No-envy in Queueing Problems

Youngsub Chun*
School of Economics
Seoul National University
Seoul 151-742, Korea
and
Department of Economics
University of Rochester
Rochester, NY 14627, USA
E-mail: ychun@plaza.snu.ac.kr

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ABSTRACT

We explore the implications of *no-envy* (Foley 1967) in the context of queueing problems. We identify an easy way of checking whether a rule satisfies *efficiency* and *no-envy.* The existence of such a rule can easily be established. Next, we ask whether there is a rule satisfying *efficiency* and *no-envy* together with an additional solidarity requirement: every agent should be affected in the same direction as a consequence of changes in the waiting costs. However, there is no rule satisfying *efficiency, no-envy,* and either one of two *cost monotonicity* axioms. To remedy the situation, we propose two modifications of *no-envy, backward/forward no-envy* and *adjusted no-envy.* Also, we discuss whether three fairness requirements, *no-envy,* the *identical preferences lower bound,* and *egalitarian equivalence,* are compatible in this context.

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Key Words: queueing problem, no-envy, cost monotonicity, identical preferences lower bound, egalitarian equivalence.

1. Introduction

Consider a group of agents who must be served in a facility. The facility can handle only one agent at a time and agents incur waiting costs. We are interested in finding the order in which to serve agents and the (positive or negative) monetary compensations they should receive. We assume that an agent's waiting cost is constant per unit of time, but that agents differ in their waiting costs. Each agent's utility is equal to his monetary compensations minus his total waiting cost. This queueing problem has been studied extensively from the incentive perspective (Dolan 1978, Suijs 1996, Mitra 2001, 2002). However, it has received only a limited attention from the normative perspective. The only exceptions are recent works, Maniquet (2003) and Chun (2004), which discuss the properties of rules obtained by applying the Shapley value (1950).

In this paper, we investigate the implications of *no-envy* in queueing problems. *No-envy,* introduced by Foley (1967), requires that no agent should end up with a higher utility by consuming what any other agent consumes. Although its implications have been studied for a wide class of problems, it has not been the object of any study in queueing problems.

We identify an easy way of checking whether a rule satisfies *efficiency* and *no-envy.* It can be described in a simple way: choose any efficient queue, and then check the difference of transfers between any two neighboring agents. If the difference is not greater than the higher waiting cost of the two agents and is not smaller than the lower waiting cost of the two agents, then it passes the *no-envy* test. Of course, it is an immediate consequence of *no-envy* that an agent served earlier should receive a smaller transfer than an agent served later. The existence of such a rule can easily be established.

Next, we investigate whether there is a rule satisfying *efficiency* and *no-envy* together with an additional solidarity requirement: all agents should gain together or lose together as a consequence of changes in the waiting costs. *Negative cost monotonicity* (Maniquet 2003) requires that an increase in an agent's waiting cost should cause other agents to weakly lose together. On the other hand, *positive cost monotonicity* (Chun 2004) requires that it should cause other agents to weakly gain together. We show that if the society consists of more than two agents, then there is no rule satisfying *efficiency, no-envy,* and either *negative cost monotonicity* or *positive cost* *monotonicity.*[1]

Faced with the impossibility results, we suggest two modifications of *no-envy* which can be applied to queueing problems. Our first modification of *no-envy* requires that an agent should not envy the other agents at least in one direction. More specifically, *backward no-envy* requires that an agent should not envy the agents with lower waiting costs, whereas *forward no-envy* requires that an agent should not envy the agents with higher waiting costs. Our second modification of *no-envy* considers that when one agent is evaluating the other agent's consumption bundle, he should have in mind that the rule will assign different transfers if he is in another agent's position since their waiting costs are different. *Adjusted no-envy* requires that an agent should not envy the other agents after accommodating this differences in transfers.

For each of these modifications, we show that the impossibility results do not hold any more. In fact, the Maniquet rule (2003) satisfies *efficiency, negative cost monotonicity, backward no-envy,* and *adjusted no-envy,* and the reverse rule (Chun 2004) satisfies *efficiency, positive cost monotonicity, forward no-envy,* and *adjusted no-envy.*

Other fairness requirements widely discussed in the literature are: the *identical preferences lower bound* requires that each agent should be at least as well off as he would be, under *efficiency* and *equal treatment of equals,* if all other agents had the same preferences, and *egalitarian equivalence* requires that there should be a reference bundle such that each agent enjoys the same welfare between his bundle and that reference bundle. We investigate whether the three requirements are compatible in the current context. First, it is easy to show that *efficiency* and *no-envy* together imply the *identical preferences lower bound.* Also, we can show an existence of a rule satisfying *efficiency, egalitarian equivalence,* and the *identical preferences lower bound.* However, if we have more than three agents, then there is no rule satisfying *efficiency, egalitarian equivalence,* and *no-envy* together.

The paper is organized as follows. Section 2 contains some preliminaries and introduces rules. Section 3 explores the implications of *efficiency*

[1]As we show later, if the society consists of only two agents, then the Maniquet rule satisfies *efficiency, no-envy,* and *negative cost monotonicity,* and the reverse rule satisfies *efficiency, no-envy,* and *positive cost monotonicity.* Moreover, the rules can be characterized by these axioms if *Pareto indifference* is additionally imposed. See Remarks 2 and 3 for details.

and *no-envy,* and Section 4 studies their implications together with cost monotonicity axioms. Section 5 introduces two modifications of *no-envy, backwarad/forward no-envy* and *adjusted no-envy,* and investigates their implications. Section 6 discusses whether three fairness requirements, *no-envy,* the *identical preferences lower bound,* and *egalitarian equivalence,* are compatible in this context. Concluding remarks follow in Section 7.

2. Preliminaries

Let $I \equiv \{1, 2, \ldots\}$ be an (infinite) universe of "potential" agents, and let $\mathcal{N}$ be the family of non-empty subsets of I. Each agent $i \in I$ is characterized by his *unit waiting cost,* $\theta_i \geq 0$. Given $N \in \mathcal{N}$, each agent $i \in N$ is assigned a position $\sigma_i \in \mathbf{N}$ in a queue and a positive or negative transfer $t_i \in \mathbf{R}$. The agent who is served first incurs no waiting cost. If agent $i \in N$ is served in the σ_i^{th} position, his waiting cost is $(\sigma_i - 1)\theta_i$. Each agent $i \in N$ has a quasi-linear utility function, so that his utility from consuming the bundle (σ_i, t_i) is given by $u(\sigma_i, t_i; \theta_i) = t_i - (\sigma_i - 1)\theta_i$.

A *queueing problem* is defined as a list $q = (N, \theta)$ where $N \in \mathcal{N}$ is the set of agents and θ is the vector of unit waiting costs. Let $\mathcal{Q}^N$ be the class of all problems for N and $\mathcal{Q} = \cup \mathcal{Q}^N$. An *allocation* for $q \in \mathcal{Q}$ is a pair $z = (\sigma, t)$, where for each $i \in N$, σ_i denotes agent i's position in the queue and t_i the monetary transfer to him. An allocation is *feasible* if no two agents are assigned the same position and the sum of the transfers is not positive. Thus, the set of feasible allocations $Z(q)$ consists of all pairs $z = (\sigma, t) \in \mathbf{N} \times \mathbf{R}^N$ such that for all i, $j \in N$, $i \neq j$ implies $\sigma_i \neq \sigma_j$ and $\sum_{i \in N} t_i \leq 0$.

Given $q = (N, \theta) \in \mathcal{Q}^N$, an allocation $z = (\sigma, t) \in Z(q)$ is *queue-efficient* if it minimizes the total waiting cost, that is, for all $z' = (\sigma', t') \in Z(q)$, $\sum_{i \in N}(\sigma_i - 1)\theta_i \leq \sum_{i \in N}(\sigma'_i - 1)\theta_i$. As noted in Maniquet (2003), the efficient queue of a problem does not depend on the transfers. Moreover, it is unique except for agents with equal waiting costs, who will be next to each other in the queue and can be permuted. The set of efficient queues for $q \in \mathcal{Q}^N$ is denoted by $Eff(q)$. Also, an allocation $z = (\sigma, t) \in Z(q)$ is *budget balanced* if $\sum_{i \in N} t_i = 0$, and *efficient* if it is *queue-efficient* and *budget balanced.* A *rule* is a mapping $\varphi : \mathcal{Q} \to \cup_{N \in \mathcal{N}} Z(q)$, which associates with each problem $q \in \mathcal{Q}^N$ a non-empty subset $\varphi(q)$ of feasible allocations. The pair $\varphi_i(q) = (\sigma_i, t_i)$ represents the position of i in the queue and his transfer in q. Given

$q = (N, \theta) \in \mathcal{Q}^N$, $z = (\sigma, t) \in Z(q)$, and $i \in N$, let $P_i(\sigma)$ be the set of agents preceding agent i and $F_i(\sigma)$ the set of agents following him.

Now we introduce axioms which we will impose on rules. *Efficiency* requires that the rule should choose an *efficient* allocation. *Pareto indifference* requires that if an allocation is chosen by a rule, then all other allocations which assign the same utilities to each agent should be chosen by the rule. Finally, *equal treatment of equals* requires that two agents with the same waiting cost should end up with the same utilities.

Efficiency: For all $q = (N, \theta) \in \mathcal{Q}^N$ and all $z = (\sigma, t) \in \varphi(q)$, $\sigma \in Eff(q)$ and $\sum_{i \in N} t_i = 0$.

Pareto indifference: For all $q = (N, \theta) \in \mathcal{Q}^N$, all $z = (\sigma, t) \in \varphi(q)$, and $z' = (\sigma', t') \in Z(q)$, if for all $i \in N$, $u(\sigma'_i, t'_i; \theta_i) = u(\sigma_i, t_i; \theta_i)$, then $z' \in \varphi(q)$.

Equal treatment of equals: For all $q = (N, \theta) \in \mathcal{Q}^N$, all $z = (\sigma, t) \in \varphi(q)$, and all i, $j \in N$, if $\theta_i = \theta_j$, then $u(\sigma_i, t_i; \theta_i) = u(\sigma_j, t_j; \theta_j)$.

Next are two rules studied in Maniquet (2003) and Chun (2004). The Maniquet rule selects an efficient queue and transfers to each agent a half of his unit waiting cost from each of his predecessors minus a half of the unit waiting cost of each of his followers.

Maniquet rule, φ^M: For all $q = (N, \theta) \in \mathcal{Q}^N$,

$$\varphi^M(q) = \{(\sigma^M, t^M) \in Z(q) |\ \sigma^M \in Eff(q), \text{ and } \forall i \in N,$$

$$t_i^M = (\sigma_i^M - 1)\frac{\theta_i}{2} - \sum_{j \in F_i(\sigma^M)} \frac{\theta_j}{2}\}.$$

Chun (2004) introduces the rule that selects an efficient queue and transfers to each agent a half of the unit waiting cost of each of his predecessors minus a half of his waiting cost to each of his followers.

Reverse rule, φ^R: For all $q = (N, \theta) \in \mathcal{Q}^N$,

$$\varphi^R(q) = \{(\sigma^R, t^R) \in Z(q) |\ \sigma^R \in Eff(q), \text{ and } \forall i \in N,$$

$$t_i^R = \sum_{j \in P_i(\sigma^R)} \frac{\theta_j}{2} - (|N| - \sigma_i^R)\frac{\theta_i}{2}\}.$$

As shown in Maniquet (2003) and Chun (2004), the Maniquet rule and the reverse rule can be obtained by applying the Shapley value (1950). To do this, the queueing problems should be mapped into cooperative games in which the worth of a coalition is appropriately defined. For the Maniquet rule, the worth of a coalition is defined to be the minimum waiting cost incurred by its members under the assumption that they are served before the non-coalitional members. For the reverse rule, it is defined to be the minimum waiting cost incurred by its members under the assumption that they are served after the non-coalitional members.

If some agents have equal waiting costs, then the efficient queue is not unique, and consequently, the allocations specified by the rule are not unique either. However, the Maniquet and reverse rules are *essentially single-valued* in the sense that all agents end up with the same utilities at the allocations that they select.

3. Efficiency and no-envy

Now we explore the implications of *no-envy* in queueing problems. *No-envy,* introduced by Foley (1967), requires that no agent should end up with a higher utility by consuming what any other agent consumes. It is a standard requirement in the studies of fairness for a wide class of problems (Thomson and Varian 1985, Thomson 2003b). Given $q = (N, \theta) \in \mathcal{Q}^N$, an allocation $z = (\sigma, t) \in Z(q)$ is *envy-free* if for all i, $j \in N$, $u(\sigma_i, t_i; \theta_i) \geq u(\sigma_j, t_j; \theta_i)$. Let $F(q)$ be the set of all envy-free allocations for $q \in \mathcal{Q}^N$.

No-envy: For all $q = (N, \theta) \in \mathcal{Q}^N$ and all $z = (\sigma, t) \in \varphi(q)$, $z \in F(q)$.

We present a simple way of checking whether a rule satisfies *efficiency* and *no-envy.* First, *efficiency* requires that the sum of total waiting costs be minimized and that the sum of transfers be zero, that is, for all $q = (N, \theta) \in \mathcal{Q}^N$ such that $N = \{1, \ldots, n\}$ and all $z = (\sigma, t) \in Z(q)$, $\theta_{\sigma_1} \geq \cdots \geq \theta_{\sigma_n}$ and $\sum_{i \in N} t_i = 0$.

Theorem 1. *A rule φ satisfies efficiency and no-envy if and only if for all $q = (N, \theta) \in \mathcal{Q}^N$ such that $N = \{1, \ldots, n\}$ and all $z = (\sigma, t) \in \varphi(q)$, $\sigma \in Eff(q)$, $\sum_{i \in N} t_i = 0$, and for all $i = 1, \ldots n-1$, $\theta_{\sigma_i} \geq t_{\sigma_{i+1}} - t_{\sigma_i} \geq \theta_{\sigma_{i+1}}$.*

Proof. Let φ be a rule satisfying the two axioms. Let $q = (N, \theta) \in \mathcal{Q}^N$ and $z = (\sigma, t) \in \varphi(q)$ be such that $N = \{1, \ldots, n\}$. By *efficiency,* $\sigma \in Eff(q)$ and $\sum_{i \in N} t_i = 0$.

To simplify notation, we assume that $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_n$ and for all $i \in N$, $\sigma_i = i$. Let $i, j \in N$. We may also assume, without loss of generality, that $j = i + k$ for $k \in \mathbf{N}$.

First, for i not to envy j,

$$u(\sigma_i, t_i; \theta_i) \geq u(\sigma_j, t_j; \theta_i),$$

which is equivalent to

$$t_i - (i-1)\theta_i \geq t_j - (j-1)\theta_i$$

or

$$k\theta_i \geq t_j - t_i.$$

In particular, if $j = i + 1$, this inequality becomes

$$\theta_i \geq t_{i+1} - t_i,$$

the desired expression.

Applying this inequality recursively, for all $\ell = i, \ldots, i + k - 1$, we have

$$\theta_\ell \geq t_{\ell+1} - t_\ell.$$

Summing these inequalities,

$$\sum_{\ell=i}^{i+k-1} \theta_\ell \geq t_{i+k} - t_i.$$

Since *efficiency* implies that $k\theta_i \geq \sum_{\ell=i}^{i+k-1} \theta_\ell$, we have

$$k\theta_i \geq t_j - t_i.$$

Therefore, it is sufficient to check the inequality between neighboring agents.

Similarly, for j not to envy i,

$$t_j - t_i \geq k\theta_{i+1}.$$

In particular, if $j = i + 1$, this inequality becomes

$$t_{i+1} - t_i \geq \theta_{i+1}.$$

Once again, by the same reasoning, we obtain the desired conclusion. ∎

Example 1: *The Maniquet and reverse rules do not satisfy no-envy.* Let $q = (N, \theta) \in \mathcal{Q}^N$ be such that $N = \{1, 2, 3\}$ and $\theta = (6, 4, 2)$. Then, $\varphi^M(q) = (\sigma^M, t^M)$ is obtained by setting $\sigma^M = (1, 2, 3)$ and $t^M = (-3, 1, 2)$. Note that agent 3 envies agent 2 since $u(\sigma_2^M, t_2^M; \theta_3) = -1 > u(\sigma_3^M, t_3^M; \theta_3) = -2$. On the other hand, $\varphi^R(q) = (\sigma^R, t^R)$ is obtained by setting $\sigma^R = (1, 2, 3)$ and $t^R = (-6, 1, 5)$. Note that agent 1 envies agent 2 since $u(\sigma_2^R, t_2^R; \theta_1) = -5 > u(\sigma_1^R, t_1^R; \theta_1) = -6$.

Example 2: *Rules satisfying efficiency and no-envy.* Let $q = (N, \theta) \in \mathcal{Q}^N$ be such that $N = \{1 \ldots, n\}$. To simplify our notation, we suppose that $\theta_1 \geq \cdots \geq \theta_n$ and $\sigma_i = i$. First, let $t_1 = \alpha_1$. We will determine α_1 after considering the budget constraint. Now, for $i = 2, \ldots, n$, we choose $\alpha_i \in [\theta_i, \theta_{i-1}]$ and $t_i = \sum_{j=1}^{i} \alpha_j$. Finally, we choose α_1 such that $\sum_{i \in N} t_j = 0$. An alternative rule can be given starting from n. Once again, let $t_n = -\beta_n$. We will determine β_n after considering the budget constraint. Now, for $i = n - 1, \ldots, 1$, we choose $\beta_i \in [\theta_{i+1}, \theta_i]$ and $t_i = -\sum_{j=i}^{n} \beta_j$. Finally, we choose β_n such that $\sum_{i \in N} t_j = 0$. Clearly, these processes lead to rules satisfying *efficiency* and *no-envy.*

Remark 1: As shown in Svensson (1983), in economies with indivisible goods, *no-envy* implies *efficiency.* Also, *no-envy* is equivalent to *group no-envy,*[2] and the set of *envy-free* allocations coincides with the set of *equal*

[2]Given two groups of the same size, suppose that a group redistributes among its members what is available to the other group. If a rule selects an allocation which is impossible to make every agent in the group better off, with at least one agent strictly better off, even after considering the possibility of redistribution, then the rule satisfies *group no-envy.*

income Walrasian allocations.[3] Similar observations can be made for queueing problems. If we restrict our attention to rules selecting *budget balanced* allocations, then *no-envy* implies *efficiency.* Also, *no-envy* is equivalent to *group no-envy,* and the set of *envy-free* allocations coincide with the set of *equal income Walrasian allocations.*

4. No-envy and cost monotonicity

We investigate whether there is a rule satisfying *efficiency* and *no-envy* together with an additional solidarity axiom: as a consequence of changes in the external environment, all agents should gain together or lose together. In our model, the axiom can be formulated in the following way. Suppose that the waiting cost of an agent increases. One could take two positions with regards to how the allocation should be affected by this change: (i) one may feel that he deserves greater compensation for having to wait if he is served at the same time, which will affect the other agents in a negative direction. Alternatively, (ii) one may feel that he should be required to pay more if he is served earlier, which will affect the other agents in a positive direction. *Negative cost monotonicity* (Maniquet 2003) requires that this should cause all other agents to weakly lose together. On the other hand, *positive cost monotonicity* (Chun 2004) requires that the increase should cause all other agents to weakly gain together. The Maniquet rule satisfies *negative cost monotonicity,* and the reverse rule satisfies *positive cost monotonicity.*

Negative cost monotonicity: For all $q = (N, \theta)$ and $q' = (N, \theta') \in \mathcal{Q}^N$, all $z = (\sigma, t) \in \varphi(q)$, all $z' = (\sigma', t') \in \varphi(q')$, and all $k \in N$, if for all $i \in N \backslash \{k\}$, $\theta_i = \theta'_i$ and $\theta_k < \theta'_k$, then for all $i \in N \backslash \{k\}$, $u(\sigma_i, t_i; \theta_i) \geq u(\sigma'_i, t'_i; \theta'_i)$.

Positive cost monotonicity: For all $q = (N, \theta)$ and $q' = (N, \theta') \in \mathcal{Q}^N$, all $z = (\sigma, t) \in \varphi(q)$, all $z' = (\sigma', t') \in \varphi(q')$, and all $k \in N$, if for all $i \in N \backslash \{k\}$, $\theta_i = \theta'_i$ and $\theta_k < \theta'_k$, then for all $i \in N \backslash \{k\}$, $u(\sigma_i, t_i; \theta_i) \leq u(\sigma'_i, t'_i; \theta'_i)$.

Now we ask whether there is a rule satisfying *efficiency, no-envy,* and *negative cost monotonicity* together. The answer is no.

[3] Allocations that can be supported as Walrasian equilibrium with the equal implicit income.

Theorem 2. *Let $|N| \geq 3$. Then, there is no rule satisfying efficiency, no-envy, and negative cost monotonicity.*

Proof. Let φ be a rule satisfying the three axioms. Let $q = (N, \theta) \in \mathcal{Q}^N$ and $z = (\sigma, t) \in \varphi(q)$ be such that $N = \{1, \ldots, n\}$ with $n \geq 3$ and $\theta_1 > \theta_2 > \cdots > \theta_n$. By *efficiency,* for all $i \in N$, $\sigma(i) = i$. Moreover, from Theorem 1, for all $i = 1, \ldots, n-1$, $\theta_i \geq t_{i+1} - t_i \geq \theta_{i+1}$.

Case 1: There exists $i \in N \backslash \{n\}$ such that $\theta_i \geq t_{i+1} - t_i > \theta_{i+1}$. Let $\alpha \in]0,1]$ be such that $t_{i+1} - t_i = \alpha\theta_i + (1-\alpha)\theta_{i+1} = \theta_{i+1} + \alpha(\theta_i - \theta_{i+1})$. Let $\varepsilon > 0$ be such that $\varepsilon < \alpha(\theta_i - \theta_{i+1})$ and $\theta'_i = \theta_{i+1} + \varepsilon$. Let θ' be the waiting cost vector obtained from θ by replacing θ_i with θ'_i, $q' = (N, \theta')$, and $z' = (\sigma', t') \in \varphi(q')$.

By *negative cost monotonicity,* all agents except i weakly gain. Since the decrease of θ_i to θ'_i does not change the efficient queue, that is, for all $i \in N$, $\sigma'(i) = i$, this is possible only if for all $j \neq i$, $t'_j \geq t_j$. By *efficiency,* $t'_i \leq t_i$. Altogether, $t'_{i+1} - t'_i \geq t_{i+1} - t_i = \theta_{i+1} + \alpha(\theta_i - \theta_{i+1}) > \theta_{i+1} + \varepsilon = \theta'_i > \theta_{i+1}$, which contradicts the conclusion of Theorem 1.

Case 2: For all $i = 1, \ldots, n-1$, $t_{i+1} - t_i = \theta_{i+1}$. Let $i \in N \backslash \{1, n\}$ and θ'_i be such that $\theta_i > \theta'_i > \theta_{i+1}$. Let θ' be the waiting cost vector obtained from θ by replacing θ_i with θ'_i, $q' = (N, \theta')$, and $z' = (\sigma', t') \in \varphi(q')$. By *negative cost monotonicity,* all agents except i weakly gain. Since the decrease of θ_i to θ'_i does not change the efficient queue, this is possible only if for all $j \neq i$, $t'_j \geq t_j$. By *efficiency,* $t'_i \leq t_i$. Altogether, $t'_{i+1} - t'_i \geq t_{i+1} - t_i = \theta_{i+1}$. If $t'_{i+1} - t'_i > \theta_{i+1} = \theta'_{i+1}$, then we go back to Case 1 and obtain the desired conclusion. If $t'_{i+1} - t'_i = \theta_{i+1}$, by *efficiency,* we deduce that for all $j \in N$, $t'_j = t_j$. In particular, $t'_i - t'_{i-1} = \theta_i$. Since $\theta_{i-1} = \theta'_{i-1} > t'_i - t'_{i-1} > \theta'_i$, we go back to Case 1 and obtain the desired conclusion. ∎

Remark 2: For $n = 2$, it is clear from the proof that there is only one rule satisfying *efficiency, Pareto indifference, no-envy,* and *negative cost monotonicity.*[4] It is obtained by setting $t_1 = -\frac{\theta_2}{2}$ and $t_2 = \frac{\theta_2}{2}$, which is the allocation chosen by the Maniquet rule. The conclusion follows by noting that for $n = 2$, the Maniquet rule satisfies the four axioms.

Next, we ask whether a possibility result can be obtained by replacing

[4]If *Pareto indifference* is not imposed, then it is possible to choose only one *efficient* queue when two agents have equal waiting costs.

negative cost monotonicity with *positive cost monotonicity.* Once again, the answer is no.

Theorem 3. *Let $|N| \geq 3$. Then, there is no rule satisfying efficiency, no-envy, and positive cost monotonicity.*

Proof. Let φ be a rule satisfying the three axioms. Let $q = (N, \theta) \in \mathcal{Q}^N$ and $z = (\sigma, t) \in \varphi(q)$ be such that $N = \{1, \ldots, n\}$ with $n \geq 3$ and $\theta_1 > \theta_2 > \cdots > \theta_n$. By *efficiency,* for all $i \in N$, $\sigma(i) = i$. Moreover, from Theorem 1, for all $i = 1, \ldots, n-1$, $\theta_i \geq t_{i+1} - t_i \geq \theta_{i+1}$.

Case 1: There exists $i \in N \backslash \{n\}$ such that $\theta_i > t_{i+1} - t_i \geq \theta_{i+1}$. Let $\alpha \in [0, 1[$ be such that $t_{i+1} - t_i = \alpha\theta_i + (1-\alpha)\theta_{i+1} = \theta_{i+1} + \alpha(\theta_i - \theta_{i+1})$. Let $\varepsilon > 0$ be such that $\theta_i > \theta_{i+1} + \varepsilon > \theta_{i+1} + \alpha(\theta_i - \theta_{i+1})$ and $\theta'_{i+1} = \theta_{i+1} + \varepsilon$. Let θ' be the waiting cost vector obtained from θ by replacing θ_{i+1} with θ'_{i+1}, $q' = (N, \theta')$, and $z' = (\sigma', t') \in \varphi(q')$.

By *positive cost monotonicity,* all agents except $i+1$ weakly gain. Since the increase of θ_{i+1} to θ'_{i+1} does not change the efficient queue, that is, for all $i \in N$, $\sigma'(i) = i$, this is possible only if for all $j \neq i+1$, $t'_j \geq t_j$. By *efficiency,* $t'_{i+1} \leq t_{i+1}$. Altogether, $t'_{i+1} - t'_i \leq t_{i+1} - t_i = \theta_{i+1} + \alpha(\theta_i - \theta_{i+1}) < \theta_{i+1} + \varepsilon = \theta'_{i+1} < \theta_i$, which contradicts the conclusion of Theorem 1.

Case 2: For all $i = N \backslash \{n\}$, $t_{i+1} - t_i = \theta_i$. Let $i = N \backslash \{1, n\}$, and θ'_i be such that $\theta_{i-1} > \theta'_i > \theta_i$. Let θ' be the waiting cost vector obtained from θ by replacing θ_i with θ'_i, $q' = (N, \theta')$, and $z' = (\sigma', t') \in \varphi(q')$. By *positive cost monotonicity,* all agents except i weakly gain. Since the increase of θ_i to θ'_i does not change the efficient queue, this is possible only if for all $j \neq i$, $t'_j \geq t_j$. By *efficiency,* $t'_i \leq t_i$. Altogether, $t'_i - t'_{i-1} \leq t_i - t_{i-1} = \theta_{i-1}$. If $t'_i - t'_{i-1} < \theta_{i-1}$, then we go back to Case 1 and obtain the desired conclusion. On the other hand, if $t'_i - t'_{i-1} = \theta'_i = \theta_{i-1}$, by *efficiency,* we deduce that for all $j \in N$, $t'_j = t_j$. In particular, $t'_{i+1} - t'_i = t_{i+1} - t_i = \theta_i$. Since $\theta'_i > t'_{i+1} - t'_i > \theta_{i+1}$, we go back to Case 1 and obtain the desired conclusion. ∎

Remark 3: For $n = 2$, it is clear from the proof that there is only one rule satisfying *efficiency, Pareto indifference, no-envy,* and *positive cost monotonicity.* It is obtained by setting $t_1 = -\frac{\theta_1}{2}$ and $t_2 = \frac{\theta_1}{2}$, which is the allocation chosen by the reverse rule. The conclusion follows by noting that for $n = 2$, the reverse rule satisfies the four axioms.

Next, we introduce two independence requirements: a change in an agent's waiting cost should not affect the agents following him or preceding him. More specifically, suppose that an agent's waiting cost changes. One could take two positions with regards to how the allocation should be affected by this change: (i) if his waiting cost increases, then one may feel that he deserves greater compensation for having to wait, which is more likely not to affect the agents following him, and (ii) if his waiting cost decreases, then one may feel that he deserves less payment for having the service, which is more likely not to affect the agents preceding him.

Independence of preceding costs (Maniquet 2003) requires that an increase in an agent's waiting cost should not affect the agents following him. On the other hand, *independence of following costs* (Chun 2004) requires that a decrease in an agent's waiting cost should not affect the agents preceding him. The Maniquet rule satisfies *independence of preceding costs,* and the reverse rule satisfies *independence of following costs.*

Independence of preceding costs: For all $q = (N, \theta)$ and $q' = (N, \theta') \in \mathcal{Q}^N$, all $z = (\sigma, t) \in \varphi(q)$, all $z' = (\sigma', t') \in \varphi(q')$, and all $k \in N$, if for all $i \in N \backslash \{k\}$, $\theta_i = \theta'_i$ and $\theta_k < \theta'_k$, then for all $j \in N$ such that $\sigma_j > \sigma_k$, $u(\sigma_j, t_j; \theta_j) = u(\sigma'_j, t'_j; \theta'_j)$.

Independence of following costs: For all $q = (N, \theta)$ and $q' = (N, \theta') \in \mathcal{Q}^N$, all $z = (\sigma, t) \in \varphi(q)$, all $z' = (\sigma', t') \in \varphi(q')$, and all $k \in N$, if for all $i \in N \backslash \{k\}$, $\theta_i = \theta'_i$ and $\theta_k > \theta'_k$, then for all $j \in N$ such that $\sigma_j < \sigma_k$, $u(\sigma_j, t_j; \theta_j) = u(\sigma'_j, t'_j; \theta'_j)$.

We ask whether there is a rule satisfying either one of two independence requirements together with *efficiency* and *no-envy.* Once again, we obtain negative results.

Theorem 4. *Let $|N| \geq 3$. Then, there is no rule satisfying efficiency, no-envy, and independence of preceding costs.*

Proof. Let φ be a rule satisfying the three axioms. Let $q = (N, \theta) \in \mathcal{Q}^N$ and $z = (\sigma, t) \in \varphi(q)$ be such that $N = \{1, \ldots, n\}$ with $n \geq 3$ and $\theta_1 > \theta_2 > \cdots > \theta_n$. By *efficiency,* for all $i \in N$, $\sigma(i) = i$. From Theorem 1, for all $i = 1, \ldots, n-1$, $\theta_i \geq t_{i+1} - t_i \geq \theta_{i+1}$.

Now let θ' be such that $\theta'_1 = \theta'_2$ and that for all $i = 3, \ldots, n$, $\theta'_i = \theta_i$. Let $q' = (N, \theta')$ and $z' = (\sigma', t') \in \varphi(q')$. Furthermore, we assume that $\theta'_2 >$

$2(t_3 - \theta_3) - (t_1 + t_2)$. By *independence of preceding costs,* for all $i = 3, \ldots, n$, $t'_i = t_i$. First, we consider the case $\sigma_1 = 1$ and $\sigma_2 = 2$. By *efficiency* and *no-envy,* $t'_2 - t'_1 = \theta'_2$. Altogether, $t'_2 - t'_3 = \theta'_2 > 2(t_3 - \theta_3) - (t_1 + t_2)$. Since $t'_1 + t'_2 = t_1 + t_2$, we have $\theta_3 > t_3 - t'_2 = t'_3 - t'_2$, contradicting the conclusion of Theorem 1.

The case $\sigma_1 = 2$ and $\sigma_2 = 1$ can be handled in a similar way. ∎

Theorem 5. *Let $|N| \geq 3$. Then, there is no rule satisfying efficiency, no-envy, and independence of following costs.*

Proof. Let φ be a rule satisfying the three axioms. Let $q = (N, \theta) \in \mathcal{Q}^N$ and $z = (\sigma, t) \in \varphi(q)$ be such that $N = \{1, \ldots, n\}$ with $n \geq 3$ and $\theta_1 = \theta_2 \geq \theta_3 \geq \cdots \geq \theta_n$, and that $\theta_3 > 0$. By *efficiency,* for all $i \in N \backslash \{1, 2\}$, $\sigma(i) = i$. First, we consider the case $\sigma_1 = 1$ and $\sigma_2 = 2$. From Theorem 1, for all $i = 1, \ldots, n-1$, $\theta_i \geq t_{i+1} - t_i \geq \theta_{i+1}$. In particular, $t_2 - t_1 = \theta_1$ and $t_n \geq \cdots \geq t_3 > t_2$.

Now let θ' be such that $\theta'_1 = \theta_1$ and $\theta'_2 = \cdots = \theta'_n = 0$. By *independence of following costs,* $t'_1 = t_1$. By *efficiency* and *no-envy,* for all $i = 2, \ldots, n$, $t'_i = \frac{1}{n-1} \sum_{j=2}^{n} t_j$, which implies that $t'_2 > t_2$. Altogether, $t'_2 - t'_1 > t_2 - t_1 = \theta_1 = \theta'_1$, which contradicts the conclusion of Theorem 1.

The case $\sigma_1 = 2$ and $\sigma_2 = 1$ can be handled in a similar way. ∎

Remark 4: As before, these impossibility results do not hold if the society consists of only two agents. For $|N| = 2$, the Maniquet rule satisfies *efficiency, no-envy,* and *independence of preceding costs,* and the reverse rule satisfies *efficiency, no-envy,* and *independence of following costs.* Moreover, by imposing *Pareto indifference* additionally, the rule can be characterized. In fact, the proofs can be obtained from Maniquet (2003) and Chun (2004) by noting that *no-envy* implies *equal treatment of equals.*

5. Backward/forward no-envy and adjusted no-envy

Given the negative results presented in the previous section, we propose two modifications of *no-envy,* which can be imposed in the context of queueing problems. As we show here, these modifications have a significant effect since we can recover positive results.

First, we propose a weakening of *no-envy*, which require that an agent should not envy other agents at least in one direction. *Backward no-envy* requires that an agent should not envy the agents with lower waiting costs (therefore, following him in the efficient queue with the possible exception of agents with the same waiting cost), whereas *forward no-envy* requires that an agent should not envy the agents with higher waiting costs (therefore, preceding him in the efficient queue with the possible exception of agents with the same waiting cost).

Backward no-envy: For all $q = (N, \theta) \in \mathcal{Q}^N$, all $z = (\sigma, t) \in \varphi(q)$, and all i, $j \in N$, if $\theta_i \geq \theta_j$, then $u(\sigma_i, t_i; \theta_i) \geq u(\sigma_j, t_j; \theta_i)$.

Forward no-envy: For all $q = (N, \theta) \in \mathcal{Q}^N$, all $z = (\sigma, t) \in \varphi(q)$, and all i, $j \in N$, if $\theta_i \leq \theta_j$, then $u(\sigma_i, t_i; \theta_i) \geq u(\sigma_j, t_j; \theta_i)$.

In the language of Theorem 1, *backward no-envy,* together with *efficiency,* requires that for all $i = 1, \ldots, n-1$, $\theta_i \geq t_{i+1} - t_i$. On the other hand, *forward no-envy,* together with *efficiency,* requires that $t_{i+1} - t_i \geq \theta_{i+1}$. As it turns out, the Maniquet rule satisfies *backward no-envy,* while the reverse rule satisfies *forward no-envy.*

Proposition 1. *The Maniquet rule satisfies backward no-envy and the reverse rules satisfies forward no-envy.*

Proof. Let $q = (N, \theta) \in \mathcal{Q}^N$ and $z = (\sigma, t) \in \varphi(q)$ be such that $N = \{1, \ldots, n\}$ and $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_n$. To simplify notation, for all $i \in N$, we set $\sigma_i^M = i$. Also, we do not attach the superscript M to σ and t. Since the Maniquet rule satisfies *efficiency* and *equal treatment of equals,* it is enough to show that for $i = 1, \ldots, n-1$, if $\theta_i > \theta_{i+1}$, i does not envy $i+1$. From the definition of the Maniquet rule,

$$u(\sigma_i, t_i; \theta_i) = -(\sigma_i - 1)\theta_i + t_i = -(i-1)\theta_i + (i-1)\frac{\theta_i}{2} - \sum_{j=i+1}^{n} \frac{\theta_j}{2},$$

and

$$u(\sigma_{i+1}, t_{i+1}; \theta_i) = -(\sigma_{i+1} - 1)\theta_i + t_{i+1} = -i\theta_i + i\frac{\theta_{i+1}}{2} - \sum_{j=i+2}^{n} \frac{\theta_j}{2}.$$

Therefore,

$$u(\sigma_i, t_i; \theta_i) - u(\sigma_{i+1}, t_{i+1}; \theta_i) = (i+1)(\frac{\theta_i}{2} - \frac{\theta_{i+1}}{2}) \geq 0,$$

as desired.

Similarly, we can show that the reverse rule satisfies *forward no-envy.* ∎

Now we introduce our second modification of *no-envy.* For i, $j \in N$, if i and j interchange their positions, then the transfers assigned by the rule to each agent will be revised accordingly, since their waiting costs are different. After accommodating the changes in transfers, if i and j do not envy each other, then we say that the rule satisfies *adjusted no-envy.* To state the requirement formally, we introduce some notation. Given $q \in \mathcal{Q}$, $z = (\sigma, t) \in \varphi(q)$, and i, $j \in N$, let σ^{ij} be the queue obtained from σ by interchanging σ_i and σ_j, and t^{ij} be the transfer vector obtained when the rule is applied to σ^{ij}. Since this queue is not efficient in general, strictly speaking, we need to generalize our notion of a rule so that it be applicable to any, not necessarily efficient, queue. For simplicity, we abuse our definition and apply the rule to any queue.

Adjusted no-envy: For all $q \in \mathcal{Q}^N$, all $z = (\sigma, t) \in \varphi(q)$, and all i, $j \in N$, $u(\sigma_i, t_i; \theta_i) \geq u(\sigma_i^{ij}, t_i^{ij}; \theta_i)$.

It is interesting to note that both the Maniquet and the reverse rules satisfy this requirement.

Proposition 2. *The Maniquet rule satisfies adjusted no-envy.*

Proof. Let $q = (N, \theta) \in \mathcal{Q}^N$ and $z = (\sigma, t) \in \varphi(q)$ be such that $N = \{1, \ldots, n\}$ and $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_n$. To simplify notation, for all $i \in N$, we set $\sigma_i^M = i$. Also, we do not attach the superscript M to σ and t. Let i, $j \in N$. We may assume, without loss of generality, that $j = i + k$ for some $k \in \mathbf{N}$.

First, we show that i does not envy $j = i + k$ if the adjustment is made. Note that

$$u(\sigma_i, t_i; \theta_i) = (i-1)\frac{\theta_i}{2} - \sum_{\ell=i+1}^{n} \frac{\theta_\ell}{2} - (i-1)\theta_i,$$

and

$$u(\sigma_i^{ij}, t_i^{ij}; \theta_i) = (i+k-1)\frac{\theta_i}{2} - \sum_{\ell=i+k+1}^{n} \frac{\theta_\ell}{2} - (i+k-1)\theta_i.$$

Therefore,

$$u(\sigma_i, t_i; \theta_i) - u(\sigma_i^{ij}, t_i^{ij}; \theta_i) = k\frac{\theta_i}{2} - \sum_{\ell=i+1}^{i+k} \frac{\theta_\ell}{2} \geq 0,$$

as desired.

Similarly, we can show that $j = i + k$ does not envy i if the adjustment is made. ∎

Proposition 3. *The reverse rule satisfies adjusted no-envy.*

Proof. Let $q = (N, \theta) \in \mathcal{Q}^N$ and $z = (\sigma, t) \in \varphi(q)$ be such that $N = \{1, \ldots, n\}$ and $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_n$ be given. To simplify notation, for all $i \in N$, we set $\sigma_i^R = i$. Also, we do not attach the superscript R to σ and t. Let i, $j \in N$. We may assume, without loss of generality, that $j = i + k$ for some $k \in \mathbf{N}$.

First, we show that i does not envy $j = i + k$ if the adjustment is made. Note that

$$u(\sigma_i, t_i; \theta_i) = \sum_{\ell=1}^{i-1} \frac{\theta_\ell}{2} - (n-i)\frac{\theta_i}{2} - (i-1)\theta_i,$$

and

$$u(\sigma_i^{ij}, t_i^{ij}; \theta_i) = \sum_{\ell=1}^{i+k-1} \frac{\theta_\ell}{2} - \frac{\theta_i}{2} + \frac{\theta_j}{2} - (n-i-k)\frac{\theta_i}{2} - (i+k-1)\theta_i.$$

Therefore,

$$u(\sigma_i, t_i; \theta_i) - u(\sigma_i^{ij}, t_i^{ij}; \theta_i) = -\frac{\theta_j}{2} - \sum_{\ell=i+1}^{i+k-1} \frac{\theta_\ell}{2} + k\frac{\theta_i}{2} \geq 0,$$

as desired.

Similarly, we can show that $j = i + k$ does not envy i if the adjustment is made. ∎

Note that each of these modifications, *backward no-envy, forward no-envy,* and *adjusted no-envy,* implies *equal treatment of equals.* Consequently, we could impose these axioms instead of *equal treatment of equals* in the characterizations of the Maniquet rule (2003) and the reverse rule (Chun 2004).

6. Other fairness requirements

Although no-envy plays an important role in the literature on the fairness, there are other interesting concepts. The main ones are: the identical preferences lower bound and egalitarian equivalence. The *identical preferences lower bound* (Moulin 1990) requires that each agent should be at least as well off as he would be, under *efficiency* and *equal treatment of equals,* if all other agents had the same preferences. *Egalitarian equivalence* (Pazner and Schmeidler 1978) requires that there should be a reference bundle such that each agent enjoys the same utility between his bundle and that reference bundle. Now we formally introduce these axioms. Given $q = (N, \theta) \in \mathcal{Q}^N$, an allocation $z = (\sigma, t) \in Z(q)$ satisfies the *identical preferences lower bound* if for all $i \in N$, $u(\sigma_i, t_i; \theta_i) \geq -\frac{|N|-1}{2}\theta_i$. It is *egalitarian equivalent* if there is a reference bundle $z_0 = (\sigma_0, t_0)$ such that for all $i \in N$, $u(\sigma_i, t_i; \theta_i) = u(\sigma_0, t_0; \theta_i)$. Let $B_{id}(q)$ be the set of all allocations meeting the *identical preferences lower bound* and $EE(q)$ be the set of all egalitarian equivalent allocations for $q \in \mathcal{Q}^N$.

Identical preferences lower bound: For all $q = (N, \theta) \in \mathcal{Q}^N$ and all $z = (\sigma, t) \in \varphi(q)$, $z \in B_{id}(q)$.

Egalitarian equivalence: For all $q = (N, \theta) \in \mathcal{Q}^N$ and all $z = (\sigma, t) \in \varphi(q)$, $z \in EE(q)$.

Remark 5: In economies with indivisible goods, when there are as many objects as agents, *no-envy* implies the *identical preferences lower bound* (Bevia 1996, 1998). Moreover, if there are only two agents, then *efficiency* and *identical preferences lower bound* together imply *no-envy.* A similar observation can be made for queueing problems if *budget balance* is additionally imposed.

Now we investigate whether a rule can satisfy *efficiency, egalitarian equivalence,* and *no-envy* together. If there are only two agents, then any rule satisfying *efficiency* and *egalitarian equivalence* satisfies *no-envy.* Moreover, if there are only three agents, then by choosing the middle position as a part of the reference bundle, we can establish the existence of a rule satisfying *efficiency, egalitarian equivalence,* and *no-envy.* However, the positive result does not generalize to problems with more than three agents, as shown in the following.

Proposition 4. *Let* $|N| \geq 4$. *Then, there is no rule satisfying efficiency, egalitarian equivalence, and no-envy together.*

Proof. The proof is by means of an example. Let $N \equiv \{1, 2, 3, 4\}$ and $\theta \equiv (10, 8, 4, 2)$. In this problem, there are four efficient and egalitarian equivalent allocations:

(i) $z^1 \equiv ((\sigma_1, -5.5), (\sigma_2, 2.5), (\sigma_3, 2.5), (\sigma_4, .5))$ with reference bundle $(\sigma_1, -5.5)$;
(ii) $z^2 \equiv ((\sigma_1, -9.5), (\sigma_2, .5), (\sigma_3, 4.5), (\sigma_4, 4.5))$ with reference bundle $(\sigma_2, .5)$;
(iii) $z^3 \equiv ((\sigma_1, -12.5), (\sigma_2, -1.5), (\sigma_3, 6.5), (\sigma_4, 8.5))$ with reference bundle $(\sigma_3, 6.5)$;
(iv) $z^4 \equiv ((\sigma_1, -17.5), (\sigma_2, -3.5), (\sigma_3, 8.5), (\sigma_4, 12.5))$ with reference bundle $(\sigma_4, 12.5)$.

However, none of these allocations is *envy-free.* At z^1, agent 3 envies agent 2 (and agent 4 envies agents 2 and 3); at z^2, agent 4 envies agent 3; at z^3, agent 1 envies agent 2; and at z^4, agent 1 envies agents 2 and 3 (and agent 2 envies agent 3). The generalization of the example to more than four agents is obvious. ∎

In economies with the indivisible goods, there is no rule satisfying *efficiency, egalitarian equivalence,* and the *identical preferences lower bound* (Thomson 2003a). However, in queueing problems, we can construct a rule satisfying *efficiency, egalitarian equivalence,* and the *identical preferences lower bound.*

Proposition 5. *If there is an odd number of agents, then there is at least one efficient and egalitarian equivalent allocation meeting the identical preferences lower bound. If there is an even number of agents, then there are at least two efficient and egalitarian equivalent allocations meeting the identical preferences lower bound.*

Proof. Let $N \equiv \{1, \ldots, n\}$ be such that n is an odd number, and $\theta \equiv (\theta_i)_{i \in N}$ be such that $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_n$. To simplify notation, for all $i \in N$, we set $\sigma_i = i$. Let $i \equiv \frac{n+1}{2}$ and $z_i \equiv (\sigma_i, t_i)$. We will determine the value of t_i later after considering the budget constraint.

For each $j \in N$, let $z_j \equiv (\sigma_j, t_i + (j-i)\theta_j)$. Now we calculate t_i by solving $nt_i + \sum_{j \in N}(j-i)\theta_j = 0$. It gives:

$$t_i = -\frac{1}{n}\sum_{j \in N}(j-i)\theta_j.$$

Since $\theta_1 \geq \theta_2 \geq \cdots \theta_n$, then $t_i \geq 0$. It is obvious that $z \equiv (z_j)_{j \in N}$ is efficient. Since for all $j \in N$, $z_j I_j z_i$, it is also egalitarian equivalent.

To prove that z satisfies the *identical preferences lower bound,* we need to show that for each $j \in N$, $u(z_j; \theta_j) \geq -\frac{n-1}{2}\theta_j$. For each $j \in N$,

$$\begin{aligned} u_j(z_j; \theta_j) &= -(j-1)\theta_j + t_i + (j-i)\theta_j \\ &\geq -(i-1)\theta_j \\ &= -(\frac{n+1}{2} - 1)\theta_j \\ &= -\frac{n-1}{2}\theta_j, \end{aligned}$$

as desired.

On the other hand, if $N \equiv \{1, \ldots, n\}$ is an even number, then we choose either $\frac{n}{2}$ or $\frac{n}{2}+1$ as reference positions. If $i = \frac{n}{2,}$, we can show that $t_i \geq -\frac{1}{2}\theta_n$. Therefore, for each $i \in N$,

$$\begin{aligned} u_j(z_j; \theta_j) &= -(j-1)\theta_j + t_i + (j-i)\theta_j \\ &\geq -(i-1)\theta_j - \frac{1}{2}\theta_n \\ &\geq -(i-1)\theta_j - \frac{1}{2}\theta_j \end{aligned}$$

$$
\begin{aligned}
&= -(\frac{n}{2} - 1 + \frac{1}{2})\theta_j \\
&= -\frac{n-1}{2}\theta_j,
\end{aligned}
$$

as desired.

If $n = \frac{n}{2} + 1$, we can show that for each $j \in N$, $t_i \geq \frac{1}{2}\theta_1 \geq \frac{1}{2}\theta_j$. From a similar calculation, we obtain the desired conclusion. ∎

7. Concluding remarks

By investigating the implications of *no-envy* in queueing problems, we establish various results. Our main negative results are: there is no rule satisfying *efficiency, no-envy,* and either one of two *cost monotonicities.* These results should be compared with the impossibility result in Moulin and Thomson (1988): in the classical economies, there is no rule satisfying *Pareto optimality,*[5] *no-envy,* and *resource monotonicity.*[6] Since our problem is very different from theirs, there is no direct logical implications between two results. However, at least conceptually, we are faced with the same difficulties: axioms of *efficiency, no-envy,* and *monotonicity* are not compatible.

To remedy this situation, two modifications of *no-envy* are proposed as fairness requirements in queueing problems. Although the implications of *backward* and *forward no-envy* are clear, it is an open question what the implications of *adjusted no-envy* in queueing problems are. In particular, its relation to *no-envy* needs to be analyzed.

[5] *Pareto optimality* requires that there is no feasible allocation which makes every agent better off and at least one agent strictly better off.

[6] *Resource monotonicity* requires that an increase in resources should not hurt any agent.

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