COLLECTIVE HOLD-UP

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Abstract. We consider a dynamic process of coalition formation in which a principal bargains sequentially with a group of agents. This problem is at the core of a variety of applications in economics and politics, including a lobbyist seeking to pass a bill, an entrepreneur setting up a start-up, or a firm seeking the approval of corrupt bureaucrats. We show that when the principal's willingness to pay is high, reallocating bargaining power from the principal to the agents generates delay and reduces agents' welfare. This occurs in spite of the lack of informational asymmetries or discriminatory offers. When this collective action problem is severe enough, agents prefer to give up considerable bargaining power in favor of the principal.

keywords: bargaining, delay, contracting externalities, political economy, vote buying.

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1. Introduction

In this paper, we study how bargaining power affects the efficiency of collective decisions in dynamic processes of coalition formation with externalities.

We focus on problems in which a principal bargains bilaterally with members of a group to obtain their agreement. Consider for example the President seeking to influence legislators of his own party to pass a policy proposal, a firm negotiating with buyers the adoption of a new technology with network externalities, an entrepreneur seeking to form a start-up, a raider attempting to takeover a target firm, or a firm or political candidate seeking the endorsement of opinion leaders.

A salient feature of these problems is that the principal typically bargains with agents sequentially. As a result, the offers the principal makes to, or receives from, an agent, will generally depend on how advanced the negotiation process is. This consideration becomes important when agents are farsighted, or fully rational, because the principal's ability to successfully transact with each agent depends on their expectations about the nature of future trades.

In this context, we study how the allocation of bargaining power between principal and agents affects whether good proposals are undertaken and bad ones rejected, whether good proposals are adopted with or without delay, and how rents are distributed between principal and agents. Would legislators be better off if each of them has a stronger bargaining position against a lobbyist or the executive? Would this lead to a less efficient policy-making process? Would markets with network externalities in which buyers have a stronger bargaining position delay innovation to better technologies?

Conventional wisdom suggests that in the absence of asymmetric information, increasing agents' bargaining power relative to the principal would improve agents' welfare and have no impact on the efficiency of collective decisions. We show, however, that both of these assertions are generally false. In particular, we show that when the principal's willingness to pay is high, redistributing bargaining power from the principal to the agents first induces and then increases delay, and reduces agents' welfare. As a result, agents are better off conceding substantial bargaining power to the principal.

In our model, a principal negotiates with a group of n agents bilaterally and sequentially.[1] There is no deadline for reaching an agreement, and no asymmetric information. In each meeting, the principal bargains with an agent over the terms by which the agent would commit his support to the principal. If an agreement is reached, the agent commits his support to the principal and exits negotiations. Otherwise, the agent remains uncommitted. The principal needs to obtain the agreement of $q < n$ agents to implement a reform, action, or policy change which affects the payoffs of all agents.[2] When this happens, the principal obtains a payoff $v > 0$, agents who committed their support to the principal obtain $z > 0$, and agents who remained uncommitted obtain $w \in \mathbb{R}$, where $w > 0$ ($w < 0$) implies that there are positive (negative) externalities on uncommitted agents, and $w = 0$ implies that there are no externalities on uncommitted agents. All players have a discount factor $\delta \in (0, 1)$.

To consider arbitrary allocations of bargaining power between the principal and each agent while maintaining the structure of the game fixed, we assume that in a bilateral meeting the principal makes an offer with probability $\phi \in [0, 1]$, and the agent makes an offer with probability $1-\phi$.[3] We then solve for Markov perfect equilibria of the game for each allocation of bargaining power $\phi \in [0, 1]$, where strategies depend on the number of agents the principal still needs in order to win. A well known feature in models of this kind is that if the principal can use discriminatory contracts, one can generate equilibria in which the principal obtains a large profit by exploiting coordination failures among agents (this point is made in Genicot and Ray (2006); see Segal and Whinston (2000), Cai (2000), Chowdhury and Sengupta (2012)). To rule this out, we focus on symmetric equilibria of the game.

We prove existence and uniqueness of equilibrium outcomes, and provide a complete characterization for the case in which the principal's willingness to pay is large. We show that, irrespective of the direction of the externalities, if the principal has enough bargaining power in bilateral negotiations the equilibrium is efficient, and agents' welfare increases with their bargaining power in bilateral negotiations. When agents have enough bargaining power,

[1]This contrasts with decentralized processes of coalition formation in the absence of a principal, a-la Gul (1989), Baron and Ferejohn (1989), Banks and Duggan (2000) or Gomes (2005).

[2]This can be taken as a particularly simple way to micro-found the source of potential externalities, but also appears quite literally in some applications, such as the approval of a bill requires a majority of senators (or some supermajority of a party), takeovers requires a majority of shares, or technology adoption with increasing returns to scale requires some q buyers to be on board.

[3]This formulation is formally equivalent to nesting an infinite horizon bilateral bargaining in our game, where one of the sides decides whether to enter in negotiations or not, and in any period of the negotiation phase after a proposal is rejected the principal (agent) makes offers with probability ϕ (respectively, $1 - \phi$).

instead, the equilibrium is inefficient, and agents' welfare decreases with their bargaining power in bilateral negotiations.

The inefficiency we identify is due to what we call a *collective hold-up* problem. When agents have the upper hand over the principal in bilateral negotiations, the principal anticipates that agents trading late in the process will extract a large fraction of the surplus, and is not willing to pay much to agents trading early on. This first part of the mechanism we identify corresponds to the standard hold-up problem, in which trading partners negotiate to divide their trade surplus after making relationship specific investments, in this case obtaining the support of other agents (Williamson (1979), Klein, Crawford, and Alchian (1978), Grossman and Hart (1986)).[4]

The key point here, however, is that by refusing to trade, agents can meet the principal later. This gives agents who meet early with the principal an incentive to hold-out, which more than compensates for the loss induced by delaying completion when the principal's willingness to pay is high. Holding out indefinitely cannot be an equilibrium, however, for this would destroy the incentives to hold out in the first place. But consistency is restored if the agent negotiating with the principal expects the *right* amount of delay in the event of not reaching an agreement with the principal. This is because delay in a given state of the bargaining process affects the value of holding out, but not the continuation value *after* agreement. In this way, the collective action problem among agents complements the principal's hold-up problem and creates delay and inefficiency.

Equilibrium outcomes for large v have three interesting features. First, we show that for a given allocation of bargaining power ϕ inducing delay, delay is front-loaded, in the sense that it occurs in the first k transactions. In these first k transactions, the expected delay for each deal increases as the process moves forward. But once the principal obtains the support of k agents, the remaining transactions occur without delay. Second, the number of transactions with positive expected delay is decreasing in ϕ, so that redistributing bargaining power from the principal to the agents expands the number of states in which transactions fail with positive probability. Third, we show that both the number of states in which there is delay with positive probability and the expected length of delay are increasing in the number of

[4]The hold-up problem led to a large literature in modern contract and organization theory, exploring institutional remedies against hold-up (see the articles cited in Che and Sákovics (2004)). In many situations, however, investments must be sunk before agents meet (e.g. Acemoglu and Shimer (1999), Cole, Mailath, and Postlewaite (2001)), or contracting is limited, as is often the case in political economy.

agents q whose approval is required for completion. This conforms to the intuition that more stringent majority rules are costly because they induce more delay.

For any given allocation of bargaining power for which there is delay in more than one transaction, the expected delay is increasing in the principal's willingness to pay. In fact, in the limit as v goes to infinity, the expected time for completion goes to infinity, and agents' payoffs go to zero. Thus, when agents anticipate that the collective hold-up problem would be severe, they prefer to grant considerable bargaining power to the principal, to the point of reducing significantly the number of transactions with delay.

It is important to clarify that delay can arise with positive externalities, no externalities, or even negative externalities on uncommitted agents. All else equal, a larger negative externality on uncommitted agents lowers the value of the lottery induced by holding-out. But when the principal's willingness to pay is sufficiently large and agents have sufficient bargaining power, holding out can still be attractive, and the main logic for delay is unaltered.

In the second part of the paper we consider the case in which an agent who committed his support to the principal obtains a non-positive payoff, as in corporate takeovers ($z = 0$) or vote buying with audience costs ($z < 0$). We show that when there are positive externalities on uncommitted agents, in any equilibrium in which the project is completed with positive probability there can only be delay when the principal is trying to obtain the support of the very first agent. With this exception, equilibrium either has no delay, or is such that there are no transactions in the initial state. Moreover, we show that when agents have large bargaining power, the equilibrium is inefficient, and there are no transactions even when agreement would be optimal.

The rest of the paper is organized as follows. In section 2 we place our paper in the context of the literature, and in section 3 we describe the model. In section 4 we present the result in a simplified setting. We fix $z = w > 0$ and analyze the two extreme cases in which either the principal or the agents have full bargaining power in bilateral meetings. In section 5 we present the main result in the general model. We conclude in section 6. All proofs are in the Appendix.

2. Related Literature

Our paper builds on the literature on *contracting with externalities* (see Grossman and Hart (1980), Rasmusen, Ramseyer, and Wiley Jr (1991), Rasmusen and Ramseyer (1994), Segal (1999, 2003), Segal and Whinston (2000), Genicot and Ray (2006)). These papers explore problems in which a single principal contracts with a group of agents in the presence of externalities among agents (e.g., corporate takeovers, exclusive contracts, public goods, lobbying).

A standard assumption in the literature is that the principal has full bargaining power.[5] As a result, we know very little about how the allocation of bargaining power affects equilibrium outcomes in this context. The one exception we are aware of is Galasso (2008), who considers a problem in which there are negative externalities across agents and trade is inefficient, but the principal benefits from trading. In this context, Galasso shows that when agents are sufficiently patient, the principal prefers to enter a finite horizon bargaining game in which she is the last mover, to a one shot game in which she makes a take-it-or-leave-it offer to agents. This happens because the repeated game allows the principal to profit from breaking coordination among agents.[6]

Our contribution is to consider arbitrary allocations of bargaining power between the principal and the agents while maintaining the structure of the game fixed. To do this, we build on the single principal version of Iaryczower and Oliveros (2017). We show that when the principal's willingness to pay is high, giving more power to the agents induces delay and reduces agents' welfare.

The emergence of delay in this context is noteworthy. Although delay with incomplete information is well studied, the possibility of inefficient delay in bargaining models with complete information is rare. Chatterjee, Dutta, Ray, and Sengupta (1993), Ray and Vohra (1999), Banks and Duggan (2006) and Gomes (2005) provide examples featuring delay in general bargaining models, and Iaryczower and Oliveros (2016) show existence of an equilibrium with delay in a model of decentralized legislative bargaining, where one agent emerges endogenously as an intermediary.[7]

Jehiel and Moldovanu (1995b) consider a model in which a seller tries to sell a single object to one of several potential buyers, where non-buyers suffer a negative externality that is dependent on the identity of the actual buyer. The seller meets agents randomly, and has

[5]This is also true in Iaryczower and Oliveros (2017), where we consider the effect of competition among principals on agents' welfare.

[6]With negative externalities, each agent wants to avoid being the last agent left to receive an offer from the principal, and as a result has an incentive to trade early at more favorable terms for the principal.

[7]In a general version of the Baron Ferejohn model, Banks and Duggan (2006) show that a stationary equilibrium with delay can only exist if the status quo is in the core, which is generally empty in multidimensional policy spaces, or when transfers are possible.

to sell the good to a buyer in less than $T < \infty$ periods. Jehiel and Moldovanu show that under some conditions there is a unique equilibrium with delay, in which transactions take place only a few stages before the end of the game. Delay appears here because as the deadline approaches, the threat that the seller sells the object to the agent who induces a larger negative externality on other agents increases (because the seller has to get rid of the object). This makes it optimal for the seller to wait in order to extract high prices from other agents, willing to avoid this negative outcome (note this is not necessarily inefficient).[8]

Closer to our paper, Cai (2000) considers a model in which a principal bargains with n agents sequentially, meeting the agents in a pre-specified order. The principal needs to get unanimous support from agents, and the bargaining protocol in each bilateral meeting is a single round of alternating offers. Cai shows that when players are sufficiently patient, there is a multiplicity of subgame perfect NE, including equilibria with and without delay. These two equilibria remain even after imposing the refinement that offers cannot depend on previously rejected offers. Differently than in our paper, delay here appears as a result of discriminating offers (Segal and Whinston (2000), Genicot and Ray (2006)) which can be constructed using the predetermined order of meetings. We explicitly rule this out by focusing on symmetric MPE, and show that delay appears (uniquely) when agents have enough bargaining power in bilateral meetings, even in the absence of discriminating contracts.

Other explanations for delay with complete information, less directly related to this paper, have been proposed. Fershtman and Seidmann (1993) show that if a player that rejects an offer is subsequently committed not to accept any poorer proposal, deadlines can lead to delay in bilateral bargaining (with large discount factor). Ma and Manove (1993) show that deadlines can also lead to delay if we assume that (i) a player is permitted to postpone the implementation of his move without losing his turn and (ii) after each offer is made, a random length of time elapses before the other player can respond. Merlo and Wilson (1995) show that efficient delay can emerge when the size of the surplus to be divided evolves stochastically over time. Yildiz (2004) and Ali (2006) show delay in bargaining with heterogeneous priors, and Acharya and Ortner (2013) show that delay can arise in bargaining over multiple issues with partial agreements.

[8] Jehiel and Moldovanu (1995a) extend the model to allow for positive externalities, and an infinite horizon. They show that with negative externalities, delay can also arise without deadlines, but that with positive externalities (as in our paper) delay can only arise in equilibria of the finite horizon model. In our paper, delay can emerge independently of the nature of externalities on non-traders.

3. The Model

There is a principal and a group of n agents who interact in an infinite horizon, $t = 1, 2, \ldots$. We say the principal wins if and when she obtains the support of $q < n$ agents. In each period t before the principal wins, any one of the $k(t)$ agents who remain uncommitted at time t meets the principal with probability $1/k(t) > 0$. In a meeting between the principal and an agent, principal and agent bargain over the terms of a deal by which i would support the principal. With probability $\phi \in [0, 1]$ the principal makes an offer $p \in \mathbb{R}$ to the agent, and with probability $1 - \phi$ the agent makes an offer $b \in \mathbb{R}$ to the principal. In both cases, the offer is a transfer from the principal to the agent (which can be positive or negative). If the recipient of the offer accepts it, i commits his support for the principal and the transfer takes place; if the offer is rejected, i remains uncommitted. Upon completion, the principal gets a payoff $v \in \mathbb{R}_+$, committed agents get $z \in \mathbb{R}_+$, and uncommitted agents get $w \in \mathbb{R}$. In any period before completion, all players get a payoff of zero, not including any transfer they have received or paid. Principal and agents have a discount factor $\delta \in (0, 1)$.

The solution concept is symmetric Markov perfect equilibria (MPE). The restriction to symmetric MPE rules out discriminatory contracts, in the spirit of Genicot and Ray (2006). In particular, the strategies of principal and agents only condition on the number of agents $m \leq q$ the principal still needs to obtain for completion. We let the state space be $M \equiv \{1, \ldots, q\}$. Offers when the principal and agents propose in state m are $p(m)$ and $b(m)$, respectively. We let $w(m)$ and $w_{out}(m)$ denote the continuation values of an uncommitted and a committed agent in state $m \in M$, and $v(m)$ denote the principal's continuation value in state $m \in M$.

Although quite simple, the model has a number of applications. To fix ideas, we sketch some of these here.

Corruption. We consider a simple model of bribes to corrupt bureaucrats, in the spirit of Olken and Barron (2009). Olken and Barron observe bribes paid by truck drivers to police, soldiers, and weigh station attendants in Indonesia. They model checkpoints as a chain of vertical monopolies, where the sequence of meetings is given, and the agreement of each checkpoint is needed for completion. In our model, instead, a firm needs to get the approval of q out of n bureaucrats, and does not have to get these approvals in a given sequence. This is as in McMillan and Zoido (2004), who document the details of corruption in Peru in the

1990s under President Alberto Fujimori, observing the bribes Fujimori's secret police chief Vladimiro Montesinos paid to bribe judges, politicians and the news media.[9] An interesting fact that emerges from both of these papers is that there is substantial bargaining for bribes. Olken and Barron (2009) show that prices are in part set through ex post bargaining rather than being fully determined ex ante, while Montesinos' videotapes show him haggling with the bribe takers (McMillan and Zoido (2004)). We assume that if the project is greenlighted, the firm gets an expected payoff $v > 0$, and the bureaucrat who supports the project obtains $z > 0$ (possibly due to more benefits down the line), while $w \geq 0$ or $w \leq 0$ depending on whether the project benefits or hurts the population at large.

New Technologies with Increasing Returns to Scale. Consider exclusive deals contracts for the introduction of a new product with network externalities (Katz and Shapiro (1992), Segal and Whinston (2000)). Suppose there are n buyers and an incumbent producing with an old technology, in a market that can accommodate at most one supplier due to increasing returns to scale or network externalities. Under the incumbent supplier, buyers obtain a per period payoff which we normalize to zero. A challenger P can supply the market with a new technology, but entry is profitable only if it can serve at least q buyers. In each period, the challenger negotiates with a potential buyer an exclusive deal contract, which can include some advantage in service or tailored design. If q buyers sign exclusive deals, the challenger enters and the incumbent drops out. In this case the challenger firm gets a payoff $v > 0$, buyers who didn't sign get $w > 0$ and buyers who signed agreements get $z \geq w$.

Start-Ups. A firm needs to hire q specialized workers to produce a new product. Upon starting production, the firm obtains an expected payoff of $v > 0$, while each of the workers gets profit participation leading to an expected value $z > 0$. To sign the workers to the company, the firm negotiates with each worker a sign-up bonus. Workers that are not hired by the firm do not benefit (or suffer) from the company's activities, so $w = 0$.

Organization of a Protest. A national union wants to set up a large protest against the government. To do this, the union leadership can offer concessions to convince each of the local or industry chapters to mobilize their members against the government. The protest is successful if the national leadership secures the agreement of q of the local bosses. If organized, the union gets $v > 0$, the local chapters participating get $z > 0$ and the chapters not participating get $w > z$ (a similar logic holds with the organization of a coup, with the

[9]As this paper documents, to form a winning coalition in the Congress, Montesinos had to bribe only some of the opposition politicians. Similarly, "In the Supreme Court, decisions are made by majority vote, so three of the five Supreme Court judges were enough." (McMillan and Zoido (2004))

rebel elite bargaining with the commander of each regiment).

In Section 5.3 we consider the case $z \leq 0$. This allows us to extend our analysis to other applications, including corporate takeovers ($z = 0$) or vote buying with audience costs ($z < 0$).

4. Benchmarks

To convey the key insights of the paper in the simplest way possible, we begin by analyzing the two extreme cases in which either the principal or the agents have full bargaining power in bilateral meetings ($\phi = 1$ and $\phi = 0$ respectively), with $z = w > 0$.

4.1. **Principal has Full Bargaining Power.**

The case in which the principal has all the bargaining power ($\phi = 1$) was analyzed in Iaryczower and Oliveros (2017). Here we briefly review the equilibrium characterization.

Consider an arbitrary state $m \in M$. Note that the agent meeting the principal will accept an offer $p(m)$ only if $\delta w_{out}(m-1) + p(m) \geq \delta w(m)$, and will accept the offer with probability one if this inequality holds strictly. Thus, whenever the principal makes an offer to agent i in state m, she offers

$$p(m) = -\delta[w_{out}(m-1) - w(m)]. \tag{1}$$

The principal is willing to make the offer $p(m)$ in state m if $p(m) \leq \delta[v(m-1) - v(m)]$, or, substituting, if and only if the bilateral surplus of moving forward is nonnegative; i.e.,

$$s(m) \equiv [v(m-1) - v(m)] + [w_{out}(m-1) - w(m)] \geq 0 \tag{2}$$

Equilibrium strategies can differ in the probability of trade in each state. Let $\gamma_m \in [0,1]$ and $\alpha_m \in [0,1]$ denote, respectively, the probability that the principal makes an offer $p(m)$ and that the agent accepts the offer $p(m)$ in state m, and let $\lambda_m \equiv \gamma_m \alpha_m$ denote the probability of trade in state m. We argue that the strategy profile in which there is trade in every state, or $\lambda_m = 1$ for all $m \leq q$, is an equilibrium (we call this a full trading equilibrium, of FTE for short). Moreover, this equilibrium is unique.

Proposition 4.1. *The game with $\phi = 1$ and $z = w$ has a unique MPE, a full trading equilibrium. In this equilibrium, the payoff of an agent is given by*

$$w^P(m) = \left[\prod_{k=1}^{m} r(k)\right] \delta^m w \quad \text{with} \quad r(k) \equiv \left(\frac{n-q+k-1}{n-q+k-\delta}\right) \tag{3}$$

The intuition for why the FTE exists is as follows. First, fix the proposed equilibrium. Since $v > 0$ and $w > 0$, when the principal needs to collect the support of only one additional agent ($m = 1$), principal and agent can create a positive surplus by moving forward. Thus, given full information, there is a price at which this transaction occurs. And since the principal makes the proposal, the solution gives the principal a positive rent. Now consider the situation in which there are m agents remaining. Since in equilibrium there is trade whenever the principal needs to secure the support of $t < m$ additional agents, in state m the principal and the selected agent can also realize a positive surplus if they move forward. Therefore there is again a price at which trade can occur, which gives the principal a positive rent.

To check more formally whether a particular strategy profile is a MPE we need to check consistency with (2).[10] In a FTE, in particular, we need to verify that $s(m) \geq 0$ for all $m \leq q$ when $w(\cdot), w_{out}(\cdot)$ and $v(\cdot)$ are computed for a FTE.

Consider the value of an uncommitted agent i in state m in a FTE, $w(m)$. With probability $\beta(m) \equiv 1/(n - q + m)$, i meets the principal, who offers him a transfer satisfying (1), leading to a continuation payoff $\delta w(m)$. With probability $1 - \beta(m)$, some agent $j \neq i$ meets the principal and ultimately agrees to support her, leaving i with a continuation value $\delta w\,(m-1)$. Solving the difference equation with initial condition $w(0) = w$ gives (3). If instead the agent is already committed in state m, $w_{out}\,(m) = \delta w_{out}\,(m-1)$, and thus

$$w^P_{out}\,(m) = \delta^m w. \tag{4}$$

Using (3) and (4) we can compute the equilibrium transfer as a function of primitives, and then solve for the principal's value function, which obeys the recursive representation $v(m) = \delta v(m-1) - p(m)$. Using $v^P(\cdot), w^P(\cdot)$ and $w^P_{out}(\cdot)$ in the surplus condition (2), we have that

$$s(m) = (1-\delta)\delta^{m-1}\left[v + w\sum_{l=1}^{m}\left(1 - \delta\prod_{k=1}^{l} r(k)\right)\right] \geq 0 \quad \forall m \leq q.$$

Uniqueness can be established via an induction argument. Consider any MPE and suppose that there is trade whenever the principal needs to secure the support of $t < m$ additional agents. If in equilibrium there were no trade with positive probability, principal and agent

[10]In particular, we must have $s(m) \geq 0$ when $\lambda_m = 1$, $s(m) \leq 0$ when $\lambda_m = 0$, and $s(m) = 0$ when $\lambda_m \in (0, 1)$. To see this last point, suppose that in equilibrium $s(m) > 0$. Then it must be that $\alpha_m = 1$, for otherwise the principal could obtain a discrete gain in payoffs by increasing her offer slightly, as any such offer would be accepted. And then it must be that $\gamma_m = 1$. It follows that if $\lambda_m \in (0, 1)$ then $s(m) = 0$.

would obtain a lower combined payoff in state m than in the FTE. Hence the gain from moving forward would be higher than in the FTE, and thus positive. It follows that the principal will make an offer, which the agent will accept (for a formal proof of uniqueness, see Iaryczower and Oliveros (2017)).

Proposition 4.1 implies that in equilibrium the principal cannot extract all surplus from the agents. The reason for this is similar to the logic behind under-provision of a public good. Note that since the agents benefit from implementing the alternative to the status quo and they cannot obtain a higher terminal payoff by remaining uncommitted ($z = w$), the principal actually charges them to move on. By rejecting the offer, however, an agent can rely on others to pay the bill. This generates an outside option that gives each agent some bargaining power over the principal. Since the cost of deferring implementation of the proposal decreases with δ, the value of the outside option is increasing in δ, and so is agents' equilibrium payoff. In fact, as δ approaches 1, $r(m) \to 1$ and $w^P(m) \to w$.[11]

4.2. **Agents have Full Bargaining Power.** We now consider the case in which agents have full bargaining power in bilateral negotiations with the principal; i.e., $\phi = 0$. We show that this shift in bargaining power dramatically changes the nature of equilibrium outcomes, inducing inefficient delay, due to the fact that agents trading later with the principal can extract a disproportionate fraction of the surplus generated by the completion of the project. We call this a *collective hold-up* problem.

The nature of the collective hold-up problem is particularly transparent when agents have full bargaining power. Note that as in the previous case, the offer $b(m)$ that an agent makes to the principal in state m is uniquely pinned down: since the principal accepts an offer $b(m)$ in state $m \in M$ iff $\delta v(m-1) - b(m) \geq \delta v(m)$, whenever the agent makes an offer to the principal he offers

$$b(m) = \delta[v(m-1) - v(m)]. \tag{5}$$

By (5), the continuation value of the principal in state m is $\delta v(m)$ independently of whether the agent makes an offer or not. It follows that $v(m) = \delta v(m)$, which implies that $v(m) = 0$ for all $m \geq 1$. Because the critical agent trading in state $m = 1$ extracts all the principal's

[11] An indirect consequence of the assumption that $w = z > 0$ is that when $\phi = 1$, transfers from principals to agents are negative, a feature that can be unappealing in some applications. In the general version of the model where we allow payoffs to depend on whether each agent supported the principal or remained uncommitted, most relevant applications involve positive transfers from the principal to agents under reasonable assumptions on parameters, even when the principal has full bargaining power.

surplus from completion of the project with his offer $b(1) = \delta v$, the principal is not willing to pay in previous states to move the process forward. Substituting in (5), equilibrium transfers from the principal to the agents are

$$b(m) = \begin{cases} \delta v & \text{if } m = 1 \\ 0 & \text{for } m \geq 2. \end{cases} \tag{6}$$

The fact that agents meeting the principal in earlier states $m > 1$ cannot extract additional rents from the principal gives them an incentive to hold out their support, which increases with v. This individual behavior, of course, is not consistent with equilibrium. If the agent meeting the principal in state m never agrees to give his support to the principal the process does not move forward, and the critical state $m = 1$ is never reached in the first place. As we show below, consistency is restored with delay in reaching agreements.

Let $\hat{\lambda}_m$ denote the probability of trade in state m when the agents have full bargaining power.[12] In equilibrium $s(m) \geq 0$ when $\hat{\lambda}_m = 1$, $s(m) \leq 0$ when $\hat{\lambda}_m = 0$, and $s(m) = 0$ when $\hat{\lambda}_m \in (0,1)$, where $s(m)$ is given by (2) as before. Note that since $v(m) = 0$ for all $m \in M$, for any state $m > 1$, $s(m) \geq 0$ if and only if $w_{out}(m-1) \geq w(m)$. In the absence of side payments, the probability of trade depends on the relative value for an agent of moving the process along supporting the principal for free, $w_{out}(m-1)$, versus holding out support with the goal of extracting the rent δv in late trading, $w(m)$.

The values $w_{out}(m)$ and $w(m)$ for a committed and uncommitted agent depend on expectations of the probability of trade in states $m' \leq m$. Since once committed, agents do not engage in further negotiations, the payoff of a *committed* agent i in state m is

$$w_{out}(m) = \hat{\lambda}_m \delta w_{out}(m-1) + (1 - \hat{\lambda}_m) \delta w_{out}(m),$$

and solving recursively,

$$w_{out}(m) = \left[\prod_{j=1}^{m} \frac{\delta \hat{\lambda}_j}{1 - \delta(1 - \hat{\lambda}_j)} \right] w, \tag{7}$$

On the other hand, the payoff of an *uncommitted* agent in state $m > 1$ is

$$w(m) = \hat{\lambda}_m \left[\beta(m) \delta w_{out}(m-1) + (1 - \beta(m))\, \delta w(m-1) \right] + (1 - \hat{\lambda}_m) \delta w(m),$$

where as before $\beta(m) \equiv 1/(n - q + m)$ denotes the probability that an agent i meets the principal in state $m \in M$. Whether agent i or another agent $j \neq i$ negotiates with the

[12] We let $\hat{\gamma}_m \in [0,1]$ denote the probability that an agent makes the offer $b(m)$ in state m, $\hat{\alpha}_m \in [0,1]$ the probability that the principal accepts the offer $b(m)$ in state m, and let $\hat{\lambda}_m \equiv \hat{\gamma}_m \hat{\alpha}_m$ denote the probability of trade in state m.

principal, an unsuccessful meeting implies that the system stays put at m, which gives i a discounted continuation payoff $\delta w(m)$. A successful meeting, instead, moves the process to state $m-1$ with the agent being committed to the principal with probability $\beta(m)$. Solving recursively,

$$w(m) = \left[\prod_{j=1}^{m} \frac{\delta\hat{\lambda}_j}{1-\delta(1-\hat{\lambda}_j)}\right](w+\beta(m)v)\,. \tag{8}$$

Given $v(1)=0$, the surplus in the critical state $m=1$ is given by $s(1)=v+w-w(1)$. Now, note that $w(1)$ is maximized at $\hat{\lambda}_1=1$, where it attains the value $\tilde{w}(1)=\delta\,(w+\beta(1)v) < v+w$. Thus $s(1)>0$ for any $\hat{\lambda}_1 \in [0,1]$, and in equilibrium there is no delay in $m=1$. Using (7) and (8), the condition for trade with positive probability at $m>1$ that $w_{out}(m-1) \geq w(m)$ boils down to

$$w \geq \left[\frac{\delta\hat{\lambda}_m}{1-\delta(1-\hat{\lambda}_m)}\right](w+\beta(m)v) \tag{9}$$

For delay to occur with positive probability at m, we need (9) to hold with equality. Now, note that since $w(m)$ is increasing in the probability of trade in state m, $\hat{\lambda}_m$, while $w_{out}(m-1)$ is independent of $\hat{\lambda}_m$, the right hand side is a continuous increasing function $f(\cdot;m)$ of $\hat{\lambda}_m$ such that $f(0;m)=0$ and $f(1;m)=\delta\,(w+\beta(m)v)$.

Since (9) is satisfied with $\hat{\lambda}_m=0$, this implies that in equilibrium there is always trade with positive probability in all states $m>1$: if an agent i who is in the position to trade with the principal in state m anticipates that no agent would ever trade with the principal in that position, i has incentives to trade. Thus no trading in m cannot be part of an equilibrium. On the other hand, there exists a (unique) solution $\hat{\lambda}_m \in (0,1)$ satisfying (9) with equality if and only if

$$w < \delta\,(w+\beta(m)v) \quad\iff\quad m < \frac{\delta}{(1-\delta)}\frac{v}{w} - (n-q) \equiv \overline{m} \tag{10}$$

It follows immediately from this that:

Lemma 4.2. *There exists a unique cutpoint $\overline{m} \in \{2,\ldots,q+1\}$ such that, in equilibrium, there is delay in each state $m : 2 \leq m < \overline{m}$, and trade with probability one in any $m \geq \overline{m}$.*

From eq. (10), the set of states in which there is delay is weakly increasing in v/w, which captures the relative value of holding out, and for any $m \in M$ there is a v/w large enough such that $m < \overline{m}$. The ratio v/w also increases the probability of delay in states below the cutpoint. In fact, note that when there is delay in state m, the probability of trade is given by the $\hat{\lambda}_m \in (0,1)$ solving $w_{out}(m-1) = w(m)$, or

$$\hat{\lambda}_m = \left(\frac{1-\delta}{\delta}\right)\frac{w}{v}\frac{1}{\beta(m)} \tag{11}$$

Note that from (11), the probability of trade is increasing in m. Therefore, we expect transactions to occur at a faster pace initially, with the process of negotiations slowing down as it goes along. Similarly, note that both the set of states in which there is delay and the probability of delay in states below the threshold are increasing in the size of the coalition required to win. This corresponds well to the intuition that more stringent supermajority requirements are costly because they induce delay.

Remark 1. The expected time for completion increases with the stringency of the q-rule. □

The previous discussion fully characterizes equilibria of the game in which agents have all the bargaining power up to the precise determination of the threshold $\overline{m}$. This threshold, in turn, is pinned down uniquely for given parameters by (10). We are interested in particular in equilibrium for large v, where the collective hold-up problem is severe. The next proposition summarizes our discussion focusing on this case.

Proposition 4.3. *Consider the game with $\phi = 0$ and $z = w > 0$. Suppose $v \geq \frac{(1-\delta)}{\delta} nw \equiv \overline{v}$. Then there is an (essentially) unique equilibrium with trading at $m = 1$ and delay in all $m : 2 \leq m \leq q$, given by (11).*[13]

Agents' payoffs are

$$w^A(q) = \left(\prod_{j=2}^{q-1} \frac{w}{w+\beta(j)v}\right)\delta w, \quad \textit{and} \quad \lim_{v\to\infty} w^A(q) = 0$$

As Proposition 4.3 shows, when the collective hold-up problem is severe, there is delay in all but the critical state $m = 1$. Moreover, delay is increasing in v. This poses a tradeoff for agents' welfare: a larger v increases the total surplus from transacting, but also leads to larger delay. As the proposition shows, in equilibrium the larger delay more than compensates for the increase in total surplus and leads to a loss of welfare for the agents. Proposition 4.3

[13] We say that the equilibrium is essentially unique because any pair $(\hat{\gamma}_m \hat{\alpha}_m)$ such that $\hat{\lambda}_m = \hat{\gamma}_m \hat{\alpha}_m$ satisfies (11) is an equilibrium. What matters for equilibrium is the expectation of delay, and not whether this occurs because of a lower probability that the agent makes a proposal, $\hat{\gamma}_m$ or a lower probability that the principal accepts this offer, $\hat{\alpha}_m$.

thus has the direct implication that when the principal's valuation for winning is sufficiently large, if able to choose, agents would prefer the situation in which the principal has full bargaining power to that in which the agents have full bargaining power.

Corollary 4.4. *For v sufficiently large, agents are better off when the principal has full bargaining power than when agents have full bargaining power: $w^P(q) > w^A(q)$.*

We should point out that the result above (Corollary 4.4) does *not* hold under unanimity, which is the classic railroad-farmers example considered by Coase (see Cai (2000), Olken and Barron (2009), Chowdhury and Sengupta (2012)). With unanimity, all the analysis of the game in which agents have full bargaining power remains unchanged, and results are essentially unaltered. While the analysis of the game in which the principal has full bargaining power also remains unchanged, agents' equilibrium payoffs in this case are zero, $w^P_{\text{un}}(q) = 0$. To see this, note that with $q = n$, $\beta(1) = 1$, so in the critical state the agent cannot free ride on others. Thus $w(1) = \delta w(1)$, which implies $w(1) = 0$. But then, recursively, $w(m) = 0$ for all $m \in M$. Since $w^A_{\text{un}}(n)$ approaches 0 as $v \to \infty$ but doesn't attain zero, with unanimity, $w^P_{\text{un}}(n) < w^A_{\text{un}}(n)$.

To sum up our analysis so far, we have shown that when principal and agents interact sequentially with bilateral binding agreements and agents are the only ones making offers, a collective hold-up problem emerges, which gives agents incentives not to trade with the principal. When the potential rent extraction is too tempting, the incentives to hold out generate delay in equilibrium, reducing agents' welfare. In this situation, agents are better off structuring negotiations so that the principal has sole bargaining power.

5. Main Results

In the previous section we studied how outcomes and welfare change in the two polar cases in which either the principal or the agents have full bargaining power in bilateral meetings. These extreme allocations of bargaining power are useful as a benchmark, but often unrealistic. Moreover, the extreme allocations of bargaining power are special in the sense that one side extracts the entire surplus generated by the transaction, while in general principal and agent both benefit from the exchange. This implies that in general the system of difference equations characterizing equilibrium payoffs can not be decoupled as we have done in the previous sections.

To tackle this problem more generally, we introduce counteroffers in a way that allows us to vary smoothly the power of agents: we assume that with probability $\phi \in (0, 1)$ the principal makes an offer to the selected agent, and with probability $1 - \phi$ the agent makes an offer to the principal. Note that this is formally equivalent to nesting an infinite horizon bilateral bargaining in our game, where one of the sides decides whether to enter in negotiations or not, and in any period of the negotiation phase after a proposal is rejected the principal (agent) makes offers with probability ϕ (respectively, $1-\phi$). In fact, in any meeting between the principal and an agent in a state $m \in M$, both games give the principal a value $\phi s(m)$, and the agent a value $(1 - \phi)s(m)$, conditional on trading. We also allow the payoffs of committed and uncommitted agents upon completion to be different. In particular, we allow arbitrary $z \in \mathbb{R}_+$ and $w \in \mathbb{R}$, as in the applications we described.[14]

5.1. **Preliminaries.** We begin by establishing some key properties of equilibrium payoffs. Recall that $\hat{\lambda}_m \equiv \hat{\gamma}_m \hat{\alpha}_m$ and $\lambda_m \equiv \gamma_m \alpha_m$ denote the probability of trade when the agents propose and when the principal proposes, respectively, and let $\mu_m \equiv \phi\lambda_m + (1-\phi)\hat{\lambda}_m$ denote the ex ante probability of trade in state m.

Consider the value of the principal in state m, $v(m)$. With probability $\phi\lambda_m$, the principal has agenda setting power and makes an offer that is accepted by the agent, getting a payoff $\delta v(m - 1) - p(m)$. With probability $1 - \phi\lambda_m$ either there is no transaction in m or there is a transaction following a proposal by the agent, and the principal obtains a discounted continuation value $\delta v(m)$. Thus

$$v(m) = \phi\lambda_m \left(\delta v(m - 1) - p(m)\right) + (1 - \phi\lambda_m)\delta v(m).$$

Using (1), and subtracing $\phi\lambda_m \delta v(m)$ on both sides, we have

$$v(m) = \left(\frac{\delta}{1 - \delta}\right) \phi s^+(m), \tag{12}$$

where $s^+(m) = \max\{s(m), 0\}$. Equation (12) says that the value of the principal in state m is proportional to the surplus in state m whenever this is positive, and zero otherwise. The expression eliminates the dependency on the probability of trade λ_m using the fact that if $s(m) > 0$ then $\lambda_m = 1$, if $s(m) < 0$ then $\lambda_m = 0$, and that $s(m) = 0$ when $\lambda_m \in (0, 1)$. A key implication of (12) is that the principal's equilibrium payoff in state m is proportional to the surplus $s(m)$ by a factor that increases with the principal's nominal bargaining power ϕ. Because delay can only occur in equilibrium if $s(m) = 0$, this means that if there is delay in state m in equilibrium, then $v(m) = 0$.

[14]In Section 5.3 we consider the case of $z \leq 0$. We show that in this case there is a breakdown of negotiations.

Consider instead the value of an uncommitted agent i in state m, $w(m)$, recalling that $\beta(m) \equiv 1/(n+m-q)$ denotes the probability that agent i meets the principal. With probability $\beta(m)(1-\phi)\hat{\lambda}_m$, agent i meets the principal, has agenda setting power, and makes an offer $b(m)$ (which is accepted), leading to a payoff $\delta w_{out}(m-1)+b(m)$. With probability $(1-\beta(m))\,\mu_m$ another agent $j \neq i$ meets the principal, and the meeting results in a transaction, leading to a payoff $\delta w(m-1)$ for player i. In all other cases (i meets the principal but either the principal has agenda setting power or the transactions falls through, or some other agent $j \neq i$ meets the principal but the transaction falls through), agent i gets a continuation payoff $\delta w(m)$:

$$\begin{aligned} w(m) = {} & \beta(m)(1-\phi)\hat{\lambda}_m\left[\delta w_{out}(m-1)+b(m)\right]+(1-\beta(m))\,\mu_m \delta w(m-1) \\ & + \Big[\beta(m)[\phi+(1-\phi)(1-\hat{\lambda}_m)]+(1-\beta(m))\,(1-\mu_m)\Big]\,\delta w(m) \end{aligned}$$

Using (5) for the transfer $b(m)$ and simplifying, we have that for all $m \geq 2$,[15]

$$w(m) = \left[\frac{\delta\beta(m)}{1-\delta\beta(m)}\right](1-\phi)s^{+}(m)+\left[1+\left(\frac{1-\delta}{1-\beta(m)}\right)\frac{1}{\delta\mu_m}\right]^{-1} w(m-1). \tag{13}$$

The agent's equilibrium payoff in state m has two components. The first is proportional to the surplus $s(m)$ whenever this is positive, by a factor that increases with the agents' bargaining power $1-\phi$. Differently to the principal's value, the agent's value $w(m)$ is positive even when $s(m)=0$, with the second component being a positive fraction (increasing in the probability of trade in state m, μ_m) of the state $m-1$ value $w(m-1)$.

The value of a committed agent, instead, only depends on the probability that the process moves forward or not: if there is a transaction (with probability μ_m) the committed agent gets a continuation payoff $\delta w_{out}(m-1)$, and otherwise gets $\delta w_{out}(m)$. Solving recursively, we obtain

$$w_{out}(m) = \left[\prod_{k=1}^{m}\left(\frac{\delta\mu_k}{1-\delta(1-\mu_k)}\right)\right] z \tag{14}$$

We can now prove our first result of this section. We show that equilibrium exists, and is unique up to the probability of trade μ. Moreover, we show that in equilibrium trade never collapses, in the sense that in all non-terminal states $m \in M$, the principal transacts with an agent with positive probability; i.e., $\mu_m > 0$ for all $m \leq q$. We also characterize

[15]As before, we have used the fact that if $s(m) > 0$ then $\hat{\lambda}_m = \mu_m = 1$, if $s(m) < 0$ then $\hat{\lambda}_m = \mu_m = 0$, and that $s(m) = 0$ when $\mu_m \in (0,1)$.

the probability of trade in each state $m \leq q$ as a function of continuation values $w_{out}(m-1), v(m-1)$ and $w(m-1)$. For any $m \geq 1$, let $\Gamma(m) \equiv w(m)/(v(m)+w_{out}(m))$. Then

Proposition 5.1. *There exists an essentially unique equilibrium, characterized by trade probabilities*

$$\mu_m = \min\left\{1, \left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right)\left(\frac{1}{\Gamma(m-1)-1}\right)\right\} > 0 \quad \forall m \in M. \tag{15}$$

To see the logic for this result, note first that with $v, z > 0$, a critical meeting ($m = 1$) must have trade with positive probability (Lemma A.2), and thus $v(1) + w_{out}(1) > 0$. Now suppose that for all $k < m$ there are transactions with positive probability, and take the implied continuation values $w_{out}(m-1), v(m-1)$ and $w(m-1)$ as given. Note that since $\mu_k > 0$ for all $k < m$, then $w_{out}(m-1) > 0$ and $v(m-1) > 0$. Thus inaction at m is not an equilibrium, for then $v(m) = w(m) = 0$ and $s(m) = v(m-1) + w_{out}(m-1) > 0$, giving principal and agent an incentive to trade. We then show that the "one-shot" game in state m has a unique SPE, which has no delay if $\Gamma(m-1)$ is sufficiently small, and otherwise has delay with positive probability.[16] The result then follows by induction.

5.2. **Bargaining Power and Equilibrium Outcomes.** We now turn to our main goal of studying how the allocation of bargaining power among principal and agents affects delay and agents' welfare. A natural starting point is to study the conditions under which the equilibrium involves no delay. To do this, we begin by characterizing agents' equilibrium payoffs in a FTE as a function of primitives. We then use the characterization of FTE payoffs $w^\dagger(m)$ to obtain a necessary and sufficient condition for no delay in equilibrium.

Differently to the polar cases we analyzed before, the fact that here both principal and agents make proposals with positive probability means that principal and agents can mutually extract rents from one another. This implies that the system of difference equations characterizing equilibrium payoffs can not be decoupled as in Section 4.1, where we could solve for agents' values independently, use these values to express transfers as a function of primitives, and then solve for the principal's equilibrium payoffs. To tackle this difficulty, in the proof we use a transformation to express the system of difference equations as a second order difference equation, which we then solve. Proposition 5.2 is in fact a corollary

[16]The ratio of $w(m-1)$ to $w_{out}(m-1)+v(m-1)$ is relevant for delay because both $v(m)$ and the first component of $w(m)$ vanish with $s(m) = 0$. This means that for any given value of $w_{out}(m-1)+v(m-1)$, we can only have $s(m) = 0$ for some $\mu_m \in (0,1)$ if $w(m-1)$ (the only source of positive value for the agents when $s(m) = 0$) is large enough so that $w(m) = w_{out}(m-1)+v(m-1)$ for such $\mu_m \in (0,1)$.

of Lemma A.3 in the Appendix, where we solve the agents' value function for any trade probabilities $\vec{\mu}$.

Proposition 5.2. *In a FTE of the subgame starting at $m \le q$, $w(m) = w^\dagger(m)$, where letting $\overline{\theta}_{km} \equiv \prod_{j=k}^{m} \left(\frac{\delta\phi}{1-\delta+\delta\phi(1-\beta(j))}\right)$, for any $m \le q$, $w^\dagger(m)$ is given by*

$$\frac{w^\dagger(m)}{\beta(m)} \equiv \overline{\theta}_{1m}(n-q)w + \sum_{k=1}^{m} \frac{1-\delta}{\delta}\frac{1-\phi}{\phi}\overline{\theta}_{km}\delta^k\left(v + kz + (n-q)w\right) \tag{16}$$

and

$$v^\dagger(m) = \left(\frac{\delta\phi}{1-\delta(1-\phi)}\right)^m v - \left(\sum_{r=1}^{m}\left(\frac{\delta\phi}{1-\delta(1-\phi)}\right)^r\right)\left(w^\dagger(m) - \delta^{m-1}z\right)$$

The FTE value $w^\dagger(m)$ is increasing in the terminal payoffs of the principal v, as well as committed and uncommitted agents, (z, w). Establishing the effect of ϕ on $w^\dagger(m)$ from (16) is more involved because for any $m > 1$, increasing ϕ has two distinct effects on $w^\dagger(m)$. On the one hand, reducing ϕ directly increases the fraction of the surplus that the agent trading with the principal in m can obtain. On the other hand, precisely because of this direct effect, reducing ϕ reduces the principal's value in earlier states, and thus the surplus available in states $m' > m$. However, we can show that $w^\dagger(m)$ is in fact decreasing in ϕ whenever the equilibrium of the m-subgame is a FTE. Note that from (12), $v(m) = \left(\frac{\delta}{1-\delta}\right)\phi s^+(m)$. Since in a FTE the surplus $s(m)$ does not change with ϕ, it follows that $v^\dagger(m)$ is increasing in ϕ. Now, total welfare in state m is

$$J(m) \equiv v(m) + (n-q+m)w(m) + (q-m)w_{out}(m)$$

Since in a FTE both $J(m)$ and $w_{out}(m)$ are constant on ϕ, it follows that

$$v^\dagger(m;\phi) - v^\dagger(m;\phi') = -(n-q+m)[w^\dagger(m;\phi) - w^\dagger(m;\phi')]$$

and thus $w^\dagger(m)$ is decreasing in ϕ whenever the equilibrium of the m-subgame is a FTE.

Using the characterization of FTE payoffs $w^\dagger(m)$, we obtain a necessary and sufficient condition for no delay in equilibrium. Moreover, we show that for large v (when the hold-up problem has bite), the unique equilibrium of any m-subgame is efficient if the principal has sufficient bargaining power, but exhibits delay if the agents have sufficient bargaining power. Here and in the rest of the paper, we write the statement "for v large, [A] is true" to mean that for fixed parameters other than v, there exists a $\overline{v} > 0$ such that if $v \ge \overline{v}$, [A] is true.

Proposition 5.3 (No Delay). *(i) There exists a FTE in the subgame starting in state m', with agents' equilibrium payoffs $w^\dagger(m')$ iff*

$$T^\dagger(m) \equiv \frac{w^\dagger(m)}{\beta(m)} - \delta^m \left(v + mz + (n-q)w\right) \leq 0 \quad \forall m \leq m'. \tag{17}$$

Moreover, for large v the following is true: for any $m \leq q$, (ii) there exists $\overline{\phi}(m) \in (0,1)$ such that if $\phi > \overline{\phi}(m)$, the unique MPE of the m-subgame is a FTE, and (iii) there exists $\underline{\phi}(m) \in (0,1)$ such that if $\phi < \underline{\phi}(m)$, the unique MPE of the m-subgame entails delay.

Part (ii) of the proposition generalizes the result of Proposition 4.1, which showed that the unique equilibrium with $w = z$ and $\phi = 1$ is a FTE. This is true as long as the principal has enough bargaining power, for any $z \in \mathbb{R}_+$ and $w \in \mathbb{R}$.

When instead the agents have enough bargaining power, the equilibrium involves delay. In fact, part (iii) of the proposition says that for any $m \in M \setminus \{1\}$ there is a sufficiently low ϕ such that the unique MPE of the m-subgame entails delay. Taking the intersection of these conditions, this implies that there is a ϕ low enough for which there is delay in all but the critical state, as in Proposition 4.3 of the benchmark model.

Proposition 5.3 indicates when we can expect delay along the bargaining process, but is silent about the features of expected delay. We could have cycles of trade, as in Jehiel and Moldovanu (1995a), or the probability of trade could be monotonic, as it is the case for all $m \geq 2$ in Proposition 4.3. We could have delay at the beginning of the bargaining process, at the end, or in some interior subset of states. Our next result sheds some light on the first of these questions. We show that in *any* equilibrium in which there is delay in states $\{\underline{m}, \ldots, \overline{m}\}$, for any state $m \geq \underline{m} + 1$, the probability of trade is monotonic. In particular, we show that the probability of trade grows with m at a rate equal to $\beta(m)$, independent of the allocation of bargaining power ϕ, the discount factor δ, or the valuations v, w and z.

Lemma 5.4. *Suppose in equilibrium $\mu_m \in (0,1)$ for all $m \in \{\underline{m}, \ldots, \overline{m}\}$. Then*

$$\frac{\mu_{m+1} - \mu_m}{\mu_m} = \beta(m) \quad \forall m \in \{\underline{m}+1, \ldots, \overline{m}-1\},$$

and thus in particular $\mu_{m+1} > \mu_m$ for all $m \in \{\underline{m}+1, \ldots, \overline{m}-1\}$.

As we saw in Proposition 5.3, when agents have enough bargaining power, there is delay in all states $m \geq 2$. In this case the previous lemma characterizes equilibrium completely, with the exception of the rate of delay at $\underline{m} = 2$. Delay at $m = 2$, in turn, is given by the expression $0 = s(2) = v(1) + \delta z - w(2)$, or using (13) with $s(2) = 0$,

$$\left(\frac{\delta \mu_2}{\left(\frac{1-\delta}{1-\beta(2)}\right) + \delta \mu_2} \right) = \frac{v(1) + \delta z}{w(1)}.$$

Since in equilibrium with large v there is full trading in $m = 1$, it is easy to compute $v(1)$ and $w(1)$ and show that $0 < \frac{\partial v(1)}{\partial v} < \frac{\partial w(1)}{\partial v}$, so that μ_2 (and then all μ_k) is decreasing in v, and goes to zero as $v \to \infty$.[17]

Lemma 5.4 pins down the growth rate of the probability of trade in any equilibrium in which delay occurs in a connected set of states $\{\underline{m}, \ldots, \overline{m}\}$. Our next result assures that for large v, all equilibria with delay have this property. In fact, we show that if in equilibrium there is delay in a state $m' < q$, then there is delay in all $m > m'$. Thus, for large v, delay is frontloaded.

Lemma 5.5. *For large v, $\mu_{m'} \in (0, 1)$ for $m' < q \Rightarrow \mu_m \in (0, 1)$ $\forall m > m'$.*

The proof of Lemma 5.5 involves three steps:

(1) First, in Lemma A.3, we characterize agents' payoffs in each state m as a function of primitives, for any given probability of trade in each state in the continuation, $(\mu_1, \ldots, \mu_{m-1})$.

(2) Using this result, in Lemma A.4 we provide a necessary and sufficient condition for full trade in any state m for an arbitrary probability of trade of the $m - 1$ subgame. In particular, we obtain an expression $T(m)$ which generalizes $T^\dagger(m)$ in Proposition 5.3, and show that $s(m) \geq (\leq) 0$ given $\mu_m \in [0, 1]$ if and only if $T(m) \leq (\geq) 0$.

(3) In the proof of Lemma 5.5 we then show that for large v, $T(m) \leq 0 \Rightarrow T(m-1) < 0$.

Using Lemmas 5.4 and 5.5 (together with uniqueness of equilibrium outcomes, Proposition 5.1), we provide a complete characterization of equilibria for large v.

Theorem 5.6 (Characterization for large v). *For any $\phi \in [0, 1]$ there exists a unique cutpoint $\overline{m}(\phi) \in M$ such that, in equilibrium, there is delay in each state $m \in M$ s.t. $m > \overline{m}(\phi)$, and*

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[17] Note also that for large v and small ϕ, $w(1) > 0$ even if $w < 0$. Since delay at q and $q - 1$ implies that $w(q) = w_{out}(q - 1)$, by (14) in this equilibrium $w(q)$ is decreasing in v, and $w(q) \to 0$ as $v \to \infty$. □

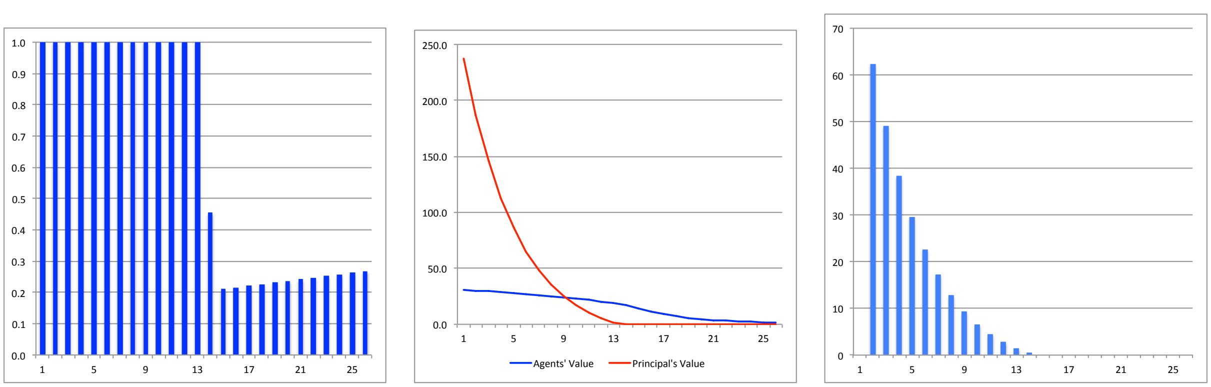
full trading in any $m \leq \overline{m}(\phi)$. *The cutpoint* $\overline{m}(\cdot)$ *is weakly increasing in* ϕ *and has range* M. *Moreover, for any* $m > \overline{m}(\phi) + 1$,

$$\mu_m = \left(\frac{n+m-q}{n+\overline{m}+1-q}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{\delta^{\overline{m}}z}{(1-\beta(\overline{m}+1))(w^{\dagger}(\overline{m})-\delta^{\overline{m}}z)+\beta(\overline{m}+1)(v^{\dagger}(\overline{m}))}\right)$$

is decreasing in v *and goes to zero as* $v \to +\infty$.

Theorem 5.6 unifies our previous results when the collective hold-up problem is severe. For a given allocation of bargaining power ϕ inducing delay, delay is front-loaded, in the sense that it happens in the first $q - \overline{m}$ transactions. In the first $q - \overline{m} - 1$ of these transactions, the expected delay for each transaction increases as we move further along the process. But once the principal obtains the support of $q - \overline{m}$ agents, the remaining transactions occur without delay.

The number of transactions with positive expected delay is decreasing in ϕ and increasing in q, so that giving more power to the agents or raising the number of agents needed for success increases the number of states in which transactions fail with positive probability. Moreover, for any given ϕ for which there is delay in more than one state, expected delay grows continuously with v and in the limit with $v \to \infty$, the expected time for completion goes to infinity.



trade probability values surplus

FIGURE 1. Trade probability, equilibrium payoffs, surplus in an example ($v = 300$, $z = w = 30$, $\delta = 0.95$, $n = 51$, $q = 26$, $\phi = 0.2$.)

Using (13) we can express the equilibrium payoff of an uncommitted agent as

$$w(q) = \left[\prod_{k=\overline{m}+1}^{q}\left(1+\left(\frac{1-\delta}{1-\beta(k)}\right)\frac{1}{\delta\mu_k}\right)^{-1}\right]w^{\dagger}(\overline{m})$$

Thus, for large enough v, any ϕ such that $\overline{m}(\phi) \leq q-2$ leads to lower equilibrium payoffs for agents than giving complete bargaining power to the principal, $\phi = 1$. In turn, since $w^{\dagger}(q)$ is decreasing in ϕ when a FTE exists, agents prefer the smallest ϕ such that a FTE exists to $\phi = 1$. It follows that for large enough v, agents prefer ϕ such that either $\overline{m}(\phi) = q-1$ or $\overline{m}(\phi) = q$, granting considerable bargaining power to the principal.

Remark 2 (Low v)*.* Most of our results apply generically, for all values of v. In particular, there is still trade with positive probability in all states, the equilibrium exists and is still essentially unique, the characterization of values is unchanged, as is the condition for no delay, and the growth of the probability of trade in contiguous states. The result that holds for large v but *does not* hold in general is Lemma 5.5, which says that if there is delay in a state $m' \in M$ there is delay in any state $m > m'$. In fact, we have constructed examples in which, for low v, there is delay in an intermediate set of states $D = \{\underline{d}, \dots, \overline{d}\}$, with $\underline{d} > 1$ and $\overline{d} < q$. This complements the results of Lemma 4.2 in Section 4, which showed that when agents have full bargaining power there is delay in all but the critical state for high v, but for low v, delay is *backloaded.* □

Remark 3 (Equilibrium for $\delta \to 1$)*.* In Theorem 5.6, we characterized equilibrium outcomes for fixed $\delta < 1$, and sufficiently large v. A natural question is how does equilibrium behave for fixed v as frictions vanish. From the expression for the trading probability μ_m in the theorem one might be tempted to conclude that as $\delta \to 1$, the probability of trade goes to zero, so negotiations slow down almost to a halt. This would be incorrect. In fact, making the dependence of each $\overline{m}(\phi)$ on δ explicit, as long as $z \geq w$, $\overline{m}_{\delta}(\phi) \to q$ as $\delta \to 1$. Thus for any given $\phi \in (0, 1]$ and $v > 0$ there is a $\overline{\delta} > 0$ such that if $\delta \geq \overline{\delta}$, the unique equilibrium is a FTE. Note that from Proposition 5.2, for any $m \in M$,

$$\lim_{\delta \to 1} w^{\dagger}(m) = \beta(m) \left[\prod_{j=1}^{m} \left(\frac{1}{1-\beta(j)} \right) \right] (n-q)w = w,$$

and

$$\lim_{\delta \to 1} v^{\dagger}(m) = v - m(w^{\dagger}(m) - z) = v + m(z-w).$$

Thus, from proposition 5.3, the condition for existence of a FTE boils down to

$$v \geq -m(z-w) \quad \forall m \in M.$$

Consider the critical state $m = 1$. Note that $p(1) = -[z - w(1)] = -(z-w)$, so that when the principal can make an offer, she keeps v and can extract the differential $z - w > 0$. But even when the agent proposes, he gets $b(1) = v - v(1) = -(z - w) = p(1)$. Thus, the critical agent cannot extract δv from the principal, and there are no incentives to hold out, and no collective hold-up problem. The result is due to simple economics. When both principal and agents do not discount the future, both principal and agents are willing to wait to get a better deal, but the principal is a monopolist, while the agent faces competition from other agents. This means that the critical agent cannot extract any surplus from the principal. Because agents are willing to wait, all agents are guaranteed w. But the principal, being the short side of the market, gets the differential $z - w$ entirely. And once this happens in $m = 1$, then by the same logic $b(m) = p(m) = -(z - w)$ for all $m \in M$, and thus $w^\dagger(m) = w$ and $v^\dagger(m) = v + m(z - w)$, independently of ϕ, provided that $\phi > 0$. □

5.3. **Breakdown of Negotiations.** Up to this point, we maintained the assumption that in the event the principal obtains the support of q agents, an agent who committed his support to the principal obtains a positive payoff $z > 0$. In some applications, however, it is reasonable to assume that $z = 0$ (e.g., corporate takeovers) or even $z < 0$ (e.g., vote buying with audience costs). Here we consider the case $z \leq 0$.

Consider for example a dynamic version of corporate takeovers model of Grossman and Hart (1980) (GH). GH analyze a problem in which a company (the raider) acquires shares of a target company to control its board of directors. It is assumed that the raider can improve the value of the company. To capture this feature, we assume that under the raider's control, the value of a share is $w > 0$, and we normalize the value of a share under the incumbent management to zero. We distinguish the payoff that a shareholder obtains when the raider wins if the shareholder does not sell to the raider ($w > 0$) from the payoff he obtains if he does sell to the raider ($z = 0$).[18]

We show that whenever there are positive externalities on uncommitted agents ($w > 0$), the condition $z > 0$ is necessary for robust delay. In particular, we show that when contracting with the principal leads to a negative payoff for the agent when the principal wins, in equilibrium there can only be delay in the initial state $m = q$, a result which holds for a "small" (but not measure zero) set of parameter values. With this exception, equilibrium is either a FTE or is such that there are no transactions in the initial state and thus $w(q) = 0$.

The result follows from Proposition 5.7 below. In it we establish two results. First we show that if $z \leq 0 < w$, there cannot be cycles of trade with probability one and trade failure

[18]As in GH and Segal (2003), we assume that shareholders are homogeneous. Unlike GH, we suppose that shareholders are fully aware of the effect of their action on the outcome of the raid attempt.

with positive probability; in fact, if in equilibrium there is trade with probability one in a state m', then this also has to be the case in all states $m < m'$. The second part of the proposition establishes that there cannot be delay in two contiguous states m and $m+1$. Together, the two results imply that with the exception of possibly mixing in the initial state, the equilibrium is either a FTE, involves no transactions in any state, or has a FTE in a m'-subgame off the equilibrium path for some $m' < q$, with no trade for $m > m'$, which implies that the process of transactions never starts.

Proposition 5.7. *Suppose $z \leq 0 < w$. Then (i) $s(m-1) \leq 0 \Rightarrow s(m) \leq 0$. Moreover, (ii) if $s(m') \leq 0$ for some $m' < q$, then $\mu_m = 0$ for all $m > m'$ and $w(q) = v(q) = 0$.*

Why no delay in contiguous states? Suppose there is delay in m' in equilibrium. Since $s(m'+1) \leq 0$, either trade collapses in $m'+1$ or again there is delay. If there is delay in both m' and $m'+1$, $v(m') = v(m'+1) = 0$, so $s(m'+1) = 0$ if and only if $w(m'+1) = w_{out}(m')$. But $w(m'+1) \geq 0$, while $w_{out}(m') = \left[\prod_{k=1}^{m'} \left(\frac{\delta\mu_k}{1-\delta(1-\mu_k)}\right)\right] z < 0$, so this is impossible. With no possible payments from the principal, all incentives to trade have to come from diminishing the value of holding out through delay. But delay can only lower the value of not trading, and thus by itself is insufficient to induce agents to trade when $z \leq 0$.

The next proposition provides a necessary and sufficient condition for the emergence of delay with $z \leq 0$. To do this, we first show – using a similar argument as in the proof of uniqueness in the benchmark model with $\phi = 1$ – that if there exists a FTE, this is the unique MPE. We then provide a necessary and sufficient conditions for existence of a FTE. The second part of the proposition follows as a corollary of previous results. Indeed, we know from Proposition 5.7 that when $z \leq 0$, $s(m) > 0 \Rightarrow s(m-1) > 0$. Thus, a necessary and sufficient condition for a FTE is that at the FTE profile, $s(q) > 0$. From Proposition 5.3, this is if and only if the condition in (17) holds for $m = q$.

Proposition 5.8. *Suppose $z \leq 0 < w$. There exists a FTE iff*

$$w^{\dagger}(q) \leq \frac{\delta^q}{n}\left(v + qz + (n-q)w\right), \tag{18}$$

and in this case this equilibrium is unique.

From Proposition 5.7 we know that if (18) doesn't hold there are two possibilities: either trade stops at some $m < q$ and then $w(q) = v(q) = 0$, or there is a FTE in the $(q-1)$-subgame and delay in the initial state q. The first case holds if condition (17) does not hold for $(q-1)$, and the latter in the intermediate case in which (17) holds for $q-1$ but not for q. This completely characterizes equilibria with $z \leq 0$.

Note that it is efficient to form a coalition in support of the project iff $v+qz+(n-q)w \geq 0$. We know from Proposition 5.3 that for large v, there is a $\overline{\phi} < 1$ such that if $\phi > \overline{\phi}$, the unique MPE of the m-subgame is a FTE. So here the coalition should form, and it does form in equilibrium for high ϕ. In general, however, outcomes are not efficient. In fact, we know from the same proposition that for large v there exists $\underline{\phi} \in (0,1)$ such that if $\phi < \underline{\phi}$ a FTE does not exist, even when this would be efficient.

The main point of the GH paper is that externalities across shareholders can prevent takeovers that add value to the company. The idea is that since shareholders that do not sell can capture the increase in value brought by the *raider*, no shareholder will tender his shares at a price that would allow the raider to profit from the takeover. GH work with a static model, and assume that shareholders ignore the impact of their actions on the outcome of the bid. In our version of the GH model – where the principal buys shares one at a time and shareholders are fully forward looking and strategic – efficient takeovers are not prevented by externalities when $\delta < 1$ *as long as the raider has enough nominal bargaining power*.[19] But when agents do have enough bargaining power, efficient takeovers can fail to occur due to the collective hold-up problem: with $z \leq 0$ the collective hold-up problem still exists, but leads not to delay but to breakdown of negotiations.

6. Conclusion

In this paper, we consider a dynamic process of coalition formation in which a principal bargains sequentially with a group of agents. We study how institutional changes affecting the allocation of bargaining power between principal and agents affect the distribution of rents and the efficiency of collective decisions. We give a complete characterization of equilibrium outcomes when the principal's willingness to pay is high. We show that redistributing bargaining power from the principal to the agents generates delay and reduces agents' welfare, even in the absence of informational asymmetries or discriminatory offers, and even with negative externalities on uncommitted agents. Concentrating bargaining power on the

[19] Holmstrom and Nalebuff (1992) show that when shareholdings are divisible the free-riding problem does not prevent the takeover process in the GH model. In our model with $\phi = 1$, the raider's profit goes to zero as $\delta \to 1$. Thus, with fixed costs, efficient raids would be prevented in the limit. This result is similar to that of Harrington and Prokop (1993), who consider a dynamic version of GH in which the raider can re-approach the shareholders who have not sold (taking all offers at the posted price in each period).

principal, instead, leads to efficient collective decision-making and, for any non-unanimous decision rule, does not lead to complete rent extraction by the principal.

Our results have implications for a number of diverse applications in economics and politics, including lobbying, exclusive deals, start-ups, endorsements and corruption. While the model abstracts away from some of the details pertinent to each application, the results shed light on a common idea behind these apparently diverse problems: bargaining institutions that decentralize power to agents can be detrimental to agents' welfare by making the coalition formation process inefficient.

The source of the inefficiency has two parts. The first is a form of the traditional hold-up problem: when agents have significant bargaining power relative to the principal, the principal anticipates that agents trading late in the process will extract a large fraction of the surplus, and as a result is not willing to pay much to agents trading early on. This is similar to Blanchard and Kremer (1997) and Olken and Barron (2009), where sequential bargaining under unanimity leads to increasing prices. The point we make here, however, is that given this hold-up problem, *competition among agents leads to delay* if and only if agents have too much bargaining power. This is what we call a collective hold-up problem.

The collective hold-up problem emerges in our model in the absence of discriminatory contracts or asymmetric equilibria, and do not require a particular form of externalities on uncommitted agents (non-traders). While we do not allow the principal to bargain with multiple agents simultaneously, we can show that this is not crucial for our results. In fact it is sufficient to assume that the principal cannot contract with q agents at once. Other extensions of the model are more challenging. In particular, our model does not allow for more general payoff structures in which payoffs depend on the size of the coalition that supports the principal, and can accrue before the coalition is formed. For example, in industries in which new technologies have a component of learning by doing, earlier sales affect later payoffs. Here the incentives to hold out compete with the benefits of joining early. This presents an interesting problem, where the principal may optimally front payments and sell at a loss. In that sense, collective hold-up may manifest itself in delayed learning. We leave this problem for future work.

Appendix A. Proofs

We begin with an observation and a lemma which will be useful later.

Remark A.1. *In section 5.1 we showed that*

$$w(m) = \left[\frac{\delta\beta(m)}{1-\delta\beta(m)}\right](1-\phi)s^{+}(m) + \left[1 + \left(\frac{1-\delta}{1-\beta(m)}\right)\frac{1}{\delta\mu_m}\right]^{-1} w(m-1). \quad (eq.13)$$

Using (13), we can express the current value for an uncommitted agent as a function of the final payoff w and the sequence of surpluses $[s_k]$ for $k \leq m$:

$$w(m) = (1-\phi)\sum_{k=1}^{m}\left(\frac{\beta(k)}{1-\beta(k)}\right)e_{km}s^{+}(k) + e_{1m}w \quad \forall m \geq 1, \tag{19}$$

where we have defined

$$e_{km} \equiv \left[\prod_{j=k}^{m}\left(1 + \left(\frac{1-\delta}{1-\beta(j)}\right)\frac{1}{\delta\mu_j}\right)\right]^{-1}$$

Lemma A.2 (Equilibrium Trade in state $m=1$). *The equilibrium probability of trade in state $m=1$ obeys the following characterization:*

1. *If $v+z \leq 0$, $\mu_1 = 0$ (no trade at $m=1$),*
2. *If $0 < v+z < \delta\left(\frac{n-q}{n-q+(1-\delta)}\right)w$, $\mu_1 \in (0,1)$ (probabilistic trade at $m=1$),*
3. *If $v+z \geq \delta\left(\frac{n-q}{n-q+(1-\delta)}\right)w$, $\mu_1 = 1$ (trade w.p. 1 at $m=1$).*

Proof of Lemma A.2. Fix a MPE σ. Since the principal only makes an offer if $s(m) \geq 0$, (12) implies $v(m) \geq 0$ for all m, and in particular $v(1) \geq 0$. Similarly, since the agent only makes an offer if $s(m) \geq 0$, if $s(m) < 0$ then $\hat{\lambda}_m = 0$. Therefore (19) implies $w(m) \geq 0$, and in particular $w(1) \geq 0$. Since $s(1) = v + z - v(1) - w(1)$, $w(1), v(1) \geq 0$ imply $s(1) \leq v+z$. It follows that if $v+z < 0$ then $s(1) < 0$ and there is no trade in equilibrium at $m=1$. Now suppose $v+z=0$. Then $s(1) = -[v(1)+w(1)]$. If $\mu_1 > 0$, then $v(1), w(1) > 0$, and thus $s(1) < 0$, which implies $\mu_1 = 0$, a contradiction. Thus $\mu_1 = 0$ and $v(1) = w(1) = 0$. It follows that if $v+z \leq 0$, in equilibrium there is no trade in state $m=1$.

Now suppose $v+z > 0$. If $\mu_1 = 0$ (no trade), then $v(1) = w(1) = 0$ and $s(1) > 0$, which implies $\lambda_1 > 0$, a contradiction. Suppose $\mu_1 = 1$. Then (12) gives $v(1) = \frac{\delta}{1-\delta}\phi s(1)$ and (19) gives

$$w(1) = \frac{\delta(1-\phi)\beta(1)}{(1-\delta\beta(1))}s(1) + \frac{\delta(1-\beta(1))}{(1-\delta\beta(1))}w$$

Substituting,

$$s(1)\left[1 + \frac{\delta\phi}{1-\delta} + \frac{\delta(1-\phi)}{(1-\delta\beta(1))}\beta(1)\right] = v + z - \frac{\delta(1-\beta(1))}{(1-\delta\beta(1))}w$$

Thus $s(1) \geq 0$, consistent with equilibrium, iff

$$v + z \geq \frac{\delta(1-\beta(1))}{(1-\delta\beta(1))}w$$

If instead

$$0 < v + z \leq \frac{\delta(1-\beta(1))}{(1-\delta\beta(1))}w = \delta\left(\frac{n-q}{n-q+(1-\delta)}\right)w \tag{20}$$

we have $\mu_1 \in (0,1)$. Note that with $s(1) = 0$, (12) implies $v(1) = 0$, and (19) implies that

$$w(1) = \left(\frac{\delta\mu_1}{\left(\frac{1-\delta}{1-\beta(1)}\right) + \delta\mu_1}\right)w \tag{21}$$

Substituting in (2), the equilibrium probability of trade is given by

$$\mu_1 = \left(\frac{1-\delta}{\delta}\right)\frac{1}{1-\beta(1)}\left(\frac{v+z}{w-(v+z)}\right) \tag{22}$$

Note that the RHS of (22) $\in (0,1)$ iff (20) holds. □

Proof of Proposition 5.1. Fix an equilibrium in the subgame starting in state $m-1$. This produces continuation values $\tilde{v}(m-1)$, $\tilde{w}(m-1)$ and $\tilde{w}_{out}(m-1)$. Given these continuation values, let $v(m;\mu_m)$ and $w(m;\mu_m)$ denote the values of the principal and uncommited agent in state m when transaction probability μ_m, and let $s(m;\mu_m)$ denote the surplus in state m when transaction probability μ_m.

From (12) and (13), $v(m;0) = w(m;0) = 0$. Thus $s(m;0) \equiv [\tilde{v}(m-1) - v(m;0)] + [\tilde{w}_{out}(m-1) - w(m;0)] = \tilde{v}(m-1) + \tilde{w}_{out}(m-1)$. It follows that if $\tilde{v}(m-1) + \tilde{w}_{out}(m-1) \geq 0$, inaction at m is not an equilibrium. But note that $\tilde{v}(m-1) \geq 0$, and by (14), if $z > 0$ and $\mu_k > 0$ for all $k < m$, then $w_{out}(m) = \left[\prod_{k=1}^{m}\left(\frac{\delta\mu_k}{1-\delta(1-\mu_k)}\right)\right]z > 0$. Thus $\mu_m = 0$ is not part of an equilibrium if $\mu_k > 0$ for all $k < m$.

Suppose $\mu_m = 1$. Using the expression for the principal's value (12) and the expression for the uncommitted agent's value (13) in the definition of the surplus (2), we have

$$s(m)\left[1+\frac{\delta}{1-\delta}\phi+\frac{\left(\frac{\delta}{1-\delta}\right)\beta(m)(1-\phi)}{\left(1+\left(\frac{\delta}{1-\delta}\right)(1-\beta(m))\right)}\right]=\tilde{w}_{out}(m-1)+\tilde{v}(m-1)-\frac{1}{\left[1+\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right)\right]}\tilde{w}(m-1)$$

Equilibrium requires that $s(m) > 0$. From the previous expression, $s(m) > 0$ iff

$$1+\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right)>\frac{\tilde{w}(m-1)}{\tilde{w}_{out}(m-1)+\tilde{v}(m-1)}. \tag{23}$$

Next, suppose $\mu_m \in (0,1)$. Equilibrium then requires $s(m) = 0$, which in turn implies $v(m) = 0$ and then $w(m) = \tilde{v}(m-1) + \tilde{w}_{out}(m-1)$. Also with $s(m) = 0$, (13) gives

$$w(m)=\left(\frac{\delta\mu_m}{\left(\frac{1-\delta}{1-\beta(m)}\right)+\delta\mu_m}\right)\tilde{w}(m-1)$$

Substituting in $w(m) = \tilde{v}(m-1) + \tilde{w}_{out}(m-1)$, and then solving for μ_m gives

$$\mu_m=\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right)\left(\frac{\tilde{v}(m-1)+\tilde{w}_{out}(m-1)}{\tilde{w}(m-1)-(\tilde{v}(m-1)+\tilde{w}_{out}(m-1))}\right), \tag{24}$$

which is the statement in the proposition. This is less than one iff (23) doesn't hold.

We have shown that if $\mu_k > 0$ for all $k < m$, equilibrium play in state m is uniquely determined, and is either $\mu_m = 1$ if (23) holds or $\mu_m \in (0,1)$ given in (24) if (23) doesn't hold. Finally note that by Lemma A.2, if $v, z > 0$ then $\mu_1 > 0$. An induction argument then completes the proof. □

Proof of Proposition 5.2. Follows from Lemma A.3 below, making $\mu_j = 1$ for all $j \in M$. □

Lemma A.3. *Let* $\theta_{km} \equiv \prod_{j=k}^{m}\left(\frac{\delta\phi\mu_j}{1-\delta+\delta\mu_j\phi(1-\beta(j))}\right)$. *In a MPE with trade probabilities* μ*, the agents' equilibrium payoff in each state* $m \in M$ *is given by*

$$\frac{w(m)}{\beta(m)}=\theta_{1m}(n-q)w+\sum_{k=1}^{m}\theta_{km}\left(\frac{1-\phi}{\phi}\right)\left(\frac{1-\delta}{\delta}\right)\frac{1}{\mu_k}\left(\prod_{j=1}^{k}\frac{\delta\mu_j}{1-\delta(1-\mu_j)}\right)(v+kz+(n-q)w) \tag{25}$$

Proof of Lemma A.3. The value functions of the principal and agents satisfy

$$v(m)=\mu_m\frac{\delta}{1-\delta}\phi s(m) \tag{26}$$

and

$$w(m)=\frac{\delta\beta(m)(1-\phi)\mu_m}{1-\delta+\delta(1-\beta(m))\mu_m}s(m)+\frac{\delta(1-\beta(m))\mu_m}{1-\delta+\delta(1-\beta(m))\mu_m}w(m-1) \tag{27}$$

Substituting (26) in the surplus condition (2) and using that $\frac{1-\beta(m)}{\beta(m)} = \frac{1}{\beta(m-1)}$ we have the system of difference equations:

$$(1-\phi)s(m) = \left(\frac{1-\delta}{\delta\mu_m} + 1 - \beta(m)\right)\frac{w(m)}{\beta(m)} - \frac{w(m-1)}{\beta(m-1)} \tag{28}$$
$$\frac{1-\delta+\delta\phi\mu_m}{1-\delta}s(m) = \mu_{m-1}\frac{\delta}{1-\delta}\phi s(m-1) + w_{out}(m-1) - w(m)$$

Solving the first equation for $s(m)$ and substituting in the second equation, we transform the system of first order difference equations into a second order difference equation. Letting $\alpha_m \equiv \frac{\delta\mu_m}{1-\delta(1-\mu_m)}$, and defining

$$H(m) \equiv \frac{\phi}{1-\phi}\frac{\delta}{1-\delta}\left[\left(\frac{1-\delta}{\delta\phi} + \mu_m(1-\beta(m))\right)\frac{w(m)}{\beta(m)} - \mu_m\frac{w(m-1)}{\beta(m-1)}\right], \tag{29}$$

we can write this recursion as

$$H(m) = \alpha_m H(m-1) + \alpha_m w_{out}(m-1) \quad \text{for } m : 3 \leq m \leq m' \tag{30}$$

Solving recursively, and using that $w_{out}(m) = \alpha_m w_{out}(m-1)$ we have

$$H(m) = \left(\prod_{j=3}^{m}\alpha_j\right)H(2) + (m-2)w_{out}(m)$$

Therefore, letting $\tau_m = \frac{1-\delta}{1-\delta+\delta\mu_m\phi(1-\beta(m))}$ for convenience,

$$\frac{w(m)}{\beta(m)} = \frac{1-\delta(1-\mu_m)}{1-\delta}\phi\tau_m\alpha_m\frac{w(m-1)}{\beta(m-1)} + \tau_m(1-\phi)\left[\left(\prod_{j=3}^{m}\alpha_j\right)H(2) + (m-2)w_{out}(m)\right]$$

The boundary conditions follow by (28) for $m = 1, 2$ and (29) for $H(2)$, which give

$$H(2) = \alpha_2\alpha_1\left(v + 2z + \frac{w}{\beta(0)}\right)$$
$$\frac{w(2)}{\beta(2)} = \tau_2\left(\alpha_2\frac{1}{\tau_1} + \frac{\delta}{1-\delta}\mu_2\phi\right)\frac{w(1)}{\beta(1)} - \alpha_2\tau_2\mu_1\frac{\delta}{1-\delta}\phi\frac{w}{\beta(0)} + \alpha_2\tau_2(1-\phi)w_{out}(1)$$
$$\frac{w(1)}{\beta(1)} = \tau_1\phi\left[\frac{\delta}{1-\delta}\mu_1\frac{w}{\beta(0)} + \alpha_1\frac{1-\phi}{\phi}\left(v + z + \frac{w}{\beta(0)}\right)\right]$$

Using these initial conditions together with $w_{out}(m) = \left(\prod_{j=1}^{m}\alpha_j\right)z$, we obtain a simple recursive representation of the value functions

$$\frac{w(m)}{\beta(m)} = \frac{1-\delta(1-\mu_m)}{1-\delta}\phi\tau_m\alpha_m\frac{w(m-1)}{\beta(m-1)} + \tau_m(1-\phi)\left(\prod_{j=1}^{m}\alpha_j\right)(v + mz + (n-q)w) \tag{31}$$

Solving recursively, we obtain

$$
\begin{aligned}
(32)\quad \frac{w(m)}{\beta(m)} = & \left(\prod_{j=1}^{m}\alpha_j\right)\left[\prod_{j=1}^{m}\left(\frac{1-\delta(1-\mu_j)}{1-\delta}\phi\tau_j\right)\right](n-q)w \\
& + (1-\phi)\left(\prod_{j=1}^{m}\alpha_j\right)\sum_{k=1}^{m-1}\left\{\left[\prod_{j=k+1}^{m}\left(\frac{1-\delta(1-\mu_j)}{1-\delta}\phi\tau_j\right)\right]\tau_k\left(v+kz+(n-q)w\right)\right\} \\
& + \tau_m(1-\phi)\left(\prod_{j=1}^{m}\alpha_j\right)\left(v+mz+(n-q)w\right),
\end{aligned}
$$

which is equivalent to (25). □

Proof of Proposition 5.3. Part (i) of the Proposition follows immediately from Lemma A.4, specializing for the case of a FTE. Part (ii) follows from Lemmas A.5 and A.6. □

Lemma A.4. *Consider any $m \leq q$. For any equilibrium $\mu_1, \ldots, \mu_{m-1}$ of the $m-1$ subgame, $s(m) \geq (\leq) 0$ given $\mu_m \in [0,1]$ if and only if*

$$
T(m) \equiv \frac{w(m)}{\beta(m)} - \left(\prod_{j=1}^{m}\frac{\delta\mu_j}{1-\delta(1-\mu_j)}\right)\left(v+mz+(n-q)w\right) \leq (\geq) 0
$$

Proof of Lemma A.4. Using (31) we get that the surplus condition (28) is equivalent to

$$
(28b)\qquad \left(\frac{\delta}{1-\delta}\right)\phi\mu_m s(m) = \left(\prod_{j=1}^{m}\frac{\delta\mu_j}{1-\delta(1-\mu_j)}\right)\left(v+mz+(n-q)w\right) - \frac{w(m)}{\beta(m)}
$$

Therefore $s(m) > (<) 0$ if and only if

$$
\left(\prod_{j=1}^{m}\frac{\delta\mu_j}{1-\delta(1-\mu_j)}\right)\left(v+mz+(n-q)w\right) > (<)\frac{w(m)}{\beta(m)}
$$

□

Lemma A.5. *Consider $m \leq q$, and suppose $v \geq m(w-z)$. Then there is a $\overline{\phi}(m) \in (0,1)$ such that if $\phi > \overline{\phi}(m)$, the unique MPE of the m-subgame is a FTE.*

Proof of Lemma A.5. From expression (16),

$$
\lim_{\phi\to 1}\frac{w^{\dagger}(m)}{\beta(m)} = \left(\prod_{j=1}^{m}\frac{\delta}{1-\delta\beta(j)}\right)(n-q)w
$$

So in the limit $T^\dagger(m) \leq 0$ iff

$$\left(\prod_{j=1}^{m} \frac{\delta}{1-\delta\beta(j)}\right)(n-q)w \leq \delta^m \left(v + mz + (n-q)w\right)$$

or iff

$$\left(\prod_{j=1}^{m} \frac{n+j-q}{n+j-q-\delta}\right) \leq \left(\frac{v+mz+(n-q)w}{(n-q)w}\right)$$

Expanding the product, the LHS is smaller than $\frac{n+m-q}{n+1-\delta-q} < \frac{n+m-q}{n-q}$, so it is sufficient that $v \geq m(w-z)$. Thus, for large ϕ, a sufficient condition for a FTE in the m-subgame is $v \geq q(w-z)$. □

Lemma A.6. *For any $m \leq q$, there exists $\underline{\phi}(m) \in (0,1)$ and $\overline{v}(m) > 0$ such that if $\phi < \underline{\phi}(m)$ and $v > \overline{v}(m)$, the unique MPE of the m-subgame entails delay.*

Proof of Lemma A.6. From expression (16),

$$\frac{w^\dagger(m)}{\beta(m)} = \sum_{j=1}^{m}\left(\prod_{k=j}^{m} \frac{1}{\frac{1-\delta}{\delta\phi}+1-\beta(k)}\right)\frac{1-\delta}{\delta}\frac{(1-\phi)}{\phi}\delta^j\left(v+jz+(n-q)w\right)$$
$$+\left(\prod_{j=1}^{m} \frac{1}{\frac{1-\delta}{\delta\phi}+1-\beta(j)}\right)(n-q)w$$

Note that all terms are positive. Dropping the first $m-2$ terms of the summation, and the last term, we have

$$\frac{w^\dagger(m)}{\beta(m)} > \left(\frac{\delta\phi}{1-\delta+\delta\phi(1-\beta(m-1))}\right)\left(\frac{(1-\delta)(1-\phi)}{1-\delta+\delta\phi(1-\beta(m))}\right)\delta^{m-1}\left(v+(m-1)z+(n-q)w\right)$$
$$+\left(\frac{(1-\delta)(1-\phi)}{1-\delta+\delta\phi(1-\beta(m))}\right)\delta^m\left(v+mz+(n-q)w\right)$$

So $T^\dagger(m) \equiv \frac{w^\dagger(m)}{\beta(m)} - \delta^m\left(v+mz+(n-q)w\right) > 0$ iff

$$\left(\frac{(1-\delta)(1-\phi)}{[1-\delta\beta(m)]}\right)\left(\frac{1}{1-\delta+\delta\phi(1-\beta(m-1))}\right)\left(v+(m-1)z+(n-q)w\right) \geq \left(v+mz+(n-q)w\right)$$

Taking derivatives of both sides with respect to v, the LHS increases faster than the RHS iff

$$\phi \leq \frac{(1-\delta)\delta\beta(m)}{(1-\delta)+\delta(1-\delta\beta(m))(1-\beta(m-1))} \equiv \overline{\phi}(m)$$

It follows that if $\phi < \overline{\phi}$, for v large enough $T^\dagger(m) > 0$. □

Lemma A.7. *Suppose $T^{\dagger}(m') \leq 0$ for all $m' \leq m$ (i.e., there exists a FTE in the m-subgame). Then $w^{\dagger}(m')$ is decreasing in ϕ for all $m' \leq m$.*

Proof of Lemma A.7. For any $m \leq q$, let

$$\Omega(m;\phi) \equiv \frac{w^{\dagger}(m)}{\beta(m)} = \overline{\pi}_{1m}(n-q)w + \sum_{k=1}^{m} \frac{1-\delta}{\delta}\frac{1-\phi}{\phi}\overline{\pi}_{km}\delta^{k}\left(v+kz+(n-q)w\right)$$

Note that

$$\Omega(m;\phi) = \left(\frac{1-\delta}{\delta\phi}+1-\beta(m)\right)^{-1}\left[\Omega(m-1;\phi)+\frac{1-\delta}{\delta}\frac{1-\phi}{\phi}\delta^{m}\left(v+mz+(n-q)w\right)\right]$$

Differentiating, and noting that $T^{\dagger}(m) \equiv \Omega(m;\phi) - \delta^{m}\left(v+mz+(n-q)w\right)$,

$$\frac{\partial\Omega(m)}{\partial\phi} = \left(\frac{1-\delta}{\delta\phi}+1-\beta(m)\right)^{-1}\left[\frac{1-\delta}{\delta\phi^{2}}\left(\Omega(m)-\delta^{m}\left(v+mz+(n-q)w\right)\right)+\frac{\partial\Omega(m-1)}{\partial\phi}\right]$$

or equivalently, since $T(m) \equiv \Omega(m) - \delta^{m}\left(v+mz+(n-q)w\right)$,

$$\frac{\partial\Omega(m;\phi)}{\partial\phi} = \left(\frac{1-\delta}{\delta\phi}+1-\beta(m)\right)^{-1}\left[\frac{1-\delta}{\delta\phi^{2}}T(m)+\frac{\partial\Omega(m-1;\phi)}{\partial\phi}\right] \tag{33}$$

Note that since

$$\Omega(1;\phi) = \left(\frac{1-\delta}{\delta\phi}+1-\beta(1)\right)^{-1}\left((n-q)w+\frac{1-\delta}{\delta}\frac{1-\phi}{\phi}\delta\left(v+z+(n-q)w\right)\right)$$

then

$$\frac{\partial\Omega(1;\phi)}{\partial\phi} = \frac{1-\delta}{\delta\phi^{2}}\left(\frac{1-\delta}{\delta\phi}+1-\beta(1)\right)^{-1}T^{\dagger}(1).$$

Note $T(m) \leq 0$ implies $T(m') \leq 0$ for any $m' < m$, and thus $T(1) \leq 0$, which from the above equation then that $T^{\dagger}(1) \leq 0$ implies $\partial\Omega(1)/\partial\phi < 0$. This together with the fact that $T^{\dagger}(m') \leq 0$ for any $m' < m$ implies by (33) that $\partial\Omega(m';\phi)/\partial\phi \leq 0$ for all $m' \leq m$. $\square$

Proof of Lemma 5.4. Here we prove that if in equilibrium $\mu_m \in (0,1)$ for all $m \in J \equiv \{m_\ell,\ldots,m_u\}$, then

$$\frac{\mu_{m+1}-\mu_m}{\mu_m} = \beta(m) \quad \forall m \in \{m_\ell+1,\ldots,m_u-1\},$$

and moreover

$$\mu_m = \left(\frac{n+m-q}{n+m_\ell-q}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{w(m_\ell)/w_{out}(m_\ell)-1}\right) \quad \forall m \in \{m_\ell+1,\ldots,m_u\}$$

(This second result will be useful in the proof of Theorem 5.6).

Suppose in equilibrium $\mu_m \in (0,1)$ for all $m \in J \equiv \{m_\ell, \dots, m_u\}$. Then $s(m) = v(m) = 0$ for all $m \in M$. Since $s(m) = 0$ for all $m \in J$, by (13),

$$w(m) = \left(\frac{\delta \mu_m}{\left(\frac{1-\delta}{1-\beta(m)}\right) + \delta \mu_m} \right) w(m-1) \quad \forall m \in J \tag{34}$$

Note that for all $m \in \{m_\ell + 1, \dots, m_u\}$, $v(m) = v(m-1) = 0$, and then $s(m) = 0$ implies $w(m) = w_{out}(m-1)$. Then

$$\frac{w(m)}{w(m-1)} = \frac{w_{out}(m-1)}{w_{out}(m-2)} \quad \forall m \in \{m_\ell + 2, \dots, m_u\},$$

Using (34) and (14), this is

$$\left(\frac{\delta \mu_m}{\left(\frac{1-\delta}{1-\beta(m)}\right) + \delta \mu_m} \right) = \left(\frac{\delta \mu_{m-1}}{(1-\delta) + \delta \mu_{m-1}} \right) \quad \forall m \in \{m_\ell + 2, \dots, m_u\}, \tag{35}$$

which implies that

$$\mu_m = \left(\frac{1}{1-\beta(m)} \right) \mu_{m-1} \quad \forall m \in \{m_\ell + 2, \dots, m_u\}, \tag{36}$$

This gives the first result using the definition of $\beta(m)$. This result directly implies

$$\mu_m = \left[\prod_{k=m_\ell+2}^{m} \left(\frac{1}{1-\beta(k)} \right) \right] \mu_{m_\ell+1} \quad \forall m \in \{m_\ell + 2, \dots, m_u\}.$$

Now, by (15), and noting that $v(m_\ell) = 0$,

$$\mu_{m_\ell+1} = \left(\frac{1-\delta}{\delta} \right) \left(\frac{1}{1-\beta(m_\ell+1)} \right) \left(\frac{w_{out}(m_\ell)}{w(m_\ell) - w_{out}(m_\ell)} \right).$$

Substituting gives

$$\mu_m = \left[\prod_{k=m_\ell+1}^{m} \left(\frac{1}{1-\beta(k)} \right) \right] \left(\frac{1-\delta}{\delta} \right) \left(\frac{1}{w(m_\ell)/w_{out}(m_\ell) - 1} \right) \quad \forall m \in \{m_\ell + 1, \dots, m_u\}$$

Noting that $1 - \beta(k) = \frac{n+m-q-1}{n+m-q}$, and simplifying, gives the result in the lemma. □

Proof of Lemma 5.5. In Lemma A.4 we showed that for any $m \le q$, and for *any equilibrium* $\mu_1, \dots, \mu_{m-1}$ of the $m-1$ subgame, $s(m) \ge (\le) 0$ given $\mu_m \in [0,1]$ if and only if $T(m) \le (\ge) 0$. We now show that for large v, $T(m) \le 0 \Rightarrow T(m-1) < 0$. Note that

$$\begin{aligned} T(m-1) = {} & \left(\frac{1-\delta+\delta\mu_m \phi(1-\beta(m))}{\phi[1-\delta(1-\mu_m)]} \right) T(m) + \left(1 - \frac{\delta \beta(m) \mu_m}{1-\delta(1-\mu_m)} \right) (v + mz + (n-q)w) \\ & - (v + (m-1)z + (n-q)w) \end{aligned}$$

so if $T(m) \leq 0$, we have

$$T(m-1) \leq \left(1 - \frac{\delta\beta(m)\mu_m}{1-\delta(1-\mu_m)}\right)(v + mz + (n-q)w) - (v + (m-1)z + (n-q)w)$$

Since the RHS is decreasing in v for any $\mu_m \in [0,1]$ and goes to $-\infty$ as $v \to \infty$, it follows that for sufficiently large v, $T(m) \leq 0 \Rightarrow T(m-1) < 0$. □

Proof of Theorem 5.6. By Lemma A.6 for any $m \leq q$, there exists $\underline{\phi}(m) > 0$ and $\overline{v}(m) > 0$ such that if $\phi < \underline{\phi}(m)$ and $v > \overline{v}(m)$, then $T^\dagger(m|\phi) > 0$. From Lemma A.5, assuming $v \geq m(w-z)$, we have that for any $m \leq q$ there is a $\overline{\phi}(m) < 1$ such that if $\phi > \overline{\phi}(m)$, $T^\dagger(m|\phi) \leq 0$, and the unique MPE of the m-subgame is a FTE. Since $T^\dagger(m|\phi)$ is continuous in ϕ, for any m there is a $c_m \in (0,1)$ such that $T^\dagger(m|c_m) = 0$ (for v large, fixed). By Lemma 5.5, for large v in equilibrium $T(m'|c_m) > 0$ for all $m' > m$. It follows that in the unique MPE for $\phi = c_m$, we have $\mu_k = 1$ for all $k \leq m$ and (provided $m < q$), $\mu_k \in (0,1)$ for $k > m$.

We have shown before that $T(m'|c_m) > 0 = T^\dagger(m|c_m)$ for all $m' > m$. In addition, by Lemma A.7, if $T^\dagger(m|\phi) \leq 0$ then $T^\dagger(m'|\phi)$ is decreasing in ϕ for all $m' \leq m$ (also for v large). This implies that $c_{m+1} > c_m$ for all $m \leq q-1$, and that for any $\phi \in (c_m, c_{m+1})$, $T^\dagger(m+1|\phi) > 0$ and $T^\dagger(m|\phi) \leq 0$. It follows that the equilibrium characterization above for $\phi = c_m$ applies unchanged to all $\phi \in [c_m, c_{m+1})$.

Now take $\phi \in [0,1]$ given, and let $\overline{m} \in M$ denote the cutpoint such that, in equilibrium, there is delay in each state $m \in M$ s.t. $m > \overline{m}$, and full trading in any $m \leq \overline{m}$. In the proof of Lemma 5.4 we show that if in equilibrium $\mu_m \in (0,1)$ for all $m \in J \equiv \{m_\ell, \ldots, m_u\}$, then

$$\mu_m = \left(\frac{n+m-q}{n+m_\ell-q}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{w(m_\ell)/w_{out}(m_\ell)-1}\right) \quad \forall m \in \{m_\ell+1, \ldots, m_u\}$$

It follows that here (with $m_\ell = \overline{m}+1$ and $m_u = q$), we have

$$\mu_m = \left(\frac{n+m-q}{n+\overline{m}+1-q}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{w(\overline{m}+1)/w_{out}(\overline{m}+1)-1}\right) \quad \forall m > \overline{m}+1 \tag{37}$$

Note that the probability of trade in each state where there is delay is decreasing in the ratio $w(\overline{m}+1)/w_{out}(\overline{m}+1)$. We now argue that this ratio is increasing in v, and that $\mu_m \to 0$ as $v \to \infty$. Note that by (14),

$$\frac{w_{out}(\overline{m}+1)}{w_{out}(\overline{m})} = \left(\frac{\delta\mu_{\overline{m}+1}}{1-\delta(1-\mu_{\overline{m}+1})}\right),$$

and by Proposition 5.1,

$$\delta\mu_{\overline{m}+1} = (1-\delta)\left(\frac{1}{1-\beta(\overline{m}+1)}\right)\left(\frac{v(\overline{m})+w_{out}(\overline{m})}{w(\overline{m})-(v(\overline{m})+w_{out}(\overline{m}))}\right).$$

Substituting,

$$\frac{w_{out}(\overline{m}+1)}{w_{out}(\overline{m})} = \left(\frac{(v(\overline{m}) + w_{out}(\overline{m}))}{\beta(\overline{m}+1)(v(\overline{m}) + w_{out}(\overline{m})) + (1-\beta(\overline{m}+1))w(\overline{m})}\right).$$

Now, since $\mu_{\overline{m}+1} \in (0,1)$, then $v(\overline{m}) + w_{out}(\overline{m}) = w(\overline{m}+1)$. Substituting, and noting that the equilibrium of the $\overline{m}$ subgame is a FTE,

$$\frac{w_{out}(\overline{m}+1)}{w(\overline{m}+1)} = \frac{w^{\dagger}_{out}(\overline{m})}{(1-\beta(\overline{m}+1))w^{\dagger}(\overline{m}) + \beta(\overline{m}+1)[(v^{\dagger}(\overline{m}) + w^{\dagger}_{out}(\overline{m}))]}.$$

It follows that for $m > \overline{m}+1$,

$$\mu_m = \left(\frac{n+m-q}{n+\overline{m}+1-q}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{w^{\dagger}_{out}(\overline{m})}{(1-\beta(\overline{m}+1))(w^{\dagger}(\overline{m}) - w^{\dagger}_{out}(\overline{m})) + \beta(\overline{m}+1)(v^{\dagger}(\overline{m}))}\right)$$

Now, $w^{\dagger}_{out}(\overline{m}) = \delta^{\overline{m}} z$ is independent of v, while both $v^{\dagger}(\overline{m})$ and $w^{\dagger}(\overline{m})$ are increasing in v, and unbounded. Thus for $m > \overline{m}+1$, μ_m is decreasing in v and goes to zero as $v \to +\infty$. This completes the proof. □

Proof of Proposition 5.7. Using (12), (19), (14) in (2) we obtain, for all $m \geq 2$

$$\left(1 + \phi\left(\frac{\delta}{1-\delta}\right) + (1-\phi)\frac{\delta\beta(m)}{1-\delta\beta(m)}\right)s(m) \tag{38}$$
$$= \phi\left(\frac{\delta}{1-\delta}\right)s(m-1) + \left[\prod_{k=1}^{m-1}\left(\frac{\delta\mu_k}{1-\delta(1-\mu_k)}\right)\right]z - \pi(1)w$$
$$-(1-\phi)\sum_{k=1}^{m-1}\left(\frac{\beta(k)}{1-\beta(k)}\right)\pi(k)s(k)$$

Since $w > 0$, $z \leq 0$, and $\pi(k)s(k) \geq 0$, (38) implies

$$\left(1 + \phi\left(\frac{\delta}{1-\delta}\right) + (1-\phi)\frac{\delta\beta(m)}{1-\delta\beta(m)}\right)s(m) \leq \phi\left(\frac{\delta}{1-\delta}\right)s(m-1) \tag{39}$$

It follows that in any equilibrium, $s(m-1) \leq 0 \Rightarrow s(m) \leq 0$. So suppose $s(m') < 0$ for some $m' < q$. Then $\mu_{m'} = 0$, and thus $w(m) = v(m) = 0$ for all $m \geq m'$ with no transactions in equilibrium for $m \geq m'$. Suppose instead $s(m') = 0$ for some $m' < q$. If $\mu_{m'} = 0$, the same conclusion holds, so suppose in equilibrium $\mu_{m'} \in (0,1)$. Because $s(m'+1) \leq 0$, in equilibrium either $\mu_{m'+1} = 0$ or $s(m'+1) = 0$ and $\mu_{m'+1} \in (0,1)$. If $\mu_{m'+1} \in (0,1)$, then $v(m') = v(m'+1) = 0$, and then $s(m'+1) = 0$ implies $w(m'+1) = w_{out}(m')$. But $w(m'+1) \geq$

0, while $w_{out}(m') = \left[\prod_{k=1}^{m'} \left(\frac{\delta\mu_k}{1-\delta(1-\mu_k)}\right)\right] z < 0$ by (14), which is a contradiction. It follows that if $s(m') \leq 0$ for some $m' < q$, then $\mu_m = 0$ for all $m > m'$ and $w(q) = v(q) = 0$. □

Corollary A.8. *Suppose $z \leq 0$. If $v + z < \delta \left(\frac{n-q}{n-q+(1-\delta)}\right) w$, then $s(m) = w(m) = w_{out}(m) = v(m) = 0$ for all $m \geq 2$.*

Proof of Corollary A.8. In Lemma A.2 we showed that a necessary condition for trade with probability one at $m = 1$ is that $v + z \geq \delta \left(\frac{n-q}{n-q+(1-\delta)}\right) w$. Thus, when this condition is violated, $\mu_1 < 1$. The result then follows from Proposition 5.7. □

Proof of Proposition 5.8. Fix an equilibrium in the subgame starting in state $m - 1$. This produces continuation values $\tilde{v}(m-1)$ and $\tilde{w}_{out}(m-1)$. Given these continuation values, let $v(m; \mu_m)$ and $w(m; \mu_m)$ denote the values of the principal and uncommited agent in state m when transaction probability μ_m, and let $s(m; \mu_m)$ denote the surplus in state m when transaction probability μ_m. From (12) and (13), if $\tilde{v}(m - 1) > 0$ and $\tilde{w}_{out}(m - 1) > 0$, then $v(m; \mu_m)$ and $w(m; \mu_m)$ are both increasing in μ_m, and therefore $s(m; \mu_m) = [\tilde{v}(m - 1) - v(m; \mu_m)] + [\tilde{w}_{out}(m - 1) - w(m; \mu_m)]$ is decreasing in μ_m. It follows that if $s(m; 1) > 0$, then $s(m; \mu_m) > 0$ for any $\mu_m \in (0, 1)$, and as a result, any such $\mu_m \in (0, 1)$ would not be consistent with equilibrium.

We finish the proof with an induction argument. First, note that if the conditions for existence of a FTE are met, then by Lemma A.2 $\mu_1 = 1$ (the unique MPE of the subgame starting at $m = 1$ is a FTE). Second, we argue that if the unique MPE of the subgame starting in state $m - 1$ is a FTE, then $\mu_m = 1$. The two conditions establish the result. To prove the induction step, note that if the unique MPE of the subgame starting in state $m-1$ is a FTE, then existence of a FTE in $m \leq q$ (guaranteed by Lemma 5.7 given the existence of a FTE) implies that $s(m; 1) > 0$. Then our previous argument implies that $s(m; \mu_m) > 0$ for any $\mu_m \in (0, 1)$, and as a result we must have $\mu_m = 1$. □

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