Rationalization of Collective Choice Functions by Games with Perfect Information

by

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Abstract: Collective choices are often cyclic and cannot be rationalized by a well-defined linear ordering. In this paper we identify conditions under which collective choices, potentially cyclic, can be rationalized by games with perfect information.

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1. Introduction

In this paper we study collective decision-making in a choice-based theoretic model. Suppose X is a (finite) set of all potential alternatives and C is a choice function on X that chooses a unique alternative $C(A)$ for every nonempty subset A of X. The interpretation of $C(A)$ is that it is the final alternative chosen by a group of individuals when the set of available alternatives is A. The choice function C is a (highly) reduced form of collective choices that contains all information we, as outside observers, have. We do not know who these individuals are, let alone the specific decision-making process they might use in reaching their choices.

We need to say a few words regarding the relevance of our model before we proceed. When a collective body is a complete democracy, we know everything about its membership and rules under which it operates. In this case, our model is not applicable. However, there are many other cases where decisions have been made behind the doors: for example, the political bureau of the former Soviet Communist Party, or the board of directors of Tyco under Mr. Kozlowski's reign. Even though the nominal members of these two examples were known, it was unclear who really had powers in decision-making and how they exercised their powers. The model in this paper deals with these types of situations. (Two recent papers by Sprumont (2000) and Ray and Zhou (2001) have studied collective choices under well-defined membership and decision-making procedures.)

There are two tasks ahead of us: the first is to define in what sense a choice function C is considered collectively rationalizable; and the second is to find conditions under which a choice function C is indeed collectively rationalizable.

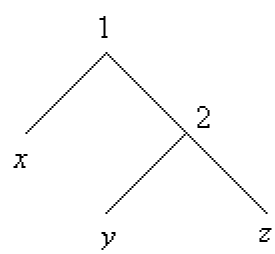
To motivate our definition of collective rationalizability, we start with a brief discussion of individual rationalizability. When C is considered an individual choice function, the rationalizability of C is often identified with the representability of C by a linear ordering R. Formally, a choice function C can be individually rationalized by a linear ordering R if and only if

(IR) $C(A) = B(A;\, R)$ for all $A \subseteq X$ with $A \neq \varnothing$,

in which $B(A; R)$ is the best element of R on A. Conditions for a choice function to be individually rationalizable have been developed by many authors, including Houthakka (1950), Arrow (1959), Richter (1966), and Sen (1971), etc. In particular, an individually rationalizable choice function C must be acyclic, i.e., there should be no triple x, y, and z that form a cycle with

(CYC) $C(\{x, y\}) = x$, $C(\{y, z\}) = y$, and $C(\{z, x\}) = z$.

However, collective choices often involve cycles. For example, consider a simple decision rule with three alternatives $X = \{x, y, z\}$ and two agents. For any set of alternatives $A \subseteq X$, agent 1 has the option to pick x first whenever $x \in A$. If agent 1 does not pick x, then x is eliminated and agent 2 can pick any alternative from the remaining set. This rule can be represented by the following tree:



Moreover, suppose agent 1's linear ordering is: $z\ R_1\ x\ R_1\ y$, and agent 2's linear ordering is: $y\ R_2\ x\ R_2\ z$. When both agents act rationally in the sense that they always play subgame perfect Nash equilibria, this decision rule, under the designated preferences, generates outcomes that are consistent with (CYC). (In addition to choices specified in (CYC), x is the outcome when all alternatives are available.) As a result, this game tree rationalizes a collective choice function with cycles.

This example leads us to adopt the following notion of the collective rationalizability of a choice function C. Suppose C is a choice function defined for a finite set X. First, we construct a tree G that has alternatives in X as terminal nodes. Second, we designate linear orderings R's for all agents in this game. We say that the choice function C is collectively rationalized by G if

$$\text{(CR)} \qquad C(A) = SPNE(G|A; R), \text{ for all } A \subseteq X \text{ with } A \neq \emptyset,$$

in which $G|A$ is the reduced game of G that is derived from G by retaining only paths that lead to terminal nodes in A, and $SPNE$ stands for subgame perfect Nash equilibrium.[1]

Compared with (IR), (CR) allows us more freedom when we try to rationalize a choice function. First, we can introduce more agents with each of them having a different linear ordering; second, we can construct game trees that have much richer structures than the plain individual utility maximization process. Yet we are not totally free with (CR). For example, any choice function that satisfies (CR) must respect unanimity, i.e.,

$$C(\{x, y\}) = x \text{ for all } y \in A \;\Rightarrow\; C(\{x\} \cup A) = x.$$

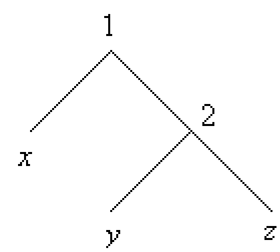
While there are many other conditions we can go through, the challenge we face is to find conditions that are both necessary and sufficient for a choice function to be rationalized by a game tree. The main contribution of this paper is to identify a pair of conditions, which together characterize choice functions that can be rationalized by game trees. The first condition -- the weak separability -- is a type of path independence condition (see Plott (1973)) and the second condition – the divergence consistency -- deals with choices when cycles are intertwined with each other. In Section 2 we provide the formal definitions of these conditions and discuss why they are necessary for a choice function to be rationalized by a game tree. In Section 3 we prove the main result of the paper that these two conditions are sufficient for a choice function to be rationalized by a game tree. We conclude the paper with more discussions and remarks in Section 4.

[1] Note that a reduced game $G|A$ of G is not necessarily a subgame of G: It is a subgame of G only when A consists of terminal nodes for a particular subgame of G.

2. The Basic Set-Up

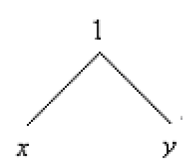
The (finite) set of alternatives is $X = \{x, y, z, \ldots\}$. A choice function C is a mapping from $2^X \backslash \varnothing$, the set of all non-empty subsets of X, to X with $C(A) \in A$ for all $A \in 2^X \backslash \varnothing$.

In this paper we use extensive games of perfect information, or game trees for short, to rationalize choice functions. For a given choice function C defined on X, we consider any game tree G that has a one-to-one mapping from all its terminal nodes to X. Without loss of generality, we shall identify G's terminal nodes with X. For any $A \in 2^X \backslash \varnothing$, $G|A$ is the reduced game tree of G that retains only branches that lead to terminal nodes in A. For example, let $X = \{x, y, z\}$ and G is the game tree we described in the introduction:

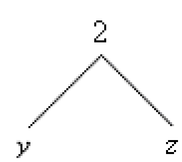


then, for $A_1 = \{x, y\}$ and $A_2 = \{y, z\}$, the reduced game trees $G|A_1$ and $G|A_2$ are:

$G|A_1$



$G|A_2$



We say that a choice function C is *rationalized* by a game tree G if there is a game tree G that has all alternatives in X as terminal nodes and

(CR) $$C(A) = SPNE(G|A)\,, \quad \text{for all } A \in 2^X \backslash \varnothing,$$

in which $SPNE(G|A)$ stands for the subgame perfect Nash equilibrium of the reduced game $G|A$.

If a choice function C is *rationalized* by some game tree G, we also say that C is *collectively rationalizable.*

There is a considerable degree of freedom in constructing a game tree to rationalize a choice function C: we are free to choose the number of players, the structure of the tree, and the preference relation of each player on X. However, there are so many possible choice functions that only a small number of choice functions are collectively rationalizable. To see it from a different angle, let us investigate some restrictions that collective rationalizability imposes on choice functions.

First, when a choice function C is rationalized by a game G of perfect information, the choice function must be in some sense weakly separable. When we split the game tree G at the root by separating any particular initial branch from the rest of the tree, we also partition X into two non- degenerate disjoint sets Y and Z. The fact that C is collectively rationalizable by G implies that $C(X) = C(Y \cup Z) = C(\{C(Y), C(Z)\})$. Moreover, this property should hold for all subsets of X, Y, and Z. Hence, any choice function C that is collectively rationalizable must satisfy:

Weak Separability. For any $A \in 2^X \backslash \varnothing$ with $|A| > 1$, there exist a non-degenerate partition $A = B \cup D$ ($B \cap D = \varnothing$, $B \neq \varnothing$, and $D \neq \varnothing$) such that

(WS) $\quad C(S \cup T) = C(\{C(S), C(T)\})$, for all $S \subseteq B$ and $T \subseteq D$ with $S \neq \varnothing$, and $T \neq \varnothing$.

If we strengthen WS by requiring that (WS) should hold for all partitions of A, we obtain strong separability, which is both necessary and sufficient for rationalizability of a choice function by a single linear ordering on X. The weak separability is also associated with the single preference rationalization in another way as the following proposition shows.

Proposition. If a choice function C satisfies weak separability and is acyclic, then C can be rationalized by a single linear ordering on X, and vice versa.

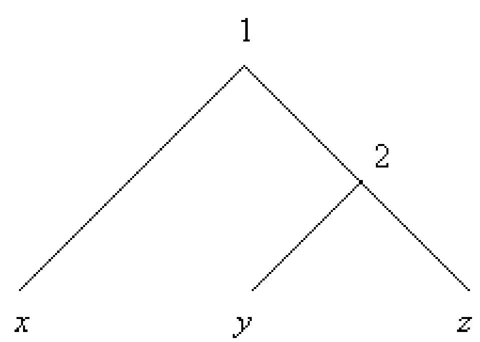
The proof of the proposition is straightforward. First, we define a binary relation R_C on X by: $x \; R_C \; y$ iff $C(\{x, y\}) = x$. Obviously, R_C is complete, reflexive, and for all $x, y \in X$: [(x

$R_C\, y$ *and* $y\, R_C\, x) \Rightarrow x = y$]. Since C is acyclic, R_C is also transitive. Therefore, R_C is a linear ordering. We then use weak separability to show that $C(A) = B(A; R_C)$ for all $A \subseteq X$ (by induction on the number of alternatives contained in A).

It is clear we need to drop acyclicity in our inquiry. The question is what condition can replace acyclicity, which together with weak separability will enable us to obtain collective rationalizability. To answer this question, let us study cycles induced by a choice function, in particular, cycles that consist of three alternatives.

For any triple $x, y,$ and z, we say x, y, and z form a 3-cycle if $C(\{x, y\}) = x$, $C(\{y, z\}) = y$, and $C(\{z, x\}) = z$, or if similar conditions hold for a permutation of x, y, and z. (We may study cycles that contain more alternatives. However, if a choice function induces a cycle of any number of alternatives, it must induce a cycle of three alternatives. This explains why we choose to focus on 3-cycles.)

When x, y, and z form a 3-cycle, choices over $\{x, y, z\}$ and its subsets can be represented by a game tree that depends on $C(\{x, y, z\})$. For example, when $C(\{x, y\}) = x$, $C(\{y, z\}) = y$, and $C(\{z, x\}) = z$, and $C(\{x, y, z\}) = x$, the choice function can be represented by the following tree:



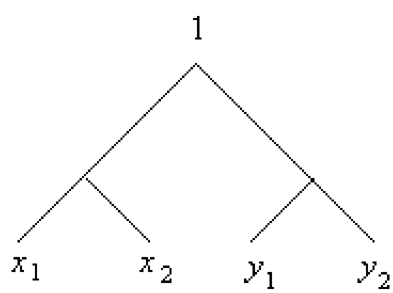
<u>1</u>	<u>2</u>
z	
x	y
y	z

Other than re-labeling of players or terminal nodes, this game tree is (almost) unique. The player who chooses first can opt for x, or pass x and let the other player choose between y and z. Also the first player must rank x between y and z, and the second player's ranking of y and z must be the opposite of that of the first player.

Since we shall encounter 3-cycles repeatedly, we adopt a convenient terminology: For any triple x, y, z, we say that *x diverges before y and z*, if x, y, and z form a 3-cycle and $C(\{x, y, z\}) = x$.

When there are more than three alternatives, a choice function may have several 3-cycles. Since each 3-cycle uniquely determines the structure of a branch of any potential game tree that represents the choice function, these 3-cycles must overlap properly for a choice function to be rationalized by a game tree.

Consider a situation in which $X = \{x_1, x_2, y_1, y_2\}$. If a choice function C is such that x_1 diverges before y_1 and y_2, and y_1 diverges before x_1 and x_2, then for C to be rationalizable by a game tree, the branches of the game must look like the following.



In addition, player 1 must rank x_1 between y_1 and y_2, and rank y_1 between x_1 and x_2. Now there are two possible cases:

First, x_1 is ranked above y_1, or more precisely, $C(\{x_1, y_1\}) = x_1$. Then $C(\{x_1, y_2\}) = y_2$ (since x_1 is between y_1 and y_2) and $C(\{x_2, y_1\}) = y_1$ (since y_1 is between x_1 and x_2). In this case, player 1's linear ordering R_1 must be: $y_2 \; R_1 \; x_1 \; R_1 \; y_1 \; R_1 \; x_2$. Hence, $C(\{x_2, y_2\}) = y_2$.

Second, x_1 is ranked below y_1, or $C(\{x_1, y_1\}) = y_1$. Then $C(\{x_1, y_2\}) = x_1$ (since x_1 is between y_1 and y_2) and $C(\{x_2, y_1\}) = x_2$ (since y_1 is between x_1 and x_2). In this case, player 1's linear ordering R_2 must be: $x_2 \; R_2 \; y_1 \; R_2 \; x_1 \; R_2 \; y_2$. Hence, $C(\{x_2, y_2\}) = x_2$.

To summarize, if a choice function C can be rationalized by a game tree, then it must satisfy the following condition:

Divergence Consistency. For any four alternatives $x_1, x_2, y_1, y_2 \in X$, if x_1 diverges before y_1 and y_2, and y_1 diverges before x_1 and x_2, then $C(\{x_1, y_1\}) = x_1$ iff $C(\{x_2, y_2\}) = y_2$.

It turns out that divergence consistency and weak separability together are sufficient for a choice function to be collectively rationalizable.

Theorem. *A choice function C can be rationalized by a game tree if and only if it satisfies weak separability and divergence consistency.*

The proof is given in the next section. Before moving on, we present two examples showing that the conditions of weak separability and divergence consistency are independent when |X| > 3.

Example 1. Let $X = \{x_1, x_2, y_1, y_2\}$. Partition it into two sets $\{x_1, x_2\}$ and $\{y_1, y_2\}$. Consider a choice function C_1 with

(1a) $C_1(\{x_1, y_1\}) = x_1$, $C_1(\{x_1, y_2\}) = y_2$, $C_1(\{x_2, y_1\}) = y_1$, $C(\{x_2, y_2\}) = x_2$;

(1b) $C_1(\{x_1, x_2\}) = x_2$, $C_1(\{y_1, y_2\}) = y_1$;

(1c) $C_1(\{x_1, x_2, y_1\}) = C_1(\{C_1(\{x_1, x_2\}), y_1\}) = C_1(\{x_2, y_1\}) = y_1$,

$C_1(\{x_1, x_2, y_2\}) = C_1(\{C_1(\{x_1, x_2\}), y_2\}) = C_1(\{x_2, y_2\}) = x_2$,

$C_1(\{x_1, y_1, y_2\}) = C_1(\{x_1, C_1(\{y_1, y_2\})\}) = C_1(\{x_1, y_1\}) = x_1$,

$C_1(\{x_2, y_1, y_2\}) = C_1(\{x_2, C_1(\{y_1, y_2\})\}) = C_1(\{x_2, y_1\}) = y_1$;

(1d) $C_1(\{x_1, x_2, y_1, y_2\}) = C_1(\{C_1(\{x_1, x_2\}), C_1(y_1, y_2\})) = C_1(\{x_2, y_1\}) = y_1$.

It is clear by construction that C_1 satisfies weak separability. However, C_1 does not satisfy divergence consistency since x_1 diverges before y_1 and y_2, and y_1 diverges before x_1 and x_2, yet $C_1(\{x_1, y_1\}) = x_1$ and $C(\{x_2, y_2\}) = x_2$. ♦

Example 2. Let $X = \{x_1, x_2, y_1, y_2\}$. Fix a linear ordering R on X with $x_1Rx_2R\,y_1Ry_2$. Consider a choice function C_2 that maximizes R on any subset of X with three alternatives or less but $C_2(X) = y_2$. It is clear that C_2 does not satisfy weak separability for $A = X$ since y_2 can never be chosen in a pairwise comparison. On the other hand, C_2 trivially satisfies divergence consistency since C_2 has no 3-cycles. ♦

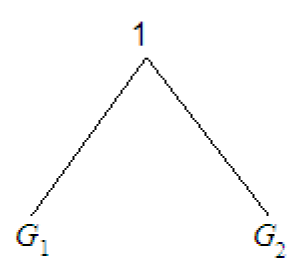
3. The Proof of the Main Result

We have already demonstrated the necessity of these two conditions in Section 2. We now show that these two conditions together are sufficient for a choice function to be rationalizable by a game tree. We prove this by induction on the number of the alternatives X contains.

The result is trivial for $|X| = 2$. Now assume that for some n, any choice function that satisfies weak separability and divergence consistency on a set X with $|X| \leq n$ can be rationalized by a game tree with no more than $|X|$ -1 players. Consider choice functions defined on some set X with $|X| = n + 1$. When a choice function C satisfying weak separability and divergence consistency on X, we can find a non-degenerate partition X_1 and X_2 of X such that

$$C(S \cup T) = C(\{C(S), C(T)\}) \quad \text{for all } S \subseteq X_1 \text{ and } T \subseteq X_2 .$$

By the induction hypothesis, there are two game trees, G_1 for X_1 and G_2 for X_2, such that C is rationalized by G_1 on X_1 and by G_2 on X_2. Then, construct the following game tree G:



The number of players needed in G_1 and G_2 is no more than $|X|$ -1. Here we just let player 1 be the one who is not in G_1 or G_2. Notice that preferences on X_2 by players in G_1

are immaterial for rationalization of C, and vice versa. So, as long as we can construct a linear ordering for player 1 that is consistent with C, then we are done. There are three possibilities: (a) $|X_1| = 1$; (b) $|X_2| = 1$; and (c) $|X_1| > 1$ and $|X_2| > 1$.

(a) $|X_1| = 1$, say $X_1 = \{x\}$. We define player 1's preference relation as follows. For all $y \in X_2$, xR_1y if $C(x, y) = x$, and yR_1x if $C(x, y) = y$. Clearly, this preference relation is incomplete. However, it has no cycles. Hence, it can be extended to a linear ordering R_1^* on X.

For any set $A \subseteq X$ with $x \in A$, $A = \{x\} \cup B$ with $B \subseteq X_2$. Then,

$$\begin{aligned} C(A) &= C(\{x\} \cup B) \\ &= C(\{x, C(B)\}) && \text{(by weak separability)} \\ &= C(\{x, SPNE(G_2|B)\}) && \text{(by induction hypothesis)} \\ &= SPNE(G|A)\ . \end{aligned}$$

Similarly, for any set $A \subseteq X$ with $x \notin A$,

$$\begin{aligned} C(A) &= SPNE(G_2|A)) && \text{(by induction hypothesis)} \\ &= SPNE(G|A)\ . \end{aligned}$$

(b) $|X_2| = 1$. This case can be dealt with similarly as case (a) above.

(c) $|X_1| > 1$ and $|X_2| > 1$. Define player 1's preference relation R_1 for pairs with one alternative in X_1 and another in X_2 as follows: for all $x \in X_1$, all $y \in X_2$, xR_1y if $C(\{x, y\}) = x$, and yR_1x if $C(\{x, y\}) = y$. For pairs with both alternatives in X_1 or both in X_2, R_1 is not yet defined.

We now show that R_1 does not have cycles. Suppose to the contrary that R_1 has a cycle. Given the nature of R_1, there must exist distinct $x_1, \ldots, x_k \in X_1$, and distinct $y_1, \ldots, y_k \in X_2$ such that $x_1R_1y_1$, $y_1R_1x_2$, $x_2R_1y_2$, $\ldots$, $x_kR_1y_k$, $y_kR_1x_1$. Moreover, we may assume there is a cycle with $k = 2$. (If $y_2R_1x_1$, then this is it. If not, then $x_1R_1y_2$ and we can drop x_2 and y_1 from the cycle and reduce its length.

This is repeated until a cycle with $k = 2$ is found.) Hence, we have $x_1, x_2 \in X_1$, and $y_1, y_2 \in X_2$ such that $x_1 R_1 y_1, y_1 R_1 x_2, x_2 R_1 y_2, y_2 R_1 x_1$, or

(0) $C(\{x_1, y_1\}) = x_1$, $C(\{y_1, x_2\}) = y_1$, $C(\{x_2, y_2\}) = x_2$, $C(\{y_2, x_1\}) = y_2$.

There are four possible subcases concerning $C(\{x_1, x_2\})$ and $C(\{y_1, y_2\})$:

(i) $C(\{x_1, x_2\}) = x_1$ and $C(\{y_1, y_2\}) = y_1$;
(ii) $C(\{x_1, x_2\}) = x_1$ and $C(\{y_1, y_2\}) = y_2$;
(iii) $C(\{x_1, x_2\}) = x_2$ and $C(\{y_1, y_2\}) = y_1$;
(iv) $C(\{x_1, x_2\}) = x_2$ and $C(\{y_1, y_2\}) = y_2$.

Consider (i). Together with (0), $C(\{x_1, x_2\}) = x_1$ implies x_1, x_2, y_2 are a 3-cycle. Also, $C(\{x_1, x_2, y_2\}) = C(\{C(\{x_1, x_2\}), y_2\}) = C(\{x_1, y_2\}) = y_2$. Hence y_2 diverges before x_1 and x_2. Together with (0), $C(\{y_1, y_2\}) = y_1$ implies x_1, y_1, y_2 are a 3-cycle. Also, $C(\{x_1, y_1, y_2\}) = C(\{x_1, C(\{y_1, y_2\})\}) = C(\{x_1, y_1\}) = x_2$. Hence x_1 diverges before y_1 and y_2. Then, by divergence consistency, $C(\{x_1, y_2\}) = y_2$ should lead to $C(\{x_2, y_1\}) = x_2$. But this contradicts (0).

Next consider (ii). Again $C(\{x_1, x_2\}) = x_1$ and (0) imply y_2 diverges before x_1 and x_2. Together with (0), $C(\{y_1, y_2\}) = y_2$ implies x_2, y_1, y_2 are a 3-cycle. Also, $C(\{x_2, y_1, y_2\}) = C(\{x_2, C(\{y_1, y_2\})\}) = C(\{x_2, y_2\}) = x_2$. Hence x_2 diverges before y_1 and y_2. Then, by divergence consistency, $C(\{x_2, y_2\}) = x_2$ should lead to $C(\{x_1, y_1\}) = y_1$. But this again contradicts (0).

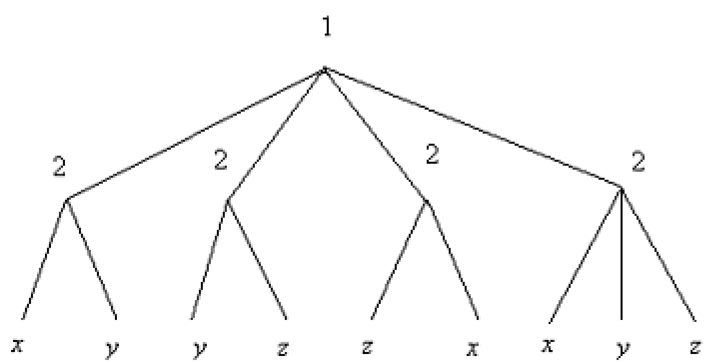
We can repeat the same argument for (iii) and (iv) and demonstrate contradictions there. Therefore, R_1 cannot have cycles.

Since R_1 has no cycles, from the definition of R_1, it can be extended to a preference relation that is linear on the entire X. Finally, we can use an argument similar to that in (a) to show that the game tree G rationalizes C on X. This completes the induction.

4. Some Remarks

In this paper we have derived conditions that are necessary and sufficient for choice functions to be rationalized by extensive games with perfect information. There are several possible extensions of the main result.

We have assumed that a choice function C defined on X is to be rationalized by such a game tree G that each alternative $x \in X$ appears as a terminal node of G once and once only. We can modify this assumption and allow each alternative to appear possibly multiple times. Of course, this modification calls for a reinterpretation of the reduced game $G|A$. One interpretation is that $G|A$ retains *all* branches of G that lead to alternatives in A. How much will this modification change our result? Does this modification allow more choice functions to be collectively rationalizable? The short answer is yes. For example, let us consider the choice function with three alternatives that is generated by a "voting by veto" game G. In this game, player 1 first has the option of vetoing at most one alternative, then player 2 picks an alternative from those that remain. Assume players' linear orderings are as given next to the tree in the following graph.



1	2
x	z
z	y
y	x

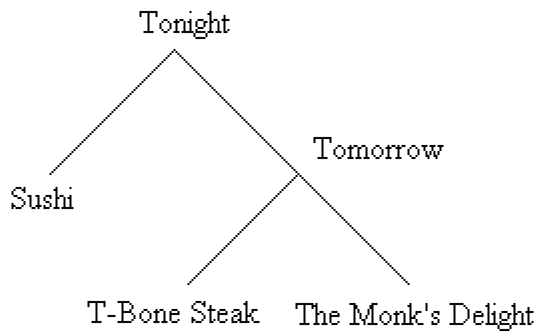
Now we can calculate the subgame perfect Nash equilibrium outcome of $G|A$ for each $A \subseteq X$ with $A \neq \varnothing$. This generates a choice function C with:

$$C(\{x, y\}) = x,\ C(\{x, z\}) = x, C(\{y, z\}) = z, \text{ and } C(\{x, y, z\}) = z .$$

It is easy to verify that C does not satisfy weak separability. In fact, with three alternatives, any choice function can be rationalized by some game tree when we allow each alternative to be terminal nodes of a tree multiple times. The interesting question is whether this is true in general. While we would like to identify a set of conditions that are necessary and sufficient for choice functions to be collectively rationalized, it is also conceivable that any choice function might be rationalized in this extended setting.

There is another possible extension in a different direction. Here we will maintain the assumption that each alternative appears on the game tree only once. However, we relax the assumption that all choice functions are single-valued by allowing the possibility of multi-valued choice correspondences. Now we have to allow players' preferences to be indifferent for certain pairs of alternatives in order to have multiple subgame perfect Nash equilibria. What conditions are necessary and sufficient for multi-valued choice correspondences to be collectively rationalizable? Obviously, weak separability continues to be necessary. So does divergence consistency. Yet together they are not sufficient for a multi-valued choice correspondence to be collectively rationalizable. Consider a choice correspondence with three alternatives: $X = \{x, y, z\}$, $C(\{x, y\}) = \{x, y\}$, $C(\{x, z\}) = \{x, z\}$, $C(\{y, z\}) = \{y, z\}$, and $C(\{x, y, z\}) = \{x\}$. Clearly, this choice correspondence satisfies weak separability (any partition of $\{x, y, z\}$ is fine), as well as divergence consistency (there are no cycles). However, C cannot be rationalized by any game tree. If C were rationalized by some game tree G, then $C(\{x, y\}) = \{x, y\}$, $C(\{x, z\}) = \{x, z\}$, $C(\{y, z\}) = \{y, z\}$ would imply $C(\{x, y, z\}) = \{x, y, z\} \neq \{x\}$. Hence, this choice correspondence is not collectively rationalizable. We do not have a complete solution for this extension yet.

Finally, we can also give our model an individual decision theory interpretation. Many researchers have reported cases in which individual choices exhibit cycles. Particularly, such cycles are common occurrence when an individual uses different criteria in evaluating various alternatives at different points. For example, Katie plans to go out for dinner tomorrow. She cares about both the healthiness and the taste of the food. The restaurant she plans to go to has three dishes are on the menu: T-bone steak, sushi, the monk's delight (a vegetarian special). In terms of healthiness, Katie ranks the monk's delight the highest, and T-bone steak the lowest. In terms of taste, however, Katie ranks T-bone steak the highest, and the monk's delight the lowest. If she wants sushi, she has to order one day in advance since fresh seafood has to be pre-ordered. But she can decide until she arrives at the restaurant if she wants T-bone or the monk's delight. Hence her decision tree is:



Katie knows that her sensible choice should be made based on the healthiness of the food. Yet she anticipates that her preference for healthy food will succumb to her preference for tasty food once she steps in the restaurant and sits down at the dinner table. This leads to a situation that is exactly the same game tree in the introduction with player 1 and player 2 there being replaced by Katie's split personalities of tonight and tomorrow.[2] In general, we can replace different players in a game tree by one player's different preferences at different decision nodes, then our result in this paper also provides potential insight to the nature of cycles of individual choices.

[2] A similar example in a slightly different context is included in Ok and Masatlioglu (2003).

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