MISTAKES IN COOPERATION:
the Stochastic Stability of Edgeworth's Recontracting*

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Abstract. In an exchange economy with a finite number of indivisible goods, we analyze a dynamic trading process of coalitional recontracting where agents may make mistakes with small probability. We show first that the recurrent classes of the unperturbed (mistake-free) process consist of (i) all core allocations as absorbing states, and (ii) cycles of non-core allocations. Next, we introduce a perturbed process, where the resistance of each transition is a function of the number of agents that make mistakes –do not improve– in the transition and of the seriousness of each mistake. If preferences are always strict, we show that the unique stochastically stable state of the perturbed process is the Walrasian allocation. In economies with indifferences, non-core cycles are sometimes stochastically stable, while some core allocations are not.

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1 Introduction

The problem of what outcomes will arise in decentralized trade amongst arbitrary groups of agents dates back to Edgeworth (1881). The solution is given by what today we refer to as the core, i.e., the set of allocations that are immune to any coalitional move that improves upon them. Edgeworth's verbal description of contracting and recontracting was inherently dynamic, and it received elegant formalizations in the analyses of Feldman (1974) and Green (1974).[1] In contrast to these papers, ours considers the possibility of mistakes in agents' decision-making within the context of dynamic coalitional exchange.[2] Applying the tools developed in evolutionary game theory, our results will point out that the core may sometimes not correspond to the right notion of stability, i.e., stochastic stability, which incorporates the small probability of mistakes as a persistent feature of the system.

Edgeworth (1881) proposed the core as an alternative to the competitive equilibrium allocations, identified by Walras (1874), and was the first to notice the connection between the two in large economies. This observation gave rise to the important core convergence/equivalence literature (Debreu and Scarf (1963), Aumann (1964)) as one of the leading game theoretic justifications of Walrasian equilibrium.[3]

In Feldman (1974) and Green (1974), a dynamic random process was imposed on a coalitional game. Starting from an arbitrary feasible allocation, the process allows each coalition to meet with positive probability in every period. When a coalition meets, they can choose to stay at the original allocation or move to a new allocation feasible for them if they all improve as a result. When this happens, the complement coalition is sent back to their individual endowments (in Feldman (1974)) or to a Pareto efficient allocation of their resources (in Green (1974)).[4] The adjustment of resources of the complement coalition ensures that the path followed in utility space by the process is not monotonic, and renders the convergence question interesting and non-trivial. Both Feldman (1974) and Green (1974) are able to identify sufficient conditions under which Edgeworth's recontracting process converges to a core allocation.

In the present paper we analyze a dynamic recontracting process similar to those in Feldman (1974) and Green (1974).[5] The major difference, however, between our analysis

[1] One other aspect of dynamics and the core is provided by its dynamic non-cooperative implementation (e.g., Perry and Reny (1994), Dagan *et al.* (2000)).

[2] Indeed, papers in cooperative game theory have no mistakes. We shall depart from this noble tradition.

[3] See Anderson (1992) and Aumann (1987) for surveys. Although the robustness of the equivalence result is remarkable, several violations thereof have been identified, from which one can learn the role of certain frictions in markets. These references include Anderson and Zame (1997) for infinite dimensional commodity spaces, Manelli (1991) for instances of satiation in preferences, and Serrano, Vohra and Volij (2001) for asymmetric information.

[4] One interpretation of these dynamic processes is in terms of formation of trading blocks in an international trade context. One can then identify patterns of autarchy, followed by the establishment of long-lasting or short-lived trading unions, and so on.

[5] One small difference is that when a coalition recontracts, the complement is sent back to their indi-

and that of Feldman-Green is that we allow agents to make "mistakes" in their recontracting. When a coalition meets and engages in conversations regarding the possible improvement upon the prevailing status quo, each agent may, with small probability, agree to a coalitional trade from which he will not benefit.

We investigate the stochastic stability of this recontracting process. This methodology, based on the techniques developed for stochastic dynamical systems by Freidlin and Wentzell (1984), was introduced to evolutionary game theory by Foster and Young (1990), Kandori, Mailath and Rob (1993) and Young (1993); see also Young (1998) for a clear exposition.[6] The idea is to study the long run behavior of a dynamical system subject to persistent random shocks. Persistent randomness ensures that the system does not get stuck at any given state. Instead, it keeps transitting all the time from one state to the next. Stochastic stability then tells us which states are the ones visited by the system a positive proportion of time in the very long run.

In most applications to evolutionary biology and game theory, randomness in the system takes the form of mutations or instances of experimentation. The stochastic stability results are then interpreted as the patterns of behavior selected by evolution in the long run. In this paper, as in Ben-Shoham *et al.* (2000), randomness is identified with the small probability of mistakes in decision making. Thus, an agent's preferences do not mutate or even change over time, but are part of the exogenously specified primitives. Fixing the preferences of the economy allows us to compare the stochastically stable states to sets of allocations prescribed by classical solution concepts. Therefore, the interpretation of the exercise performed here is simply to get a sense of which allocations will be visited by Edgeworth's recontracting process a positive proportion of time in the long run if mistakes are small probability events that all agents make all the time.

Given the finite structure imposed by the methodology based on Markov chains, it is convenient, and we shall do so here, to work with finite assignment or *housing economies*, as introduced in Shapley and Scarf (1974). When we analyze the recontracting process free of mistakes, we find a large class of recurrent classes: each core allocation constitutes an absorbing state, and in addition, we find recurrent classes consisting of cycles of non-core allocations.

In economies where preferences are strict, we are able to get a remarkable refinement of this set of predictions when we perturb the system with mistakes. Our first main result is that the unique stochastically stable state of the recontracting process with mistakes is the Walrasian allocation, provided that serious mistakes (those where agents end up worse off as a result) are sufficiently more costly than minor ones (those where the agent joins a coalition to end up indifferent to how he started). At the heart of this result is

vidual endowments only if necessary for the recontracting move (that is, only if the redistribution of the blocking coalition's endowments is not also a redistribution of the commodities they get at the blocked allocation.

[6]Other references for evolutionary game theory in general are Weibull (1995), Vega Redondo (1996) and Samuelson (1997).

the property of "global dominance" of the Walrasian allocation for this case, uncovered in Roth and Postlewaite (1977): indeed, for every feasible allocation of the economy, there exists a coalition that can (weakly) improve upon the status quo with their components of the Walrasian allocation. This makes it easy to get to the Walrasian allocation from any other. In the process of coalitional recontracting, the blocking coalition will meet with positive probability and get their components of the Walrasian allocation, while the rest of the agents will reach it as an application of trading cycles (see Shapley and Scarf (1974)). In doing this, the only "mistakes" to be made are agreeing to trades that leave one exactly indifferent. On the other hand, to go from the Walrasian allocation to any other is much harder, as it turns out that at least one agent will have to make a serious mistake and agree to a trade that will make him worse off.

The conclusions of our analysis are quite different in economies with indifferences. We provide a series of examples to illustrate that the predictions of stochastic stability will not coincide with any of the classical solution concepts. In particular, we regard our last example (Example 4) as the other main result of the paper: it shows that non-core cycles are sometimes stochastically stable, whereas some core allocations are not. Thus, in recontracting with mistakes, the economy may frequently visit coalitionally unstable cycles, while entire regions of the core will not be reached but a zero proportion of time in the very long run. To the best of our knowledge, this also appears to be one of the first examples in the evolutionary literature where a non-singleton recurrent class turns out to be stochastically stable.

The paper is organized as follows. Section 2 presents the model and basic definitions. Section 3 introduces the unperturbed recontracting process. Its perturbed version with mistakes is found in Section 4. Section 5 contains our result when preferences are strict. Section 6 focuses on economies with non-singleton indifference sets, and goes through a series of examples. Section 7 concludes.

2 A Housing Economy

A *housing economy* is a 4-tuple $\mathcal{E} \equiv \langle N, H, (\succeq_i, e_i)_{i\in N}\rangle$, where N is a finite set of individuals, H is a finite set of houses with $|H| = |N|$, and for each individual $i \in N$, $\succeq_i$ is a complete and transitive preference relation over H, with $\succ_i$ denoting its associated strict preference relation and $\sim_i$ its indifference relation. Finally, $(e_i)_{i\in N}$ is the individual endowment.

A *coalition* S of agents is a non-empty subset of N. The complement coalition of S, $N\backslash S$, will be sometimes denoted by $-S$. A *feasible allocation* for coalition S is a one-to-one function that assigns to the coalition S a redistribution of the coalitional endowment $(e_i)_{i\in S}$. Denote the set of feasible allocations for coalition S by A_S. We simply write A for A_N.

An allocation $x \in A$ is *individually rational* if there is no individual $i \in N$ for whom $e_i \succ_i x_i$.

An allocation $x \in A$ is a *core allocation* if there is no coalition S and no feasible allocation for S, $y \in A_S$, such that $y_i \succ_i x_i$ for all $i \in S$.

An allocation $x \in A$ is a *strong core allocation* if there is no coalition S and no feasible allocation for S, $y \in A_S$, such that $y_i \succeq_i x_i$ for all $i \in S$ and $y_j \succ_j x_j$ for some $j \in S$.

An allocation $x \in A$ is a *Walrasian allocation* if there exists $p \in \mathbb{R}^H_+ \backslash \{0\}$ such that for all $i \in N$ and for all $h \in H$, $h \succ_i x_i$ implies $p_h > p_{e_i}$.

It has been shown (see Shapley and Scarf (1974)) that an allocation x is Walrasian if and only if it can be obtained as a result of trading cycles. That is, there exists a partition of the set of agents $\{S_1, S_2, \ldots, S_m\}$ such that:

$x_{S_1} \in A_{S_1}$, and for every $j \in S_1$, $x_j \succeq_j x_i$ for every $i \in N$; and

for every $k = 2, \ldots, m$, $x_{S_k} \in A_{S_k}$, and for every $j \in S_k$, $x_j \succeq_j x_i$ for every $i \in S_k \cup \ldots S_m$.

In other words, the agents in S_1 redistribute their endowments and get their most preferred houses; the agents in S_2 redistribute their endowments and get their most preferred houses out of the endowments of $S_2 \cup \ldots S_m$, etc.

Given a housing economy, we shall define a perturbed Markov process as in Kandori, Mailath and Rob (1993) and Young (1993). The states of the process are the allocations of the housing economy. In each period a coalition of agents is selected at random and the system moves from one state to another when the matched agents trade. In the unperturbed Markov process $\mathcal{M}^0$ of Section 3, agents do not make mistakes in their coalitional meetings: they trade if and only if there is a strictly beneficial coalitional recontracting opportunity. In the perturbed process $\mathcal{M}^\epsilon$ of Section 4, agents will make mistakes with a small probability, and sign a contract with a coalition even when they do not improve as a result.

It is often the case that an unperturbed Markov process (and it will certainly be the case for $\mathcal{M}^0$) has many stationary distributions. On the other hand for all $\epsilon \in (0, 1)$, the perturbed process $\mathcal{M}^\epsilon$ is ergodic, which implies that it has a unique stationary distribution. Denote the unique stationary distribution of $\mathcal{M}^\epsilon$ by μ^ϵ. This stationary distribution, which is independent of the initial state, represents the proportion of time that the system will spend on each of its states in the long run. It also represents the long run probability that the process will be at each allocation. In order to define the stochastically stable states, we check the behavior of the stationary distribution μ^ϵ as ϵ goes to 0. It is known that $\lim_{\epsilon \to 0} \mu^\epsilon$ exists and further it is one of the stationary distributions of the unperturbed process $\mathcal{M}^0$. The *stochastically stable states* of the system $\mathcal{M}^\epsilon$ are defined to be those states that are assigned positive probability by this limit distribution. We are interested in these allocations because they are the one that are expected to be observed in the long run "most of the time."

3 An Unperturbed Trading Process

Consider the following unperturbed Markov process $\mathcal{M}^0$, adapted from Feldman (1974) and Green (1974). In each period t, if the system is at the allocation $x(t)$, all coalitions are chosen with arbitrary, but positive, probability. Suppose coalition S is chosen.

(i) If there exists an S-allocation $y_S \in A_S$ such that $y_i \succ_i x_i(t)$ for all $i \in S$, the coalition moves to each such y with positive probability in that period. Then, the new state is either

$$x(t+1) = (y_S, x_{-S}(t)) \text{ if } x_{-S}(t) \in A_{-S}, \text{ or}$$
$$x(t+1) = (y_S, e_{-S}) \text{ if } x_{-S}(t) \notin A_{-S}.$$

(ii) Otherwise, $x(t+1) = x(t)$.

The interpretation of the process is one of coalitional recontracting. Following a status quo, a coalition can form and modify the status quo if all members of the coalition improve as a result. When this happens, upon coalition S forming, the complement coalition $N \backslash S$ continues to have the same houses as before if this is feasible for them. Otherwise, $N \backslash S$ breaks apart and each of the agents in it receives his individual endowment. If after coalition S gets together, all its agents cannot find any strict improvement, the original status quo persists.

It is clear that the absorbing states of this unperturbed process are precisely the core allocations of the economy. However, the absorbing states are not the only recurrent classes of $\mathcal{M}^0$, as shown by the following example.

Example 1 Let $N = \{1, 2, 3\}$ and denote by (e_1, e_2, e_3) the individual endowment allocation. Let the agents' preferences be as follows:

$$e_3 \succ_1 e_2 \succ_1 e_1;$$

$$e_1 \succ_2 e_3 \succ_2 e_2;$$

$$e_2 \succ_3 e_1 \succ_3 e_3.$$

Consider the following three allocations: $x = (e_1, e_3, e_2)$, $y = (e_2, e_1, e_3)$ and $z = (e_3, e_2, e_1)$. These three allocations constitute a recurrent class: if the system is at x, the state changes only when coalition $\{1, 2\}$ meets, yielding y. At y, the system can move only to z, when coalition $\{1, 3\}$ meets. Finally, the system will move out of z only by going back to x, when coalition $\{2, 3\}$ meets.

Note that the unique Walrasian allocation $w = (e_3, e_1, e_2)$ also constitutes a singleton recurrent class.

We can prove the following result, characterizing the recurrent classes of the unperturbed process $\mathcal{M}^0$:

Proposition 1 The recurrent classes of the unperturbed process $\mathcal{M}^0$ take the following two forms:

(i) Singleton recurrent classes, each of which containing each core allocation.

(ii) Non-singleton recurrent classes: in each of them, the allocations are individually rational but are not core allocations.

Proof: It is clear that each core allocation constitutes an absorbing state of $\mathcal{M}^0$, and that every absorbing state must be a core allocation.

For the second form of recurrent class, note that, by construction of the system, no state in a recurrent class can ever be non-individually rational: if at state x, $e_i \succ_i x_i$, the coalition $\{i\}$ is chosen with positive probability. Then, the system moves to the individual endowment e, never to return to a non-individually rational allocation. It is also clear that each of the states in the recurrent class cannot be absorbing, i.e., a core allocation. ■

Thus, each core allocation is an absorbing state of the unperturbed Markov process $\mathcal{M}^0$, and in principle there may be additional non-singleton recurrent classes, as that in Example 1. Note also that as soon as the economy has more than one core allocation, the system $\mathcal{M}^0$ has many stationary distributions.

4 A Perturbed Trading Process

Next we introduce the perturbed Markov process $\mathcal{M}^\epsilon$ for an arbitrary $\epsilon \in (0,1)$, a perturbation of $\mathcal{M}^0$. Suppose the state of the system is the allocation x and that coalition S meets. We shall say that a member of S makes a "mistake" when he signs a contract that either leaves him indifferent to the same house he already had or he becomes worse off upon signing. Each of the members of S may make one of these "mistakes" with a small probability, as a function of $\epsilon > 0$, independently of the others. Specifically, for a small fixed $\epsilon \in (0,1)$, we shall postulate that an agent's probability of agreeing to a new allocation that leaves him indifferent is ϵ, while the probability of agreeing to an allocation that makes him worse off is ϵ^λ for a sufficiently large positive integer λ. That is, the latter mistakes are much less likely than the former, while both are rare events in the agent's decision-making process.

Before we define the perturbed process, we need some notation and definitions. Consider an arbitrary pair of allocations z and z'. Let $T(z,z') \subseteq 2^N \backslash \{\emptyset\}$ be the set of coalitions such that, if chosen, can induce the perturbed system to transit from z to z' in one step. Note that it is always the case that $N \in T(z,z')$ for any z and z'.

In the direct transition from z to z' and for each $S \in T(z,z')$, define the following numbers:

$$\begin{aligned} n_I(S,z,z') &= |\{i \in S : z_i \sim_i z'_i\}|, \\ n_W(S,z,z') &= |\{i \in S : z_i \succ_i z'_i\}|, \\ n(S,z,z') &= \lambda n_W(S,z,z') + n_I(S,z,z'). \end{aligned}$$

In the perturbed Markov process $\mathcal{M}^\epsilon$ the transition probabilities are calculated as follows. Suppose that the system is in allocation z. All coalitions are chosen with a fixed positive probability. Assume coalition S is chosen. If $S \notin T(z,z')$, then S moves to z' with probability 0. If $S \in T(z,z')$ and $n(S,z,z') > 0$, then coalition S agrees to move to z' with probability $\epsilon^{n(S,z,z')}$. If $S \in T(z,z')$ and $n(S,z,z') = 0$, coalition S moves to those z' with some probability bigger than some $\delta > 0$.

For all $\epsilon \in (0,1)$ small enough, the system $\mathcal{M}^\epsilon$ is a well-defined irreducible Markov process. As such, it has a unique invariant distribution. This distribution gives the probability that the system is in each of the allocations in the long run. We are interested in the limit of the corresponding invariant distributions as ϵ tends to 0. More precisely, we are interested in the allocations that are assigned positive probability by this limiting distribution. These particular allocations are called the stochastically stable allocations.

In order to obtain our results, we will use the techniques developed by Kandori, Mailath and Rob (1993) and Young (1993). We need some more definitions. These concepts will be used in the proof of our main result and in the analysis of the examples of Section 6.

Note that by the definition of the perturbed Markov process $\mathcal{M}^\epsilon$, for every two allocations z and z', the direct transition probability $\mu_{z,z'}(\epsilon)$ converges to the limit transition probability $\mu_{z,z'}(0)$ of the unperturbed process $\mathcal{M}^0$ at an exponential rate. In particular, for all allocations z, z' such that $\mu_{z,z'} = 0$, the convergence is at a rate $r(z,z') = \min_{S \in T(z,z')} \lambda n_W(S,z,z') + n_I(S,z,z')$. We call the value $r(z,z')$ the *resistance of the direct transition* from allocation z to allocation z'.

For any two allocations z, z', a (z,z')*-path* is a sequence of allocations $\xi = (i_0, i_1, \ldots, i_n)$ such that $i_0 = z$, $i_n = z'$. The *resistance of the path* ξ is the sum of the resistances of its transitions.

Let $Z^0 = \{E^0, E^1, \ldots, E^Q\}$ be the set of recurrent classes of the unperturbed process $\mathcal{M}^0$ and consider the complete directed graph with vertex set Z^0, which is denoted by Γ. We want to define the resistance of each one of the edges in this graph. For this, let E^i and E^j be two elements of Z^0. The *resistance of the edge* (E^i, E^j) in Γ, $r(E^i, E^j)$, is the minimum resistance over all the resistances of the (z^i, z^j)-paths, where $z^i \in E^i$ and $z^j \in E^j$. A spanning tree rooted at E^j is a set of Q directed edges such that from every recurrent class different from E^j, there is a unique directed path in the tree to E^j. The *resistance of a spanning tree rooted at* E^j is the sum of the resistances of its edges. The *stochastic potential* of the recurrent class E^j is the minimum resistance attained by a spanning tree rooted at E^j. As shown in Young (1993), the set of stochastically stable states of the perturbed process $\mathcal{M}^\epsilon$ consists of those states included in the recurrent classes with minimum stochastic potential.

5 Economies with Singleton Indifference Sets

In this section we shall assume that for every agent $i \in N$ the preference relation $\succeq_i$ is antisymmetric, which implies that all indifference sets are singletons. Making such an assumption, Roth and Postlewaite (1977) proved the following result:

Lemma 1 Let $\mathcal{E}$ be a housing economy where all preferences are strict. Then,

(i) There is a unique Walrasian allocation w.

(ii) The allocation w is the only strong core allocation.

(iii) For every allocation $x \in A$, $x \neq w$, there exists a coalition S such that w_S is feasible for S and satisfies $w_i \succeq_i x_i$ for all $i \in S$ and $w_j \succ_j x_j$ for some $j \in S$.

Lemma 1 will be useful in proving our first main result, to which we turn now.

Theorem 1 Let $\mathcal{E}$ be a housing economy where all preferences are strict. Suppose that $\lambda > |N| - 2$. Then, the unique stochastically stable allocation of the perturbed process $\mathcal{M}^\epsilon$ is the Walrasian allocation w.

Proof: Since when $|N| = 2$, the core consists of the singleton w and there are only two feasible allocations, it is clear that the statement holds, by Proposition 1. Thus, assume that $|N| \geq 3$. Recall that we denote the set of recurrent classes of the unperturbed process $\mathcal{M}^0$ by $\{E^0, E^1, \ldots, E^Q\}$, and let E^0 be the singleton recurrent class containing the Walrasian allocation w (again recall Proposition 1 and Lemma 1, parts (i) and (ii)). Denote the stochastic potential of E^0 by $\text{sp}(E^0)$. We show that the stochastic potential of any other recurrent class E^k, $k = 1, \ldots, Q$, is greater than $\text{sp}(E^0)$.

Let $E^k \neq E^0$ be an arbitrary recurrent class. Consider a E^k-tree of stochastic potential $\text{sp}(E^k)$. Introduce in it the following two modifications:

(i) Delete the edge that connects the class E^0 to its successor E^j on the path to E^k.

(ii) Add a directed edge going from E^k to E^0.

Note that the resulting graph is an E^0-tree. Moreover, the resistance of this new E^0-tree $r(T)$ equals

$$r(T) = \text{sp}(E^k) - r(E^0, E^j) + r(E^k, E^0).$$

To finish the proof, we show in the following two lemmas that $r(E^0, E^j) > r(E^k, E^0)$. This means that we would have constructed an E^0-tree whose resistance is less than $\text{sp}(E^k)$, thereby showing that $\text{sp}(E^0) < \text{sp}(E^k)$.

Lemma 2 Consider the edge $E^0 \to E^j$ that is deleted from the E^k-tree. Then, $r(E^0, E^j) \geq \lambda$.

Proof: Consider a path that attains the resistance $r(E^0, E^j)$, of the edge that connect the Walrasian allocation to the recurrent class E^j, and let x^1 be the first allocation in this path.

We claim that, in the direct transition from w to x^1, at least one agent becomes worse off. Let the coalition involved in this transition be S^1. If it were the case that $x_i^1 \succeq_i w_i$ for every $i \in S^1$, we would be saying that w is not a strong core allocation, contradicting Lemma 1, part (ii). Therefore, at least one agent becomes worse off in this direct transition, from which it follows that $r(E^0, E^j) \geq \lambda$. ■

Lemma 3 Consider the edge $E^k \to E^0$ that is added to the E^k-tree. Then, $r(E^k, E^0) \leq |N| - 2$.

Proof: We calculate an upper bound for $r(E^k, E^0)$ as follows. Let $x^0 \in E^k$. By Lemma 1, part (iii), there exists a coalition S such that $(w_i)_{i \in S} \in A_S$, $w_i \succeq_i x_i$ for all $i \in S$ and $w_j \succ_j x_j$ for some $j \in S$. In the next paragraphs, we refer to S as one of the maximal (in the sense of set inclusion) such coalitions . We can have two cases.

Case 1: $S = N$. In this case, the maximum possible resistance associated with the direct transition from x to w is $(|N| - 2)$, i.e., the one given by the highest number of indifferences that can occur in N.

Case 2: $S \neq N$. This case admits two subcases:

Subcase 2.1: Suppose that $x_{-S} \notin A_{-S}$. Then, when coalition S meets, the system moves to $y = (w_S, e_{-S})$ with positive probability. The resistance of this transition cannot be greater than $(|N| - 2)$ because, within S, one can have at most $(|S| - 2)$ indifferences. But note that from y, the system can move to w with a resistance no bigger than $|N \backslash S| - 2$: if necessary, the coalition $T \subseteq N \backslash S$ of agents who are not receiving their Walrasian house at y will be partitioned in subsets (according to the trading cycles), each of which to perform the necessary trade so that the final result is w. Therefore, since $|S| \leq |N| - 2$, the number of indifferences found in this transition is at most $(|N| - 4)$.

Subcase 2.2: $x_{-S} \in A_{-S}$. In this case, coalition S meets and the system moves to $y = (w_S, x_{-S})$ with positive probability. But then, by our choice of S and Lemma 1, part (iii) applied to the subeconomy consisting of agents $N \backslash S$, it must necessarily be the case that $x_{-S} = w_{-S}$. Therefore, $N \backslash S = \emptyset$ because otherwise S would not be a maximal blocking coalition. But in this case $S = N$ and we are back in case 1.

Therefore, the resistance of the transition $E^k \to E^0$ is bounded above by the maximum of the two expressions involved in the two cases analyzed, which is $(|N| - 2)$. ■

In consequence, it follows from our assumption on the size of λ that $r(E^k, E^0) < r(E^0, E^j)$, which concludes the proof. ■

6 Economies with Non-Singleton Indifference Sets

In this section we explore how the stochastic process of recontracting with mistakes, $\mathcal{M}^\epsilon$, performs over the class of economies that allow non-singleton indifference sets for some agents. Over this larger class of economies, recall that Proposition 1 still holds. However, the conclusions of Lemma 1 do not extend. First, although existence is guaranteed, there may be multiple Walrasian allocations. Second, the strong core may also contain multiple allocations, while it may sometimes be empty. And third, the "global dominance" property of Walrasian allocations as specified in Lemma 1, part (iii), is also lost.

In economies with only strict preferences, the Walrasian allocation correspondence and the strong core coincide, and under our assumption on λ, the same allocation is the only one that passes the test of stochastic stability. It is convenient, therefore, to examine the larger class of economies to disentangle the different forces and understand what conclusions emerge from our dynamic analysis based on agents' mistakes. The trading system with mistakes gives rise to complicated dynamics, and no general result of equivalence can be established.

We shall present three examples. We arrange them by increasing difficulty and relevance. Indeed, we regard Example 4 as the other main result of the paper. In all three examples, one agent has a completely flat indifference map, but this is only for simplicity of exposition. Also, in the examples we shall use the notation $z \to_S^r z'$ to express that the transition of least resistance from z to z' takes place through coalition S at a resistance r.

We begin by showing that the set of stochastically stable allocations is not the strong core. As we just pointed out, the strong core may be empty in these economies, while stochastic stability always selects at least one allocation; but even when the strong core is not empty, one can generate examples where it does not coincide with the set of stochastically stable states of $\mathcal{M}^\epsilon$.

Example 2 In this example, a non-empty strong core is strictly contained in the set of stochastically stable allocations. Let $N = \{1, 2\}$ and agents' preferences be described as follows:

$$e_1 \sim_1 e_2;$$

$$e_1 \succ_2 e_2.$$

Both allocations are Walrasian and only the allocation resulting from trade is in the strong core. Note that both allocations are stochastically stable: $(e_1, e_2) \to_N^1 (e_2, e_1)$ and $(e_2, e_1) \to_{\{1\}}^1 (e_1, e_2)$.

The next example shows that the set of stochastically stable allocations may be a strict subset of the strong core and of the set of Walrasian allocations.

Example 3 Let $N = \{1, 2, 3\}$ and the agents' preferences be given by:

$$e_1 \sim_1 e_2 \sim_1 e_3;$$

$$e_1 \sim_2 e_3 \succ_2 e_2;$$

$$e_1 \sim_3 e_2 \succ_3 e_3.$$

In this economy, all allocations except the initial endowment allocation are Walrasian and belong to the core. The strong core consists of the following three allocations: (e_2, e_3, e_1), (e_3, e_1, e_2) and (e_1, e_3, e_2).

The unique stochastically stable allocation is $x = (e_1, e_3, e_2)$. To see that x is the only allocation with minimum stochastic potential, one can construct one such x-tree as follows. First, we note that the only recurrent classes of $\mathcal{M}^0$ are the five absorbing states corresponding to each Walrasian allocation. Next, note that to go from (e_2, e_3, e_1) to x can be done with a resistance of 1 (only one indifference): $(e_2, e_3, e_1) \rightarrow^1_{\{1\}} (e_1, e_2, e_3) \rightarrow^0_{\{2,3\}} x$. The same goes for the transition (e_3, e_1, e_2) to x: $(e_3, e_1, e_2) \rightarrow^1_{\{1\}} (e_1, e_2, e_3) \rightarrow^0_{\{2,3\}} x$. As for the other transitions, we have $(e_3, e_2, e_1) \rightarrow^1_{\{2,3\}} x$ and $(e_2, e_1, e_3) \rightarrow^1_{\{2,3\}} x$. Therefore, the resistance of this x-tree is 4 and one cannot build a cheaper tree than that. On the other hand, to get out of x, the resistance will always be at least 2, i.e., at least two indifferences, which implies that, in constructing a tree for any of the other recurrent classes, its resistance must be at least 5.

The next example shows how different the conclusions one reaches in the analysis of cooperation with mistakes may be from those of standard cooperative game theory. It turns out that in a dynamic model where agents may make mistakes in decision-making, the core may not agree with the set of states that are visited by the process a positive proportion of time. In contrast, some non-core allocations may fair better in this sense than some core allocations.

Example 4 [7] This example, an outgrowth of Example 1, shows that a cycle of non-core allocations may be stochastically stable, at the same time as some core allocations having higher stochastic potential. Let $N = \{1, 2, 3, 4\}$, and the agents' preferences be as follows:

$$e_4 \succ_1 e_3 \succ_1 e_2 \succ_1 e_1;$$

[7] As communicated to us by Bob Aumann, a similar story is told in the Talmud: there is a cycle of three two-person coalitions improving the status quo. The three players involved in the cycle are the two wives of a man and a third party who buys the man's estate. The cycle occurs after the man died and left his estate. The diseased agent naturally corresponds to our agent 4, who has a flat indifference map. See also Binmore (1985) for a more recent related problem.

$$e_1 \succ_2 e_3 \succ_2 e_2 \succ_2 e_4;$$

$$e_2 \succ_3 e_1 \succ_3 e_3 \succ_3 e_4;$$

$$e_1 \sim_4 e_2 \sim_4 e_3 \sim_4 e_4.$$

Consider the following three allocations: $x = (e_1, e_3, e_2, e_4)$, $y = (e_2, e_1, e_3, e_4)$ and $z = (e_3, e_2, e_1, e_4)$. Since agent 4 cannot strictly improve, he cannot be part of any blocking coalition. In fact, as in Example 1, these three allocations constitute a recurrent class: if the system is at x, the state changes only when coalition $\{1, 2\}$ meets, yielding y. At y, the system can move only to z, when coalition $\{1, 3\}$ meets. Finally, the system will move out of z only by going back to x, when coalition $\{2, 3\}$ meets. That is, $x \rightarrow^0_{\{1,2\}} y \rightarrow^0_{\{1,3\}} z \rightarrow^0_{\{2,3\}} x$.

The core consists of the following five allocations: $c_1 = (e_3, e_1, e_2, e_4)$, $c_2 = (e_4, e_1, e_2, e_3)$, $c_3 = (e_4, e_1, e_3, e_2)$, $c_4 = (e_4, e_3, e_1, e_2)$, $c_5 = (e_4, e_3, e_2, e_1)$.

It is easy to see that these five absorbing states –i.e., the core allocations– and the cycle are the only recurrent classes of the unperturbed system: the 12 allocations where e_4 is allocated to either agent 2 or agent 3 are not even individually rational. And from each of the remaining four allocations, one gets to one of the already identified recurrent classes with 0 resistance:

$a_1 = (e_1, e_2, e_3, e_4) \rightarrow^0_{\{1,2,3\}} c_1$, $a_2 = (e_2, e_3, e_1, e_4) \rightarrow^0_{\{1,2,3\}} c_1$, $a_3 = (e_4, e_2, e_1, e_3) \rightarrow^0_{\{2,3\}} x$, $a_4 = (e_4, e_2, e_3, e_1) \rightarrow^0_{\{2,3\}} c_5$.

Let $E = \{x, y, z\}$ be the non-singleton recurrent class consisting of non-core allocations. Next, we construct an E-tree and show that it has minimum stochastic potential. This tree must have five edges, coming out of each of the five core allocations. We detail the transitions below:

$$c_1 \rightarrow^1_{\{1,4\}} a_4 \rightarrow^0_{\{2,3\}} c_5,$$

$$c_j \rightarrow^1_{\{4\}} a_1 \rightarrow^0_{\{1,2\}} y, \quad \text{for} j = 2, 3, 4, 5.$$

Therefore, the class E has minimum stochastic potential.

Note also that there are four Walrasian allocations: c_1, c_2, c_3 and c_5. Thus, the example also shows that there are non-Walrasian stochastically stable allocations.

However, apart from E, the only stochastically stable allocations are the four Walrasian allocations c_1, c_2, c_3 and c_5: there are allocations in the core that are not visited in the long run but a zero proportion of time. In particular, note how it takes at least two indifferences to get out of other recurrent classes to go to c_4. This implies that c_4 cannot be stochastically stable. As an illustration, we construct a $\{c_4\}$-tree as follows:

$$c_1 \rightarrow^1_{\{1,4\}} a_4 \rightarrow^0_{\{2,3\}} c_5,$$

$$c_j \rightarrow^1_{\{4\}} a_1 \rightarrow^0_{\{1,2\}} (y \in E), \quad \text{for} j = 2, 3, 5,$$

$$(z \in E) \rightarrow^2_N c_4.$$

7 Concluding Remarks

1. Theorem 1 uses the "global dominance" property of the Walrasian allocation, as specified in Lemma 1, part (iii). Although the models are very different, this dominance of the Walrasian allocation resembles the main driving force of the result in Vega-Redondo (1997). This paper proposes an evolutionary process based on imitation, and its Walrasian result relies on the fact that if a firm produces the competitive output in a symmetric oligopoly, its profit is always higher than that of those firms that produce any other output level.

2. Note that the sufficient condition on the cost of a serious mistake, $\lambda > |N| - 2$, used to obtain Theorem 1, is jeopardized when $|N|$ grows. Thus, for a given specification of λ, the system may get stuck at other allocations because the number of indifferences required to abandon a non-Walrasian allocation to go to the Walrasian allocation grows. If one fixes λ, making the economy large is an obstacle to the Walrasian result: other allocations could also be visited by the process a positive fraction of time in the long run. This contrasts with the core convergence literature, based on the existence of more blocking coalitions in large economies.

3. Along the same lines, if indifferences are present in the economy, the examples in Section 6 demonstrate that stochastic stability may yield a variety of patterns, and that the long run prediction may be compatible with the presence of market frictions – non-Walrasian allocations. In particular, Example 4 suggest that the core may not be telling the whole story of coalitional stability in a model where mistakes are allowed, as a complement to one of the central messages of cooperative game theory.

4. Our results are robust if the transition rule in the unperturbed process is that of coalitional weak blocking, instead of the strict blocking specified in $\mathcal{M}^0$. That is, coalition S moves from x_S to $y_S \in A_S$ if $y_i \succeq_i x_i$ for every $i \in S$ and $y_i \succ_i x_i$ for some $i \in S$. A version of Proposition 1 is obtained: the recurrent classes of this new unperturbed process are of two kinds: singletons, corresponding to each strong core allocation, and non-singletons consisting of individually rational allocations that are not in the strong core. With all preferences being strict, Theorem 1 is still obtained thanks to the "global dominance" of the Walrasian allocation, but no assumption on λ is required, because in this new process indifferences constitute no friction if they are accompanied of at least one strict improvement in the coalition. Finally, one can still sustain stochastically stable cycles in economies with indifferences: one easy way to see this is to consider an example with an empty strong core.

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