p'_D(x) - p_D(x)$ and along with the fact that

$$f_{CD}(p) + \delta V_C(q) \leq f_{CD}(p_D(x)) + \delta V_C(q_D(x))$$

gives

$$f_{CD}(p') + \delta V_C(q) < f_{CD}(p'_D(x)) + \delta V_C(q_D(x))$$

so that q cannot be part of C's equilibrium offer for default option x'.

Combining the two results, if $\{p, q\} \in A'_D(x)$ with $p > p_D(x)$ then q cannot be part of C's equilibrium proposal for x'. If $\{p, q\} \in A'_D(x)$ with $p \leq p_D(x)$ then $V_C(q_D(x)) \geq V_C(q)$ so that if $q = q_D(x')$ it has to be the case that $V_C(q_D(x)) \geq V_C(q_D(x'))$. □

Next we want to confirm our original conjecture that in equilibrium for a given default option in a given type of period P is indifferent between accepting and rejecting C's offer given that the unconstrained maximizer of C's overall utility is not in P's acceptance set. Formally, we want to show that for default option x if $\gamma_{CD} \notin A_D(x)$ then $\{p_D(x), q_D(x)\} \in A^{\partial}_D(x)$ and similarly if $\gamma_{CA} \notin A_A(x)$ then $\{p_A(x), q_A(x)\} \in A^{\partial}_A(x)$. A key complication is the fact that the V_C function can possess local maxima in the $[\pi^* + \phi\delta r_d, \pi^* + \phi(1 - \delta(1 - r_d))]$ interval and hence C's utility maximizing offer can lie in the interior of P's acceptance set, even though her unconstrained optimizer is outside of it. Denoting the problematic interval by $Z = [\pi^* + \phi\delta r_d, \pi^* + \phi(1 - \delta(1 - r_d))]$ we deal with A and D periods in the following two claims respectively. The two claims then deliver the conditions on δ specified in parts two and three of assumption 2.

claim 21. *Let x be the default policy and assumption 2 (namely its part* *one and two) holds. Then C's equilibrium proposal in A periods, provided $\gamma_{CA} \notin A_A(x)$, satisfies $\{p_A(x), q_A(A)\} \in A^{\partial}_A(x)$.*

Proof. We know by proposition 3 that for any $x \in X$, $p_A(x) = \pi^*$. Denoting by $A_A(x, y = z)$ a 'slice' through $A_A(x)$ when variable y (either p or q) is equal to z, C's optimization problem in A period for default option x can be rewritten as $\max_{q \in A_A(x,p=\pi^*)}\{\delta V_C(q)\}$ and we want to show that whenever $\gamma_{CA} \notin A_A(x)$ then $q_A(x) \in \{\min\{A_A(x, p = \pi^*)\}, \max\{A_A(x, p = \pi^*)\}\}$. It is easy to confirm that under part one of assumption 2 for any x, $A_A(x, p = \pi^*)$ is a non-empty, compact and convex subset of X.

Next note that by part eight of claim 19 if $\max\{V_C(x), V_C(y)\} \geq \max_{z\in(x,y)} V_C(z)$ for some $x \leq \pi^*+\phi\delta r_d$ and some $y \geq \pi^*+\phi(1-\delta(1-r_d))$ then $\max\{V_C(x'), V_C(y')\} \geq \max_{z\in(x',y')} V_C(z)$ for any $x' \leq x$ and any $y' \geq y$. Hence if we can show that the claim is true for $x = \pi^* + \phi\delta r_d$, which maximizes $U_{PA}(x, x)$ and hence delivers the smallest $A_A(x)$, we are done as $Z \in A_A(\pi^* + \phi\delta r_d, p = \pi^*)$.

Now minima and maxima of $A_A(\pi^*+\phi\delta r_d, p = \pi^*)$ are given respectively by $q^-_A = \pi^*+\phi r_d-\phi r_d\sqrt{1-\delta}$ and $q^+_A = \pi^*+\phi r_d+\phi r_d\sqrt{1-\delta}$ for $\{\delta, r_d\} \in \mathbb{P}$ for which $q^+_A \leq \kappa^+$ (it is easy to confirm $\kappa^- \leq q^-_A$). Then we can use part four (along with part five) of claim 19 to conclude that $V_C(q^-_A) \geq V_C(x)$ for any $x \in [q^-_A, 2(\pi^*+\phi\delta r_d)-q^-_A]$ and part seven (along with part two) of the same claim to conclude that $V_C(q^+_A) \geq V_C(x)$ for any $x \in [2(\pi^*+\phi(1-\delta(1-r_d)))-q^+_A, q^+_A]$. The condition for $2(\pi^*+\phi\delta r_d)-q^-_A \geq 2(\pi^*+\phi(1-\delta(1-r_d)))-q^+_A$ that rewrites as $\delta \geq 1-r_d^2$ then delivers the claim. For values of $\{\delta, r_d\}$ for which $q^+_A \geq \kappa^+$ the argument is similar if somewhat complicated by use of the function $z(x)$ mentioned in part six of claim 19. We do not repeat the purely algebraic argument here as it delivers a condition on $\{\delta, r_d\}$ that is strictly weaker than the condition $\delta \geq 1-r_d^2$ just derived. □

claim 22. *Let x be the default policy and assumption 2 (namely its part one and three) holds. Then C's equilibrium proposal in D periods, provided $\gamma_{CD} \notin A_D(x)$, satisfies $\{p_D(x), q_D(x)\} \in A^{\partial}_D(x)$.*

Proof. First note that for default option x if $\gamma_{CD} \notin A_D(x)$ then if $\{p_D(x), q_D(x)\}$ is strictly inside $A_D(x)$ then it has to be the case that $p_D(x) = \pi^* - \phi$. If not and $p_D(x) = p \neq \pi^* - \phi$ then there exists (not necessarily positive) ε such that C can offer $\{p-\varepsilon, q_D(x)\} \in A_D(x)$ with $U_{CD}(p-\varepsilon, q_D(x)) > U_{CD}(p, q_D(x))$. Also it has to be the case that $q_D(x) = z$ for some $z \in Z$. If not, then by part eight of claim 19, there has to exist q such that $\{p_D(x), q\} \in A^{\partial}_D(x)$ and satisfies $U_{CD}(p_D(x), q) \geq U_{CD}(p_D(x), q_D(x))$ and we can specify $\{p_D(x), q\}$ to be C's equilibrium offer satisfying the claim.

Next $\gamma_{CD} \notin A_D(x)$ implies that $x \in X^+ = (\pi^* - \phi, \pi^* + 3\phi)$ and, under assumption 2, for any $x \in (\pi^*-\phi, \pi^*+\phi(1-\delta(1-r_d))]$ there exists a unique $x' \in [\pi^*+\phi(1-\delta(1-r_d)), \pi^*+3\phi)$ such that $U_{PD}(x, x) = U_{PD}(x', x')$, which implies $A_D(x) = A_D(x')$. Denoting by $X^c = (\pi^* - \phi, \pi^* + \phi(1 - \delta(1 - r_d))]$

we focus on $x \in X^c$ since if the claim holds for any such x it has to hold for any $x' \in [\pi^* + \phi(1-\delta(1-r_d)), \pi^* + 3\phi) = X^+ \setminus X^c$.

Next assume that $p_D(x) = \pi^* - \phi$ and $q_D(x) = z$ for some $z \in Z$ are part of the equilibrium for some $x \in X^c$ such that $\{\pi^* - \phi, z\} \in A_D(x) \setminus A_D^\partial(x)$. We show that this leads to contradiction.

Observe that if $p_D(x) = \pi^* - \phi$ and $q_D(x) = z$ for some $z \in Z$ is C's equilibrium offer for $x \in X^c$ with $\{\pi^* - \phi, z\} \in A_D(x) \setminus A_D^\partial(x)$, it has to be the case that $p_D(x^c) = \pi^* - \phi$ and $q_D(x^c) = z$ is C's equilibrium offer for $x^c \in X^c$ such that $\{\pi^* - \phi, z\} \in A_D^\partial(x^c)$. Such x^c is implicitly defined by $U_{PD}(x^c, x^c) = U_{PD}(\pi^* - \phi, z)$. We denote x^c as a function of z by $x^c(z)$ for $z \in Z$. It is easy to show that $x^c(z)$ is increasing on Z^- and decreasing on Z^+ where $Z^- = [\pi^* + \phi\delta r_d, \pi^* + \phi r_d]$ and $Z^+ = [\pi^* + \phi r_d, \pi^* + \phi(1-\delta(1-r_d))]$ respectively with $Z = Z^- \cup Z^+$ and that $x^c(z) \leq z$.

Now from the fact that $p_D(x^c(z)) = \pi^* - \phi$ and $q_D(x^c(z)) = z$ is C's equilibrium offer it follows

$$f_{CD}(x^c(z)) + \delta V_C(x^c(z)) \leq f_{CD}(\pi^* - \phi) + \delta V_C(z),$$

which rewrites as

$$\begin{aligned} &-(x^c(z) - \pi^* + \phi)^2 + \delta^2 r_d V_C(q_D(x^c(z))) + \delta^2(1-r_d)V_C(q_A(x^c(z))) \\ \leq &-\delta r_d(p_D(z) - \pi^* + \phi)^2 + \delta^2 r_d V_C(q_D(z)) + \delta^2(1-r_d)V_C(q_A(z)). \end{aligned}$$

We show that this inequality fails under assumption 2.

First we show that $V_C(q_A(x^c(z))) \geq V_C(q_A(z))$ using part four of claim 19. Evaluating $x^c(z)$ at its maximum, that is for $z = \pi^* + \phi r_d$, gives $x^c(\pi^* + \phi r_d) = \pi^* + \phi(1-\delta(1-r_d)) - \phi\sqrt{(1-\delta)(4-\delta+2\delta r_d - \delta r_d^2)}$ and in order to use claim 19 we need this to be smaller than $\pi^* + \phi\delta r_d - \phi(1-\delta)$ (as $\phi(1-\delta)$ is size of the Z interval). This condition rewrites as $0 \leq \delta(1-\delta)(3-r_d)(1+r_d)$, which clearly holds for any $\{\delta, r_d\} \in \mathbb{P}$.

Next we show that $V_C(q_D(x^c(z))) \geq V_C(q_D(z))$ that follows from claim 20 along with the fact that $x^c(z) \leq z$, $x^c(z) \in X^c$ and $z \in X^c$, which implies $A_D(z) \subseteq A_D(x^c(z))$.

Finally we show that $-(x^c(z) - \pi^* + \phi)^2 \geq -\delta r_d(p_D(z) - \pi^* + \phi)^2$. As we do not know the exact value of $p_D(z)$ we replace it by the minimum value of policy in the $A_D(z)$ set. We denote this policy, as a function of z, by $p_m(z)$ and note it solves $f_{PD}(z) + \delta V_P(z) = f_{PD}(p_m(z)) + \delta V_P(\pi^* + \phi r_d)$ as $\pi^* + \phi r_d$ maximizes the V_P function under assumption 2. Similarly, $x^c(z)$ defined above by $\{\pi^* - \phi, z\} \in A_D^\partial(x^c(z))$ for some $z \in Z$ solves $f_{PD}(x^c(z)) + \delta V_P(x^c(z)) = f_{PD}(\pi^* - \phi) + \delta V_P(z)$.

In what follows we need to focus only on $z = \pi^* + \phi\delta r_d$. To see this note that using the implicit function theorem (non-differentiability of V_P poses no problem here as even at the point where V_P is not differentiable, it possesses left and right derivatives)

$$\frac{\partial p_m(z)}{\partial z} = \frac{2(\pi^* + \phi - z) + \delta \frac{\partial V_P(z)}{\partial z}}{2(\pi^* + \phi - p_m(z))}$$
$$\frac{\partial x^c(z)}{\partial z} = \frac{\delta \frac{\partial V_P(z)}{\partial z}}{2(\pi^* + \phi - x^c(z)) + \delta \frac{\partial V_P(x^c(z))}{\partial x^c(z)}}.$$

If we can prove that $\frac{\partial x^c(z)}{\partial z} \leq \sqrt{\delta r_d} \frac{\partial p_m(z)}{\partial z}$ for any $z \in Z$ then if the inequality $-(x^c(z) - \pi^* + \phi)^2 \geq -\delta r_d (p_m(z) - \pi^* + \phi)^2$ holds for $z = \pi^* + \phi\delta r_d$ it has to hold for any $z \in Z$.

For $z \in Z^+$ we have $\frac{\partial x^c(z)}{\partial z} \leq 0 \leq \sqrt{\delta r_d} \frac{\partial p_m(z)}{\partial z}$ (denominators in $\frac{\partial x^c(z)}{\partial z}$ and $\frac{\partial p_m(z)}{\partial z}$ are positive as $x^c(z) \in X^c$ and $p_m(z) \in X^c$ while nominators are positive and negative respectively). For $z \in Z^-$, $\frac{\partial x^c(z)}{\partial z} \leq \sqrt{\delta r_d} \frac{\partial p_m(z)}{\partial z}$ rewrites as (using only nominators as denominator in $\frac{\partial x^c(z)}{\partial z}$ is larger than denominator in $\frac{\partial p_m(z)}{\partial z}$ and using the fact that under assumption 2, $\kappa^- \leq z \leq \kappa^+$ for any $z \in Z$)

$$\frac{\delta - \sqrt{\delta r_d}}{1 - \delta}(\pi^* + \phi - z) - \phi \frac{\delta(1 - r_d)(1 - \sqrt{\delta r_d})}{1 - \delta} \leq 0$$

which, as straightforward algebra shows, holds for $z \in Z^-$.

We now focus on the inequality $-(x^c(z) - \pi^* + \phi)^2 \geq -\delta r_d (p_m(z) - \pi^* + \phi)^2$ evaluated at $z = \pi^* + \phi\delta r_d$. This gives us $x^c_m = x^c(\pi^* + \phi\delta r_d)$ and $p^c_m = p_m(\pi^* + \phi\delta r_d)$ that read as

$$x^c_m = \pi^* + \phi(1 - \delta(1 - r_d)) - \phi\sqrt{(1 - \delta)(4 - \delta + 2\delta r_d - \delta^2 r_d^2)}$$
$$p^c_m = \pi^* + \phi - \phi\sqrt{1 - 2\delta r_d + \delta r_d^2}$$

where the expression for x^c_m applies only as long as $\kappa^- \leq x^c_m$. We do not need to focus on the case when $\kappa^- > x^c_m$ as then $\kappa^- > x^c_m > x^c(\pi^* + \phi\delta r_d)$ (which is a direct consequence of the V_P function being the upper envelope of two quadratic functions).

At this point it is helpful to replace δr_d in expressions for x^c_m and p^c_m by k which gives

$$x^k_m = \pi^* + \phi\left(1 - \frac{k}{r_d} + k\right) - \phi\sqrt{\left(1 - \frac{k}{r_d}\right)\left(4 - \frac{k}{r_d} + 2k - k^2\right)}$$
$$p^k_m = \pi^* + \phi - \phi\sqrt{1 - 2k + k r_d}$$

where $k \in [\frac{1}{5}, 1]$ and $r_d \in [k, 1]$ under part one of assumption 2.

As a next step we prove that $\frac{\partial x^k_m}{\partial r_d} \leq k \frac{\partial p^k_m}{\partial r_d}$ for any $k \in [\frac{1}{5}, 1]$ and any

$r_d \in [k, 1]$, which implies that if $-(x_m^k - \pi^* + \phi)^2 \geq -k(p_m^k - \pi^* + \phi)^2$ holds for some $k \in [\frac{1}{5}, 1]$ and $r_d \in [k, 1]$, then it has to hold for the same k and any $r'_d \geq r_d$. To confirm $\frac{\partial x_m^k}{\partial r_d} \leq k \frac{\partial p_m^k}{\partial r_d}$ for any $k \in [\frac{1}{5}, 1]$ and any $r_d \in [k, 1]$ we rewrite the inequality $\frac{\partial x_m^k}{\partial r_d} \leq k \frac{\partial p_m^k}{\partial r_d}$ into the form $\mathbb{P}_k(r_d) \leq 0$ where $\mathbb{P}_k(r_d)$ is a complicated expression of k and r_d that we view as a polynomial in r_d with coefficients given by k. We need to confirm $\mathbb{P}_k(r_d)$ does not have a root in $[k, 1]$ for any $k \in [\frac{1}{5}, 1]$. To do so we use the Descartes rule with a substitution $r_d = \frac{\alpha + \beta y}{1+y}$ with $\alpha = k$ and $\beta = 1$ (see Prasolov, 2004, corollary to theorem 1.4.1). This gives us polynomial $\mathbb{P}_k(y)$ in y with coefficients given by k. We use Sturm's theorem (Prasolov, 2004, theorem 1.4.3) to check that all the coefficients in $\mathbb{P}_k(y)$ are negative for $k \in [\frac{1}{5}, 1]$ which, by Descartes rule, implies that $\mathbb{P}_k(y)$ does not have a positive root for any $k \in [\frac{1}{5}, 1]$, which in turn implies that $\mathbb{P}_k(r_d)$ does not have a root in $[k, 1]$ for any $k \in [\frac{1}{5}, 1]$.

The last thing we need it to find is a line through the $\{\delta, r_d\}$ space $\mathbb{P}$, expressed as $\delta = f(r_d)$, for which $-(x_m^c - \pi^* + \phi)^2 \geq -\delta r_d (p_m^c - \pi^* + \phi)^2$ holds. This will imply that the inequality holds for any $\{\delta', r'_d\}$ such that $k = \delta r_d = \delta' r'_d$ and $r'_d \geq r_d$. Combined with part one of assumption 2 $\delta \geq \frac{1}{5 r_d}$, if we can show that the inequality holds for any $\{\delta = f(r_d), r_d\}$ where $\delta \geq \frac{1}{5 r_d}$, it has to hold for any $\{\delta', r_d\}$ such that $f(r_d) \geq \delta' \geq \frac{1}{5 r_d}$. One such $f(r_d)$ is given by $f(r_d) = 1 - \frac{(1-r_d)^2}{2}$. To see this we substitute the expressions for x_m^c and p_m^c into $-(x_m^c - \pi^* + \phi)^2 \geq -\delta r_d (p_m^c - \pi^* + \phi)^2$ along with $\delta = 1 - \frac{(1-r_d)^2}{2}$, getting polynomial $\mathbb{P}(r_d)$ in r_d and we confirm that it has no root in the $[\frac{1}{5}, 1]$ interval using Sturm's theorem again. This delivers part three of assumption 2 and proves the claim. □

Claims 21 and 22 confirm our original conjecture that C's offers bring P to indifference between accepting and rejecting given the unconstrained maximizer of C's overall utility is not available. Hence C's strategy as a solution to her dynamic optimization program indeed generates P's acceptance sets conjectured in that optimization program. Therefore C's proposal strategies $\rho_C = \{p_D(x), p_A(x), q_D(x), q_A(x)\}$ generated by C's dynamic optimization problem under acceptance sets generated by V_P and P's voting strategies ρ_P generated by V_P constitute CS-MPE.

The rest of the proposition follows easily. Uniqueness of the CS-MPE in terms of associated value functions follows from the uniqueness of V_P in any CS-MPE and uniqueness of the solution to C's optimization program. Part one of the proposition follows from proposition 3, part two is trivial to establish, part three follows from claim 20 and part four follows from claims 15 and 16.

A1.5 Proof of proposition 5

Using definition 4 of S, $x \in S$ implies $q_A(x) = q_D(x) = x$ and hence stable D period policy decisions $p_D(x) = p^*$. D-efficiency and hence part one then follows from the fact that $p^* \in [\pi^* - \phi, \pi^* + \phi]$, which is easy to see. For part two denote as in the proof of proposition 4 by $X^- = X \setminus (\pi^* - \phi, \pi^* + 3\phi)$ and by $X^+ = (\pi^* - \phi, \pi^* + 3\phi)$. We know by claim 15 from the proof of proposition 4 that $p_D(x) = \pi^* - \phi$ and $p_A(x) = \pi^*$ for any $x \in X^-$. For $x \in X^+$ we know $q_A(x) \in \{\min\{A_A(x, p = \pi^*)\}, \max\{A_A(x, p = \pi^*)\}\}$ by claim 21 from the proof of proposition 4 and it is easy to show $q_A(x) = x$ possibly only for $x = \pi^*$ as $\min\{A_A(\pi^*, p = \pi^*)\} = \pi^*$. This opens the possibility that $q_A(\pi^*) = q_D(\pi^*) = \pi^*$ and hence possibly $\pi^* \in S$. This in turn would imply $p_A(\pi^*) = p_D(\pi^*) = \pi^*$, which is easy to show as well. In any case, π^* has zero measure.

For part three, from part four of proposition 4, C proposes unconstrained maximizers of her overall utility $\gamma_{CD} = \{\pi^* - \phi, z\}$ and $\gamma_{CA} = \{\pi^*, z'\}$ whenever $\gamma_{CD} \in A_D(x)$ and $\gamma_{CA} \in A_A(x)$ for some $z, z' \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)$. As a result whenever x is such that C can propose γ_{CA} (γ_{CD}) in A (D) period, we can specify proposal strategies such that the bargaining reaches S immediately. Integration intervals in the proposition are then a translation of the conditions $\gamma_{CA} \in A_A(x)$ and $\gamma_{CD} \in A_D(x)$ that can be derived easily using V_P from proof of proposition 4. The fourth part is then a direct consequence of proposition 3.

A1.6 Proof of proposition 6

To prove the proposition we prove that the policy C proposes for a given default option x under the implicit status-quo bargaining is in P's acceptance set for the same default option under the explicit status-quo bargaining. This, along with the fact that the explicit status-quo bargaining relaxes the constraint on C's optimization problem, will imply the first part. We superscript all variables from the implicit status-quo bargaining by I and all variables from the explicit status-quo bargaining by E and use the notation from the proofs of propositions 1 and 4.

For D periods notice that by feasibility of equilibrium proposals under implicit status-quo bargaining

$$f_{PD}(p_D^I(x)) + \delta V_P^I(p_D^I(x)) \geq f_{PD}(x) + \delta V_P^I(x)$$

for any $x \in X$. Adding $\pm\delta V_P^E(p_D^I(x))$ and $\pm\delta V_P^E(x)$ to the left and right hand sides, the inequality after rearrangement becomes

$$\begin{aligned} f_{PD}(p_D^I(x)) + \delta V_P^E(p_D^I(x)) \geq & f_{PD}(x) + \delta V_P^E(x) \\ & + \delta[(V_P^I(x) - V_P^E(x)) - (V_P^I(p_D^I(x)) - V_P^E(p_D^I(x)))] \end{aligned}$$

so that if we can prove that the term in the square brackets is positive for any $x \in X$, $\{p_D^I(x), p_D^I(x)\} \in A_D^E(x)$ will follow.

The difference of P's value functions under the two bargaining protocols from the proofs of propositions 1 and 4 is

$$V_P^I(x) - V_P^E(x) = \begin{cases} \phi^2 \dfrac{3\delta r_d(1-r_d)}{1-\delta} \\ \qquad \text{for } \ x \in X \setminus (\pi^* + \phi\delta r_d - \kappa, \pi^* + \phi\delta r_d + \kappa) \\ \dfrac{1-r_d}{(1-\delta)(1-\delta r_d)}(x - \pi^* + \phi\delta r_d)(x - \pi^* - 3\phi\delta r_d) \\ \qquad \text{for } \ x \in [\pi^* + \phi\delta r_d - \kappa, \pi^* - \phi\delta r_d] \cup [\pi^* + 3\phi\delta r_d, \pi^* + \phi\delta r_d + \kappa] \\ 0 \\ \qquad \text{for } \ x \in [\pi^* - \phi\delta r_d, \pi^* + 3\phi\delta r_d] \end{cases}$$

where $\kappa = \phi\sqrt{\delta r_d(3 + \delta r_d)}$ as before. Also note that $V_P^I(x) - V_P^E(x)$ is non-negative for $\forall x \in X$, which proves the second part of the proposition.

To prove

$$V_P^I(x) - V_P^E(x) - (V_P^I(p_D^I(x)) - V_P^E(p_D^I(x))) \geq 0$$

for $\forall x \in X$, first take $x \in X^-$. Then $p_D^I(x) = \pi^* - \phi$ and $V_P^I(x) - V_P^E(x) = V_P^I(\pi^* - \phi) - V_P^E(\pi^* - \phi)$ so that the inequality holds. For default options $x \in \{z | p_D^I(z) \geq \pi^* - \phi\delta r_d\}$ it is easy to show $x \in [\pi^* - \phi\delta r_d, \pi^* + \phi(2 + \delta r_d)]$ and $p_D^I(x) \leq x$ so that $\pi^* - \phi\delta r_d \leq p_D^I(x) \leq x$ and the inequality follows from the fact that $V_P^I(x) - V_P^E(x)$ is non-decreasing for $x \geq \pi^* - \phi\delta r_d$. For default options $x \in [\pi^* - \phi, \pi^* - \phi\delta r_d]$ the inequality holds as $p_D^I(x) = x$. Finally, for $x \in [\pi^* + \phi(2 + \delta r_d), \pi^* + 3\phi]$, $p_D^I(x) = 2(\pi^* + \phi) - x \in [\pi^* - \phi, \pi^* - \phi\delta r_d]$ and as $\pi^* + \phi\delta r_d + \kappa \leq \pi^* + \phi(2 + \delta r_d)$, we have $V_P^I(x) - V_P^E(x) = \phi^2 \frac{3\delta r_d(1-r_d)}{1-\delta}$, whereas $V_P^I(p_D^I(x)) - V_P^E(p_D^I(x)) \leq \phi^2 \frac{3\delta r_d(1-r_d)}{1-\delta}$, so that the inequality holds.

For A periods a similar argument shows that it suffices to show

$$V_P^I(x) - V_P^E(x) - (V_P^I(p_A^I(x)) - V_P^E(p_A^I(x))) \geq 0$$

for $\forall x \in X$ in order to show $\{p_A^I(x), p_A^I(x)\} \in A_A^E(x)$. The inequality then follows from the fact that $p_A^I(x) \in [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d]$ so that the second term in the inequality is always equal to zero, whereas the first term is always positive. The third part of the proposition then follows using straightforward algebra and results from the proofs of propositions 1 and 4.

A1.7 Proof of proposition 7

We prove two claims that together prove the proposition. The strategy of the proof borrows heavily from Riboni and Ruge-Murcia (2008).

claim 23. *The difference in utilities associated with two sequences of policy decisions is linear in ϕ (for the first condition in definition 5) and in $\nu_{i,0}$ (for the second condition in definition 5).*

Proof. For the first condition in definition 5 of an essentially two-member committee, take two general sequences of policy decisions $\mathbf{p} = \{p_0, p_1, \ldots\}$ and $\mathbf{p}' = \{p'_0, p'_1, \ldots\}$. The utility associated with these policy sequences for a committee member with preference parameter ϕ is

$$U(\mathbf{p}, \phi) = -\sum_{t=0}^{\infty} \delta^t (p_t - \pi^* - \phi \mathbb{I}_D(t))^2$$

where $\mathbb{I}_D(t)$ is D period indicator function. Taking the derivative of the difference $U(\mathbf{p}, \phi) - U(\mathbf{p}', \phi)$ with respect to ϕ gives

$$\frac{\partial [U(\mathbf{p}, \phi) - U(\mathbf{p}', \phi)]}{\partial \phi} = \sum_{t=0}^{\infty} 2\delta^t \mathbb{I}_D(t)(p_t - p'_t),$$

which does not depend on ϕ. It follows that the difference in utility between $\mathbf{p}$ and $\mathbf{p}'$ is linear in ϕ.

For the second condition in definition 5, the utility associated with the sequence of policy decisions for a member with preference shock $\nu_{i,0}$ in the current period that is already realized and hence common knowledge is

$$U(\mathbf{p}, \nu_{i,0}) = -(p_0 - \pi^* - \phi - \nu_{i,0})^2 - \sum_{t=1}^{\infty} \delta^t \left[(p_t - \pi^* - \phi \mathbb{I}_D(t))^2 + r_d \mathbb{E}[\nu_{i,t}^2] \right]$$

with derivative of the difference $U(\mathbf{p}, \nu_{i,0}) - U(\mathbf{p}', \nu_{i,0})$ with respect to $\nu_{i,0}$ equal to

$$\frac{\partial [U(\mathbf{p}, \nu_{i,0}) - U(\mathbf{p}', \nu_{i,0})]}{\partial \nu_{i,0}} = 2(p_0 - p'_0),$$

which again does not depend on $\nu_{i,0}$. □

The next claim shows that the proposal is passed if and only if it is accepted by the median member. Formally, for the first condition in definition 5 for the committee of N (odd) members denote their preference parameters $\{\phi_1, \ldots, \phi_N\}$ such that $\phi_i < \phi_j$ for every pair $1 \leq i < j \leq N$. Then the median member has the preference shock ϕ_m that satisfies $|\{i|\phi_i > \phi_m\}| = |\{i|\phi_i < \phi_m\}|$. For the second condition in definition 5 for the $N-1$ (even) members denote their preference parameters $\{\phi + \nu_{1,0}, \ldots, \phi + \nu_{N-1,0}\}$ such that $\phi + \nu_{i,0} < \phi + \nu_{j,0}$ for every pair $1 \leq i < j \leq N-1$. Then the two median members have preference shocks $\phi + \nu_{m,0}$ where $\nu_{m,0} = 0$ and $|\{i|\nu_{i,0} > \nu_{m,0}\}| = |\{i|\nu_{i,0} < \nu_{m,0}\}|$.

claim 24. *Assuming stage-undominated voting strategies, for a committee with N members with N odd, C's proposal γ is passed if and only if it is accepted by the median committee member.*

Proof. For sufficiency, assume the median member accepts, then by the preceding claim either all committee members with $\phi_i > \phi_m$ ($\nu_{i,0} > \nu_{m,0}$) accept or all committee members with $\phi_i < \phi_m$ ($\nu_{i,0} < \nu_{m,0}$) accept. In either case, γ passes.

For necessity, assume the median member does not vote for γ. Then either all members with $\phi_i > \phi_m$ ($\nu_{i,0} > \nu_{m,0}$) do not vote for γ or all members with $\phi_i < \phi_m$ ($\nu_{i,0} < \nu_{m,0}$) do not vote for γ. In either case γ is not approved. □

Using claim 24 C's proposal strategy when faced with an essentially two-member committee will take into account only median member(s) of the committee. In the A periods for the first condition in definition 5 this is a player with D period preference shock ϕ and for the second condition of the same definition those are all the remaining committee members who in D periods have preference shocks equal to ϕ on average. In the D periods we have either one or two players with preference shock equal to ϕ being median ones, depending on the exact condition used in definition 5. As a result, C's proposal strategy in the dynamic bargaining game played by any essentially two-member committee will be equal to the proposal strategy in the dynamic bargaining game played by C with only one other player with the two players having D period preference shocks $-\phi$ and ϕ respectively. The proposition then follows from the fact that C's proposal is always approved in equilibrium by the median player and hence by the whole essentially two-member committee if its members use stage-undominated strategies.

A2 Static mechanism implementation

We restrict attention to static transfer-free direct mechanisms in which the policy in period t is independent of history. In mechanism $M : \{m_C, m_P\} \to \Delta(X)$ player $i \in \{C, P\}$ submits message $m_i \in \{A, D\}$ and M implements a policy from X chosen according to some distribution, so that $\Delta(X)$ denotes the set of all distributions on X.

Because player types are perfectly correlated we can restrict attention to mechanisms that learn the type of period with certainty. Each such mechanism will be characterized by a pair of distributions, one for A periods with *cdf* F_A and one for D periods with *cdf* F_D.

It is immediate that F_A implements π^* with certainty in any Pareto efficient mechanism. For D periods, C's expected utility is equal to

$$\int_X -(x - \pi^* + \phi)^2 dF_D(x) = -\operatorname{var}(x) - (\mathbb{E}(x) - \pi^* + \phi)^2$$

and P's expected utility is equal to

$$\int_X -(x - \pi^* - \phi)^2 dF_D(x) = -\operatorname{var}(x) - (\mathbb{E}(x) - \pi^* - \phi)^2$$

so that $\operatorname{var}(x) = 0$ in any Pareto efficient mechanism.

As a result, any Pareto efficient static transfer-free direct mechanism has to involve $M(A, A) = \pi^*$ and $M(D, D) = p^*$ where $p^* \in [\pi^* - \phi, \pi^* + \phi]$. Moreover, $p^* = \pi^*$ for utilitarian (maximizing sum of expected utilities) mechanism.

A3 Numerical simulation of equilibrium under explicit status-quo bargaining

This section describes the procedure to obtain numerical estimates of the equilibrium C's value function V_C and her proposals in the model with the explicit status-quo bargaining protocol. We use the standard value function iteration method along with several results proven earlier. First of all recall that by proposition 3 we know $p_A(x) = \pi^*$. Furthermore, from the proof of proposition 4 we know the shape of P's acceptance sets A_A and A_D and equilibrium proposals for $x \in X^-$. Finally by the same proposition we know C's value function V_C is unique.

To estimate the remaining part of the equilibrium, we restrict the proposal space along each dimension to $X' = [\pi^* - 1.1\phi, \pi^* + 3.1\phi]$ and specify a grid of discrete nodes $\{d_1, \ldots, d_N\} \in X'$. Call this grid G. We use $\pi^* = 2$, $\phi = 1$ which, with the distance of the neighbouring nodes equal to 0.001, gives $N = 4201$. With the proposal space specified, we implement the following iterative procedure. At the iteration step t we solve C's optimization problem for A and D periods for each default option in G. Denote by $V_C^t(G)$ the $N \times 1$ vector of C's continuation values, each of them associated with a distinct node (default option) $d_i \in G$ at the t-th step of the iteration.

For D periods we solve for each $d_i \in G$

$$\max_{\{p,q\} \in A_D(d_i) \subseteq G^2} -(p - \pi^* + \phi)^2 + \delta V_C^t(q)$$

by searching the discretized feasible proposal space $A_D(d_i) \subseteq G^2$. This gives us two $N \times 1$ vectors of proposals for the D period, one along the policy dimension, $\mathbf{p}_D^t$, and the second along the status-quo dimension, $\mathbf{q}_D^t$, with the i-th element of each being C's proposed policy and status-quo for default option d_i.

For A periods we already know $p_A(x) = \pi^*$ hence for each $d_i \in G$ we solve

$$\max_{\{\pi^*,q\} \in A_A(d_i) \subseteq G^2} V_C^t(q)$$

again by searching the feasible proposal grid $A_A(d_i) \subseteq G^2$. This gives us one $N \times 1$ vector of status-quo proposals for the A period, $\mathbf{q}_A^t$, with the i-th element being the proposed status-quo for default option d_i.

Finally we compute $N \times 1$ vector of C's continuation values as

$$V_C^{t+1}(G) = r_d \left[-(\mathbf{p}_D^t - \pi^* + \phi)^2 + \delta V_C^t(\mathbf{q}_D^t) \right] + (1 - r_d) \left[\delta V_C^t(\mathbf{q}_A^t) \right]$$

and proceed to the iteration step $t + 1$. We stop the iteration procedure when $\max_{i \in \{1,\dots,N\}} |V_C^{t+1}(d_i) - V_C^t(d_i)| \leq 1.0 \times 10^{-6}$. As usual, for the first step of the iteration we use $V_C^1(G) = \mathbf{0}$. We experience no problems with convergence and for the simulations shown the procedure converges in about 70 iterations.

The reason why we use this rather rudimentary numerical procedure instead of some more involved one (e.g. a better optimization algorithm and functional approximation for V_C) is twofold. First, we suspect the V_C to be ill-behaved with a number of local maxima and we do not want the optimization algorithm to pick a wrong one especially as the acceptance sets are in general not convex. Second, we suspect the resulting equilibrium to involve several discontinuities and we do not want the functional approximation to 'smooth out' the problem.