Electoral Competition and Factional Sabotage *

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Abstract

Intra-party sabotage is a widespread phenomenon that undermines the strength of political parties. What brings opposing factions to engage in sabotage rather than enhancing the party image, and what strategies can parties adopt to contain it? This paper presents a model of elections in which intra-party factions can devote resources to campaign for the party or to undermine each other and obtain more power. The party redistributes electoral spoils among factions to motivate their investment in campaigning activities. The model shows that sabotage increases when the stakes of the election are low — e.g., in consensus democracies that grant power to the losing party — because the incentives to focus on the fight for internal power increase. It also suggests that the optimal party strategy for winning the election in the face of intra-party competition is to reward factions with high powered incentives when campaigning effort can be easily monitored, but treat factions equally otherwise.

Keywords: Party Organization, Electoral Competition, Party Factions, Sabotage.

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1. Introduction

Sabotage is an undeniable fact of party life. Examples abound across different times and places. Intra-party sabotage permeated one of the most highly factionalized parties of all times, the Christian Democratic party (DC) that ruled Italy from the aftermath of World War II until the 1990s, contributing to its demise in 1994.[1] This intense factional competition constituted a fundamental root of the corruption that caused the end of the Italian "First Republic" (Golden and Chang, 2001). More recently, a leaked internal report provided evidence of factionalism and sabotage that took place during Jeremy Corbyn's four-year tenure as leader of the Labour Party.[2] In the wake of the evidence, several members of Corbyn's faction maintained that the party would have won in 2017 absent sabotage.[3]

Democracy works differently in the presence of factional competition than in the absence. Warring factions take away resources that parties can otherwise devote to electoral competition, thereby affecting electoral results and final policy outcomes.[4] Despite the pervasiveness of factional competition across a variety of political systems, little is known about the conditions facilitating sabotage, or the strategies parties adopt to contain it. Identifying these conditions is necessary to understand better political parties, and ultimately the sustainability of political systems. Does an increase in party polarization alleviate or exacerbate factional competition? What are the institutional features of the electoral system that help to promote cooperation among factions? Given factions' incentives, how do party rules change to limit sabotage and foster cooperation?

To answer these questions, the paper introduces factional competition in a model of elections between two parties. The model identifies features of the electoral environment that alleviate intra-party sabotage, and shows how the party organization changes to limit it, thus maximizing the chances of winning the election. Features of the competitive environment such as ideological polarization and electoral institutions affect factions' incentives to sabotage each other instead of mobilizing towards the party's common good. Factions' incentives are taken into account by the party, which changes its organization accordingly.

What strategies can parties adopt to motivate factions? Historically, within both the Italian DC

[1] The party imploded as a consequence of "Tangentopoli," one of the biggest corruption scandals of all times (Waters, 1994).

[2] See Mason, R. (2020) 'Hostility to Corbyn curbed Labour efforts to tackle antisemitism, says leaked report,' The Guardian, 12 April. Link to original report: https://cryptome.org/2020/04/Labour-Antisemitism-Report.pdf.

[3] Link to Labour Party MP's tweet: https://twitter.com/RichardBurgon/status/1249461680834256898.

[4] Since the American Founding Fathers, several authors have regarded factions as potentially dangerous. In Federalist 10, Madison outlines the dangers that factionalist interests can pose to political unions (Madison, 1787). Similarly, V. O. Key, as cited in Boucek (2009), blamed factions for encouraging favoritism and graft among elected officials (Key, 1949).

and the Japanese Liberal Democratic Party (LDP) — identified as the most stable factionalized parties across democratic systems (Bettcher, 2005) — factions' relative power determined the distribution of electoral spoils, the main driver of factional action. In the DC case, the spoils allocation method followed an *explicit* formula according to which cabinet positions were distributed among factions in proportion to the number of party members each faction had.[5] Motivated by this evidence, this paper formalizes the concept of party organization by analyzing different incentive schemes that reward the electoral campaigning effort of factions. In the model, factions decide how much to invest in campaigning activities to support the party — e.g., constituency service that increases party valence —, and how much to sabotage each other to obtain more power within the party. Sabotage is defined as a range of activities that worsen the party's collective good but may increase a faction's relative power within it.[6]

The goal of each party is to win the general election. Party platforms are fixed, and parties incentivize factions to invest in campaigning activities to increase the odds of winning. Each party controls electoral spoils (cabinet positions, assignment to committees) and distributes them among factions without leaving resources on the table. The share of electoral spoils obtained by each party is determined by the amount of power-sharing of the electoral environment (Lijphart, 1984): majoritarian democracies concentrate power in the hands of the winning parties, while in consensual democracies resources are more evenly shared with minority parties.

Factional investment in campaigning activities is only imperfectly observable. Often parties need to make organizational decisions based on imperfect measures of factional performance (e.g., the party vote share in a given district or the number of members each faction brings to the party). These performance indicators result in an internal ranking of factions, based on which the party distributes electoral spoils. The ranking depends on both campaigning effort and sabotage: the faction ranking higher could be the one that worked more for the party or the one that focused on undermining the other faction. For instance, having factions tied to electoral strongholds makes it easy for a party to reward campaigning effort, by observing electoral performance in a given area. In this case, campaigning is "more effective" than sabotage in achieving a high ranking. In the absence of such measures, the party might be constrained to rely on other indicators that lend themselves to sabotage, such as the number of members brought to the party by each faction.[7]

[5] The method refers to the "Cencelli Manual", which since then became a common political idiom (Venditti, 2016). Section 2 illustrates the relevance of the manual, and describes in more detail the Italian and Japanese factions.

[6] As such, the term sabotage encompasses actions that directly undermine the other faction's standing, as well as the investment in internal activities that, by benefiting one faction, create negative externalities on the other faction's welfare.

[7] In order to increase their relative number of party memberships, factions of the Italian Democratic Party used to engage in sabotaging activities such as impeding subscriptions to competing factions.

To maximize the chances of winning, each party specifies how much of the party's spoils are distributed to factions according to the internal ranking. In line with the empirical evidence, I assume that parties commit to the internal ranking for rewarding factions.[8] The rewards that the party can choose span from low-powered — i.e., both factions are equally rewarded, independently of the ranking indicator — to high-powered — i.e., the faction ranking higher obtains all the party spoils. When campaigning effort is more effective than sabotage to achieve a high ranking, choosing high-powered incentives amounts to reward the faction that probabilistically invested more in campaigning activities, while low-powered incentives discourage campaigning. Conversely, when sabotage is more effective than campaigning activities, high powered incentives encourage sabotage.

The first set of results show that sabotage varies with inter-party power-sharing. In equilibrium, factions work more for the party as the political system resembles a majoritarian democracy (that is, as inter-party power-sharing decreases). Conversely, the more the system reflects a consensus democracy, granting power to the losing party, the more factions sabotage each other, as the incentives to focus on the intra-party contest increase. Several constitutional design scholars warn against certain features of winner-take-all electoral systems (Tsebelis, 1995; Powell Jr, 2000; Golder and Ferland, 2017). This result underscores the overlooked element of intra-party incentives generated by institutions when comparing different democratic systems.

Factions' equilibrium behavior changes with ideological polarization as well. I distinguish between polarization *across parties* and polarization *across factions* within the same party, and show that the two have different implications. When parties' platforms are distant from each other and the median voter is moderate, factions in the more extreme party sabotage more than those in the moderate party, which instead campaign more to win the election. Intuitively, a higher probability of victory is associated with a higher expected payoff for factions in the moderate party, which invest more resources in campaigning. However, when factions in the same party are ideologically distant, the extreme faction invests more in campaigning than the moderate one. This happens because the stakes of the election are greater for the more extreme faction, which suffers a higher ideological cost from losing than the moderate one, which is ideologically closer to the opposing party. This prediction resonates with the UK Labour party in the 2017 campaign, where Corbyn's faction devoted substantial resources to campaigning for the party, while the moderate Labour MPs engaged in public hostility against Corbyn's faction and its policies.

Given factions' incentives, which rewards does the party choose in equilibrium? Intuitively, when

[8] This assumption closely reflects portfolio allocation in both the Italian DC and the Japanese LDP, as Section 2 illustrates.

campaigning effort is more effective than sabotage to obtain a high internal ranking, high powered incentives are optimal: the faction with the highest internal ranking obtains more electoral spoils. The method of allocation of cabinet positions in the Japanese Liberal Democratic Party (LDP) before the 1994 electoral reform is consistent with this prediction.[9] Yet, sabotage could be more efficient than campaigning effort to obtain a high internal ranking in some situations. In the case of the Italian DC for instance, it was easier for factions to deny cards to the opposing faction than to bring new members to the party (Venditti, 2016). When this is the case, the party knows that the better-placed faction is the one that (probabilistically) sabotaged more, and in equilibrium incentives are low powered.

The model produces several empirical implications on the effect of electoral institutions on intra-party competition. First, it suggests that an increase in inter-party power-sharing can exacerbate competition within parties. Such change might refer to the electoral system (e.g., from winner-take-all to proportional), or an institutional change holding fixed the electoral system's proportionality (e.g., from executive dominance to legislative-executive balance). Second, features of the electoral systems such as the use of preference votes — where voters can indicate a preference for candidates on the ballot — can increase the visibility of factional campaigning effort and its relative effectiveness to rank higher within the party.[10] While the literature on personal vote states that the use of preference votes increases competition at the *individual candidate* level (Carey and Shugart, 1995), this model suggests that competition could be reduced at the *factional* level, thus uncovering potential omitted variable bias in the correlation between weak parties and open list PR systems.

This paper provides a novel theoretical framework to understand how factional competition shapes the life of a party — its internal institutions, campaigning capacity, and policy platforms. As such, it relates to the theoretical literature analyzing the role of factions within parties (Persico, Pueblita and Silverman, 2011; Dewan and Squintani, 2016; Izzo, 2018). In addition to providing a new framework for the analysis of intra-party organization, the model advances this literature by studying intra-party sabotage, which empirically is often driven by factional divisions (Zariski, 1965; Brass, 1966; Cox and Rosenbluth, 1994; Mershon, 2001; Balán, 2011; Nellis, 2019).

Most of the existing theoretical models on intra-party organization focus on primaries and their effects on outcomes such as public good provision (Ting, Hirano and Snyder Jr, 2018), candidates' quality (Serra, 2011) and chances of winning the general election (Adams and Merrill, 2008). Others

[9] The method consisted in dividing proportionally the electoral spoils among factions and give a premium to the "mainstream faction" with the highest share of votes (Browne and Kim, 2003; Ramseyer and Rosenbluth, 2009).

[10] As argued in Section 6, by observing candidates' preference votes the party could condition factions' rewards on the number of preference votes that factions' candidates get. This in turn could incentivize factions to campaign for the party.

compare primaries to hierarchical internal organizations and study their effect on electoral effectiveness in majoritarian elections (Caillaud and Tirole, 2002; Crutzen, Castanheira and Sahuguet, 2010). In the context of proportional representation systems, Buisseret et al. (2017) study how political parties structure candidate selection by ranking candidates on lists. I contribute to this literature in the following ways. First, the model formalizes with contract theory tools the relation of agency among the central party leadership and factions within parties. Second, I embed this framework in a probabilistic voting model of electoral competition, where electoral imbalance is endogenously provided by intra-party competition among factions. Thus, the model conceptualizes a party internal organization as the degree of *power-sharing* among its factions, where the optimal internal organization is determined in equilibrium by the electoral environment.

The remainder of the paper is organized as follows. Section 2 provides a historical account of factional dynamics in the Italian Christian Democratic party and the Japanese Liberal Democratic Party. Section 3-5 describe the model and its results. Section 6 proposes some empirical implications and Section 7 concludes.

2. Portfolio Allocation *Within* Parties: Factions in Italy and Japan

This section provides a brief historic account of the Italian Christian Democratic Party (DC) and the Japanese Liberal Democratic party (LDP). Both parties based the distribution of cabinet portfolios on factions' relative power: in the DC case, the method of allocation followed an explicit rule named *"Cencelli manual"* (Venditti, 2016).

2.1. Italian Christian Democratic Party

The Cencelli manual is a weight calculation method invented in 1968 by Massimiliano Cencelli. The method was adopted to calculate how many ministries and undersecretaries the DC would get in the upcoming election, and it was used to assign offices to factions for decades. The calculations of all the governments formed after 1968 are collected in hundreds of pages with factional denominations as they form, break up and recompose, their absolute and percentage weight, the final number of ministries and under secretaries, and finally the names of ministries and under secretaries of all the formed governments.

Figure 1, taken from the original manual, provides an illustration of the method.[11] The baseline for calculating a faction's relative weight is given by the percentage of party members belonging to

[11] The original table is reported in the Appendix.

each faction (in the picture, the bars).[12] Based on the percentage obtained, the party assigns cabinet positions to each faction (triangles). Thus, it is clear that factions are incentivized to obtain as many memberships as possible and, as history shows, *by any means.*

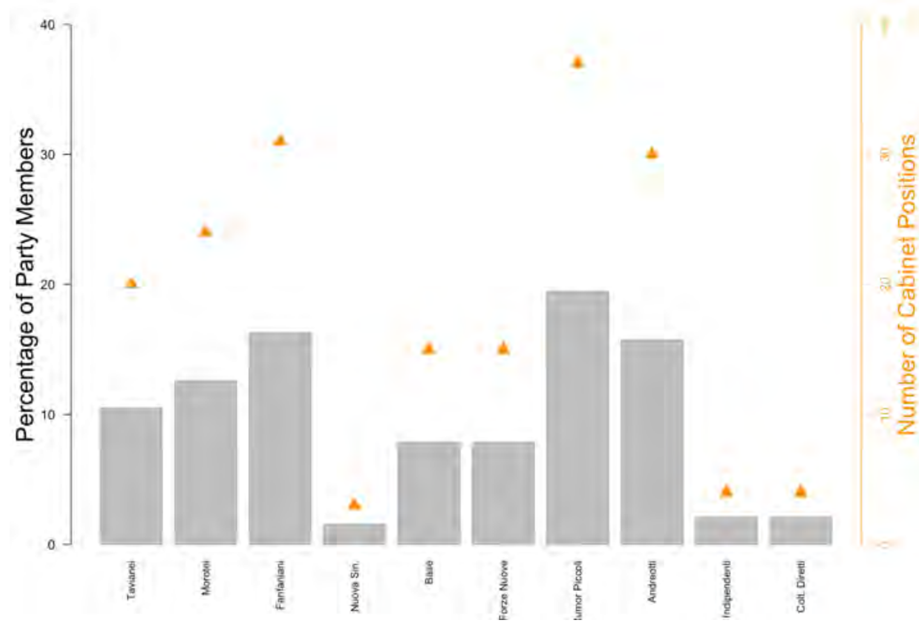

Figure 1 – Portfolio Allocation Rule in Italian Christian Democracy (1973). The bars represent the percentage of party members belonging to each faction composing the DC in 1973. Based on this percentage, the total number of cabinet positions is divided among the party factions. The triangles show the total number of members of each faction obtaining a cabinet position.

The Cencelli Manual reflects the phase of highest internal fractionalization of the DC (1960s-70s), with factions fighting each other to obtain more seats (Sartori, 1971). Historical accounts of sabotage inside the party were closely related to the widespread phenomenon of "membership card inflation" (Venditti, 2016), which plagued the party since the 1960s. A common way factions used to inflate membership cards was to assign memberships to individuals who either were unaware, dead, or had emigrated.[13] In some localities, the total number of votes was even lower than the DC members. By diverting factions' investment of resources from constituent service valued by voters in the attempt to increase members, these activities fall under the definition of sabotage. Another — perhaps more explicit — sabotaging technique frequently adopted by factions was to impede subscription to competing factions with delays and procedural complications. These sabotaging activities affected the electoral

[12] In the model, this percentage corresponds to the outcome of an internal ranking of factions, which depends on their choice of effort and sabotage.

[13] In 1976, two DC senators revealed that more than 50% of membership cards were false, "corresponding to people who either did not exist or never asked to be members of the party" (Venditti, 2016).

performance of the DC, which saw its powered gradually reduced until the party's demise in 1994.

To understand how widespread these phenomena were, it is worthwhile to look at what happened in 1976, when the party took two measures: the first measure forced all party representatives to immediately accept requests of registration to the party, the second one increased the membership fee. As a consequence, the members of the DC that year dropped by $360,000$ units (21.2%), and the newly registered members increased from 6.1% to 15.1%. This event provides evidence that there was indeed a membership card inflation, and that many got for the first time a card that was before denied by competing factions.

One important member of the old faction Dorotea, Giuseppe Zamberletti, defined the Manual as "the organization chart of the Christian Democratic company". The party is described as a holding company, where who has more membership cards has the right to more power, regardless of other factors such as valence, honesty, and quality of administration. All the governments closely followed the Cencelli Manual in assigning office positions. If the proportions were not correct (i.e., not following the manual's predictions) the penalized factions would jeopardize the government, while officially pretending to be loyal. Indeed, there have been occurrences of "franchi tiratori"- MPs voting against the government in secret votes because excluded from power sharing in violation of the Cencelli manual (Venditti, 2016).

2.2. Factions in the Japanese Liberal Democratic Party

While the manual employed by the Italian DC is a unique case of portfolio distribution that relies on *explicit* mathematical calculations, other parties rely on similar measures to quantify the value of cabinet posts and assign them to internal factions. One prominent example is the Japanese Liberal Democratic Party (LDP): before the electoral reform of 1993, government formation and portfolio allocation were internally decided among factions (Sartori, 2005; Kohno, 1992; Ramseyer and Rosenbluth, 2009).

The early LDP factions were personalistic organizations aimed at electing party leaders, and factional size fluctuated considerably with the leaders' retirement or death. In time, turnover decreased until defections from factions almost ceased: after 1972 politicians' fates were tied to the chosen faction until retirement and changing faction was a very rare event. Moreover, from the 1980s onwards factions were not personalistic anymore; rather, the leader came to be seen increasingly as an agent for the faction (Bettcher, 2005). Being cohesive and stable, factions acted as unitary actors and the party leadership as a selectorate maximizing the collective good of factions.

The method of allocation of cabinet positions in the LDP consisted in dividing proportionally the electoral spoils among factions and give a premium to the largest factions (Browne and Kim, 2003;

Ramseyer and Rosenbluth, 2009). Since the late 1960s, the number of ministerial positions obtained by each faction has corresponded closely to their relative strength within the party (measured by factions' membership in the Diet), and by the mid-1980s this method of portfolio allocation was strictly applied. Despite having the opportunity to revise the proposal of cabinet posts (by asking for a leadership change, leaving the party, or calling for a vote of no-confidence), no disagreement ever happened (Adachi and Watanabe, 2007). Moreover, historically no cabinet formation required more than 3 days, which is a surprisingly short period given that the average length of government formation process in Western Europe is 28 days (Ecker and Meyer, 2015). This evidence is suggestive of how portfolio allocation was a contract stipulated among LDP factions before the realization of the electoral outcome.

Intra-party conflict happened frequently within the LDP as well, especially before the electoral system's reform in 1994 which decreased the district magnitude to a single-member district. Before 1994 too many candidates from different factions were selected to run in each district, and competition was so destructive that the term *tomodaore* (going down together) was coined to refer to the problem of overnomination, which often led to failure to elect as many representatives as a unified party could have elected (Cox and Rosenbluth, 1996; Nemoto, Pekkanen and Krauss, 2014).

3. The Model

Consider a probabilistic model of electoral competition between two parties, *left* and *right*. There are three players in the left party: two factions denoted by L_1, L_2 and a party leader L (*she*). Players in the right party are denoted by R_1, R_2 and R respectively. Denote *left*'s preferred platform as $x^L \in \mathbb{R}$, which is implemented when the party wins the election (the same holds for *right*). Both platforms (x^L, x^R) are common knowledge and fixed.[14] There exists a representative voter, denoted by V, who votes for *left* or *right*. The voter's ideal point is denoted by x^V. Without loss of generality, let $x^V = 0$, and $x^L < x^V < x^R$. For exposition purposes in what follows I will refer to party *left* (the description of actors in *right* is analogous).

A faction is a unitary team of politicians who share the same ideological preferences: denote the policy preferred by faction L_i as x_i^L, where $i = 1, 2$. I start by assuming that factions in the same party share the same ideological preferences ($x_1^L = x_2^L = x^L$), and relax this assumption in Section 5. Each faction can use its resources to increase the party electoral chances by exerting campaigning effort, denoted by e_i^L, and/or to sabotage the other faction, a_i^L. Both actions are costly: $C(e_i^L) = (e_i^L)^2/2$ and $C(a_i^L) = (a_i^L)^2/2$. The assumption of convex costs reflects the decreasing returns associated to each

[14] The latter assumption is relaxed in Subsection 5.1, which analyses endogenous platforms.

activity: while initially it is easy to find compromising material to sabotage the other faction and good slogans to convince voters, at the margin more resources are needed to have a substantial impact on the campaign.

The goal of the model is to understand which circumstances encourage campaigning effort and which encourage sabotage: in order to do so in a tractable way, the total amount of sabotage is assumed to be equal to $a_i^L = 1 - e_i^L$ — that is, all the resources that are not used to promote the party are invested in damaging the rival faction (this assumption is relaxed in the Appendix). The binding budget constraint together with convex costs implies that a positive level of sabotage is cost efficient: the focus will be on how the equilibrium investment in effort and sabotage changes with parameters of interest.

The party leader — who stands for the party organization — only cares about winning the election.[15] The leader incentivizes factions to invest in campaigning for the party by promising rewards contingent on the electoral outcome. Rewards depend on the total amount of electoral spoils, normalized to one: the winning party gets a share of spoils α, where $\alpha \in [1/2, 1]$, and the losing party gets $(1 - \alpha)$. The parameter α refers to the degree of inter-party power-sharing (Lijphart, 1984; Herrera, Morelli and Palfrey, 2014). I use the short-hand "majoritarian democracies" to refer to systems with high α, and "consensus democracies" to refer to systems with low α.

The party leader does not directly observe factions' campaigning effort: L observes a relative ranking indicator that can take two values, $s^L \in \{1, 2\}$. When $s^L = 1$ faction L_1 is ranked higher than L_2, and vice-versa when $s^L = 2$. Both campaigning effort and sabotage help factions to rank higher. Formally, the probability that faction L_1 obtains a higher rank than L_2 is:

$$\rho_1^L = \Pr\{s^L = 1\} = \frac{1}{2} + \frac{(e_1^L - e_2^L) + \gamma(a_1^L - a_2^L)}{\phi}. \tag{1}$$

The parameter $\gamma \in \mathbb{R}_+$, which is common knowledge, reflects the relative effectiveness of sabotage to rank higher: when $\gamma > 1$ sabotage is more effective than effort in achieving a high internal ranking, while for $\gamma < 1$ investing in campaigning effort is more effective than sabotage.[16] The parameter ϕ is a normalization ensuring that the probability is bounded. The probability that faction L_2 obtains a higher rank is simply $\rho_2^L = 1 - \rho_1^L$.

Based on the indicator s^L, the leader distributes among factions the share of electoral spoils obtained

[15] The party leader can be thought of representing the central party organization. As such, the leader does not have ideological preferences and is *above factions*. Subsection 5.1 discusses how the leader would tailor the party policy platform to factions' preferences if she belonged to a particular faction.

[16] The parameter γ refers to institutional and non-institutional factors that make factions' campaigning effort more difficult to monitor and reward by the party. Section 6 presents some examples.

contingent on the party winning (α) or losing the election ($1-\alpha$) by assigning a *premium to the faction that ranks higher* within the party (e.g., faction L_1 if $s^L = 1$). Formally, L assigns π_v^L to L_1 if the left party wins, and π_d^L if it loses (the subscript v stands for victory, while d for defeat). Premia are assumed to be non–negative — that is, the leader can "punish" factions at most with zero incentives. Importantly, this assumption is motivated by real-world instances where party leaders are *de facto* constrained by the existing measures of factional performance and have no other choice but to abide by them.[17,18] Rewards for factions in *left* satisfy the following budget constraint:

$$\begin{aligned} \pi_v^L + 2b_v^L &= \alpha \\ \pi_d^L + 2b_d^L &= 1-\alpha, \end{aligned} \tag{2}$$

where b_v^L and b_d^L are baseline prizes offered to both factions in case of party victory and defeat respectively. Given the leader's budget constraint assumption (2), the value of the baseline prizes is equal to $b_d^L = (1-\alpha-\pi_d^L)/2$, $b_v^L = (\alpha - \pi_v^L)/2$ and the only relevant choice for L is the vector of premia (π_d^L, π_v^L).[19] The leader chooses (π_d^L, π_v^L) to maximize the probability of winning the election, which is increasing in factions' campaigning effort and determined in equilibrium by the voter's choice.

The voter's payoff has two components: the first is a standard quadratic loss from the distance from parties' platform, the second depends on party's valence, which refers to all those attributes that are valued independently of ideology. Campaigning activities by the factions such as constituency service increase the party's appeal to voters by increasing its valence. Formally, V's realized payoff if the left party wins is:

$$u^V(e_1^L, e_2^L; x^L) = -\Big(x^V - x^L\Big)^2 + e_1^L + e_2^L. \tag{3}$$

Finally, before the election an exogenous shock that favors party R affects the voter's payoff, where ξ is uniformly distributed in $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$. I assume that ψ is small enough to ensure a bounded probability of victory for both parties. The parameter ψ can be interpreted as the importance of the electoral campaign: as ψ increases, the support of the shock shrinks and factional campaigning activities (as well

[17] In the Italian DC case for instance, the leadership knew that a faction's relative higher ranking — corresponding to more membership cards brought to the party — was a symptom of its higher investment in sabotaging activities, but still could not punish factions for bringing more members to the party.

[18] The Appendix relaxes this assumption showing that results are robust to a setup where leaders can punish the higher-ranked faction with a negative premium (i.e., rewarding the lower-ranked faction).

[19] Given the budget constraint assumption, L's incentive scheme can also be interpreted as the share of spoils offered to each faction under the event of party victory and defeat. I distinguish between baseline prizes and premia to reflect real instances of incentives used by parties. For instance, Browne and Kim (2003) note that in the Japanese LDP *"the more numerous positions, notably in the cabinet, were allocated in close proportion to a faction's membership in the Diet, whereas scarcer positions [...] were balanced among the very largest factions"*.

as party policy platforms) become more salient to the voter.

Let $\boldsymbol{\pi}^L = (\pi_d^L, \pi_v^L)$ and $\boldsymbol{e}^L = (e_1^L, e_2^L)$. The payoff of faction L_1 can be expressed as:

$$u_1^L(\boldsymbol{e}^L, \boldsymbol{\pi}^L; x_1^L) = R_1^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L) - \left(x_1^L - x^*\right)^2 - C(e_1^L) - C(1 - e_1^L), \tag{4}$$

where $x^* \in \{x^L, x^R\}$ is the winning party's platform, and the reward $R_1^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L)$ is a function of the incentive scheme $\boldsymbol{\pi}^L$ (which depends on factions' effort $\boldsymbol{e}^L$).[20] The expected rewards from electoral victory and defeat are, respectively:

$$\begin{aligned} R_1^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L)|_v &= b_v^L + \rho_1^L(\boldsymbol{e}^{\boldsymbol{L}})\pi_v^L, \\ R_1^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L)|_d &= b_d^L + \rho_1^L(\boldsymbol{e}^{\boldsymbol{L}})\pi_d^L. \end{aligned} \tag{5}$$

That is, L_1 is rewarded with π_v^L (π_d^L) only when it ranks higher than L_2 (which happens with probability ρ_1^L), and the size of the reward is determined by the electoral institutions ($b_v^L + \pi_v^L > b_d^L + \pi_d^L$, as $\alpha > 1/2$).

The timing of the game is as follows: first, leaders announce an incentive scheme contingent on the electoral outcome. Second, factions decide how much resources to invest in campaigning and sabotage. Finally, elections are held, and prizes are distributed according to the contract. A strategy for L maps from the internal ranking s^L to an incentive scheme (π_d^L, π_v^L). For L_1 and L_2, a strategy is a mapping from the set of incentives to an allocation decision e_1^L, e_2^L (and analogously for R_1 and R_2). The voter votes for the party that gives her the higher payoff. The solution concept is Subgame Perfect Equilibrium.

4. Equilibrium Analysis

In what follows I start by computing the voter's decision, which determines the probability of each party winning the election. Given this winning probability, the expected payoff of each faction is derived as a function of the other factions' decision, and for each possible incentive scheme offered by the leader. Finally, I compute the incentive scheme chosen by the party leader, and characterize the equilibrium of the game.

The voter prefers party *left* if:

$$u^V(e_1^L, e_2^L; x^L) \geq u^V(e_1^R, e_2^R; x^R) + \xi.$$

[20] Notice that the reward function $R_1^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L)$ also depends on $\boldsymbol{\pi}^{\boldsymbol{R}}$ and $\boldsymbol{e}^{\boldsymbol{R}}$, which affect the incentive scheme $\boldsymbol{\pi}^L$ (this is omitted from the notation above for parsimony.

The probability that *left* wins the election is the probability that V prefers *left*. By the uniform assumption, this is simply a function of factions' effort ($\boldsymbol{e}$):

$$p^L(\boldsymbol{e}) = \frac{1}{2} + \psi\left[u^V\left(e_1^L, e_2^L; x^L\right) - u^V\left(e_1^R, e_2^R; x^R\right)\right]. \tag{6}$$

Factions choose how many resources to allocate in campaigning and sabotage. Formally, L_1 solves:

$$\max_{e_1^L \in [0,1]} p^L(\boldsymbol{e})\left[b_v^L + \rho_1^L \pi_v^L\right] + \left(1 - p^L(\boldsymbol{e})\right)\left[b_d^L + \rho_1^L \pi_d^L - (x^L - x^R)^2\right] - \frac{\left(e_1^L\right)^2}{2} - \frac{\left(1 - e_1^L\right)^2}{2},$$

where the first term (expected payoff from winning the election) does not include an ideological cost because $x_1^L = x^L$. Notice that e_1^L has two effects on factions' expected payoff: first, campaigning increases the party's electoral chances via higher $p^L(\boldsymbol{e})$. Second, e_1^L enters the probability of ranking higher within the party (ρ_1^L): this component reflects the strategic tension faced by factions.

Depending on the value of γ, the probability of ranking higher could be increasing or decreasing in campaigning: this follows from (1) and the assumption $a_i^L = 1 - e_i^L$. When $\gamma < 1$, campaigning effort helps winning the election and improves the odds of being assigned a positive premium by the leader. The trade-off between campaigning effort and sabotage arises when $\gamma > 1$: while sabotage increases the odds of a high internal ranking (hence a higher share of spoils), campaigning helps the party to win the election. This trade-off is apparent in the faction's first-order condition, which can be written as:

$$\underbrace{\frac{\partial p^L(\boldsymbol{e})}{\partial e_1^L}\left[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\right]}_{\text{External Incentive}} + \underbrace{\frac{\partial \rho_1^L}{\partial e_1^L}\left[p^L \pi_v^L + (1 - p^L)\pi_d^L\right]}_{\text{Internal Incentive}} + 1 - 2e_1^L = 0, \tag{7}$$

where the first term represents the marginal return of effort on winning the election while keeping the competition inside the party fixed, and the second term corresponds to the marginal return of effort on ranking higher within the party holding the electoral incentives fixed. The first term is always positive — i.e., factions' campaigning always improves the party's electoral chances in the election. The sign of the internal incentive term depends on whether sabotage is more effective than campaigning: when $\gamma < 1$ the internal incentive term is positive (campaigning helps towards achieving a high ranking), whereas it is negative when $\gamma > 1$ (sabotage is more effective than campaigning for ranking higher).

The Appendix shows that each faction's objective is concave, which implies that the first-order condition above identifies the solution to the faction's maximization problem. The solution of the system of first-order conditions, one for each faction in each party, determines factional effort as a best reply to the other factions' efforts, for both incentives schemes (π_d^L, π_v^L) and (π_d^R, π_v^R). For ease of notation, the following analysis refers to equilibrium effort as e_1^{L*} (and analogously for the other factions).

Substituting the value of the voter's realized payoffs $u^V(e_1^L, e_2^L; x^L)$ and $u^V(e_1^R, e_2^R; x^R)$ into the probability of victory expression (6) we can formally express L's expected payoff as the probability that the left party wins, which is indirectly affected by R's premia (via e_1^R, e_2^R):

$$\max_{\pi_d^L, \pi_v^L} \quad \frac{1}{2} + \psi\left[e_1^{L*} + e_2^{L*} - e_1^{R*} - e_2^{R*} - (x^L)^2 + (x^R)^2\right]. \tag{8}$$

Recall the first-order condition of L_1, which can be expressed as the following implicit equation in e_1^L:

$$e_1^L = \frac{1}{2} + \frac{\psi}{2}\left[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L)\right] + \left(\frac{1-\gamma}{4}\right)\left[p^L(\boldsymbol{e})\pi_v^L + \left(1 - p^L(\boldsymbol{e})\right)\pi_d^L\right]. \tag{9}$$

Differentiating with respect to π_v^L yields:

$$\frac{\partial e_1^{L*}}{\partial \pi_v^L} = \frac{\psi}{2}\Big[\rho_1^L - \frac{1}{2}\Big] + \Big(\frac{1-\gamma}{4}\Big)\Big[p^L(\boldsymbol{e}) + \psi\Big(\frac{\partial e_1^{L*}}{\partial \pi_v^L} + \frac{\partial e_2^{L*}}{\partial \pi_v^L} - \frac{\partial e_1^R *}{\partial \pi_v^L} - \frac{\partial e_2^R}{\partial \pi_v^L}\Big)\big(\pi_v^L - \pi_d^L\big)\Big], \tag{10}$$

which follows from the budget constraint assumption (2), the fact that $\partial p^L/\partial e_i^{L*} = \psi$ and that $\partial \rho_1^L/\partial \pi_v^L = 0$ in equilibrium. The Appendix shows that — after performing the same for e_2^{L*}, e_1^{R*}, e_2^{R*} — the resulting partial derivative of L's objective is positive for $\gamma < 1$ and negative for $\gamma > 1$, for both π_v^L and π_d^L. As a result, the equilibrium incentive scheme features maximum premia when $\gamma < 1$, and zero premia when $\gamma > 1$, as the first result below summarizes.

First, I derive the equilibrium when no party has an ex-ante electoral advantage over the other due to policy platforms. Since $x^L < 0 < x^R$, the assumption of no electoral advantage means that either both parties' platforms coincide with x^V (i.e., $x^L = x^R = 0$), or that platforms are equidistant from it ($x^L = -x^R$). The assumption will be relaxed in the next section, which analyses the equilibrium for any policy platforms such that $x^L < 0 < x^R$. While less general, this first result is valuable because it yields a simple closed-form solution for the equilibrium effort and intuitive comparative statics.

Proposition 1. Equilibrium without ex-ante Electoral Advantage. *Suppose $x^L = -x^R$. Then, the optimal incentives offered by L in equilibrium (and, symmetrically, by R) are $(\pi_d^{L*}, \pi_v^{L*}) = (0, 0)$ if*

$\gamma > 1$*, and* $(\pi_d^{L*}, \pi_v^{L*}) = (1-\alpha, \alpha)$ *if* $\gamma < 1$*. The unique level of campaigning effort for both factions is:*

$$e^{L*} = \begin{cases} \dfrac{2 + \psi\big(2\alpha - 1 + 2\big(x^L - x^R\big)^2\big)}{4} & \text{if} \quad \gamma > 1 \\ \dfrac{5 - \gamma + 2\psi\big(2\alpha - 1 + 2\big(x^L - x^R\big)^2\big)}{8} & \text{if} \quad \gamma < 1 \end{cases} \tag{11}$$

Proof. Unless otherwise stated, all proofs are collected in the Appendix. □

When sabotage is more effective than campaigning effort in achieving primacy within the party ($\gamma > 1$) the leader's optimal strategy is not to reward it, hence incentives are low powered — i.e., the equilibrium incentive scheme has zero premia. Conversely, when $\gamma < 1$ the equilibrium incentive scheme features high powered incentives. Intuitively, in this case the incentives to exert campaigning effort arise from both the election and the internal ranking indicator, while sabotage only hurts factional welfare. Hence, the party leader designs the internal contest such that the faction with the higher internal ranking is rewarded with the highest feasible premium (all the electoral spoils) — contingent on the electoral outcome — and the second faction is not rewarded. By doing so, the leader increases factions' incentives to mobilize. This result is in line with standard intuition from contest theory suggesting that the effort-maximizing incentive scheme is a winner-take-all contest — i.e., high powered incentives are optimal in contests where the probability of winning is increasing in effort (Nalebuff and Stiglitz, 1983).

The expression for the equilibrium campaigning effort e^{L*} in Proposition 1 allows us to directly check how factions' campaigning effort changes in equilibrium with (i) the amount of power sharing of the institutional setting $(1-\alpha)$, (ii) the importance of the electoral campaign (ψ), and (iii) parties' ideological extremism (x^L, x^R).

Corollary 1. Electoral Environment. *For all* γ*:*

(i) factions' campaigning effort (e^{L*}) *increases with the importance of the electoral campaign* (ψ)*,*

(ii) e^{L*} *decreases with the proportionality — or amount of power granted to minorities — of the institutional setting* $(1-\alpha)$*.*

The proof follows by inspection of the closed-form solution for equilibrium effort in Proposition 1. Intuitively, when the support of the aggregate shock gets smaller (higher ψ), the electoral outcome depends less on the random component and more on factional campaigning effort. As a consequence, factional effort is more effective in influencing the voter's decision. Perhaps less intuitively, factional campaigning effort in equilibrium is strictly increasing in α (or alternatively, factional sabotage is strictly decreasing in α): as the power granted to minority parties increases (as α goes down), in equilibrium factions invest more resources in sabotaging each other and less in mobilizing for the party. In the limit ($\alpha \to 1/2$), winning the election provides the same electoral spoils as losing, which leads factions to focus on the competition *within* the party.

The effect of α on campaigning suggests a simple yet neglected relation between inter-party and intra-party competition. Constitutional design scholars typically focus on the incentives that institutions produce at the party level (Lijphart, 1984; Powell Jr, 2000): majoritarian democracies are associated with adversarial fights for power, consensual democracies with bargaining and compromise across parties. Proposition 1 suggests that an institutional change in the electoral stakes can also affect competition within parties. Notice that a change in α might refer to a change in the electoral system (e.g., from winner-take-all to proportional), or to an institutional change holding fixed the electoral system's proportionality (e.g., from executive dominance to legislative-executive balance). As such, the result applies to two-party as well as multi-party systems.

A similar mechanism could also arise through an increase in polarization, for which I resort to the following working definition: polarization increases if x^L decreases and x^R increases by the same amount, thus holding $x^L + x^R$ constant.[21] This ensures that any increase in polarization does not change the identity of the (ex-ante) advantaged party, and allows us to focus exclusively on the level of divergence between party platforms ($x^L - x^R$). By increasing the stakes of the election, party polarization can motivate factions to campaign for the party.

Corollary 2. Polarization. *For all γ, factions' campaigning effort (e^{L*}) increases with ideological polarization: as the distance between x^L and x^R increases, factions invest more resources in campaigning effort and less in sabotaging activities.*

To see how polarization can affect factional behavior consider factions' payoff from losing the election in the following two cases. First, when $x^L = x^R = 0$, the ex-ante probability of victory for each party is the same and factions do not suffer any ideological cost from losing the election. When $x^L = -x^R$ and $x^L, x^R \neq 0$, parties' ex-ante winning probability does not change but now factions suffer a cost from losing the election — which is increasing in $x^L - x^R$. This increasing cost in turn implies that, as ideological polarization increases, factions invest more resources in campaigning effort (refraining from sabotaging each other), in order to avoid a costly unfavorable electoral outcome.

[21] Notice that this condition is trivially satisfied when no party has an ex-ante electoral advantage.

4.1. Introducing Electoral Imbalance

One question that arises when parties are heterogeneous in their ex-ante winning probability is which factions campaign more for the party between those in the leading and trailing party. To answer this question, I relax the assumption that party platforms are equidistant from the voter's preferred platform, thus allowing for ex-ante electoral imbalance in parties' electoral prospects.

The next result establishes the consequences of party ideological extremism on factional campaigning effort and parties' electoral prospect, for any platform x^L, x^R such that $x^L < 0 < x^R$. The first part of Proposition 2 generalizes the effect of polarization to the case of general platforms, showing that the comparative statics highlighted by Corollary 2 continue to hold.

The second part of the result focuses on the difference between total campaigning effort in *left* and *right* $(2e^{L*} - 2e^{R*})$. When platforms are not equidistant from the median, an increase in platforms' extremism affects campaigning effort in equilibrium through an additional channel: the change in the odds of winning the election. The result shows that, when $\gamma < 1$, an increase in a party's extremism — defined as the distance between the party platform and the voter's preferred platform — leads its factions to campaign less than the other party's factions. The next result assumes without loss of generality that the ex-ante winning probability of *left* is lower than *right*: $|x^L| > |x^R|$.[22]

Proposition 2. Ideological Extremism.

(i) For all γ, factional campaigning effort in both parties (e^{L*}, e^{R*}) increases with polarization.

(ii) When $\gamma < 1$, $\partial(2e^{L*} - 2e^{R*})/\partial|x^L| < 0$ for all x^R, and $\partial\big(2e^{R*} - 2e^{L*}\big)/\partial|x^R| < 0$ for all x^L. When $\gamma > 1$, factions in both parties campaign equally.

The intuition for the first part of Proposition 2 is analogous to that of Corollary 2: when polarization $(x^L - x^R)$ increases, factions' expected payoff from losing decreases because of the ideological loss they suffer. This in turn increases the marginal return from exerting campaigning effort, to avoid the unfavorable event of an electoral defeat.

Proposition 2(ii) shows that, when $\gamma < 1$, factions in the moderate party campaign more than those in the extreme party. Because $\gamma < 1$, in equilibrium incentives are high powered ($\pi_d^{L*} = \pi_d^{R*} = 1 - \alpha$, $\pi_v^{L*} = \pi_v^{R*} = \alpha$). Since $\alpha > 1/2$, the expected payoff from the election is lower for L_1, L_2: i.e.,

[22] In the Appendix I derive the vector of equilibrium effort choices as the unique solution (in closed form) of the system of factions' first-order conditions. The closed form solution is omitted from the main text as it does not provide further intuition than (11). The Appendix also shows that the equilibrium incentive scheme is the same as the one derived in Proposition 1, when factions are equidistant from x^V.

the *internal incentive* term in the faction's first-order condition is lower for L_1, L_2 as in equilibrium $p^L(\boldsymbol{e}^*) < 1/2$. As x^L moves away from x^V, the left party's expected payoff from the election decreases (via a lower winning probability), which induces its factions to campaign less for the party. Conversely, factions in the more moderate party campaign more in equilibrium, and the difference in parties' total effort is increasing in the extremism of the trailing party's platform. Hence, trailing parties are more likely to be hornets' nests, with factions investing in sabotage rather than working for the party.

Proposition 2(ii) also suggests that factional incentives to sabotage increase as the party weakens. This prediction offers an unexplored explanation of observed empirical patterns of intra-party competition. Golden and Chang (2001) identify political fights within the Italian DC party as one of the main causes of the corruption scandals involving the party deputies. Plausibly, the increased political competition resulting from the steady rise of the left and the associated loss of spoils faced by DC factions contributed to increasing factional sabotage: as the stakes of the election decreased, the appeal of securing internal power became more important to factions.[23] This explanation is consistent with the high levels of intra-party competition, and resulting corruption scandals, that doomed the DC party and contributed to end the Italian "First Republic".

Finally, when $\gamma > 1$ all the premia are set to zero in equilibrium, and $e^{L*} = e^{R*}$ regardless of the distance between policy platforms. With zero premia, the *internal incentive* term in the faction's first-order condition is equal to zero, and the incentive to campaing exclusively arises from the *external incentive*. The latter depends only on the difference between the two party platforms and is the same for factions in *left* and *right*. Thus, when $\gamma > 1$ and $|x^L| > |x^R|$ both parties' factions exert the same amount of campaigning in equilibrium — even though *left*'s ex-ante winning probability is lower than the one of *right* — and the expression $\partial\big(e^{L*} - e^{R*}\big)/\partial|x^L|$ in equilibrium is equal to zero.[24]

Finally, notice that belonging to an underdog party is always costly for factions. Let $\mathcal{W}^L$ be the welfare of factions in the left party, where

$$\mathcal{W}^L(x^L) = p^L\alpha + (1-p^L)\big[1-\alpha-2(x^L-x^R)^2\big] - \frac{e_1^2+e_2^2}{2} - \frac{(1-e_1)^2+(1-e_2)^2}{2}. \tag{12}$$

Remark 1. *Factions welfare is strictly decreasing in $|x^L|$.*

Ideological extremism hurts factional welfare via two channels: it decreases the probability of

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[23] In particular, the rise of the left was triggered by the Socialist Party becoming more moderate in the 1970s and 1980s.

[24] One extension in the Appendix shows that, when allowing for negative premia, the result $\partial\big(e^{L*} - e^{R*}\big)/\partial|x^L| < 0$ generalizes for all values of γ. By allowing the party to punish the faction ranking higher (rewarding the faction ranking lower) when $\gamma > 1$, negative premia create the same incentive to campaign as high powered incentives in the case $\gamma < 1$.

succeeding in the election and it increases the stakes of losing via a higher ideological cost.

So far I assumed that factions in the same party — having the same ideological preferences — suffer the same ideological cost for losing the election, and Proposition 2 shows how campaigning effort in equilibrium changes across parties with ideological distance. The next section relaxes this assumption and shows how campaigning effort changes as a function of each faction's ideological extremism.

5. Factions' Ideological Heterogeneity

This section relaxes the assumption of factions' homogeneous preferences within the party. Without loss of generality, let faction L_1 be more extreme than L_2 ($x_1^L < x_2^L < 0$). I assume that the policy platform implemented by a party corresponds to the simple average of its factions' ideological bliss points, i.e., $x^L = (x_1^L + x_2^L)/2$ (and symmetrically for x^R). Later in this section I relax this assumption and consider policy platforms as weighted averages of factions' bliss points, where the weights depend on factions' relative power.[25] This assumption is based on the empirical observation that, comparing factions' ideal points with the overall party position, factions bound the party in its platform choice (Ceron, 2012).

The next result illustrates the equilibrium when factions of the same party differ in ideology, showing how polarization *within* parties affects factional incentives to campaign for the party.

Proposition 3. Heterogeneous Factions. *In equilibrium, ideologically extreme factions campaign more than moderate ones, which instead devote more resources to sabotage. The equilibrium incentive scheme is analogous to the homogeneous case:* $(\pi_d^{L*}, \pi_v^{L*}) = (0, 0)$ *if* $\gamma > 1$, *and* $(\pi_d^{L*}, \pi_v^{L*}) = (1 - \alpha, \alpha)$ *if* $\gamma < 1$.

Proposition 3 demonstrates that when factions in the same party do not share the same ideological position, the extreme faction invests more in the electoral campaign than the moderate one, which sabotages more instead. Intuitively, the moderate faction L_2 is ideologically closer to *right*'s bliss point than the extreme one L_1, thus suffering a lower cost for losing the election. As a consequence, L_2 invests more resources into sabotaging the rival faction.

This result underscores the different effect that ideological polarization produces *within* and *across* parties. Proposition 2 has shown that, when parties are polarized, factions in the more extreme party sabotage more than factions in the leading, moderate party. In contrast, Proposition 3 shows that, when *factions* within the same party are polarized, the more extreme faction invests more in campaigning

[25] In this extension weights are decided by the party leader, who can reward the higher ranking faction with "policy concessions" (granting more influence in the party platform).

than the moderate faction. Intuitively, as a faction becomes more extreme the expected payoff of both factions decreases (via a lower winning probability), leading to a reduction in campaigning by both factions. Yet, campaigning decreases asymmetrically: the extreme faction campaigns more because the stakes of the election are greater, suffering a higher ideological loss from losing.

This logic is consistent with the different behavior of the extreme and moderate factions of the Labour Party during the 2017 UK electoral campaign. There is evidence that the Labour Left "largely relied on positive campaigning and mobilized grassroots activism to an extent rarely seen before, ensuring that it inspired new voters" (Bell, 2018). For example, the grassroots movement "Momentum" helped the Labour Party win 32 new seats in the 2017 election, even supporting moderate candidates.[26] Conversely, moderate Labour MPs extensively engaged in sabotage against Corbyn's campaign, as recently described in an internal report of the Labour Party.[27]

The reader might wonder why the leader would commit to using the ranking indicator at all in this case, given that she would be strictly better off rewarding the faction which exerted more effort (the more extreme faction, in equilibrium). That is, L would want to design a non-anonymous contract that punishes sabotage more for the moderate faction. However, non-anonymous incentive schemes are typically not feasible: this assumption is not realistic in all those cases where leaders are constrained by party legal rules and formal procedures, which are the same for all factions and are decided ex-ante. Yet, if the leader could decide over the party policy platform, she would be able to reward the moderate faction through a policy concession, that is, setting the party platform closer to the moderate faction's preferred position. The next section analyses this possibility, showing that leaders could tailor policies to factional preferences to increase the party's electoral chances.

5.1. Policy Concessions as Incentives to Factions

How can parties tailor policies to factional preferences in order to increase electoral chances? Typically, party manifestos weigh factions' preferred platforms based on their share of votes gained during congresses (Levy, 2004; Ceron, 2012; Lo, Proksch and Slapin, 2016; Dewan and Squintani, 2016). The following extension adds this feature to the model.

While the baseline model assumes that both factions have the same weight in determining the party platform, this section endogenizes the weight of each faction, asking which policy weight the party

[26] See Lott, R. (2019) 'Inside Momentum, Labour's Secret Weapon', Vice, 18 November.

[27] The report describes a "hyper-factional atmosphere" where more right-wing senior Labour staff actively seek to sabotage the work of those on the party's left, referring to them contemptuously as "Trots" (2020 Labour Antisemitism Report, Section 2.1.3.i).

adopts in equilibrium. Let the party leader L choose — in addition to premia — how much to weigh the preferred policy of the faction ranking higher, where the policy weight is denoted by $\lambda \in [1/2, 1]$. Formally, a strategy for L is now defined by an incentive scheme (π_d^L, π_v^L) and a policy weight (λ).

The timing of the game remains unchanged: first, L announces an incentive scheme $(\pi_d^L, \pi_v^L, \lambda)$. Second, factions decide how much resources to invest in campaigning and sabotage. Finally, elections are held, and premia (π_d^L, π_v^L) as well as policy concessions (determined by λ) are distributed. Importantly, policy concessions are meted out once the internal ranking is revealed, which happens *after* the electoral outcome is known. This assumption implies that λ affects factions' decision only through their expected reward.[28] To see how, it is convenient to express faction L_1's expected reward *from electoral victory* as

$$R_1^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L, \lambda)|_v = b_v^L + \rho_1^L \pi_v^L - \;\; x_1^L - x^L(\lambda, \boldsymbol{e^L})\big)^2, \tag{13}$$

where

$$x^L(\lambda, \boldsymbol{e^L}) = \begin{cases} \lambda x_1^L + (1-\lambda) x_2^L & \text{if} \quad s^L(\boldsymbol{e^L}) = 1 \\ (1-\lambda) x_1^L + \lambda x_2^L & \text{if} \quad s^L(\boldsymbol{e^L}) = 2. \end{cases} \tag{14}$$

That is, the policy incentive consists in a lower policy cost of ranking higher (and a higher cost of ranking lower), conditional on winning the election. Conversely, L_1's expected reward *from losing the election* does not depend on λ — and is therefore equivalent to the baseline model (5) — because the implemented policy platform in the event of electoral defeat is chosen by *right*.

Recall from Proposition 3 that the ideologically extreme faction campaigns more than the moderate one in equilibrium, as the latter suffers a lower ideological cost from losing the election. This difference is crucial for the next result, which shows that under certain conditions the leader might reward sabotage with policy concessions.

Proposition 4. Policy Concessions. *Let $|x_1^L - x_2^L| > 0$. When $\gamma > 1$, in equilibrium the leader rewards sabotage contingent on electoral victory by setting $\lambda^* = 1$, and premia are $(\pi_d^{L*}, \pi_v^{L*}) = (0, 0)$. When $\gamma < 1$, the equilibrium premia are $(\pi_d^{L*}, \pi_v^{L*}) = (1-\alpha, \alpha)$, and there exists d' such that if $|x_1^L - x_2^L| < d'$, then $\lambda^* = 1$; if $|x_1^L - x_2^L| \geq d'$, then $\lambda^* = 1/2$.*

Proposition 4 states that, when $\gamma > 1$, L rewards the strongest faction by setting the party platform equal to the faction's preferred policy. When sabotage is more effective than campaigning to achieve

[28] The fact that policy concessions can only be post-electoral implies that λ affects the probability of electoral victory only indirectly, through factions' campaigning effort. That is, the voter compares the same party platforms — i.e., simple averages of the factional platforms — as in the baseline model. To study the case of policy concessions directly affecting the voter decision it would be necessary to analyze a repeated-game framework, which is outside the scope of this paper.

a higher internal ranking, the faction that ranks higher is the one that (probabilistically) sabotages more (Proposition 3). In this case, setting a positive premium corresponds to rewarding sabotage, and the equilibrium premia (π_d^{L*}, π_v^{L*}) are set to zero as in the baseline model. In equilibrium, the leader promises a policy concession *contingent on victory* to the faction for which the ranking is higher, by setting $\lambda^* = 1$. This motivates the moderate faction L_2 — which in equilibrium is more likely to obtain the policy concession — to campaign more for the party and less against the other faction.

To understand why this is the case, it is key to note that, when the relative ranking indicator rewards sabotage, the more extreme faction campaigns more in equilibrium ($e_1^{L*} > e_2^{L*}$). This in turn implies that $\rho_2^L > \rho_1^L$. That is, the internal contest among factions is not a coin flip anymore: the moderate faction L_2 has more chances to win the premium than L_1. In this case, a policy concession incentivizes L_2's equilibrium effort, to increase the party's chances of victory. Crucially, the extreme faction's effort is always greater than the moderate one in equilibrium. This implies that, even if e_2^{L*} increases under λ^*, $\rho_2^L > \rho_1^L$ and L_2 still ranks higher in equilibrium.

When $\gamma < 1$, the extreme faction (which campaigns more in equilibrium) is more likely to rank higher than the moderate one, as campaigning is more effective than sabotage. In this case, a high λ incentivizes factions to campaign in order to rank higher thus moving the party platform closer to their bliss point. This clearly helps the party win the election via higher campaigning. Indeed, when the ideological distance between L_1 and L_2 is low enough, the leader sets $\lambda^* = 1$ to maximize total effort.

Suppose now that $|x_1^L - x_2^L|$ is high enough: in this scenario, a high λ reduces the appeal of electoral victory to the moderate L_2 by shifting the party platform to the extreme x_1^L. When the distance between factions' bliss point is high enough, the loss from the moderate faction's sabotage outweighs the gain in campaigning of the extreme faction, and the leader sets low powered incentives, choosing not to reward any faction with a policy concession.

Finally, Proposition 4 suggests how the equilibrium incentive scheme would vary if the leader had ideological preferences. Suppose that L shares the same ideological preferences of the extreme faction L_1, and suppose that $\gamma > 1$. In this case, the leader would trade-off a lower ideological cost by decreasing λ — because in equilibrium the moderate L_2 is more likely to rank higher — and a higher probability of victory by increasing λ, via higher total effort. Thus, by allowing leaders to share ideological preferences with factions, the forces highlighted in Proposition 4 would still be at work, but the leader would have to weigh the incentive to increase the probability of victory of the party with her ideological cost of rewarding with policy concessions a platform distant from her own bliss point.

6. Discussion and Empirical Implications

This section discusses the empirical implications of the model's findings for the study of factions and party organizations. I first analyze the implications of institutional minority rights' protection and polarization on factional behavior. I then turn to the implications for the internal organization parties should adopt. Finally, I discuss potential operationalizations of sabotage.

Institutional System and Sabotage. Corollary 1 predicts that intra-party sabotage should increase as the system of government tends to a consensus democracy, granting more power to losing parties. As minority parties obtain a higher share of electoral spoils, factions become more incentivized to sabotage each other (rather than investing their resources to promote the party in the general election) in order to obtain a higher share of the spoils. Hence, we should expect sabotage to be empirically associated with parties' representation in government, with whether seats are reserved for small parties and how easy it is to start a new party, with electoral thresholds for parliamentary representation, with the electoral system (proportional vs. majoritarian), and other institutional constraints such as whether the system is unicameral vs. bicameral, or centralized vs. federalist (Lijphart, 1984).

Polarization and Electoral Security. Proposition 2 suggests that an increase in party polarization leads to less sabotage, as factions face a higher ideological cost from losing the election. Moreover, when parties have different ex-ante electoral chances, intra-party sabotage should be more pervasive in trailing parties. While polarization increases campaigning effort for both parties' factions via an increased cost of losing the election, the amount of campaigning in equilibrium varies with parties' electoral security. In particular, campaigning is less valuable to factions in trailing parties, who are less likely to win the election. Hence, we should expect factions in trailing parties to sabotage more.

Variation in Sabotage Relative Effectiveness. The model shows that the party choice of incentives changes with γ, the relative effectiveness of sabotage to achieve a higher ranking within the party. The parameter γ can refer to institutional and non-institutional features of the environment that make factional campaigning effort harder to reward by the party: the electoral system (e.g., list flexibility) is an example of the former, factional geographical dispersion of the latter.

An increase in factional *geographic dispersion* (e.g., a shift from factions' geographical separation to their overlap) could be represented in the model by an increase in γ. If factions are associated with geographic strongholds, the party can attribute its vote share in a given district to the local faction's campaigning activities, and consider it when designing the incentive scheme. Ceteris paribus, a change to factions that overlap geographically corresponds to an increase in sabotage's relative effectiveness, as it becomes harder to associate campaigning effort to each region's vote share.

The parameter γ can also capture features of the electoral system, such as the use of preference votes. There is considerable variation in *list flexibility* among proportional representation systems, with a majority of countries adopting a closed list system. A shift from closed to open list — which can be represented in the model as a decrease in γ — could make sabotage less effective than campaigning. This happens when candidates mainly obtain preference votes by campaigning effort rather than by sabotaging activities. Intuitively, the party observes its candidates' preference votes, and knows each candidate's faction. The party equilibrium incentive scheme can then move from low-powered to high-powered incentives — rewarding factions for their preference votes — reducing factions' equilibrium investment in sabotage as a result.

The literature on personal vote suggests that moving from a closed list to an open list PR system should increase competition among candidates in a party: looking at the individual candidates' incentives, an open list system implies the need to obtain preference votes, often fighting rivals within the party (Carey and Shugart, 1995; Bräuninger, Brunner and Däubler, 2012). While the incentive to compete within the party increases at the *individual candidate* level, the model shows that the incentive to sabotage could be reduced at the *faction* level. Hence, by considering factions instead of individual candidates as unit of analysis, empirical scholars might uncover potential omitted variable bias in the correlation between weak parties and open list PR systems, controlling for a novel moderating variable — that is, the equilibrium party organization which affects factions' decision to sabotage.

Finally, observed party organizations are consistent with the model's implications. The method of allocation of cabinet positions in the LDP (before the electoral reform in 1994) was to divide the electoral spoils proportionally among factions and give a premium to the "mainstream faction" (Browne and Kim, 2003; Ramseyer and Rosenbluth, 2009). This method corresponds in the model to high powered incentives, which are optimal when effort is more effective than sabotage. Similarly to preference votes, the Japanese SNTV electoral system allowed the party to condition the faction's reward on the the elected party members' identity — the mainstream faction obtaining the premium was, in fact, the faction with the most winning candidates.

Empirical Challenge: Measuring Intra-Party Sabotage. Existing empirical research has focused on negative campaigning against opposing parties, but rarely on measuring intra-party dissent. One implication that can be derived from the model is the emergence of political scandals triggered internally as a product of factional sabotage. Besides resulting from the opposition's attacks (Dziuda and Howell, 2020), political scandals can emerge due to intra-party competition, where party insiders leak information on co-partisans' misdeeds in order to gain power within the party (Balán, 2011). If factional competition can trigger the outbreak of scandals, then Proposition 2 suggests that these are more likely to emerge when a party weakens electorally.

A possible way to measure political scandals is to consider charges of malfeasance against parliament members, often resulting in corruption scandals. In most democracies, before proceeding with a judicial investigation of a legislator, public prosecutors need to ask official permission from the legislative body to lift the immunity of the involved deputy. For instance, in Italy, these requests to proceed — *richieste di autorizzazione a procedere (RAP)* — are sent to the Chamber of Deputies, and from 1948 the Italian judiciary made more than 5000 requests to parliament to proceed with MPs' investigation.

Golden and Chang (2001) find that the number of RAPs against DC deputies is positively related to intra-party competition, proxied by the number of preference votes received by DC candidates in a district and divided by the total number of list votes received by the party in the same district. Furthermore, the public availability of the requests allows to delve deeper into the political motives of the investigations. Indeed, several requests include the "leaker" identity, who is often another politician. Analyzing RAPs from 1983 to 2019, Invernizzi and Ceron (2020) identify the leaker's political affiliation, and provide evidence of a political use of denunciations. The paper shows that when a party weakens, the likelihood that political enemies denounce past misbehavior of members of the weakened party increases, suggesting that the political use of denunciation is elastic to changes in the electoral odds. In some cases, they show, these political enemies belong to the same party — but to a different faction — of the accused MPs. These findings provide further evidence in favor of the hypothesis that factional sabotage should increase as parties weaken.

7. Conclusion

In their efforts to win office, political parties strategically change their internal organization. One potent tool used by parties is the allocation of electoral spoils among party members, who typically form factions to achieve their policy positions. This paper captures with a formal model the relation of agency among the party leadership and factions. The model formalizes with contract theory tools the allocation of electoral spoils among competing factions. This agency framework is embedded in a general equilibrium model of elections, which allows studying how electoral stakes affect intra-party competition.

The baseline model shows that factions' contests over electoral spoils can be positive or destructive depending on several features of the competitive environment. First, as the power granted to minority parties increases, factions invest more resources into sabotaging each other and less in mobilizing for the party. Conversely, when the stakes of the election increase — via polarization or institutional changes — factions invest more in campaigning for the party. This finding improves our understanding of alternative democratic systems by highlighting the often neglected effect of different electoral institutions on intra-party competition.

The model also shows the effect of ideological polarization on intra-party competition. As polarization increases, factions in both parties campaign more for the party to avoid a costly electoral defeat. While factions in the moderate party campaign more, those in the more extreme party engage in sabotage. Thus, the model suggests that — in the presence of electoral imbalance — intra-party competition should be more severe in trailing parties and when parties weaken electorally. The latter result is consistent with empirical evidence of political use of denunciation against Italian MPs belonging to weakened parties.

Anticipating factions' incentives, the party can limit sabotage by rewarding factions for their campaigning effort. When factions' campaigning effort can be monitored and rewarded easily, the party encourages competition among factions through a winner-take-all contest for electoral spoils. When, on the other hand, sabotage is more effective than campaigning effort to achieve internal power, the party distributes electoral spoils among factions in an egalitarian way to discourage destructive competition. An extension endogenizes party platforms as part of the leader strategy, showing that the leader might want to reward moderate factions with policy concession to increase the party electoral chances in the latter case.

The model shows how incentives change when parties have to cope with imperfect signals of effort, in the presence of sabotaging activities. The same approach can be extended to compare the efficiency of different incentive schemes within political parties. In particular, the analysis of a proportional contest function — that is, a proportional allocation of electoral spoils relative to each faction's own performance rather than factions' relative performance — is perhaps one of the most promising research avenues that emerge from this model's findings. This would shed light on the question of which is the optimal party structure to win elections.

Part

Appendix

Table of Contents

A. Preliminaries

A.1. Faction's Problem

Faction L_1's problem is:

$$\max_{e_1^L \in [0,1]} \quad p^L \left[b_v^L + \rho_1^L \pi_v^L \right] + (1 - p^L) \left[b_d^L + \rho_1^L \pi_d^L - (x^L - x^R)^2 \right] - \frac{e_1^L)^2}{2} - \frac{1 - e_1^L)^2}{2}, \tag{15}$$

where $\rho_1^L = (1 - \gamma)(e_1^L - e_2^L)/2$ and $p^L = \frac{1}{2} + \psi[-(x^L)^2 + (x^R)^2 + e_1^L + e_2^L - e_1^R - e_2^R]$.

All the other factions solve their respective maximization problem. The first-order condition associated to L_1 is:

$$2e_1^L = 1 + \psi\Big[(b_v^L - b_d^L) + \Big(\frac{1}{2} + (1-\gamma)\frac{e_1^L - e_2^L}{2}\Big)(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\Big] + \\ \Big(\frac{1-\gamma}{4}\Big)(\pi_v^L - \pi_d^L) + \psi\Big(\frac{1-\gamma}{2}\Big)\Big[-(x^L)^2 + (x^R)^2 + e_1^L + e_2^L - e_1^R - e_2^R\Big], \tag{16}$$

and likewise for L_2, R_1 and R_2.

First, notice that the factions' objective function is concave when the support of the shock is wide enough, i.e., $\psi < 2/\alpha$. The second-order condition can be expressed as

$$\psi(1-\gamma)(\pi_v^L - \pi_d^L) - 2,$$

which implies that the first-order conditions of the factions' problem identify a maximum when $\psi < 2/\alpha$.

Lemma 1. *There exists an equilibrium, i.e., a solution to the system of first-order conditions of each faction. The equilibrium is unique.*

Proof. Solving the system of four first-order conditions yields a unique closed-form solution for effort exerted by factions in both parties. The solution is symmetric for factions in the same party, i.e., $e_1^{L*} = e_2^{L*} = e^{L*}$, and equal to

$$e^{L*} = \frac{\big[(\gamma-1)(\pi_d^R - \pi_v^R)\psi - 2\big]\big[2 + (2\alpha-1)\psi + 2\psi(x^L - x^R)^2\big] + (\gamma-1)(\pi_v^L + \pi_d^L) - \theta(\pi_v^R)\pi_v^L + \theta(\pi_d^R)\pi_d^L}{4\big[-2 + \psi(\gamma-1)(\pi_d^R + \pi_d^L - \pi_v^R - \pi_v^L)\big]},$$

$$e^{R*} = \frac{\big[(\gamma-1)(\pi_d^L - \pi_v^L)\psi - 2\big]\big[2 + (2\alpha-1)\psi + 2\psi(x^L - x^R)^2\big] + (\gamma-1)(\pi_v^R + \pi_d^R) - \theta(\pi_v^L)\pi_v^R + \theta(\pi_d^L)\pi_d^R}{4\big[-2 + \psi(\gamma-1)(\pi_d^L + \pi_d^R - \pi_v^L - \pi_v^R)\big]},$$

where $\theta(\pi)$, for $\pi = \{\pi_d^L, \pi_v^L, \pi_d^R, \pi_v^R\}$, equals

$$\theta(\pi) = \psi\big[\pi(1-\gamma) + 2 + 2[(x^L)^2 - (x^R)^2] + \psi(1-2\alpha) + 2\psi(x^L - x^R)^2\big],$$

and has the following properties:

- If $\gamma < 1$ ($\gamma > 1$), $\theta(\pi)$ is increasing (decreasing) in π,
- If $(x^L)^2 > (x^R)^2$ (i.e., if party L is electorally disadvantaged) and $\gamma < 1$, $\theta(\pi)$ is positive.

□

Claim 1 (**Interior Effort.**). *The following condition ensures that* $e^{L*} \in (0,1)$*:*

$$\alpha + (x^L - x^R)^2 < \frac{1}{2\psi} \tag{17}$$

Proof. (I) Condition for $e^{L*} < 1$**.**

Using the expression for the faction's first-order condition (16), we can evaluate when the first-order condition evaluated at $e^{L*} = 1$ is negative:

$$\psi\Big[(b_v^L - b_d^L) + \frac{1 + (1-\gamma)(1-e_2^L)}{2}(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\Big] + \frac{1-\gamma}{4}(\pi_v^L + \pi_d^L)$$
$$+ \psi\Big(\frac{1-\gamma}{2}\Big)\Big[-(x^L)^2 + (x^R)^2 + 1 + e_2^L - e_1^R - e_2^R\Big] < 1,$$

for which a sufficient condition is

$$\alpha + (x^L - x^R)^2 < \frac{1}{2\psi}. \tag{18}$$

(II) Condition for $e^{L*} > 0$ **.** Using the expression for the faction's first-order condition (16) we can evaluate when the first-order condition evaluated at $e^{L*} = 0$ is positive:

$$\psi\Big[(b_v^L - b_d^L) + \frac{1 + (1-\gamma)(-e_2^L)}{2}(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\Big] + \frac{1-\gamma}{4}(\pi_v^L + \pi_d^L)$$
$$+ \psi\Big(\frac{1-\gamma}{2}\Big)\Big[-(x^L)^2 + (x^R)^2 + e_2^L - e_1^R - e_2^R\Big] > 0,$$

where the LHS is greater than

$$\psi\Big(\frac{\alpha - 1}{2}\Big) + 1.$$

The threshold (19) implies that the following is a sufficient condition for campaigning effort to be positive in equilibrium:

$$(1-\alpha) < \frac{2}{\psi}. \tag{19}$$

Since effort is continuous, the conditions identified in (18) and (19) ensure that the unique level of effort exerted by factions in equilibrium must be interior. □

A.2. Party Leader's Problem

Each party leader maximizes the probability of winning the election with respect to the party's two premia. By Lemma 1, we know that when L_1, L_2 face a symmetric problem, in equilibrium

$e_1^{L*} = e_2^{L*} = e^{L*}$. This allows us to re-write L's objective as:

$$\max_{\pi_d^L,\pi_v^L} \quad u^L(\pi_d^L,\pi_v^L) = \frac{1}{2} + \psi\big[2e^{L*}\ \ \pi_d^L,\pi_v^L\big) - 2e^{R*}\ \ \pi_d^L,\pi_v^L\big) - (x^L)^2 + (x^R)^2\big]. \tag{20}$$

Since party P's premia affect the probability of winning only through the effort level of party P's factions as well as party Q's factions, we can re-express the leader's objective (20) as the maximization of the following transformed payoff:

$$\max_{\pi_d^L,\pi_v^L} \quad \tilde{u}^L(\pi_d^L,\pi_v^L) = 2e^{L*}\ \ \pi_d^L,\pi_v^L\big) - 2e^{R*}\ \ \pi_d^L,\pi_v^L\big).$$

Notice that the game between the two party leaders is a zero-sum game, as each leader wins what the other party loses:

$$\tilde{u}^L(\pi^L,\pi^R) + \tilde{u}^R(\pi^L,\pi^R) = 2e^{L*} - 2e^{R*} + \ \ 2e^{R*} - 2e^{L*}\big) = 0. \tag{21}$$

Given Equation 21, we can define the payoff function of this zero-sum game as $\tilde{u}^L(\pi^L,\pi^R) = u$, with $\tilde{u}^R(\pi^L,\pi^R) = -u$.

Party L's leader maxmin value is given by

$$\underline{v}^L = \max_{\{\pi_d^L,\pi_v^L\}} \quad \min_{\{\pi_d^R,\pi_v^R\}} \tilde{u}^L(\pi_d^L,\pi_v^L,\pi_d^R,\pi_v^R) \tag{22}$$

and party R's leader maxmin value is given by

$$\underline{v}^R = \max_{\{\pi_d^R,\pi_v^R\}} \quad \min_{\{\pi_d^L,\pi_v^L\}} -\tilde{u}^L(\pi_d^L,\pi_v^L,\pi_d^R,\pi_v^R) = \min_{\{\pi_d^R,\pi_v^R\}} \quad \max_{\{\pi_d^L,\pi_v^L\}} \tilde{u}^L(\pi_d^L,\pi_v^L,\pi_d^R,\pi_v^R).$$

Substituting the values of equilibrium efforts into (21), we can express the payoff function of the game in closed-form as follows:

$$u = \frac{(\gamma-1)\Big[\pi_d^L - \pi_d^R + \pi_v^L - \pi_v^R + 2\psi\ \ \pi_d^L + \pi_d^R - \pi_v^L - \pi_v^R\big)\big(x_L^2 - x_R^2\big)\Big]}{-4 + 2(\gamma-1)\psi\ \ \pi_d^L + \pi_d^R - \pi_v^L - \pi_v^R\big)}, \tag{23}$$

where $\tilde{u}^L(\pi^L,\pi^R) = u$, and $\tilde{u}^R(\pi^L,\pi^R) = -u$. Hence, L (R) maximizes (minimizes) the payoff function (23) with respect to $\boldsymbol{\pi^L}$ ($\boldsymbol{\pi^R}$).

Lemma 2. *The payoff function of the game u is*

- *increasing in π_v^L for $\gamma < 1$*

- *decreasing in π_v^L for $\gamma > 1$*

Proof. Recall that the payoff function of the zero-sum game among party leaders is defined by $u = 2e^{L*} - 2e^{R*}$. By symmetry across factions in the same party, we can rewrite the first-order condition of L_1, L_2 as

$$2e = 1 + \psi\ \ b_v^L + \rho_1^L \pi_v^L) + p^L\Big(\frac{1-\gamma}{2}\Big)\pi_v^L - \psi\ \ b_d^L + \rho_1^L \pi_d^L) + (1-p^L)\Big(\frac{1-\gamma}{2}\Big)\pi_d^L \tag{24}$$

Taking the partial derivative of the faction's equilibrium effort (24) with respect to π_v^L yields

$$\frac{\partial 2e^{L*}}{\partial \pi_v^L} = \Big(\frac{1-\gamma}{2}\Big)\Big[p^L + \psi\Big(\frac{\partial 2e^{L*}}{\partial \pi_v^L} - \frac{\partial 2e^{R*}}{\partial \pi_v^L}\Big)\ \ \pi_v^L - \pi_d^L)\Big] + \psi\Big[-\frac{1}{2} + \rho_1^L\Big], \tag{25}$$

which follows from the budget constraint assumption on b_v^L, the fact that $\partial p^L/\partial e^{L*} = 2\psi$ and that $\partial \rho_1^L/\partial \pi_v^L = 0$ in equilibrium. Define $\mathcal{X}_1$ as

$$\mathcal{X}_1 := 2\Big(\frac{\partial e^{L*}}{\partial \pi_v^L} - \frac{\partial e^{R*}}{\partial \pi_v^L}\Big),$$

and notice that in equilibrium $\rho_1^L = 1/2$. Then, the partial derivative of effort with respect to π_v^L (25) can be simplified to

$$\frac{\partial 2e^{L*}}{\partial \pi_v^L} = \Big(\frac{1-\gamma}{2}\Big)\Big[p^L + \psi\mathcal{X}_1\ \ \pi_v^L - \pi_d^L)\Big], \tag{26}$$

where $2e^{L*}$ is the first component of L's payoff function. Next, we need to characterize $\partial 2e^{R*}/\partial \pi_v^L$. To do so, observe that

$$\frac{\partial 2e^{L*}}{\partial \pi_v^R} = \Big(\frac{1-\gamma}{2}\Big)\Big[\psi\Big(\frac{\partial 2e^{L*}}{\partial \pi_v^R} - \frac{\partial 2e^{R*}}{\partial \pi_v^R}\Big)\ \ \pi_v^L - \pi_d^L)\Big].$$

Hence, by symmetry

$$\frac{\partial 2e^{R*}}{\partial \pi_v^L} = \Big(\frac{1-\gamma}{2}\Big)\Big[\psi\Big(\frac{\partial 2e^{R*}}{\partial \pi_v^L} - \frac{\partial 2e^{L*}}{\partial \pi_v^L}\Big)\ \ \pi_v^R - \pi_d^R)\Big],$$

which is equivalent to

$$\frac{\partial 2e^{R*}}{\partial \pi_v^L} = \Big(\frac{1-\gamma}{2}\Big)\psi(-\mathcal{X}_1)\ \ \pi_v^R - \pi_d^R). \tag{27}$$

Using (26) and (27) we can re-express $2\Big(\frac{\partial e^{L*}}{\partial \pi_v^L} - \frac{\partial e^{R*}}{\partial \pi_v^L}\Big)$ as

$$\mathcal{X}_1 = \Big(\frac{1-\gamma}{2}\Big)p^L + \Big(\frac{1-\gamma}{2}\Big)\psi\mathcal{X}_1\ \ \pi_v^L - \pi_d^L) + \Big(\frac{1-\gamma}{2}\Big)\psi\mathcal{X}_1\ \ \pi_v^R - \pi_d^R),$$

which rearranged yields

$$\mathcal{X}_1\Big[1-\frac{1-\gamma}{2}\psi\ \ \pi_v^L-\pi_d^L+\pi_v^R-\pi_d^R)\Big]=\Big(\frac{1-\gamma}{2}\Big)p^L.$$

Substituting back $\mathcal{X}_1=2\Big(\frac{\partial e^{L*}}{\partial\pi_v^L}-\frac{\partial e^{R*}}{\partial\pi_v^L}\Big)$ allows us to evaluate how the payoff function changes with π_1:

$$2\Big(\frac{\partial e^{L*}}{\partial\pi_v^L}-\frac{\partial e^{R*}}{\partial\pi_v^L}\Big)=\frac{\Big(\frac{1-\gamma}{2}\Big)p^L}{2-(1-\gamma)\psi\ \ \pi_v^L-\pi_d^L+\pi_v^R-\pi_d^R)}.$$

Notice that $\ \pi_v^L-\pi_d^L+\pi_v^R-\pi_d^R)\in[0,2\alpha]\subset[0,2]$. Hence the denominator is positive either when $\gamma>1$, or for $\gamma<1$ and ψ small enough. When this is the case, the sign of $\mathcal{X}_1$ is driven by the numerator, which is positive when $\gamma<1$ and negative when $\gamma>1$, which completes the proof. □

Lemma 3. *The payoff function of the game u is*

- *increasing in π_d^L for $\gamma<1$*
- *decreasing in π_d^L for $\gamma>1$*

Proof. Taking the partial derivative of the faction's equilibrium effort (24) with respect to π_d^L yields

$$\frac{\partial 2e^{L*}}{\partial\pi_d^L}=\Big(\frac{1-\gamma}{2}\Big)\Big[\psi\Big(\frac{\partial 2e^{L*}}{\partial\pi_v^L}-\frac{\partial 2e^{R*}}{\partial\pi_v^L}\Big)\ \ \pi_v^L-\pi_d^L)+(1-p^L)\Big]+\psi\Big[-\frac{1}{2}+\rho_1^L\Big],\tag{28}$$

Let $\mathcal{X}_0:=2\Big(\frac{\partial e^{L*}}{\partial\pi_d^L}-\frac{\partial e^{R*}}{\partial\pi_d^L}\Big)$. Then, (28) simplifies to

$$\frac{\partial 2e^{L*}}{\partial\pi_d^L}=\Big(\frac{1-\gamma}{2}\Big)\Big[2\psi\mathcal{X}_0\ \ \pi_v^L-\pi_d^L)+1-p^L\Big]-\psi,\tag{29}$$

which is the first component of the payoff function.

Next, we need to characterize $\frac{\partial 2e^{R*}}{\partial\pi_d^L}$. To do so, observe that

$$\frac{\partial 2e^{L*}}{\partial\pi_d^R}=\Big(\frac{1-\gamma}{2}\Big)\Big[\psi\Big(\frac{\partial 2e^{L*}}{\partial\pi_d^R}-\frac{\partial 2e^{R*}}{\partial\pi_d^R}\Big)\ \ \pi_v^L-\pi_d^L)\Big].$$

Hence, by symmetry

$$\frac{\partial 2e^{R*}}{\partial\pi_d^L}=\Big(\frac{1-\gamma}{2}\Big)\Big[\psi\Big(\frac{\partial 2e^{R*}}{\partial\pi_d^L}-\frac{\partial 2e^{L*}}{\partial\pi_d^L}\Big)\ \ \pi_v^R-\pi_d^R)\Big],$$

which is equivalent to

$$\frac{\partial 2e^{R*}}{\partial \pi_v^L} = \Big(\frac{1-\gamma}{2}\Big)\psi(-\mathcal{X}_0)\ \ \pi_v^R - \pi_d^R\big). \tag{30}$$

Using (29) and (30) we can re-express $2\Big(\frac{\partial e^{L*}}{\partial \pi_d^L} - \frac{\partial e^{R*}}{\partial \pi_d^L}\Big)$ as

$$\mathcal{X}_0 = \Big(\frac{1-\gamma}{2}\Big)\psi\mathcal{X}_0\ \ \pi_v^L - \pi_d^L\big) + \Big(\frac{1-\gamma}{2}\Big)(1-p^L) - \psi + \Big(\frac{1-\gamma}{2}\Big)\psi\mathcal{X}_0\ \ \pi_v^R - \pi_d^R\big),$$

which rearranged yields

$$\mathcal{X}_0\Big[1 - \frac{1-\gamma}{2}\psi\ \ \pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R\big)\Big] = \Big(\frac{1-\gamma}{2}\Big)(1-p^L) - \psi.$$

Substituting back $\mathcal{X}_0 = 2\Big(\frac{\partial e^{L*}}{\partial \pi_d^L} - \frac{\partial e^{R*}}{\partial \pi_d^L}\Big)$ allows us to evaluate how the payoff function changes with π_d:

$$2\Big(\frac{\partial e^{L*}}{\partial \pi_d^L} - \frac{\partial e^{R*}}{\partial \pi_d^L}\Big) = \frac{\Big(\frac{1-\gamma}{2}\Big)(1-p^L) - \psi}{2 - (1-\gamma)\psi\ \ \pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R\big)}.$$

As in the previous case, the denominator is positive either when $\gamma > 1$, or for $\gamma < 1$ and ψ small enough. When this is the case, the sign of $\mathcal{X}_0$ is driven by the numerator, which is positive when $\gamma < 1$ (for ψ small enough) and negative when $\gamma > 1$, which completes the proof. □

B. Main Results

B.1. Proof of Proposition 1

Proof. To derive the equilibrium incentive scheme, notice that by Lemma 2 and Lemma 3 the following is true when $\gamma < 1$:

$$\frac{\partial e^{L*}}{\partial \pi_d^L} - \frac{\partial e^{R*}}{\partial \pi_d^L} > 0$$
$$\frac{\partial e^{L*}}{\partial \pi_v^L} - \frac{\partial e^{R*}}{\partial \pi_v^L} > 0.$$

Thus, in equilibrium L sets $\pi_d^{L*} = (1-\alpha)$, $\pi_v^{L*} = \alpha$. When $\gamma > 1$, the inequality is reversed, therefore the optimal premia are $\pi_d^{L*} = \pi_v^{L*} = 0$.

Given the payoff of the game (23), the right party faces a problem that is symmetric to L's one, i.e., $\max_{\{\pi_d^R, \pi_v^R\}}\{-u\}$. Therefore, R sets in equilibrium $\pi_d^R = (1-\alpha)$, $\pi_v^R = \alpha$ when $\gamma < 1$, and $\pi_d^R = \pi_v^R = 0$ when $\gamma > 1$.

When no party has an ex-ante electoral advantage over the other, faction L_1 solves the following problem:

$$\max_{e_1^L\in[0,1]}\left\{\begin{array}{c}\left[\frac{1}{2}+\psi(e_1^L+e_2^L-e_1^R-e_2^R)\right]\left[b_v^L+\left(\frac{1}{2}+\frac{(1-\gamma)(e_1^L-e_2^L)}{2}\right)\pi_v^L\right]+\\ \left[\frac{1}{2}-\psi(e_1^L+e_2^L-e_1^R-e_2^R)\right]\left[b_d^L+\left(\frac{1}{2}+\frac{(1-\gamma)(e_1^L-e_2^L)}{2}\right)\pi_d^L-\ \ x^L-x^R\big)^2\right]\\ -\frac{(e_1^L)^2}{2}-\frac{(1-e_1^L)^2}{2}\end{array}\right\}.$$

The first-order condition associated to faction L_1 is:

$$\psi\Big[b_v^L+\Big(\frac{1}{2}+\frac{(1-\gamma)(e_1^L-e_2^L)}{2}\Big)\pi_v^L\Big]-\psi\Big[b_d^L+\Big(\frac{1}{2}+\frac{(1-\gamma)(e_1^L-e_2^L)}{2}\Big)\pi_d^L-(x^L-x^R)^2\Big]+$$
$$\frac{1-\gamma}{2}\pi_v^L\Big[\frac{1}{2}+\psi(e_1^L+e_2^L-e_1^R-e_2^R)\Big]+\frac{1-\gamma}{2}\pi_d^L\Big[\frac{1}{2}-\psi(e_1^L+e_2^L-e_1^R-e_2^R)\Big]+1-2e_1^L=0,$$

and likewise for faction L_2 in party L and factions R_1 and R_2 in party R. Solving the system of four first order conditions, one for each faction, yields a unique solution for e_1^L,e_2^L,e_1^R,e_2^R:

$$e^{L*}=\frac{\begin{array}{c}[(\gamma-1)(\pi_d^R-\pi_v^R)\psi-2][2+(2\alpha-1+2(x_L-x_R)^2)\psi]+(\gamma-1)\pi_v^L(1+\psi)(-2+(\gamma-1)\pi_v^R\\ +(2\alpha-1-2(x_L-x_R)^2)\psi)+(\gamma-1)\pi_d^L(1+\psi(2+\pi_d^R-\gamma\pi_d^R+[2\alpha-1+2(x_L-x_R)^2)\psi)]\end{array}}{4\psi(\gamma-1)(\pi_d^L+\pi_d^R-\pi_v^L-\pi_v^R)-8} \tag{31}$$

and, analogously, effort chosen by factions in party R is equal to

$$e^{R*}=\frac{\begin{array}{c}[(\gamma-1)(\pi_d^L-\pi_v^L)\psi-2][2+(2\alpha-1+2(x_L-x_R)^2)\psi]+(\gamma-1)\pi_v^R(1+\psi)(-2+(\gamma-1)\pi_v^L\\ +(2\alpha-1-2(x_L-x_R)^2)\psi)+(\gamma-1)\pi_d^L(1+\psi(2+\pi_d^L-\gamma\pi_d^R+[2\alpha-1+2(x_L-x_R)^2)\psi)]\end{array}}{4\psi(\gamma-1)(\pi_d^R+\pi_d^L-\pi_v^R-\pi_v^L)-8} \tag{32}$$

Substituting the equilibrium premia yields

$$e^{L*}=\begin{cases}\dfrac{2+\psi\ \ 2\alpha-1+2\ \ x^L-x^R\big)^2\big)}{4} & \text{if}\quad \gamma>1\\[2ex] \dfrac{5-\gamma+2\psi\ \ 2\alpha-1+2\ \ x^L-x^R\big)^2\big)}{8} & \text{if}\quad \gamma<1,\end{cases}$$

which completes the proof. □

B.2. Proof of Proposition 2

Proof. (i) The proof simply follows by inspection of the first-order conditions and by observing that an increase in polarization $\ \ x^L-x^R\big)$ only increases the external incentive term.

(ii) The following first-order condition identifies equilibrium effort of factions in party L:

$$\psi \ \ b_v^L + \rho_1^L \pi_v^L\big) + p^L\Big(\frac{1-\gamma}{2}\Big)\pi_v^L - \psi\big[b_d^L + \rho_1^L \pi_d^L - \ \ x^L - x^R\big)^2\big] + (1-p^L)\Big(\frac{1-\gamma}{2}\Big)\pi_d^L + 1 - 2e^L = 0,$$

and can be re-expressed as

$$2e^L = 1 + \Big(\frac{1-\gamma}{2}\Big)\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] + \psi\Big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + \ \ x^L - x^R\big)^2\Big].$$

Differentiating with respect to $|x^L|$ yields

$$\begin{aligned}\frac{\partial 2e^L}{\partial |x^L|} &= \Big(\frac{1-\gamma}{2}\Big)\frac{\partial p^L}{\partial |x^L|}(\pi_v^L - \pi_d^L) + 2\psi \ \ x^L - x^R\big) \\ &= -(1-\gamma)\psi|x^L|(\pi_v^L - \pi_d^L) + 2\psi \ \ x^L - x^R\big),\end{aligned} \tag{33}$$

where the second equality follows from $\partial p^L/\partial|x^L| = -2\psi|x^L|$.

Differentiating with respect to $|x^R|$ yields

$$\begin{aligned}\frac{\partial 2e^L}{\partial |x^R|} &= \Big(\frac{1-\gamma}{2}\Big)\frac{\partial p^L}{\partial x^R}(\pi_v^L - \pi_d^L) - 2\psi \ \ x^L - x^R\big) \\ &= (1-\gamma)\psi x^R(\pi_v^L - \pi_d^L) - 2\psi \ \ x^L - x^R\big).\end{aligned}$$

By symmetry:

$$\frac{\partial 2e^R}{\partial |x^L|} = -(1-\gamma)\psi|x^L|(\pi_v^R - \pi_d^R) + 2\psi \ \ x^L - x^R\big).$$

Differentiating $(2e^L - 2e^R)$ with respect to $|x^L|$ yields

$$\frac{\partial(2e^L - 2e^R)}{\partial|x^L|} = -(1-\gamma)\psi|x^L|(\pi_v^L + \pi_v^R - \pi_d^L - \pi_d^R). \tag{34}$$

In equilibrium, when $\gamma < 1$ the optimal contract offered by both leaders is $\pi_v^{L*} = \pi_v^{R*} = \alpha$, and $\pi_d^{L*} = \pi_d^{R*} = 1-\alpha$. Substituting the optimal contract yields

$$\frac{\partial(2e^{L*} - 2e^{R*})}{\partial|x^L|} = -2(1-\gamma)\psi|x^L|(2\alpha - 1),$$

which is always negative. That is, the difference in equilibrium efforts $(2e^{L*} - 2e^{R*})$ decreases in L's ideological extremism. The proof of $\frac{\partial(2e^{R*} - 2e^{L*})}{\partial|x^R|} < 0$ is analogous therefore omitted.

Finally, when $\gamma > 1$, the optimal contract offered by both leaders is $\pi_v^{L*} = \pi_v^{R*} = \pi_d^{L*} = \pi_d^{R*} = 0$, which substituted into (34) yields zero for every x^L. $\square$

B.3. Proof of Remark 1

Proof. Differentiating $\mathcal{W}^L(x^L)$ with respect to $|x^L|$ yields

$$\frac{\partial \mathcal{W}^L(x^L)}{\partial |x^L|} = \frac{\partial p^L}{\partial |x^L|}\Big[2\alpha - 1 + 2\ \ x^L - x^R)^2\Big] - 4|x^L - x^R|\ \ 1 - p^L),$$

where

$$\frac{\partial p^L(x^L)}{\partial |x^L|} = -2\psi |x^L| < 0,$$

which is always negative. $\square$

B.4. Proof of Proposition 3

Proof. When factions are heterogeneous, we can express L_1's first-order condition as

$$e_1^{L*} = \frac{1}{2} + \left(\frac{1-\gamma}{4}\right)\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] + \frac{\psi}{2}\Big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) - \ \ x_1^L - x^L)^2 + \ \ x_1^L - x^R)^2\Big], \quad (35)$$

and L_2's first-order condition as

$$e_2^{L*} = \frac{1}{2} + \left(\frac{1-\gamma}{4}\right)\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] + \frac{\psi}{2}\Big[b_v^L - b_d^L + \ \ 1-\rho_1^L)(\pi_v^L - \pi_d^L) - \ \ x_2^L - x^L)^2 + \ \ x_2^L - x^R)^2\Big], \quad (36)$$

By inspection of the last term of the first-order condition in (35) and (36), it is clear that $e_1^{L*} > e_2^{L*}$ for $|x_1^L| > |x_2^L|$.

To derive the equilibrium incentive scheme, first take the sum $e_1^{L*} + e_2^{L*}$:

$$e_1^{L*} + e_2^{L*} = 1 + \left(\frac{1-\gamma}{2}\right)\Big[\pi_d^L + 2p^L(\pi_v^L - \pi_d^L)\Big] + \frac{\psi}{2}\Big[2\alpha - 1 - \ \ x_1^L - x^L)^2 + \ \ x_1^L - x^R)^2 - \ \ x_2^L - x^L)^2 + \ \ x_2^L - x^R)^2\Big].$$

Differentiating $e_1^{L*} + e_2^{L*}$ with respect to π_v^L yields

$$\frac{\partial(e_1^{L*} + e_2^{L*})}{\partial \pi_v^L} = \frac{1-\gamma}{2}\Big[p^L + \frac{\partial p^L}{\partial \pi_v^L}(\pi_v^L - \pi_d^L)\Big].$$

Analogously:

$$e_1^{R*} + e_2^{R*} = 1 + \left(\frac{1-\gamma}{2}\right)\Big[\pi_v^R + 2(1-p^L)(\pi_d^R - \pi_v^R)\Big] + \frac{\psi}{2}\Big[2\alpha - 1 - \ \ x_1^R - x^R)^2 + \ \ x_1^R - x^L)^2 - \ \ x_2^R - x^R)^2 + \ \ x_2^R - x^L)^2\Big],$$

and

$$\frac{\partial(e_1^{R*}+e_2^{R*})}{\partial \pi_v^L} = \frac{(1-\gamma)}{2}\frac{\partial p^L}{\partial \pi_v^L}(\pi_d^R - \pi_v^R).$$

Define χ as $\frac{\partial(e_1^{L*}+e_2^{L*}-e_1^{R*}-e_2^{R*})}{\partial \pi_v^L}$. We have:

$$\chi = \frac{1-\gamma}{2}\Big[p^L + \frac{\partial p^L}{\partial \pi_v^L}(\pi_v^L - \pi_d^L - \pi_d^R + \pi_v^R)\Big]. \tag{37}$$

Since

$$\frac{\partial p^L}{\partial \pi_v^L} = \psi\frac{\partial(e_1^{L*}+e_j^{L*}-e_1^{R*}-e_2^{R*})}{\partial \pi_v^L} = \psi\chi,$$

the expression for χ (37) can be re-written as

$$\chi = \frac{\partial(e_1^{L*}+e_2^{L*}-e_1^{R*}-e_2^{R*})}{\partial \pi_v^L} = \frac{p^L}{\frac{2}{1-\gamma} - \psi(\pi_v^L - \pi_d^L - \pi_d^R + \pi_v^R)},$$

which is positive for $\gamma < 1$ and ψ small enough. This implies $\pi_v^{L*} = \alpha$. The sign of the derivative is negative for $\gamma > 1$, which implies $\pi_v^{L*} = 0$. The proof for π_d^{L*} is analogous and therefore omitted. □

B.5. Proof of Proposition 4

Proof. Faction L_1's realized payoff from winning the election and ranking high within the party is

$$\begin{aligned} u_1^L(\pi_v^L, s^L = 1) &= b_v^L + \pi_v^L - \Big(x_1^L - \ \lambda x_1^L + (1-\lambda)x_2^L\Big)\Big)^2 - \frac{(e_1^L)^2}{2} - \frac{(1-e_1^L)^2}{2} \\ &= b_v^L + \pi_v^L - (1-\lambda)^2(x_1^L - x_2^L)^2 - \frac{(e_1^L)^2}{2} - \frac{(1-e_1^L)^2}{2} \end{aligned} \tag{38}$$

Similarly, L_1's realized payoff from winning the election and ranking low is

$$u_1^L(\pi_v^L, s^L = 2) = b_v^L - \lambda^2(x_2^L - x_1^L)^2 - \frac{(e_1^L)^2}{2} - \frac{(1-e_1^L)^2}{2}. \tag{39}$$

Given these expressions, we can write L_1's first-order condition as

$$\begin{aligned} e_1^{L*} =& \frac{1}{2} + \frac{\psi}{2}\Big[b_v^L - b_d^L + \rho_1^L \ \pi_v^L - \pi_d^L - (1-\lambda)^2(x_1^L - x_2^L)^2\Big) - \ 1 - \rho_1^L\Big)\lambda^2(x_2^L - x_1^L)^2 + \ x_i^L - x^R\Big)^2\Big] \\ &+ \Big(\frac{1-\gamma}{2}\Big)\Big[\pi_d^L + p^L(\pi_v^L - \pi_d^L) - (1-\lambda)^2(x_1^L - x_2^L)^2 + \lambda^2(x_2^L - x_1^L)^2\Big]. \end{aligned} \tag{40}$$

Similarly, we can write L_1's first-order condition as

$$e_2^{L*} = \frac{1}{2} + \frac{\psi}{2}\Big[b_v^L - b_d^L + \ 1 - \rho_1^L\Big)\Big(\pi_v^L - \pi_d^L - (1-\lambda)^2(x_2^L - x_1^L)^2\Big) - \rho_1^L\lambda^2(x_1^L - x_2^L)^2 + \ x_i^L - x^R\Big)^2\Big]$$

$$+\left(\frac{1-\gamma}{2}\right)\Big[\pi_d^L + p^L(\pi_v^L - \pi_d^L) - (1-\lambda)^2(x_2^L - x_1^L)^2 + \lambda^2(x_1^L - x_2^L)^2\Big]. \tag{41}$$

Summing the two factions' equilibrium efforts after substituting the budget constraint and simplifying yields:

$$\begin{aligned} e_1^{L*} + e_2^{L*} =& 1 + \frac{\psi}{2}\Big[2\alpha - 1 - (\lambda^2 + (1-\lambda)^2)(x_2^L - x_1^L)^2 + \ \ x_1^L - x^R\big)^2 + \ \ x_2^L - x^R\big)^2\Big] \\ & + \ \ 1-\gamma\big)\Big[\pi_d^L + p^L(\pi_v^L - \pi_d^L) + p^L(x_2^L - x_1^L)^2(2\lambda - 1)\Big], \end{aligned}$$

from which we can differentiate with respect to λ to obtain

$$\frac{\partial(e_1^{L*} + e_2^{L*})}{\partial\lambda} = - \ \ 2\lambda - 1\big)\psi \ \ x_2^L - x_1^L\big)^2 + \ \ 1-\gamma\big)\left[\frac{\partial p^L}{\partial\lambda}\Big(\pi_v^L - \pi_d^L + (x_2^L - x_1^L)^2(2\lambda - 1)\Big) + 2p^L \ \ x_2^L - x_1^L\big)^2\right].$$

Similarly, summing the first-order conditions of factions in *right* yields,

$$e_1^{R*} + e_2^{R*} = 1 + \frac{\psi}{2}\Big[2\alpha - 1 - (x_1^R - x^R)^2 + \ \ x_2^R - x^R\big)^2 + \ \ x_1^R - x^L\big)^2 + \ \ x_2^R - x^L\big)^2\Big] + \ \ 1-\gamma\big)\Big[\pi_v^R + p^L(\pi_d^R - \pi_v^R)\Big],$$

from which we can differentiate with respect to λ to obtain

$$\frac{\partial(e_1^{R*} + e_2^{R*})}{\partial\lambda} = \ \ 1-\gamma\big)\frac{\partial p^L}{\partial\lambda} \ \ \pi_d^R - \pi_v^R\big).$$

Define χ_λ as

$$\chi_\lambda \equiv \frac{\partial(e_1^{L*} + e_2^{L*} - e_1^{R*} - e_2^{R*})}{\partial\lambda}.$$

We have:

$$\chi_\lambda = \ \ 1 - 2\lambda\big)\psi \ \ x_2^L - x_1^L\big)^2 + \ \ 1-\gamma\big)\psi\chi_\lambda\Big(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R + (x_2^L - x_1^L)^2(2\lambda - 1)\Big) + 2p^L \ \ x_2^L - x_1^L\big)^2,$$

which simplified yields

$$\chi_\lambda = \frac{(x_2^L - x_1^L)^2\Big[2p^L + \psi(2\lambda - 1)\Big]}{1 - \ \ 1-\gamma\big)\psi \ \ \pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R + (x_2^L - x_1^L)^2(2\lambda - 1)\big)}. \tag{42}$$

The numerator of (42) is always positive, as $\lambda \in [1/2, 1]$. The denominator is positive when

$$(1-\gamma)\big[\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R + (x_2^L - x_1^L)^2(2\lambda - 1)\big] < \frac{1}{\psi}. \tag{43}$$

When $\gamma > 1$, the LHS in (43) is always negative, which implies that in equilibrium $\lambda^* = 1$. That is, when sabotage is more effective than campaigning effort it is always optimal to set a policy concession for the faction ranking higher. Since premia are constrained to be nonnegative, in this case there is no trade-off between eliciting campaigning and setting a winning platform.

When $\gamma < 1$, the sign of LHS in (43) depends on the value of $(x_2^L - x_1^L)^2$. A sufficient condition for the denominator of χ_λ (42) to be positive is

$$2\alpha + (x_2^L - x_1^L)^2(2\lambda - 1) < \frac{1}{\psi(1-\gamma)}, \tag{44}$$

which requires the distance in factions' ideological bliss point to be low enough. A sufficient condition for the denominator to be negative is

$$(x_2^L - x_1^L)^2(2\lambda - 1) > \frac{1}{\psi(1-\gamma)} + 2(1-\alpha), \tag{45}$$

which requires the distance in factions' ideological bliss point to be high enough. That is, when the ideological distance is high enough and $\gamma < 1$, the optimal incentive scheme sets low powered incentives. When the distance is low enough, the optimal incentive scheme features high powered incentives.

Lastly, we need to derive the optimal premia. To do so, notice that

$$\frac{\partial(e_1^{L*} + e_2^{L*})}{\partial \pi_v^L} = (1-\gamma)\Big[\frac{\partial p^L}{\partial \pi_v^L}\ \ \pi_v^L - \pi_d^L + (x_2^L - x_1^L)^2(2\lambda - 1)\big) + p^L\Big], \tag{46}$$

and

$$\frac{\partial(e_1^{R*} + e_2^{R*})}{\partial \pi_v^L} = (1-\gamma)\frac{\partial p^L}{\partial \pi_v^L}\ \ \pi_d^R - \pi_v^R), \tag{47}$$

which yields

$$\frac{\partial(e_1^{L*} + e_2^{L*} - e_1^{R*} - e_2^{R*})}{\partial \pi_v^L} = \frac{p^L}{\frac{1}{1-\gamma} - \psi\Big[\pi_v^L - \pi_d^L - \pi_d^R + \pi_v^R + (x_2^L - x_1^L)^2(2\lambda - 1)\big)\Big]}. \tag{48}$$

By inspection, $\partial p^L/\partial \pi_v^L < 0$ for $\gamma > 1$, and $\partial p^L/\partial \pi_v^L > 0$ for $\gamma < 1$ and ψ small enough — which completes the proof. □

C. Extensions

C.1. Negative Premia

In the baseline model, the leader is constrained to choose an incentive scheme which rewards the faction that ranks higher according to internal monitoring device with non-negative premia. That is, when sabotage is more effective than campaigning to get a high ranking, the leader cannot "punish" the winning faction by setting a negative premium (or, alternatively, the losing faction cannot be rewarded). This assumption reflects the fact that leaders are often constrained by parties' legal rules and formal procedures, which are the same for all factions and are decided ex-ante.[29]

However, from a theoretical standpoint one might argue that the party should recognize and avoid such inefficient metrics, and be able to punish factions that mobilize less. That is, when $\gamma > 1$, negative premia should be strictly better than positive ones: by promising all the electoral spoils to the lower-ranking faction, leaders can ensure higher campaigning effort. In terms of information inferred from the ranking indicator, if negative premia are allowed, then having the ranking indicator increasing in mobilization effort is equivalent to having it increasing in sabotage. The main results of the baseline model are robust to a specification which allows leaders to punish high ranking factions.

Corollary 3 (**Equilibrium with negative premia**). *When premia can be negative, the optimal premia offered by L in equilibrium (and, symmetrically, by R) are $(\pi_d^{L*}, \pi_v^{L*}) = (\alpha - 1, -\alpha)$ if $\gamma > 1$, and $(\pi_d^{L*}, \pi_v^{L*}) = (1 - \alpha, \alpha)$ if $\gamma < 1$.*

Proof. The proof directly follows from Lemma 2 and 3, which show that the leader's expected payoff is strictly increasing (decreasing) in both premia when $\gamma < 1$ ($\gamma > 1$). $\square$

Intuitively, the highest incentive to invest in campaigning effort coincides with a punishment for ranking higher when $\gamma > 1$. When the factions' incentives are aligned to those of the leader ($\gamma < 1$), high powered incentives are optimal, as in the baseline model.

How does allowing for negative premia change the equilibrium investment decision of factions? The baseline model shows that factions in extreme parties campaign less than factions in moderate parties when $\gamma < 1$. Extending the analysis to negative premia shows that this is true for every value of γ, i.e.,

$$\frac{\partial \ e^{L*} - e^{R*})}{\partial |x^L|} < 0.$$

[29] The assumption that leaders cannot renege on contracts is supported by the evidence on historical factions presented in the paper, which shows that these set of rules can even take the form of explicit contracts, as in the Italian case.

Since it is equivalent to have the internal ranking determined by sabotage or campaigning, the effect of polarization on campaigning effort is the same whether $\gamma > 1$ or $\gamma < 1$.

The baseline model also shows that, when factions are heterogeneous, rewarding sabotage with a positive premium contingent on electoral victory increases the moderate faction's campaigning. This result is robust to a specification that allows for negative premia. In particular, in this case the difference between total effort in party L and total effort in party R is decreasing in the value of the extreme faction's preferred policy, $|x_i^L|$.

Finally, the result on policy concessions is also robust to a specification allowing for negative premia. In particular, when $\gamma > 1$ and the extreme faction's preferred policy is extreme enough, the leader sets $\lambda^* = 1$, thus rewarding the moderate faction with a policy concession.

C.2. Non-binding Resource Constraint

The baseline model assumes $a_i^L + e_i^L = 1$ — that is, effort (e_i^L) and sabotage (a_i^L) exhaust the faction's unitary budget of resources. In what follows I analyze the general case $a_i^L + e_i^L \leq 1$. I show that in equilibrium (i) effort must be positive, (ii) sabotage is either positive or zero. That is, both $a_i^{L*} > 0,\ e_i^{L*} > 0$ and $a_i^{L*} = 0,\ e_i^{L*} > 0$ are possible in equilibrium.

Consider the decision of faction L_1 in party L. There is a total budget normalized to 1, and the following condition must hold: $a_i^L + e_i^L \leq 1$, that is, doing nothing is an option for factions. Faction 1's maximization problem is

$$
\begin{aligned}
\max_{e_1^L, a_1^L} \quad & p^L\big[b_v^L + \rho_1^L \pi_v^L\big] + (1 - p^L)\big[b_d^L + \rho_1^L \pi_d^L - (x^L - x^R)^2\big] - \frac{(e_1^L)^2}{2} - \frac{(a_1^L)^2}{2} \\
\text{s.t.} \quad & e_1^L + a_1^L \leq 1, \\
& e_1^L, a_1^L \geq 0.
\end{aligned}
$$

where $p^L = \frac{1}{2} + \psi\big[-(x^L)^2 + (x^R)^2 + e_1^L + e_2^L - e_1^R - e_2^R\big]$ and $\rho_1^L = \frac{1}{2} + \frac{(e_1^L - e_2^L) + \gamma(a_1^L - a_2^L)}{\phi}$. For simplicity, but without loss of generality, let $x^L = -x^R$ (no party has an ex-ante electoral advantage).

The Lagrangean associated with L_1's problem can be expressed as

$$
\begin{aligned}
\mathcal{L}(e_1^L, a_1^L) = {} & p^L\big[b_v^L + \rho_1^L \pi_v^L\big] + (1 - p^L)\big[b_d^L + \rho_1^L \pi_d^L - (x^L - x^R)^2\big] - \frac{(e_1^L)^2}{2} - \frac{(a_1^L)^2}{2} \\
& - \lambda_1(a_1^L + e_1^L - 1) + \lambda_2(e_1^L) + \lambda_3(a_1^L)
\end{aligned}
$$

The optimization problem satisfies the constraint qualifications, hence we know that the solution of the faction's maximization is the solution of the following Karush-Kuhn-Tucker conditions:

$$
\begin{array}{ll}
(1) & \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \dfrac{1}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - e_1^L - \lambda_1 + \lambda_2 = 0 \\
(2) & \dfrac{\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - a_1^L - \lambda_1 + \lambda_3 = 0 \\
(3) & a_1^L + e_1^L - 1 \leq 0 \qquad \wedge \quad \lambda_1\big[a_1^L + e_1^L - 1\big] = 0 \\
(4) & e_1^L \geq 0 \qquad \wedge \quad \lambda_2 \;\, e_1^L\big) = 0 \\
(5) & a_1^L \geq 0 \qquad \wedge \quad \lambda_3 \;\, a_1^L\big) = 0 \\
(6) & \lambda_1 \geq 0,\; \lambda_2 \geq 0,\; \lambda_3 \geq 0.
\end{array}
$$

where conditions (1) and (2) are the first order conditions with respect to e_1^L and a_1^L.

Before proceeding with the cases to evaluate, notice that the following holds in equilibrium:

- $e_1^{L*} = e_2^{L*}$ and $a_1^{L*} = a_2^{L*}$, which implies $\rho_1^L = 1/2$
- Since $x^L = -x^R$, we have $p^L(e_1^{L*}, e_2^{L*}, e_1^{R*}, e_2^{R*}) = 1/2$

Given the inequality constraint, there are four cases to consider.

$\boxed{\text{(I)}\;\; a_1^L > 0,\; e_1^L > 0.}$ Then, $\lambda_2 = \lambda_3 = 0$ from conditions (4) and (5). We can find the value of λ_1 from condition (1) and (2):

$$
\begin{array}{ll}
(1) & \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \dfrac{1}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - e_1^L = \lambda_1 \\
(2) & \dfrac{\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - a_1^L = \lambda_1.
\end{array}
$$

Substituting (1) into (2) yields

$$
e_1^L - a_1^L = \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1-\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big].
$$

Recall that rewards are simply: $b_v^L = \frac{\alpha - \pi_v^L}{2}$ and $b_d^L = \frac{1-\alpha-\pi_d^L}{2}$. Hence we can re-write $e_1^L - a_1^L$ as

$$
e_1^L - a_1^L = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1-\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big].
$$

In equilibrium, effort is increasing (decreasing) in both premia when $\gamma < 1$ ($\gamma > 1$), hence premia are set to ($\pi_d^{L*} = 1 - \alpha$, $\pi_v^{L*} = \alpha$) when $\gamma < 1$, and to ($\pi_d^{L*} = \pi_v^{L*} = 0$) when $\gamma > 1$. Substituting in

(π_d^{L*}, π_v^{L*}), as well as $p^L = 1/2$ yields

$$a_i^{L*} = e_i^{L*} - \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] - \mathbb{1}\{\gamma < 1\}\frac{1-\gamma}{\phi}. \tag{49}$$

which substituted into condition (1) yields

$$\lambda_1 = \mathbb{1}\{\gamma < 1\}\frac{1}{\phi} + \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] - e_i^{L*},$$

where the first two terms are nonnegative because $\phi > 0$, $\psi > 0$, $\alpha \geq 1/2$. We can then find a sufficient condition for $\lambda_1 \geq 0$, which is

$$\psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] \geq 1 - \mathbb{1}\{\gamma < 1\}\frac{1}{\phi}.$$

$\boxed{\text{(II)}\ \ a_1^L > 0,\ e_1^L = 0.}$ Then, $\lambda_3 = 0$ from (5). We can find the value of λ_1 from condition (2):

$$(2) \quad \frac{\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - a_1^L = \lambda_1.$$

But then, substituting the value found for λ_1 into (1) we get

$$(1) \quad \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1-\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] + a_1^L + \lambda_2 = 0$$

which clearly contradicts $\lambda_2 \geq 0$.

$\boxed{\text{(III)}\ \ a_1^L = 0,\ e_1^L > 0.}$ Then, $\lambda_2 = 0$ from (4). We can find the value of λ_1 from condition (1):

$$(1) \quad \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - e_1^L = \lambda_1,$$

which substituted into (2) yields

$$(2) \quad \frac{\gamma-1}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + e_1^L + \lambda_3 = 0.$$

In equilibrium,

$$\lambda_3 = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1-\gamma}{\phi} - e_1^{L*}. \tag{50}$$

We can then find a sufficient condition for $\lambda_3 \geq 0$, which is

$$\psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] \geq 1 - \frac{1-\gamma}{\phi},$$

which becomes harder to satisfy as γ increases.

$\boxed{\text{(IV)} \quad a_1^L = e_1^L = 0.}$ Then, $\lambda_1 = 0$ from condition (3), and

$$\begin{aligned}
&(1) \qquad \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] + \lambda_2 \leq 0 \\
&(2) \qquad \frac{\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] + \lambda_3 \leq 0,
\end{aligned}$$

which again contradicts $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$.

C.3. Two Separate Actions - Different Cost Functions

The baseline model assumes (i) $a_1^L = 1 - e_1^L$ and (ii) separate quadratic costs for both actions. These assumptions imply that in equilibrium there is always a positive amount of sabotage (as equilibrium effort is interior) to minimize costs, even when $\gamma < 1$ — that is, even when sabotage is less effective than campaigning to achieve a high internal ranking. While Subsection C.2 relaxes (i) by analyzing the case $a_1^L + e_1^L \leq 1$, this section extends the analysis to consider two separate actions, $e_1^L \in [0,1]$ and $a_1^L \in [0,1]$, and different convex cost functions.

(I) Separate Quadratic Costs. Using the baseline model's cost function, faction L_1's maximization problem can be expressed as

$$\max_{e_1^L, a_1^L} \quad p^L\big[b_v^L + \rho_1^L\pi_v^L\big] + (1-p^L)\big[b_d^L + \rho_1^L\pi_d^L - (x^L - x^R)^2\big] - (e_1^L)^2 + (a_1^L)^2,$$

where $p^L = 1/2 + \psi(-x_L^2 + x_R^2 + e_1^L + e_2^L - e_1^R - e_2^R)$ and $\rho_1^L = 1/2 + \phi^{-1}[e_1^L - e_2^L + \gamma(a_1^L - a_2^L)]$. We start the analysis with the following observations.

Remark 2. *Factions do not invest in sabotage ($a_i^{L*} = 0$)* if and only if *both premia are set to zero. When $\pi_v^L > 0$ and/or $\pi_d^L > 0$, in equilibrium factions exert some positive level of sabotage.*

Proof. To show this, observe that

$$\frac{\partial U_1^L}{\partial a_1^L} = \frac{\gamma}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big] - 2a_1^L, \tag{51}$$

where $\partial U_1^L/\partial a_1^L \leq 0$ for every a_1^L if and only if $\pi_v^L = \pi_d^L = 0$. Hence, when premia are set to zero, $a_i^{L*} = 0$.

To show the second part of the claim, notice that when either premia is positive $\partial U_1^L/\partial a_1^L \geq 0$ at $a_1^L = 0$, implying $a_i^{L*} > 0$. □

Remark 3. *Factions always exert positive effort in equilibrium.*

Proof. The proof simply follows by inspection of

$$\frac{\partial U_1^L}{\partial e_1^L} = \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big] - 2e_1^L, \tag{52}$$

which is positive at $e_1^L = 0$ for every value of the premia. □

Given these observations it follows that factions invest in sabotage in equilibrium for any incentive scheme which features non-negative premia. The following result derives the equilibrium incentive scheme, which always features positive premia. Given this result, it is always true that factions exert positive sabotage in equilibrium with separate quadratic cost of sabotage and effort.

Lemma 4. *L's objective function is always increasing in both premia, implying $\pi_v^{L*} = \alpha$, $\pi_d^{L*} = 1 - \alpha$.*

Proof. The two first-order conditions of the faction's problem are:

$$2e_1^L = \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L)\big] + \frac{1}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big] \tag{53}$$

$$2a_1^L = \frac{\gamma}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big] \tag{54}$$

Differentiating effort with respect to π_v^L yields:

$$\frac{\partial e_1^L}{\partial \pi_v^L} = \frac{\psi}{\phi}\Big[\frac{\partial(e_1^L - e_2^L)}{\partial \pi_v^L} + \gamma\frac{\partial(a_1^L - a_2^L)}{\partial \pi_v^L}\Big](\pi_v^L - \pi_d^L) + \psi\Big[\frac{1}{2} + \frac{e_1^L - e_2^L}{2} + \gamma\Big(\frac{a_1^L - a_2^L}{2}\Big)\Big] + \frac{1}{\phi}\Big[p^L + \frac{\partial p^L}{\partial \pi_v^L}(\pi_v^L - \pi_d^L)\Big]$$

When factions are symmetric (same ideological preferences) $a_1^{L*} = a_2^{L*}$ and $e_1^{L*} = e_2^{L*}$, which implies

$$\frac{\partial e^L}{\partial \pi_v^L} = \frac{\psi}{2} + \frac{1}{\phi}\Big[p^L + \psi\frac{\partial 2(e^L - e^R)}{\partial \pi_v^L}(\pi_v^L - \pi_d^L)\Big]$$

and

$$\frac{\partial e^L}{\partial \pi_v^R} = \frac{1}{\phi}\Big[\psi\frac{\partial 2(e^L - e^R)}{\partial \pi_v^R}(\pi_v^L - \pi_d^L)\Big].$$

By symmetry:

$$\frac{\partial e^R}{\partial \pi_v^L} = \frac{1}{\phi}\Big[\psi \frac{\partial 2(e^R - e^L)}{\partial \pi_v^L}(\pi_v^R - \pi_d^R)\Big],$$

which allows us to express L's objective function as

$$\frac{\partial(e^L - e^R)}{\partial \pi_v^L} = \frac{\psi}{2} + \frac{1}{\phi}p^L + \frac{2\psi}{\phi}\frac{\partial(e^L - e^R)}{\partial \pi_v^L}(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R)$$

Rearranging, we can easily see that

$$\frac{\partial(e^L - e^R)}{\partial \pi_v^L} = \frac{\frac{\psi}{2} + \frac{1}{\phi}p^L}{1 - \frac{2\psi}{\phi}(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R)},$$

which is positive for ϕ large enough. This implies $\pi_v^{L*} = \alpha$. The proof for $\pi_d^{L*} = 1 - \alpha$ is analogous and therefore omitted. □

Hence with this functional form assumption, the optimal premia do not depend on γ and are always set at the maximum. Premia are independent of γ because L does not internalize the cost of sabotage: with separate actions and separate costs for both actions, sabotage does not imply a lower investment in campaigning activities. The next cost function restores this property of the model while keeping the two actions separate.

(II) Campaigning and Sabotage as Substitutes. Consider the following cost function

$$C(e_1^L, a_1^L) = (e_1^L + a_1^L)^2,$$

which preserves the crucial property of decreasing return of both activities, and that more investment in one activity increases the marginal cost of the other. The latter property is the fundamental reason why the leader might want to disincentivize sabotage in this setup: higher sabotage increases campaigning effort's marginal cost, thereby reducing the amount of equilibrium effort, which is what the leader seeks to maximize in order to win the election.

The two first-order conditions of the faction's problem can now be expressed as:

$$2e_1^L = \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big] - 2a_1^L, \quad (55)$$
$$2a_1^L = \frac{\gamma}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big] - 2e_1^L,$$

from which we can conclude that $e_1^{L*} = a_1^{L*} = 0$ is never a solution. This simply follows by inspection of $\partial U_1^L / \partial e_1^L$, which is always positive at $e_i^{L*} = a_i^{L*} = 0$. Similarly, when $\gamma < 1$, effort must be positive as the next result shows.

Claim 2. *If $\gamma < 1$, $e_1^{L*} = 0$, $a_1^{L*} > 0$ is not a solution.*

Proof. Since $a_1^{L*} > 0$, we can replace a_1^{L*} into the first-order condition with respect to e_1^{L*} and obtain

$$\frac{\partial U_1}{\partial e_1^L} = \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L)\big] + \frac{1-\gamma}{\phi}\big[p^L \pi_v^L + (1-p^L)\pi_d^L\big],$$

which is clearly positive, hence $e_1^{L*} > 0$. □

On the other hand, it is possible to have $e_1^{L*} > 0$, $a_1^{L*} = 0$. This happens in equilibrium when sabotage is less effective than effort, as the next result shows.

Claim 3. *If $\gamma < 1$, $a_1^{L*} = 0$.*

Proof. Suppose $a_1^L = 0$. Because of the budget constraint assumption and the fact that in equilibrium $e_1^{L*} = e_2^{L*}$, $a_1^{L*} = a_2^{L*}$, we can re-write the first-order condition with respect to e_1^L as

$$2e_1^L = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1}{\phi}\Big[p^L \pi_v^L + (1-p^L)\pi_d^L\Big],$$

which substituted into $\partial U_1^L / \partial a_1^L|_{a_1^L=0}$ yields the following condition:

$$\frac{\gamma - 1}{\phi}\Big[p^L \pi_v^L + (1-p^L)\pi_d^L\Big] - \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] < 0. \tag{56}$$

When the inequality holds, $a_1^{L*} = 0$. By inspection of (56), a sufficient condition for $\partial U_1^L / \partial a_1^L|_{a_1^L=0} < 0$ to hold is $\gamma < 1$, but it is not necessary: the condition also holds when both premia are zero or when the first term of the LHS is sufficiently low. □

The case left to establish is whether $e_1^{L*} > 0$, $a_1^{L*} > 0$ can be true in equilibrium. In order to prove it, it is first necessary to show what the optimal incentive scheme is. This is not straightforward: on the one hand, high powered incentives always elicit campaigning effort, on the other, when sabotage is highly effective high powered incentives might reduce campaigning via an increase in the marginal cost of effort. Unfortunately, finding the optimal incentive scheme when $e_1^L > 0$, $a_1^L > 0$ is not tractable because of the elevated number of first order conditions (8) that depend on each other. Nevertheless, the next result shows that factions exert positive effort in equilibrium when $\pi_v^{L*} = \alpha$ and γ is large. The proof is organized as follows. I begin by assuming that $e_1^L > 0$, and $a_1^L = 0$. The proof shows that the equilibrium incentive conditional on an electoral victory is always $\pi_v^{L*} = \alpha$. Then, the optimal incentive is substituted in $\frac{\partial U_1^L}{\partial a_1^L}|_{a_1^L=0}$ to prove that, when γ is high enough, the sign of the derivative is positive for every π_d^L, which implies that $a_1^{L*} > 0$. Intuitively, when γ is large the return from sabotage is high and factions invest in sabotage in equilibrium when incentives are high powered.

Claim 4. *When $\pi_v^{L*} = \alpha$ and γ is large enough, $e_1^{L*} > 0,\ a_1^{L*} > 0$.*

Proof. Consider the case $e_1^L > 0$, $a_1^L = 0$. Since $e_1^{L*} = e_2^{L*}$ and $a_1^{L*} = a_2^{L*}$, and by the budget constraint assumption $b_v^L + \pi_v^L/2 = \alpha/2$ and $b_d^L + \pi_d^L/2 = (1-\alpha)/2$, the first-order condition of L_1 with respect to e_1^L simplifies to

$$2e_1^L = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big] \tag{57}$$

To find the optimal value of π_v^L, plug in the expression for p^L, and consider the symmetric first-order condition of faction R_1:

$$2e_1^L = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1}{\phi}\Big[\pi_d^L + \Big(\frac{1}{2} + \psi(-x_L^2 + x_R^2 + 2e_1^L - 2e_1^R)\Big)(\pi_v^L - \pi_d^L)\Big],$$
$$2e_1^R = \psi\Big[\alpha - \frac{1}{2} + (x^R - x^L)^2\Big] + \frac{1}{\phi}\Big[\pi_d^R + \Big(\frac{1}{2} - \psi(-x_L^2 + x_R^2 + 2e_1^L - 2e_1^R)\Big)(\pi_v^R - \pi_d^R)\Big].$$

Subtracting $2(e_1^L - e_1^R)$ yields

$$2(e_1^L - e_1^R) = \frac{1}{\phi}\big[\pi_d^L - \pi_d^R + \Big(\frac{1}{2} + \psi(-x_L^2 + x_R^2)\Big)(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R) + \psi(2e_1^L - 2e_1^R)(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R)\big],$$

which can be re-expressed as

$$2(e_1^L - e_1^R) = \frac{(\frac{1}{2} + \psi(-x_L^2 + x_R^2))(\pi_v^L + \pi_v^R) + (\frac{1}{2} - \psi(-x_L^2 + x_R^2))(\pi_d^L + \pi_d^R)}{\phi - \psi(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R)},$$

which is increasing in π_v^L: in the numerator π_v^L is pre-multiplied by the probability of victory of the left, and as π_v^L increases the negative term in the denominator becomes smaller. This is intuitive: increasing the power of the incentives increases equilibrium campaigning effort, which is what the leader wants to maximize. Hence in equilibrium $\pi_v^{L*} = \alpha$. Substituting π_v^{L*} into $\partial U_1^L/\partial e_1^L = 0$ yields

$$2e_1^{L*} = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1}{\phi}\big[p^L\alpha + (1-p^L)\pi_d^{L*}\big],$$

where the optimal level of π_d^{L*} is unknown. Recall that $\frac{\partial U_1^L}{\partial a_1^L}|_{a_1^L=0}$ is

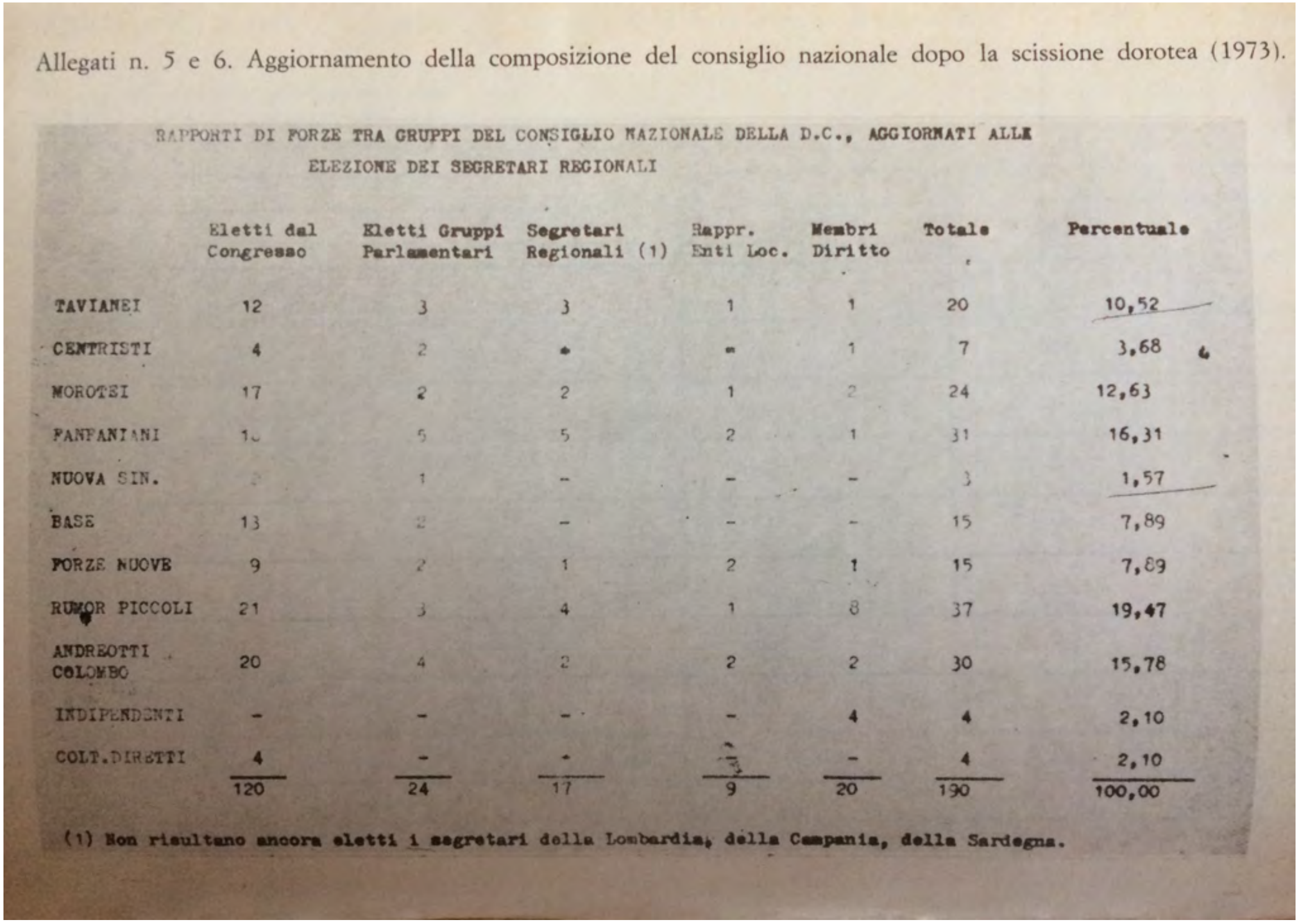
$$\frac{\gamma}{\phi}\left[p^L\pi_v^L + (1-p^L)\pi_d^L\right] - 2e_1^L. \tag{58}$$

We want to show that $\frac{\partial U_1^L}{\partial a_1^L}|_{a_1^L=0}$ can be positive. To find a sufficient condition, consider $\pi_d^L = 0$. Substituting the equilibrium values expressions for e_1^{L*} and π_v^{L*} yields

$$\frac{\gamma-1}{\phi}p^L\alpha - \psi\Big(\alpha - \frac{1}{2}\Big),$$

which is clearly positive for γ high enough. Hence it must be that, for $\gamma > \phi\psi/2$, in equilibrium $a_1^{L*} > 0$. □

D. Portfolio Allocation in Historical DC: From the Original Manual



Allegati n. 5 e 6. Aggiornamento della composizione del consiglio nazionale dopo la scissione dorotea (1973).

RAPPORTI DI FORZE TRA GRUPPI DEL CONSIGLIO NAZIONALE DELLA D.C., AGGIORNATI ALLE ELEZIONE DEI SEGRETARI REGIONALI

	Eletti dal Congresso	Eletti Gruppi Parlamentari	Segretari Regionali (1)	Rappr. Enti Loc.	Membri Diritto	Totale	Percentuale
TAVIANEI	12	3	3	1	1	20	10,52
CENTRISTI	4	2	-	-	1	7	3,68
MOROTEI	17	2	2	1	2	24	12,63
FANFANIANI	[illegible]	5	5	2	1	31	16,31
NUOVA SIN.	2	1	-	-	-	3	1,57
BASE	13	2	-	-	-	15	7,89
FORZE NUOVE	9	2	1	2	1	15	7,89
RUMOR PICCOLI	21	3	4	1	8	37	19,47
ANDREOTTI COLOMBO	20	4	2	2	2	30	15,78
INDIPENDENTI	-	-	-	-	4	4	2,10
COLT.DIRETTI	4	-	-	-	-	4	2,10
	120	24	17	9	20	190	100,00

(1) Non risultano ancora eletti i segretari della Lombardia, della Campania, della Sardegna.

Figure 2 – Portfolio Allocation Rule in Italian Christian Democracy (1973) Factional division of seats following the method in the Cencelli manual. The left column displays the names of the different factions composing the DC in 1973. The second column displays the total number of elected members of each faction, as a function of the total percentage obtained in the party congress (last column on the right).

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