Title: The Role of the Mean and of the Median in Social Choice Theory

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ABSTRACT In the Mean Voting System (MVS), each voter in some finite set votes for his or her favorite point in R^n, and the social choice is taken to be the mean location of all points cast as votes. It is easy to see that if some voter casts the last vote, and she knows how the others voted, she can vote strategically and make the social choice be any point she wishes – the mean is "totally manipulable."

Extending work of Saari and Merlin, we show that many standard social welfare functions f (for which the outcome is a *ranking* of alternatives) actually make use of the MVS; these f have provably equivalent definitions in which

  i) Each possible ranking v is pre-associated with a certain point A(v) in R^n,
  ii) one calculates the mean location q of all points A(v) for v in some profile P,
  iii) and the outcome f(P) is the ranking v_0 for which A(v_0) is closest (using
       standard, Euclidean distance) to q.

Such "mean social welfare functions" include the Borda count, approval voting, Condorcet's method, and (surprisingly) the Kemeny Rule. Speaking loosely, the differences between these systems arise only from their choice of "plotting function" A.

Do all such voting systems inherit a degree of manipulability from their reliance on the mean? If so, would a less manipulable substitute for the mean, replacing step (ii) above, yield alternative systems that are preferable in this respect?

The *mediancentre* of set S of points of R^n is the point minimizing the sum of the distances to the points. It is one of several generalizations of the median to the multivariable context. We compare the axiomatic basis for the mediancentre with that for the mean, and show that any voting system that replaces mean with mediancentre in step (ii) inherits some of these axiomatic differences.

For example, the *Mediancentre Borda* fails to have the consistency property, but it has the interesting property that when a majority of the voters rank candidates similarly, their ranking will prevail. We compare the manipulability of the standard Borda count system with its mediancentre variant.